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Phosphorus Conservation, Eutrophication Reduction and Social Welfare Improvement: Taxation of Extracted Phosphorus or Subsidy of Recycled Phosphorus?

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Abstract

This paper aims at analyzing the role of an environmental tax or an environmental subsidy as instruments for preserving phosphate reserves, for improving water quality by reducing eutrophication, and for increasing social welfare. Toward these goals, we use a duopoly model "à la Stackelberg", assume the presence of a benevolent government that takes into account the beneficial effect of recycling in the social welfare function and refunds the revenue of the tax to the society. First, we find that taxing extracted phosphorus or subsidizing recycled phosphorus contributes to the postponement of the depletion of the resource and to the reduction of eutrophication. Second, we find that taxing extracted phosphorus reduces consumers'surplus, whereas subsidizing recycled phosphorus increases it. Third, we show that the subsidy set by the regulator is higher than the marginal benefit of recycling, whereas the level of the tax with respect to the marginal social damage of pollution is ambiguous. Fourth, we state that the tax and the subsidy increase social welfare. Fifth, by way of comparison, we find that if the regulator aims at saving phosphorus, reducing eutrophication and improving social welfare simultaneously, subsidizing recycled phosphorus is the best policy, but if he aims only at saving phosphorus and reducing eutrophication, taxing extracted phosphorus is more optimal than subsidizing recycled phosphorus.

JEL Classification:

Keywords: Environmental Tax, Subsidy, Phosphorus, Eutrophication, Recycling, Competition "à la Stackelberg".

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1 Introduction

A basic economic insight is that a competitive economy, under ideal conditions, will generate a socially efficient or a Pareto optimal allocation of private goods, meaning that it is not possible to reallocate resources in such a way that everyone becomes better off (Sandmo, 2003). One element of the ideal conditions requirement is the absence of external effects. In other words, if one agent generates externalities, the allocation is no longer socially optimal if the market is not regulated. Externalities may be both positive or negative. In this paper, we focus on both types of externalities because extracted phosphorus creates eutrophication phenomenons by polluting water, whereas recycling prevents phosphorus from polluting water. Accordingly, extraction generates a negative externality, while recycling generates a positive externality. Since Pigou (1920), it is well known that negative externalities caused by pollution would be internalized by the market if polluters paid a tax equal to the marginal social cost of polluting emissions (Nimubona and Sinclaire-Desgagné, 2005), while several economists stress that it would be desirable to subsidize a polluter in order to induce him to abate pollution or to subsidize green products, which generate a positive externality.

In this paper, the polluter is associated with a firm that extracts phosphorus, whereas the environmentally friendly firm refers to the recycler, as mentioned above. It is widely recognized that unrecycled phosphorus[1] pollutes waters. In fact, primary[2] phosphorus ends up into the water due to water run-off, soil erosion, drainage from agricultural land, excreta from livestock, municipal and industrial effluents. Consequently, this situation creates eutrophication phenomenons. As stated above, recycling[3] will reduce water pollution by preventing phosphorus from ending up into the water. In addition to the reduction of eutrophication, recycling contributes to create energy and to save phosphorus. For instance, Li and al. (2015) argue that the implementation of a factory in China (in Chongqing town), which treats wastewater and sludge, allows to save $67,000$ KWh per day and enables to generate 1 tonne of phosphorus per day. Recall that the term "Eutrophication" refers to an unwanted explosion of living aquatic-based organisms in lakes and estuaries that results in oxygen depletion, which can destroy an aquatic ecosystem (Liu and al., 2008). Significant eutrophication took place in the $1950s$ in the Great Lakes of North America, in Cayuga Lake, which is in Central New York (Jacobs and Casler, 1979), in the Poyang Lake watershed that is in China (Deng and al, 2011), in the Norfolk broads of United Kingdom (Philipps, 1984). It has been also prevalent in many

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[1] There are other elements like nitrogen, carbon and trace, which create eutrophication (Lee, 1973).

[2] We distinguish primary phosphorus to secondary phosphorus (which is also called green phosphorus). The former refers to extracted phosphorus, whereas the latter corresponds to recycled phosphorus. It is taken as green phosphorus because it reduces water pollution.

[3] It is noteworthy to mention that if recycled phosphorus ends up into the water it yields the same effect which is triggered by extracted phosphorus. But to focus on the benefit of recycling in the reduction of eutrophication, we assume, in the world of this model, that recycled phosphorus does not end up into the water after its consumption.

other lakes and estuaries around the world (International Lake Environment Committee Foundation, 2003).

One way to reduce[4] eutrophication consists of taxing virgin phosphorus or subsidizing recycled phosphorus, as mentioned above. This would diminish (increase) extracted phosphorus (recycled phosphorus) and would reduce (boost) extracted phosphorus (recycled phosphorus). Owing to the strategic substitutability of both types of phosphorus, the increase of one triggers the decrease of the other, and vice versa. Aiming at reducing eutrophication, taxation of extracted phosphorus has been applied in some countries, including the United States of America[5] (see Jacobs and Casler, 1979; Shakhramanyan and al**.**, 2012) and China. As well as reducing pollution of waters, taxation or subsidy would contribute to prolong the lifetime of phosphorus. The exhaustion[6] of which is predicted in a near future. Although subsidy can be seen as a cost for the government in question, taxation would give him the opportunity to collect some funds in order to finance several goals (see Gersbach and Requate, 2004).

The consideration of positive or negative external effects in the decisions of production of the firms and in the social welfare function generates a number of interesting questions. Does the tax or the subsidy contribute to the prolongation of the lifetime of phosphorus and the reduction of water pollution ? Is the level of the tax (or of the subsidy) set above or below the marginal social damage of pollution (the marginal social benefit of recycling) ? What is the effect of each of these policies on consumers' surplus and on social welfare ? Is taxation more optimal in saving phosphorus and reducing eutrophication than subsidy ? What is the best policy in terms of the improvement of social welfare ? In the present paper, we address these and related questions.

In connection with the questions above, the aim of this paper is first to analyze the effect of the tax or the subsidy on the depletion of phosphorus and on water pollution. Second, we will see what is the level of the tax or of the subsidy, respectively with respect to the marginal social damage of pollution and with respect to the marginal social benefit of recycling. Third, we aim at comparing the two policies in terms of optimality. Toward these ends, we use a duopoly model in which two firms compete "à la Stackelberg" for two consecutive steps. In the first step, firm A chooses the quantity it extracts, whereas in the second step firm B chooses the quantity it recycles. At the

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[4] The policy of reduction of eutrophication took place in many countries, including the United States and Canada, via the the 1978 Great Lakes Water Quality Agreement which states that the effluents from 400 municipal treatment plants discharging to the lakes should contain a maximum of 1.0 mg litre^{-1} of phosphorus in the upper lakes and 0.5 mg litre^{-1} of phosphorus in the lower lakes (Harrington-Hughes, 1978), China and United Kingdom (Philipps, 1984).

[5] With the aim of reducing eutrophication of Cayuga Lake (United States), Jacobs and Casler (1979) compare an effluent tax policy with a uniforme reduction policy. They stress that it is less costly to reduce eutrophication by taxing effluents than by aiming at reducing it uniformely. For instance, they estimate that the reduction of phosphorus discharge at a level of 10 percent costs $32,065$\$ in the case of the tax on effluents, whereas it costs $37,177$\$ in the case of the uniforme reduction policy.

[6] Cordell and al., 2009 highlight that phosphate reserves may be depleted in $50-100$ years.

very begining of the game, we assume the presence of a benevolent[7] government which sets the level of the tax or that of the subsidy, and both firms produce accordingly.

Summarizing some of our findings, we can state the following effects. First, we find that taxing extracted phosphorus or subsidizing recycled phosphorus contributes to delay the depletion of the resource and to reduce eutrophication. Second, we find that taxing extracted phosphorus reduces consumers'surplus, whereas subsidizing recycled phosphorus increases it. Third, we show that the subsidy set by the regulator is higher than the marginal benefit of recycling, whereas the level of the tax with respect to the marginal social damage of pollution is ambiguous. Fourth, we show that the tax and the subsidy increase social welfare. Fifth, by way of comparison, we find that if the regulator aims at saving phosphorus, reducing eutrophication and improving social welfare simultaneously, subsidizing recycled phosphorus is the best policy, but if he aims only at saving phosphorus and reducing eutrophication, taxing extracted phosphorus is more optimal than subsidizing recycled phosphorus.

This paper is based on several strands of the literature: the first one is related to the analysis of the effect of taxation on environmental pollution and on social welfare. Recalling that the pigovian conclusion that the level of the tax must equal the marginal social cost of polluting emissions is made within a context of perfect competition. In fact, when the market is imperfectly competitive, the tax should be set lower than the marginal social cost of pollution, because it trades off the desire to provide incentives for abatement and the necessity to prevent a greater contraction of output (Nimubona and Sinclaire-Desgagné (2005), see also Levin (1985)). Buchanan (1969) echoes the previous conclusion and gives the following intuition: setting a tax lower than the marginal social damage solves the tendency of imperfectly competitive firms to underproduce. The same conclusion is highlighted by Barnett (1980) in the case where the elasticity of demand is finite. Nevertheless, there are other authors who show that the corrective tax can be higher than the marginal social cost. Using a Cournot duopoly model, Simpson (1995) finds that the optimal tax rate may exceed the marginal damage and provides the following intuition: "Cournot duopolists will not, in general, produce their outputs efficiently. To the extent that a pollution tax may shift production output from the less efficient firm to its more efficient rival, higher tax rates may be called for". David and Sinclaire-Desgagné (2005) state that, under some conditions, an optimal emission tax should be set higher than the marginal social cost of pollution. The intuition underlying this idea is that imperfect competition between environment firms results in abatement prices larger than the marginal social cost of abatement; emission taxes must then be raised in order to make polluters reduce their emissions sufficiently. Barnett (1980) finds that the tax can be higher than the marginal social damage if demand is more price elastic. Using a Bertrand duopoly model with differentiated products, Kurtyka and Mahenc (2011) state

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[7] We prefer to keep the term "benevolent" in the sense that the government cares about the whole society, which consists here of the two firms and consumers.

that the corrective tax can be higher than the marginal damage. The intuition behind their result is that heavy taxation of the polluting good will encourage consumers to switch toward the green product.

In spite of the lack of consensus on the level of the tax with respect to the marginal social damage inflicted by the polluter, the overall conclusion of this line of research is that taxation reduces pollution. However, there are some situations where the effect of the environmental tax may induce an undesirable effect. Levin (1985) analyzes this issue within a Cournot oligopoly model. He assumes that a tax is imposed on each seller at a uniform rate per unit of output and shows if firms are sufficiently different, pollution increases in the tax rate. The intuition he provides is that if firms are asymmetric and one taxes them symmetrically, there is a chance that the heavy polluter will benefit, resulting in the rise of pollution[8]. Even if our two firms are symmetric in terms of costs of production, which are zero, we show, contrary to Levin (1985), that the tax always reduces pollution.

It is noteworthy to mention that several authors argue that taxation reduces social welfare. Buchanan (1969) stresses that, under an imperfectly competitive organization[9], the corrective tax will often reduce[10] than increase welfare. He shows that any levy of a per unit tax on the monopolist's output will decrease total welfare. Barnett (1980) echoes the same conclusion. In contrast to them, we find that the corrective tax can increase social welfare. Using a Cournot Oligopoly model, Ushio[11] (2000) highlights that the tax reduces social welfare if the market structure is symmetric or if the inverse demand function is linear or convex. Albeit the fact that the inverse demand function is linear in our model, social welfare is improved here by the implementation of the taxation scheme.

The second strand of the literature concerns the relationship between a subsidy and pollution control. Baumol and Oates (1995) argue that although a subsidy will tend to reduce the emission of the firm, it is apt to increase the emissions of the industry beyond what they would be in the absence of the fiscal incentives. Mestelman (1982) uses a general equilibrium model and analyzes the effects of taxes and subsidies in a competitive economy which is characterized by a production externality. He shows that the use of a subsidy is inefficient. Mestelman (1984) shows that a pollution tax is consistently preferred to a subsidy by majority of individuals. Diamond and Mirrlees (1971) find that the efficient combination of abatement and outpout requires the pigovian tax alone, because the pollution abatement subsidy distorts the price the input used for reducing pollution, resulting thus in a non-achievement of efficiency. Fredriksson (1997) highlights that pollution abatement subsidies are inefficient instruments

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[8] Also, he argues that if output and pollution levels are positively correlated, then sharp concavity in the demand curve, given by $P^{''} < 0$, will result in increased pollution. This curvature leads small firms to cut output and large firms to increase output as a result of the tax, which can lead to increased pollution even though industry output falls.

[9] Specifically, under a monopolistic organization.

[10] He highlights that if the industry generating the external diseconomy is competitively organized, the corrective tax can be unambiguously hailed as welfare-improvement.

[11] Note that he indicates that the tax increases social welfare if the market structure is asymmetric and the inverse demand function is sufficiently concave at the equilibrium.

for pollution control. He stresses as follows the reasons for which subsidies reduce social welfare. He considers the benchmark as the social optimum situation where the government sets a tax and not a subsidy and argues that if pollution is decreasing in the subsidy rate, the subsidy benefits the environmentalists. Also, the industrialists always gain from receiving the subsidy. Conversely, the remaining groups in society pay a share of the subsidy, but derive not utility from it. Even if aggregate payoffs of the industrialists and the government rise, total welfare declines when one moves away from the social optimum. In contrast to Diamond and Mirrlees (1971), Mestelman (1982), and Fredriksson (1997), we show that the subsidy is efficient in pollution control in the sense that it leads to the reduction of pollution. Unlike Fredriksson (1997), we show clearly that the subsidy improves social welfare even if one moves away from a situation where no tax is applied. Contrary to Mestelman (1984), we find that a subsidy is preferred to a tax in terms of the improvement of social welfare.

Clearly, the innovation introduced in this paper is that it is one of the few, if not the only, which takes simultaneously into account the damage of pollution and the benefit of a non-polluting good/resource in a social welfare function. This consideration is notably due to the specificity of the considered resource that presents a variety which is not polluting, at least in the short run. To the best of our knowledge, there is not another paper which makes this consideration. In terms of results, the main contributions are as follows. First, in contrast to the earlier literature which states that the corrective tax always reduces social welfare (See for instance Buchanan (1969) and Barnett (1980)), the present paper shows that taxation improves social welfare. Second, some authors (see for instance Simpson[12] (1995), David and Sinclaire-Desgagné (2005), etc.) have shown that the optimal pollution tax on an imperfectly competitive industry is higher than the marginal damage if and only if firms are asymmetric in terms of costs of production. In the present paper, we find that, even if firms are symmetric, the tax may be higher than the marginal damage. Third, in contrast to the bulk of the literature which prefers absolutely taxation to subsidy, we show that the subsidy is more optimal than the tax in terms of the improvement of social welfare.

The remainder of the paper is as follows. The next section introduces the concept of "eutrophication". Section 3 presents the model and the results. The main conclusions and some further research lines are given in section 4 and all proofs are relegated to the appendix in section 5.

2 Eutrophication

Bodies of water can be categorized as being in one of two states on the basis of their nutrient content. Low nutrient oligotrophic waters are clear and have

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[12] We have used a quantity competition model like him and find the same result but under opposite assumptions.

relatively little animal and plant life, whereas the high nutrient content of eutrophic waters encourages the development of fauna and flora (Salerno, 2009). Eutrophication denotes the enrichment in nutrients of lakes and rivers that leads to this state of abundant life and therefore sounds like a positive development for a natural habitat. This enrichment can disrupt the natural balance of the natural system and lead to a complete transformation of the habitat (Ricklefs, 1979). The new altered state is often characterized by rapid plant and algae growth. When the density of the vegetation becomes such the ecosystem can no longer support it, it dies and begins to decay (Salerno, 2009). Since the rate of decomposition enhances, the process consumes so much oxygen that fish and other aquatic animals suffocate (Ricklefs, 1979). In addition, the growth of non-toxic algae results in shade and an rise of the water pH[13], which then favors the abundance of the cyanobacteria or blue-green algae, a bacterium that can produce lethal toxins (Scheffer, 1998). Algae can also affect treatment of water for potable supply, by blocking filters or passing through them causing bad odeur and taste (Collingwood, 1977).

Eutrophication manifests in four stages:

(*i*) Increasing pollution: phosphorus ends up into waters, due to water runoff, soil erosion, etc. At the beginning, the oxygen content favors aquatic life. Fish are not affected.

(*ii*) Algae growth: phosphorus leads to the development of algae which consume so much oxygen. the oxygen content increases at the surface of the waters but diminishes significantly in the depths of the waters. Some species die.

(*iii*) Anaerobic decomposition: sediments rich in organic matter accumulate more. Aerobic bacteria multiply in order to degrade organic matter and consume oxygen. The oxygen content is strongly weakened on the whole water column.

(*iv*) Extreme degradation of the environment dystrophy stage : The oxygen content has significantly fallen. There is an absence of oxygen in the aquatic environment. The depletion of oxygen favors the formation of sulfuric acid and ammoniac in the water, leading to the death of fish. At this stage, there is a health risk for fauna and for humanity that use this water, because some cyanobacteria produce toxins.

3 The model

The economy we consider consists of a benevolent government, consumers and two firms, named firm A and firm B . Firm A holds phosphate rocks, extracts them and transforms them into phosphorus which is used as a fertilizer. This, phophorus is what is, widely, commercialized. Firm B, after consuming phosphorus, recycles it and sells it. Therefore, both firms compete. Note that it is technically impossible to recycle the whole phosphorus extracted previously, resulting in $r < q$. Phosphorus which is directly extracted from phosphate rocks

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[13] The pH measures the acidity or the basicity of a solution. A solution with a pH of 7 is considered to be neutral. If $pH < 7$, the solution is considered to be acid and basic if $pH > 7$ (https://fr.wikipedia.org/wiki/Potentiel_hydrog%C3%A8ne).

is called primary phosphorus and that which is recycled is called secondary or green phosphorus. As mentioned above, the primary phosphorus is considered as a polluting resource (Cordell and al., 2011), because it ends up into the water due to water run-off, soil erosion, etc. and, consequently, creates eutrophication phenomenons. Recycling of phosphorus prevents it from ending up into the waters, therefore, reduces water pollution (Weikard and Seyhan, 2009; Cordell and al., 2011; Cogoye, 2009; Beir and Girmens, 2009; Ridder and al., 2012). Another way or an additional means of reducing water pollution would consist of applying a tax to the primary phosphorus or subsidizing recycled phosphorus. Thus, since taxation of virgin phosphorus or subsidy of recycled phosphorus discourages the extraction and encourages the recycling activity, each of these policies reduces eutrophication phenomenons. We assume the presence of a benevolent government that applies this taxation or subsidizes the recycler. In order to compare these two policies, we assume that the benevolent government implements both policies separately. This analysis could be extended to an international scale if there was a supranational government that regulates pollution.

Both firms compete with the quantities of phosphorus they put in the market. Let q denote the quantity which is extracted by Firm A and r be the quantity that is recycled by firm B. For the sake of simplicity, we assume that the inverse demand function is linear and is given by $p(Q) = a - Q$, where Q is the total quantity of phosphorus which is sold and a may be interpreted as the size of the market or, the maximum price at which phosphorus can be sold or also as a choke price (Sweeney, 1992; Baksi and Long, 2009). We, also, consider only one tax applied to the polluting resource instead of having another tax applied to the emissions (for this case, see Cremer and Firouz, 2003). It is also considered that there are no extraction and recycling costs. We, also, assume that the tax-revenue (τq) is refunded to the society.

The timing of the game between the regulator and the firms can be described as follows. In the first step, the regulator sets the level of the tax or the level of the subsidy and refunds all the revenue of the tax to the society (in the case of taxation). In the second step, firm A chooses the quantity it extracts. In the third step, firm B chooses the amount it recycles, after consuming phosphorus.

3.1 Phosphorus conservation and eutrophication reduction

Since Pigou (1920), it is well known that taxation is a means of reducing pollution. Recent studies have also shown that subsidy can yield a similar result, at least within a static context. In this section, we aim at investigating the effects of each of these policies on the reduction of eutrophication. As the reduction of eutrophication coincides here with the decrease of extracted phosphorus, reaching the former goal enables to delay the depletion of phosphorus.

First, we will analyze the effect of the tax on the lifetime of phosphorus and on the reduction of eutrophication. Second, the same issue will be explored within the context of a subsidy.

3.1.1 Taxation of extracted phosphorus

As stated above the timing of the game can be described as follows. In the first step, the regulator sets the level of the tax rate τ and refunds the whole revenue of the tax to the society. In the second step, firm A chooses the quantity it extracts. In the third step, firm B chooses the amount it recycles. Knowing the level of the tax charged by the benevolent government, each firm maximizes its own programme. In order to find the subgame-perfect Nash equilibrium of the whole game, we will solve the latter by backward induction. Thus,

Step 2: The profit maximization for firm B is:

$$\max_{r>0} \pi^B = (a - q - r)r \tag{1}$$

$$s.t.\ r < q \tag{2}$$

Condition (2) indicates the fact that recycling is never complete (see Weikard and Seyhan, 2009). The first-order condition for the programme above is given by:

$$r(q) = \frac{a-q}{2} \tag{3}$$

Step 1: Since the game is solved by backward induction, firm A inserts the reaction function of firm B in its decision of production and maximizes the following programme:

$$\max_{q>0} \pi^A = (a - q - \frac{1}{2}(a-q))q - \tau q \tag{4}$$

Solving the two previous programmes yields the optimal quantities $\hat{q}(\tau)$ and $\hat{r}(\tau)$ whose the comparative statics are summed up through the following proposition:

Proposition 1 *The tax levied by the authority decreases (increases) extracted phosphorus (recycled phosphorus). Formally, we have:*

(*i*)

$$\frac{\partial \hat{q}(\tau)}{\partial \tau} < 0 \tag{5}$$

(*ii*)

$$\frac{\partial \hat{r}(\tau)}{\partial \tau} > 0 \tag{6}$$

Proof. See Appendix II ■

The comparative static results deriving from proposition (1) are quite intuitive. The point (*i*) states that the environmental tax curbs the quantity which is extracted directly from phosphate rocks. This decreasing effect will delay the exhaustion of phosphorus and enables, therefore, the resource to be saved. In fact, the imposition of a tax will make extracted phosphorus more expensive in that the extractor will set higher price, leading consumers to switch towards recycled phosphorus which remains cheaper. Thus, this environmental tax creates a switching effect which consists of boosting the recycling activity, resulting in $\frac{\partial \hat{r}(\tau)}{\partial \tau} > 0$. Since the primary resource is polluting, its decrease reduces environmental pollution. The environmental pollution reduction is also strengthened by recycling. Indeed, as mentioned above, recycling prevents phosphorus from ending up into the waters, avoids, therefore, eutrophication phenomenons.Thus, the tax plays a twofold role.

For $a = 4$, this result is illustrated through the following figure:

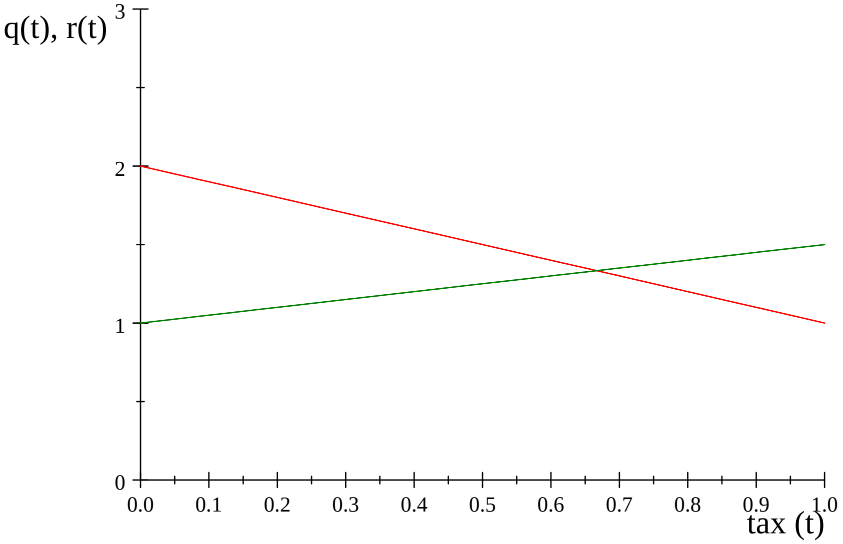

Figure 1: Effect of the tax on the optimal quantities

$$\text{Legend:}\begin{cases} -\text{ Red: Curve of extraction of phosphorus} \\ -\text{ Green: Curve of recycling of phosphorus} \end{cases}$$

This figure shows that the tax reduces extracted phosphorus while it boosts recycled phosphorus. As $r < q$, the graphic is valid only if $\tau \in (0; \frac{2}{3})$. Otherwise, the recycling curve is above the extraction curve (one can also verify it through (63) in appendix II). For this level of the tax, pollution diminishes but does not entirely disappear. This figure indicates also that taxation reduces polluting phosphorus from 2 to $\frac{4}{3}$. In addition, it increases green phosphorus from 1 to $\frac{4}{3}$. The variation of extracted phosphorus is given by$-\frac{2}{3}$, whereas the variation of recycled phosphorus is $\frac{1}{3}$. Clearly, we can state that the tax reduces consumers' surplus in the sense that its decreasing effect is higher than its increasing effect.

Now, let us analyze the impact of the subsidy on the optimal quantities.

3.1.2 Subsidy of recycled phosphorus

We now turn to the equilibrium outcome under the presence of a subsidy. The timing of the game can be described as follows. In the first step, the regulator sets the level of the subsidy rate s that it levies from the society and that it pays to the recycling firm. In the second step, firm A chooses the quantity it extracts. In the third step, firm B chooses the amount it recycles. Knowing the level of the subsidy paid by the benevolent government, each firm maximizes its own programme. As in the case of taxation, we solve the game by backward induction. Thus,

Step 2: The profit maximization for firm B is:

$$\max_{r>0} \pi^B = (a-q-r)r + sr \tag{7}$$

$$s.t.\ r < q \tag{8}$$

The first-order condition deriving from the programme above is then given by:

$$r(q) = \frac{a-q+s}{2} \tag{9}$$

Step 1: Firm A inserts the best-reply function of firm B in its decision of production and maximizes the following programme:

$$\max_{q>0} \pi^A = (a-q-\frac{1}{2}(a-q+s))q \tag{10}$$

Solving the two previous programmes yields the optimal quantities $\hat{r}(\tau)$ and $\hat{q}(\tau)$ whose the comparative statics are summed up through the following proposition:

Proposition 2 *The subsidy increases (decreases) recycled phosphorus (extracted phosphorus). Formally, we have:*

(i)

$$\frac{\partial \hat{r}(s)}{\partial s} > 0 \tag{11}$$

(ii)

$$\frac{\partial \hat{q}(s)}{\partial s} < 0 \tag{12}$$

Proof. For the detail of calculations, see appendix III ■

The intuition underlying proposition (2) can be explained as follows. Subsidizing recycled phosphorus increases the profit of the recycling firm, resulting in the rise of the quantity it recycles. Since recycled phosphorus and extracted phosphorus are strategic substitutes, the increase of the former induces mechanically the slowdown of the latter.

For $a = 4$, the previous result is depicted through the following figure:

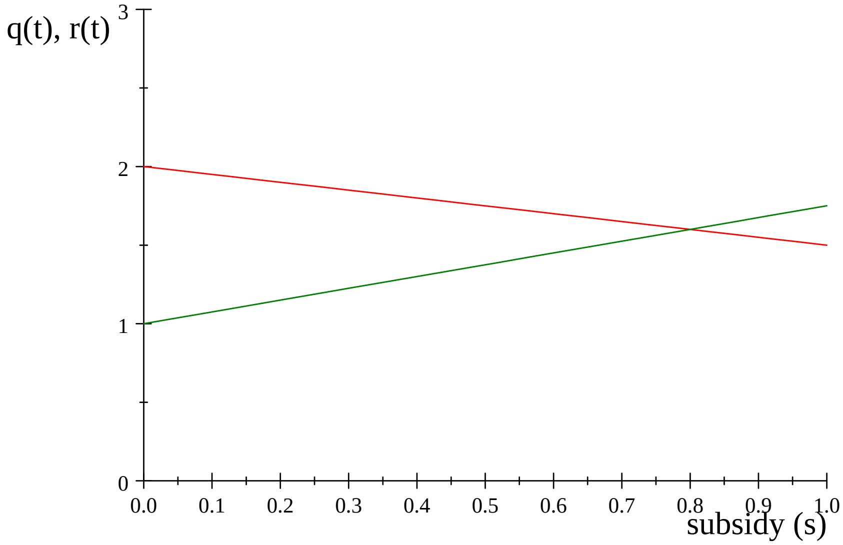

Figure 2: Effect of the subsidy on the optimal quantities

$$\text{Legend:}\begin{cases} -\text{ Red: Curve of extraction of phosphorus} \\ -\text{ Green: Curve of recycling of phosphorus} \end{cases}$$

Figure 2 shows that the subsidy increases recycled phosphorus, whereas it reduces extracted phosphorus. As $r < q$, the graphic is valid only if $s \in (0;\frac{4}{5})$. Otherwise, the recycling curve is above the extraction curve. Through the figure, it appears also that the subsidy reduces eutrophication and contributes to delay the depletion of the resource (in that it reduces extracted phosphorus from 2 to $\frac{8}{5}$) but it does not eliminate the former and will not enable to avoid the exhaustion of phosphorus in the sense that extraction still occurs. The figure shows also that the subsidy increases recycled phosphorus from 1 to $\frac{8}{5}$. The variation of extracted phosphorus is-0.4 and that of recycled phosphorus is 0.6. Since the increasing effect is higher than the decreasing effect, the implementation of subsidy scheme increases consumers'surplus.

The two previous propositions highlight that both the tax on extracted phosphorus and the subsidy on recycled phosphorus yield the same results in qualitative terms. The question which obviously arises is whether or not they yield identical results in quantitative terms. In other words, which policy is more optimal in terms of phosphorus saving and in terms of the reduction of eutrophication ? In the next section, we will address this related issue.

3.1.3 Tax on extracted phosphorus or subsidy on recycled phosphorus ?

In order to know which policy is more optimal in terms of phosphorus saving and in terms of the reduction of eutrophication, we will see the extent of the variation of the optimal quantities. In other words, does the tax reduce (increase) extracted phosphorus (recycled phosphorus) more than the subsidy ? To answer this question, let us consider the two following cases:

case 1: Taxation: the variation of the optimal quantities in moving from the benchmark (zero-tax scenario) to the situation where the market is regulated (presence of a pigovian tax) is:

▶ For extracted phosphorus:

$$\overbrace{\Delta q^{\tau}}^{-} = -\tau \tag{13}$$

▶ For recycled phosphorus:

$$\overbrace{\Delta r^{\tau}}^{+} = \frac{1}{2}\tau \tag{14}$$

For the **proof** of the calculus above, see appendix II. It is straightforward to see through (13) and (14) that the tax decreases extracted phosphorus more than it increases recycled phosphorus. This explains why the total quantity sold by the industry decreases due to the introduction of this taxation scheme, resulting therefore in the decline of consumers' surplus.

case 2: Subsidy: in this case, the variation of the optimal quantities in moving from the benchmark (zero-subsidy scenario) to the situation where the market is regulated (presence of a subsidy) is:

▶ For extracted phosphorus:

$$\overbrace{\Delta q^{s}}^{-} = -\frac{1}{2}s \tag{15}$$

▶ For recycled phosphorus:

$$\overbrace{\Delta r^{s}}^{+} = \frac{3}{4}s \tag{16}$$

For the **proof** of the calculus above, refer to appendix III. Conditions (15) and (16) show clearly that the subsidy increases recycled phosphorus more than it decreases extracted phosphorus. That is why the subsidy increases the total quantity sold by the two firms, resulting then in the rise of consumers' surplus.

Let us turn now to the comparison of the two policies: to get a clearer picture, we sum up (13), (14), (15) and (16) in the following table:

$\Delta\setminus$policies	tax	subsidy
Δq	$-\tau$	$-\frac{1}{2}s$
Δr	$\frac{1}{2}\tau$	$\frac{3}{4}s$

In order to obtain a standard for a comparison, we will set the tax rate (τ) equal to the subsidy rate (s) as in Ballard and Medema (1993). This choice can be justified as follows. Since both policies are implemented separately, the government can use the whole revenue of the tax in order to subsidize the recycling firm, under the assumption that the implementation of the tax scheme is earlier than that of the subsidy scheme. The analysis of the results of this table yields the upcoming proposition.

Proposition 3 *If the amount of the tax per unit of pollution is equal to the amount of the subsidy per unit of the recycled product:*

(i) *The tax is better than the subsidy in terms of the reduction of eutrophication and in terms of the postponement of the depletion of extracted phosphorus. Formally, we have:*

$$\overbrace{\Delta q^{\tau}}^{-} > \overbrace{\Delta q^{s}}^{-} \tag{17}$$

(ii) *Conversely, if the government aims only at boosting the recycling activity, it is more optimal to subsidize recycled phosphorus than to tax extracted phosphorus. Formally, we have:*

$$\overbrace{\Delta r^{s}}^{+} > \overbrace{\Delta r^{\tau}}^{+} \tag{18}$$

Point (i) of proposition (3) states that the reduction of extracted phosphorus is rather greater in the situation where taxation is applied than in the case where the subsidy scheme is implemented. Equivalently, this means that taxation of virgin phosphorus is more optimal than subsidy of recycled phosphorus in terms of the reduction of eutrophication and in terms of the postponement of the depletion of extracted phosphorus. A possible explanation for this scenario is that taxation reduces directly extracted phosphorus by increasing its price, whereas subsidy of recycled phosphorus reduces it indirectly by boosting recycling, first, before discouraging extraction, accordingly, due to the strategic substitutability of both types of phosphorus. In addition, by subsidizing recycled phosphorus, the rise of recycling cannot exceed a certain threshold, due to the fact that recycling is never complete, i.e. $r < q$. Such a constraint indicates that the enhancement of recycled phosphorus in the subsidy rate is limited. Consequently, this will limit the reduction of extracted phosphorus, whereas in the case of taxation, the latter can decrease continuously in the tax rate.

The intuition behind point (ii) is as follows. Subsidizing recycled phosphorus gives more power to the recycling firm than taxing extracted phosphorus in the sense that it increases directly its revenue. Even if taxation enhances its revenue by augmenting the quantity it recycles, due to the strategic substitutability, everything happens as if the revenue that it earns directly in the case of the subsidy is higher than that it earns indirectly in the case of taxation.

3.2 Comparison of taxation and subsidy in terms of social welfare improvement

In this section, we will explore whether taxing extracted phosphorus is more optimal than subsidizing recycled phosphorus or not in terms of social welfare.

3.2.1 Taxation of extracted phosphorus

Consider that the market is regulated, i.e. a tax is applied to extracted phosphorus and assume that the revenue of the tax is returned to the society and the positive externality generated by recycling is taken into account by the regulator. To maximize welfare by taxation, the benevolent government must find some tax rate τ per unit of pollution, which will maximize:

$$w(\tau) = CS(\tau) + \pi^A(\tau) + \pi^B(\tau) - D(q(\tau)) + B(r(\tau)) + \tau q(\tau) \tag{19}$$

We use a linear inverse demand function $p(Q) = a - Q$, a linear damage function, i.e. $D(q) = \varepsilon q$ and a linear benefit function, i.e. $B(r) = \delta r$ (where δ is the marginal benefit of recycling). For the sake of simplicity and in order to focus only on the level of the tax $\hat{\tau}$ with respect to the marginal damage of pollution ε, let us assume that $\delta = 1$. After making some simplications, the social welfare function writes:

$$w(\tau) = r(\tau) - \varepsilon q(\tau) + \frac{1}{2}aq(\tau) + ar(\tau) - \frac{1}{2}(r(\tau))^2 \tag{20}$$

Inserting in (20) the optimal quantities established in appendix II and making some simplications yield:

$$w(\tau) = \frac{-4\tau^2 - [2\,(2a - 21\varepsilon) + 1]\tau + a[15a - 8\,(2\varepsilon - 1)]}{32} \tag{21}$$

It has been clearly shown in appendix II that $w(\tau) > 0$. Clearly, it appears that the marginal damage of pollution decreases social welfare, in the sense that $\frac{\partial w(\tau)}{\partial \varepsilon} = \tau - \frac{1}{2}a < 0$, because $\tau < \frac{1}{6}a$, as shown by equation (63) established in appendix II. This result is quite intuitive. Before establishing the final result in this case, let us make the following assumptions:

Assumption 1.

$$\frac{a-4}{8} < \varepsilon < \frac{a-3}{6} \tag{22}$$

Assumption 2.

$$a > 3 \tag{23}$$

Assumption 1 garantees to have a positive optimal tax rate $\hat{\tau}$, a postive quantity $\hat{q}$ and the condition $\hat{r} < \hat{q}$, whereas assumption 2 garantees to have $\varepsilon > 0$. Solving $\frac{\partial w(\tau)}{\partial \tau} = 0$, yields the results summarized in the following preliminary result:

Lemma 4 *The government selects an equilibrium pollution abatement tax* $\hat{\tau}$ *and an optimal social welfare* $\hat{w}^{\hat{\tau}}$ *which satisfy:*

(*i*)

$$\hat{\tau} = 4\varepsilon - \frac{(a-4)}{2} \tag{24}$$

(*ii*)

$$\hat{w}^{\hat{\tau}} = \frac{4\varepsilon^2 - 2\,(a-2)\,\varepsilon + a^2 + 1}{2} \tag{25}$$

Proof. See appendix II ■

Under (22) and (23), point (*i*) states that the level of the tax with respect to the marginal social damage is ambiguous and depends on the level of pollution. In fact, if $\frac{a-4}{8} < \varepsilon < \frac{a-4}{6}$, the tax is set below the marginal social damage of pollution. Conversely, if $\frac{a-4}{6} < \varepsilon < \frac{a-3}{6}$, the tax is set above the marginal social damage of pollution. In connection with some purposes of this paper, which consist of saving phosphorus and of reducing eutrophication, it is reasonable for the benevolent government to set the corrective tax above the marginal social damage. This will lead the extracting firm to underproduce more, resulting in the rise of recycled phosphorus that he aims also at fostering. In contrast to the conventional wisdom that the tax is set above the marginal damage only and only if the two firms have different costs of production, we show that the tax may be higher than the marginal damage even if firms are symmetric.

In order to explore the effect of the tax on social welfare, derivation of the latter with respect to the tax rate yields:

$$\frac{dw(\tau)}{d\tau} = -\frac{a + 2\tau - 8\varepsilon - 4}{8} \tag{26}$$

From (25) and given our assumptions, we can state the following result:

Proposition 5 *The optimal tax set by the regulator increases social welfare. If the government has the incentive to deviate from the optimal tax, for whatever reason, we have:*

(*iii*) *If* $\hat{\tau}_1 < \tau < \hat{\tau}$*, social welfare increases in the tax rate, resulting in:*

$$\frac{dw(\tau)}{d\tau} > 0 \tag{27}$$

(*iv*) *Conversely, if* $\hat{\tau} < \tau < \hat{\tau}_2$*, social welfare decreases in the tax rate, resulting in:*

$$\frac{dw(\tau)}{d\tau} < 0 \tag{28}$$

Proof. See appendix III ■

The general intuition underlying the increase of social welfare in the tax rate is the following. Owing to the tax applied by the benevolent government, two effects working in opposite directions emerge. On the one hand, the tax enhances the cost incurred by the extracting firm. This curbs the equilibrium extracted quantity, which in turn reduces consumers' surplus and the profit of the extracting firm. On the other hand, the tax increases the quantity which is recycled by the recycling firm, resulting in the increase of the profit of the latter. As mentioned above, this result can be explained by the fact that extracted and recycled phosphorus are strategic substitutes. Then, the decline of the extracted quantity leads, mechanically, the recycling firm to increase the quantity it recycles. The rise of recycled phosphorus may improve the consumers' benefits compared to the situation where the whole demand was met by one supplier. As well as improving quantitatively social welfare, the increase of the recycled quantity improves it qualitatively because it prevents phosphorus from ending up into the waters. Consequently, it contributes to the improvement of the quality of the waters, thanks to the reduction of the eutrophication phenomenons. The enhancing effect of the tax on the recycling firm's profit and the improvement of the quality of the waters outweigh the decreasing effect of the tax on the sum of consumers' surplus and the extracting firm's profit, resulting in higher social welfare.

3.2.2 Subsidy of recycled phosphorus

Consider that the market of recycled phosphorus is regulated, i.e. the recycling firm receives a subsidy rate s from the government. This rate of the subsidy comes obviously from the budget of the society which is managed by the government. Thus, the amount of the subsidy $sr(s)$ will be removed from the social welfare function. To maximize welfare with respect to the subsidy, the government must find some subsidy rate s per unit of recycled product, which will maximize:

$$w(s) = SC(s) + \pi^A(s) + \pi^B(s) - D(q(s)) + B(r(s)) - sr(s) \tag{29}$$

We use a linear inverse demand function, a linear damage function, i.e. $D(q) = \varepsilon q$ (where ε is the marginal damage of pollution) and a linear benefit function, i.e. $B(r) = \delta r$ (where δ is the marginal benefit of recycling). For the sake of simplicity and in order to focus only on the level of the subsidy $\hat{s}$ with respect to the marginal benefit of recycling δ, let us assume that $\varepsilon = 1$. After making some simplications, the social welfare function writes:

$$w(s) = \frac{(a - s - 2)\, q(s) + (2a - r + 2\delta)\, r(s)}{2} \tag{30}$$

Inserting in (30) the optimal quantities given in appendix III yields:

$$w(s) = \frac{-s^2 + 2[a + 4\,(3\delta + 2)]s + a[15a + 8\,(\delta - 2)]}{32} \tag{31}$$

We have clearly proved in appendix III that $w(s) > 0$. Clearly, it appears that the marginal benefit of recycling increases social welfare, in the sense that $\frac{\partial w(s)}{\partial \delta} = \frac{a+3s}{4}$. This result is quite intuitive. Solving $\frac{\partial w(s)}{\partial s} = 0$, yields the results summarized in the following lemma:

Lemma 6 *The government selects an equilibrium pollution abatement subsidy rate* $\hat{s}$ *and optimal social welfare* $\hat{w}^{\hat{s}}$ *which satisfy:*

(i)

$$\hat{s} = a + 12\delta + 8 \tag{32}$$

(ii)

$$\hat{w}^{\hat{s}} = \frac{\delta[2a + 3(3\delta + 4)] + a^2 + 4}{2} \tag{33}$$

Proof. See appendix III ■

Under our assumptions, we show that the optimal level of the subsidy rate is higher than the marginal benefit generated by recycling. The intuition is similar to that established in the case of taxation. In fact, the benevolent government sets the level of the subsidy above the marginal benefit of recycling in order to boost the recycling activity. The rise of recycled phosphorus induces the decline of extracted phosphorus, resulting then in the postponement of the depletion of the resource and on the reduction of eutrophication.

Given our assumptions, we are able to establish the following proposition:

Proposition 7 *The optimal subsidy rate set by the regulator increases social welfare. If the government has the incentive to deviate from the optimal subsidy, for whatever reason, we have:*

(iii) If $\hat{s}_1 < s < \hat{s}$*, social welfare increases in the subsidy rate, resulting in:*

$$\frac{dw(s)}{ds} > 0 \tag{34}$$

(iv) Conversely, if $\hat{s} < s < \hat{s}_2$*, social welfare decreases in the subsidy rate, resulting in:*

$$\frac{dw(s)}{ds} < 0 \tag{35}$$

Proof. See appendix III ■

Proposition (7) indicates that the subsidy improves social welfare. Owing to the subsidy paid by the benevolent government to the recycling firm, two effects working in opposite directions emerge. In fact, the subsidy enhances the profit of the recycling firm, resulting in the rise of recycled phosphorus. Since recycled phosphorus and extracted phosphorus are strategic substitutes, this leads to the decline of extracted phosphorus. But as the increasing effect is higher than the decreasing effect (i.e. $\Delta r^s > \Delta q^s$), the total quantity of the industry increases in subsidy, resulting in the rise of consumers' surplus. As well as improving quantitatively social welfare, the increase of the recycled quantity improves it qualitatively because it prevents phosphorus from ending up into the waters. Consequently, it contributes to the improvement of the quality of the waters, thanks to the reduction of the eutrophication phenomenons. The increasing effect of the subsidy on the sum of consumers' surplus and the recycling firm's profit outweighs the decreasing effect of the subsidy on the extracting firm's profit, resulting in higher social welfare.

3.2.3 Comparison of both policies

Clearly, we observe that taxation of extracted phosphorus and subsidy of recycled phosphorus play the same role, in that both improve social welfare. The remaining question consists naturally of investigating what policy is more optimal in quantitative terms. In order to compare both policies, let us assume that $\varepsilon = \delta = 1$. Then, optimal social welfares are respectively given by:

▶ In the case of taxation:

$$\hat{w}^{\hat{\tau}} = \frac{a^2 - 2a + 9}{2} \tag{36}$$

▶ In the case of subsidy:

$$\hat{w}^{\hat{s}} = \frac{a^2 + 2a + 25}{2} \tag{37}$$

The comparison of (36) and (37) yields the next proposition:

Proposition 8 *If the regulator has the incentive to maximize social welfare, subsidy is the best policy. Formally, we have:*

$$\hat{w}^{\hat{s}} > \hat{w}^{\hat{\tau}} \tag{38}$$

Proof. See appendix III ■

As a conclusion, we can highlight that the subsidy is more optimal than the tax from a social welfare standpoint.

4 Conclusion

In the search for an efficient control for negative externalities, economists, generally, use taxes or/and subsidies as instruments of pollution reduction. This paper analyzes separately the effect of each of these two environmental policies on the lifetime of phosphorus, on the reduction of eutrophication phenomenons and on the improvement of social welfare. Considering two firms, a polluter and a green firm, competing "à la Stackelberg" and assuming the presence of a benevolent government that taxes the polluter or subsidizes the recycler, we state the following results. First, we show that taxing polluting phosphorus (subsidizing recycled phosphorus) permits to reach this target, in the sense that each of them enables to reduce eutrophication phenonemons. Second, we find that taxation or subsidy allows for saving this resource by fostering the recycling activity. Indeed, the latter contributes to postpone the extraction of primary phosphorus. Third, we show that the subsidy set by the regulator is higher than the marginal benefit of recycling, whereas the level of the tax with respect to the marginal social damage of pollution is ambiguous. Fourth, we show that the tax and the subsidy increase social welfare. Fifth, by way of comparison, we find that if the regulator aims at saving phosphorus, reducing eutrophication and improving social welfare simultaneously, subsidizing recycled phosphorus is the best policy, but if he aims only at saving phosphorus and reducing eutrophication, taxing extracted phosphorus is more optimal than subsidizing recycled phosphorus.

We may extrapolate by concluding that the implementation of a specific policy depends on the power of the agent in terms of lobbying. Also, it is not unreasonable to suspect that the government will have the incentive to implement subsidy rather than taxation in order to enjoy the favour of consumers, on the eve of the elections.

Since, in this paper, we have addressed the issues by considering taxation and subsidy separately, the first natural challenge for the future consists of investigating whether the combination of both policies will yield the same results or not.

The second challenge is to set up a dynamic model which allows to take into account the problem that entry causes on pollution. In fact, several authors have stated that subsidy gives to the potential entrants the incentive to enter the market. They have argued that global pollution increases in the subsidy rate even if the individual pollution may decrease. It is noteworthy to mention that these studies are based on the assumption that subsidized firms are polluters. Since in this paper we consider that the subsidized firm is green in that its activity reduces pollution, we may expect that the subsidy will continue to reduce pollution even if we consider a dynamic contex.

Another interesting issue is related to the level of the subsidy that the government can set to maintain the polluter in the market or to drop it out of the market. As subsidy reduces pollution and improves social welfare, higher is the level of the subsidy, lower is the level of pollution and higher is social welfare. One can think that there may be a level of subsidy, named $\tilde{s}$ (higher than the optimal level $\hat{s}$ obtained when pollution occurs) which will drop the polluter out of the market. Then, if $s < \hat{s}$, the government gives to the polluter more importance[14]. If $\hat{s} < s < \tilde{s}$, the polluter remains on the market but its importance reduces with respect to the previous situation. If $s \geq \hat{s}$, the polluter is brought out of the market and only the green firm produces. Accordingly, there is no pollution and social welfare increases due to the high level of the subsidy.

For the sake of simplicity, we have not addressed the problem as the issue of an exhaustible resource. Imposing a capacity constraint to the resource would be interesting and is another challenge for the future.

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[14] This can be the case because the polluter is very strong in terms of lobbying. The government subsidizes only in order to reduce pollution slightly.

5 Appendix

5.1 Appendix I: the market is unregulated

Detailed calculus for the optimal quantities: the timing of the game between the extractor (firm A) and the recycler (firm B) is as follows. In the first step, firm A extracts a quantity of phosphorus $q > 0$. After observing the level of firm A's extraction, firm B recycles a quantity $r > 0$, in the second step. We solve this game by backward induction. The programmes of the two firms are:

Step 2: firm B maximizes

$$\max_{r>0} \pi^B = (a - q - r)r \tag{39}$$

$$s.t. \; r < q \tag{40}$$

The lagrangian for this programme is defined as follows:

$$L(r, \lambda) = (a - q - r)r + \lambda(q - r) \tag{41}$$

The first-order conditions are given by:

$$\frac{dL(r, \lambda)}{dr} = a - q - 2r - \lambda = 0 \tag{42}$$

$$\lambda(q - r) = 0; \; \lambda \geq 0 \tag{43}$$

▶ case 1: from (43), we know if $\lambda = 0$, $r < q$, then (42) writes:

$$a - q - 2r = 0 \tag{44}$$

which results in:

$$r(q) = \frac{a - q}{2} \tag{45}$$

▶ case 2: from (43), we know if $\lambda > 0$, $r = q$. It is impossible to have $r = q$ because recycling is never complete. Then, λ remains always positive and the best-reply function of the recycling firm is then given by (45).

Step 1: the programme of firm A is:

$$\max_{q>0} \pi^A = (a - q - \frac{1}{2}(a - q))q \tag{46}$$

The first-order condition is then given by:

$$\frac{1}{2}a - q = 0 \tag{47}$$

Which results in the following optimal quantities:

$$q^* = \frac{1}{2}a \tag{48}$$

$$r^* = \frac{1}{4}a \tag{49}$$

The total quantity put in the market is given by

$$Q^* = q^* + r^* = \frac{3}{4}a \tag{50}$$

5.2 Appendix II: Detailed calculus when the market is regulated

In this section, we assume that only virgin phosphorus is taxed. The possibility of subsidizing recycled phosphorus is not taken into account here. It will be considered later.

5.2.1 Detailed calculus for the optimal quantities

Proof of proposition 1: Under the tax τ applied by the regulator[15], each firm maximizes its own payoff. We solve this game by backward induction. Thus,

Step 2: the programme of firm B is defined as follows:

$$\max_{r>0} \pi^B = (a - q - r)r \tag{51}$$

$$s.t.\ r < q \tag{52}$$

The lagrangian for this programme can be established as follows:

$$L(r, \lambda) = (a - q - r)r + \lambda(q - r) \tag{53}$$

The first-order conditions are given by:

$$\frac{dL(r, \lambda)}{dr} = a - q - 2r - \lambda = 0 \tag{54}$$

$$\lambda(q - r) = 0; \lambda \geq 0 \tag{55}$$

▶ case 1: from (55), we know if $\lambda = 0$, $r < q$, then (54) writes:

$$a - q - 2r = 0 \tag{56}$$

which results in:

$$r(q) = \frac{a - q}{2} \tag{57}$$

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[15] It is noteworthy to mention that the timing of the game between the regulator and the firms is as follows. In the first step, the regulator sets the level of the tax. In the second step, firm A extracts $q > 0$. In the third step, firm B recycles $r > 0$.

► case 2: from (55), we know if $\lambda > 0$, $r = q$. This is impossible because recycling is never complete. Then, λ remains always positive and the best-reply function of the recycling firm is then given by (57).

Step 1: the programme of firm A is defined as follows:

$$\max_{q>0} \pi^A = (a - q - \frac{1}{2}(a-q))q - \tau q \tag{58}$$

The first-order condition for this programme yields the optimal extracted quantity:

$$\hat{q}(\tau) = \frac{1}{2}a - \tau \tag{59}$$

With

$$\frac{d\hat{q}(\tau)}{d\tau} = -1 \tag{60}$$

And the optimal recycled qunatity:

$$\hat{r}(\tau) = \frac{a+2\tau}{4} \tag{61}$$

With

$$\frac{d\hat{r}(\tau)}{d\tau} = \frac{1}{2} \tag{62}$$

Since $\tau > 0$, $\hat{q}(\tau) \geq 0$ and $\hat{q}(\tau) < \hat{r}(\tau)$ if and only if:

$$0 \leq \tau < \frac{1}{6}a \tag{63}$$

Under our assumptions, (63) explains why in figure 1 the graphic is valid only if $\tau < \frac{2}{3}$.

The total quantity sold par the two firms is:

$$\hat{Q}(\tau) = \frac{3a - 2\tau}{4} \tag{64}$$

It is straightforward to see that

$$\frac{d\hat{Q}(\tau)}{d\tau} = -\frac{1}{2} \tag{65}$$

(65) clearly signals that the tax decreases consumers's surplus in that it reduces the global quantity.

5.2.2 Variation of the quantities

$$\Delta q^{\tau} = q(\tau) - q^* = -\tau \tag{66}$$

$$\Delta r^{\tau} = r(\tau) - r^* = \frac{1}{2}\tau \tag{67}$$

5.2.3 Detailed calculus for the optimal welfare

Proof of lemma 5: we assume that the benevolent regulator refunds the revenue of the tax to the society, takes into account the external effects (positive for r and negative for q) in the damage function . The tax τ maximizes the following social welfare function which is the difference between the sum of producer's profits and consumer's surplus and any technological external costs which are not accounted for in firm A's profits :

$$w(\tau) = CS(\tau) + \pi^A(\tau) + \pi^B(\tau) - D(q(\tau)) + B(r(\tau)) + \tau q(\tau) \tag{68}$$

We use a linear inverse demand function $p(Q) = a - Q$, a linear damage function, i.e. $D(q) = \varepsilon q$ and a linear benefit function, i.e. $B(r) = \delta r$ (where δ is the marginal benefit of recycling). For the sake of simplicity and in order to focus only on the level of the tax $\hat{\tau}$ relatively to the marginal damage of pollution ε, let us assume $\delta = 1$. After making some simplications, the social welfare function writes:

$$w(\tau) = r(\tau) - \varepsilon q(\tau) + \frac{1}{2}aq(\tau) + ar(\tau) - \frac{1}{2}(r(\tau))^2 \tag{69}$$

Inserting the optimal quantities in the social welfare function and making some simplications yield:

$$w(\tau) = \frac{-4\tau^2 - [4a - 42\varepsilon + 1]\tau + a[15a - 8\,(2\varepsilon - 1)]}{32} \tag{70}$$

Derivation of the social welfare function with respect to the tax τ yields the first-order condition below:

$$8\varepsilon + 4 - a - 2\tau = 0 \tag{71}$$

Which, in turn, yields the optimal level of the tax given by:

$$\hat{\tau} = 4\varepsilon - \frac{a-4}{2} \tag{72}$$

Inserting the optimal tax in (59) and (61) yields the following optimal quantities:

$$\hat{q} = a - 4\varepsilon - 2 \tag{73}$$

And

$$\hat{r} = 2\varepsilon + 1 > 0 \tag{74}$$

$\hat{q} > 0$ if

$$\varepsilon < \frac{a-2}{4} \tag{75}$$

Notice that, for technical reasons, recycling is never complete, i.e. $\hat{r} < \hat{q}$. Then,

$$\varepsilon < \frac{a-3}{6} \tag{76}$$

The combination of (75) and (76) gives:

$$\varepsilon < \frac{a-3}{6} \tag{77}$$

ε must be positive. Then

$$a > 3 \tag{78}$$

As $\hat{\tau} > 0$, we have:

$$\varepsilon > \frac{a-4}{8} \tag{79}$$

The combination of (77) and (79) yields:

$$\frac{a-4}{8} < \varepsilon < \frac{a-3}{6} \tag{80}$$

Let us show that the above interval is not empty, i.e. $\frac{a-4}{8} < \frac{a-3}{6}$. This holds if and only if

$$a > \frac{1}{2} \tag{81}$$

This is true under (78). Now let us investigate whether the tax is above or below the marginal damage. To do so, we proceed as follows:
$\hat{\tau} > \varepsilon$ if and only if

$$\varepsilon > \frac{a-4}{6} \tag{82}$$

But since $\frac{a-4}{6} > \frac{a-4}{8}$, the position of the tax with respect to the marginal damage is ambiguous:
(*i*) If $\varepsilon > \frac{a-4}{6}$, as seen above, the tax is higher than the marginal social damage, resulting in:

$$\hat{\tau} > \varepsilon \tag{83}$$

(*ii*) Conversely, if $\varepsilon < \frac{a-4}{6}$, the tax is lower than the marginal social damage, resulting in:

$$\hat{\tau} < \varepsilon \tag{84}$$

In contrast to the conventional wisdom that the tax is set above the marginal damage only and only if the two firms have different costs of production, we show that the tax may be greater than the marginal damage when firms are symmetric (point (*i*)).

Now, let us show that $w(\tau) > 0$. Solving $w(\tau) = 0$ with respect to τ gives the two following roots:

$$\hat{\tau}_1 = \overbrace{4\varepsilon - \frac{a-4}{2}}^{\hat{\tau}} - 2\sqrt{4\varepsilon^2 - 2\left(a-2\right)\varepsilon + a^2 + 1} < \hat{\tau} \tag{85}$$

$$\hat{\tau}_2 = \overbrace{4\varepsilon - \frac{a-4}{2}}^{\hat{\tau}} + 2\sqrt{4\varepsilon^2 - 2\left(a-2\right)\varepsilon + a^2 + 1} > \hat{\tau} \tag{86}$$

Since $w(\tau)$ is a second degree equation in τ, $w(\tau) > 0$ if $\hat{\tau}_1 < \hat{\tau} < \hat{\tau}_2$, which is true. Consequently, $w(\tau) > 0$. Inserting the optimal level of the tax in (70) yields the following optimal social welfare:

$$\hat{w}^{\hat{\tau}} = \frac{\overbrace{4\varepsilon^2 - 2\,(a-2)\,\varepsilon + a^2 + 1}^{M}}{2} \tag{87}$$

For warrantly reasons, let us verify that $M > 0$. Since $M = 0$ is a second degree equation in ε, let us calculate the discrminant. Calculations yield:

$$\Delta = -4a\,(3a+4) < 0 \tag{88}$$

Then, M is positive because it has the same sign as 4. Hence, $\hat{w}^{\hat{\tau}} > 0$.

Let us now investigate the effect of the marginal damage on the social welfare. Thus,

$$\frac{d\hat{w}^{\hat{\tau}}}{d\varepsilon} = 4\varepsilon - a + 2 < 0 \tag{89}$$

Resulting in:

$$\varepsilon < \frac{a-2}{4} \tag{90}$$

This result holds under (75). Hence, as one may expect, the marginal social damage of pollution decreases social welfare.

Now let us investigate the effect of the tax on social welfare. It is obvious that for $\hat{\tau}$, social welfare is optimal and is then given by (87). In order to see the effect of the tax on social welfare, derivation of (70) with respect to the tax rate yields:

$$\frac{dw(\tau)}{d\tau} = -\frac{a + 2\tau - 8\varepsilon - 4}{8} \tag{91}$$

Our calculations show clearly that $\frac{dw(\tau)}{d\tau} > 0$ if $\hat{\tau}_1 < \tau < \hat{\tau}$. In this situation the tax increases social welfare, whereas the latter reduces in the tax rate if $\hat{\tau} < \tau < \hat{\tau}_2$, in that $\frac{dw(\tau)}{d\tau} < 0$ in such a case. One can reasonably expect that the regulator will never set a tax which is different from the optimal tax, i.e. $\tau \neq \hat{\tau}$ but if he has the incentive to deviate from the optimal tax for whatever reason, the tax rate can increase or decrease social welfare.

5.3 Appendix III: Subsidizing recycled phosphorus

In this section, we assume that virgin phosphorus is not taxed. Only recycled phosphorus can be subsidized. We distinguish the following cases:

5.3.1 case 1: benchmark: there is no subsidy

The programmes of the two firms are given by:

Step 2: profit of firm B

$$\max_{r>0} \pi^B = (a - q - r)r \tag{92}$$

$$s.t.\ r < q \tag{93}$$

$$r(q) = \frac{a - q}{2} \tag{94}$$

Step 1: profit of firm A

$$\max_{q>0} \pi^A = (a - q - \frac{1}{2}(a - q))q \tag{95}$$

$$q^* = \frac{1}{2}a \tag{96}$$

And

$$r^* = \frac{1}{4}a \tag{97}$$

5.3.2 case 2: Implementation of a subsidy scheme

Proof of proposition 2

Step 2: the programme of firm B is defined as follows:

$$\max_{r>0} \pi^B = (a - q - r)r + sr \tag{98}$$

$$s.t.\ r < q \tag{99}$$

The reaction function deriving from this programme is:

$$r(q) = \frac{a - q + s}{2} \tag{100}$$

Step 1: the programme of firm A is defined as follows:

$$\max_{q>0} \pi^A = (a - q - \frac{1}{2}(a - q + s))q \tag{101}$$

The optimal quantity resulting from this programme is:

$$q^s = \frac{a - s}{2} \tag{102}$$

With

$$\frac{dq^s}{ds} = -\frac{1}{2} \tag{103}$$

And the optimal quantity recycled by firm B is given by:

$$r^s = \frac{a + 3s}{4} \tag{104}$$

With

$$\frac{dr^s}{ds} = \frac{3}{4} \tag{105}$$

$q^S > 0$ and $r^S < q^S$ imply:

$$a > 5s \tag{106}$$

The total quantity is then given by:

$$Q^s = \frac{3a + s}{4} \tag{107}$$

With

$$\frac{\partial Q^s}{\partial s} = \frac{1}{4} \tag{108}$$

5.3.3 Variation of quantities

$$\underbrace{\Delta q^s}_{-} = q^s - q^* = -\frac{1}{2}s \tag{109}$$

$$\underbrace{\Delta r^s}_{+} = r^s - r^* = \frac{3}{4}s \tag{110}$$

But

$$\Delta r^s > \Delta q^s \tag{111}$$

Resulting in:

$$\Delta Q > 0 \tag{112}$$

Or in:

$$\frac{dQ^s}{ds} > 0 \tag{113}$$

The subsidy increases the recycled quantity whereas it decreases the extracted quantity. But, the increasing effect is higher than the decreasing effect, resulting in the rise of the total quantity $(q + r)$ with respect to the increase of the subsidy. Consequently, the subsidy increases consumers' surplus.

5.3.4 Comparison of social welfares

Implementation of subsidy scheme: **Proof of lemma** 6: the social welfare function is given by:

$$w(s) = CS + \pi^A + \pi^B - D(q) + B(r) - sr \tag{114}$$

We use a linear inverse demand function, a linear damage function, i.e. $D(q) = \varepsilon q$ (where ε is the marginal damage of pollution) and a linear benefit function, i.e. $B(r) = \delta r$ (where δ is the marginal benefit of recycling). For the sake of simplicity and in order to focus only on the level of the subsidy $\hat{s}$ relatively to the marginal benefit of recycling δ, let us assume $\varepsilon = 1$. After making some simplications, the social welfare function writes:

$$w(s) = \frac{(a - s - 2)\, q(s) + (2a - r + 2\delta)\, r(s)}{2} \tag{115a}$$

Inserting $q(s)$ and $r(s)$ in (115a) yields:

$$w(s) = \frac{-s^2 + 2[a + 4\,(3\delta + 2)]s + a[15a + 8\,(\delta - 2)]}{32} \tag{116}$$

With:

$$\frac{\partial w(s)}{\partial s} = \frac{a - s + 12\delta + 8}{16} \tag{117}$$

Derivation the social welfare function with respect to the subsidy s yields the first-order condition below:

$$a - s + 12\delta + 8 = 0 \tag{118}$$

Which results in the following optimal level of the subsidy:

$$\hat{s} = a + 12\delta + 8 \tag{119}$$

Let us explore if $a + 12\delta + 8 > \delta$. This results in: $a + 11\delta + 8 > 0$, which is true. Then,

$$\hat{s} > \delta \tag{120}$$

The optimal subsidy rate is above the marginal benefit of recycling. It is straightforward to show that the marginal benefit increases the subsidy. This results in:

$$\frac{d\hat{s}}{d\delta} = 12 \tag{121}$$

Let us show that $w(s) > 0$. Solving $w(s) = 0$ with respect to s gives the two following roots:

$$\hat{s}_1 = \overbrace{a + 12\delta + 8}^{\hat{s}} - 4\sqrt{a^2 + 2a\delta + 9\delta^2 + 12\delta + 4} < \hat{s} \tag{122}$$

And

$$\hat{s}_2 = \overbrace{a + 12\delta + 8}^{\hat{s}} + 4\sqrt{a^2 + 2a\delta + 9\delta^2 + 12\delta + 4} > \hat{s} \tag{123}$$

Since $w(s)$ is a second degree equation in s, $w(s) > 0$ if $\hat{s}_1 < \hat{s} < \hat{s}_2$, which is true. Consequently, $w(s) > 0$. Inserting $\hat{s}$ in (116) yields the next optimal social welfare:

$$\hat{w}^{\hat{s}} = \frac{\delta\,(2a + 3\,(3\delta + 4)) + a^2 + 4}{2} > 0 \tag{124}$$

Let us investigate the influence of the subsidy on social welfare. To do so, we proceed as follows:

$$\frac{dw(s)}{ds} = \frac{\overbrace{a + 12\delta + 8}^{\hat{s}} - s}{16} \tag{125}$$

From (125), it is easy to show that $\frac{dw(s)}{ds} > 0$ if $\hat{s}_1 < s < \hat{s}$, meaning that the subsidy increases social welfare in such a situation, whereas the latter decreases in the subsidy rate if $\hat{s} < s < \hat{s}_2$. As in the case of taxation, it is not obvious that the regulator will set a subsidy rate different from the optimal subsidy. But if, for a whatever reason, he has the incentive to increase or to decrease the subsidy with respect to the optimal subsidy, social welfare may increase or decrease in the subsidy rate.

Let us now turn to the comparison of taxation and subsidy: for a standart of comparison, assume that $\delta = \varepsilon = 1$. Then, optimal social welfares are respectively given by:

▶ In the case of taxation:

$$\hat{w}^{\hat{\tau}} = \frac{a^2 - 2a + 9}{2} \tag{126}$$

▶ In the case of subsidy:

$$\hat{w}^{\hat{s}} = \frac{a^2 + 2a + 25}{2} \tag{127}$$

It is straightforward to observe that:

$$\hat{w}^{\hat{s}} > \hat{w}^{\hat{\tau}} \tag{128}$$

The result established in (128) is the **proof of proposition** 8.

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