Recycling of an exhaustible resource and hotelling's rule

Bocar Samba Ba, Philippe Mahenc

HAL Id: hal-02798470

https://hal.inrae.fr/hal-02798470v1

Submitted on 5 Jun 2020

Recycling of an Exhaustible Resource and Hotelling's Rule

Bocar Samba Ba[*]   Philippe Mahenc[†]

January 30, 2015

Abstract

We examine the strategic interplay between the extraction of a virgin resource and the recycling of this resource. For this, we analyze a two-period model that covers the whole spectrum of market power in the two sectors of extraction and recycling, assuming they are independent. The choice of prior extraction creates potential future competition between the two sectors. We show that prior extraction is designed in a way that either accommodates, ignores or deters recycling. The general insight is the following. While recycling increases the socially efficient level of prior extraction, the possibility of recycling makes the extraction sector too parsimonious in setting the level of prior extraction whether the strategic motive is to soften competition with recycling or to discourage recycling.

JEL : .

[*]University Montpellier I (LAMETA), Avenue Raymond Dugrand, Site de Richter, C. S. 79606, 34960 Montpellier Cedex 2 - France - Email: bacarbo84@yahoo.fr

[†]University Montpellier I (LAMETA), Avenue Raymond Dugrand, Site de Richter, C. S. 79606, 34960 Montpellier Cedex 2 - France - Email: philippe.mahenc@univ-montp1.fr

1 Introduction

Since the early work of Smith (1972), recycling waste or scrap into production is viewed as an alternative to undesirable littering. It is also commonly recognized that recycling helps save natural resources through conservation. This basic idea however needs to be addressed regarding those of exhaustible resources such as phosphorus or aluminium, that can be partially recovered after use. Surprisingly enough, the economic literature on exhaustible resources has very little considered the recycling possibility. Although developments in recycling techniques raise effective resource stocks (see Dasgupta, 1993), it is not clear why recycling should curb the extraction of resources that tend to run out.

The purpose of this paper is to examine the strategic interplay between the extraction of a virgin resource and the recycling of this resource, assuming that the two sectors of extraction and recycling are independent. As recycling yields a substitute to the virgin resource, prior extraction is potentially the source of later competition between the sectors of extraction and recycling. Thus, the extraction sector must anticipate how its initial choice of extraction affects not only the present and future demands for the resource, but also the intensity of future competition against the recycling sector. Regarding recycling as an opportunity or a threat, the extraction sector may find it worthwile to accommodate recycling, or, on the contrary, to make recycling unprofitable. We analyze here which strategic decision prevails under different regimes of intrinsic competition within the two sectors of extraction and recycling. When recycling is accommodated, the arbitrage rule of Hotelling (1931) that the marginal revenue from extraction rises at the rate of interest must be amended to take into consideration the extrinsic competition with the recycling sector. Otherwise, prior extraction may serve the strategic goals of either deterring or ignoring recycling under circumstances where it would be socially desirable to recycle the exhaustible resource. The strategy of ignoring recycling maintains prior extraction at the same level whether or not recycling is possible, whereas the strategy of deterring recycling reduces prior extraction below the level

that would obtain with no recycling possibility.

The history of exhaustible resources shows evidence that the extraction sector goes through various regimes of competition and the recycling market is often ill-organized. Martin (1982) recognizes that "many of the industries currently practicing recycling are highly concentrated".

One interesting example is the phosphate extraction together with phosphorus recycling. The majority of global phosphate rock reserves are located in Morocco, providing this country with a monopoly position in supplying the virgin resource (see Cordell et al., 2009). Thus, one may expect governmental regulation in Morocco to play a leading role in choosing the quantity of virgin phosphate to be extracted. In turn, this regulation may be more or less benevolent depending on various factors such as the pressure put on the government by shareholders of the extraction company, or the share of the consumer surplus that escapes the government's jurisdiction. At the same time, the sector of phosphorus recycling has no institutional or organizational home (Cordell, 2006; Livingston et al., 2005). Phosphorus recycling throughout the world is mainly based on the reuse of nutrient flows stemming from food production and consumption[1]. While the sanitation sector in cities, e. g. waste water treatment or sewage sludge plants, plays a key role in phosphorus recycling[2], this service is scarcely high on the agenda of stakeholders. In addition, the process of recovering phosphorus from sewage or waste water requires a specific infrastructure and high levels of technical skills, which only developed countries can afford. According to Weikard and Seyhan (2009), phosphorus recycling is mainly undertaken by developed countries not only because they have advanced wastewater treatment technologies, but also because, unlike developing countries, they have phosphorus-saturated soils[3]. Finally, the degree of competition in the

---

[1]There are various methods available to recover phosphorus such as ploughing crop residues back into the soil, composting food waste from households, using human and animal excreta, etc.

[2]"Around 41% of phosphorus from sewage sludge across the European Union is currently recovered and reused in agriculture" from the European Commission's expert seminar on the sustainability of phosphorus resource (2011, http://ec.europa.eu/environment/natres/pdf/conclusions_17_02_2011.pdf.)Already According to Ensink et al. (2004), more than 25% of urban vegetables are being fertilized with wastewater from cities in Pakistan.

[3]These authors show that developing countries benefit in the short and medium run from phosphorus

4

sector of phosphorus recycling, hence the welfare impact the extraction of virgin resource has on recycling, may highly depend on the ability of the concerned countries to organize this competition.

Another example of recyclable exhaustible resource is aluminium. It is now well documented because aluminum has been recovered since the early 1900s[4]. The monopoly nature of virgin aluminium production in 1945 was acknowledged by the famous Alcoa case (Swan, 1980[5]). By contrast, the recycling sector of the industry is generally considered as competitive throughout the literature. In the view of Friedman (1967), the competitive recycling sector would tend to push the aluminium price down to the marginal cost of virgin aluminum production. Martin (1982) disputes this statement in a model where Alcoa is treated as a monopolist faced with an independent recycling sector. Assuming that a fixed proportion of scrap is discarded by consumers, this author shows that the long run price sold by the monopolist is strictly greater than the marginal cost of virgin aluminium. Suslow (1986) argues that Alcoa's market power was hardly eroded by the very competitive nature of recycling because virgin and recovered aluminium were not perfect substitutes. This view conflicts with Swan (1980)'s intuition that the monopolist in the aluminium extraction sector had a strong strategic control over the recycling sector. Building on the assumption that the two sectors of extraction and recycling were independent in the Alcoa case, Grant (1999) provides empirical evidence that, first, recycling mattered to Alcoa, second, the producer of the virgin resource enjoyed a significant degree of market power, and third, aluminium recycling was not efficient although the sector was competitive. Since then, the aluminium industry has gone through different regimes of imperfect competition, both in the extraction and the recycling

---

recycling in developed countries, but face stronger competition for the resource in the long-term.

[4]In 1989, about 28 percent of the total aluminum supply in the United States came from recovered aluminum (see http://www.epa.gov/osw/nonhaz/municipal/pubs/sw90077a.pdf).

[5]In 1945, Alcoa was judged to enjoy a strong monopoly position which was supported rather than threatened by competition from secondary aluminium, produced by recycling scrap aluminium. Swan (1980) provides empirical evidence that the price charged by Alcoa is only slightly below the pure monopoly price but is well above the purely competitive price. The question of whether Alcoa had maintained its monopoly position by strategically controlling the supply of scrap aluminum ultimately available to secondary producers is debated at length in the economic literature. Grant (1999) provides a nice survey on this debate.

sectors. Again, the benevolence of governmental regulations or— which is equivalent —the conflicting pressures of consumer and corporate lobbies, do play an important part in the welfare impact of recycling.

To focus on the strategic interplay between prior extraction and recycling, we investigate a simple two-period model that covers the whole spectrum of market power, from monopoly to perfect competition, both in the two sectors of extraction and recycling. The extraction sector faces a rather specific intertemporal extraction scheduling problem. The extraction decision in the first period determines both what remains to be extracted and what could be recycled in the second period. In particular, the decison to accommodate recycling creates extrinsic competition between the two sectors in the second period. Our model sheds light on two effects of prior extraction on recycling, which coexist when the extraction sector has some degree of market power. The first effect is a balance effect between the two periods. Recycling gives rise to a valuable extension of the available stock of the resource, which disrupts the arbitrage rule of Hotelling by increasing the marginal revenue from later extraction. The second effect is a strategic effect due to the threat of extrinsic competition with the recycling sector.When the extraction sector finds it optimal to accommodate recycling, prior extraction represents a commitment to soften future competition against recycling. This results in an inefficiently low level of prior extraction. Besides, prior extraction may also be chosen in a strategic way designed to discourage recycling. The extraction sector achieves this goal again with an inefficiently low level of prior extraction.

Our general insight can be summarized as follows. While recycling increases the socially efficient level of prior extraction, the possibility of recycling makes the extraction sector too parsimonious in setting the level of prior extraction whether the strategic motive is to soften competition with recycling or to discourage recycling.

The early literature related to this paper examines how market power in the extraction sector affects the Hotelling rule. Hotelling (1931) shows that the monopolist has a tendency to be more resource-conservative than "competition... or maximizing of social value would

require". Stiglitz (1976) adds that the monopoly parsimony depends on the elasticity of demand and extraction costs. Except the case where the elasticity of demand is constant and extraction costs equal zero, the result that the monopolist extracts the resource at a lower rate than that of the competitive firm seems rather robust (see also Tullock,1979, for the case of inelastic demand). Lewis (1975) however finds out conditions on the price elasticity of demand for which the monopolist depletes the resource faster than required by social efficiency. Furthermore, a growing number of Cournot competitors on the market for an exhaustible resource tends to increase early extraction (see Lewis and Schmalensee, 1980). Hoel (1978) analyzes a situation in which the monopolist in the extraction sector faces perfect competition with a perfect substitute for the exhaustible resource. This author shows that the monopolist reduces initial extraction compared to the case where the monopolist controls both resource extraction and substitute production. In the present analysis, substitute production results from prior extraction, hence the extraction sector determines the amount of input available for substitute production.

The issue of recycling an exhaustible resource has developed more recently in the economic literature with the aforementioned debate on the Alcoa case. Besides that, Hollander and Lasserre (1988) investigate the case of a monopolist in the extraction sector which recycles the scrap from its own production. The monopolist has monopsony power in the scrap market and faces a fringe of price-taking recyclers. These authors show that the extraction sector finds it profitable to preempt market entry by competitive recyclers when the cost of recycling is sufficiently high. Unlike this work, we analyze the competition between the virgin resource and the recycled product that occurs after prior extraction, assuming that the extraction sector does not recycle its own output. Gaudet and Van Long (2003) examines how market power in the recycling sector affects the primary production of a non-exhaustible resource. They show that the possibility of recycling may increase the market power of the extraction sector. Clearly, this cannot occur in the present model since extrinsic competition between the exhaustible resource and its recycled output mitigates the extraction sector's market

power. Lastly, Fisher and Laxminarayan (2004) demonstrate that a monopolist may extract the exhaustible resource faster than a competitive company when the resource is sold at different prices on two separate markets with different iso-elastic demands and no arbitrage possibility between markets.

The article proceeds as follows. Section 2 introduces the two-period model. Section 3 presents the socially optimal solution. In section 4, we analyse the case of a monopolist in the resource extraction facing Cournot competition with the recycling sector. Concluding remarks appear in section 5.

2   The two-period model

One extraction sector, indexed by $i = 1$, holds the stock of an exhaustible natural resource, equal to s. This sector can extract the resource and transport it to market at no cost. Exploration does not occur and s is the single known stock of the resource in the world of this model. The exhaustible resource market is characterized by the inverse demand function $P(q)$, hence the consumers' gross surplus is $S(q) = \int_0^q P(x)dx$. An independent recycling sector, indexed by $i = 2$, has the technology and skill to recover part of the resource from used quantities[6]. The buyers of the virgin resource dispose of the used resource within sector 2, e. g. because it cannot be used again without being recycled. The recycled resource is viewed by consumers as a perfect substitute for the extracted resource. Recycling the amount of extracted resource, q, yields the output r that falls short of q due to depreciation and shrinkage which charaterize every recovery process[7]. The recycling cost $c(r)$ reflects the value of the used virgin resource together with the prices of all the factors needed to produce the recovered substitute of the resource.We assume both that the recycling process exhibits decreasing returns to scale[8], and that it is impossible to recover fully the prior amount of the

---

[6]Regarding phosphorus, for instance, sector 2 may be viewed as the group of developed countries with phosphorus-saturated soils and advanced wastewater treatment technologies (see Weikard and Seyhan, 2009).
[7]See Martin (1982) for aluminium scrap recovery and Weikard and Seyhan (2009) for phosphorus recovery from sewage sludge.
[8]Swan (1980) and Martin (1982) provide convincing arguments that it is indeed the case for aluminium.

virgin resource.

Moreover, we assume for simplicity that the resource becomes worthless after two periods. The extraction sector divides the resource stock between both periods. Supply in the first period determines what is left to be sold in the second period. The game unfolds as follows. In the first period, the extraction sector chooses quantity q and the market clears at price $P(q)$. In the second period, the recycling sector attends the market with quantity r and, simultaneously, the remaining stock of the resource, $s - q$, is sold by the resource sector; the market then clears at price $P(s - q + r)$.

The objective of a sector is to maximize a weighted sum of profits and consumers' surplus which places less fixed weight on the latter surplus. Thus we may write the objective functions as

$$W^1 = \alpha(S_1 - \pi_1^1) + \pi_1^1 + \delta\left[\alpha(S_2 - \pi_2^1) + \pi_2^1\right] \tag{1}$$

for the extraction sector, where δ is the discount factor, $S_1 = S(q), \pi_1^1 = P(q)q, S_2 = S(s - q + r), \pi_2^1 = P(s - q + r)(s - q)$ and $0 \leq \alpha \leq 1$; and

$$W^2 = \beta(S_2 - \pi_2^2) + \pi_2^2 - c \tag{2}$$

for the recycling sector, where $\pi_2^2 = P(s - q + r)r$ and $0 \leq \beta \leq 1$.

In the economies we have in mind, both the extraction and the recycling sectors enjoy market power, and so they depart somewhat from the objective of maximizing welfare. The parameter α (β) measures the degree of "intrinsic" competition in the extraction (recycling) sector. Competition is called intrinsic to distinguish it from the extrinsic competition between the two sectors. As α increases, the objective of the extraction sector moves from the benchmark of the intrinsic monopolist ($\alpha = 0$) to the full benevolence benchmark ($\alpha = 1$) in which the extraction sector behaves as a price-taking firm. The case $\alpha = 0$ will allow straightforward comparison with the well-known rule of Hotelling (1931).When $\alpha < 1$, the extraction sector's objective function then accounts for a Leviathan motive similar to that specified by Wirl and Dockner (1995) for the oil market. This is also consistent with real-world features

of the market for an exhaustible resource such as phosphorus, mainly characterized by high concentrations of phosphate reserves in few countries such as Morocco and China (see Cordell et alii, 2009, or Weikard and Seyhan, 2009). Similarly, as β increases, the objective of the recycling sector moves from the Cournot duopoly benchmark which accounts for extrinsic competition between the recycling and the extraction sectors ($\beta = 0$), to the full benevolence benchmark in which the recycling sector is intrinsincally similar to a fringe of small price-taking firms ($\beta = 1$). The case $\beta = 1$ is closely related to Swan (1980)'s study of the market for aluminium, where the monopolist "Alcoa" is confronted by an independent competitive recycling sector (see also Martin, 1982).[9]

We first make the following assumptions:

Assumption 1.

- $c(r)$ is a non-decreasing, twice continuously differentiable and convex function on $[0, q)$ with $c(0) = 0$ and $c(r) = +\infty$ for $r \geq q$.

- $P(q)$ is twice continuously differentiable with $P'(q) < 0$.

- $P(q)$ is log-concave.[10]

- $P(s) > c'(0)$.

- $\lim_{r \to s} W_r^2(s, r) < 0$.[11]

As will be shown later, these conditions guarantee the existence of a recycled amount of the resource. They are very general. Regarding point 3, $LogP$ is concave if P is concave, linear or $P(q) = Aq^{\gamma-1}$ with $0 < \gamma < 1$ so that $1/(1 - \gamma)$ is the elasticity of demand. Most commonly used demand functions are, in fact, log-concave. The limiting case is $P(q) = Ae^{-q}$ which is strictly convex and log-linear (hence log-concave). Point 4 means that the least amount of the recovered resource would be purchased, were it sold at marginal cost. Point 5

---

[9]An alternative interpretation is that the owners of the firms in both sectors are more successful than consumers at exerting political pressure on governments. Under this interpretation, the corporation's interest is represented by a lobby capable of offering political contributions to governments (see Grossman and Helpman, 1994) or capturing the regulatory process. The parameters $1 - \alpha$ and $1 - \beta$ represent the gap between corporations and consumers in terms of lobbying.

[10]$P(q)$ is log-concave if $P''(.)P(.) - P'(.)^2 < 0$.

[11]Partial derivative is denoted by $W_x^i(q, r) \equiv \partial W^i(q, r)/\partial x, x \in \{q, r\}$ throughout the article.

ensures that, when the whole stock is exhausted before recycling, what is worth recycling is also technically recyclable.

Since the first-period resource extraction determines what is left to be sold in the second period, the size of the stock constrains the extraction sector which thus takes no strategic decisions in the second period. The recycling sector observes the first-period extraction q before acting. Thus, a (pure) strategy for the extraction sector is a choice q, and a strategy for the recycling sector is a mapping $R : [0, +\infty) \to [0, +\infty)$. It follows that the equilibrium of the two-period game reduces to a pair (q^*, r^*) of Nash equilibrium with sequential move defined as follows:

Definition 1:

1. $W^1(q^*, R(q^*)) \geq W^1(q, R(q)) \ \forall q \in [0, s]$

2. $W^2(q^*, R(q^*)) \geq W^2(q^*, r) \ \forall r \in [0, s]$

The first step is to derive the subgame reaction function of the recycling sector to the second-period extraction, given the first-period extraction q. Neglect that the recycling sector cannot recover 100 percent of the virgin resource and denote by $\widetilde{r}(q)$ the solution to equation $W_r^2(q, r) = 0$. Under Assumption 1, $\widetilde{r}(q)$ exists and is unique, as shown below. Function $\widetilde{r}(q)$ represents the part of reaction function such as the recycling sector is producing an output strictly lower than q. Now introduce the assumption of recycling shrinkage in the form of $c(r) = +\infty$ for $r \geq q$. The welfare function $W^2(q, r)$ reaches zero at the point $r = q$, and the optimum response for the recycling sector is no longer given by $\widetilde{r}(q)$ if $\widetilde{r}(q) \geq q$. The recycling sector is better off securing zero profit by staying out at the point $\widetilde{q}$ defined implicitily by $\widetilde{r}(q) = q$. Hence, the reaction function of the recycling sector is discontinuous at $\widetilde{q}$, and made up of two parts: one part is zero on the interval $[0, \widetilde{q}]$, and the other part is given by $\widetilde{r}(q)$ on $(\widetilde{q}, s]$. The position of the discontinuity depends on the underlying parameters of demand, cost and intrinsic competition in the two sectors.

The following proposition states the existence of $\widetilde{r}(q)$ and establishes that this function has a positive slope, strictly lower than 1.

Proposition 1 *Given a market for an exhaustible natural resource with inverse demand $P(.)$ and recycling cost $c(r)$ satisfying Assumption 1, there exists a local maximum $\widetilde{r}(q)$ strictly lower than q, that is implicitly defined by*

$$\widetilde{r}(q) = -\frac{P(s-q+r) - c'(r)}{(1-\beta)P'(s-q+r)},$$

such that $0 < \widetilde{r}'(q) < 1$.

Proof: see Appendix 1.

As $\widetilde{r}(q)$ proves to be upward sloping, extracting more resource in the first period induces the recycling sector to produce more in the next period. Prior extraction creates the recycling activity, which yields a perfect substitute to the virgin resource produced by future extraction. Thus, increasing prior extraction of the virgin resource expands the future market share for the recycled substitute, which in turn reduces the future market share for the virgin resource.

To define the overall subgame reaction function of the recycling sector, we now examine the intersection $\widetilde{q}$ between $\widetilde{r}(q)$ and the identity function. As $P(s) > c'(0)$ by assumption, it may happen that $\widetilde{r}(0) = -\frac{P(s+\widetilde{r}(0)) - c'(\widetilde{r}(0))}{(1-\beta)P'(s+\widetilde{r}(0))}$ exceeds 0. Moreover, Point 5 of Assumption 1 implies that $\widetilde{r}(s) = -\frac{P(s) - c'(s)}{(1-\beta)P'(s)} < s$. Since $\widetilde{r}(q)$ is upward sloping, we have $\widetilde{r}(q) < s$ for all $q \le s$. Thus, there may exist $\widetilde{q}$ inside $[0, s]$ such that $\widetilde{q} = \widetilde{r}(\widetilde{q})$. Hence,

$$\widetilde{q} = -\frac{P(s-\widetilde{q}+r) - c'(r)}{(1-\beta)P'(s-\widetilde{q}+r)}. \tag{3}$$

Quantity $\widetilde{q}$ is the minimum threshold of prior extraction above which the recycling strategy can be effective. Indeed, for all $q \le \widetilde{q}$, we have $\widetilde{r}(q) \ge q$. In that case, the best response of the recycling sector to prior extraction is zero due to the impossibility of fully recovering the virgin resource. Formally, the overall subgame reaction function of the recycling sector is

$$R(q) = \left\{ \begin{array}{l} 0, \text{ for } q \le \widetilde{q}, \\ \widetilde{r}(q), \text{ for } \widetilde{q} < q. \end{array} \right. \tag{4}$$

Anticipating the recycling sector's reaction, the extraction sector chooses q to maximize $W^1(q, R(q))$. This function is discontinuous at $\widetilde{q}$ where there is a jump of the same sign as $W^1_r(q, r)$ at $(q, r) = (\widetilde{q}, 0)$. Thus, $W^1(q, R(q))$ is not concave in q, and it may achieve two local maxima, of which one accommodates recycling and the other does not.

Let q^r denote the optimal level of prior extraction that accommodates recycling. This output must satisfy the first-order condition

$$W^1_q(q^r, \widetilde{r}(q^r)) + W^1_r(q^r, \widetilde{r}(q^r))\widetilde{r}'(q^r) = 0, \tag{5}$$

where $\widetilde{r}'(q) > 0$. The total derivative of the extraction sector's welfare in the left-hand side of (5) gives the incentive to extract the resource prior to recycling. It can be decomposed into two effects. The first effect is W^1_q. This is a "balance effect" between the first and the second period: any welfare improvement produced in the first period by the extraction of the virgin resource is offset by a welfare deterioration in the second period. The balance effect would exist even if prior extraction of the resource were not recovered, and therefore recycling could not affect future extraction. The second effect captured by $W^1_r\widetilde{r}'$ is a "strategic effect" that results from the influence of prior extraction on the recycling decision. This dependence of recycling on extraction was pointed out by Judge Hand in the Alcoa case and debated at length in the economic literature.

Further calculations yield

$$W^1_r(q, r) = \delta\left[\alpha P(s - q + r) + (1 - \alpha)P'(s - q + r)(s - q)\right] \tag{6}$$

Observe that $W^1_r(q, r) > 0$ (< 0) when $\alpha = 1$ (0). When the extraction sector is intrinsically competitive, welfare increases with the recycled quantity due to valuable stock extension, whereas the monopoly revenue of the extraction sector decreases with the recycled quantity because the market price decreases in the second period. The right-hand side of (6) shows that the strategic effect of prior extraction through recycling is the sum of two opposite forces when the extraction sector is neither perfectly competitive, nor a pure monopoly ($\alpha \in (0, 1)$).

13

On the one hand, recycling expands the stock of the natural resource sold in the second period, which enhances the second-period revenue of the extraction sector by $\alpha P(.)$ for all the units of resource the recycling sector is selling. Clearly, this revenue enhancing effect exists only if $\alpha > 0$, that is, the extraction sector is somewhat competitive.

On the other hand, recycling puts a downward pressure on the second-period market price, reflected by $(1 - \alpha) P'(.)$, which applies to $s - q$, i. e., all the units of the virgin resource left to be sold by the extraction sector. The negative effect the drop in price has on the extraction sector's revenue occurs only when $\alpha < 1$, that is, the extraction sector has intrinsic market power.

Comparing the perfect competition and imperfect competition cases for (6), it is apparent that the difference is $(1 - \alpha) P'(s - q + r)(s - q)$. It follows that the strategic effect due to the drop in market price is negative since $\widetilde{r}(q)$ is upward sloping. Using the competitive regime as the socially efficient benchmark, one can argue that there is a tendency of an imperfectly competitive resource sector to extract "too little" of the resource prior to recycling.

Proposition 2 *Under Assumption 1, the extraction sector endowed with intrinsic market power accommodates recycling with a level of prior extraction lower than that implied by the socially efficient outcome.*

The second-order condition for q^r is

$$W_{qq}^1(q^r, \widetilde{r}(q^r)) + 2W_{qr}^1(q^r, \widetilde{r}(q^r))\widetilde{r}'(q^r) + W_{rr}^1(q^r, \widetilde{r}(q^r))\widetilde{r}'(q^r)^2 + W_r^1(q^r, \widetilde{r}(q^r))\widetilde{r}''(q^r) < 0. \quad (7)$$

The latter condition shows that existence and uniqueness of q^r can be a problem. Even if a priori we restrict ourselves to demand and cost functions satisfying Assumption 1, the inequality does not necessarily follow.

Proposition 3 *Given a market for an exhaustible natural resource with inverse demand $P(.)$ and recycling cost $c(r)$ satisfying Assumption 1, the level of prior extraction q^r that*

accommodates recycling is a local maximum of $W^1(q, R(q))$ *if*

$$\widetilde{r}'\left(q^r\right) = -\frac{W_{rq}^2(q^r, \widetilde{r}\left(q^r\right))}{W_{rr}^2(q^r, \widetilde{r}\left(q^r\right))} = -\frac{W_q^1(q^r, \widetilde{r}\left(q^r\right))}{W_r^1(q^r, \widetilde{r}\left(q^r\right))}$$

and (7) *holds.*

Proof: see Appendix 2.

Further comparative static analysis proves difficult at the level of generality used so far. To provide a clearer insight into the recycling possibilities,we will specialize the model as follows:

Assumption 2.

- $S(q) = aq - q^2/2$, which yields the demand function $P(q) = a - q$,

- $c(r) = c\frac{r^2}{2}$ on $[0, q)$ with $c(r) = +\infty$ for $r \geq q$,

- $\delta = 1$.

Under Assumption 2, the second-order condition (7) writes

$$-\frac{(1 + c - \beta)}{(2 + c - \beta)^2} \left(4 - 3\alpha + (\alpha - 2)(\beta - c)\right) < 0, \tag{8}$$

which clearly holds whatever c, α and β.

Corollary 4 ?? *Under Assumption 2, if there exists* $q^r \in (\widetilde{q}, s]$ *satisfying* (5), *it is a local maximum of* $W^1(q, R(q))$.

Figure 1 shows how to find a geometric solution to the game with sequential move under Assumption 2. The figure depicts the isowelfare curves and the reaction functions of the two sectors in (q, r) space. The dotted line $q_0 A$ represents the extraction sector's reaction function. This function cuts each of the extraction country's isowelfare curves at its maximum. In particular, W^1 is maximized at the point q_0, given $r = 0$, hence q_0 is the optimal output when the extraction sector exercises unrestrained market power, thereby ignoring the recycling

sector. Holding q fixed, the extraction sector does better when r is lower provided that $W_r^1(q, r) < 0$, which writes $r > a + \frac{q-s}{\alpha}$ under Assumption 2. This inequality holds for sufficiently low values of α, in which case lower isowelfare curves represent higher welfare levels for the extraction sector. Whether the extraction sector's reaction function is upward-sloping or downward-sloping depends on the sign of $-\frac{W_{qq}^1}{W_{qr}^1}$, which is equal to $2(2 - \alpha)(> 0)$ under Assumption 2. The recycling sector's reaction function $R(q)$ is made up of the two segments $[0, \widetilde{q}]$ and $(B, C]$. In the figure, the isowelfare curve for the extraction sector is tangent to $(B, C]$ at M, and this curve meets the q-axis at $\overline{q}$. Output q^r on the q-axis vertically below M is the optimal extraction that accommodates recycling. The figure illustrates the case where q^r lies to the right of q_0, which occurs provided that $\widetilde{r}(q)$ is not very steep. The calculations made in the proof of Proposition 1 produce

$$\widetilde{r}'(q) = -\frac{W_{rq}^2(.)}{W_{rr}^2(.)} = \frac{P'(.) + (1 - \beta)P''(.)r}{(2 - \beta)P'(.) + (1 - \beta)P''(.)r - c''(.)}, \tag{9}$$

which reduces to $\widetilde{r}'(.) = \frac{1}{2+c-\beta}$ under Assumption 2. This suggests that $c''(.)$ must be sufficiently high and/or β sufficiently low to have $q_0 < q^r$. Moreover, from (3), the position of the point $\widetilde{q}$ where $R(q)$ is discontinuous, both depends on $c'(r)$ and β. If $c'(r)$ is so high and/or β is so low that $\widetilde{q}$ lies to the left of $\overline{q}$ as depicted in the figure, the best choice for the extraction sector is q^r, and therefore the extraction sector accommodates recycling. By contrast, it may happen that $c'(r)$ is so low and/or β is so large that $\widetilde{q}$ lies to the right of q_0. Then, the extraction sector finds it worthwhile to extract q_0 and ignore recycling. In the intermediate case where $\widetilde{q}$ lies between $\overline{q}$ and q_0, the extraction sector prefers extracting the level $\widetilde{q}$ of virgin resource over q^r to get the highest possible welfare, which prevents recycling. This is a deterring strategy in the sense that the extraction sector keeps its first-period output lower than the level q_0 of virgin resource that would be optimally extracted without recycling. If $\widetilde{q}$ is the optimal level of prior extraction, then the first-period price $P(\widetilde{q})$ exceeds the price $P(q_0)$ charged by the extraction sector with unrestrained market power, and residual demand in the second period results from $P(s - \widetilde{q})$, which leaves no profits to the recycling sector. As

16

$P(s - \widetilde{q})$ falls short of $P(s - q_0)$, $P(s - \widetilde{q})$ is similar to a limit price that deters the entry of the recycling sector.
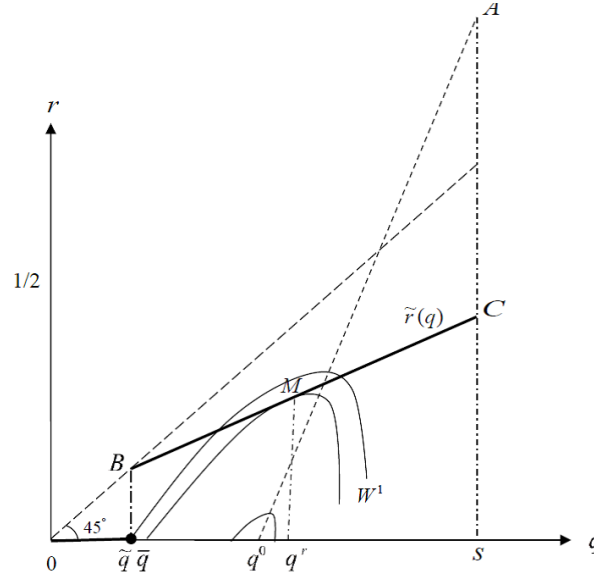

Figure 1

This discussion leads to the following classification of recycling possibilities.

Corollary 5 *Depending on the position of $\widetilde{q}$, the extraction sector has three different strategies in the face of recycling:*

(1) $q_0 < \widetilde{q}$. Prior extraction is q_0. The extraction sector ignores recycling.

(2) $\overline{q} < \widetilde{q} < q_0$. Prior extraction is $\widetilde{q}$. The extraction sector deters recycling.

(3) $\widetilde{q} < \overline{q}$. Prior extraction is q^r. The extraction sector accommodates recycling.

In the classification above, Case 1 is the standard situation in which the best policy for the extraction sector is to produce in the first period as if recycling were irrelevant. Cases 2 and 3 show how this standard behavior is modified in a strategic way designed to influence recycling. In Case 2, the possibility of recycling induces the extraction sector to reduce the level of prior extraction and correspondingly charge a second-period limit price that deters recycling. By contrast, in case 3, the extraction sector chooses the level of prior extraction so as to accommodate the recycling business.

17

3 The socially efficient solution ($\alpha = \beta = 1$)

We now restrict attention to the socially efficient solution in which the extraction and the recycling sectors both behave as price-taking firms. The recycling sector maximizes the following welfare function

$$W^2(q, r) = S(s - q + r) - c(r). \tag{10}$$

In the second period, the recycling sector chooses quantity r given quantity q previously chosen by the extraction country, so that market price equals the marginal cost of recycling

$$P(s - q + r) = c'(r). \tag{11}$$

From Proposition 1, we know that the second-order condition for the recycling sector's maximization problem is satisfied under Assumption 1.

In the first period, the extraction sector problem chooses q to maximize

$$W^1(q, R(q) = S(q) + \delta S(s - q + R(q)). \tag{12}$$

Let q_e^r denote the socially efficient extraction that accommodates recycling. The first-order condition at the local maximum for q_e^r is

$$P(q_e^r) - \delta P(s - q_e^r + \widetilde{r}(q_e^r)) = -\delta P(s - q_e^r + \widetilde{r}(q_e^r))\widetilde{r}'(q_e^r). \tag{13}$$

As previously seen, the welfare effect of prior extraction can be decomposed into the balance effect captured by the left-hand side of (13) and the indirect welfare effect due to recycling reflected by the right-hand side of (13). In this section, the latter effect is not called "strategic" as previously, because the price-taking extraction sector is not meant to play strategically. Condition (13) can be interpreted as a variant of the "Hotelling rule" for a non-renewable and recyclable resource. Indeed, this condition tells that the extraction sector must be indifferent between selling a unit of resource today or tomorrow, given that the tomorrow resource is both extracted and recycled. As the natural stock size s is increased by the recycled amount

$\widetilde{r}(q_e^r)$ in the second period, the value $P(q_e^r)$ of a unit of resource extracted in the first period must be the same as the present value $\delta P(s - q_e^r + \widetilde{r}(q_e^r))$ of a unit of resource sold in the second period, corrected by the recycling effect $\delta P(s - q_e^r + \widetilde{r}(q_e^r))\widetilde{r}'(q_e^r)$. Clearly, this is the spirit of Hotelling rule. As $\widetilde{r}(q)$ is upward sloping, the second-period welfare is improved by $P(.)\widetilde{r}'(.)$ because recycling creates a valuable extension of the resource stock. As a result, recycling increases prior extraction in the context of perfect competitition.

Proposition 6 *Under Assumption 1, recycling pushes up the socially efficient level of prior extraction.*

To obtain further insight into the existence and social desirability of recycling, we now turn to the specialized version of the model given by Assumption 2. The minimum threshold for recycling to be effective is then $\widetilde{q}_e = \frac{a-s}{c}$. Solving (11) for $\widetilde{r}(q)$, we obtain the reaction function of the recycling sector

$$R(q) = \begin{cases} 0 \text{ if } q \leq \widetilde{q}_e, \\ \frac{a-s+q}{1+c} \text{ if } \widetilde{q}_e < q. \end{cases} \tag{14}$$

Furthermore, solving (13) for q_e^r yields

$$q_e^r = \min\{\frac{a(1+2c)+sc^2}{1+2c+2c^2}, s\}. \tag{15}$$

The extraction sector chooses q to maximize

$$W^1(q, R(q)) = \begin{cases} aq - q^2/2 + [a(s-q) - (s-q)^2/2] \text{ if } q \leq \widetilde{q}_e, \\ aq - q^2/2 + [a(s-q+(a-s+q)/(1+c)) - (s-q+(a-s+q)/(1+c))^2/2] \text{ if } \widetilde{q}_e < q \end{cases} \tag{16}$$

This function is piecewise concave with an upward jump at $\widetilde{q}_e$ whenever $\widetilde{q}_e < s$, which occurs for parameter values such that $c > \frac{a-s}{s}$. If, moreover, $c \leq 2\frac{a-s}{s}$, the function achieves two local maxima: one maximum is $q_0^e = \frac{s}{2}$, which proves lower than $\widetilde{q}_e$, and thus precludes recycling; the other one is s, which accommodates recycling. The comparison between $W^1(q_0^e, 0)$ and $W^1(s, \widetilde{r}(s))$ shows that the extraction sector is better off choosing s[12]. If $2\frac{a-s}{s} < c$, then

---

[12]Indeed, calculations show that $W^1(s, \widetilde{r}(s)) - W^1(q_0^e, 0) = \frac{a^2(2+4c)-(1+c)^2 s^2}{4(1+c)^2} > 0$ when $c \leq 2\frac{a-s}{s}$.

$W^1(q, R(q))$ has a unique maximum either at s or at $q_e^r < s$, with $\widetilde{q}_e < q_e^r$, which therefore accommodates recycling. Lastly, if $c \leq \frac{a-s}{s}$, recycling is not feasible whatever the resource quantity extracted in the first period, and so q_0^e is the unique maximum.

The next proposition summarizes this result.

Proposition 7 *Under Assumption 2, if $cs > a - s$, the socially efficient solution is to set q_e^r, thereby accommodating recycling; otherwise, it is socially efficient to ignore recycling and set q_0^e.*

Recycling is socially desirable only if the marginal cost cs of recycling the whole stock of the resource exceeds the marginal social value $a - s$ of this stock. That is the case when the resource is abundant enough. If, conversely, the resource is too scarce, then the shrinkage of the virgin resource during the recovering process makes recycling worthless given the available technology. Indeed, inequality $cs \leq a - s$ means that social efficiency would require to recover more than the available amount of the resource stock and sell it at the marginal cost of recycling. Since this is technically impossible, it is socially efficient to ignore recycling.

4 A monopolist in the resource extraction ($\alpha = 0$)

In this section, we focus on a situation in which a monopolist in the resource extraction is confronted by an independent recycling sector with a degree β of intrinsic competition. This will provide useful comparison with the monopoly analysis of a non-recyclable exhaustible resource in Stiglitz (1976).

The recycling sector maximizes the welfare function given by (2). From Proposition 1, there exists a unique solution to this problem. The recycling sector's reaction function to prior extraction is given by (4).

When the extraction sector's problem in the first period is

$$\max_q P(q)q + \delta\left[P(s - q + R(q))(s - q)\right] \tag{17}$$

Assume the existence of a local maximum q_m^r that accommodates recycling. Focusing on q_m^r, the first-order condition for the maximization program above is given by

$$P(q_m^r) + P'(q_m^r)q_m^r - \delta P(s - q_m^r + \widetilde{r}(q_m^r)) + P'(s - q_m^r + \widetilde{r}(q_m^r))(s - q_m^r)) \quad (18)$$

$$= -\delta P'(s - q_m^r + \widetilde{r}(q_m^r))\widetilde{r}'(q_m^r)(s - q_m^r).$$

Condition (18) has a familiar interpretation in the economics of exhaustible resources (see for instance Stiglitz, 1976). The extraction monopolist compares the marginal revenue today with the discounted marginal revenue he would obtain by postponing the extraction until tomorrow. The difference with previous literature is that recycling the resource both augments the stock size and gives rises to a perfect substitute for further quantity of extracted resource. The left-hand side of (18) measures the aforementioned balance effect of prior extraction on the expected revenue in both periods, given that the available stock is $s + \widetilde{r}(q_m^r)$. Were this effect set equal to zero, it would correspond to the Hotelling rule in the case investigated by Stiglitz (1976), where the monoply power is unrestrained by recycling. Bearing in mind the possibility of recycling, the monopolist strategically anticipates the impact of prior extraction on the competition between recycling and further extraction. As previously shown, this strategic effect strengthens competition in the second period, because $R(q)$ is upward sloping: increasing prior extraction triggers a more agressive reaction by the recycling sector, which reduces the second-period price by $P'(.)\widetilde{r}'(.)$. The resulting downward pressure on price scales down the second-period marginal revenue of the extraction sector. This provides the monopolist with an incentive to look "friendly" from the start and extract less resource than the Hotelling rule would require in the absence of strategic effect. Such a strategy has the flavor of the so-called "puppy-dog" profile in the terminology of business strategies (see Fudenberg and Tirole, 1984). The extraction sector commits the recycling sector to soften competition between recycling and further extraction. However, the "puppy-dog" strategy obeys here the unescapable logic of the extraction rule that the marginal revenue must rise at the rate of interest.

Proposition 8 *Under Assumption 1, the possibility of recycling reduces the level of prior extraction set by the monopolist to accommodate recycling.*

Let us now investigate the model under Assumption 2. The minimum threshold for recycling to be effective is $\widetilde{q}_m = \frac{a-s}{1+c-\beta}$. There are some degrees of intrinsic competition in the recycling sector of particular interest to our analysis. Table 1 provides notations and Figure 2 displays the degrees of intrinsic competition in (c, β) space.

Table 1	**Degrees of intrinsic competition in the recycling sector**
β_f	maximum degree of intrinsic competition for which recycling is feasible (subscript f means *feasible recycling*)
β_d	maximum degree of intrinsic competition for which prior extraction deters recycling (subscript d means *deterring recycling*)
β_m	maximum degree of intrinsic competition for which recycling arises from a maximum (subscript m means *recycling at a maximum*)
β_a	maximum degree of intrinsic competition for which recycling is accommodated (subscript a means *accommodating recycling*)

where $\beta_f = c + \frac{2s-a}{s}, \beta_d = c + \frac{3s-2a}{s}, \beta_m = c + \frac{8s-3a-\sqrt{a(9a-8s)}}{4s}$ and $\beta_a = c + \frac{(2\sqrt{2}+10)s - (2\sqrt{2}+5)a - s\sqrt{(20\sqrt{2}+3}}{4s}$

Under Assumption 2, the recycling sector's objective function is

$$W^2(q, r) = \beta\left[a(s - q + r) - (s - q + r)^2/2 - (a - s + q - r)r\right] + (a - s + q - r)r - cr^2/2, \quad (19)$$

which yields the following reaction function

$$R(q) = \begin{cases} 0 \text{ if } q \leq \widetilde{q}_m, \\ \frac{a-s+q}{2+c-\beta} \text{ if } \widetilde{q}_m < q. \end{cases} \quad (20)$$

Furthermore, solving (18) for q_m^r yields

$$q_m^r = \min\{\frac{a + 2s(1 + c - \beta)}{2(3 + 2c - 2\beta)}, s\}. \quad (21)$$

The extraction sector chooses q to maximize

$$W^1(q, R(q)) = \begin{cases} (a - q)q + (a - s + q)(s - q) \text{ if } q \leq \widetilde{q}_m, \\ (a - q)q + (1 + c - \beta)(s - q)(a - s + q)/(2 + c - \beta) \\ \qquad\qquad \text{if } \widetilde{q}_m < q. \end{cases} \quad (22)$$

Function (22) may achieve a local maximum at q_0^m, which precludes recycling provided that $q_0^m \leq \widetilde{q}_m$. Recall that prior extraction at q_0^m is the optimal output of the monopolist unrestrained by recycling. It turns out that q_0^m is not higher than $\widetilde{q}_m$ when $\beta \geq \beta_d$, otherwise

22

$W^1(q, 0)$ is increasing at $\widetilde{q}_m$, where it jumps downward to $W^1(\widetilde{q}_m, \widetilde{r}(\widetilde{q}_m))$[13]. Hence, β_d is the maximum degree of intrinsic competition for which prior extraction deters recycling rather than ignoring it, in the sense that the monopolist makes recycling unprofitable by keeping its output at $\widetilde{q}_m$ lower than q_0^m.

Function (22) may also achieve a local maximum at q_m^r provided that $q_m^r \in (\widetilde{q}_m, s)$. This interval is not empty as long as $\widetilde{q}_m < s$, which occurs for all $\beta < \beta_f$. If, conversely, $\beta \geq \beta_f$, $W^1(q, R(q))$ reaches a unique maximum at q_0^m because $\beta_d < \beta_f$ for a given c, as illustrated in Figure 2. Hence, β_f is the maximum degree of intrinsic competition for which recycling proves feasible. On the other hand, we have that $\widetilde{q}_m < q_m^r$ for all $\beta < \beta_m$. Straightforward calculations show that, for all $\beta < \beta_m$, we also have $q_m^r < s$. In addition, one can easily check that $\beta_f > \beta_m$ when $a > s$. Thus, for all $\beta < \beta_m$, q_m^r is inside the non-empty interval $(\widetilde{q}_m, s)$, and therefore β_m is the maximum degree of intrinsic competition for which recycling arises from a maximum.

Further calculations yield that $\beta_d < \beta_m < \beta_f$ for a given c, as depicted in Figure 2. It follows that function (22) achieves two local maxima when $\beta < \beta_m$. Consider first that $\beta \in [\beta_d, \beta_m]$. One local maximum is q_0^m and the other one is q_m^r. It turns out that $W^1(q_0^m, 0) > W^1(q_m^r, \widetilde{r}(q_m^r))$ provided that β falls short of $c - \frac{a^2 - 8as + 4s^2}{2s(2a-s)}$ which proves higher than β_m. Thus, the extraction sector is better off choosing q_0^m when $\beta \in [\beta_d, \beta_m]$, which precludes recycling. Consider now that $\beta < \beta_d$. It is still true that function (22) achieves two local maxima, of which one is q_m^r, but the other one is now $\widetilde{q}_m$. Equation $W^1(\widetilde{q}_m, 0) - W^1(q_m^r, \widetilde{r}(q_m^r)) = 0$ has four solutions in β, only one of which proves lower than β_d, that is, β_a. For all $\beta \in [\beta_a, \beta_d]$, the recycling sector prefers $\widetilde{q}_m$ to q_m^r, thereby deterring recycling. On the contrary, for all $\beta \in [0, \beta_a)$, the recycling sector prefers to accomodate recycling by choosing q_m^r rather than $\widetilde{q}_m$.

Lastly, we turn to the case where $\beta \in [\beta_m, \beta_f]$. Then, function (22) achieves a local maximum at q_0^m, jumps downward to $W^1(\widetilde{q}_m, \widetilde{r}(\widetilde{q}_m))$ at $\widetilde{q}_m$, and decreases in $q > \widetilde{q}_m$. Thus,

---

[13]One can check that $W_r^1(\widetilde{q}, 0) = \frac{a - s(2 + c - \beta)}{1 + c - \beta} < 0$ only if $\beta < \beta_f$ (or equivalently, $\widetilde{q} < s$, as shown below).

we have $W^1(q_0^m, 0) > W^1(\widetilde{q}_m, \widetilde{r}(\widetilde{q}_m))$ in this parameter configuration, and q_0^m is again the optimal choice for the extraction sector.

The next proposition summarizes the previous discussion.

Proposition 9 *Under Assumption 2, the monopolist sets the level of prior extraction at:*

(1) q_0^m and ignores recycling when $\beta \in [\beta_d, 1]$;

(2) $\widetilde{q}_m$ and deters recycling when $\beta \in [\beta_a, \beta_d)$;

(3) q_m^r and accommodates recycling when $\beta \in [0, \beta_a)$.

We can now compare the monopolist's optimal behavior to the socially efficient outcome stated in Proposition 7 for every parameter configuration. Not surprisingly, the best extraction strategy for the monopolist is never efficient. From Proposition 2, we already know that the monopolist extracts an inefficiently low level of the resource when the monopolist optimally accommodates recycling. This in fact occurs when the recycling sector is poorly competitive (Case 3). In that case, one can easily check that $q_m^r < q_e^r$: compared to the socially efficient solution, the monopolist adopts a puppy-dog strategy to relax future competition with the recycling sector. Confronted with a more competitive recycling sector (Cases 1 and 2), the monopolist chooses prior extraction in a way designed to discourage recycling. In order to achieve this purpose, the extraction sector commits to a low level of prior extraction which both increases the first-period price and decreases the second-period price, so that the prospective profit of the recycling sector falls down to zero. For the highest degrees of intrinsic competition in the recycling sector (Case 1), the best extraction strategy is to maintain the first-period output at the same level whether or not recycling is possible, thereby producing q_0^m which is then strictly below the limit quantity $\widetilde{q}_m$. For intermediate degrees of intrinsic competition in the recycling sector (Case 2), $\widetilde{q}_m$ then falls below q_0^m, and thus setting q_0^m would now allow recycling. In that case, the monopolist finds it more profitable to make a preemptive commitment at $\widetilde{q}_m$ and deter recycling, which is an inefficient strategy whether or not recycling is socially desirable.

5 Conclusion

In this paper, we have proposed a theoretical approach to the recycling possibility on the market of an exhaustible resource, which encompasses various regimes of competition in two independent sectors respectively devoted to extraction and recycling. The extraction sector divides the resource stock between two periods, facing competition from the recycling sector in the second period. Hence, the first-period extraction determines what can be possibly recycled thereafter, under the constraint that the virgin resource cannot be fully recovered. As the extraction sector creates its own competition whenever recycling proves feasible, the choice of the level of prior extraction is of great strategic importance.

Our analysis sheds light on three strategic motives of the extraction sector bearing in mind the reaction of the recycling sector. The extraction sector chooses prior extraction in a way that either accommodates, ignores or deters recycling, depending on the underlying parameters of demand, cost and intrinsic competition in each sector.

When the extraction sector finds it optimal to accommodate recycling, a novel version of Hotelling's arbitrage rule obtains. It states that the traditional balance of the marginal revenues between the first and the second period must be corrected by the strategic impact of recycling. In the first period, the extraction sector anticipates how the recycling sector will respond in quantity to the initial extraction of the virgin resource. It turns out that the recycling reaction function is upward sloping, regardless of the degree of intrinsic competition in the recycling sector. This results in a valuable extension of the resource stock when both the extraction and the recycling sectors are perfectly competitive, and so recycling increases the socially efficient level of prior extraction. In contrast, when the extraction sector is imperfectly competitive, recycling increases the intensity of competition between the two sectors. Then, the extraction sector makes the recycling sector's reaction less aggressive and softens competition by reducing prior extraction. This puppy-dog strategy results in an inefficiently low level of prior extraction.

Instead of accommodating recycling, the extraction sector may also want to prevent re-cycling. For this purpose, the extraction sector ought to commit to levels of prior extraction that push the second-period price down enough that the recycling sector stays out of business. We show that the monopolist in the extraction sector makes such preemptive commitments when the degree of intrinsic competition in the recycling sector is sufficiently high. Then, either the monopolist deters recycling with a limit output lower than would obtain without recycling, or the monopolist behaves as if recycling were irrelevant when intrinsic competition in the recycling sector is very tough.

6 Appendix

6.1 Appendix 1: Proof of Proposition 1

The objective function of the recycling sector is

$$W^2(q,r) = \beta\left[S(s-q+r) - P(s-q+r)r\right] + P(s-q+r)r - c(r) \tag{23}$$

The recycling sector's marginal welfare is

$$W_r^2(q,r) = P(s-q+r) + (1-\beta)P'(s-q+r)r - c'(r), \tag{24}$$

If the equation $W_r^2(q,r) = 0$ has an interior solution $\tilde{r}(q)$, then $\tilde{r}(q) = -\frac{P(s-q+r)-c'(r)}{(1-\beta)P'(s-q+r)}$.

Further calculations yield

$$W_{rr}^2(q,\tilde{r}(q)) = (2-\beta)P'(s-q+\tilde{r}(q)) + (1-\beta)P''(s-q+\tilde{r}(q))\tilde{r}(q) - c''(\tilde{r}(q)) \tag{25}$$

Substitute $\tilde{r}(q) = -\frac{P(s-q+r)-c'(r)}{(1-\beta)P'(s-q+r)}$ in the expression above.

$$
\begin{aligned}
W_{rr}^2(q,\tilde{r}(q)) &= (2-\beta)P'(s-q+\tilde{r}(q)) - (1-\beta)\frac{P(s-q+\tilde{r}(q))-c'(\tilde{r}(q))}{P'(s-q+\tilde{r}(q))}P''(s-q+\tilde{r}(q)) - c''(\tilde{r}(q)) \tag{26}\\
&= (1-\beta)P'(.) - \frac{P''(.)P(.) - P'^2(.)}{P'(.)} - c''(r) + \frac{c'(.)}{P'(.)}P''(.) \tag{27}
\end{aligned}
$$

As $P(.)$ is log-concave, we know that $P''(.)P(.) - P'(.)^2 < 0$. Thus,

$$W_{rr}^2(q,\tilde{r}(q)) < (1-\beta)P'(.) - \frac{P''(.)P(.) - P'^2(.)}{P'(.)} - c''(r) + \frac{c'(.)}{P'(.)}\frac{P'^2(.)}{P(.)} < 0, \tag{28}$$

and so, $\tilde{r}(q)$ is a local maximum.

Furthermore, differentiating $W_r^2(q,\tilde{r}(q)) = 0$, we get

$$\tilde{r}'(q) = -\frac{W_{rq}^2(.)}{W_{rr}^2(.)} = \frac{P'(.) + (1-\beta)P''(.)r}{(2-\beta)P'(.) + (1-\beta)P''(.)r - c''(.)} \tag{29}$$

Substitute $\tilde{r}(q) = -\frac{P(s-q+r)-c'(r)}{(1-\beta)P'(s-q+r)}$ in the expression above. We know that $W_{rr}^2(q,\tilde{r}(q)) < 0$.

Moreover, using the log-concavity of $P(.)$, we obtain

$$W_{rq}^2(q,\tilde{r}(q)) = (-P'^2(.) + P''(.)P(.) - c'(.)P''(.))/P'(.) > 0 \tag{30}$$

It follows that $\tilde{r}'(q) > 0$. Further calculations yield

$$\tilde{r}'(q) - 1 = \frac{(\beta-1)P'(.) + c''(.)}{(2-\beta)P'(.) + (1-\beta)P''(.)r - c''(.)}, \tag{31}$$

which turns to be strictly negative due to the log-concavity of $P(.)$.

6.2   Appendix 2: Proof of Proposition 3

The extraction sector's objective function is the discounted welfare equal to

$$W^1(q,r) = \alpha(S(q)-P(q)q)+P(q)q+\delta\left[\alpha(S(s-q+r)-P(s-q+r)(s-q))+P(s-q+r)(s-q)\right]$$

(32)

The first-order condition for an interior equilibrium q^r is given by (5). Together with(29),

equation (5) yields

$$\tilde{r}'\left(q^r\right) = -\frac{W_q^1(q^r,\tilde{r}\left(q^r\right))}{W_r^1(q^r,\tilde{r}\left(q^r\right))} = -\frac{W_{rq}^2(q^r,\tilde{r}\left(q^r\right))}{W_{rr}^2(q^r,\tilde{r}\left(q^r\right))}.$$

(33)

The second-order condition at q^r is

$$W_{qq}^1(q^r,\tilde{r}\left(q^r\right))+W_{rq}^1(q^r,\tilde{r}\left(q^r\right))\tilde{r}'\left(q^r\right)+W_{qr}^1(q^r,\tilde{r}\left(q^r\right))\tilde{r}'\left(q^r\right)+W_{rr}^1(q^r,\tilde{r}\left(q^r\right))\tilde{r}'\left(q^r\right)+W_r^1(q^r,\tilde{r}\left(q^r\right))\tilde{r}''\left(q^r\right) < 0,$$

(34)

which amounts to (7).

References

[1] **Cordell, D., Drangert J.-O. and S. White**. 2009. The Story of Phosphorus: Global Food Security and Food for Thought, *Global Environmental Change*, 19, 292–305

[2] **Dasgupta, P**. 1993. Natural Resources in an Age of Substituability, in Kneese, A. V. and J. L. Sweeney (eds) *Handbook of Natural resource and Energy Economics*, Vol. 3, Elsevier science, Amsterdam, chapter 23.

[3] **Ensink, J.H.J., Mahmood, T., Hoek, W.v.d., Raschid-Sally, L. and F. P. Amerasinghe.** 2004. A Nationwide Assessment of Wastewater Use in Pakistan: an Obscure Activity or a Vitally Important One?, *Water Policy*, 6, 197–206.

[4] **Fisher, F. and R. Laxminarayan.** 2004. Monopoly Extraction of an Exhaustible Resource with Two Markets, *Canadian Journal of Economics / Revue canadienne d'Economie*, 37, 1, 178-188.

[5] **Fudenberg, D and J. Tirole.** 1984. The Fat-Cat Effect, the Puppy-Dog, and the Lean and Hungry Look, *American Economic Review*, 74, 2, 361-366.

[6] **Friedman, M.** 1967. Price Theory. Aldine Publishing Co., Chicago.

[7] **Gaudet, G. and N. Van Long.** 2003. Recycling Redux: A Nash–Cournot Approach, *Japanese Economic Review*, 54, 4, 409–419.

[8] **Grant, D.** 1999. Recycling and Market Power: A More General Model and Re-Evaluation of the Evidence, *International Journal of Industrial Organization*, 17, 59–80.

[9] **Grossman, G. and E. Helpman.** 1994. Protection for Sale, *American Economic Review*, 84, 833-850.

[10] **Hoel, M.** 1978. Resource Extaction, Substitute Production, and Monopoly, *Journal of Economic Theory*, 19, 28-37.

[11] **Hollander, A. and P. Lasserre.** 1988. Monopoly and the Pre-Emption of Competitive Recycling, *International Journal of Industrial Organization*, 6, 489–497.

[12] **Hotelling, H.** 1931. The Economics of Exhaustible Resources, *Journal of Political Economy*, 39, 2, 137-175.

[13] **Lewis, T. R.** 1975. Monopoly Exploitation of an Exhaustible Resource, *Journal of Environmental Economics and Management*, 3, 198-204.

[14] **Lewis, T. R. and R. Schmalensee.** 1980. On Oligopolistic Markets for Nonrenewable Natural Resources, *Quarterly Journal of Economics*, 95, 3, 475-491.

[15] **Martin, R.** 1982. Monopoly Power and the Recycling of Raw Materials, *Journal of Industrial Economics*, 30, 4. , 405-419.

[16] **Stewart, M. B.** 1980. Monopoly and the Intertemporal Production of a Durable Extractable Resource, *Quarterly Journal of Economics*, 94, 1, 99-111.

[17] **Smith, V.L.** 1972. Dynamics of Waste Accumulation: Disposal versus Recycling, *Quarterly Journal of Economics*, 86, 4, 600-16.

[18] **Stiglitz, J.** 1976. Monopoly and the Rate of Extraction of Exhaustible Resources, *American Economic Review*, 66, 4. , 655-661.

[19] **Suslow, V.** 1986. Estimating Monopoly Behavior with Competitive Recycling: An application to Alcoa, *RAND Journal of Economics*, 17, 3, 389-403.

[20] **Swan, P. L**. 1980. The Influence of Recycling on Monopoly Power, *Journal of Political Economy*, 88, 1, 76-99.

[21] **Tullock, G**. 1979. Monopoly and the Rate of Extraction of Exhaustible Resources, *American Economic Review*, 69, 1, 231-233.

[22] **Weikard, H. P. and D. Seyhan**. 2009. Distribution of Phosphorus Resources Between Rich and Poor Countries: The effect of Recycling, *Ecological Economics*, 1749-1755.

[23] **Wirl, F. and E. Engelbert**. 1995. Leviathan Governments and Carbon Taxes: Costs and Potential Benefits, *European Economic Review*, 39, 1215-1236.