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Probability Weighting in Recursive Evaluation of Two-Stage Prospects

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Outline

1/ Introductory remarks

2/ Conceptual framework

3/ Experimental study

4/ Concluding remarks

Why investigating the evaluation of Two-stage prospects?

Two-stage prospect and reduction of compound lotteries

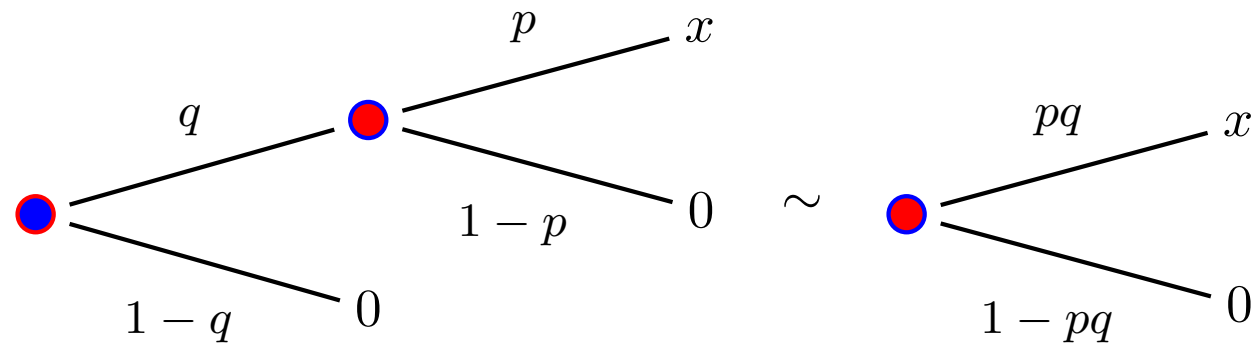


Table 1: compound risk and its reduced one-stage lottery

- Rational Decision makers (DMs) reduce compound risks, represented by compound lotteries, into single stage lotteries by using the Reduction of compound prospects axiom (RCP).

\Rightarrow Rational DMs should exhibit a perfect neutrality toward compound risk.

However...

Three observations on compound risk

1. Reduction of compound prospects have been descriptively challenged in many empirical investigations:
 - Bar Hillel (1973), Bernasconi & Loomes (1992), Budescu & Fisher (2001), Abdellaoui, Klibanoff & Placido (2014), Nebout & Dubois (2014)...
2. Following Becker & Brownson (1964) and Yates and Zukowski (1976), Segal (1987, 1990) represented ambiguous bets as two-stage prospects.
3. Prospect Theory (PT) is the most successful descriptive model of decision making under risk and ambiguity.

⇒ Would it still be the case when dealing with attitudes toward two-stage prospects?

Theoretical and Empirical Background

Background: One-stage prospects

- $(x, p; y)$ denotes the one-stage *prospect* resulting in outcome x with probability p and in outcome y with probability $1 - p$ with $x \geq y \geq 0$.
 - Probability p is generated using a known urn containing 100 balls numbered from 1 to 100, i.e. drawing a ball which has a number between 1 and $p \times 100$.

- $(x, E_p; y)$ denotes the corresponding ambiguous prospect. The probability $P(E_p)$ is unknown to the DM.
 - We use an unknown containing 100 balls numbered from 1 to 100 in unknown proportions, i.e. drawing a ball which has a number between 1 and $p \times 100$. Symmetry arguments imply $P(E_p) = p$. (Chew & Sagi, 2006, 2008)



Background: Evaluation of one-stage prospects

- Under Expected Utility (EU), prospects are evaluated as follows:

$$EU(x, p; y) = pu(x) + (1 - p)u(y)$$

- Where u is the utility function (and a risk attitude index).
- Violations of EU popularized by Kahneman & Tversky.

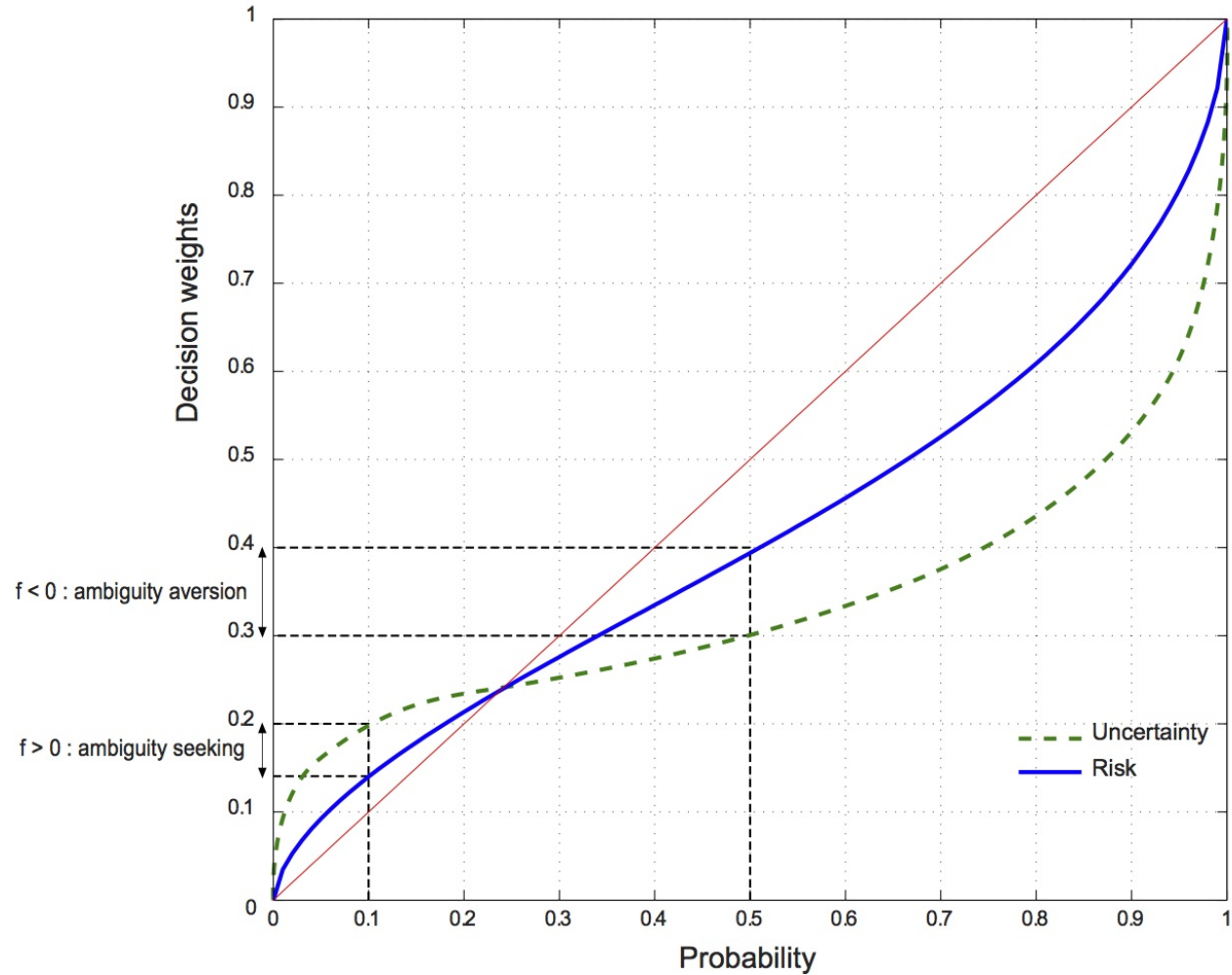
- Under Prospect Theory (PT), prospects are evaluated as follows in the gain domain:

$$PT(x, p; y) = w(p)u(x) + (1 - w(p))u(y)$$

- u is the utility function.
- w is the probability weighting function. w is strictly increasing and satisfies $w(0) = 0$ and $w(1) = 1$.

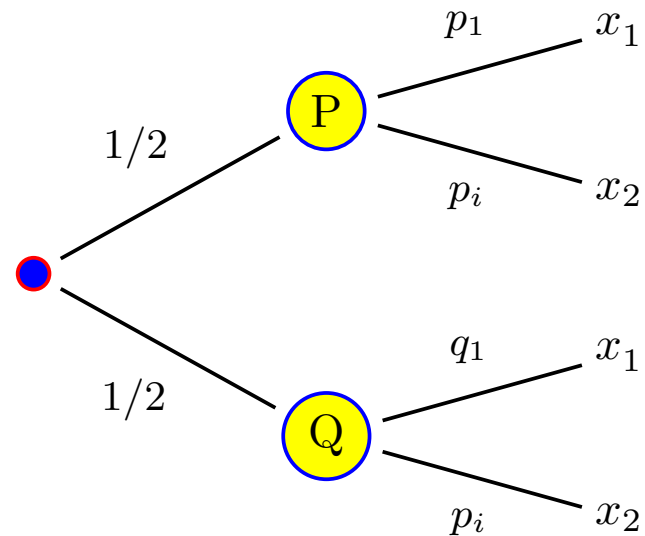
⇒ Many experimental evidence on RDU under risk and ambiguity.

Probability weighting under risk and ambiguity



⇒ Ambiguity increases **likelihood insensitivity**.

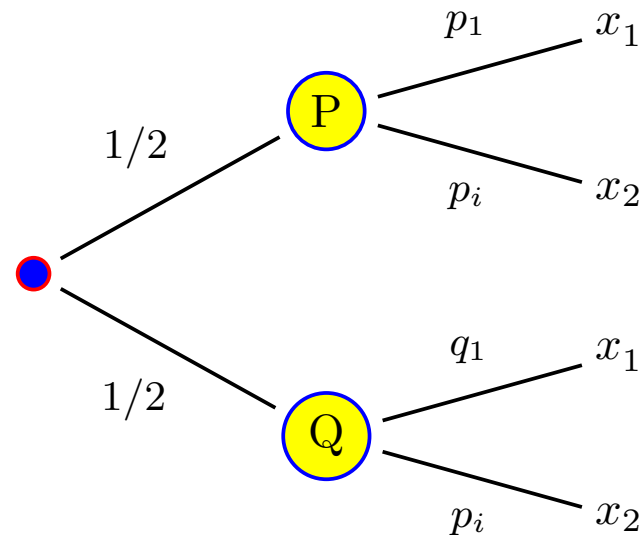
How to evaluate two-stage prospects?



- Traditional Recursive Expected Utility (TREU):

$$\Rightarrow \frac{1}{2} \times EU(P) + \frac{1}{2} \times EU(Q)$$

How to evaluate two-stage prospects?

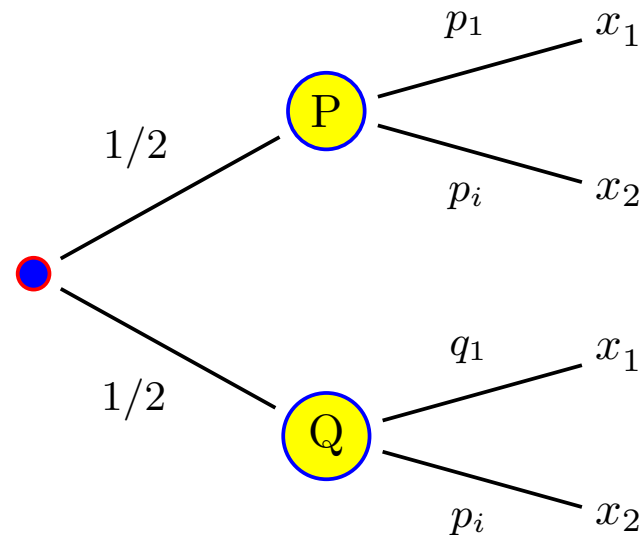


- Recursive Expected Utility without RCP (REU):

$$\Rightarrow \frac{1}{2} \times \phi [EU(P)] + \frac{1}{2} \times \phi [EU(Q)]$$

- Kreps & Porteus (1978) introduced this transformed EU functional to account for delayed resolution of uncertainty.
- Klibanof & al. (2005) used the same preference functional to model ambiguity. (Seo, 2009, Ergin & Gul (2008),...)

How to evaluate two-stage prospects?



- Recursive Prospect Theory (RPT):

$$\Rightarrow \pi_1 \times \phi [PT(P)] + \pi_2 \times \phi [PT(Q)]$$

- Segal (1987) suggested this recursive form of RDU to model ambiguity attitudes through second-order probabilities.
- Abdellaoui & Zank (2014) provide the first axiomatization of this general form of Prospect Theory.

Experimental Design

Experiment: outline

62 subjects:

- Parts 1 & 2: certainty equivalents for known and unknown Ellsberg urns (2×13 equivalents).
- Parts 3 & 4: matching probabilities for two-stage prospects (2×10 equivalents).
- Payment: show up 10 euros + RIS (max 50 euros).
- Individual interviews (about 45 minutes).

Under RPT, we aim to elicit the following functionals:

	Elicitation 1: Risk	Elicitation 2: Ambiguity
Attitudes towards 1-stage prospects	u w	\tilde{u} \tilde{w}
Attitudes towards 2-stage prospects	w^*	\tilde{w}^*

Certainty equivalents for one-stage prospects

	$c \sim (x, p; y)$ and $\tilde{c} \sim (x, E_p; y)$						$c_p \sim (x, p; y)$ and $\tilde{c}_p \sim (x, E_p; y)$						
x	50	40	50	50	25	10	50	50	50	50	50	50	50
y	25	20	10	35	5	0	0	0	0	0	0	0	0
p	0.30	0.30	0.30	0.30	0.30	0.30	0.02	0.06	0.17	0.33	0.50	0.67	0.94
E_p	E_{30}	E_{30}	E_{30}	E_{30}	E_{30}	E_{30}	E_2	E_6	E_{17}	E_{33}	E_{50}	E_{67}	E_{94}

- Under risk, we elicit c and c_p following Abdellaoui & al. (2008).
 - Under ambiguity, we elicit \tilde{c} and \tilde{c}_p following Abdellaoui & al. (2011).
- ⇒ We can compare our results to these benchmark studies.

Alternative A



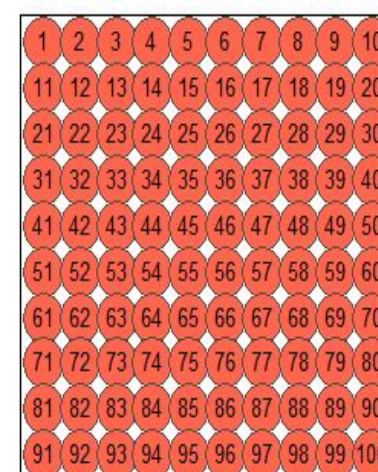
n° 1 à 50 vous gagnez : 50 €

sinon 0 €

Exemples

- | | | | |
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| <input type="radio"/> | <input type="checkbox"/> | <input type="radio"/> | 0 € |
| <input type="radio"/> | <input type="checkbox"/> | <input type="radio"/> | 5 € |
| <input type="radio"/> | <input type="checkbox"/> | <input type="radio"/> | 10 € |
| <input type="radio"/> | <input type="checkbox"/> | <input type="radio"/> | 15 € |
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| <input type="radio"/> | <input type="checkbox"/> | <input type="radio"/> | 40 € |
| <input type="radio"/> | <input type="checkbox"/> | <input type="radio"/> | 45 € |
| <input type="radio"/> | <input type="checkbox"/> | <input type="radio"/> | 50 € |

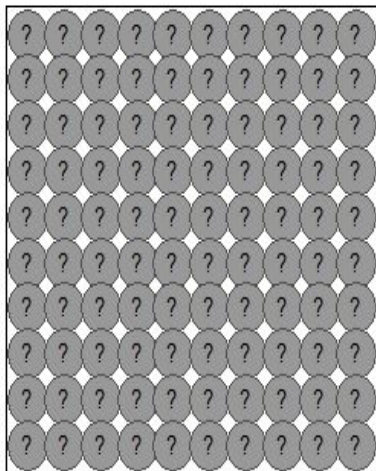
Alternative B



n° 1 à 100 vous gagnez : 25 €

OK

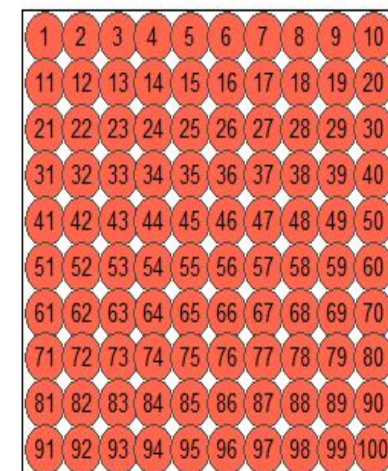
Alternative A



n° 1 à 50 vous gagnez : 50 €

sinon 0 €

Alternative B



n° 1 à 100 vous gagnez : 18 €

- | | | | |
|-----------------------|-------------------------------------|-----------------------|--------|
| <input type="radio"/> | <input type="checkbox"/> | <input type="radio"/> | 15 € |
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| <input type="radio"/> | <input type="checkbox"/> | <input type="radio"/> | 20 € |

OK

Matching probabilities for two-stage prospects

$$((\bar{x}, p), q) \sim (\bar{x}, r) \text{ and } ((\bar{x}, E_p), q) \sim (\bar{x}, \tilde{r})$$

q	1/3	1/3	1/3	1/3	1/3	2/3	2/3	2/3	2/3	2/3
p	0.06	0.17	0.33	0.50	0.94	0.06	0.17	0.50	0.75	0.94
E_p	E_6	E_{17}	E_{33}	E_{50}	E_{94}	E_6	E_{17}	E_{50}	E_{75}	E_{94}

- $\bar{x} = 50$.
- 10 matching probabilities for risky second stage stage, r and 10 for ambiguous second stage \tilde{r} .
- 2 first stage probability levels: 1/3 and 2/3.

⇒ Elicitation, comparison and test of 4 second stage probability weighting functions.

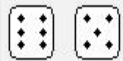
Alternative A

(Deux tirages)

Tirage Préliminaire



Rien n'est gagné



Vous tirez une boule dans l'urne



n° 1 à 30 vous gagnez : 50 €

sinon 0 €

Exemples

- 0 boules
- 10 boules
- 20 boules
- 30 boules
- 40 boules
- 50 boules
- 60 boules
- 70 boules
- 80 boules
- 90 boules
- 100 boules

Alternative B

(Un seul tirage)



n° 1 à 10 vous gagnez : 50 €

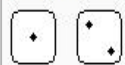
sinon 0 €

OK

Alternative A

(Deux tirages)

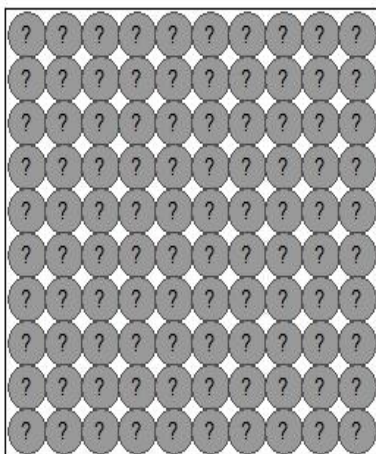
Tirage Préliminaire



Rien n'est gagné



Vous tirez une boule dans l'urne



n° 1 à 50 vous gagnez : 50 €

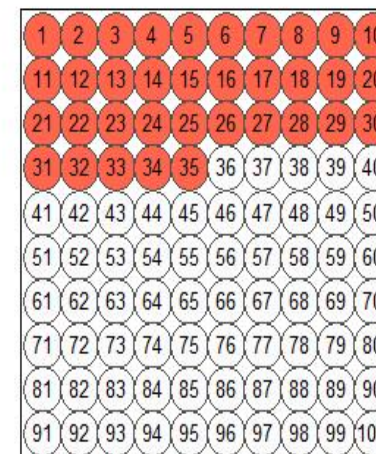
sinon 0 €

Exemples

- 30 boules
 31 boules
 32 boules
 33 boules
 34 boules
 35 boules
 36 boules
 37 boules
 38 boules
 39 boules
 40 boules

Alternative B

(Un seul tirage)



n° 1 à 35 vous gagnez : 50 €

sinon 0 €

OK

Model Specification

Model specification: TREU and REU

Using the equivalence revealed by the elicitation of the matching probability r :

$$((\bar{x}, p), q) \sim (\bar{x}, r)$$

we infer the following equalities:

1. Under TREU, we have:

$$q \times p = r$$

2. Under REU, we have:

$$q \times \phi(p) = \phi(r)$$

Where ϕ is a transformation function.

Parametric specification: $\phi(x) = x^{1/\theta}$.

Model specification RPT

- Under RPT, we have:

Setup	RPT- r RPT for "risk-risk"	RPT- a RPT for "risk-ambiguity"
One-stage	$c \sim (x, p; y)$ $u(c) = w(p)u(x) + (1 - w(p))u(y)$	$c \sim (x, p; y)$ $u(c) = w(p)u(x) + (1 - w(p))u(y)$
		$\tilde{c} \sim (x, E_p; y)$ $u(\tilde{c}) = \tilde{w}(p)u(x) + (1 - \tilde{w}(p))u(y)$
Two-stage	$((\bar{x}, p), q) \sim ((\bar{x}, m), 1)$ $w(q)w^*(p) = w^*(\tilde{m})$	$((\bar{x}, E_p), q) \sim ((\bar{x}, \tilde{m}), 1)$ $w(q)\tilde{w}^*(p) = w^*(\tilde{m})$

- Parametric specifications:

$$u(x) = x^\alpha \text{ and } w(p) = \exp(-(-\ln(p)^\gamma)^\delta).$$

Results

Results: RCP and TREU

p	$q = 1/3$		$q = 2/3$	
	$\#(\Delta \geq 0)$	t -test	$\#(\Delta \geq 0)$	t -test
0.06	60/2	8.42**	48/14	2.12**
0.17	38/24	3.65**	27/35	-0.22 ^{ns}
0.33	26/28	2.26*	-	-
0.50	39/23	3.39**	24/38	-1.96*
0.75	-	-	24/25	0.33 ^{ns}
0.94	27/35	0.69 ^{ns}	40/22	1.99**

Table 2: RCP ($\Delta = (r - pq)/pq$)

- RCP is globally violated, thus TREU is not descriptively valid for evaluating two-stage prospects.
- Overall, we observe preference for the compound prospect, especially for $q = 1/3$.

Results: REU under risk

Here, we present the descriptive statistics of the 3 OLS estimations of θ , assuming $\phi(x) = x^{1/\theta}$ under risk (i.e. only using the matching probabilities, r)

	Probability q		
	1/3	2/3	{1/3, 2/3}
Mean	0.89	1.03	0.91
Median	0.90	0.98	0.91
Std	0.18	0.35	0.18
IQR	0.75-0.98	0.83-1.23	0.79-1.01

- At an aggregate level, ϕ is convex for $q = 1/3$ but exhibits linearity for $q = 2/3$.
 - Impact of probability q on the shape of function ϕ is confirmed by a paired t -test ($p < 0.01$).
- ⇒ Inconsistent with REU in which the probabilistic structure of the two-stage prospect should not impact the shape of ϕ .

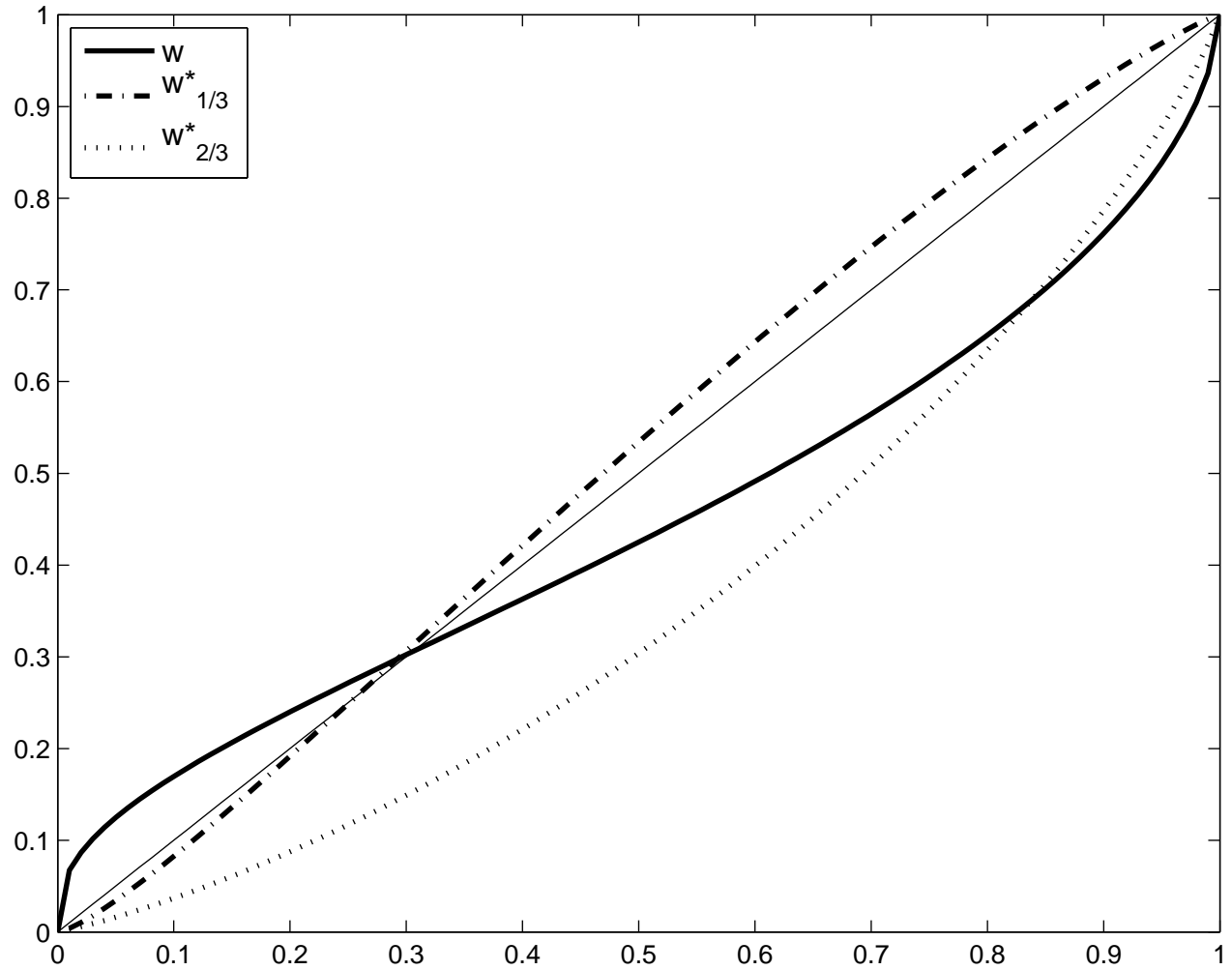
Results: RPT under risk

Stage	func	param	Estimates			
			Mean	Median	Std	IQR
First	u	α	0.94	0.93	0.26	0.74-1.10
		γ	1.06	1.07	0.32	0.85-1.22
	w	δ	0.62	0.61	0.20	0.48-0.77
Second	$w_{1/3}^*$	$\gamma_{1/3}^*$	1.01	0.96	0.42	0.74-1.18
		$\delta_{1/3}^*$	1.20	1.15	0.29	1.03-1.35
	$w_{2/3}^*$	$\gamma_{2/3}^*$	1.66	1.63	0.52	1.25-2.04
		$\delta_{2/3}^*$	0.90	0.85	0.28	0.69-1.07

□ Function, w^* , depends on probability q . While it is close to linearity for $q = 1/3$, it is convex for $q = 2/3$.

⇒ Inverse than for REU but same problem (differences both for elevation and curvature between $w_{1/3}^*$ and $w_{2/3}^*$).

Results: RPT under risk



Additional results

- Adding ambiguity (first and second stage) does not change our main results i.e.
 - Impact of probability q on the shape of function ϕ .
 - Stage dependent pwf.
 - Dependence of the second stage pwf on the first stage probability.
- Benchmark results are found for the single stage pwf under risk and ambiguity.
- No association between RCP and ambiguity attitudes (\neq from Halevy, 2008 and Segal).

Concluding remarks

1. Recursive evaluation of two-stage prospects is more complex than allowed by any existing recursive model.
2. In Kreps and Porteus (or KMM) integral, function ϕ is sensitive to the first-stage probability of winning.
3. Second-stage probability weighting is very sensitive to the first-stage winning probability.

⇒ Abdellaoui & Zank (2014) first axiomatized a RPT model that could account for our experimental findings.

Thank you for your attention!