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Nicolas QUEROU
Antoine SOUBEYRAN
Raphaël SOUBEYRAN

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Unite de Formation et de Recherche d'Economie
Avenue Raymond DUGRAND C.S. 79606
3 4 9 6 0 MONTPELLIER Cedex 2

E-mail : lameta@lameta.univ-montp1.fr
nontp1.fr

Université Montpellier 1
agro Montpellier Ecole Nationale Supérieure Agronomique
cnrs Centre National de la Recherche Scientifique
INRA

Moral Hazard and Capability*

Nicolas QUÉROU † Antoine SOUBEYRAN‡ Raphael SOUBEYRAN§

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Abstract. We consider a moral hazard problem where the agent has limited wealth which restricts his possible actions. In such cases, the lower the level of wealth is, including transfer from or to the principal, the lower the maximum effort level that can be provided. We show then that the optimal contract is, in some cases, a sharing contract and the optimal up-front transfer is a payment from the principal to the agent. Moreover, whereas incentives and monetary transfer to the agent are substitutes in the case where the agent has sufficient wealth, they are complements when the agent has limited wealth. This implies that monetary aid and incentives are complements when providing monetary aid to the agent is optimal. We discuss the implications of our findings in a variety of settings, including payments for ecosystem services, venture capital, and a current debate on wealth and cognitive functions.

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†CNRS, UMR5474 LAMETA, F-34000 Montpellier, France. Email: querou@supagro.inra.fr
‡GREQAM-AMSE, Aix-Marseille University, France. E-mail: antoine.soubeyran@gmail.com
§INRA, UMR1135 LAMETA, F-34000, Montpellier, France. Email: soubeyra@supagro.inra.fr

1 Introduction

Economic agents respond to incentives. However, agents with limited wealth may not be able to respond to incentives. Providing incentives to an agent who has not the means to reach a desirable outcome or providing means to an agent who faces no incentives may lead to inefficient economic activity. For example, payment for ecosystem services (PES) are paid to developing countries forest owners at the end of a given period of time if they have kept their forest intact. A major drawback of this mechanism is that it does not take into account the short-term large opportunity costs that liquidity constrained forest owners face (Jayachandran, 2013). In the present paper, we argue that, if the agent has limited wealth, it can be in the interest of the project owner to provide both means and incentives to the agent.

In our model, as in standard moral hazard models, the agent makes an unobserved costly action, which produces a stochastic output. The principal provides incentives by paying the agent on observed output and by paying (or asking) an up-front payment to the agent. However, unlike in standard moral hazard models, the agent has a limited wealth that links the up-front payment and the action. This assumption is consistent with different situations where: the opportunity cost can be monetary, the effort provided by the agent can actually be an investment, or the agent can invest in training activities in order to improve his capability (for instance, his maximum potential effort). The principal and the agent are both risk neutral, so that the only distortion comes from the budget constraint which limits possible actions. When the budget constraint binds, the up-front payment and the effort become rivals. The agent cannot pay a large up-front payment and make a large effort.

We show that providing aid is sometimes optimal, but it prevents the project owner to get the full return of the project. Indeed, when the budget constraint has intermediate levels, the principal optimally chooses a sharing contract, knowing that the agent will exert a sub-optimal effort level. However, this enables the principal to pay a lower up-front payment (or to ask a smaller up-front payment) to the agent. In such a case, the agent benefits from having a limited budget and gets some positive rents, whereas he gets no rent in the benchmark case without asymmetric information.

We show a major difference between the case where the agent has sufficient wealth and the case in which his wealth is sufficiently limited. Indeed, when the agent has sufficient wealth, incentives and monetary transfer to the agent are substitutes whereas they become complements when the agent has sufficient but limited wealth. Specifically, when the wealth of the agent is sufficiently large, the bonus paid to the agent increases whereas the transfer received from the principal decreases with the value of the project. In contrast, when the agent has sufficiently limited wealth (but not too much, otherwise no contract is signed) the bonus and the transfer either both increase or both decrease with the value of the project.

We also show that, if the budget of the agent is endogenous, the agent can get the full surplus of the relationship. Indeed, assuming that the agent can consume some of his wealth before the contract is signed, we show that the agent keeps the minimum level of wealth so that the principal offers him a contract. He does not keep more wealth than this level in order to induce the principal to pay the largest (or to ask the smaller) possible up-front payment.

The economic literature has studied the effect of financial constraints in a variety of settings, but it has not considered the effect of limited wealth that limits the agent's possible actions. Indeed, our analysis differs from Lewis and Sappington (2000b,a, 2001) who focus on constraints on the up-front payment, but not on the level of investment of the agent. More specifically, in all these contributions agents are assumed to make bond payments at the outset of the principal-agent relationship, even though the principal can promise to return some portion of the initial bond in Lewis and Sappington (2001). This differs from the situations we focus on, where it is sometimes optimal for the principal to make a positive transfer to the agent up front. In Lewis and Sappington (2000b) the authors briefly consider a situation where the principal can provide a productive input. However, there is an observable and known relationship between the quantity of input provided and the increase in productivity. Again, this contrasts with the present analysis, where the moral hazard problem may actually be related to an investment (see a related application to venture capital in Section 3.2). Lewis and Sappington (2001) consider an adverse selection problem in which the agent is privately informed about his wealth and ability, and analyze the role of these two fundamentals on the power of the incentive scheme designed by the principal.[1] By contrast, we focus on a situation where the agent's ability is endogenous, and analyze how this influences the cases where the principal offers a contract and the power of the incentive scheme. The present paper contributes to the literature by analyzing the optimal trade-off between aid (enhancing the agent's ability to supply effort) and incentives (inducing the agent to supply effort). It complements Lewis and Sappington (2000b) as our model contributes to providing further explanation, in the context of venture capital contracts, about why venture capitalists typically maintain a significant ownership stake in the ventures they finance (Sahlman, 1990) and why venture capitalists will not always provide large share of proceeds to capable entrepreneurs. Laffont and Matoussi (1995) consider a moral hazard problem in which the agent has limited budget for a verifiable input. There is no potential relationship between wealth and effort level, while this relationship is central in our contribution. More specifically, these authors consider two polar cases. In the first one, the agent supplies effort (say labor), there is no investment, and the agent cannot manipulate his resource constraint. In the second one, there is no labor supplied, the agent can make an investment and possibly manipulate the resource constraint, but there is no provision of incentives by the principal. Thus, this contribution cannot consider the optimal mix between aid and incentives, as in the present paper. Che and Gale (2000) study selling mechanisms with limited budget under adverse selection and Burkett (2015) consider the case with budget manipulation by the agent. In the present paper, we consider a moral hazard problem and consider the possibility that the agent spends its budget for consumption in anticipation of the project. This leads to the interesting result that an agent may get the full surplus of the relationship.

Our analysis is also related to the literature on aid. Azam and Laffont (2003) study aid contracts and Cordella and Dell'Ariccia (2007) focus on the comparison of budget support and project aid, in a situation in which the principal can (imperfectly) control the receiver inputs but cannot provide incentives based on the output level. As mentioned above, we focus on the optimal mix between aid and incentives in the present analysis.

[1] The power of an incentive scheme is the rate at which the agent's payoff increases as the surplus he generates increases. In the present paper, the power of the incentive scheme under which the agent operates is the share of the value of the project the agent receives in case of a success.

One of the applications of the model (in Section 3.3) is related to the problem of aiding an agent to raise his capabilities, that we model as an individual potential. In many situations an agent can improve his potential, for instance this can be achieved by education or training. Such situations can be accounted for by the present model. Then a financial aid may help the agent to improve his potential in a particular task by removing other constraints that exist up-front at the outset of the contractual relationship, such as consumption constraints, health-related constraints, or technological constraints.

The topic of the present paper is also related to a current ongoing debate on the relationship between wealth and cognitive abilities. There is no consensus on this question at the moment. Many et al. (2013) provide experimental and field evidence on the fact that low levels of wealth may impede cognitive capacities (focusing on the case of sugarcane farmers in India), while Carvhalo et al. (2015) do not find such a relationship in the case of a group of low-income US households. Even though they provide different conclusions, these two studies suggest that more research is required regarding the effect of the feeling of (material) scarcity on cognitive functions. While we do not take a stance on this debate, we would like to stress that this topic might constitute a potential application of the present work.

The remainder of the paper proceeds as follows. The basic model is introduced in Section 2. Three applications of the model, related to payment for ecosystem services, venture capital, and the issue of raising individual potential, respectively, are developed in Section 3 in order to highlight that our setting can be applied in a variety of situations. The analysis of the basic model is provided in Section 4. The possibility that the agent consumes part of his wealth at the outset of the contractual relationship is considered and analyzed in Section 5. Policy implications are discussed in Section 6. Finally, Section 7 concludes. All proofs are provided in an appendix at the end of the paper.

2 The Model

We consider a principal agent model in which a principal may contract with an agent in order to complete a project. The budget of the agent is B. The principal may choose to propose a contract to the agent or to get his outside option, $\phi^P \geq 0$. The probability of success of the project p is an increasing and concave function of an unobservable effort of the agent $e \geq 0$, that is $p(e)$, which is assumed to be twice continuously differentiable, strictly increasing and strictly concave, $p'(e) > 0$ and $p''(e) < 0$. If the project is a success, the principal gets a positive return $V > 0$. The principal do not observe the effort of the agent, thus the contract is only based on the success or failure of the project. The principal asks an up-front payment T to the agent (which may be negative or positive) and a bonus paid to the agent, w, in case of success. If the project fails, the agent receives no bonus. If the agent refuses the contract, he gets $\phi^A \geq 0$ and he consumes is wealth, B. If he accepts the contract, the principal pays him the up-front payment T, which is possibly a payment from the agent to the principal. However, it cannot exceed the budget of the agent, i.e. $-T \leq B$. Then, the agent chooses an effort $e \in [0, \overline{e}]$,[2] with cost e. We assume that the effort is exerted before the project is fulfilled and it is limited by

[2] We assume that $\overline{e}$ is large enough so that it never constrains the optimal solution.

the budget constraint of the agent, which writes:

$$e \leq B + T. \tag{1}$$

This assumption is consistent with various situations: either the opportunity cost is monetary, or the effort provided by the agent is actually an investment, or the agent can invest in training activities in order to improve his potential. These various settings are illustrated by several applications in Section 3. If the project succeeds, the agent receives the bonus and consumes its remaining wealth, $u^A = w + B + T - e$. The principal gets the return of the project, pays the bonus to the agent, and pays the up-front payment: $u^P = V - w - T$. If the project fails, the agent receives no bonus, but he consumes its remaining wealth, $u^A = B + T - e$. The principal gets no return from the project, pays no bonus to the agent, but gets the up-front payment: $u^P = -T$. The expected payoff of the agent is then:

$$Eu^A = p(e)\,w - e + B + T, \tag{2}$$

and the expected payoff of the principal is:

$$Eu^P = p(e)\,(V - w) - T. \tag{3}$$

Notice that the outside option of the principal is ϕ^P while the outside option of the agent is $\phi^A + B$.

Assumption (A1): *The project is not worthwhile if the effort is zero:*

$$p(0)V < \phi^A + \phi_P. \tag{4}$$

If the principal does not proposes a contract, he gets his outside option, ϕ^P. If he prefers to propose a contract to the agent, his programme is the following:

$$\underset{T,w,e}{Max}\ Eu^P = p(e)\,(V - w) - T, \tag{5}$$

such that the incentive constraint holds:

$$e \in \arg \max_{e \in [0,\bar{e}]} \{p(e)\,w + B + T - e\}, \tag{6}$$

the participation constraint holds:

$$p(e)\,w + B + T - e \geq B + \phi^A, \tag{7}$$

and the budget constraint holds:

$$B + T \geq e. \tag{8}$$

Expression (5) reflects the fact that the principal's expected net return is the difference between the expected surplus from the project and the rent that accrues to the agent, in the form of the bonus w and the up-front payment, T. Condition (6) is the incentive constraint, which ensures that the effort level provides the maximal payoff to the agent. Condition (7) ensures the participation of the agent, and (8) guarantees that the agent is not asked to provide an effort that exceeds his budget and the up-front payment.

Example 1: *Throughout the paper, we will use the following example to illustrate the general results. The probability of a success is* $p(e) \equiv \frac{1}{\gamma}e^{\gamma}$ *with* $\gamma \in (0,1)$*,* e *belongs to* $[0,1]$ *and* V *belongs to* $(0,1]$*.*

In the next section we highlight the genericity of our approach by providing several applications and showing how they are consistent with the present model.

3 Applications

We provide three applications of problems that have attracted increasing interest among economists in different fields. Specifically, we first present an application related the conservation of natural resources, that is, the design of payment for ecosystem services (see Alix-Garcia and Wolf 2014 for a review on payments for forest conservation). Then an application is provided on the issue of venture capital, a problem that is the object of a large number of contributions (see Hellman 1998 for a related application). Finally, while the first two applications are mainly focused on the notion of financial aids, we provide a general application on the idea of motivating an agent to raise maximal effort, which has intuitive meaning and is somehow related to the current debate on the relationship between wealth and cognitive functions (Many et al., 2013; Carvhalo et al., 2015). In each case we first describe the problem at hand, and then highlight how it can be stated using a model similar to the one presented in Section 2.

3.1 Payment for Ecosystem Services (PES)

A landlord contracts with the government on a given share of land over a given period of time. The government values the plot of land and $V > 0$ is the government benefit if the plot of forest is conserved during the period of time and 0 if the plot of forest is not. Ex-ante, there are N trees on this land. This is (imperfectly) measured thanks to satellite imagery or to an expert visual evaluation. The contract stipulates that the landlord receives an up-front payment T and gets an additional payment w if the plot of land under contract is detected as conserved at the end of the period.[3] Some trees can be destroyed because of accidental fire or unexpected tree disease. Some trees can also be cut by the landlord.[4] It may be difficult and costly to asses whether trees were destroyed because of natural hazard or because of the landlord will. The landlord chooses the number of trees, $N - n$, he cuts during the given period of time and keeps n trees. The landlord gets a benefit, $N - n$, from cutting $N - n$ trees. The probability that the plot of land is declared to be "conserved" is $p(n)$. However, the landlord is financially constrained: he has wealth, R, and needs to use a minimal amount of money in order to live during the length of the contract. This minimal amount is denoted $\underline{c}$. If the landlord does not accept the contract, he cuts the forest and consumes his wealth, he then gets $B + N$. In such

[3] See Munoz-Pina et al. (2008) for a description of PES implemented in Mexico

[4] For instance, in Brazil, farmers convert forests into cattle pasture land (Simonet et al., 2015).

a situation, the problem between the government (the principal) and the landlord (the agent) can be stated as follows:

$$Max_{(T,w,n)} Eu^P = p(n)(V-w) - T \tag{9}$$

$$e \in \arg\max_{n\in[0,N]} \{p(n)w + R + T + N - n\} \tag{10}$$

$$p(n)w + R + T + N - n \geq R + N \tag{11}$$

$$R + T + N - n \geq \underline{c} \tag{12}$$

This program is easily checked to be consistent with our model. Indeed, denote $B = R + N - \underline{c}$, then equations (9), (11) and (12) correspond to those of our model using $\phi^A = 0$. Using the expression of B, inequality (10) becomes $e \in \arg\max_{n\in[0,N]} \{p(n)w + B + T - n + \underline{c}\}$. The only difference with our model is due to parameter $\underline{c}$, which is a constant. Thus, the solution to this problem and to ours are identical.

3.2 Venture Capital

We next present a problem related to the issue of venture capital, where funding is provided by investors to startup firms and small businesses often characterized by difficult access to capital markets and long-term growth potential, such as startups producing innovative technologies. The problem here is one of providing money to an entrepreneur or startup business by a venture capital firm at an early stage of development. By doing so the venture capitalist expects sufficient returns on its investment. The interested reader is referred to Metrick and Yasuda (2010) for an extensive overview of issues related to venture capital.

We here consider the case of a venture capitalist (the principal) who is willing to provide added financial means T to allow an entrepreneur (the agent) to make an investment in a project. The entrepreneur can also invest his own wealth in the project, B. The entrepreneur starts a business which, if successful, generates benefit V. The moral hazard problem comes from the fact that the entrepreneur can choose the level of investment $I \leq B + T$. Instead of retaining an adequate stake in the business, the entrepreneur may shirk or invest in perks at the expense of productive investments (see Mehta 2004 for a related discussion). If the entrepreneur accepts the contract offered by the venture capitalist, he receives w if the project is successful and 0 otherwise. The probability $p(I)$ is the probability that the entrepreneur succeeds in starting the firm. If the agent rejects the contract offered, he gets the outside option payoff $B + \phi^A$, which corresponds to the sum of his initial wealth and a given extra profit ϕ^A that he gets from another economic activity. With this description of the problem, the venture capitalist's program becomes

$$Max_{(T,w,I)} Eu^P = p(I)(V-w) - T \tag{13}$$

$$I \in \arg\max_{I\geq 0} \{p(I)w + B + T - I\} \tag{14}$$

where the excess sum of money $B+T-I \geq 0$ is spent in added consumption, and yields the extra utility $B+T-I$. The participation constraint of the entrepreneur writes:

$$p(I)w + B + T - I \geq B + \phi^A, \tag{15}$$

and the budget constraint writes:

$$B + T \geq I. \tag{16}$$

It is easily checked that this problem is identical to the one described in Section 2, where I is the action of the agent. The main question is thus to discuss why I is not observable by the principal. An explanation goes as follows. The agent spends money ($I \leq B+T$) in order to both buy new means (a new equipment) and learn how to use them. The principal may know how much the equipment costs, but does not know how much (learning) effort the agent exerts in order to know how to use this new equipment. In such a case, the moral hazard problem is on the learning effort, rather than on the execution effort.

3.3 Aiding to Raise the Agent's Capability

In this section, we discuss a fairly general application that refers to the idea of aiding the agents to raise his maximal potential effort. This application is actually related to a current debate in economics and psychology going on the relationship between wealth and cognitive functions. Here we want to highlight the potential trade-off that may emerge between providing incentives to an agent to supply effort and aiding him to develop the maximal effort he can potentially supply.

This application also refers to the problem of how the potentially endogenous development of an agent's capabilities,[5] which may result both from the principal, via aids, or from the agent, via learning or training activities, can act on the power of the incentive scheme. Sen (1999) defines capabilities as a subset of possibilities. In this application, the agent's capability corresponds to the different levels of effort he can supply. It is measured by the length of the interval starting from no effort to a maximal effort level. The larger this maximal effort is, the higher is the agent's capability.

As mentioned above, we here consider that the agent's capability corresponds to the maximum level of effort he can supply. Let $\overline{e}(H) \geq 0$ be the maximum effort that the agent can spend in the project. Suppose that it depends on training or other activities H, which costs $I = I(H)$, in monetary units. For simplification take $I = H$. The cost of effort is ce, where $c \geq 0$.

A principal may provide money T to an agent to enable him to undertake training activities $H = I$ so that he can supply a higher maximal effort. It is assumed that the principal does not know I nor $\overline{E}(I)$. The other assumptions are the same as in the model presented in Section 2. The principal's problem can then be written as follows:

[5] The notion of capability comes from a large literature. There are three well-known strands of literature developing theories of capability: the capability approach of development in philosophy and economics (Sen, 1985, 1989, 1999), the social cognitive theory of human behavior in psychology, which introduces five related types of capability and the self efficacy concept (Bandura, 1977, 2001), and the resource based theory of the firm (Wernerfelt, 1984; Conner and Prahalad, 1996) in management sciences.

$$Max_{(T,w,e,I)} Eu^P = p(e)(V - w) - T \tag{17}$$

such that the agent chooses the level of effort and the level of investment in his potential:

$$(e, I) \in \arg \max_{0 \leq e \leq \overline{E}(I)} \{p(e)w + B + T - I - ce\}, \tag{18}$$

and such that the agent chooses to participate:

$$p(e)w + B + T - I - ce \geq B + \phi^A, \tag{19}$$

such as the agent's budget constraint holds:

$$B + T \geq I, \tag{20}$$

and such that the effort does not exceed the agent's potential:

$$0 \leq e \leq \overline{E}(I). \tag{21}$$

In this application, it is straightforward to show that the agent will optimally choose the maximal effort level. First, inspecting the incentive constraint yields the conclusion that the investment has a decreasing effect on the agent's payoff. Second, the constraint stating that the effort level is at most as large as the potential maximal effort acts as the only lower bound on the investment level. Combining these two points, one deduces that the agent necessarily chooses to supply his potential maximal effort level. The problem can then be rewritten as follows:

$$Max_{(T,w,I)} Eu^P = \pi(I)(V - w) - T \tag{22}$$

$$I \in \arg \max_{I \geq 0} \{\pi(I)w + B + T - \psi(I)\}, \tag{23}$$

$$\pi(I)w + B + T - \psi(I) \geq B + \phi^A, \tag{24}$$

$$B + T \geq I, \tag{25}$$

where $\pi(I) \equiv p(\overline{E}(I))$ and $\psi(I) \equiv I + c\overline{E}(I)$.

If the cost of effort is negligible, that is $c = 0$, we have $\psi(I) \equiv I$, and the problem is identical to the one described in Section 2, with I replacing e and π replacing p.[6]

4 Analysis of the Model

We now provide the analysis of the model presented in Section 2. Let us first provide the benchmark of the first best situation.

[6] The case where the cost of effort is positive $c > 0$ is an extension of the present paper that is left for future research.

4.1 First Best

The first best effort is the level of effort that maximizes the joint payoff of the agent and the principal. If the project is launched, the joint payoff is the expected return of the project, $p(e)V$, net of the cost of effort, e:

$$\underset{\in[0,\overline{e})}{Max}\ W = p(e)V - e \tag{26}$$

The solution of this programme is given by

$$e^{FB} = g(V), \tag{27}$$

where $g(x) = (p')^{-1}(1/x)$, with $g' > 0$. The effort level, e^{FB}, is thus increasing in the value of the project, V.[7]

In order to exclude cases in which the first best situation leads to no contract, we make the following assumption:

Assumption (A2): *The project is worthwhile if the effort level is* $e = e^{FB}$:

$$p\left(e^{FB}\right)V - e^{FB} > \phi^A + \phi^P \tag{28}$$

This condition also ensures that the first best level of effort is e^{FB}, as defined by (27).

4.2 Optimal Contract: Motivating versus Aiding

Proposition 1: *If the wealth of the agent is sufficiently large,* $\overline{\overline{B}} < B$*, the optimal contract has the following properties:*
(i) Full returns, i.e. the bonus for success equals the value of the project:

$$\widehat{w} = V,$$

(ii) No aid, i.e. the up-front payment is always a payment from the agent to the principal:

$$\widehat{T} = \phi^A - \left(p\left(e^{FB}\right)V - e^{FB}\right) < 0,$$

(iii) No rent, i.e. the agent gets no surplus:

$$\widehat{Eu^A} - \left(\phi^A + B\right) = 0,$$

(iv) Efficiency, i.e. the agent's effort equals the first best level:

$$\widehat{e} = e^{FB},$$

and $\overline{\overline{B}} = p\left(e^{FB}\right)V - \phi^A$.

Proposition 1 states that if the agent has sufficient wealth, the principal can sustain efficiency

[7] Indeed, using condition (27) and the implicit function theorem, we have $\partial e^{FB}/\partial V = -p'(e^{FB})/p''(e^{FB}) > 0$.

and get the full expected surplus of the project. He can reach this outcome by making the agent the residual claimant for all returns from the project after having received an up-front payment from the agent equal to the expected surplus of the project.

In the following we use the elasticity of the probability function p and the elasticity of its derivative p' that are functions of the effort e:

$$\epsilon_p(e) = ep'(e)/p(e) \text{ and } \epsilon_{p'}(e) = ep''(e)/p'(e) \tag{29}$$

In order to ensure the uniqueness of the optimal contract in the cases we analyze next,[8] we make the following assumption:

Assumption (A3): *The ratio of the elasticity of the marginal probability of success and the elasticity of the probability of success, $\epsilon_{p'}(e)/\epsilon_p(e)$, is non increasing.*

We can then show the following result:

Lemma 1: *If the wealth of the agent is intermediate, $\underline{B} \leq B < \overline{B}$, the optimal contract is such that: the bonus for success, w^*, is unique and characterized by*

$$1 = \frac{V}{2}p'\left(g\left(w^*\right)\right) + \frac{1}{2}\frac{\epsilon_{p'}(g\left(w^*\right))}{\epsilon_p(g\left(w^*\right))}, \tag{30}$$

the up-front payment from the principal to the agent is

$$T^*\left(B\right) = g\left(w^*\right) - B,$$

and the agent's effort is

$$e^* = g\left(w^*\right),$$

and $\underline{B} = \phi^P - p(g\left(w^*\right))\left(V - w^*\right) + g\left(w^*\right) < \overline{B} = p(g\left(w^*\right))w^* - \phi^A < \overline{\overline{B}}$, where $g(x) = \left(p'\right)^{-1}(1/x)$.

Lemma 1 characterizes the optimal contract for intermediate values of the agent's wealth. Without more structure on the probability p, we cannot explicitly characterize the optimal bonus, w^*. If one assumes, for instance, that $p\left(e\right) \equiv \frac{1}{\gamma}e^{\gamma}$ with $\gamma \in (0,1)$, e belongs to $[0,1]$ and V belongs to $(0,1]$, then one can find the explicit expression of the optimal bonus,[9] $w^* = \frac{\gamma}{1+\gamma}V$.

For the ease of exposition, we make the following assumption that ensures the existence of all feasible cases:

Assumption (A4): *The budget thresholds $\underline{B}$ and $\overline{B}$ are such that $0 < \underline{B} < \overline{B}$.*

[8] We also need the following Inada conditions. $lim_{e \to 0}\frac{V}{2}p'\left(e\right) + \frac{1}{2}\frac{\epsilon_{p'}(e)}{\epsilon_p(e)} > 1$ and $lim_{e \to \overline{e}}\frac{V}{2}p'\left(e\right) + \frac{1}{2}\frac{\epsilon_{p'}(e)}{\epsilon_p(e)} < 1$. These conditions hold for instance in the case of Example 1, since $lim_{e \to 0}\frac{V}{2}p'\left(e\right) + \frac{1}{2}\frac{\epsilon_{p'}(e)}{\epsilon_p(e)} = +\infty > 1$ and $lim_{e \to \overline{e}}\frac{V}{2}p'\left(e\right) + \frac{1}{2}\frac{\epsilon_{p'}(e)}{\epsilon_p(e)} = \frac{V}{2} + \frac{\gamma - 1}{2} < 1$ because V and γ are smaller than 1.

[9] For this example, the optimal effort of the agent is $e^* = \left(\frac{\gamma}{1+\gamma}V\right)^{\frac{1}{1-\gamma}}$.

We maintain this assumption in the rest of the paper. We can now derive the properties of the optimal contract:

Proposition 2: *If the wealth of the agent is intermediate, $\underline{B} \leq B < \overline{B}$, the optimal contract has the following properties:*
(i) Shared returns, i.e. the bonus for success, is strictly positive and smaller than (one half of) the value of the project:

$$0 < w^* < \frac{V}{2}.$$

(ii) Aid, i.e. the up-front payment can be a payment from the principal to the agent when the agent's wealth is small enough. Formally, if $T^*(\underline{B}) \leq 0$ then $T^*(B) \leq 0$ for all $B \in [\underline{B}, \overline{B})$ and if $T^*(\underline{B}) > 0$ there exists $\tilde{B} \in (\underline{B}, \overline{B}]$ such that

$$T^*(B) > 0 \Leftrightarrow B < \tilde{B}.$$

(iii) Positive rents, i.e. the agent gets a strictly positive surplus:

$$Eu^{A*} - \left(\phi^A + B\right) > 0.$$

(iv) Inefficiency, i.e. the effort of the agent is strictly lower than the first best effort level:

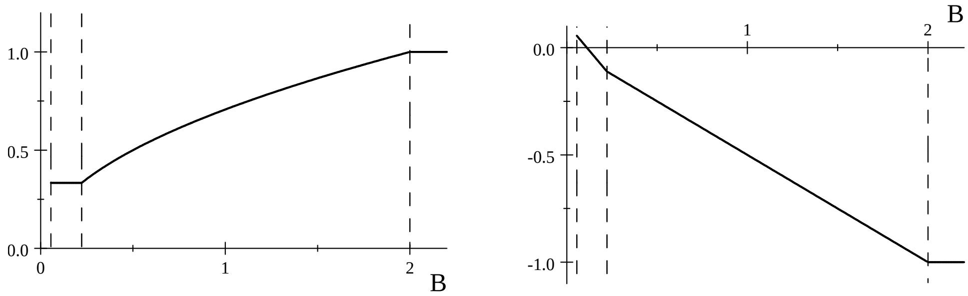
$$e^* < e^{FB}.$$

Proposition 2 states that if the agent's wealth is intermediate, the principal cannot sustain efficiency and he cannot get the full expected surplus of the project. If the principal chooses to let the agent being the residual claimant of the full returns of the project, $w = V$, the latter makes the first best level effort, the principal cannot ask an up-front payment larger than $T = e^{FB} - B < 0$, and then the agent gets a large rent $Eu^{A*} = p\left(e^{FB}\right)w^*$. Instead, the principal can reach a second best by letting a share - smaller than one half - of the returns of the project to the agent after having paid an up-front payment to the agent in order to enable him to reach the second best effort level. Notice that point (iv) ensures that $\overline{B} < \overline{\overline{B}}$. Figure 1 illustrates the up-front payment from the principal to the agent as a function of the agent's wealth, in the case where $T^*(\underline{B}) > 0$. While it is always a payment from the agent to the principal ($T^* < 0$) when the agent's wealth is large (when $\overline{\overline{B}} \leq B$, see Proposition 1), it is a payment from the principal to the agent ($T^* > 0$) when a sharing contract is signed and the agent's wealth is sufficiently small (when B is between $\underline{B}$ and $\tilde{B}$, see Proposition 2).

Proposition 3: *If the wealth of the agent is sufficiently small, $B < \underline{B}$, the principal does not propose a contract, both the principal and the agent get their outside option, ϕ^P and $\phi^A + B$, respectively.*

Proposition 3 states that, if the agent's wealth is sufficiently small, it is too costly for the principal to provide aid to the agent, even if the project is worthwhile a priori (see Assumption A2). For instance, if the principal chooses to let the agent being the residual claimant of the full returns of the project, $w = V$, the latter makes the first best level effort. However, the principal

Figure 1: Optimal bonus and transfer to the agent



Optimal bonus (w)

Optimal transfer (T)

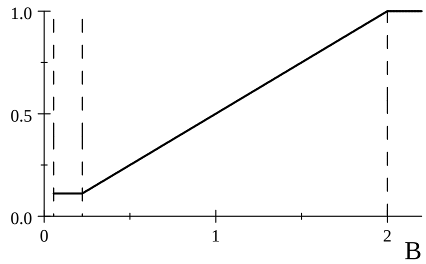
Note: This Figure displays the optimal bonus and transfer as functions of B, in the case of Example 1. We set $\phi^A = 0$, $\phi^P = 7/18$, $V = 1$ and $\gamma = 1/2$. The dashed vertical lines correspond to the thresholds, $\underline{B} = 1/18$, $\overline{B} = 2/9$ and $\overline{\overline{B}} = 2$.

cannot ask an up-front payment larger than its outside option $T = B - e^{FB} < \underline{B} - e^{FB} = \phi^P$, and then he prefers not to propose a contract to the agent.

Proposition 4: *If the wealth of the agent is intermediate, $\overline{B} \leq B < \overline{\overline{B}}$, the principal offers a contract such that both the budget constraint and the participation constraint are binding. Moreover, the optimal effort is $\widetilde{e} = g(\widetilde{w})$, the optimal transfer is $\widetilde{T} = g(\widetilde{w}) - B$ where the optimal bonus is characterized by $p(g(\widetilde{w}))\widetilde{w} = B + \phi^A$.*

Proposition 4 states that for agent's wealth levels above those for which the optimal contract is characterized by Proposition 2 and below those for which the optimal contract is characterized by Proposition 1, the optimal values of the bonus and the transfer are fully characterized by the (binding) participation constraint and the budget constraint. Moreover, one can show that the optimal bonus, the optimal transfer and the optimal effort for this case lie in between the respective optimal values for the case of Proposition 2 and the case of Proposition 1. As illustrated in Figure 1, we have $w^* \leq \widetilde{w} \leq \widehat{w} = V$ and $\widehat{T} \leq \widetilde{T} \leq T^*$. As illustrated in Figure 2, we have $e^* \leq \widetilde{e} \leq \widehat{e} = e^{FB}$.

Figure 2: Optimal effort of the agent



Note: This Figure displays the optimal effort of the agent as a function of B, in the case of Example 1. We set $\phi^A = 0$, $\phi^P = 7/18$, $V = 1$ and $\gamma = 1/2$. The dashed vertical lines correspond to the thresholds, $\underline{B} = 1/18$, $\overline{B} = 2/9$ and $\overline{\overline{B}} = 2$.

The surplus of the principal and the agent are given in the following Corollary.

Corollary 1: *The optimal values of the bonus w, the transfer T, and the $effort$ as well as the surplus of the agent are continuous with respect to B over $[\underline{B}, +\infty)$. The surplus of the principal* *is continuous with respect to* B *over* $[0,+\infty)$*. The surplus of the agent writes:*

$$Eu^{A*}-\left(\phi^{A}+B\right)=\begin{cases}0 \text{ if } B<\underline{B}\\ \overline{B}-B \text{ if } \underline{B}\leq B<\overline{B}\\ 0 \text{ if } \overline{B}\leq B\end{cases},$$

the surplus of the principal writes

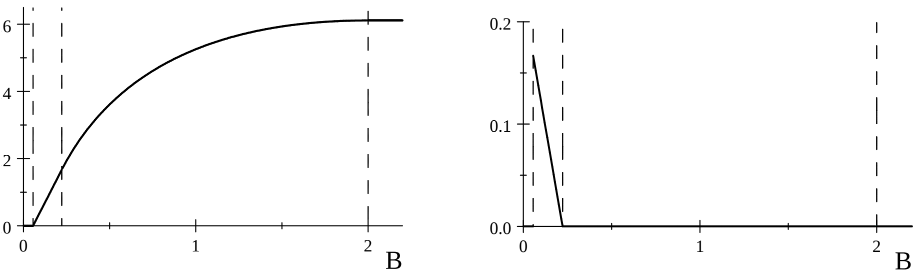
$$Eu^{P*}-\phi^{P}=\begin{cases}0 \text{ if } B\leq\underline{B}\\ B-\underline{B} \text{ if } \underline{B}\leq B\leq\overline{B}\\ p\left(\widetilde{e}\right)V-\widetilde{e}-\left(\phi^{A}+\phi^{P}\right) \text{ if } \overline{B}\leq B\leq\overline{\overline{B}}\\ p\left(e^{FB}\right)V-e^{FB}-\left(\phi^{A}+\phi^{P}\right) \text{ if } \overline{\overline{B}}\leq B\end{cases}$$

and the total surplus of the relationship is given by

$$Eu^{A*}+Eu^{P*}-\left(\phi^{A}+\phi^{P}\right)=\begin{cases}0 \text{ if } B\leq\underline{B}\\ \overline{B}-\underline{B} \text{ if } \underline{B}\leq B\leq\overline{B}\\ p\left(\widetilde{e}\right)V-\widetilde{e}-\left(\phi^{A}+\phi^{P}\right) \text{ if } \overline{B}\leq B\leq\overline{\overline{B}}\\ p\left(e^{FB}\right)V-e^{FB}-\left(\phi^{A}+\phi^{P}\right) \text{ if } \overline{\overline{B}}\leq B\end{cases}$$

Figure 3 illustrates the expected surpluses as functions of the agent's wealth B. The expected surplus of the agent is zero for $B\leq\underline{B}$, since he gets his outside option. At $B=\underline{B}$, the expected surplus of the agent jumps upward and decreases up to $B=\overline{B}$, because its outside option increases with B. This is because the optimal contract is such that the budget constraint of the agent is saturated. Over this interval, an additional unit of wealth does not benefit to the agent. After $\overline{B}$, the agent gets its outside option, first because the optimal contract is such that both the budget constraint and the participation constraint are binding (when B is between $\overline{B}$ and $\overline{\overline{B}}$) and then because the optimal contract reaches the first best and leaves him no rent (when B is above $\overline{\overline{B}}$). The expected payoff of the principal equals his outside option when the agent's wealth is low ($B\leq\underline{B}$), then his surplus is zero. His surplus then increases because the (second-best) contract is implemented (when B is between $B=\underline{B}$ and $\overline{B}$) and the up-front payment is bounded by the wealth of the agent. At $B=\overline{B}$, it increases at a lower rate because the participation constraint is binding $\overline{\overline{B}}$. For larger values of the agent's wealth, the budget constraint is no more binding, the optimal contract enables to reach the first best situation, and the principal gets the full surplus of the relationship, which does not depend on the agent's wealth.

Figure 3: Expected surplus of the principal and the agent



Expected surplus of the principal

Expected surplus of the agent

Note: This Figure displays the expected surplus of the principal and the agent as functions of B, in the case of Example 1. We set $\phi^A = 0$, $\phi^P = 7/18$, $V = 1$ and $\gamma = 1/2$. The dashed vertical lines correspond to the thresholds, $\underline{B} = 1/18$, $\overline{B} = 2/9$ and $\overline{\overline{B}} = 2$.

4.3 Complementarity between Incentives and Transfer to the Agent

Proposition 5: *(i) If the wealth of the agent is sufficiently large, $\overline{\overline{B}} \leq B$, that is when the contract is efficient, the optimal contract is such that incentives and transfer to the agent are* <u>*substitute*</u> *as regards the value of the project. Formally, $\frac{\partial \widehat{w}}{\partial V} \geq 0$ and $\frac{\partial \widehat{T}}{\partial V} \leq 0$.*
(ii) If the wealth of the agent is intermediate, $\underline{B} \leq B < \overline{B}$, the optimal contract is such that incentives and aid are <u>*complementary*</u> *as regards the value of the project. Formally, $\frac{\partial w^*}{\partial V} \geq 0$ if and only if $\frac{\partial T^*}{\partial V} \geq 0$.*
(iii) Otherwise, that is if $B < \underline{B}$ or $\overline{B} \leq B \leq \overline{\overline{B}}$, the optimal contract is such that incentives and aid are independent of the value of the project. Formally, $\frac{\partial w^*}{\partial V} = 0 = \frac{\partial T^*}{\partial V}$.

This result reveals a striking difference between the case in which the principal can reach an efficient contract, as in the standard moral hazard model when the agent and the principal are risk neutral, and the case in which the principal has to implement a sharing contract and the participation is not binding (case ii). In the former case, the bonus increases while the transfer from the principal to the agent decreases with the value of the project. In the latter case, the bonus and the transfer either both increase or both decrease with the value of the project. This implies that, when aid is optimal (see Proposition 2, point ii), then aid and incentives are complements.

5 Endogenous Agent's Budget

Now assume that there are two periods. An initial period 0 followed by period 1, period 1 being the game described and studied in Section 2. At period 0, the agent, who has an initial limited budget $B_0 \geq 0$, may choose to consume part of his initial budget $c_0 \in [0, B_0]$ and gets a payoff $u_0^A = c_0$. At the beginning of period 1, the budget of the agent is the remaining $B = B_0 - c_0$ and the game played is the same as in Section 2.

At period 0, the agent may choose to consume his initial wealth, B_0, or to save it for period

1. The agent maximizes the sum of his expected gains:

$$\underset{c_0 \in [0,B_0]}{Max} U^A = u_0^A + \delta E u^{A*},$$

where $u_0^A = c_0$ and $B = B_0 - c_0$. Parameter δ denotes the agent's discount factor, and it is assumed to strictly lie between 0 and 1.

The programme can be written in substituting $c_0 = B_0 - B$:

$$\underset{B \in [0,B_0]}{Max} U^A = B_0 - B + \delta E u^{A*},$$

This problem is well defined if Eu^{A*} is uniquely defined, i.e. (30) characterizes a unique bonus w^*. In the following, we assume that the optimal bonus w^* is unique. Notice that, as already mentioned, w^* is, for instance, unique when $p(e) \equiv \frac{1}{\gamma} e^{\gamma}$ with $\gamma \in (0,1)$. We also assume that $\underline{B} > 0$. We then obtain the following result:

Proposition 6: *(i) If the agent is sufficiently wealthy initially, i.e. $B_0 \geq \underline{B}$, then he chooses $B^* = \underline{B}$ when $\delta \geq \underline{B}/\overline{B}$ and $B^* = 0$ otherwise. (ii) If the agent is not sufficiently wealthy initially, i.e. $B_0 < \underline{B}$, then he chooses $B^* = 0$.*

When the agent is sufficiently wealthy initially (case (i)) and sufficiently patient ($\delta \geq \underline{B}/\overline{B}$), he consumes his initial wealth as long as his remaining budget is sufficient for the principal to offer a sharing contract in period 1. When the agent is not sufficiently patient ($\delta < \underline{B}/\overline{B}$), since he heavily discounts the future, he prefers to consume all his initial wealth during period 0, and then no contract is signed in period 1. When the agent is not sufficiently wealthy initially ($B_0 < \underline{B}$), he anticipates that no contract will be signed in period 1, and since he discounts the future, he consumes all his initial wealth during period 0.

Proposition 7: *When the budget of the agent is endogenous, as long as the agent is sufficiently wealthy initially ($B_0 \geq \underline{B}$) and sufficiently patient $\left(\delta \geq \underline{B}/\overline{B}\right)$, the equilibrium has the following properties:*
(i) Sharing Contract, i.e. if a contract is signed, it is necessarily the sharing contract described in Lemma 1 and it has the properties described in Proposition 2.
(ii) Agent's Full Bargaining Power, i.e. the agent always gets the full surplus of the relationship: $Eu^{A*} - (\underline{B} + \phi^A) = \overline{B} - \underline{B}$ and $Eu^{P*} - \phi^P = 0$.

Proposition 6 provides two striking results that diverge from the standard properties of the optimal contract under moral hazard with a risk neutral agent which is described in Proposition 1. Considering endogenous limited wealth modifies the main properties of the optimal contract drastically. The agent is no more the full residual claimant of the returns of the project, he gets a share of the returns of the project. The up front payment can be a payment from the principal to the agent (if $T^*(\underline{B}) > 0$), instead of being a payment from the agent to the principal ($T^*(\underline{B}) < 0$). Moreover, the agent gets all the surplus of the relationship instead of getting no rent. The intuition of this result is as follows. The agent has incentives to keep a sufficiently large budget (larger than $\underline{B}$) so that the principal will not choose its outside option. He also have incentives to keep a sufficiently small budget (smaller than $\overline{\overline{B}}$) so that the principal will not choose an efficient contract which leaves no rent to the agent. Since all the contracts that meets these two conditions generate the same total surplus (see Corollary 1), the agent keeps the level of budget that maximizes the up-front payment he receives from the principal. He then chooses the lowest level, $\underline{B}$, which implies that the principal gets its outside option while the agent gets all the surplus.

6 Policy Implications

We come back to the three applications introduced in Section 3. More specifically, we discuss the implications of our findings in the settings that correspond to these three cases.

6.1 Payments for Ecosystem Services

As mentioned before, the payments for ecosystem services (PES) are provided to developing countries forest owners at the end of a given period of time if they have kept their forest intact. As explained in (Jayachandran, 2013), a major drawback of this mechanism is that it does not take into account the short-term large opportunity costs that liquidity constrained forest owners may face. However, Alix-Garcia and Wolff (2014) argue that contracts vary from 5 years in Mexico to 20 years in Ecuador and that PES programmes tend to pay at the end of each contract year and not at the end of the contract. As explained in Section 3.1, this case is consistent with our analysis. Indeed, since forest owners face monetary opportunity costs, their level of wealth is constrained and may restrict the range of conservation actions they can undertake. Our analysis highlights that it is actually optimal to offer a contract stipulating revenue sharing on one hand and, on the other side, a positive transfer at the outset of the relationship from the principal to agents characterized by sufficiently low levels of wealth.

6.2 Venture Capital

The implications of our findings allow to provide a new perspective on venture capital contracts. They actually complement Lewis and Sappington (2000b), as our model provides further potential explanations why venture capitalists typically maintain a significant ownership stake in the ventures they finance (Sahlman, 1990), and why venture capitalists will not always provide large share of proceeds to capable entrepreneurs. In such a situation, when the entrepreneur has sufficiently limited wealth and may need to make extra (unobservable) investment to improve his productivity, then a sharing contract will emerge provided that the entrepreneur's wealth is sufficiently (but not too) low. The venture capitalist will then provide both an up-front payment and a revenue sharing contract, but will retain significant stake in the venture.

6.3 Aiding to Raise the Agent's Capability

Implications can be drawn at different levels. First, the analysis interpreted in the context of application 3.3 highlights very clearly that the principal's interest is to aid the agent to increase his capability. A first implication is thus that the present findings provide support for the existence of regular training activities on the job, and highlight that incentives and aid are sometimes complementary. Proper attention should be paid to these aspects.

Second, we can come back to the more specific application provided by the analysis of Many et al. (2013). They focus on the case of sugarcane farmers in India, and their analysis suggests that, in this specific setting, low levels of wealth may impede cognitive capacities. This case is then somehow consistent with the model presented here. Our findings offer a new explanation for the prevalence of sharecropping contracts in agriculture (for other explanations see, among others, Eswaran and Kotwal 1985 and Allen and Lueck 2004). In our setting, revenue sharing will emerge every time that the agent's wealth is sufficiently (but not too) low.

7 Conclusion

We introduce in this article a moral hazard problem where the agent has limited wealth, which limits his possible range of actions. This is consistent with several situations where, among other: the opportunity cost is monetary, the effort provided by the agent actually consists in an investment, or the agent can invest in training activities in order to improve his capability. In such cases, the lower the level of wealth is (including transfer from or to the principal), the lower the maximum effort level that can be provided. Limited wealth and its limiting effect on the set of possible actions is the distortion compared to the standard model.

In this setting we show that the optimal contract is, in some cases, a sharing contract and the optimal up-front transfer is a payment from the principal to the agent. Moreover, whereas incentives and monetary transfer to the agent are substitutes in the case where the agent has sufficient wealth, they are shown to be complements when the agent has limited wealth. Finally it is also shown that, if the agent can consume his wealth at the outset of the contractual relationship, he gets all the surplus of the relationship.

These findings yield a number of policy implications in the context of (among others) natural resource conservation or venture capital, and provides several insights related to a current debate on the potential effect of wealth on cognitive functions. For instance, they suggest the use of payment for ecosystem services stipulating both revenue sharing and a positive transfer from the principal at the outset of the relationship when agents' income is low. This article provides a number of interesting predictions, and constitutes a first step in this research agenda. Further analysis of the problem in a dynamic setting seems to be a natural next step.

Appendix A

We first prove the following preliminary result that will be useful in some of our proofs.

Lemma 2: *If the principal offers a contract to the agent, i.e.* $p(e)(V-w)-T \geq \phi^P$*, then the optimal effort and the optimal bonus are strictly positive* $e^* > 0$ *and* $w^* > 0$.

Proof of Lemma 2: Since the principal's payoff increases with the up-front payment, the principal has an incentive to increase the level of the up-front payment as long as the participation constraint (7) and the budget constraint (8) are not binding. Thus, the optimal up-front payment is given by

$$T^* = e - \min\{B, p(e)\, w - \phi_A\}. \tag{31}$$

The programme of the principal can then be rewritten as follows

$$\underset{w,e}{Max}\ \widetilde{Eu^P} = p(e)(V-w) - e + \min\left\{B, p(e)w - \phi^A\right\}, \tag{32}$$

such that

$$e \in \arg\max_{e\in[0,\overline{e}]} \{p(e)w - e\}. \tag{33}$$

We cannot have $w^* \leq 0$ or $e^* = 0$. Indeed, if $w^* \leq 0$ condition (33) implies $e^* = 0$. If $e^* = 0$ the payoff of the principal becomes $\widetilde{Eu^P} = p(0)(V - w^*) + \min\left\{B, p(0)w^* - \phi^A\right\}$. Since $\phi^A \geq 0$, we have $\widetilde{Eu^P} = p(0)V - \phi^A$. Using Assumption (A1), we conclude that $\widetilde{Eu^P} < \phi^P$ and the principal will not offer a contract to the agent.

Proof of Proposition 1: Assume that $p(e)(V-w) - T \geq \phi^P$ and $B > p(e)w - \phi^A$. Since $e^* > 0$, the programme of the principal can be written as

$$\underset{e,w}{Max}\ \widetilde{Eu^P} = p(e)(V-w) + p(e)w - \phi^A - e = p(e)V - \phi^A - e,$$

such that $p'(e)w = 1$.

We must have $p'(e)V = 1 = p'(e)w$. Hence, $\widehat{w} = V$ and $\widehat{e} = g(V)$. The corresponding payoff of the principal is $\widetilde{Eu^P} = p(g(V))V - \phi^A - g(V)$. The principal prefers to offer a contract than not because, thanks to Assumption (A2), we have $\widetilde{Eu^P} = -\widehat{T} = p(g(V))V - g(V) - \phi^A > \phi^P$. We must also have $B > p(g(V))V - \phi^A = \overline{\overline{B}}$. Notice that the principal cannot reach a larger payoff if $B > p(e)w - \phi^A$ does not hold.

Proof of Lemma 1: Assume that $p(e)(V-w) - T \geq \phi^P$ and $B < p(e^*)w^* - \phi^A$. We know from the proof of Lemma 2 that $e^* > 0$, the programme of the principal can then be written as

$$\underset{e,w}{Max}\ \widetilde{Eu^P} = p(e)(V-w) - e + B, \tag{34}$$

such that $p'(e)w = 1$. Since $w^* > 0$ from Lemma 2, we can write $e = g(w)$. Substituting $e = g(w)$ into the principal's objective, we have

$$\widetilde{Eu^P} = p(g(w))(V-w) - g(w) + B, \tag{35}$$

Let us show that $w^* < V$. Assume it is not the case, that is $w \geq V$. Since $g' > 0$, the objective $\widetilde{Eu^P}$ is decreasing in w. This is not compatible with Lemma 2. Hence, $w^* < V$.

The principal's problem can then be rewritten as follows:

$$\max_{w\in[0,V]} \widetilde{Eu^P} = p(g(w))(V-w) + B - g(w). \tag{36}$$

Given that $\widetilde{Eu^P}$ is continuous and differentiable over the compact $[0, V]$ and that the maximum is not reached for 0 or V, the necessary condition writes:[10]

$$1 = p'\left(g\left(w^*\right)\right)\left(V - w^*\right) - \frac{p\left(g\left(w^*\right)\right)}{g'\left(w^*\right)} \tag{37}$$

Notice that $p'\left(g\left(w\right)\right) = \frac{1}{w}$. Differentiating this condition, we find $g'(w) = -1/\left(w^2 p''\left(g\left(w\right)\right)\right)$. We can then rewrite condition (37) as follows:

$$1 = \frac{V}{2} p'\left(g\left(w^*\right)\right) + \frac{1}{2} \frac{p\left(g\left(w^*\right)\right) p''\left(g\left(w^*\right)\right)}{\left(p'\left(g\left(w^*\right)\right)\right)^2}, \tag{38}$$

which in turn, since $g' > 0$ enables to deduce that $w^* < V/2$. Condition 38 can be rewritten as follows:

$$1 = \frac{V}{2} p'\left(g\left(w^*\right)\right) + \frac{1}{2} \frac{\epsilon_{p'}(g\left(w^*\right))}{\epsilon_p(g\left(w^*\right))}, \tag{39}$$

Such a w^* exists. Indeed, w^* is necessarily in $[\epsilon, V]$, with $\epsilon \in (0, V)$ (see the proof of Lemma 1) and since p is twice continuously differentiable, all the terms in condition (30) are continuous. Hence, using Brouwer's fixed point theorem, we conclude that condition (39) has a solution as long as the Inada conditions hold. Here, these conditions are $lim_{e\to 0} \frac{V}{2} p'\left(e\right) + \frac{1}{2}\frac{\epsilon_{p'}(e)}{\epsilon_p(e)} > 1$ and $lim_{e\to \overline{e}} \frac{V}{2} p'\left(e\right) + \frac{1}{2}\frac{\epsilon_{p'}(e)}{\epsilon_p(e)} < 1$. These conditions hold for instance in the case of Example 1, since $lim_{e\to 0} \frac{V}{2} p'\left(e\right) + \frac{1}{2}\frac{\epsilon_{p'}(e)}{\epsilon_p(e)} = +\infty > 1$ and $lim_{e\to \overline{e}} \frac{V}{2} p'\left(e\right) + \frac{1}{2}\frac{\epsilon_{p'}(e)}{\epsilon_p(e)} = \frac{V}{2} + \frac{\gamma-1}{2} < 1$ because V and γ are smaller than 1.

The payoff of the principal is $\widetilde{Eu^{P*}} = p(g\left(w^*\right))\left(V - w^*\right) - g\left(w^*\right) + B$, which is larger than ϕ^P if and only if $B \geq \underline{B}$.

The initial assumption that $B < p\left(e^*\right) w^* - \phi^A = \overline{B}$ is without loss of generality. First remark that, if $B > p\left(e^*\right) w^* - \phi^A$, we can use the proof of Proposition 1 to show that we must have $B > \overline{\overline{B}}$, which is a contradiction. Second, let us show that we cannot have an optimal contract such that $B = p\left(e^*\right) w^* - \phi^A$. We have to show that the expected surplus of the principal is larger when he chooses contract (w^*, T^*) rather than contract $(\widetilde{w}, \widetilde{T})$ defined in Proposition 4. Notice that $\widetilde{Eu^{P*}} = B - \underline{B}$, where $\underline{B}$ does not depend on B. If the principal chooses contract $(\widetilde{w}, \widetilde{T})$ then his surplus is $p(\widetilde{e})V - \widetilde{e} - \phi^A - \phi^P$ with $\widetilde{e} = g(\widetilde{w})$ and $p(\widetilde{w})\widetilde{w} = B + \phi^A$. In the proof of Proposition 4, we show that the surplus of the principal is continuous at $B = \overline{B}$. It is then sufficient to prove that $p(\widetilde{e})V - \widetilde{e}$ derivative with respect to B is strictly larger than the derivative of $\widetilde{Eu^{P*}} = B - \underline{B}$ with respect to B. Using $p(\widetilde{w})\widetilde{w} = B + \phi^A$ and $p'(\widetilde{w})\widetilde{w} = 1$, we have $\frac{d\widetilde{w}}{dB} = \frac{1}{p(\widetilde{w}) + g'(\widetilde{w})}$. Differentiating $p(\widetilde{e})V - \widetilde{e}$ with respect to B, we find $\frac{(p'(\widetilde{w})V-1)g'(\widetilde{w})}{p(\widetilde{w})+g'(\widetilde{w})}$. After some computation, we find that this derivative is strictly larger than one if and only if

$$\frac{V}{2} p'\left(\widetilde{e}\right) + \frac{1}{2} \frac{\epsilon_{p'}(\widetilde{e})}{\epsilon_p(\widetilde{e})} > 1. \tag{40}$$

This inequality holds for $B < \overline{B}$. Indeed, when $B = \overline{B}$, $\widetilde{e} = e^*$ and then the left hand side equals 1. Thanks to Assumption (A3) and the fact that $p'' < 0$, we know that the left hand side in 40 is strictly decreasing in $\widetilde{e}$ which is strictly increasing in B.

Hence, if $\underline{B} \leq B < \overline{B}$, then the principal proposes w^* characterized by (38) and $T^* =$

[10] In the example where $p\left(e\right) \equiv \frac{1}{\gamma} e^{\gamma}$ with $\gamma \in (0, 1)$, e and V belong to $[0, 1]$, the objective is concave with respect to w.

$g(w^*) - B$.

Proof of Proposition 2: We know from the proof of Proposition 1 that $w^* > 0$. Using condition (30) and the fact that $g' > 0$, we conclude that $w^* < V/2$, which proves point (i). Now let us prove point (ii). From the proof of Lemma 1, we know that $T^* = g(w^*) - B$, which is strictly positive if and only if $g(w^*) > B$. Since we must have $\underline{B} \leq B < \overline{B}$, we conclude that $T^* \leq 0$ when $g(w^*) \leq \underline{B}$, i.e. $0 \leq \phi^P - p(e^*)(V - w^*)$. When $\phi^P - p(e^*)(V - w^*) < 0$, we have $\underline{B} < g(w^*)$, which is sufficient to prove point (ii). Now let us consider point (iii). The payoff of the agent is $Eu^{A*} = p(e^*)w^* + B + T^* - e^* = p(e^*)w^*$. From the proof of Proposition 2, we know that the participation constraint (7) is not binding, thus we have $p(e^*)w^* > B + \phi^A$. Let us prove point (iv). We have $e^* = g(w^*)$ with $w^* < V$. Since $g' > 0$, we conclude that $e^* = g(w^*) < g(V) = e^{FB}$.

Proof of Proposition 3: We have $B < \underline{B}$. Assume that $p(e)(V - w) - T \geq \phi^P$. If $B < p(e^*)w^* - \phi^A$. From the proof of Lemma 1, we know that the payoff of the principal is $\widetilde{Eu^{P*}} = p(g(w^*))(V - w^*) - g(w^*) + B$, and it is less than ϕ^P since $B < \underline{B}$. This is a contradiction. If $B \geq p(e^*)w^* - \phi^A$, we know from the proof of Proposition 1 that we must have $B \geq \overline{\overline{B}}$. Since $\overline{\overline{B}} > \overline{B}$, we have $B > \overline{B}$, a contradiction.

Proof of Proposition 4: We have $\overline{B} \leq B \leq \overline{\overline{B}}$. Assume that $p(e)(V - w) - T \geq \phi^P$. If $B < p(\widetilde{e})\widetilde{w} - \phi^A$, we know from the proof of Lemma 1 that we must have $B < \overline{B}$, which is a contradiction. If $B > p(\widetilde{e})\widetilde{w} - \phi^A$, we know from the proof of Proposition 1 that we must have $B > \overline{\overline{B}}$, which is a contradiction. Now assume that $p(\widetilde{e})\widetilde{w} = \phi^A + B$. Using the incentive constraint, we have $\widetilde{e} = g(\widetilde{w})$ where $g(x) = (p')^{-1}(1/x)$. Using the budget constraint, we have $\widetilde{T} = g(\widetilde{w}) - B$. Moreover, in this case $\widetilde{w}$ is characterized by $p(g(\widetilde{w}))\widetilde{w} = \phi^A + B$. Such a $\widetilde{w}$ exists. Indeed, $\phi^A + B \geq 0$ and when $\widetilde{w}$ goes to 0, $p(g(\widetilde{w}))\widetilde{w}$ goes to 0 and when $\widetilde{w}$ goes to $+\infty$, then $p(g(\widetilde{w}))\widetilde{w}$ goes to $+\infty$. It remains to show that the surplus of the principal is non negative, i.e. $p(\widetilde{e})(V - \widetilde{w}) - \widetilde{T} \geq \phi^P$. Let us first show that the payoff of the principal is continuous at $B = \overline{B}$. When $B = \overline{B}$, we have $p(g(\widetilde{w}))\widetilde{w} = \overline{B} + \phi^A$, which is equivalent to $p(g(\widetilde{w}))\widetilde{w} = p(g(w^*))w^*$. Since p and g are strictly increasing functions, we must have $w^* = \widetilde{w}$ and then $T^* = \widetilde{T}$, $e^* = \widetilde{e}$ and then the payoff of the principal is continuous at $B = \overline{B}$. Now let us show that the payoff of the principal is continuous at $B = \overline{\overline{B}}$. In this case, we have $p(g(\widetilde{w}))\widetilde{w} = \overline{\overline{B}} + \phi^A$, which is equivalent to $p(g(\widetilde{w}))\widetilde{w} = p(g(V))V$. Hence $\widetilde{w} = V$, $\widetilde{e} = e^{FB}$, $\widetilde{T} = T^{FB}$ and then the payoff of the principal is continuous at $B = \overline{\overline{B}}$. Now, we let us show that $\widetilde{w}$ is strictly increasing in B. Is is sufficient to notice that $p(g(\widetilde{w}))\widetilde{w} = \phi^A + B$ where the left hand side is strictly increasing in $\widetilde{w}$. We then have $w^* \leq \widetilde{w} \leq V$. Finally, let us show that the payoff of the principal is strictly increasing in B. The payoff of the principal can be written as $p(g(\widetilde{w}))V - g(\widetilde{w}) - \phi^A$. The total derivative of this expression with respect to B is given by $(V/\widetilde{w} - 1)g'(\widetilde{w})\frac{d\widetilde{w}}{dB}$. We then have $\frac{d\widetilde{w}}{dB} > 0$ for $B < \overline{\overline{B}}$ and $\frac{d\widetilde{w}}{dB} = 0$ for $B = \overline{\overline{B}}$. Since the surplus of the principal is strictly positive for $B = \overline{B}$, it is strictly positive for $\overline{B} \leq B < \overline{\overline{B}}$.

Proof of Corollary 1: In the proof of Proposition 4, we show that the optimal effort, the optimal bonus and the optimal transfer and the surplus of the principal are all continuous at $B = \overline{B}$ and at $B = \overline{\overline{B}}$. The surplus of the agent is then continuous at these points as well. It remains to show that these optimal values are also continuous at $B = \underline{B}$.

Proof of Proposition 5: To prove point (i), it is sufficient to notice that $p'\left(e^{FB}\right)V = 1$ and the result follows. Point (ii) is immediate from Lemma 1. Point (iii) is immediate from Propositions 3 and 4.

Proof of Proposition 6: The objective, U^A, is decreasing in B on each interval. If $\underline{B} \leq B$, the agent chooses $B = \underline{B}$ and then gets $U = B_0 - \underline{B} + \delta\left(\overline{B} + \phi^A\right)$. If $B \leq \underline{B}$, the agent chooses $B = 0$ and he gets $U = B_0 + \delta\phi^A$.

Proposition 7: Immediate from the proof of Proposition 6.

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Contact :

Stéphane MUSSARD : mussard@lameta.univ-montp1.fr