Probability Weighting in Recursive Evaluation of Two-Stage Prospects

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Outline

$\boxed{\textbf{1/ Introductory remarks}}$

$\boxed{\textbf{2/ Conceptual framework}}$

$\boxed{\textbf{3/ Experimental study}}$

$\boxed{\textbf{4/ Concluding remarks}}$

Why investigating the evaluation of Two-stage prospects?

Two-stage prospect and reduction of compound lotteries

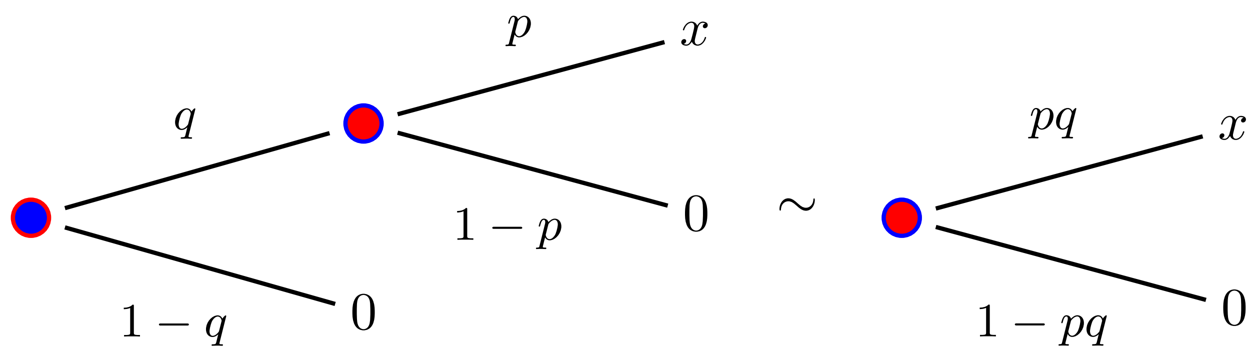


Table 1: compound risk and its reduced one-stage lottery

- ☐ Rational Decision makers (DMs) reduce compound risks, represented by compound lotteries, into single stage lotteries by using the Reduction of compound prospects axiom (RCP).

$\Rightarrow$ Rational DMs should exhibit a perfect neutrality toward compound risk.

However...

Three observations on compound risk

1. Reduction of compound prospects have been descriptively challenged in many empirical investigations:
   - Bar Hillel (1973), Bernasconi & Loomes (1992), Budescu & Fisher (2001), Abdellaoui, Klibanoff & Placido (2014), Nebout & Dubois (2014)...

2. Following Becker & Brownson (1964) and Yates and Zukowski (1976), Segal (1987, 1990) represented ambiguous bets as two-stage prospects.

3. Prospect Theory (PT) is the most succesful descriptive model of decision making under risk and ambiguity.

   $\Rightarrow$ Would it still be the case when dealing with attitudes toward two-stage prospects?

Theoretical and Empirical Background

Background: One-stage prospects

- $(x, p; y)$ denotes the one-stage *prospect* resulting in outcome x with probability p and in outcome y with probability $1 - p$ with $x \geq y \geq 0$.

  – Probability p is generated using a known urn containing 100 balls numbered from 1 to 100, i.e. drawing a ball which has a number between 1 and $p \times 100$.

- $(x, E_p; y)$ denotes the corresponding ambiguous prospect. The probability $P(E_p)$ is unknown to the DM.

  – We use an unknown containing 100 balls numbered from 1 to 100 in unknown proportions, i.e. drawing a ball which has a number between 1 and $p \times 100$. Symmetry arguments imply $P(E_p) = p$. (Chew & Sagi, 2006, 2008)

Background: Evaluation of one-stage prospects

- Under Expected Utility (EU), prospects are evaluated as follows:

$$EU(x, p; y) = pu(x) + (1 - p)u(y)$$

  – Where u is the utility function (and a risk attitude index).
  – Violations of EU popularized by Kahneman & Tversky.

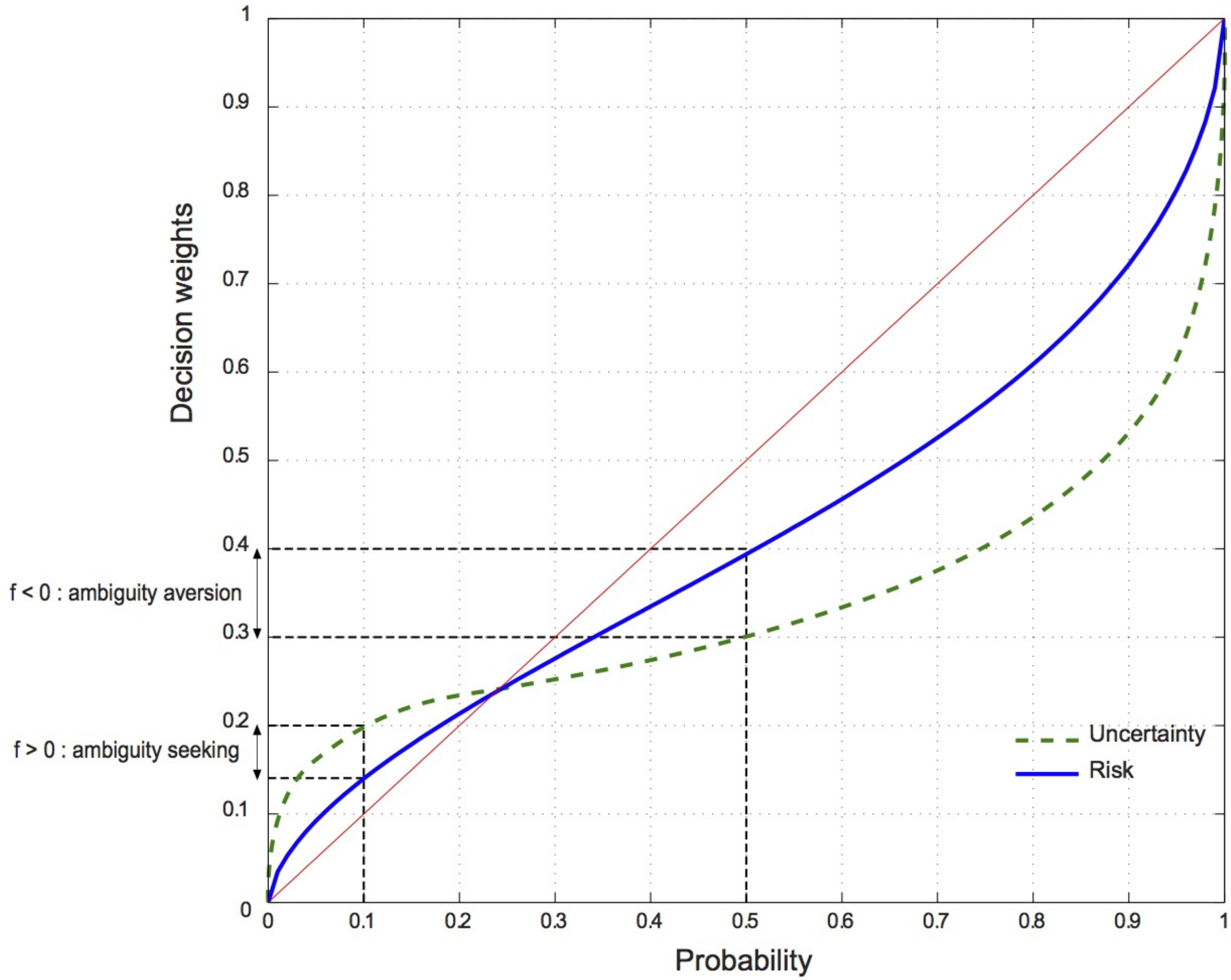
- Under Prospect Theory (PT), prospects are evaluated as follows in the gain domain:

$$PT(x, p; y) = w(p)u(x) + (1 - w(p))u(y)$$

  – u is the utility function.
  – w is the probability weighting function. w is strictly increasing and satisfies $w(0) = 0$ and $w(1) = 1$.

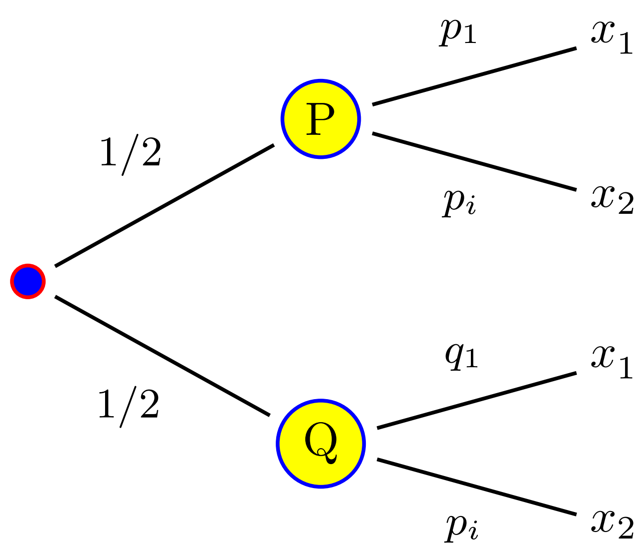
$\Rightarrow$ Many experimental evidence on RDU under risk and ambiguity.

Probability weighting under risk and ambiguity



⇒ Ambiguity increases **likelihood insensitivity.**

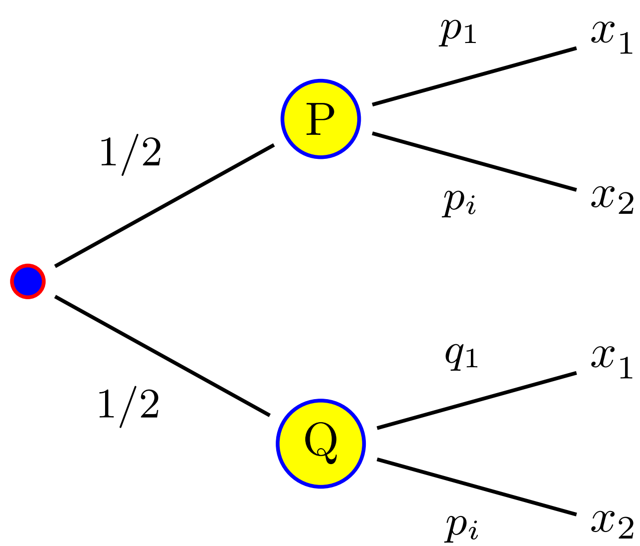
How to evaluate two-stage prospects?



- Traditional Recursive Expected Utility (TREU):

$$\Rightarrow \frac{1}{2} \times EU(P) + \frac{1}{2} \times EU(Q)$$

How to evaluate two-stage prospects?



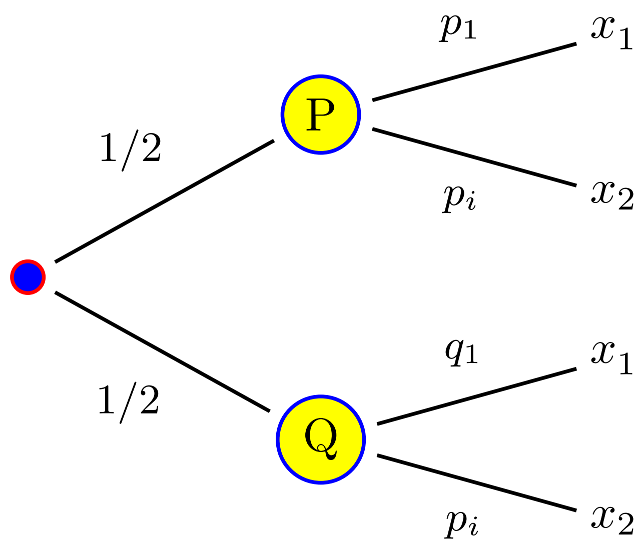
- Recursive Expected Utility without RCP (REU):

$$\Rightarrow \frac{1}{2} \times \phi\left[EU(P)\right] + \frac{1}{2} \times \phi\left[EU(Q)\right]$$

  – Kreps & Porteus (1978) introduced this transformed EU functionnal to account for delayed resolution of uncertainty.

  – Klibanof & al. (2005) used the same preference functinnal to model ambiguity. (Seo, 2009, Ergin & Gul (2008),...)

How to evaluate two-stage prospects?



□ Recursive Prospect Theory (RPT):

$$\Rightarrow \pi_1 \times \phi\left[PT(P)\right] + \pi_2 \times \phi\left[PT(Q)\right]$$

– Segal (1987) suggested this recursive form of RDU to model ambiguity attitudes through second-order probabilities.

– Abdellaoui & Zank (2014) provide the first axiomatization of this general form of Prospect Theory.

Experimental Design

Experiment: outline

62 subjects:

- Parts 1 & 2: certainty equivalents for known and unknown Ellsberg urns (2×13 equivalents).
- Parts 3 & 4: matching probabilities for two-stage prospects (2×10 equivalents).
- Payment: show up 10 euros + RIS (max 50 euros).
- Individual interviews (about 45 minutes).

Under RPT, we aim to elicit the following functionals:

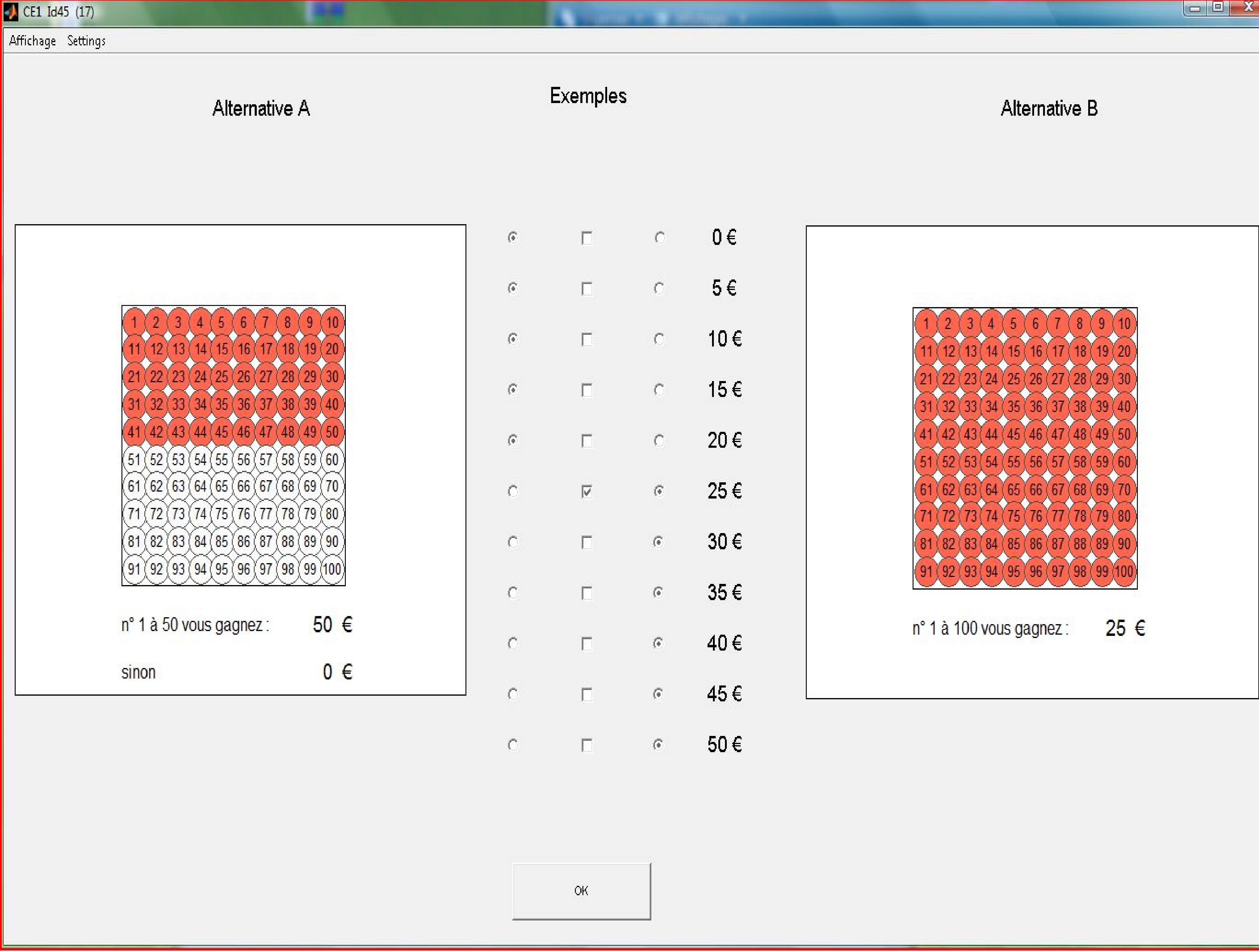
	Elicitation 1: Risk	Elicitation 2: Ambiguity
Attitudes towards 1-stage prospects	u w	$\tilde{u}$ $\tilde{w}$
Attitudes towards 2-stage prospects	w^*	$\tilde{w}^*$

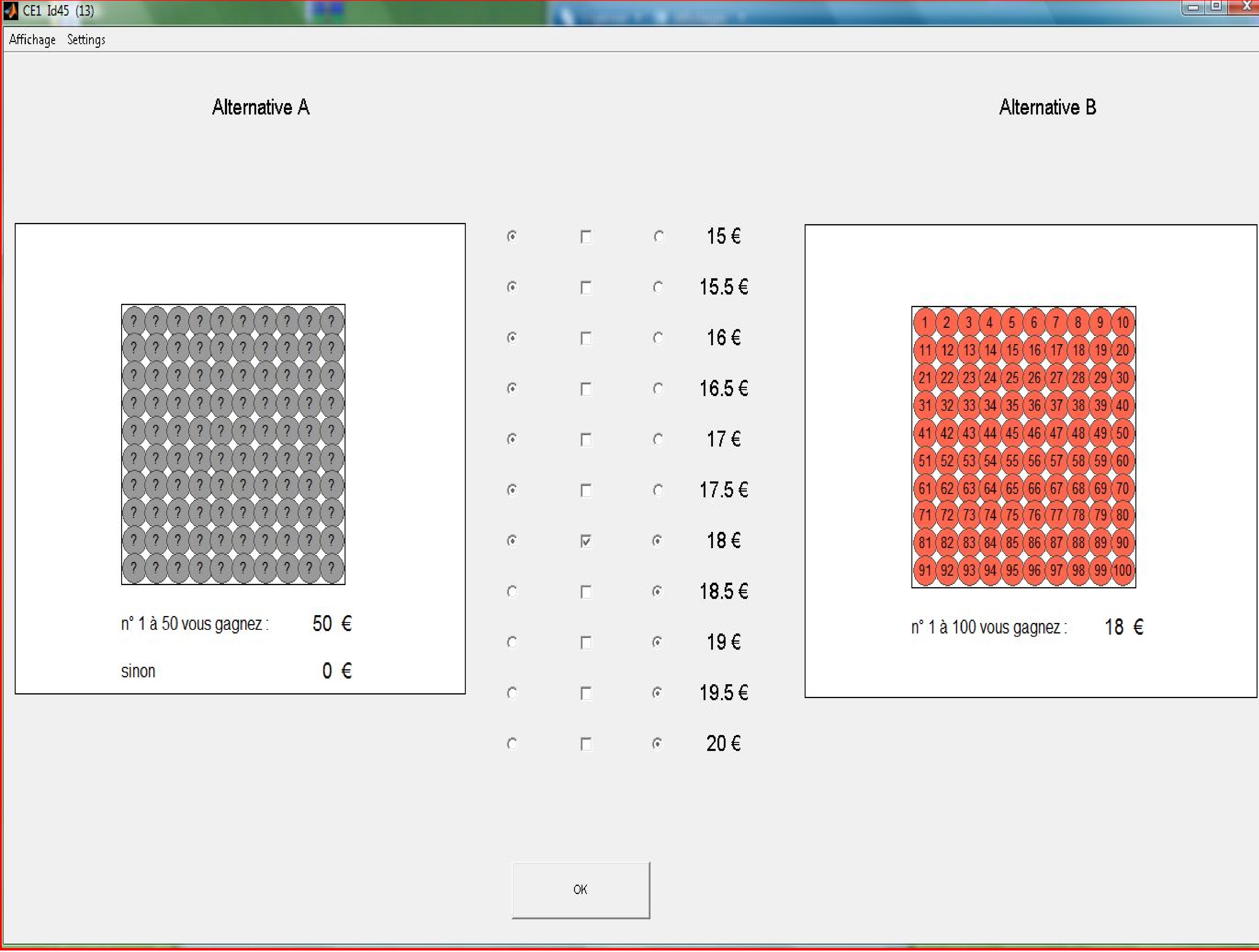
Certainty equivalents for one-stage prospects

	$c \sim (x,p;y)$ and $\tilde{c} \sim (x,E_p;y)$						$c_p \sim (x,p;y)$ and $\tilde{c}_p \sim (x,E_p;y)$						
x	50	40	50	50	25	10	50	50	50	50	50	50	50
y	25	20	10	35	5	0	0	0	0	0	0	0	0
p	0.30	0.30	0.30	0.30	0.30	0.30	0.02	0.06	0.17	0.33	0.50	0.67	0.94
E_p	E_{30}	E_{30}	E_{30}	E_{30}	E_{30}	E_{30}	E_2	E_6	E_{17}	E_{33}	E_{50}	E_{67}	E_{94}

- Under risk, we elicit c and c_p following Abdellaoui & al. (2008).
- Under ambiguity, we elicit $\tilde{c}$ and $\tilde{c}_p$ following Abdellaoui & al. (2011).

$\Rightarrow$ We can compare our results to these benchmark studies.


CE1 Id45 (17)
Affichage Settings
Alternative A
Exemples
Alternative B
0 €
5 €
10 €
15 €
20 €
25 €
30 €
35 €
40 €
45 €
50 €
n° 1 à 50 vous gagnez : 50 €
sinon 0 €
n° 1 à 100 vous gagnez : 25 €
OK

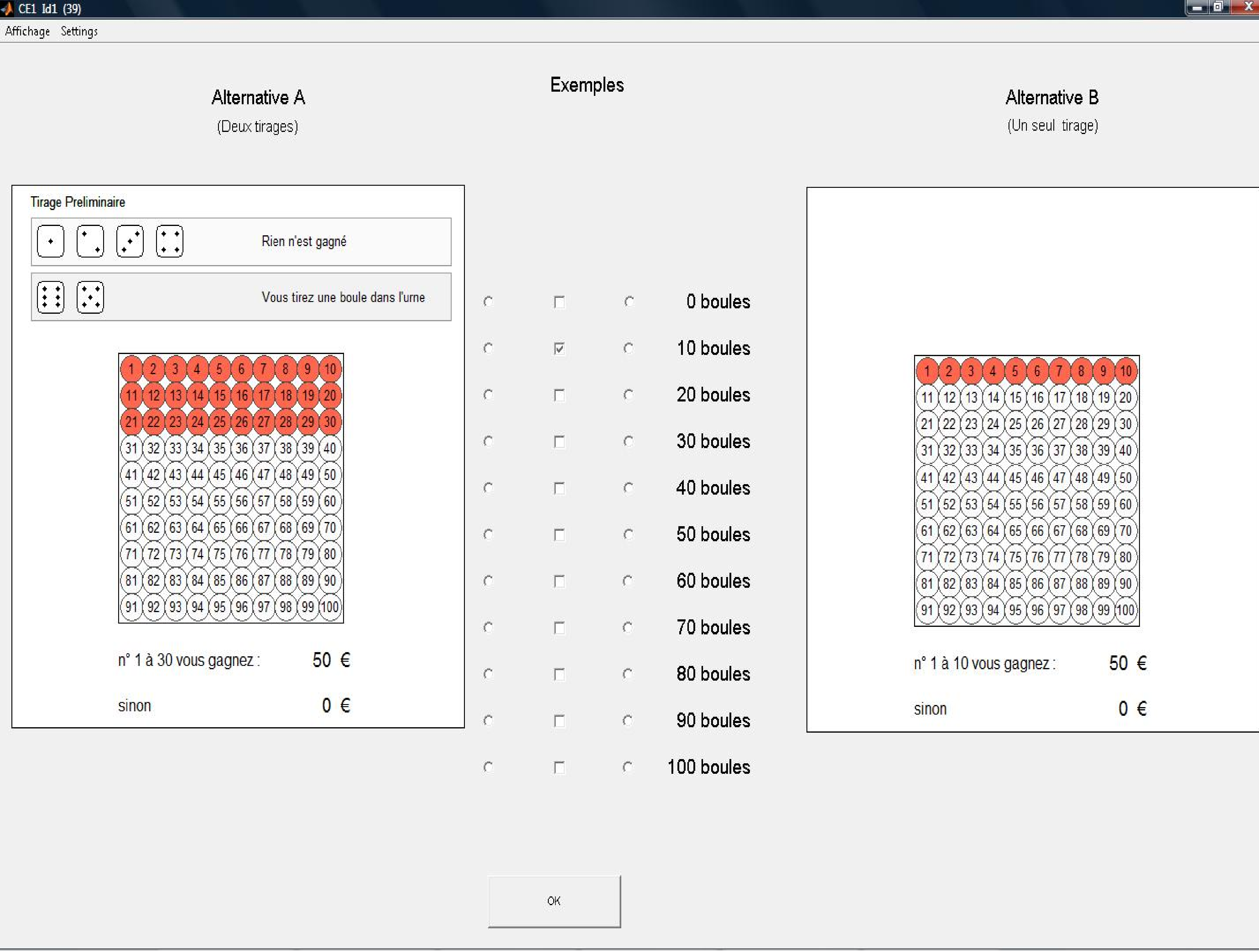

CE1 Id45 (13)
Affichage
Settings
Alternative A
Alternative B
15 €
15.5 €
16 €
16.5 €
17 €
17.5 €
18 €
18.5 €
19 €
19.5 €
20 €
n° 1 à 50 vous gagnez : 50 €
sinon 0 €
n° 1 à 100 vous gagnez : 18 €
OK

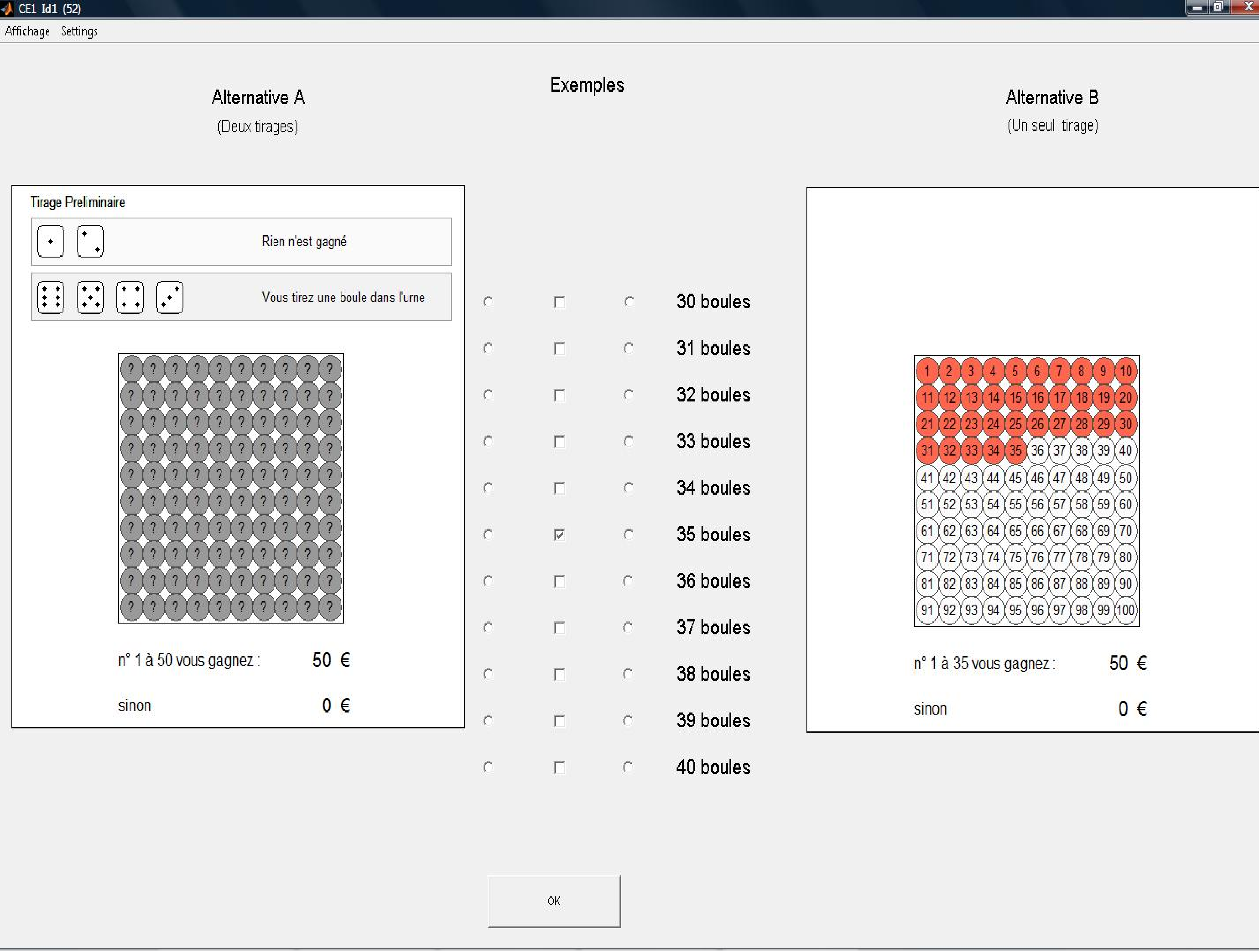
Matching probabilities for two-stage prospects

$((\bar{x},p),q) \sim (\bar{x},r)$ and $((\bar{x},E_p),q) \sim (\bar{x},\tilde{r})$										
q	1/3	1/3	1/3	1/3	1/3	2/3	2/3	2/3	2/3	2/3
p	0.06	0.17	0.33	0.50	0.94	0.06	0.17	0.50	0.75	0.94
E_p	E_6	E_{17}	E_{33}	E_{50}	E_{94}	E_6	E_{17}	E_{50}	E_{75}	E_{94}

- $\bar{x} = 50$.
- 10 matching probabilities for risky second stage stage, r and 10 for ambiguous second stage $\tilde{r}$.
- 2 first stage probability levels: 1/3 and 2/3.

$\Rightarrow$ Elicitation, comparison and test of 4 second stage probability weighting functions.


CE1 Id1 (39)
Affichage
Settings
Exemples
Alternative A
(Deux tirages)
Alternative B
(Un seul tirage)
Tirage Preliminaire
Rien n'est gagné
Vous tirez une boule dans l'urne
0 boules
10 boules
20 boules
30 boules
40 boules
50 boules
60 boules
70 boules
80 boules
90 boules
100 boules
n° 1 à 30 vous gagnez : 50 €
sinon 0 €
n° 1 à 10 vous gagnez : 50 €
sinon 0 €
OK


CE1 Id1 (52)
Affichage Settings
Exemples
Alternative A
(Deux tirages)
Tirage Preliminaire
Rien n'est gagné
Vous tirez une boule dans l'urne
n° 1 à 50 vous gagnez : 50 €
sinon 0 €
30 boules
31 boules
32 boules
33 boules
34 boules
35 boules
36 boules
37 boules
38 boules
39 boules
40 boules
Alternative B
(Un seul tirage)
n° 1 à 35 vous gagnez : 50 €
sinon 0 €
OK

Model Specification

Model specification: TREU and REU

Using the equivalence revealed by the elicitation of the matching probability r:

$$((\bar{x}, p), q) \sim (\bar{x}, r)$$

we infer the following equalities:

1. Under TREU, we have:

$$q \times p = r$$

2. Under REU, we have:

$$q \times \phi(p) = \phi(r)$$

Where ϕ is a transformation function.

Parametric specification: $\phi(x) = x^{1/\theta}$.

Model specification RPT

- Under RPT, we have:

Setup	RPT-r RPT for *"risk-risk"*	RPT-a RPT for *"risk-ambiguity"*
One-stage	$c \sim (x, p; y)$ $u(c) = w(p)u(x) + (1 - w(p))u(y)$	$c \sim (x, p; y)$ $u(c) = w(p)u(x) + (1 - w(p))u(y)$ $\tilde{c} \sim (x, E_p; y)$ $u(\tilde{c}) = \tilde{w}(p)u(x) + (1 - \tilde{w}(p))u(y)$
Two-stage	$((\bar{x}, p), q) \sim ((\bar{x}, m), 1)$ $w(q)w^\star(p) = w^\star(\tilde{m})$	$((\bar{x}, E_p), q) \sim ((\bar{x}, \tilde{m}), 1)$ $w(q)\tilde{w}^\star(p) = w^\star(\tilde{m})$

- Parametric specifications:

$u(x) = x^\alpha$ and $w(p) = exp(-(-ln(p)^\gamma)^\delta$.

Results

Results: RCP and TREU

p	$q = 1/3$		$q = 2/3$	
	$\#(\Delta \gtrless 0)$	t-test	$\#(\Delta \gtrless 0)$	t-test
0.06	60/2	8.42^{**}	48/14	2.12^{**}
0.17	38/24	3.65^{**}	27/35	-0.22^{ns}
0.33	26/28	2.26^{*}	-	-
0.50	39/23	3.39^{**}	24/38	-1.96^{*}
0.75	-	-	24/25	0.33^{ns}
0.94	27/35	0.69^{ns}	40/22	1.99^{**}

Table 2: RCP ($\Delta = (r - pq)/pq$)

- RCP is globally violated, thus TREU is not descriptively valid for evaluating two-stage prospects.
- Overall, we observe preference for the compound prospect, especially for $q = 1/3$.

Results: REU under risk

Here, we present the descriptive statistics of the 3 OLS estimations of θ, assuming $\phi(x) = x^{1/\theta}$ under risk (i.e. only using the matching probabilities, r)

	Probability q		
	$1/3$	$2/3$	$\{1/3, 2/3\}$
Mean	0.89	1.03	0.91
Median	0.90	0.98	0.91
Std	0.18	0.35	0.18
IQR	0.75-0.98	0.83-1.23	0.79-1.01

- At an aggregate level, ϕ is convex for $q = 1/3$ but exhibits linearity for $q = 2/3$.
- Impact of probability q on the shape of function ϕ is confirmed by a paired t-test ($p < 0.01$).

$\Rightarrow$ Inconsistent with REU in which the probabilistic structure of the two-stage prospect should not impact the shape of ϕ.

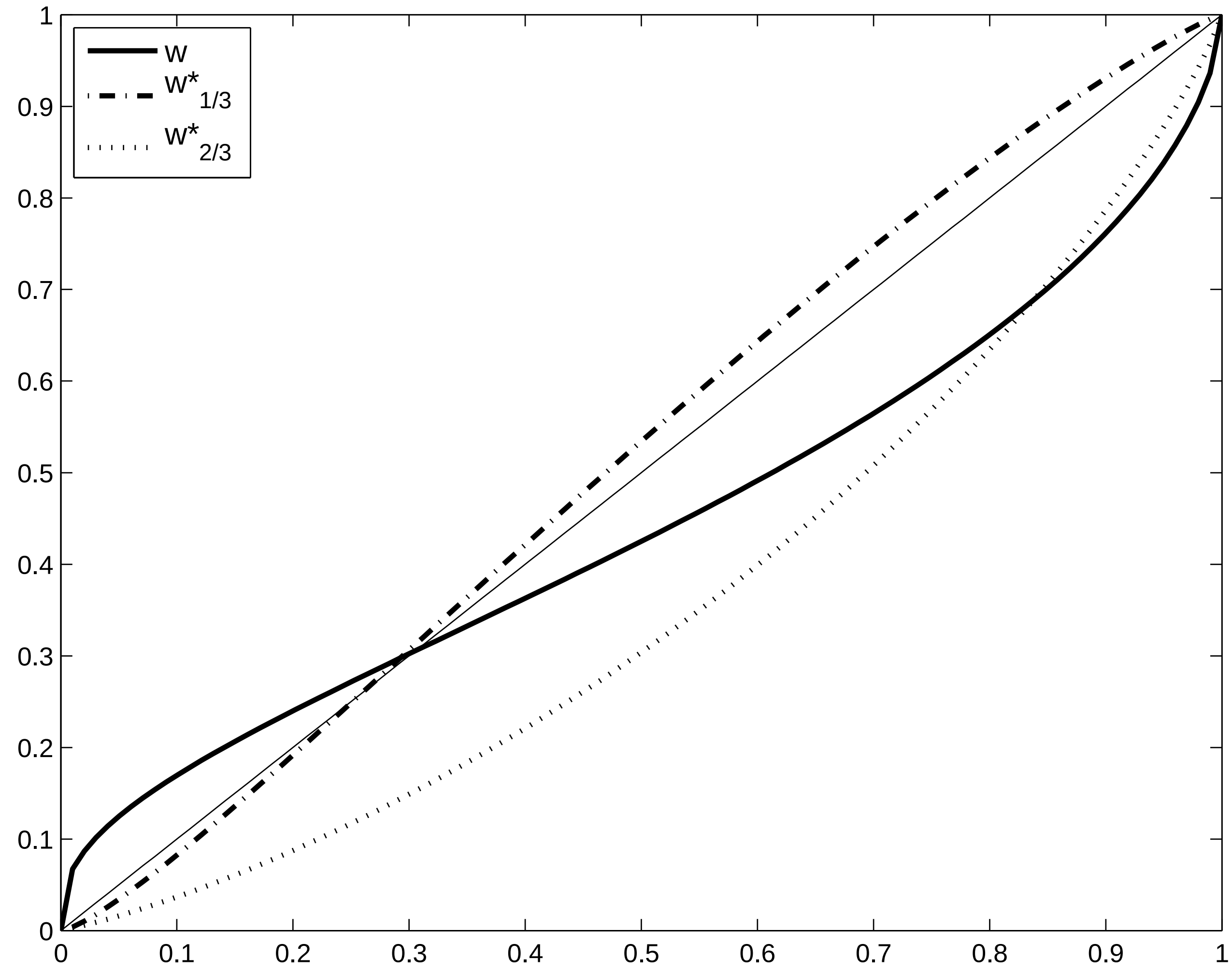
Results: RPT under risk

Stage	func	param	Estimates			
			Mean	Median	Std	IQR
First	u	α	0.94	0.93	0.26	0.74-1.10
	w	γ	1.06	1.07	0.32	0.85-1.22
		δ	0.62	0.61	0.20	0.48-0.77
Second	$w^*_{1/3}$	$\gamma^*_{1/3}$	1.01	0.96	0.42	0.74-1.18
		$\delta^*_{1/3}$	1.20	1.15	0.29	1.03-1.35
	$w^*_{2/3}$	$\gamma^*_{2/3}$	1.66	1.63	0.52	1.25-2.04
		$\delta^*_{2/3}$	0.90	0.85	0.28	0.69-1.07

- Function, $w^\star$, depends on probability q. While it is close to linearity for $q = 1/3$, it is convex for $q = 2/3$.

⇒ Inverse than for REU but same problem (differences both for elevation and curvature between $w^*_{1/3}$ and $w^*_{2/3}$).

Results: RPT under risk



$\Rightarrow$

Additional results

- Adding ambiguity (first and second stage) does not change our main results i.e.
  - Impact of probability q on the shape of function ϕ.
  - Stage dependent pwf.
  - Dependence of the second stage pwf on the first stage probability.
- Benchmark results are found for the single stage pwf under risk and ambiguity.
- No association between RCP and ambiguity attitudes ($\neq$ from Halevy, 2008 and Segal).

Concluding remarks

1. Recursive evaluation of two-stage prospects is more complex than allowed by any existing recursive model.

2. In Kreps and Porteus (or KMM) integral, function ϕ is sensitive to the first-stage probability of winning.

3. Second-stage probability weighting is very sensitive to the first-stage winning probability.

$\Rightarrow$ Abdellaoui & Zank (2014) first axiomatized a RPT model that could account for our experimental findings.

Thank you for your attention!