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ProbitSpatial: introduction for users

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ProbitSpatial
Introduction for users

Fast and accurate spatial probit estimations

Davide Martinetti
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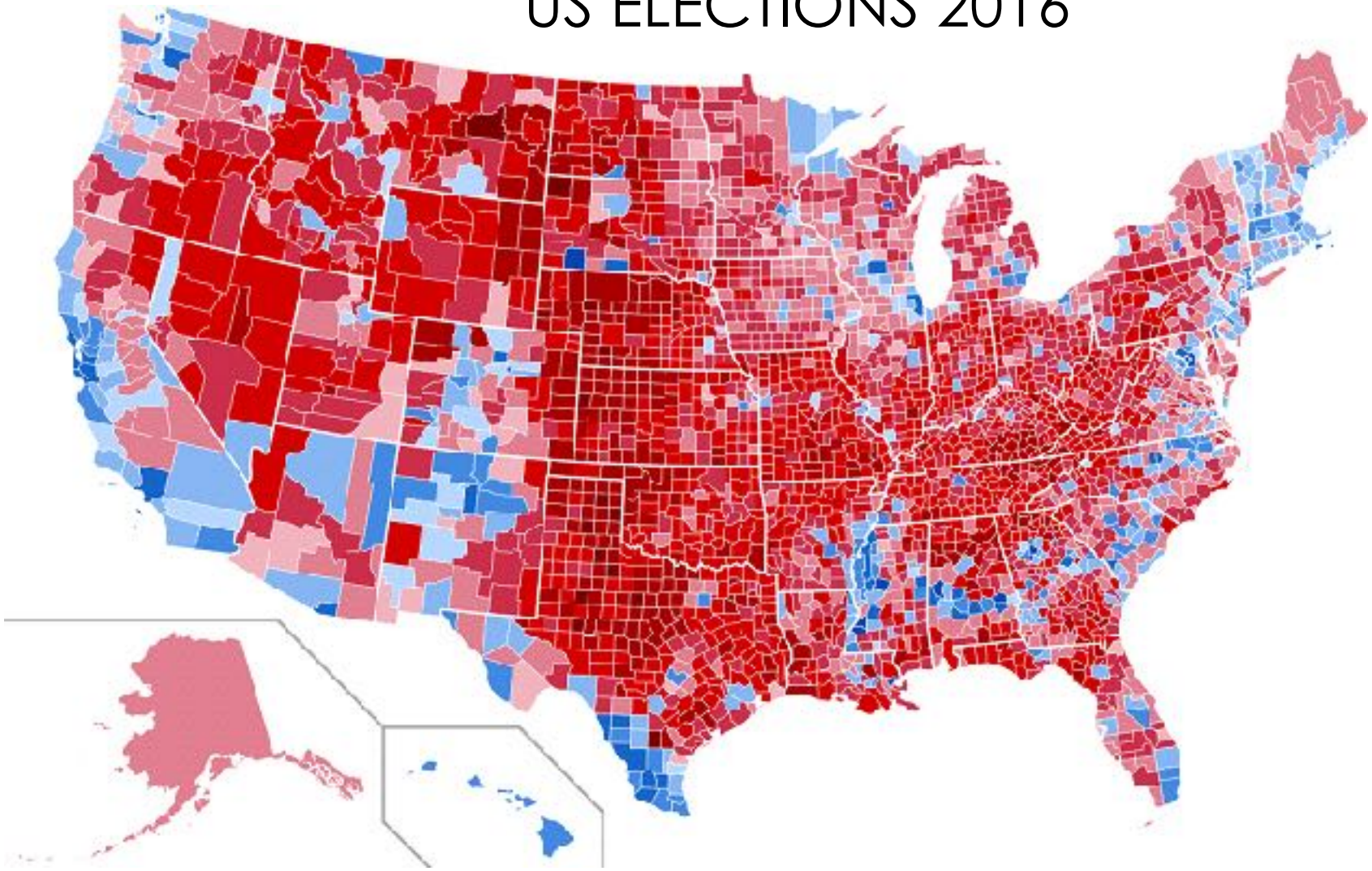
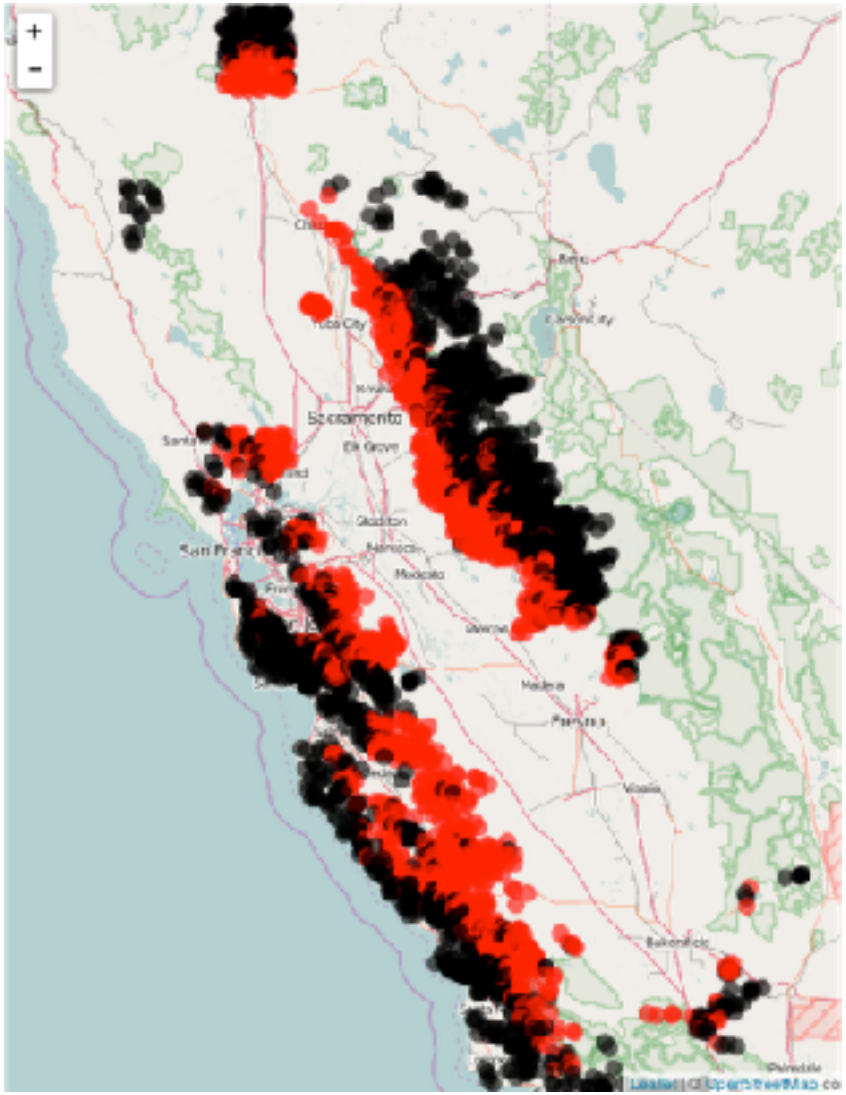
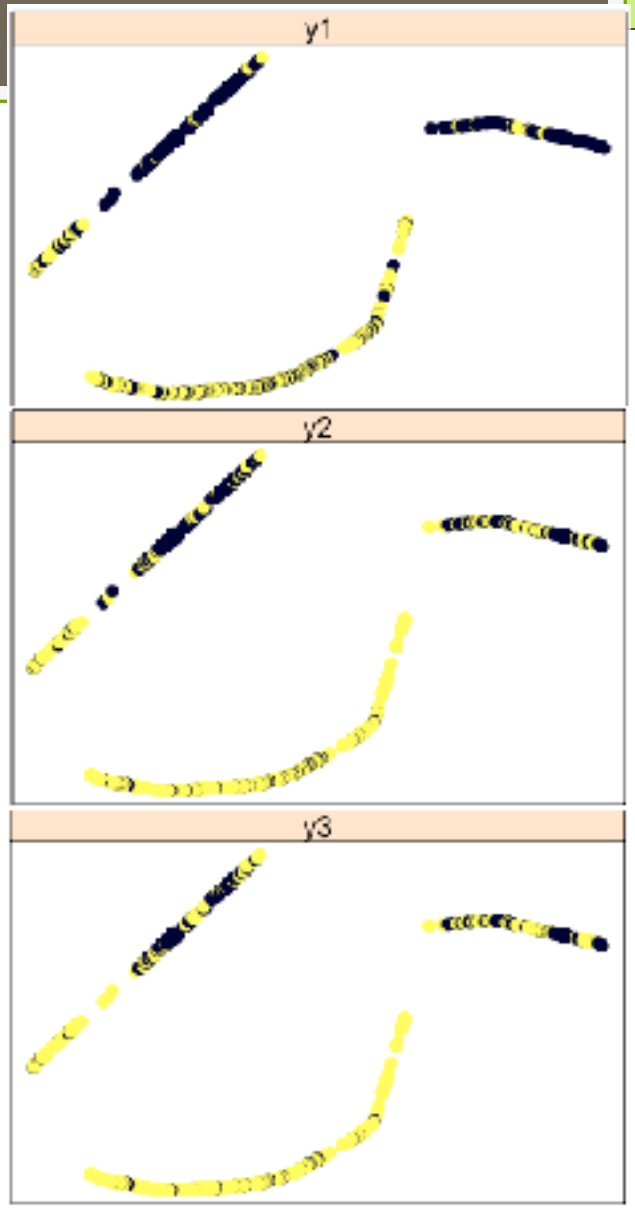
INRA - FRANCE

US ELECTIONS 2016



BLUE OAKS IN CALIFORNIA





REOPENING AFTER KATRINA

Regression of spatial data

- Spatial data tend to show some kind of **spatial pattern**
- Regression models should account for the spatial features of the data, otherwise
  - **Inefficient and inconsistent estimates (McMillen, 1992)**
  - **heteroskedastic and correlated residuals**

PROBIT model

- **Regression model** for binary dependent variable
- **Unobserved** latent variable Y* such that the observed Y=1, if Y*>0 and Y=0 otherwise

$$Y^* = X\beta + \varepsilon$$

$$P(Y^* > 0 \mid X) = \Phi(X\beta + \varepsilon)$$

- Assumptions on the disturbances ε
  - INDEPENDENT $\Phi(0, I_n)$
  - NORMALLY DISTRIBUTED

Spatial PROBIT model

SPATIAL AUTO-REGRESSIVE (SAR)

$$Y^* = \rho W Y^* + X\beta + \varepsilon$$

$$Y^* = (I_n - \rho W)^{-1}(X\beta + \varepsilon)$$

Residuals ~ N(0,Σ)with $\Sigma = (I_n - \rho W)^{-1\prime}(I_n - \rho W)^{-1}$

Spatial PROBIT model

SPATIAL ERROR MODEL (SEM)

$$Y^* = X\beta + u$$

$$u = \rho W u + \varepsilon$$

Residuals ~ N(0,Σ) with $\Sigma = (I_n - \rho W)^{-1\prime}(I_n - \rho W)^{-1}$

Spatial PROBIT model

SAR + SEM = SARAR

$$Y = \rho WY + X\beta + u$$

$$u = \lambda Mu + \varepsilon$$

Residuals ~ N(0,Σ)

$$\Sigma = (I_n - \rho W)^{-1}{}'(I_n - \lambda M)^{-1}{}'(I_n - \lambda M)^{-1}(I_n - \rho W)^{-1}$$

Spatial PROBIT estimation

Estimation problems arise due to **dependence across observations**.

ML methods implies computation of **mvn probabilities.**

- Approximate ML methods
  - E-M (`McSpatial` – McMillen '92)
  - Bayesian Gibbs Sampler (`spatialprobit`– LeSage '00)
  - RIS (a.k.a. GHK, Beron & Vijverberg '03)

- Non-likelihood methods
  - Generalised Methods of Moments (Pinkse & Slade'98)
  - Linearized GMM (Klier & McMillen '08, `McSpatial`)

Expectation-Maximization

McMillen, 1992

E-step: Y is replaced by the expected value of Y*, given estimates $(\hat{\beta}, \hat{\rho})$

M-step: new estimates by max log-likelihood

Repeat until convergence

- PROS: avoid MVN probabilities
- CONS: inversion matrix $(I - \rho W)$, slow

Gibbs sampling

LeSage, 2000

Joint distribution of Y is unknown =>

specifies the complete conditional distributions for all parameters in the model and proceeds to sample from these distributions to collect a large set of parameter

- PROS: distribution parameter => inference
- CONS: slow

Recursive importance sampling, RIS

<u>Beron and Vijverberg (2004)</u>

Direct estimation of the mvn probabilites!

Decompose Σ with Choleski, so each observation depends only on the previous one.

Same as GHK simulator.

- PROS: max-likelihood => LR tests
- CONS: not suitable for large n

GMM and LGMM

<u>GMM (Pinkse & Slade,1998)</u>

Minimization of generalized residuals via GMM with instruments $I + X + WX + W^2X + W^3X + \ldots$

<u>LGMM (Klier & McMillen, 2008)</u>

Linearization of GMM around convenient starting point.

- PROS: fast
- Inefficient compared to ML estimators

Our proposal

ML estimation with approximation of MVN probabilities

Trinh & Genz, 2013 or Mendell & Elston, 1974

Basic ideas

- n-dim integral ➔ Prod. of n 1-dim integrals
- Cholseky decomposition of $\Sigma = CC'$ and variable transformation $x = Cz \Leftrightarrow z = C^{-1}x$
- Iteratively recompute integral limits $a \leq Cz \leq b$ taking advantage of the triangular form of C
- Approximate z using its truncated expected values

Numerical considerations

- Sparse matrix algebra (# of neighs. ➔ # non-zeros)
- Reordering of obs. to minimize error propagation
- Reordering for faster Cholesky decomposition
- Taylor approx. $(I - \rho W)^{-1} = I + \rho W + \rho^2 W^2 \ldots$
- `Rcpp & RcppEigen`

One likelihood – four methods

<u>2 optimization strategies:</u>

- Full-likelihood with gradients;
- Conditional likelihood: compute $X^* = (I - \hat{\rho} W)^{-1} X$ conditional to $\hat{\rho}$ and fit standard probit to find β. Optimize w.r.t. $\hat{\rho}$.

<u>2 dependence structures:</u>

- Use Σ as explained earlier;
- Use precision matrix $P = \Sigma^{-1}$, since it is sparser. Lower accuracy, but lower computation time.

Some comparisons

$$Y = \rho WY + \beta_0 + X_1\beta_1 + \varepsilon$$

$$\beta = (4, -2), \rho = 0.45, X_1 \approx N(2, 4)$$

$$nneighs = 5, n = 1500$$

Method	Time (secs.)	Bias β1	Bias ρ
EM	+10000	0.99	-0.099
Gibbs	+10000	0.13	-0.053
RIS	8000	0.19	0.001
GMM	1000	-312	-0.014
Lin-GMM	10	1.19	0.100
ProbitSpatial	1.02	0.01	0.00

Going bigger

Size	Nneighs=2	Nneighs=3	Nneighs=10
100	0.164	0.192	0.234
1000	0.273	1.401	6.810
10000	2.575	5.717	
100000	28.467	127.105	
1000000	504.10		

`ProbitSpatial` package

- ProbitSpatial uniqueness
  - SEM and SARAR
  - Between 10 and 10000 times faster
  - Can digest sample sizes of 1 million
- Existing alternatives
  - In `R: McSpatial` or `spatialprobit`
  - In `Matlab: Spatial Toolbox`
- Impact
  - +1000 downloads from Rstudio since March
  - Presented at USER 2016 – Stanford University

Out-of-sample predictions

BLUP formula: Goulard et al., 2017

$$\hat{Y}_O^{BP} = \hat{Y}_O^{TC} - Q_{OO}^{-1} Q_{OS} \times \left(Y_S - \hat{Y}_S^{TC} \right)$$

$$Q = \frac{1}{\sigma^2}\left(I - \rho(W' + W) + \rho^2 W'W\right) = \begin{pmatrix} Q_{SS} & Q_{SO} \\ Q_{OS} & Q_{OO} \end{pmatrix}$$

$$\begin{pmatrix} \hat{Y}_S^{TC} \\ \hat{Y}_O^{TC} \end{pmatrix} = (I - \hat{\rho} W)^{-1} X\hat{\beta}$$

Marginal effects

LeSage and Pace, 2009

- **First problem**: interpretation of changes of X on Y is not direct: non-linear transformations due to the probit model
- **Second problem**: spatial spillover effects! Changes in a single observation influence all other observations.
- For each explanatory variable ➔ n × n matrix of partial derivatives.
- Scalar summary measures that average across the sample of observations:
  - Direct effects
  - Indirect effects
  - Total effects

Marginal effects

$$\frac{\partial \Pr(Y_i = 1)}{\partial X_{v,j}} = \Phi(\eta_i) S_{ij} \hat{\beta}_v$$

$$S = (I - \hat{\rho} W)^{-1}$$

$$\eta = S X \hat{\beta}$$

- Direct effects: mean of diagonal
- Total effects: mean of rows sum
- Indirect effects: total - direct