Coordination problems and the control of epidemics affecting fruit trees

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A complex management problem

- production by private owners distributed within a landscape
- economic losses due to the infection outbreak
- diffusion of pathogens intra and inter-patch
- finite horizon, multi-year production
- treatment by (partially inefficient) detection and removal of infected trees, (discrete binary choice)

Figure: Sharka example

Objective: Understand the decentralized problem

Problem often studied under the centralized perspective.

Our objective: understand better the decentralized behavior.
Emerging literature: [Atallah et al., 2017], [Fenichel et al., 2014], [Costello et al., 2017]

We analyze classical questions...
coordination issues, inefficiency characterization...
with specific modeling constraints

Modeling: infection diffusion within a period

Management options: $\rho_i \in \{0, \rho_{max}\}; \quad 0 < \rho_{max} < 1$

State variables:
I_i Quantity of infected in patch i.
S_i Quantity of uninfected trees.
Growth and diffusion of the infection: r_{ij}
Evolutionary law (discrete time model), with $I << S$:

$$(I_i^{t+1}, S_i^{t+1}) = f(S^t, I^t, \rho^t)$$

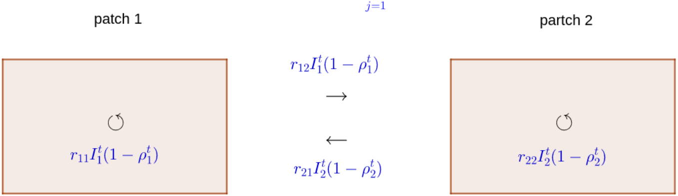
$$I_i^{t+1} = I_i^t(1 - \rho_i) + \sum_{j=1}^{N} I_j^t(1 - \rho_j) r_{ji}$$

$$S_i^{t+1} = S_i^t - \sum_{j=1}^{N} I_j^t(1 - \rho_j) r_{ji}$$

Modeling: Infection diffusion, two patches model

Diffusion in a two patches model

$$I_i^{t+1} = I_i^t(1-\rho_i) + \sum_{j=1}^{N} I_j^t(1-\rho_j) r_{ji}$$



Economic model: profit function

$$\pi_i^t(I^t, S^t, \rho^t) = \left(S_i^{t+1} v_i + I_i^{t+1} u_i - \frac{\rho_i^t}{\rho_{max}} (c_a + c_h A_i) \right)$$

subject to:

$$(I^{t+1}, S^{t+1}) = f(S^t, I^t, \rho^t).$$

v_i production value by an uninfected tree in patch i

u_i production value by an infected tree i

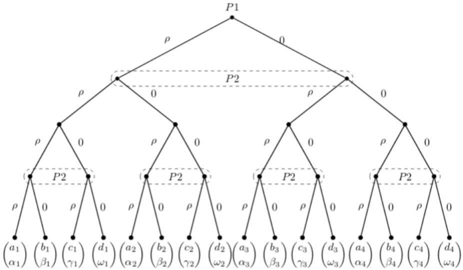
c_a access cost

c_h per ha^{-1} inspection cost

A_i patch i surface

Conceptual framework

Framework



$$V_i^T(\rho^0, \rho^1, I^0, S^0) = \pi_i^0(I^0, S^0, \rho^0) + \beta\pi_i^1(I^1, S^1, \rho^1)$$
$$I^{t+1}, S^{t+1} = f(I^t, S^t, \rho^t).$$

Resolution for the closed loop feedback-Nash equilibrium concept.
Comparison with the Pareto optimum.

Results

Impact of the initial condition in the 2 patches 2 steps model:

- An example of analytical result: zone where $(\rho_{max}, \rho_{max}, \rho_{max}, \rho_{max})$ is the unique FNE
- Multiplicity of FNE
- Characterization of inefficiency

Maximal effort as a FNE

Proposition: Within the initial condition state space, there is a zone where initial infection is sufficiently high so that both players do maximal effort:

$(\rho_{max}, \rho_{max}, \rho_{max}, \rho_{max})$ is the unique Nash equilibrium if and only if $(I_1^0, I_2^0) \in \Delta_{max}$, where Δ_{max} is defined by the set of inequalities:

$$\begin{cases} I_2^0 > \frac{\alpha_1 - I_1^0(1-\rho_{max})(1+r_{11})}{(1-\rho_{max})r_{21}} \\ I_2^0 > \frac{\alpha_2 - I_1^0(1-\rho_{max})r_{12}}{(1-\rho_{max})(1+r_{22})} \\ I_2^0 > k_2 \\ I_1^0 > k_1 \end{cases}$$

where $\alpha_i \equiv \frac{1}{F_i}(c_a + c_h \frac{1}{\rho_{max}} A_i)$ where $F_i \equiv (v_i - u_i)r_{ii} - u_i$, and k_1 and k_2 are some constants.

Private efficiency in the case of maximal effort

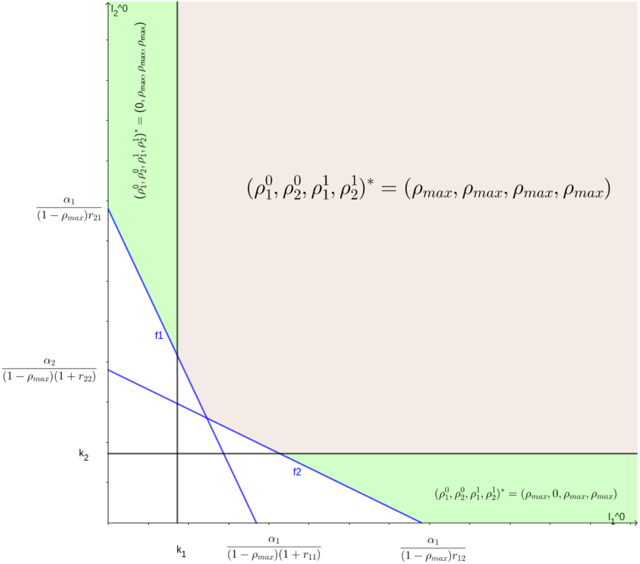
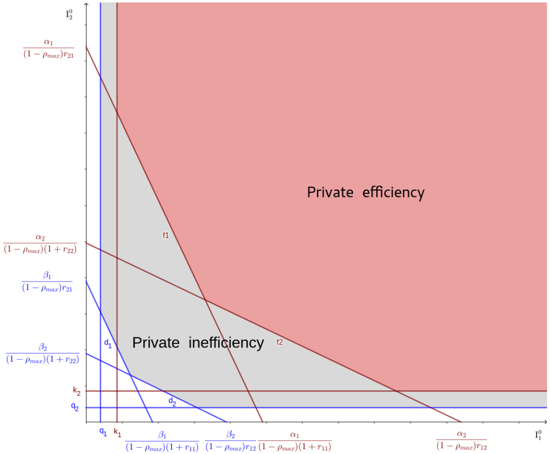


Illustration for ρ_{max} as a unique Nash equilibrium

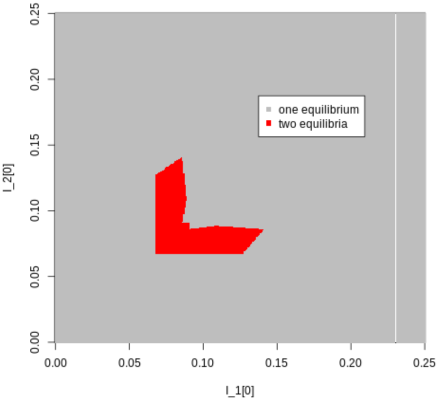


$$\beta_i \equiv \frac{1}{D_i}(c_a + c_h \frac{1}{\rho_{max}} A_i) \text{ where } D_i \equiv (v_i - u_i)r_{ii} + (v_j - u_j)r_{ij} - u_i$$

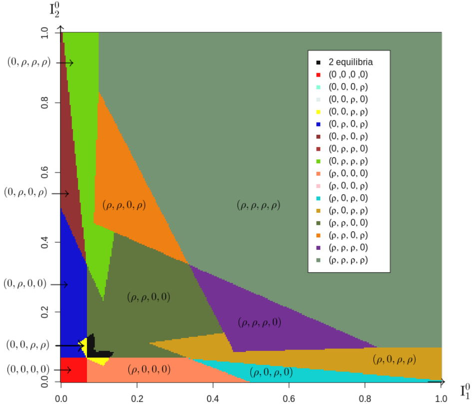
$$\alpha_i \equiv \frac{1}{F_i}(c_a + c_h \frac{1}{\rho_{max}} A_i) \text{ where } F_i \equiv (v_i - u_i)r_{ii} - u_i$$

Number of Nash equilibria

Proposition: Multiplicity might arise... even in a symmetric case (proof using an example).



Example, Nash equilibria according to the initial condition



Symmetric example, inefficiency

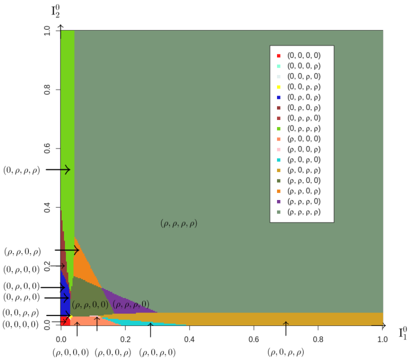


Figure: Pareto optimum, symmetric example

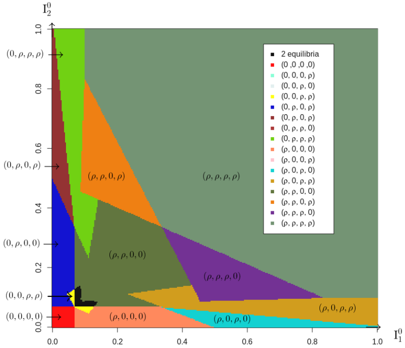
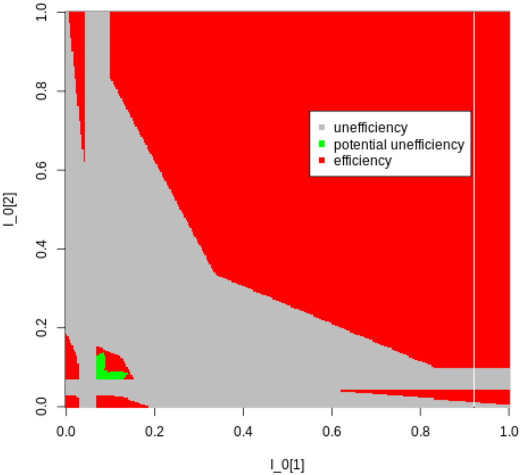


Figure: Nash equilibria, symmetric example

Symmetric example, inefficiency



Conclusion

Main results

When infection is still small, ($I << S$), and detection imperfect, and given parameters (T, R, U, V...)

- Nash feedback resolution of the game shows equilibria depending on the initial infection level
- Geometric characterization of efficiency and inefficiency zones as a function of the initial infectious state
- Coordination issues: multiplicity of equilibria for some (I_1^0, I_2^0)

Conclusion

Perspectives

- Introduce asymmetry in the case study, look at the impact of other parameters
- Study de-synchronization of production cycles and longer time horizons
- Apply this framework to analyze real life problems (find some data); question large scale management programs using known parameters
- Work on the modeling: SI model, probabilistic framework...

Thanks for listening !

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Temporal structure