Sharing a groundwater resource in a context of regime shifts

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Abstract

We consider the exploitation of a common groundwater resource for irrigation as a differential game. In particular, we use the Rubio and Casino adaptation of the Gisser and Sánchez model where we introduce a sudden change in the dynamics of the resource, namely a decrease in the recharge rate of the aquifer. We then compare the socially optimal solution with Open-loop and feedback equilibrium. First, we show analytically that different solutions (at the steady state) do not depend on the intensity of the shock, but on the value of the recharge rate upon occurrence of the shock. Moreover, we show that solutions get closer at the steady state for lower values of recharge rates. We finally apply the game to the particular case of the Western La Mancha aquifer. The aim of this application is to estimate (in terms of welfare) the inefficiency of open loop and feedback strategies with regards to the characteristics of the shock. We show that the loss of welfare due to private exploitation is maximal for low-intense or later shocks and can reach important values of 40 million of euros.

1 Introduction

In this paper, we study the exploitation of a common groundwater resource as a differential game in order to take into account the strategic and dynamic interactions between the users of the resource. Indeed, we consider a groundwater resource used for irrigation by several farmers. This type of natural resource is often exploited under a common property regime, that is the access is restricted to land owners situated over the aquifer. Numerous papers have studied this issue (for example Gisser and Sánchez (1980) [4], Negri (1989) [5], Provencher and Burt (1993) [7], Rubio and Casino (2001) [8]) and have concluded that private exploitation is inefficient (in terms of stock and welfare) in comparison to the socially optimal exploitation (or efficient solution).

This inefficiency is due to the various externalities which appear because of the sharing of this type of resource, namely the "pumping cost" externality which characterizes the fact that withdrawals made by one farmer lower the water-table level, resulting in an increase in pumping costs for the other users. On the other hand, the "stock" externality (also called strategic externality) represents the competition which appears between farmers because of the limited availability of water (the stock) (see [7]).

Gisser and Sánchez (1980) [4] showed the inefficiency of the private solution of resource exploitation for the Pecos River Basin, New Mexico. They also characterized the analytical difference between the optimal and private exploitations, and they concluded that the difference is negligible if the capacity of the aquifer is large. Nieswiadomy (1988) [6] called this consideration the Gisser-Sánchez Rule (or Gisser and Sánchez effect (GSE)). The most important policy implication derived from this study is that regulation of a common groundwater resource is not justified if the difference of welfare from private and optimal exploitations is insufficiently important. However, authors assume that farmers behave myopically in the calculation of the private solution, that is, farmers take decisions over a short period of time, without considering the impact of the other users on the available stock (the stock externality).

Other studies have used game theory to take into account the strategic and dynamic interactions between the resource users when computing the private solution (for example Negri (1989) [5], Provencher and Burt [7], Rubio and Casino [8]). Negri characterizes analytical solutions of the water-table level at the steady-state for two types of Nash equilibrium (Open-loop and feedback solutions) and for the socially optimal case. He shows that the difference between the socially optimal solution and the open-loop solution is positive and captures the pumping cost externality. Moreover, he show that the difference between the open-loop solution and the feedback solution is also positive and captures the strategic externality. The difference between the socially optimal and the feedback solutions is then positive and represents the total inefficiency of private exploitation. Provencher and Burt [7] take up Negri's ideas to prove, in a general way, that if the objective function of the problem is concave, the feedback solution is inefficient, in comparison with the socially optimal. In [8], Rubio and Casino adapt the Gisser and Sánchez model as a differential game and derive analytical solutions of socially optimal, open loop and feedback solutions over an infinite planning horizon. They also confirm Negri's result: strategic behaviour exacerbates the inefficiency of private exploitation. Moreover, they confirm the Gisser-Sánchez rule taking into account the strategic externality: the difference between optimal and private exploitations is negligible if the aquifer is relatively large.

In this context, we take Rubio and Casino´s game in [8] and we introduce a sudden change in the dynamics of the resource. Such a shock (also called regime shift) may occur due to a decrease in mean precipitation that leads to a decrease in the recharge of the aquifer, or it may correspond to the abstraction of a certain amount of water that is dedicated to other uses in the case of a drought, such as filling drinking water reservoirs. In both cases, the problem is to model an abrupt decrease of the water availability for the users of the resource. In [1], de Frutos Cachorro et al. (2014) study the effect of information about this type of shock on the optimal management of the water resource. For the deterministic case, when the date of the shock is known, they show that a regulator (the water agency) would prepare for the event by applying an incautious extraction strategy. Such a result can already be found in the literature dealing with the impact of irreversible events (see Tsur and Zemel (2014) [10]), where the phenomenon is known as the "impatience effect". Moreover, using a numerical application to the Western la Mancha aquifer, Spain, de Frutos Cachorro et al. show that a non-monotonic extraction behaviour is possible in the short term, when value of the shock is important and when the shock occurs in the medium or long run. In this paper, we combine Rubio and Casino´s game theory approach and de Frutos Cachorro et al. study on the effect of regime shifts, in order to access the difference between the socially optimal solutions and solutions with strategic interactions in presence of a shock.

Indeed, the contribution of this paper consists in the study of the inefficiency of the private solution of a dynamic game by considering a recharge rate which is not constant over time. In particular, we compare the socially optimal solution with Open-loop and feedback equilibrium considering linear strategies as Rubio and Casino in [8]. Furthermore, we propose an alternative information structure to the open-loop solution which we call "the piecewise open-loop"[1].

At first, we show that the combined effect of strategic interactions and this type of shock leads to an overexploitation of the resource in the short, medium and long run. Secondly, we study the inefficiency of private exploitation with regard to the intensity and date of occurrence of the shock. We show that cost and strategic effects are particularly important for low-intense shocks or shocks that take place in the medium term. Finally, we estimate the inefficiency of the private exploitation in terms of welfare for a particular case, the Western la Mancha aquifer. This aquifer is situated in the South of Spain, under a semi-arid climate where dry periods are frequent. Moreover, in the last decades, the aquifer has suffered from various inefficient regimes of exploitation. We prove that a regulation of the aquifer through a centralized management is even more justified in a context of regime shifts, providing efficiency gains which can reach 40 millions of euros.

This paper is organized in the following way. In section 2, we present Rubio and Casino´s game and we introduce an exogenous and deterministic shock in the game. In section 3, we describe analytical resolutions of the problem for different information structures. In section 4, we compute the socially optimal solution corresponding to the problem. In section 5, we compare the different analytical solutions at the steady-state and then, we make a numerical application of the model to the Western La Mancha aquifer. Finally, in section 6, we conclude and give some perspectives for future research.

[1]Care should be taken not to confuse this type of deterministic game with a piecewise open-loop game in the theory of stochastic games.

2 The model

First, we present the adaptation of the Gisser and Sanchez model (1980) [4] as a differential game developed by Rubio and Casino (2001, 2003) ([8], [9]).

In [4], the demand for irrigation water is a linear function,

$$g = a - bp, \quad a, b > 0, \tag{1}$$

where g represents water pumping and p, the price of water.

In [8], Rubio and Casino assume that the number of farmers is fixed and finite over time (M farmers).

The individual demand for irrigation water can be described as a linear function,

$$g_i = \theta_i(a - bp), \quad i = 1..M, \tag{2}$$

where $0 < \theta_i < 1$ and $\sum_{i=1}^{M} \theta_i = 1$. Thus,

$$\sum_{i=1}^{M} g_i = \sum_{i=1}^{M} \theta_i(a - bp) = a - bp = g. \tag{3}$$

g_i represents the rate of extraction of farmer i.

Moreover, the revenues of the farmer i is equal to

$$\int_{g_i} p(x)dx = \int_{g_i} \frac{a - \frac{g_i}{\theta_i}}{b} dx = \frac{a}{b} g_i - \frac{1}{2b\theta_i} g_i^2.$$

We assume that the marginal cost of extraction is a linear function that depends on G, the stock of the aquifer. Total costs of extraction are then

$$\bar{C} = (z - cG)g, \quad z, c > 0, \tag{4}$$

where z is the sum of fixed costs and the maximum marginal cost of extraction and c the slope of the marginal pumping cost function. As z and c does not depend on the rate of extraction, the individual pumping cost of the ith farmer is

$$\bar{C}_i = (z - cG)g_i, \quad z, c > 0. \tag{5}$$

The dynamic of the aquifer can be described as

$$\dot{G} = -(1 - \alpha)g + r = -(1 - \alpha)\sum_{i=1}^{M} g_i + r, \tag{6}$$

where r is the recharge rate and α the return coefficient, ($\alpha \in [0, 1)$) (see [1] for details).

Assuming that interactions between farmers are rational and non-cooperative, the problem of the ith farmer is to maximise welfare, defined as the present value of his future profits, where ρ is the discount rate, taking into account the dynamic of the aquifer (equation 6) and given initial conditions and positivity constraints:

$$\max_{g_i(.)} \int_0^\infty \quad F_i(G, g_i) \quad e^{-\rho t} \quad dt, \tag{7}$$

where,

$$F_i(G, g_i) = \frac{a}{b} g_i - \frac{1}{2b\theta_i} g_i^2 - (z - cG) g_i, \tag{8}$$

$$\dot{G} = -(1-\alpha) \sum_{i=1}^{M} g_i + r, \tag{9}$$

$$G(0) = G_0 \quad \text{given}, \tag{10}$$

$$g_i \geq 0 \qquad G \geq 0. \tag{11}$$

In what follows, we will introduce an exogenous shock in the system and solve different non-cooperative cases. Then, we firstly remind how Rubio and Casino solve the game defined previously when players have different information structures: open-loop and feedback[2].

In the case of an open-loop structure, every farmer i chooses at the beginning of the planning period the path of extractions that maximises the present value of the sum of their profits over the planning horizon (in this case $t \in [0, \infty)$), assuming that strategies choosen by the other farmers depend on time and knowing the initial state of the resource. The problem to be solved is then (7) constrained by equations (9), (10) and (11).

In the case of a feedback structure, the problem of the ith farmer is the same as in the open-loop case but assuming that strategies played by the other users depend not only on time but also on the state of the resource. In particular, we consider that pumping strategies are linear with respect to the state variable. The problem to be solved is then (7), constrained by the equation of motion:

$$\dot{G} = -(1-\alpha)(g_i + \sum_{j \neq i} a_j G + b_j) + r, \tag{12}$$

and conditions (10) and (11).

We are now going to disturb the system of the resource. This disturbance is an exogenous shock on the dynamic of the aquifer. It represents a sudden reduction on the recharge rate, r, at time t_a (known to the users). Thus, from t_a on, the recharge rate switches from $r = r_1$ to $r = r_2$, with $r_1 > r_2$. The problem of the ith farmer becomes then (7), constrained by the dynamic:

[2] As the problem is already solved in Rubio and Casino (2001, 2003) ([8], [9]), we are not going to detail the resolution of the various problems.

$$\dot{G} = \begin{cases} -(1-\alpha)\sum_{i=1}^{M} g_i + r_1 & \text{si} \quad t \leq t_a \\ -(1-\alpha)\sum_{i=1}^{M} g_i + r_2 & \text{si} \quad t > t_a, \end{cases} \tag{13}$$

with $r_1 > r_2$ and conditions (10) and (11).

In what follows, we describe analytical resolutions of the disturbed problem according to the various structures of information defined previously. In every case, we solve problems in two steps: firstly between t_a and ∞ and then, between 0 and t_a. We anticipate that equilibrium of the various problems will be different according to the structure of information used by players: OL (open-loop) , POL (piecewise open-loop) and FB (feedback).

3 Non-cooperative cases

3.1 Resolution of the open-loop case

We assume that farmers made a commitment about their path of extractions over time. This is an open-loop information structure. The Hamiltonian corresponding to the problem of the ith farmer is:

$$H_i = \begin{cases} F_i(G, g_i) + \pi_i(t)(r_1 - (1-\alpha)\sum_{i=1}^{M} g_i) & \text{if} \quad t \leq t_a \\ F_i(G, g_i) + \pi_i(t)(r_2 - (1-\alpha)\sum_{i=1}^{M} g_i) & \text{if} \quad t > t_a, \end{cases} \tag{14}$$

with $F_i(G, g_i)$ from equation (8), and $\pi_i(t)$, the adjoint variable. $G(t)$ and $\pi_i(t)$ are continuous functions on the interval $[0, \infty)$. We can see the analytical resolution of the open-loop game in appendix A.

3.2 Resolution of the piecewise open-loop case

We propose an alternative structure of information of the open-loop case, the piecewise open-loop case. This game is more realistic than the open-loop case for our problem because farmers can revise their strategies when the shock takes place. In other words, we suppose that farmers make a new commitment at the date t_a to follow open-loop strategies by knowing the state of the resource at the occurrence of the shock (in t=t_a). The full resolution of this type of game is given in the Appendix B.

3.3 Resolution of the feedback case

Now, a more realistic case with regards to previous propositions is the feedback information structure. Indeed, farmers observe the level of the resource during the planning period, i.e. they have information about the state (or the table-water level) of the resource over time. Thus, it is more credible for the farmers to maximize their profit assuming that actions or strategies made by the other farmers depend not only on time but on the state of the groundwater resource. We are going to solve this case on the basis of the principle of dynamic programming. The full resolution of the problem is detailed in Appendix C.

One of the objective of this paper being to estimate the inefficiency of various equilibria defined previously, we need to define the efficient solution of the problem: the social optimum.

4 The social optimum

We suppose that a social planner decides how to manage the resource. The problem for the regulator is to maximize the social welfare, defined as the present value of the sum of future revenues of the M users of the resource.

The problem for the regulator is:

$$\max_{\{g_i\}_{i=1}^{M}} \int_0^\infty \sum_{i=1}^{M} F_i(G, g_i) e^{-\rho t} \quad dt \tag{15}$$

with $F_i(G, g_i)$ described in equation (8), constrained by equation of motion (9), initial (10) and positivity conditions (11).

Now, if a shock occurs at the known date t_a, the problem for the social planner becomes (15), constrained by the equation of motion (13), where r_1 (respectively r_2) are values of the recharge rate before (respectively from) t_a, with initial and positivity conditions described in equations (10) and (11). The full resolution of this problem is detailed in Appendix D.

In what follows, we analyse and compare the socially optimal solution with the different equilibria (open-loop, piecewise open-loop and feedback) obtained when such a shock takes place.

5 Results

5.1 Theoretical Results

In this section, we compare the efficiency of the different solutions at the steady state. From equations (76), (32), (62), we obtain solutions of the stock for the social optimum, the open-loop, the piecewise open-loop and respectively the feedback case, with M, the number of symmetric farmers ($M > 1$). Thus,

$$G_\infty^{SO} = \frac{r_2}{cb(1-\alpha)} + \frac{r_2}{\rho} - \frac{a}{cb} + \frac{z}{c}, \tag{16}$$

$$G_\infty^{OL} = G_\infty^{POL} = \frac{r_2}{cb(1-\alpha)} + \frac{r_2}{M\rho} - \frac{a}{cb} + \frac{z}{c}, \tag{17}$$

and

$$G_\infty^{FB} = \frac{r_2}{2(1-\alpha)a_1^*} - \frac{b_1^*}{a_1^*} \tag{18}$$

with expressions $b_1^* < 0$ and $a_1^* > 0$ defined in equations (58) and (59) in Appendix C. Moreover, at the steady state, solutions of the pumping rate are [3]:

$$g_\infty^{FB} = g_\infty^{OL} = g_\infty^{POL} = g_\infty^{SO} = \frac{r_2}{(1-\alpha)M}. \tag{19}$$

Proposition 1 *When the value of the recharge rate upon occurrence of the shock, r_2, decreases (resp. increases), the level of the stock at the steady state decreases (resp. increases) for the different cases (SO, OL, POL and FB). Moreover, solutions of pumping rates at the steady state are the same for the different information structures and decrease (resp. increase), the lesser (resp. the greater) the value of r_2.*

Demonstration This is immediate from equations (17), (18) and (19). It is enough to prove that the derivatives of expressions described in these equations with regard to r_2 are bigger than 0.□

Proposition 1 shows that the more the final value of the recharge rate is reduced (resp. increased), the smaller (resp. higher) the optimal level of the stock of the different solutions in the long term. Furthermore, the resource is exploited less (resp. more) intensively at the steady state when the recharge rate after the date of occurrence of the shock takes a smaller (resp. greater) value. Finally, the rate of exploitation does not depend on the decision makers information structure at the steady state.

Now, we study the difference between the various solutions to estimate the inefficiency of private solutions (open-loop and feedback) in terms of stocks. Differences are calculated and described below:

$$G_\infty^{SO} - G_\infty^{OL} = \frac{r_2}{\rho}(1 - \frac{1}{M}) \tag{20}$$

and

$$G_\infty^{OL} - G_\infty^{FB} = \frac{r_2}{2}(\frac{1}{M\rho} + \frac{1}{(1-\alpha)cb} - \frac{1}{2(1-\alpha)a_1^*}) - \frac{a}{cb} + \frac{z}{c} + \frac{b_1^*}{a_1^*}, \tag{21}$$

with expressions b_1^*, a_1^* described in equations (58) and (59).

Proposition 2 *When a deterministic shock on the recharge rate takes place, the cost and strategic effects remains positives.*

[3] We remind that we assume parameters such as solutions of the stock and the rate of extraction are positive

Demonstration This is immediate from equation (20) for the cost effect. With respect to the strategic effect, we can see the detailed proof in appendix E.□

Proposition 2 confirms that the cost and strategic effects remains positives when there is a shift on the recharge rate that at a certain date. Finally, we may estimate the value of the different effects according to r_2, the value of the recharge rate from $t = t_a$.

Proposition 3 *When r_2 decreases (resp. increases), the cost and strategic effects decrease (resp. increase).*

Demonstration This is also immediate from equations (20) and (21). It is necessary to prove that the derivatives of expressions described in these equations with regard to r_2 are greater than 0.□

Proposition 3 shows that at the steady state the value of the different effects decrease (or increase), the less (more) important the value of the recharge rate after t_a. In other words, pumping strategies (at the steady state) derived from private and optimal exploitation get closer if the aquifer recharge decreases. We remind that Rubio and Casino found the same expressions (20) and (20) in [8] and show first that difference between solutions declines as the discount rate and/or the number of farmers increases. They also confirm that the same result is obtained when the storage capacity of the aquifer increases (GSE effect). In this paper, we add to Rubio and Casino´s result the importance of a recharge rate variation.

Moreover, we highlight the fact that different solutions at the steady state do not depend on the intensity of the shock $r_1 - r_2$, but on the value of the recharge rate upon occurrence of the shock, r_2. Thus, to estimate the magnitude of the inefficiency of private solutions according to the value of the shock, we have to measure externalities in terms of welfare, and not in terms of stock. To this aim, in what follows, we apply our game for the real case of the Western la Mancha aquifer.

5.2 Numerical application

In this section, we use parameter values from de Frutos Cachorro et al. (2014) [1] that are based on real parameter values from several sources (e.g. Esteban and Albiac (2011) [2], Esteban and Dinar (2012) [3]). The parameter values used are listed in table 1.

5.2.1 Comparison of the different information structures with the social optimum

In this section, we estimate the inefficiency of the various solutions open-loop and feedback by comparing with the social optimum (the efficient solution), at the steady state and at the date of occurrence of the shock. Moreover, we complete this analysis by studying the

Parameters	Description	Units	Value
b	Water demand slope	(Million Cubic Meters/Year)^2 Euros^{-1}	0.097
a	Water demand intercept	Million Cubic Meters/Year	4403.3
z	Pumping costs intercept	Euros/Million Cubic Meters	266 000
c	Pumping costs slope	Euros/(Million Cubic Meters)^2	3.162
r	Natural recharge	Million Cubic Meters/Year	360
G_0	Stock level (in volume)	Million Cubic Meters	80960
H_0	Current water table	Meters	640
S_L	Surface elevation	Meters	665
A	Aquifer area	Square Kilometers	5500
S	Storativity coefficient	*unitless*	0.023
ρ	Social discount rate	Year^{-1}	0.05
α	Return flow coefficient	*unitless*	0.2
M	Number of players	*unitless*	2

Table 1: Values of parameters of the Western la Mancha aquifer.

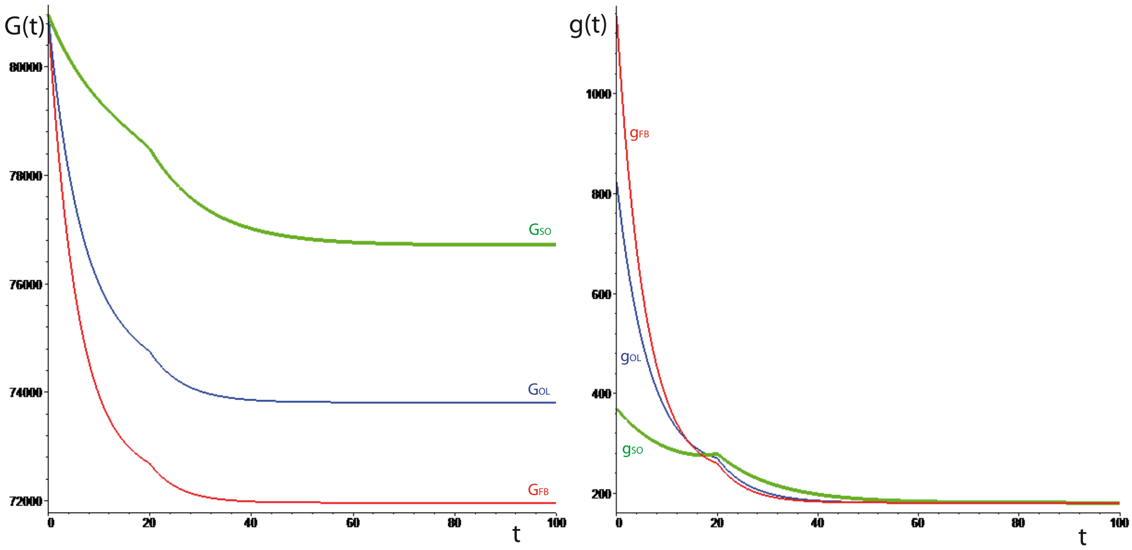

Figure 1: Solutions of $G^*(t)$ (left-land side) in millions of cubic meters and $g^*(t)$ (right-hand side) in millions of cubic meters per year : the social optimum (in green), the open-loop (in blue) and the feedback (in red) cases, when $r_1 - r_2 = 70$ and $t_a = 20$ years.

problem when the intensity and the date of occurrence of the shock vary.

In Figure 1, we observe optimal solutions of stock $G^*(t)$ (on the left) and pumping rate $g^*(t)$ (on the right), in particular the socially optimal (SO) (in green), the open loop (in blue) and the feedback solutions (in red), for a shock of mid-intensity of 70 millions of cubic meters per year (Mm^3/year) (i.e. $r_1 = 360$ and $r_2 = 290$) at the 20^{th} year of exploitation of the aquifer (i.e. $t_a = 20$ years).

Focusing on the left-hand side of the figure, we note that the most efficient solution at the steady state is the social optimum. Indeed, the stock reaches a level of 76 711 Mm^3, which is higher than levels obtained by the OL and FB solutions (of around 73 811 and 71 962 Mm^3 respectively). Thus, the difference between the social optimal and the open-loop solutions is 2 899 Mm^3 whereas the difference between the socially optimal and the feedback solutions is 4 749 Mm^3. We first confirm theoretical results proved in proposition 2: the cost and strategic effects remain positives in the long term when a shock takes place. Moreover, we observe on the right-hand side that the pumping rate at the steady state is constant for the different solutions, with a value of approximatively 181 Mm^3/year, as demonstrated in proposition 1.

We now analyse the problem for a medium-term planning horizon, between t=0 and $t = t_a = 20$ years. On the right hand, we note that the different solutions intersect before the arrival of the shock. In particular, total extractions until the arrival of the shock are higher in the feedback case (9 672 Mm^3) than in the OL (8 383 Mm^3) and SO (6 044 Mm^3) solutions. This means that the feedback strategy is also the less conservative for the resource in the medium-term. In other words, the "impatience effect", that is the increase of extractions before the occurrence of the shock is more important in the feedback case, and less important in the socially optimal solution.

Let us now calculate the well-being (described in equation (7)) associated to the different strategies, (for the numerical example of the Figure 1) at the medium and long terms. In the long term (cf. Table 5), the inefficiency of the feedback solution with regard to the socially optimal solution is estimated at approximately 37 478 thousands of euros and at 23 085 thousands of euros with regard to the open-loop solution. At the medium term (cf. Table 5), the difference of welfare between the SO and FB solutions is positive, but between the SO and OL solutions is negative, that is the open loop strategy is more profitable than the socially optimal solution until the occurrence of the shock. Additional simulations are then necessary to better understand this result.

Variation of the intensity of the shock

In this section, we compare the efficiency (in terms of stock and welfare) of the various solutions for shocks of different intensities. For example, in Figure 2 we simulate a shock of 210 Mm^3/year, which is about 140 Mm^3/year more intense than the shock described in the previous section (and illustrated in Figure 1), but takes place at the same date.

First, we analyse the problem in the long term. We note that the cost and the strategic effects, evaluated as the difference between the SO and OL solutions and respectively between the OL and FB solutions, remain positive and are twice as high as that of the mid-intense shock of 70 Mm^3/year, which is three times less intense. This means that cost and strategic effects do not vary proportionally with a change on the intensity of the

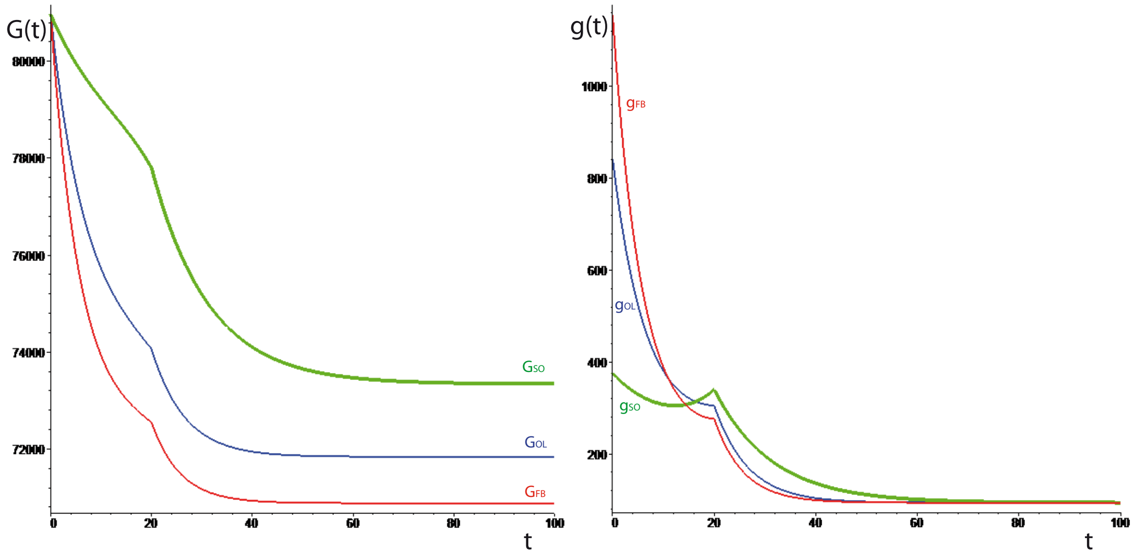

Figure 2: Solutions of $G^*(t)$ (left-land side) in millions of cubic meters and $g^*(t)$ (right-hand side) in millions of cubic meters per year : the social optimum (in green), the open-loop (in blue) and the feedback cases (in red), when $r1 - r_2 = 210$ and $t_a = 20$ years.

shock. Moreover, we made other simulations for shocks of different intensities which are illustrated in Table 2. We confirm that the cost and strategic effects decrease when r_2 decreases, as proved theoretically in proposition 3. Second, in Table 4 and 5, we can observe that the differences of welfare decrease the more intense the shock. This means that the inefficiency (in terms of stock and welfare) derived from private exploitation, evaluated as the difference between the SO and the FB solutions, is maximal for low-intense shocks. For example, the loss of welfare derived from private exploitation under competition (feedback solution) is about 6 millions euros smaller when the intensity of the shock increases by around 140 Mm^3 (see the difference between columns 5 and 7 of the last line in Table 5).

We now make the same type of analysis in the finite planning horizon $[0,t_a]$ (with $t_a = 20$), that is at medium term. We first analyse extraction behavior before the occurrence of the shock (before t_a) for the various solutions (see the right-hand side of Figure 2). In the medium term, total extractions in the feedback case (9 757 Mm^3) remain higher than in the OL (8 810 Mm^3) and SO 6 474 Mm^3) cases. Moreover, in the three cases, we observe a more intense extraction behavior in comparison with the shock of mid-intensity of 70 Mm^3/year. Indeed, total extractions increase by 85, 427 and 430 Mm^3 for the cases FB, OL ans SO respectively when the intensity of the shock increases by around 140 Mm^3/year. In addition, the impatience effect, increases the higher the shock and this increase is more important in the SO and OL cases than in the FB case. This means that the magnitude of the "impatience effect" is reduced when considering the "strategic" externality. On the other hand, the study of the differences in welfare obtained because of different extraction behaviour in the medium term (Table 5) shows that welfare obtained from SO-OL and OL-FB strategies vary in a non-monotonic way with respect to the value of the shock, reaching sometimes negative values. Hence, the social optimum is not always the efficient solution, if we analyse the problem in the short or medium terms.

$r_1 - r_2$	**30**	**70**	**210**
SO-OL	3 300	2 899	1 500
OL-FB	2 104	1 850	956
SO-FB	5 404	4 749	2 456

Table 2: Differences between solutions of stock in millions of m^3 at the steady state.

$r_1 - r_2$	**30**	**70**	**210**
SO-OL	3 744	3 743	3 738
OL-FB	2 220	2 063	1 515
SO-FB	5 964	5 806	5 253

Table 3: Differences between solutions of stock in millions of m^3 at the date of occurrence of the shock, $t_a = 20$.

$r_1 - r_2$	30		70		210	
	[0, t_a]	TOTAL	**[0, t_a]**	TOTAL	**[0, t_a]**	TOTAL
SO	110 446	150 451	111 462	146 658	114 666	136 234
OL	114 908	135 750	114 886	132 265	114 445	122 475
FB	101 074	110 879	101 039	109 180	100 900	104 637

Table 4: Welfare from SO, OL and FB strategies (in thousands of euros) for different values the shock $r_1 - r_2$ and for the date of occurrence $t_a = 20$.

$r_1 - r_2$	30		70		210	
	[0, t_a]	TOTAL	**[0, t_a]**	TOTAL	**[0, t_a]**	TOTAL
SO-OL	-4 462	14 701	-3 424	14 393	221	13 759
OL-FB	13 834	24 871	13 847	23 085	13 545	17 838
SO-FB	9 372	39 572	10 423	37 478	13 766	31 597

Table 5: Differences of welfare (in thousands of euros) for different values the shock $r_1 - r_2$ and for the date of occurrence $t_a = 20$.

Variation of the date of the shock

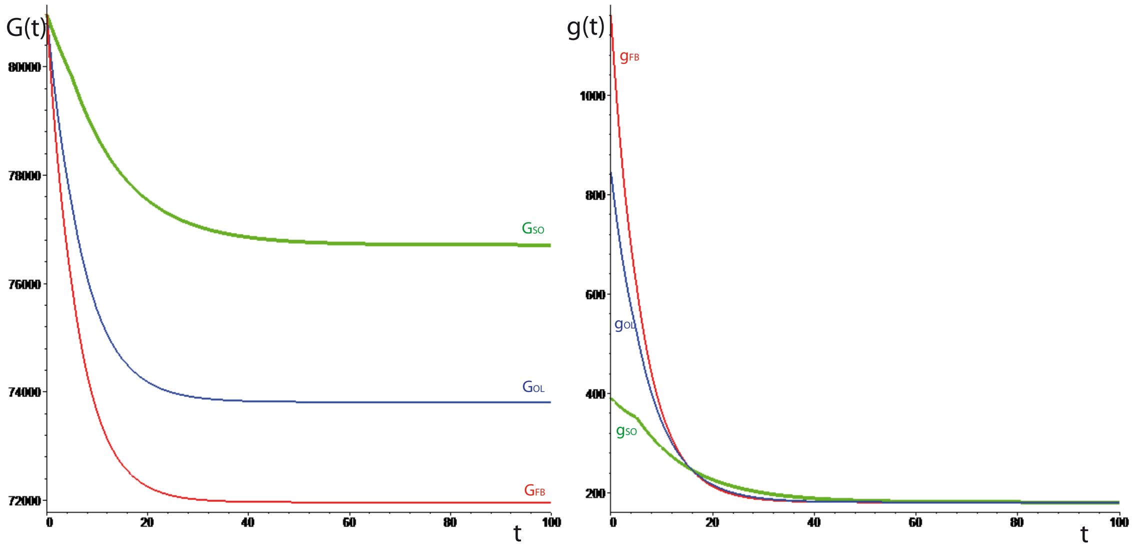

Figure 3: Solutions of $G^*(t)$ (left-land side) in millions of cubic meters and $g^*(t)$ (right-hand side) in millions of cubic meters per year : the social optimum (in green), the open-loop (in blue) and the feedback cases (in red), when $r_1 - r_2 = 70$ and $t_a = 5$ years.

After the analysis and estimation of extraction behaviour in the different cases according to the intensity of the shock, we study the different solutions with respect to the date of occurrence of the shock. In Figure 3, we observe optimal solutions of stock $G^*(t)$ (on the left) and pumping rate $g^*(t)$ (on the right), in particular the socially optimal (SO) (in green), the open loop (in blue) and the feedback solutions (in red), for a shock of mid-intensity of 70 millions of cubic meters per year (Mm^3/year) (i.e. $r_1 - r_2$= 70 Mm^3/year) at the 5^{th} year of exploitation ($t_a = 5$ years). In what follows, we compare this shock with the previous shock illustrated in Figure 1, which has the same intensity but takes place 15 years later.

	$t_a = 5$	$t_a = 20$	$t_a = 50$
SO-OL	2 373	3 743	3 625
OL-FB	1 472	2 063	2 019
SO-FB	3 845	5 806	5 645

Table 6: Differences between solutions of stock evaluated at $t = t_a$ (G_{ta}) in millions of m^3 for a shock of 70 Mm^3/an, at different dates of occurrence t_a.

In the long-run analysis, simulation results do not depend on the date of occurrence of the shock as we note in analytical solutions (equations (16), (17) and (18)). This is not true in terms of welfare (see Table 7). We note that total welfare increases the later the shock occurs. Logically, farmers better adapt to a shock which occurs later in time, procuring a gain of welfare. The same results is obtained when we compute differences between solutions (see Table 8). For example, the loss in total welfare derived from private explotation under competition (feedback solution) with respect to optimal exploitation, is greater of around 3 800 thousands of euros when the shock occurs in $t_a = 20$, instead of occurring earlier at $t_a = 5$.

However, results change if we realize a short-term analysis of efficiency in terms of stock and welfare. Once again, we confirm that until the arrival of the shock, total extractions are higher in the FB case (4 254 Mm^3) than in the OL (3 328 Mm^3) and SO (1 845 Mm^3) cases. Moreover, they are less important than in the later shock that occurs at $t_a = 20$. However, we made other simulations of shocks of different dates in Table 6 and confirm that the previous result is not monotonic in time. For example, we observe that the differences between solutions increase between $t_a = 5$ and 20, but decrease between $t_a = 20$ and 50. In terms of welfare, we can see in Table 7 that welfare obtained in $[0,t_a]$ increases the later the shock occurs. This is logical because extractions are also more important when the shock takes place later. This can be explained by the fact that for later occurring shocks, farmers take the time to better adapt to the shock before its occurrence. However, when the shock occurs at an earlier date ($t_a = 5$), differences between solutions are negative in the short term (see Table 8). The inefficiency of private exploitation (in terms of stock) is translated by a gain of welfare in the short term, in contrast with results obtained in the long-term. For example, feedback strategies entail a gain of 21,3 millions of euros with regards to the social optimum, for a shock of mid-intensity (70 Mm^3) which takes place at $t_a = 5$ years. This result means that in case of occurrence of this shock the planning horizon [0,5] is not sufficiently long to adapt to this shock.

$r_1 - r_2$	$t_a = 5$		$t_a = 20$		$t_a = 50$	
	[0, ta]	TOTAL	**$[0, t_a]$**	TOTAL	**$[0, t_a]$**	TOTAL
SO	48 912	138 348	111 462	146 658	144 321	152 021
OL	68 017	125 350	114 886	132 265	133 402	137 086
FB	70 268	104 725	101 039	109 180	109 876	111 594

Table 7: Welfare obtained from the different solutions (in thousand of euros) for a shock of $r_1 - r_2$=70 Mm^3 and for different dates of occurrence.

We conclude first, that the feedback solution is the least efficient solution (in terms of stock and welfare) over the infinite planning horizon. Moreover, the inefficiency derived from private exploitation (that is open-loop and feedback strategies) is maximal for low-intense or later occurring shocks. However, this result is not true in a shorter planning horizon. The inefficiency of private exploitation (in terms of stock) can result in a gain of welfare if the planning horizon is not sufficiently long to adapt to the shock.

$r_1 - r_2$	$t_a = 5$		$t_a = 20$		$t_a = 50$	
	[0, t_a]	TOTAL	**[0, t_a]**	TOTAL	**[0, t_a]**	TOTAL
SO-OL	- 19 105	12 998	-3424	14 393	11 219	14 935
OL-FB	- 2 251	20 625	13 847	23 085	23 526	25 492
SO-FB	-21 356	33 623	10 423	37 478	34 745	40 427

Table 8: Differences of welfare (in thousand of euros) from different strategies for a shock of value $r_1 - r_2$=70 Mm^3 and different fates of occurrence.

5.2.2 An alternative information structure: piecewise open-loop

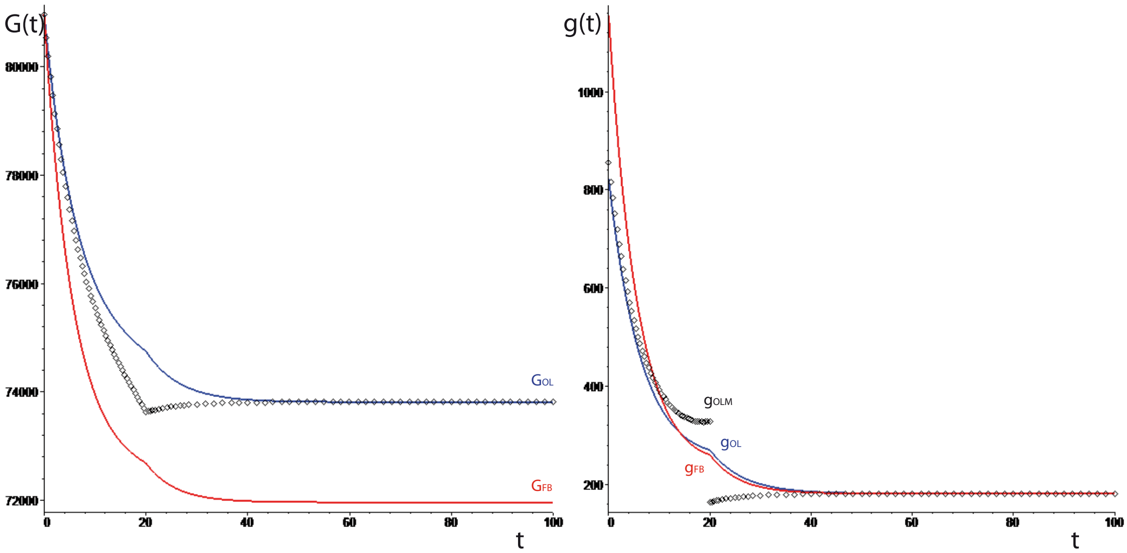

Figure 4: Solutions of $G^*(t)$ (on the left hand-side) in millions of m^3 and $g^*(t)$ (on the right hand-side) in millions of m^3/year: open-loop (in blue), piecewise open-loop (in dotted black) and feedback (in red), for a shock of $r_1 - r_2 = 70Mm^3$ that occurs at $ta = 20$ years.

Finally, a more realistic case than the information structure open-loop (OL) is the open-loop structure by parts (POL), also called the piecewise open-loop. This solution offers the possibility of redefining open-loop strategies at the time of arrival of the shock. Intuitively, this type of information would be situated between the OL and FB cases. We want to confirm our initial intuition by realizing numerical simulations.

Figure 4 depicts solutions of stock (on the left) and pumping rate (on the right) for the shock of mid-intensity ($r_1 - r_2$=70 Mm^3) that takes place at the medium term ($t_a = 20$) obtained from the different information structures: open-loop (in blue), piecewise open-loop (in dotted black) and feedback (in red).

As usual, we analyse first results at the steady state. In terms of stock, solutions are the

	[0, t_a]	TOTAL
OL	114 886	132 265
POL	113 865	128 116
FB	101 039	109 180

Table 9: Welfare obtained from the different solutions (in thousand of euros) for a shock of $r_1 - r_2$=70 Mm^3 occurring at $t_a = 20$.

same as in the open-loop case as we prove analytically in equation (17), but total welfare is greater in the open-loop case by around 4 millions of euros. This is because of different extraction behaviour at short and medium term. It´s then more interesting to analyse the different pumping strategies until the arrival of the shock. In the POL case, total extractions are around 9 084 Mm^3 and provides a level of welfare of 113 865 thousands of euros at the medium-term. As we expected, pumping rates of this alternative information structure are placed between OL and FB solutions, with 8 368 Mm^3 and respectively 9 762 Mm^3 of total water pumped before t_a. Moreover, the difference of welfare with regard to the OL case is around one million of euros at the medium term (cf. Table 9).

We conclude that the piecewise open-loop structure is a less efficient solution (in terms of stock and welfare) than the open-loop structure at medium term. At the steady state, the two types of information structures OL and POL provide the same levels of stock. However, the OL strategy is preferable in terms of welfare. This means that it is not interesting for farmers to make a new commitment and redefine open-loop strategies at the time of arrival of the shock.

6 Conclusions and discussion

We have extended the analysis of a deterministic shock made in de Frutos Cachorro et al. (2014) [1] by taking into account the different externalities which arise in the exploitation of a common groundwater resource, i.e. the dynamic and strategic interactions between users of the resource. We present different solutions: the social optimum and two Nash equilibria corresponding to classic information structures: open-loop and feedback. Moreover, we propose an alternative information structure, more realistic than the open-loop situation, the piecewise open-loop structure.

Firstly, we analyse the impact of the deterministic shock on stock levels at the steady state according to the various structures of information used by farmers. We find the same tendencies proved in [1] when they only study the socially optimal case. We show first that different solutions (at the steady state) do not depend on the intensity of the shock, but on the value of the recharge rate upon occurrence of the shock. Thus, the lesser the value of the recharge rate, the more the level of the stock of the resource decreases. Nevertheless, even if adaptation behaviour in the face of such shocks is similar for the different information structures, the magnitude of the impact of different pumping strategies on the stock is significantly different. We then analyse differences between the socially optimal, the open-loop and the feedback strategies. We remind that Negri (1989) [5] shows (at the steady state) that the difference between the solutions SO and OL is positive and captures the pumping cost externality. Furthermore, he shows that the difference between the solutions OL and FB is positive and captures the strategic externality. Finally, the difference between the solutions SO and FB is then positive, and captures both externalities and shows the inefficiency of private exploitation. In this paper, we add to Negri´s analysis the consideration of regime shifts as a variation on the recharge rate of the aquifer. We show analytically that the pumping cost and strategic effect decrease the lesser the value of the recharge rate upon occurrence. In order words, solutions get closer when the value of the recharge rate is small. However, to estimate the magnitude of the inefficiency of private solutions according to the value of the shock and not to the value of the recharge rate at the steady state, we have to measure externalities in terms of welfare. To this goal, we apply our game for the particular case of the Western la Mancha (WLM) aquifer.

In the long term (steady-state), we confirm theoretical results. When such shocks occur, private exploitation (open-loop and feedback strategies) is inefficient (in terms of stock and welfare) compared to optimal exploitation. Moreover, the consideration of the strategic externality (feedback solution) exacerbates the overexploitation of the resource with respect to open loop strategies. These results are in agreement with the existing literature (Negri (1989) [5], Rubio and Casino (2001) [8]). For example, for a shock of mid-intensity which occurs at medium term, the total loss of welfare due to the private exploitation under competition (feedback solution) is estimated to 37,4 millions of euros. However, in the short and medium term, before the occurrence of the shock, result change. We show that private exploitation remains the least efficient solution in terms of stock but this inefficiency can be translated by a gain of welfare (instead of a loss) in many situations. We therefore study the adaptation behaviour according to the characteristics (intensity or the date of occurrence).

We first study the inefficiency (in terms of stock) of different solutions with respect to the severity of the shock. We show that the "impatience effect", that is the increase in extractions before the occurrence of the shock, is more important in the FB case than in the SO and OL cases, but the magnitude of this increase with regards to the value of the shock, is more important in the SO and OL cases, that in the FB case. In other words, even if the FB strategy always entails lower levels of stock with regard to the OL and SO strategies, the FB solution is less influenced by an increase of the value of the shock because of supplementary information held by farmers. Moreover, we show that the cost and strategic effects, that is differences between solutions, decrease the more important the shock for shorter planning horizon. Next, we make the same analysis when the date of occurrence of the shock varies. At the steady state, adaptation strategies do not depend on the date of the shock. However, if we estimate the inefficiency of the private exploitation (in terms of stock) before the arrival of the shock, this one is more important for events which take place in the medium term. In the short or respectively in the long term, the cost and strategic effects are less important because the time of adaptation is insufficient or respectively too long. In conclusion, in the existing literature ([4], [5], [8]), authors show that different effects (at the steady state) decrease the more important the capacity of the aquifer and/or when the discount rate and/or the number of farmers increases. The main contribution of the paper is to show that the different solutions get closer also for lower values of recharge rates in the long term and when the shock becomes more intense or takes place in the medium term for shorter planning horizon.

Results change if we analyse the problem in terms of welfare. First, we conclude that the inefficiency of private exploitation is explained by a loss of total welfare calculated in the infinite planning horizon. Furthermore, this loss is maximal for low-intense or later shocks. For example, this inefficiency may reach about 40 millions of Euros for a low intense shock that occurs in the medium-term. The most important policy implication of this analysis is that a regulation through a centralized management of the Western la Mancha aquifer is justified. Moreover, this regulation becomes more necessary for low-intense or later shocks. However, the inefficiency from private exploitation can entail a gain of welfare in the short or medium term. For example, open-loop and feedback strategies entails a total gain of 19 and respectively 2 millions of Euros with regard to the social optimum situation when the shock arrives at a earlier date (for example at the fifth year of exploitation). This result may explain why farmers in the field adopt intensive pumping rates if they do not consider the long run.

Subsequently, we propose an alternative information structure situated between the open-loop and feedback cases, namely the piecewise open-loop (POL) case. This structure is more realistic than the OL, because it offers the possibility to restore open-loop strategies at the time of occurrence of the shock. The study of the POL information structure of information is especially interesting in the medium and short terms, because it entails more conservative strategies than in the FB case, but less preservative than in the OL case. On the other hand, at the steady state, solutions are the same for the OL and POL cases. Furthermore, the POL strategy entails a loss of welfare with regard to OL solution in the short, medium and long terms. This means that it is not interesting for the users to restore new open-loop strategies at the time of the shock.

Finally, we want to propose some possible extensions of the chapter. Firstly, we can introduce the uncertainty on the model, for example through the date of the shock, as realized in [1], or on the intensity of the shock. Secondly, it would be interesting to introduce asymmetries (or heterogeneities) between groups of farmers, or to take into account a higher number of farmers. Thirdly, solutions to the inefficiency of private exploitation could be proposed for the particular case of the WLM aquifer.

A Resolution for the open-loop case

We are going to solve the open-loop case proceeding firstly between t_a and ∞. The Hamiltonian of this problem is:

$$H_i = F_i(G, g_i) + \pi_i(r_2 - (1-\alpha)\sum_{i=1}^{M} g_i) = \frac{a}{b} g_i - \frac{1}{2b\theta_i} g_i^2 - (z - cG) g_i + \pi_i(r_2 - (1-\alpha)\sum_{i=1}^{M} g_i). \tag{22}$$

Applying the maximum principle and assuming interior solutions, we have the usual first order conditions:

$$\frac{\partial H_i}{\partial g_i} = 0 \quad \Rightarrow \quad \frac{a}{b} - z + cG - \frac{1}{b\theta_i} g_i - \pi_i(1-\alpha) = 0, \tag{23}$$

$$\dot{\pi}_i = -\frac{\partial H_i}{\partial G} + \rho\pi_i \quad \Rightarrow \quad \dot{\pi}_i = -c g_i + \rho\pi_i. \tag{24}$$

The equilibrium of the open-loop game is obtained by solving M strategies which verify the conditions (23) and (24) (i=1..M), i.e. a linear system of 2M equations. To simplify the analytical resolution of the problem, we assume that players are symmetric, $\theta_i = \frac{1}{M}$, $g = g_i$ and $\pi = \pi_i$. From (23), we find the optimal rate of extraction as a function of the resource stock and the shadow price:

$$g = \frac{1}{M}(a - zb + cbG - \pi b(1-\alpha)). \tag{25}$$

Substituting (25) in the equations of motion of the state (9) and adjoint variable (24), we have the following dynamic system:

$$\dot{G} = r_2 - (1-\alpha)(a - zb) - cb(1-\alpha)G + \pi b(1-\alpha)^2, \tag{26}$$

$$\dot{\pi} = \frac{1}{M}(-c(a - zb) - c^2 bG + cb(1-\alpha) + \rho M)\pi, \tag{27}$$

which allows us to find the roots of the characteristic polynom:

$$\beta_{1,2} = \frac{\rho M + c(1-\alpha)b(1-M)}{2M} \tag{28}$$

$$\pm \frac{\sqrt{\rho^2 M^2 + cb(1-\alpha)(-2M(1-\alpha)cb + c(1-\alpha)b(1+M^2) + 2\rho M(1+M))}}{2M}. \tag{29}$$

From equations (25), (26) and (27), with $\dot{G} = 0$ and $\dot{\pi} = 0$, we find the steady state of the problem:

$$g_\infty^{OL} = \frac{r_2}{(1-\alpha)M}, \tag{30}$$

$$\pi_\infty^{OL} = \frac{c r_2}{M\rho(1-\alpha)}, \tag{31}$$

$$G_\infty^{OL} = \frac{r_2}{cb(1-\alpha)} + \frac{r_2}{M\rho} - \frac{a}{cb} + \frac{z}{c}. \tag{32}$$

Assuming parameters are positives, g_∞ and π_∞ (equations (30) and (31)) have always positives values. Moreover, in what follows, we assume parameters such as the value of G_∞ (equation (32)) is also positive.

Finally, we find optimal extraction path with β_2, the negative root:

$$G^{OL+}(t) = e^{\beta_2(t-t_a)}(G_{t_a} - G_\infty^{OL}) + G_\infty^{OL}, \tag{33}$$

$$g^{OL+}(t) = \frac{r_2}{(1-\alpha)M} - \frac{\beta_2}{(1-\alpha)M} e^{\beta_2(t-t_a)}(G_{ta} - G_\infty^{OL}), \tag{34}$$

$$\pi^{OL+}(t) = e^{\beta_2(t-t_a)}(\pi_{ta} - \pi_\infty^{OL}) + \pi_\infty^{OL}, \tag{35}$$

and,

$$\pi_{ta} = \frac{a}{b(1-\alpha)} - \frac{z - cG_{ta}}{(1-\alpha)} - \frac{1}{b(1-\alpha)^2}(r_2 - \beta_2(G_{ta} - G_\infty^{OL})), \tag{36}$$

which is obtained from equations (9) and (23) with $r = r_2$.

In a second step, we will solve the problem between 0 and t_a. In this period, the Hamiltonian of the problem is described by:

$$H_i = F_i(G, g_i) + \pi_i(r_1 - (1-\alpha)\sum_{i=1}^{M} g_i) = \frac{a}{b} g_i - \frac{1}{2b\theta_i} g_i^2 - (z - cG)g_i + \pi_i(r_1 - (1-\alpha)\sum_{i=1}^{M} g_i). \tag{37}$$

We use the same principle of resolution than previously. We have first order conditions (equations (22), (23), (24) with $r_2 = r_1$) by applying the maximum principle. Moreover, we assume that players are symmetric.

In a finite horizon problem, we write solutions as described below:

$$G^{OL-}(t) = C_1 e^{\beta_1 t} + C_2 e^{\beta_2 t} + C_3, \tag{38}$$

$$\pi^{OL-}(t) = D_1 e^{\beta_1 t} + D_2 e^{\beta_2 t} + D_3. \tag{39}$$

Substituting $G^{OL-}(t)$ and $\pi^{OL-}(t)$ (equations (38) and (39)) in first order conditions (23), (24), and taking into account boundary conditions $G(0) = G_0$ and $\pi(t_a) = \pi^{OL+}(t_a)$, we obtain a system of 6 equations with 6 unknowns (C_i, D_i with i=1,2,3). We find the follow solutions to the system:[4]:

[4]Solutions of D_i for i=1,2,3, are not detailed here, but they are available from authors request.

$$C_1 = \frac{-(1-\alpha)b(-\rho M c r_1 + \rho M c(1-\alpha)^2 b\pi_{ta} - (1-\alpha)c^2 b r_1) - c^2 e^{\beta_2 t_a}\rho M(1-\alpha)bG_0}{D_1}$$

$$\frac{+ce^{\beta_2 t_a}\rho M(r_1 + (1-\alpha)zb - (1-\alpha)a) + cr_1 M\beta_2 - (1-\alpha)\rho M^2\pi_{ta}(\beta_2 - \rho) + c^2 e^{\beta_2 t_a} b(1-\alpha)r_1}{D_1}$$

$$\frac{+ce^{\beta_2 t_a}\rho M(r_1 + (1-\alpha)zb - (1-\alpha)a) + cr_1 M\beta_2 - (1-\alpha)\rho M^2\pi_{ta}(\beta_2 - \rho))}{D_1}, \tag{40}$$

with

$$D_1 = \rho M(-e^{\beta_1 t_a}\beta_1(\rho M + cb(1-\alpha) - M\beta_2) + (1-\alpha)cb(-(1-\alpha)cb(e^{\beta_2 t_a} - e^{\beta_1 t_a}) + Me^{\beta_1 t_a}(\beta_2 - \rho))),$$

$$C_2 = \frac{-(-\rho M - cb(1-\alpha) + \beta_2 M)(-c^2 b^2(1-\alpha)^2 r_1 - \rho M e^{\beta_1 t_a} cb(1-\alpha)G_0(\beta_1 + cb(1-\alpha))}{D_2}$$

$$\frac{+cb^2(1-\alpha)^3\rho M\pi_{ta} + e^{\beta_1 t_a}(\beta_1 + cb(1-\alpha))(\rho M(r_1 - (a - zb)(1-\alpha)) + cb(1-\alpha)r_1)}{D_2}, \tag{41}$$

with

$$D_2 = cb(1-\alpha)\rho M((\beta_2 - \rho M - cb(1-\alpha) + M\beta_2)),$$

$$+cb(1-\alpha)(e^{\beta_2 t_a} cb(1-\alpha) + e^{\beta_1 t_a}(-cb(1-\alpha)(\beta_2 - \rho)M)),$$

$$C_3 = \frac{\rho M(r_1 - (a - zb)(1-\alpha)) + cb(1-\alpha)r_1}{cb(1-\alpha)\rho M}. \tag{42}$$

with π_{ta} described in equation (36). Finally, taking into account that $G(t)$ is a continuous function ($G^{OL-}(t_a) = G^{OL+}(t_a)$), we find optimal solutions for the open loop game, that is $G^{OL}(t)$, $g^{OL}(t)$ and $\pi^{OL}(t)$.

B Resolution of the piecewise open loop case

As in the previous case, we are going to solve the problem first between t_a and ∞,

$$\max_{g_i(.)} \int_{t_a}^{\infty} \quad F_i(G, g_i) \quad e^{-\rho(t-t_a)} \quad dt, \tag{43}$$

with $F_i(G, g_i)$ (equation (8)), constrained by the dynamics (equation (9)) with $r = r_2$ and conditions (11) and $G(t_a) = G_{ta}$.

Assuming that players are symmetric and applying the maximum principle, we have to solve the same problem than in the previous section between t_a and ∞. Thus, we obtain solutions described in equations (33), (25) and (35).

In a second step, we solve the problem between 0 and t_a, that is,

$$\max_{g_i(.)} \int_{0}^{t_a} \quad F_i(G, g_i) \quad e^{-\rho t} \quad dt + e^{-\rho t_a} \phi(t_a, G_{ta}). \tag{44}$$

$$\dot{G} = -(1-\alpha) \sum_{i=1}^{M} g_i + r_1. \tag{45}$$

$$G(0) = G_0 \quad \text{given}, \tag{46}$$

with the transversality condition

$$\pi(t_a) = \frac{\partial \phi(ta, G_{ta})}{\partial G_{ta}}, \tag{47}$$

where $\pi(t)$ is the adjoint variable. $\phi(ta, G_{ta})$ represents the post event value (or "scrap value function") and is described by the following equation:

$$\phi(G_{ta}) = \sigma + \tau G_{ta} + \upsilon G_{ta}^2,^{5} \tag{48}$$

with,

$$\tau = \frac{\beta_2(1-\alpha)^2 c^2 b^2 \rho r_2 + 2\beta_2 M(1-\alpha)^2 c b^2 \rho^2 z + \beta_2^2 M(1-\alpha) b \rho^2 z - 2\beta_2^2 M(1-\alpha) c b \rho r_2}{M^2(1-\alpha)^3 c b^2 \rho(\rho^2 - 3\rho\beta_2 + 2\beta_2^2)}$$

$$\frac{+M(1-\alpha)^2 c^2 b^2 \rho^2 r_2 + 2\beta_2 M(1-\alpha) c b \rho^2 r_2 + 2\beta_2^2 M(1-\alpha)^2 c b \rho a - \beta_2^2 M(1-\alpha)\rho^2 a}{M^2(1-\alpha)^3 c b^2 \rho(\rho^2 - 3\rho\beta_2 + 2\beta_2^2)}$$

$$\frac{-2\beta_2 M(1-\alpha)^2 c b \rho^2 a + \beta_2^3 M(1-\alpha)\rho a + \beta_2^2 M \rho^2 r_2 - 2\beta_2^2 M(1-\alpha)^2 c b^2 \rho z}{M^2(1-\alpha)^3 c b^2 \rho(\rho^2 - 3\rho\beta_2 + 2\beta_2^2)}$$

[5] We do not detail the expression σ because it is not necessary for resolution, but it is available from authors on request.

$$\frac{+\beta_2^2(1-\alpha)cb\rho r_2 - 2r_2M(1-\alpha)^2c^2b^2\rho\beta_2 - \beta_2^3(M\rho r_2 + M(1-\alpha)b\rho z + (1-\alpha)cbr_2)}{M^2(1-\alpha)^3cb^2\rho(\rho^2 - 3\rho\beta_2 + 2\beta_2^2)}, \tag{49}$$

$$v = -\frac{\beta_2(\beta_2 + 2c(1-\alpha)b)}{2M(1-\alpha)^2b(\rho - 2\beta_2)} \tag{50}$$

and $\beta_2 < 0$, the negative root described in equation (29). In the period t=[0, t_a], solutions of the piecewise open loop case are solutions of the problem open loop described in equations (38) and (39)), with $\pi(t_a)$ which verifies condition (47). Finally and assuming that $G(t)$ is a continuous function, we obtain optimal solutions of the piecewise open loop problem: $G^{POL}(t)$, $g^{POL}(t)$ and $\pi^{POL}(t)$.

C Resolution of the feedback case

Now, for the feedback case, we solve as previously first the problem between t_a and ∞. The problem of player i is:

$$\max_{g_i(.)} \int_{t_a}^{\infty} F_i(G, g_i) \quad e^{-\rho(t-t_a)} \quad dt, \tag{51}$$

with $F_i(G, g_i)$ (equation (8)), constrained by the dynamics (12) with $r = r_2$ and conditions (11) and $G(t_a) = G_{ta}$.

For each player i ($i = 1..M$) the optimal value of the resource, $V^i(G)$, have to verify the Hamilton-Jacobi-Bellman equation:

$$\rho V^i(G) = \max_{g_i}(F_i(G, g_i) - V_G^i(G)(r_2 - (1-\alpha)\sum_{j=1}^{M} g_j)), \quad i = 1..M, \tag{52}$$

with $V^i(G)$ and g_j ($j \neq i$):

$$V^i(G) = AG^2 + BG + C, \tag{53}$$

$$g_j = a_jG + b_j. \tag{54}$$

To simplify the analytical resolution of the problem, we assume now $M = 2$ players (or group of farmers). First, we solve the problem for the player 1 (i.e. $i = 1$). Solving the problem on the right hand-side of (52), we find the optimal pumping rate of player 1, g_1^*:

$$g_1^* = a_1G + b_1. \tag{55}$$

with,

$$a_1 = \frac{b(c - 2(1-\alpha)A)}{2}, \tag{56}$$

$$b_1 = \frac{a - zb - b(1-\alpha)B}{2}. \tag{57}$$

Substituting now g_1^* on the right hand-side of equation (52) and equalizing the left and hand sides of the equation, we obtain optimal values of coefficients A, B and C of V^i with regards to variables a_2 and b_2 (see equation (54)), which are coefficients of pumping rate of player 2.

Moreover, assuming that players are symmetric as in the previous cases, $g_1(t)$=$g_2(t)$ for any $t = t_a..\infty$. Thus, a_1=a_2 and b_1=b_2. Substituting A^* and B^* in equations (56) and (57), and taking into account the propriety of symmetry between players, we find optimal values of coefficients of the pumping rate function of player 1, b_1^* and a_1^*:

$$b_1^* = \frac{(1-\alpha)\rho(-a+zb) + (1-\alpha)^2 a_1^*(-a+zb) + r_2(\rho + (1-\alpha)(cb+2a_1^*))}{(1-\alpha)((1-\alpha)(cb+2a_1^*) + 2\sqrt{(\rho+2(1-\alpha)a_1^*)(\rho+2(1-\alpha)(cb+a_1^*)}))}$$

$$+ \frac{r_2\sqrt{(\rho+2(1-\alpha)a_1^*)(\rho+2(1-\alpha)(cb+a_1^*))}}{(1-\alpha)((1-\alpha)(cb+2a_1^*) + 2\sqrt{(\rho+2(1-\alpha)a_1^*)(\rho+2(1-\alpha)(cb+a_1^*)}))}. \tag{58}$$

$$a_1^* = \frac{-\rho + (1-\alpha)cb + \sqrt{\rho^2 + 4\rho(1-\alpha)cb + (1-\alpha)^2c^2b^2}}{6(1-\alpha)}. \tag{59}$$

Finally, substituting b_1^* and a_1^* in the dynamics of the aquifer (12) with $r = r_2$, we may solve the differential equation (12), constrained by condition $G(t_a) = G_{ta}$. Then, we obtain optimal solutions of the feedback problem, $G^{FB+}(t)$, $g_1^{FB+}(t)$ and the optimal value fonction $V^*(G)$ in $[t_a,\infty)$:

$$G^{FB}(t) = e^{-2(1-\alpha)a_1^*(t-t_a)}(G_{ta} - G_\infty) + G_\infty, \tag{60}$$

$$g_1^{FB}(t) = \frac{r}{2} + 2(1-\alpha)a_1^* e^{-2(1-\alpha)a_1^*(t-t_a)}(G_{ta} - G_\infty),$$

and

$$V^+(G) = A^*G^2 + B^*G + C^*, \tag{61}$$

with,

$$G_\infty = \frac{r_2}{2(1-\alpha)a_1^*} - \frac{b_1^*}{a_1^*} \tag{62}$$

and b_1^*, a_1^* described in equations (58) and (59).

Next, we solve the problem between 0 and t_a. The value function of the problem of player i, $V^i(t,G)$ [6] verifies the Hamilton-Jacobi-Bellman equation:

$$\rho V^i(t,G) - V_t^i(t,G) = \max_{g_i}(F_i(G,g_i) - V_G^i(G)(r_1 - (1-\alpha)\sum_{j=1}^{M} g_j)), \quad i = 1..M, \tag{63}$$

with $V^i(G,t)$ and g_j $(j \neq i)$:

$$V^i(t,G) = A(t)G^2 + B(t)G + C(t), \tag{64}$$

$$g_j(t) = a_j(t)G + b_j(t), \tag{65}$$

and the transversality condition,

$$V^i(t_a, G_{ta}) = V^+(G_{ta}). \tag{66}$$

The value of $V^+(G_{ta})$ is obtained from the first step of resolution of the problem, and is described in equation (61). To solve the second step of the problem, we are going to use a similar process than we use previously. The challenge here lies in the fact that strategies of players depend on the stock of the resource G and on functions $a_1(t)$ and $b_1(t)$ in a independent way. So, the resolution of the problem is largely numerical.

First, assuming $M = 2$ players and solving the right part of equation (63) for $i = 1$, we find the expression (55) that is the optimal pumping rate of player 1, $g_1^*(t)$, with $a_1 = a_1(t)$ and $b_1 = b_1(t)$, described in equations (56), (57) and $A = A(t)$, $B = B(t)$, which are now functions that depend on t.

Moreover, as players are symmetric, $g_1(t)$=$g_2(t)$ for any $t = 0..t_a$, then $a_1(t)$=$a_2(t)$ and $b_1(t)$=$b_2(t)$. Now, substituting $g_1^*(t)$ in the right part of equation (63), and equalizing the right and left parts of the equation, we have to solve a system of 3 differential equations in $A(t), B(t)$ and $C(t)$, which are coefficients of the value function $V(t,G)$, between $t = 0$ and $t = t_a$, (see equation (64)), taking into account boundaries conditions:

$$A(t_a) = A^*, B(t_a) = B^*, C(t_a) = C^*,$$

derived from the transversatity condition (66):

$$V^-(G_{ta}, t_a) = A(t_a)G_{t_a}^2 + B(t_a)G_{t_a} + C(t_a) = V^+(G_{ta}).$$

At this stage, we obtain $A^*(t)$ and $B^*(t)$ by a numerical approximation method. Substituting $A^*(t)$ and $B^*(t)$ in expression $g_1^*(t)$, we find the optimal values of coefficients

[6] We remind that in this type of problem with a finite horizon planning, the value function have to be described as a function that depends on G and t independently.

$b^*(t)$ and $a^*(t)$ of the pumping rate. Next, we substitute these values in the equation of motion (12) with $r = r_1$. Finally we obtain the numerical solution of the feedback problem between 0 and t_a, that is $G^{FB-}(t)$ and $g^{FB-}(t)$, where the initial condition $G(0) = G_0$, is given.

D Resolution of the social optimum

To solve this problem, we separate it into two parts and proceed by backward induction. First, we solve the maximization between t_a and ∞. The problem of the social planner is to find $\phi^{SO}(G_{ta})$,

$$\phi(G_{ta}) = \max_{g_i(.)} \int_{t_a}^{\infty} \sum_{i=1}^{M} F_i(G, g_i) \quad e^{-\rho(t-t_a)} \quad dt, \tag{67}$$

with $F_i(G, g_i)$ (equation (8)), constrained by equation (9) with $r = r_2$ and conditions (11) and $G(t_a) = G_{ta}$.

The Hamiltonian of this problem is given by:

$$H = \sum_{i=1}^{M} (\frac{a}{b} g_i - \frac{1}{2b\theta_i} g_i^2 - (z - cG) g_i) + \lambda(-(1-\alpha) \sum_{i=1}^{M} g_i + r_2),$$

where λ is the adjoint variable. Applying the maximum principle and assuming interior solutions, we have the usual first order conditions:

$$\frac{\partial H}{\partial g_i} = 0 \quad \Rightarrow \quad \frac{a}{b} - \frac{1}{b\theta_i} g_i - (z - cG) - \lambda(1-\alpha) = 0, \quad i = 1..M, \tag{68}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial G} + \rho\lambda \quad \Rightarrow \quad \dot{\lambda} = -c \sum_{i=1}^{M} g_i + \rho\lambda, \quad i = 1..M. \tag{69}$$

We assume that players are symmetric in order to simplify the analytical resolution of the problem. Thus, $\theta_i = \frac{1}{M}$ and $g = g_i$.

From (68), we find the optimal extraction volume as a function of the resource stock and the shadow price:

$$g = \frac{1}{M}(a - zb + cbG - \lambda b(1-\alpha)). \tag{70}$$

Substituting (70) in the equations of motion of the state (9) and adjoint variable (69), we have the following dynamic system:

$$\dot{G} = r_2 - (1-\alpha)(a - zb) - cb(1-\alpha)G + \lambda b(1-\alpha)^2, \tag{71}$$

$$\dot{\lambda} = -c(a - zb) - c^2 bG + (cb(1-\alpha) + \rho)\lambda, \tag{72}$$

which allows us to find the roots of the characteristic polynom:

$$\rho_{1,2} = \frac{\rho \pm \sqrt{\rho^2 + 4cb(1-\alpha)\rho}}{2}. \tag{73}$$

From equations (70), (71) and (72), with $\dot{G} = 0$ and $\dot{\lambda} = 0$, we find the steady state of the social optimum problem:

$$g_\infty^{SO} = \frac{r_2}{(1-\alpha)M}, \tag{74}$$

$$\lambda_\infty^{SO} = \frac{cr_2}{\rho(1-\alpha)}, \tag{75}$$

$$G_\infty^{SO} = \frac{r_2}{cb(1-\alpha)} + \frac{r_2}{\rho} - \frac{a}{cb} + \frac{z}{c}. \tag{76}$$

Since we assume that all parameters are positive, g_∞ and λ_∞ (equations (74) and (75)) are always positive. Moreover, in what follows, we consider parameters such that G_∞ (equation (76)) is positive.

Finally, we have the optimal extraction paths from $t = t_a$, with ρ_2, the negative root:

$$G^{SO+}(t) = e^{\rho_2(t-t_a)}(G_{ta} - G_\infty) + G_\infty, \tag{77}$$

$$\lambda^{SO+}(t) = e^{\rho_2(t-t_a)}(\lambda_{ta} - \lambda_\infty) + \lambda_\infty, \tag{78}$$

$$g^{SO+}(t) = \frac{r_2}{(1-\alpha)M} - \frac{\rho_2}{(1-\alpha)M}(G_{ta} - G_\infty)e^{\rho_2(t-t_a)}, \tag{79}$$

with,

$$\lambda_{ta} = \frac{a}{b(1-\alpha)} + \frac{-z + cG_{ta}}{(1-\alpha)} - \frac{r_2}{b(1-\alpha)^2} + \frac{1}{b(1-\alpha)^2}\rho_2(G_{ta} - G_\infty), \quad G_{ta} \quad \text{unknown.}$$

Substituting (77) and (79) in problem (67), we can compute the scrap value, $\phi^{SO}(G_{ta})$[7]:

$$\phi(G_{ta}) = \epsilon + \kappa G_{ta} + \iota G_{ta}^2, \quad \text{with}^8 \tag{80}$$

$$\kappa = \frac{-r_2\rho(4cb(1-\alpha) + \rho) + \rho^2(1-\alpha)(a - zb) + 4cb(1-\alpha)^2\rho(a - zb)}{\rho(1-\alpha)^2 b(\eta + \rho + 4cb(1-\alpha))}$$

[7]We find that the expression $\phi(ta, G_{ta})$ does not have the independent term t_a. In what follows, we write the scrap value function, $\phi(G_{ta})$.

[8]We do not detail expression of σ because it is not necessary for the resolution of the problem, but it is available from the authors upon request

$$+\frac{(2cb(1-\alpha)+\rho)r_2\eta-\rho(1-\alpha)\eta(a-zb))}{\rho(1-\alpha)^2b(\eta+\rho+4cb(1-\alpha))}, \tag{81}$$

$$\iota=-\frac{c(-4cb(1-\alpha)-\rho+\eta)}{(1-\alpha)(\eta+\rho+4cb(1-\alpha))}, \quad \text{and} \tag{82}$$

$$\eta=\sqrt{\rho}\sqrt{\rho+4bc(1-\alpha)}. \tag{83}$$

We now turn to the second part of the problem, between 0 and t_a, considering the optimal solution for the first part. The problem for the social planner is now (44) constrained by equations (45), (46) and the transversality condition:

$$\lambda(t_a)=\frac{\partial\phi(ta,G_{ta})}{\partial G_{ta}}=2\iota G_{ta}+\kappa,.$$

with $\phi(ta,G_{ta})$ described by equation (80). The Hamiltonian can be written as:

$$H=\sum_{i=1}^{M}(\frac{a}{b}g_i-\frac{1}{2b\theta_i}g_i^2-(z-cG)g_i)+\lambda(-(1-\alpha)\sum_{i=1}^{M}g_i+r),$$

where λ is the adjoint variable. Applying the maximum principle and assuming interior solutions, we have the usual first order conditions (68) and (69). From this and equation of the motion of the state (45), with $r=r_1$, we have the system of differential equations:

$$\dot{G}=r_1-(1-\alpha)(a-zb)-cb(1-\alpha)G+\lambda b(1-\alpha)^2, \tag{84}$$

$$\dot{\lambda}=-c(a-zb)-c^2bG+(cb(1-\alpha)+\rho)\lambda. \tag{85}$$

We know that the solutions of the finite problem are now, of the shape:

$$G^{SO-}(t)=A_1e^{\rho_1 t}+A_2e^{\rho_2 t}+A_3, \tag{86}$$

$$\lambda^{SO-}(t)=B_1e^{\rho_1 t}+B_2e^{\rho_2 t}+B_3,$$

with,

$$G^{SO-}(0)=A_1+A_2+A_3=G_0, \tag{87}$$

$$\lambda(t_a)=B_1e^{\rho_1 ta}+B_2e^{\rho_2 ta}+B_3=2\iota G_{ta}+\kappa, \tag{88}$$

and ρ_1, ρ_2 described in equation (73). This constitutes a system of 6 equations and 6 unknowns, which we can solve to find optimal solutions for the problem for the first period, between 0 and t_a. We find optimal values of A_i, B_i $(i=1..3)$[9]:

[9] We do not provide detailed solutions of B_i $(i=1..3)$ because the equations are too long and they are not necessary for the proofs, however, they are available from the authors upon request.

$$A_i = C_1^i + C_2^i(2\upsilon G_r^*(t_a) + \tau), \quad \text{i=1,2,}$$

with,

$$C_1^1 = \frac{b(1-\alpha)(\rho c(r_1 + (1-\alpha)e^{\rho_2 t_a}(a - zb) - r_1 e^{\rho_2 t_a}) - \rho_2 c r_1)}{D^1}$$

$$+ \frac{b(1-\alpha)((1-\alpha)c^2(r_1 b + e^{\rho_2 t_a}\rho b G_0 - r_1 b e^{\rho_2 t_a}))}{D^1},$$

$$C_2^1 = \frac{b(1-\alpha)^2\rho(\rho_2 - \rho - cb(1-\alpha))}{D^1},$$

$$D^1 = \rho((\rho 2 - \rho)e^{\rho_1 t_a} cb(1-\alpha) + c^2 b^2 (1-\alpha)^2 (e^{\rho_2 t_a} - e^{\rho_1 t_a}) - cb(1-\alpha), \rho_1 e^{\rho_1 t_a} + \rho_1 e^{\rho_1 t_a}(\rho_2 - \rho)),$$

$$C_1^2 = \frac{-(cb(1-\alpha) + \rho - \rho_2)(\rho_1 e^{\rho_1 t_a}\rho((1-\alpha)(a - zb) - r_1)}{D^2}$$

$$+ \frac{c^2 b^2 (1-\alpha)^2 (G_0 \rho + r_1 - r_1 e^{\rho_1 t_a})}{D^2},$$

$$\frac{+cb(1-\alpha)((G_0\rho - r_1)e^{\rho_1 t_a}\rho_1 - \rho r_1 e^{\rho_1 t_a}) + cb(1-\alpha)^2 e^{\rho_1 t_a}\rho(a - zb))}{D^2},$$

$$C_2^2 = \frac{(cb(1-\alpha) + \rho - \rho_2)(cb^2(1-\alpha)^3\rho)}{D^2},$$

$$D^2 = cb(1-\alpha)\rho((\rho_2 - \rho)cb(1-\alpha)e^{\rho_1 t_a} + c^2 b^2 (1-\alpha)^2 (e^{\rho_2 t_a} - e^{\rho_1 t_a}))$$

$$+cb(1-\alpha)\rho(-cb(1-\alpha)e^{\rho_1 t_a}\rho_1 - \rho e^{\rho_1 t_a}\rho_1 + e^{\rho_1 t_a}\rho_1\rho_2),$$

and,

$$A_3 = -\frac{r_1\rho - \rho a + \rho z b + \rho\alpha a - \rho\alpha z b + c r_1 b - c r_1 b\alpha}{b\rho(\alpha - 1)c}. \tag{89}$$

Finally, considering the continuity of the function of the variable state, i.e. $G^{SO-}(t_a) = G^{SO+}(t_a)$, we obtain optimal solution of the stock $G^{SO}(t)$ and water pumping $g^{SO}(t)$ for the social optimum problem.

E Proofs of propositions

Proposition: At the steady state, the strategic effect, evaluated as the difference $G^{OL}(\infty)$-$G^{FB}(\infty)$, is always positive.

Demonstration We have to prove that $G_\infty^{FB} - G_\infty^{OL} < 0$.

From equations (17) and (18), we know that

$$G_\infty^{FB} - G_\infty^{OL} = \frac{r_2}{2}(\frac{1}{2(1-\alpha)a_1^*} - \frac{1}{M\rho} - \frac{1}{(1-\alpha)cb} + \frac{a}{cb} - \frac{z}{c} - \frac{b_1^*}{a_1^*},$$

with b_1^*, a_1^* described in equations (58) and (59).

Substituting optimal values b_1^* and a_1^* (equations (57 and 56) and simplifying, we obtain

$$G_\infty^{FB} - G_\infty^{OL} = \frac{F1}{F2}, \tag{90}$$

with F2 described in expression,

$$F2 = 2c\rho(2A^*(1-\alpha) + c),$$

and

$$F1 = \frac{E1}{E2}, \tag{91}$$

with,

$$E1 = c(\frac{a}{b} - z) + r_2\frac{2}{b}2A^* + 4A^*(\frac{a}{b} - z)(\alpha - 1)$$

and

$$E2 = -\rho\frac{2}{b} + 2c(\alpha - 1) + 6A^* - 12A^*\alpha + 6A^*\alpha^2.$$

On one hand, $F2 > 0$ because $A^* < \frac{c}{2(1-\alpha)}$ (stability condition of the feedback solution[10]).

Substituting now A^* in E1 and E2, we obtain

$$E1 = -\frac{1}{6}(2(\alpha - 1)c + \rho\frac{2}{b} - \sqrt{\omega})r_2c^2 \tag{92}$$

and

$$E2 = -\rho 2b - \frac{1}{2}\sqrt{\omega}.$$

[10]This condition is derived from Rubio and Casino [8] and it is necessary to assure the stability of the linear system of differential equations.

As ω and b have positive values by assumption, $E2 < 0$.

Now, two cases are possible. First, if $(2(\alpha-1)c+\rho\frac{2}{b}) < 0$, then the expression written in brackets in equation (92)

$$(2(\alpha-1)c+\rho\frac{2}{b})-\sqrt{\omega} \tag{93}$$

is negative and then E1 is positive.

Second, if $(2(\alpha-1)c+\rho\frac{2}{b}) > 0$ in (93), evaluating the square of expression written in brackets, then we obtain $12\rho bc(\alpha-1) < 0$. Thus, from the monotonic propriety of the square function, the value of expression (93) is less than 0 and in the two possible cases, E1>0.

Finally, if E1>0 and E2<0, then F1<0 (described in equation (91)). As F1<0 and F2>0, we find from equation (90) that $G_\infty^{FB}-G_\infty^{OL} < 0$, as required. □

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