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Katrin ERDLENBRUCH  
Raphael SOUBEYRAN  
Mabel TIDBALL  
Agnes TOMINI

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Unité de Formation et de Recherche d'Economie  
Avenue Raymond DUGRAND C.S. 79606  
34960 MONTPELLIER Cedex 2

E-mail : [lameta@lameta.univ-montp1.fr](mailto:lameta@lameta.univ-montp1.fr)  
web : [www.lameta.univ-montp1.fr](http://www.lameta.univ-montp1.fr)

# (Anti-)Coordination Problems with Scarce Water Resources\*

Katrin Erdlenbruch<sup>†</sup>      Raphaël Soubeyran<sup>‡</sup>      Mabel Tidball<sup>§</sup>  
Agnes Tomini<sup>¶</sup>

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## Abstract

This paper deals with the interactions between farmers who can choose between two water supplies (groundwater or rainwater) which are interdependent and have different productivities. Collecting rainwater reduces the amount of water that can replenish the aquifer and allows farmers to avoid the pumping cost externality (but increases the cost of pumping groundwater). We show that multiple equilibria can exist. For a policy-maker, this immediately raises the equilibrium selection issue. This problem is worsened by the fact that the number of equilibria increases with a decrease in the recharge rate. In addition, comparative statics show that, depending on the equilibrium, a policy intervention can have opposite effects. Finally, we show that asymmetric equilibria can also exist, when one group of farmers chooses to harvest rainwater to avoid the pumping cost externality and the other group chooses to pump groundwater.

**JEL Classification:** Q15, Q25, C72

**Keywords:** groundwater, rainwater, water productivity, differential game, asymmetric equilibrium, multiplicity of equilibria, drought.

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<sup>†</sup>Irstea, UMR G-EAU, F-34196 Montpellier, France; [katrin.erdlenbruch@irstea.fr](mailto:katrin.erdlenbruch@irstea.fr).

<sup>‡</sup>INRA, UMR1135 LAMETA, F-34000 Montpellier, France; [soubeyra@supagro.inra.fr](mailto:soubeyra@supagro.inra.fr).

<sup>§</sup>INRA, UMR1135 LAMETA, F-34000 Montpellier, France; [tidball@supagro.inra.fr](mailto:tidball@supagro.inra.fr).

<sup>¶</sup>UMR1135 LAMETA, F-34000 Montpellier, France; [agnes.tomini@supagro.inra.fr](mailto:agnes.tomini@supagro.inra.fr).

# 1 Introduction

Water scarcity is expected to become an ever-increasing problem in the future and one of the main issues under climate change (IPCC [8]). With the increase in the frequency of extreme weather events, one of the key climate impacts is changing precipitation patterns, which may challenge hydrological functioning, and disturb existing equilibria (IPCC [8]). We can thus expect that the growing pressure on common resources will trigger further competition among resource users, jeopardizing existing management arrangements (Ostrom [15]). In this context, it is important to study both interactions among resource users and physical interactions among water resources.

In this paper, we deal with the interactions between identical farmers who can choose between two water supplies (groundwater or rainwater). First, groundwater and rainwater have different productive properties: evaporation of rainwater may be greater than that of groundwater and the consumptive use of groundwater may consequently be higher; groundwater may be salty or contain toxic substances (e.g. chloride) leading to higher consumptive use of rainwater. Second, groundwater and rainwater are interdependent because the collection of rainwater reduces the amount of water that replenishes the aquifer. An interesting property of this game is that the farmers who collect rainwater escape the pumping cost externality but they generate a negative externality for the farmers who pump groundwater: they reduce the amount of water that can replenish the aquifer which, in turn, increases the pumping cost.

This paper builds on the literature on water resource economics (see for example Cummings [3], Gisser and Sanchez [7], Koundouri [10], Moreaux and Reynaud [12],[13], Negri [14], Roseta-Palma [18], Rubio and Casino [20]). In particular, our paper is linked to the literature that considers water as a system of different water bodies. One strand of the literature deals with optimal management of multiple groundwater resources (see Roumasset and Wada [19] or Zeitouni and Dinar [26]). Another strand studies the conjunctive use of ground and surface water (see Burt [1], Chakravorty and Umetsu [2], Gemma and Tsur [6], Knapp and Olson [9], Krulce, Roumasset and Wilon [11], Pongkijvorasin and Roumasset [16], Stahn and Tomini [22], [23] or Tsur and Graham-Tomasi [24]). Roumasset and Wada [19] showed that optimal management of several *independent* groundwater resources depends on their marginal opportunity cost: only the resource with the lowest

marginal opportunity cost is used initially, whereas in the steady state, all resources are used. However, in the present paper, we consider two interrelated resources. Zeitouni and Dinar [26] studied the case of two *interrelated aquifers*: depending on the relative height of the water tables, water will flow from one aquifer to the other. This could lead to the contamination of the aquifer with the better water quality. Optimal water management is then defined by the joint management of these interrelated resources, the threat of contamination representing an additional externality that has to be taken into account. In the present paper, we do not deal with two interrelated aquifers but instead the interaction between rainwater and groundwater resources. Rainwater and groundwater are physically linked because rainwater (partially) infiltrates the soil and replenishes the aquifer. Stahn and Tomini ([22], [23]) also considered the joint use of groundwater and rainwater and focused on the optimal management of these resources. They showed that, in the long-run, the introduction of rainwater harvesting may lead to a greater depletion of the groundwater aquifer. This result was obtained by extending the standard groundwater model to include the connection between two water supplies. Especially, in [22], the decrease in the water table first occurs because of the negative effect of rainwater harvesting on the groundwater recharge rate and, second, because the efficiency of water use depends on the relative rate of evapotranspiration in the storage reservoir and infiltration to the groundwater aquifer. In contrast to Stahn and Tomini ([22], [23]), we focus on the strategic interactions between several farmers (see Dockner et al. [4], Rubio and Casino [21], Negri [14]) and the problem of sharing a common resource.

In sum, we model the management of the interrelated groundwater-rainwater water resource system, in the context of strategic interactions between homogeneous resource users, i.e. farmers who share a common resource. We first assume that farmers make consistent commitments (open-loop game). We then deal with the feedback game, in which each farmer's irrigation strategy is a function of the resource stock. We focus on the role of the cost and productivity differential between the use of rainwater and groundwater. Because resources are physically interlinked, we cannot simply compare marginal extraction and user costs of separate use, as proposed by Pongkijvorasin and Roumasset [16], but we compare all possible equilibria of joint or separate use of the resource.

We report three main results. First, we show that multiple equilibria can exist. For a policy maker, this immediately raises the equilibrium selection issue. Second, we show that this problem is worsened by the fact that the number of equilibria increases with an increase in the scarcity of water (with a lower recharge rate). In addition, comparative statics show that variations in one parameter of the model can have opposite effects depending on the equilibrium concerned. Third, we show that asymmetric equilibria can exist in our symmetric player dynamic game (see Vives [25] for conditions that determine the non-existence of asymmetric equilibria in a static framework). For instance, one group of farmers chooses to harvest rainwater to avoid the pumping cost externality while the other group pumps groundwater. This specialization is due to the pumping cost externality: pumping becomes more costly when the water-table is reduced by another resource-user and some farmers consequently avoid this externality by choosing to harvest rainwater instead of pumping groundwater.

In terms of policy implications, our results suggest that it may be difficult to define a policy for a resource with low recharge rates, because many different equilibria can co-exist. In addition, with varying parameter values, the fundamental parameters of each equilibrium do not vary in the same sense.

The paper is organized as follows. In section 2, we present the model. In section 3, we describe the necessary conditions required for the existence of symmetric and asymmetric stationary Nash equilibria, emphasizing the interactions between the two water sources. In section 4 we discuss the co-existence of both symmetric and asymmetric stationary Nash equilibria (SNE), highlighting the role of water availability through the level of the recharge rate. Finally, in section 5, we discuss our results and draw some conclusions.

## 2 The Model

We consider a continuous time strategic interaction problem where a fixed number  $N \geq 2$  of farmers uses water as an input and can use rainwater and/or groundwater.

## 2.1 Groundwater Dynamics

We consider a single-cell, unconfined and “bathtub” type aquifer with flat bottom and perpendicular sides. In this aquifer, the water table increases because some part of rain (net of rainwater harvesting) soaks into the soil and reaches the ground to replenish the aquifer and it decreases because of farmers’ withdrawals. We denote  $R$  the quantity of rain and  $\rho \in (0, 1)$  the infiltration rate. In line with the wider part of literature,<sup>1</sup> the natural recharge is exogenously determined (i.e. not stock dependent).<sup>2</sup>

Farmer  $i$  pumps a quantity of groundwater  $g_i(t)$  in the aquifer at time  $t$  and therefore the decline of the level of water table results from the total pumping :  $\sum_{i=1}^N g_i(t)$ . He can also directly collect a quantity  $r_i(t)$  from the recharge at the surface, before rainwater seeps into the ground. Consequently, rainwater harvesting reduces the amount of water that replenishes the aquifer by the total quantity of rainwater that resource users have harvested, that is  $\sum_{i=1}^N r_i(t)$ . Thus, when the farmers collect rainwater, the quantity of water that reaches the aquifer is  $\rho \left( R - \sum_{i=1}^N r_i(t) \right)$ .

Combining these assumptions all together, we assume that the groundwater dynamics is characterized by the following differential equation:

$$\dot{h}(t) = \rho \left( R - \sum_{i=1}^N r_i(t) \right) - \sum_{i=1}^N g_i(t), \quad (1)$$

where  $h(t)$  is the level of the water table at time  $t$ .

This simple formulation allows us to account for the connection between the two water supplies and emphasizes the hydrological aspect.

## 2.2 Net Farmers’ Benefits

Farmers use a combination of the two water supplies,  $g_i(t) \geq 0$  and  $r_i(t) \geq 0$  at period  $t$ , for production.<sup>3</sup> We assume that the two water supplies impact the output differ-

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<sup>1</sup>See, among others, Gisser and Sanchez [7], Koundouri [10], Rubio and Casino [21].

<sup>2</sup>For simplification, we do not take into account the local percolation and discharge. When the water table is near the ground surface, there is little opportunity for recharge and shallow aquifers are recharged by local percolation of surface water and discharged by crops that use the water out of the ground. However, large aquifers run deep and are highly dependent on rain and melting snow.

<sup>3</sup>As we focus on irrigation strategies, water is the only input considered.

ently. Namely, the productivity of groundwater is denoted  $\mu > 0$  and the productivity of rainwater is denoted  $\theta > 0$ . We assume that the two water supplies are substitutes:

$$W_i(t) = \mu g_i(t) + \theta r_i(t). \quad (2)$$

The quantity of water,  $W_i(t)$ , is the unique input and  $F(\cdot)$  is an increasing and concave production function. The price of one unit of output is normalized to 1, thus  $F(W_i(t))$  represents farmer  $i$ 's contemporaneous benefit. The cost of extraction of groundwater,  $C(\cdot, \cdot)$ , is a function of the level of the water table,  $h(t) \geq 0$ , and of the quantity of water pumped,  $g_i(t)$ .<sup>4</sup> This cost increases with the quantity of groundwater pumped and decreases with the level of the water table:

$$C_g(h(t), g_i(t)) \geq 0, \quad C_h(h(t), g_i(t)) \leq 0. \quad (3)$$

We also assume that  $C_h(h, 0) = 0$ .

The cost of collection of rainwater,  $D(r(t))$ , (e.g the transport cost from the point of the reservoir to the irrigation area) depends on the quantity collected and does not depend on the level of the water table. This cost is increasing,  $D' > 0$ . We make usual assumptions on the convexity of the cost functions,  $C_{gg}(h(t), g_i(t)) \geq 0$ ,  $D''(r_i(t)) \geq 0$ . We also assume that the cross-derivative of the cost of groundwater extraction is negative,  $C_{gh}(h(t), g_i(t)) < 0$ , i.e. the marginal cost of groundwater extraction decreases when the level of the water table increases.

We assume that it is practically impossible to collect all the recharge through rainwater harvesting:

$$R > \sum_i^N r_i(t). \quad (4)$$

Farmer  $i$ 's net benefit at time  $t$  is then:

$$F(\mu g_i(t) + \theta r_i(t)) - C(h(t), g_i(t)) - D(r_i(t)). \quad (5)$$

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<sup>4</sup>As aforementioned, we consider a “bathtub” type aquifer, and then the cost is the same at each point of the aquifer.



The farmers behave non-cooperatively. They maximize the present value of their stream of profits given the extraction path of others with a common discount rate  $\delta$ . The  $i$ th farmer faces the following dynamic optimization problem:<sup>5</sup>

$$\begin{aligned} & \max_{\{g_i, r_i\}} \int_0^\infty (F(W(t)) - C(h(t), g_i(t)) - D(r_i(t))) \exp^{-\delta t} dt \\ & \text{w.r.t} \left\{ \begin{array}{l} \dot{h} = \rho \left( R - \sum_i^N r_i(t) \right) - \sum_i^N g_i(t) \\ g_i(t) \geq 0 \\ r_i(t) \geq 0 \\ h(t) \geq 0 \\ R > \sum_i^N r_i(t) \\ h(0) \text{ given and } h(\infty) \text{ free} \end{array} \right. \end{aligned} \quad (6)$$

### 3 Stationary Nash Equilibria

The farmers solve their dynamic problem simultaneously. We first focus on the open-loop Stationary Nash equilibria (SNE). We then show how our main results are affected when the farmers use feedback strategies (see Remark 1 at the end of the present section). We can define the current-value Hamiltonian function of farmer  $i$  as follows:

$$H_i = F(W_i(t)) - C(h(t), g_i(t)) - D(r_i(t)) + p_i \left[ \rho \left( R - \sum_{i=1}^N r_i(t) \right) - \sum_{i=1}^N g_i(t) \right], \quad (7)$$

where  $p_i(t)$  is the shadow price of groundwater for farmer  $i$ .

and the corresponding Lagrangian function:<sup>6</sup>

$$L_i = H_i + \lambda_{gi} g_i(t) + \lambda_{ri} r_i(t), \quad (8)$$

where  $\lambda_{gi}$  and  $\lambda_{ri}$  are the Lagrangian multipliers.

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<sup>5</sup>We assume that this problem has a solution.

<sup>6</sup>We compare the various steady states in different situations where the aquifer is not depleted,  $h > 0$ .

The first order conditions for a *stationary Nash equilibria (SNE)* are:

$$\frac{\partial L_i}{\partial g_i} = \mu F'(W_i) - C_g(h, g_i) - p_i + \lambda_{gi} = 0, \quad (9)$$

$$\lambda_{gi} \geq 0, \quad \lambda_{gi} g_i = 0, \quad (10)$$

$$\frac{\partial L_i}{\partial r_i} = \theta F'(W_i) - D'(r_i) - \rho p_i + \lambda_{ri} = 0, \quad (11)$$

$$\lambda_{ri} \geq 0, \quad \lambda_{ri} r_i = 0, \quad (12)$$

$$\dot{p}_i = 0 = \delta p_i - \frac{\partial L_i}{\partial h} = \delta p_i + C_h(h, g_i), \quad (13)$$

$$\dot{h} = 0 = \rho R - \rho \sum_{i=1}^N r_i - \sum_{i=1}^N g_i, \quad (14)$$

where  $W_i = \mu g_i + \theta r_i$ .

Condition (9) states that the marginal benefit of one additional unit of groundwater must be equal to the total marginal cost (that is the sum of costs, extraction or storage, with the opportunity cost of removing one unit of water from the ground), if the farmers indeed use groundwater. Condition (11) states that the marginal benefit of one additional unit of rainwater must be equal to the total marginal cost (that is the sum of costs, extraction or storage, with the opportunity cost of removing one unit of water from the ground), if the farmers indeed use rainwater. Condition (13) characterizes the time variation of shadow price of player  $i$ ,  $p_i$  that represents the effect that the depletion of the water table in the current period has on future profits. It is positively affected by the discount rate, the current price and the marginal effect of the water table depletion on pumping cost. Conditions (10) and (12) are the complementary slackness conditions.

In the following, the marginal costs of groundwater and rainwater play an important role in the optimal choice of the farmers. The marginal costs of groundwater and rainwater are essentially different because the marginal cost of groundwater,  $C_g(h, g)$ , depends on the level of the water table  $h$  while the short run marginal cost of rainwater,  $D'(r)$ , does not. In a SNE, the user cost of groundwater,  $p_i = -\frac{1}{\delta} C_h(h, g)$ , is the marginal effect of a decrease in the water table weighted by the discount rate  $\delta$  (see condition (13)). It represents the increase in the future marginal costs of groundwater use due to contemporaneous use of groundwater. The user cost of rainwater,  $\rho p_i$ , is a fraction of

the user cost of groundwater, because the recharge rate of the aquifer from rainwater is  $\rho < 1$ . It represents the increase in the future marginal costs of groundwater use due to contemporaneous use of rainwater.

Let us define the full marginal cost of groundwater,  $MC$ . It is the sum of the short run marginal cost,  $C_g(h, g)$ , and the user cost of groundwater,  $p_i = -\frac{1}{\delta}C_h(h, g)$ :

$$MC(h, g) \equiv C_g(h, g) - \frac{1}{\delta}C_h(h, g), \quad (15)$$

Similarly, we define the full marginal cost of rainwater,  $MD$ . It is the sum of the short run marginal cost  $D'(r)$  and the user cost of rainwater,  $\rho p_i = -\rho \frac{1}{\delta}C_h(h, g)$ :

$$MD(h, r, g) \equiv D'(r) - \rho \frac{1}{\delta}C_h(h, g). \quad (16)$$

At the SNE, farmers may choose a different mix of groundwater and rainwater. In particular, they can use groundwater or rainwater only but they can also use both water sources simultaneously. However, because all farmers are identical, we can easily show that if a group of farmers uses groundwater they pump the same amount of groundwater and if they collect rainwater, they collect the same amount of rainwater.

In the following, we successively consider necessary conditions for the existence of symmetric and asymmetric stationary Nash equilibria.

### 3.1 Symmetric Stationary Nash Equilibria

A symmetric SNE is a SNE where all the farmers choose the same mix of groundwater and rainwater. Three situations may happen: (i) the farmers use rainwater only, (ii) the farmers use groundwater only or (iii) the farmers use both water supplies.

In a symmetric SNE, for any two farmers  $i$  and  $j$ , we have  $g_i = g_j = g$  and  $r_i = r_j = r$  (and then  $p_i = p$ ,  $\lambda_{gi} = \lambda_g$  and  $\lambda_{ri} = \lambda_r$ ). Consequently, the aggregate amount of groundwater used is  $\sum_{i=1}^N g_i = Ng$  and the total amount of rainwater used is  $\sum_{i=1}^N r_i = Nr$ .

In the following, we use superscripts  $RW$ ,  $GW$  and  $c$  to denote the SNE values at the (i) “rainwater harvesting SNE”, (ii) the “groundwater pumping SNE” and (iii) “the

conjunctive use SNE", respectively.

We first show that there is no SNE such that all the farmers use rainwater only and then we turn to the characterization of the other symmetric SNE.

### 3.1.1 Farmers use rainwater only

Let us first consider the SNE where the farmers use rainwater only. This implies that the slackness condition is :  $g^{RW} = 0$ ,  $\lambda_r^{RW} = 0$  and  $\lambda_g^{RW} \geq 0$ . Taking into account that at the SNE we must have  $\dot{h}^{RW} = \dot{p}^{RW} = 0$ , from equation (13) it is straightforward that:

$$p^{RW} = -\frac{1}{\delta} C_h(h^{RW}, 0) = 0. \quad (17)$$

Hence, when the farmers use rainwater only, their user cost is null. We now show the following result:

**Proposition 1** *There is no SNE where the farmers use rainwater only.*

**Proof of Proposition 1:** Using  $\dot{h}^{RW} = 0$ , condition (9) becomes  $Nr^{RW} = R$  and this is contradictory with  $\sum_i r_i < R$ .  $\square$

The intuition of this result is simple: if the farmers use rainwater only, they harvest all the recharge, which contradicts our assumption that this is not practically feasible (see condition (4)).

### 3.1.2 Farmers use groundwater only

Let us now consider the SNE where all the farmers withdraw groundwater only. This implies that  $r^{GW} = 0$  and, from the slackness condition (12), we have  $\lambda_r^{GW} \geq 0$  while the slackness condition (10) leads to  $\lambda_g^{GW} = 0$ .

Taking into account that  $\dot{h}^{GW} = \dot{p}^{GW} = 0$ , equations (13) and (14) can be used to find the characterization of the extraction rate and the shadow price in the SNE:

$$g^{GW} = \frac{\rho R}{N} \quad (18)$$

$$p^{GW} = -\frac{1}{\delta} \cdot C_h(h^{GW}, g^{GW}). \quad (19)$$

In the long-run, farmers use an identical share of the recharge of the aquifer, as we can see in condition (18).

Substituting  $\lambda_g^{GW} = 0$  and condition (19) into condition (9), we obtain an implicit characterization of the SNE level of the water table,  $h^{GW}$  :

$$\mu F'(\mu g^{GW}) = MC(h^{GW}, g^{GW}). \quad (20)$$

Substituting  $r^{GW} = 0$  and (19) into condition (11), we obtain the SNE value for the Lagrangian multiplier,

$$\lambda_r^{GW} = MD(h^{GW}, 0, g^{GW}) - \theta F'(\mu g^{GW}). \quad (21)$$

Condition (21) and  $\lambda_r^{GW} \geq 0$  imply that the full marginal rainwater cost in the long-run must be greater than the long-run marginal productivity of rainwater.

$$MD(h^{GW}, 0, g^{GW}) \geq \theta F'(\mu g^{GW}) \quad (22)$$

Finally, combining (20) and (22), we obtain the following necessary condition for the existence of a groundwater pumping SNE:

**Proposition 2** *If a groundwater pumping SNE exists, then the ratio of the marginal productivities of the two water sources is higher than the ratio of the full marginal costs:*

$$\frac{\mu}{\theta} \geq \frac{MC(h^{GW}, g^{GW})}{MD(h^{GW}, 0, g^{GW})}.$$

**Proof of Proposition 2:** Combine conditions (20) and (22).  $\square$

Proposition 2 shows that when a groundwater pumping SNE exists, the relative productivity of groundwater exceeds the relative full marginal cost of groundwater (compared to rainwater). In contrast with Roumasset and Wada [19], the optimal extraction is not only driven by extraction costs, but also by the difference of productivity ( $\mu$  and  $\theta$ ).

### 3.1.3 Farmers use rainwater and groundwater

Now, we focus on SNE in which each farmer uses both rainwater and groundwater. In this case, the complementary slackness conditions (10) and (12) require that the two Lagrangian multipliers are zero,  $\lambda_r^c = \lambda_g^c = 0$ .

Using  $\dot{p}^c = \dot{h}^c = 0$ , we deduce that the rainwater harvesting level and the shadow price depend on the quantity of groundwater pumped:

$$p^c = \frac{g^c}{\delta}, \quad (23)$$

and,

$$r^c = \frac{\rho R - N g^c}{N \rho}. \quad (24)$$

Then, substituting these expressions into conditions (9) and (11), we obtain the following system of equations:

$$\begin{cases} \mu F'(W^c) = MC(h^c, g^c), \\ \theta F'(W^c) = MD(h^c, r^c, g^c), \end{cases} \quad (25)$$

where  $W^c = \mu g^c + \theta r^c$ .

These conditions enable us to characterize the conjunctive use SNE. The following proposition provides a necessary condition:

**Proposition 3** *If a conjunctive SNE exists, then the ratio of the marginal productivities is equal to the ratio of the full marginal costs:*

$$\frac{\mu}{\theta} = \frac{MC(h^c, g^c)}{MD(h^c, r^c, g^c)}. \quad (26)$$

**Proof of Proposition 3:** Combine the two conditions in (25).  $\square$

This condition means that farmers are indifferent between the two water sources. They will use rainwater or groundwater indifferently.

### 3.2 Asymmetric Stationary Nash Equilibria

To characterize the various possible types of asymmetric SNE, it is convenient to define three different groups of farmers: (i)  $n_G \geq 0$  farmers are fully specialized in groundwater pumping (group  $G$ ), (ii)  $n_R \geq 0$  farmers are fully specialized in rainwater harvesting (group  $R$ ), and (iii)  $n_B \geq 0$  farmers use both groundwater and rainwater (group  $B$ ).<sup>7</sup> We use superscript  $a$  to denote the (asymmetric) SNE values.

We need to distinguish the first order conditions for these three groups of farmers. Formally, the optimal choice of a farmer who uses groundwater only, i.e.  $r_G^a = 0$  and  $g_G^a > 0$ , is characterized by the following necessary conditions (with the slackness condition,  $\lambda_{gG}^a = 0$ ):

$$\mu F'(\mu g_G^a) - C_g(h^a, g_G^a) - p_G^a = 0, \quad (27)$$

$$\theta F'(\mu g_G^a) - D'(0) - \rho p_G^a(t) + \lambda_{rG}^a = 0, \quad (28)$$

$$\dot{p}_G^a = 0 = \delta p_G^a + C_h(h^a, g_G^a), \quad (29)$$

where  $p_G^a$  denotes the shadow price of the farmers who use groundwater only.

A farmer who uses rainwater only, i.e.  $r_R^a > 0$  and  $g_R^a = 0$ , is characterized by the following necessary conditions (with the slackness condition,  $\lambda_{rR}^a = 0$ ):

$$\mu F'(\theta r_R^a) - C_g(h^a, 0) - p_R^a + \lambda_{gR}^a = 0, \quad (30)$$

$$\theta F'(\theta r_R^a) - D'(r_R^a) - \rho p_R^a = 0, \quad (31)$$

$$\dot{p}_R^a = 0 = \delta p_R^a + C_h(h^a, 0), \quad (32)$$

where  $p_R^a$  is the shadow price for the farmers who use rainwater only.

The choice of a farmer who uses both rainwater and groundwater, i.e.  $r_B^a > 0$  and  $g_B^a > 0$ , is characterized by the following necessary conditions (with the two slackness

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<sup>7</sup>Asymmetric equilibria are such that at least two of these three groups have at least one member. Otherwise the equilibrium would be a symmetric equilibrium.

conditions,  $\lambda_{rB}^a = \lambda_{gB}^a = 0$ ):

$$\mu F'(\theta r_B^a) - C_g(h^a, g_B^a) - p_B^a = 0, \quad (33)$$

$$\theta F'(\theta r_B^a) - D'(r_B^a) - \rho p_B^a = 0, \quad (34)$$

$$\dot{p}_B^a = 0 = \delta p_B^a + C_h(h^a, g_B^a), \quad (35)$$

where  $p_B^a$  is the shadow price for the farmers who use both rainwater and groundwater.

Finally, the water table dynamics is affected by the resource use of all the farmers (14):

$$\dot{h}^a = 0 = \rho R - [n_G g_G^a + n_B g_B^a] - \rho [n_R r_R^a + n_B r_B^a]. \quad (36)$$

First consider the group of farmers who use rainwater only, group  $R$  and remark that  $p_R = 0$  because  $C_g(h(t), 0) = 0$ .

**Proposition 4** *In any asymmetric SNE, the shadow price for the farmers who use groundwater only or both groundwater and rainwater is strictly positive,  $p_G^a, p_B^a > 0$  whereas the equilibrium shadow price for the farmers who use rainwater only is null,  $p_R^a = 0$ .*

**Proof of Proposition 4:** The proof of this result is straightforward using  $C_g(h(t), 0) = 0$ ,  $C_{gh} < 0$  and the necessary conditions.  $\square$

The farmers who use groundwater suffer from a negative externality generated by groundwater and rainwater use, because the use of groundwater and rainwater decreases the level of the water table, which in turn increases the groundwater pumping costs. Differently, the user cost of the farmers of group  $R$  is null and then they do not care about the level of the water table. They escape the externality because they harvest rainwater instead of pumping groundwater.

Now, let us further analyse the necessary conditions for each group of farmer. Using (31) and  $p_R^a = 0$ , we find that the marginal benefit of rainwater must equal its marginal cost:

$$\theta F'(\theta r_R^a) = D'(r_R^a) \quad (37)$$

and using (30),  $p_R^a = 0$ , and  $\lambda_{gR}^a \geq 0$ , we find that the marginal cost of groundwater must



be larger than its marginal benefit:

$$C_g(h^a, 0) \geq \mu F'(\theta r_R^a) \quad (38)$$

We use equations (37) and (38) and obtain a necessary condition for the choice of the group who uses rainwater only. This condition is:

$$\frac{\mu}{\theta} \leq \frac{C_g(h^a, 0)}{D'(r_R^a)}. \quad (39)$$

This condition states that the productivity ratio has to be smaller than the marginal cost ratio. In fact, the choice of the farmers of group  $R$  is driven by short term effects only. Indeed, because they do not use groundwater, they do not take into account any future increase in groundwater pumping, and their shadow price is nil.

Now consider group  $G$  (the farmers who use groundwater only). Using equations (27) and (29), we find that the marginal benefit of groundwater must be equal to the long term marginal cost of groundwater:

$$\mu F'(\mu g_G^a) = MC(h^a, g_G^a), \quad (40)$$

and using condition (28) and  $\lambda_{rG}^a \geq 0$ , we find that the full marginal cost of rainwater must be larger than the marginal benefit of rainwater:

$$MD(h^a, 0, g_G^a) \geq \theta F'(\mu g_G^a) \quad (41)$$

Further, combining equation (40) and (41) leads to the following necessary condition:

$$\frac{\mu}{\theta} \geq \frac{MC(h^a, g_G^a)}{MD(h^a, 0, g_G^a)}. \quad (42)$$

This condition states that the relative marginal productivity of groundwater is larger than its relative full marginal costs. This condition is similar to the condition for the groundwater pumping SNE but the equilibrium values differ. Indeed, in the present case, the level of the water table depends on the choice of the three groups of farmers (group  $G$ , group  $R$  and group  $B$ ) and they use different mix of groundwater and rainwater whereas

the level of the water table depends on the choice of the unique group of farmer, group  $G$ , at the groundwater pumping SNE.

Finally, considering the necessary conditions (33), (34) and (35) regarding the choice of farmers in group  $B$ , we find the following necessary condition:

$$\frac{\mu}{\theta} = \frac{MC(h^a, g_B^a)}{MD(h^a, r_B^a, g_B^a)}. \quad (43)$$

Thus, the farmers of group  $B$  must be indifferent between using rainwater or groundwater: the relative productivity must be equal to the relative full marginal cost. This condition is similar to the necessary condition for a conjunctive use SNE but the equilibrium values differ.

The following proposition summarizes our results:

**Proposition 5** *If an asymmetric SNE exists, then we must have:*

$$\frac{MC(h^a, g_G^a)}{MD(h^a, 0, g_G^a)} \underset{(I1)}{\leq} \frac{\mu}{\theta} \underset{(E)}{=} \frac{MC(h^a, g_B^a)}{MD(h^a, r_B^a, g_B^a)} \underset{(I2)}{\leq} \frac{C_g(h^a, 0)}{D'(r_R^a)},$$

and at least two groups are non empty. The left-hand-side inequality (I1) is necessary only if  $n_G > 0$ , condition (E) is necessary only if  $n_B > 0$  and the right-hand side inequality (I2) is necessary only if  $n_R > 0$ .

**Proof of Proposition 5:** The proof lies in the reasoning above the Proposition.  $\square$

To provide more intuition on the possible existence of asymmetric equilibria, let us consider the case where some farmers use rainwater only. We know that they face no user cost and they do not take into account the depletion of the aquifer. In this context, an asymmetric SNE exists only if some farmers still pump groundwater. The following result provides a necessary condition:

**Corollary 1:** *Assume that the marginal cost of rainwater is constant,  $D'(r) = K$ . If an asymmetric SNE such that the group of farmers using rainwater only is non empty ( $n_R > 0$ ) exists, then:*

$$\frac{\mu}{\theta} \geq \frac{1}{\rho}. \quad (44)$$

**Proof of Corollary 1:** In the Appendix.  $\square$

This result states that when some farmers use rainwater only, it is still optimal for some other farmers to pump groundwater if the relative productivity ratio of groundwater is larger than  $1/\rho$ , that is the relative groundwater recharge rate of the aquifer.

### 3.3 Feedback strategies

Let us now describe how the main results of the present section are affected when the farmers use feedback strategies. Assuming that the farmers use feedback strategies means that they choose strategies,  $(g_i, w_i)$ , that depend on the level of the water table,  $h$ . Each farmer knows that the other farmers base their strategies on the level of the water table. Consider the case of two farmers (denoted 1 and 2), the SNE groundwater and rainwater use levels of farmer  $i$  are of the form  $g_i = g_i(h)$ ,  $r_i = r_i(h)$  and the Hamiltonian can be written as:

$$H_i = F(W_i) - C(h, g_i) - D(r_i) + p_i \left[ \rho \left( R - r_i - \sum_{j \neq i} r_j(h) \right) - (g_i + \sum_{j \neq i} g_j(h)) \right].$$

The first order conditions still write (9–14) except condition (13). This will affect the full marginal costs as follows:

*Remark 1 :* When the farmers use feedback strategies and  $N = \{1, 2\}$ , the results of Propositions 1 to 5 are unchanged if the full marginal costs of player  $i$  are (re)defined as follows:

$$MC(h, g_i) = C_g(h, g_i) - \frac{1}{\delta + \frac{\partial g_j}{\partial h} + \rho \frac{\partial r_j}{\partial h}} C_{hq}(h, g_i), \quad (45)$$

and,

$$MD(h, r_i, g_i) = D'(r) - \rho \frac{1}{\delta + \frac{\partial g_j}{\partial h} + \rho \frac{\partial r_j}{\partial h}} C_h(h, g), \quad (46)$$

with  $i \neq j$  and  $i = 1, 2$ .

As each farmer knows that the other farmers base their strategies on the level of the water table, they take into account the induced change in the water use of the other farmers. The only difference with the openloop case is the additional term,  $\frac{\partial g_j}{\partial h} + \rho \frac{\partial r_j}{\partial h}$ , that affect the full marginal costs.

## 4 Anti-coordination and Coordination Problems

The negative externality generated by the use of groundwater and rainwater on the groundwater users (through the decrease of the water table and the increase in the pumping cost) raises the problem of the regulation of this economy. We do not want to discuss the nature of the objective and the instruments of the regulator, but we want to address some potential strong difficulties that a regulator may face in this context. We use an example and show four results: (i) that several symmetric and asymmetric equilibria may coexist (when the recharge is small), (ii) that these equilibria are stable, (iii) that the level of the water table differs across these equilibria, and (iv) that the equilibrium levels of rainwater and groundwater use varies in opposite directions with respect to the fundamentals of the model.

We assume the following functional forms. The production function is quadratic:

$$F(W_i(t)) = W_i(t) - \frac{1}{2}(W_i(t))^2, \quad (47)$$

and the two cost functions, the pumping cost and the rainwater harvesting cost, are specified as follows:

$$C(h(t), g_i(t)) = (c - h(t))g_i(t) \text{ with } c > 0, \quad (48)$$

$$D(r_i(t)) = Kr_i(t) \text{ with } K > 0. \quad (49)$$

Note that the water table is supposed to be upper-bounded (in this example), that is if the aquifer reaches its maximum height,  $c = h$ , groundwater pumping is no more costly. To avoid unrealistic cases where the net benefit is always decreasing in the amount of rainwater used, we assume that  $\theta > K$ , i.e. the marginal contribution of rainwater in the production process must be higher than its marginal cost.

The first order conditions for farmer  $i$  (conditions 9–14) can be rewritten:

$$\frac{\partial L_i}{\partial g_i} = \mu - \mu W_i - (c - h) - p_i + \lambda_{gi} = 0, \quad (50)$$

$$\lambda_{gi} \geq 0, \quad \lambda_{gi} g_i = 0, \quad (51)$$

$$\frac{\partial L_i}{\partial r_i} = \theta - \theta W_i - K - \rho p_i + \lambda_{ri} = 0, \quad (52)$$

$$\lambda_{ri} \geq 0, \quad \lambda_{ri} r_i = 0, \quad (53)$$

$$\dot{p}_i = 0 = \delta p_i - \frac{\partial L_i}{\partial h} = \delta p_i - g_i, \quad (54)$$

$$\dot{h} = 0 = \rho R - \rho \sum_{i=1}^N r_i - \sum_{i=1}^N g_i, \quad (55)$$

where  $W_i = \mu g_i + \theta r_i$ .

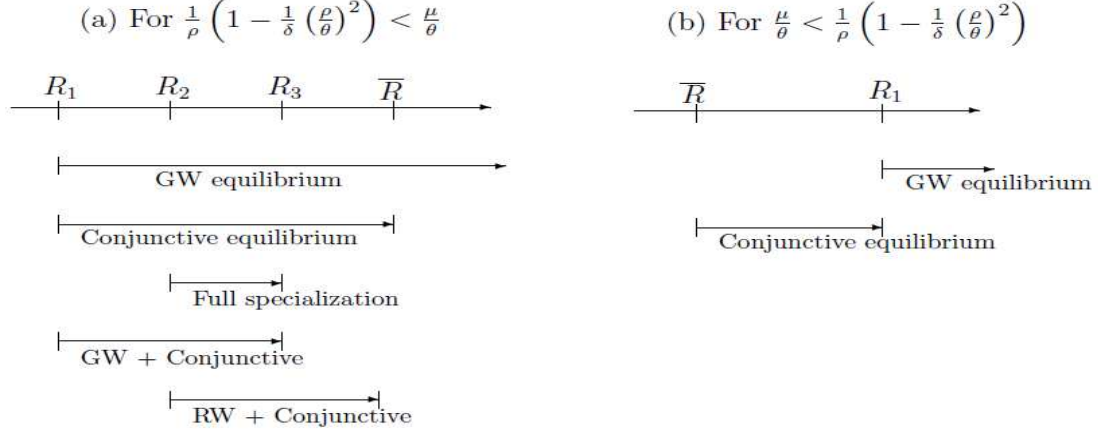
In the following, we proceed as follows: we analyze the necessary conditions (50–55) and distinguish the various types of SNE in our example. For each type of SNE, we can translate the necessary conditions into intervals to which the recharge rate must belong to. We can then use the conditions on the recharge rate to check whether two different equilibria may coexist (we check whether the intervals overlap). Figure 1 summarizes the findings discussed below about the existence of different equilibria when the recharge rate varies.<sup>8</sup> The graph on the left hand side summarizes the conditions under which multiple equilibria may exist, including asymmetric equilibria. The graph on the right hand side summarizes the conditions under which the SNE is unique.

To construct the graphs, we first derived the conditions for the existence of a ground-water pumping SNE. We can show that this type of equilibria exists (see the Appendix) only if the productivity of groundwater  $\mu$  is sufficiently large and/or the productivity of rainwater  $\theta$  is sufficiently small ( $R > \frac{N\delta(\theta-K)}{\rho(\mu\delta\theta+\rho)} \equiv R_1$ ). We then derive the conditions for the existence of a conjunctive use SNE. We show that farmers may use the two water sources conjunctively only in two specific situations: when the recharge level is lower than in the previous case ( $R < R_1$ ) and the relative productivity of groundwater is sufficiently low, as shown on the right hand side of the graph of Figure 1; or when the recharge level is larger ( $R_1 < R$ ) and the relative productivity of groundwater is high, as shown on the graph on

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<sup>8</sup>Figure 1 represents the case where in which there are two (active) groups at the asymmetric SNE.

Figure 1: Co-existence of various stationary Nash equilibria



the left hand side of Figure 1. We also show (in the Appendix) that when the relative productivity of groundwater is low, the groundwater use and the conjunctive use SNE cannot coexist (see the graph on the right hand side of Figure 1). This means that there are no coordination problems in that situation. We then consider three types of asymmetric SNE: equilibria in which the farmers fully specialize in the use of one of the two resources (“Full specialization”,  $n_G, n_R > 0$  and  $n_B = 0$ ); equilibria in which a group of farmers use both groundwater and rainwater and one group of farmers uses groundwater only (“GW+Conjunctive”,  $n_G, n_B > 0$  and  $n_R = 0$ ); and equilibria in which one group of farmers uses both sources and one group uses rainwater only (“RW+Conjunctive”,  $n_R, n_B > 0$  and  $n_G = 0$ ).<sup>9</sup> We show that they can exist only if the recharge is sufficiently low ( $R < \bar{R} \equiv \frac{N(\theta-K)}{\theta^2}$ ).<sup>10</sup>

The following proposition summarizes the main results derived through the example:

**Proposition 6** *Consider the example (equations 47–49): If a SNE exists*

(a) *If the recharge  $R$  is sufficiently large, the SNE is unique and it is the groundwater use SNE.*

(b) *If  $\frac{\mu}{\theta} < \frac{1}{\rho} \left(1 - \frac{1}{\delta} \left(\frac{\rho}{\theta}\right)^2\right)$ , the SNE is unique and symmetric.*

<sup>9</sup>Another type of asymmetric equilibrium may also exist. For necessary conditions, see “RW+GW+Conjunctive” in Appendix (Material for the example).

<sup>10</sup>The exact levels of the thresholds  $R_2$  and  $R_3$  are given in the Appendix (Material for the example).

(c) If the recharge  $R$  is small and  $\frac{\mu}{\theta} > \frac{1}{\rho} \left(1 - \frac{1}{\delta} \left(\frac{\rho}{\theta}\right)^2\right)$ , there are at least two symmetric and one asymmetric SNE.

Hence, our results indicate that farmers choose heterogenous levels of rainwater and groundwater in a SNE and that this arises for small recharge levels. Another implication is that many SNE may coexist.

Coordination and anti-coordination problems are only determinant if identified equilibria are stable. We hence examine the types of SNE that may exist in the example.

**Proposition 7** *The SNE exist and are stable. The groundwater use SNE and the specialized asymmetric SNE are saddle points and the other SNE are degenerated (one of the eigenvalues is null).*

**Proof of Proposition 7:** In the Appendix.  $\square$

Hence, the coordination and anti-coordination problems we highlighted relate to stable equilibria.

There exist numerical examples for which all possible SNE illustrated in figure 1 can coexist, which is illustrative of an (anti-)coordination problem (for instance, this is true for the following set of numerical values:  $c = 50$ ,  $\delta = 0.02$ ,  $K = 2.2$ ,  $M = 1$ ,  $\mu = 27$ ,  $N = 2$ ,  $R = 0.1$ ,  $\rho = 0.2$ ,  $\theta = 2.7$ ). We can compute the level of the aquifer, the groundwater and rainwater used by each group of farmers and the corresponding shadow-prices for all equilibria. In particular, we can analyze the levels of the stationary equilibrium water table in the different cases. In this example, we can show that the SNE water table level is lower and lower as more and more groups of farmers use rainwater rather than groundwater. Namely, the level of the water table is higher at the groundwater pumping SNE than at the “GW+conjunctive” SNE, which is higher than the water table level at the “Full specialization” SNE, which is higher than the water table level at the conjunctive use SNE, which is higher than the water table level at the “RW+conjunctive” SNE.

For any regulator, the problem of coordination and anti-coordination could be less crucial if policy interventions (that correspond to changing the parameter values of our model) induce changes in the same direction, whatever the equilibrium considered. We consider changes in the infiltration rate,  $\rho$ , and the recharge,  $R$ . Indeed, some policies may favour plantations which in turn favour infiltration rates (increase  $\rho$ ). Other policies

may divert water from other uses to the agricultural sector (increase  $R$ ). The following proposition present our comparative statics results:

**Proposition 8** *At the groundwater use and at the full specialization asymmetric SNE, an increase in the recharge  $R$  or in the infiltration rate  $\rho$  induces an **increase** in groundwater pumping:*

$$\begin{aligned}\frac{\partial g^{GW}}{\partial \rho} &> 0, & \frac{\partial g^{GW}}{\partial R} &> 0, \\ \frac{\partial g^a}{\partial \rho} &> 0, & \frac{\partial g^a}{\partial R} &> 0.\end{aligned}$$

*However, the signs of the comparative statics are opposite in the conjunctive use SNE. In particular, if  $R < \bar{R}$ , an increase in the recharge  $R$  or in the infiltration rate  $\rho$  induces a **decrease** in groundwater pumping:*

$$\frac{\partial g^c}{\partial \rho} < 0 \quad \frac{\partial g^c}{\partial R} < 0.$$

**Proof of Proposition 8:** In the Appendix (Computations for Figure 1).  $\square$

The effect of policies that affect the infiltration rate,  $\rho$ , and the recharge,  $R$  on the level of groundwater pumping may be difficult to anticipate in light of Proposition 8.<sup>11</sup> Indeed, if the regulator does not know at which equilibrium the economy is, he cannot anticipate the effect of a policy. This result highlights a supplementary difficulty for regulating the use of water.

## 5 Concluding Remarks

In this paper, we considered possible interactions between farmers who can choose between two water supplies (groundwater or rainwater) which have different productive properties and are interdependent. An interesting property of this game is that the farmers who

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<sup>11</sup>We can also check the variations of the water-table level as a function of  $R$  and  $\rho$ . We can show that:  $\frac{\partial h^{GW}}{\partial \rho} > 0$  and  $\frac{\partial h^{GW}}{\partial R} > 0$ . Likewise,  $\frac{\partial h^a}{\partial \rho} > 0$  and  $\frac{\partial h^a}{\partial R} > 0$ . However the variation for the conjunctive use water-table,  $h^c$ , has undetermined signs.



collect rainwater escape the pumping cost externality but generate a negative externality for farmers who pump groundwater.

We obtained three interesting results. First, we show that multiple equilibria can co-exist. For a policy maker, this immediately raises the equilibrium selection issue. This problem is worsened by the second cornerstone of our study. The number of equilibria increases as water becomes scarcer; in other words, with a lower recharge rate. In the context of climate change and increased risk of drought, this model sheds light on the difficulties we may face in terms of water uses and public policy. In addition, comparative statics show that a policy intervention can have opposite effects depending on the equilibrium concerned. The last result of this study is the possible occurrence of asymmetric equilibria in which agents split into different groups in the same way as in anti-coordination games. In particular, it can happen that one group of farmers chooses to harvest rainwater to avoid the pumping cost externality, while the other group still uses groundwater because of better productivity (in spite of an increase in the marginal pumping cost).

Obviously, a number of questions remain to be explored. For instance, we assume that there is no irrigation return flow to the aquifer. It would be interesting to see what happens when water not consumed by crops percolates to the aquifer and, as a result, may modify (i.e. clean a brackish aquifer) the qualitative productive properties of groundwater. It would also be interesting to see if the water table is higher in the open-loop strategy than under the feedback rules, as shown by Negri [14], when agents can choose between two water supplies. This would provide a new opportunity to discuss the externalities involved in such a commons game.

## References

- [1] Burt O.R., (1964), The economics of conjunctive use of ground and surface water, *Hilgardia*, 36(2), p.31-111.
- [2] Chakravorty U., and Umetsu C., (2003), Basinwide water management: a spatial model, *Journal of Environmental Economics and Management*, 45, p.1-12.

- [3] Cummings R. G., (1971), Optimum exploitation of groundwater reserves with salt-water intrusion, *Water Resources Research*, 7(6), p.1415–1424.
- [4] Dockner E.J., Jorgensen S., Long N. V. and Sorger G., (2000), *Differential games in economics and management science*, Cambridge University Press, 350p.
- [5] Engwerda J.C., (1998), On the open-loop Nash equilibrium in LQ-games, *Journal of Economic Dynamics and Control*, 22, p.729–762.
- [6] Gemma M. and Tsur Y., (2007), The stabilization value of groundwater and conjunctive water management under uncertainty, *Review of Agricultural Economics*, 29(3), p.540–548.
- [7] Gisser M. and Sanchez D.A., (1980), Competition versus optimal control in groundwater pumping, *Water Resources Research*, 31, p.638–642.
- [8] IPCC, Climate Change 2007, Synthesis Report, Geneva, Switzerland 2008, 104p.
- [9] Knapp K.C. and Olson L.J., (1995), The economics of conjunctive groundwater management with stochastic surface supplies, *Journal of Environmental Economics and Management*, 28, p.340–356.
- [10] Koundouri P., (2004), Current issues in the economics of groundwater resource management, *Journal of Economic Surveys*, 18(5), p.703–740.
- [11] Krulce D. L., Roumasset J.A. and Wilson T., (1997), Optimal management of a renewable and replacable Resource: The case of coastal groundwater, *American Journal of Agricultural Economics*, 79, p.1218–1228.
- [12] Moreaux M. and Reynaud A., (2004), Optimal management of a coastal aquifer and a substitute resource, *Water Resources Research*, 40, p.1–10.
- [13] Moreaux M. and Reynaud A., (2006), Urban freshwater needs and spatial cost externalities for coastal aquifers: A theoretical approach, *Regional Science and Urban Economics*, 36, p.163–186.

- [14] Negri D.H., (1989), The common property aquifer as a differential game, *Water Resources Research*, 25(1), p.9–15.
- [15] Ostrom, E.: Governing the Commons - The Evolution of Institutions for Collective Action, Cambridge University Press, New York 1990.
- [16] Pongkijvorasin S. and Roumasset J., (2007), Optimal conjunctive use of surface and groundwater with recharge and return flows: dynamic and spatial patterns, *Review of Agricultural Economics*, 29(3), p.531–539.
- [17] Provencher B. and Burt O., (1993), The externalities associated with the common property exploitation of groundwater, *Journal of Environmental Economics and Management*, 24, p.139–158.
- [18] Roseta-Palma C., (2003), Joint quantity/quality management of groundwater, *Environmental and Resource Economics*, 26, p.89–106.
- [19] Roumasset J.A. and Wada C.A., (2012), Ordering the extraction of renewable resources: The case of multiple aquifers, *Resource and Energy Economics* 34, p.112–128.
- [20] Rubio S.J. and Casino B., (2001), Competitive versus efficient extraction of a common property resource: The groundwater case, *Journal of Economic Dynamics and Control*, 25, p.1117–1137.
- [21] Rubio S.J. and Casino B., (2003), Strategic behavior and efficiency in the common property extraction of groundwater, *Environmental and Resource Economics*, 26(1), p.73–87.
- [22] Stahn H. and Tomini A., (2009), A drop of rainwater against a drop of groundwater: does rainwater harvesting really spare groundwater?, DT GREQAM 2009-43.
- [23] Stahn H. and Tomini A., (2011), Rainwater harvesting under endogenous capacity of storage: a solution to aquifer preservation?, DT GREQAM 2011-37.

- [24] Tsur Y. and Graham-Tomasi T., 1991, The buffer value of groundwater with stochastic surface water supplies, *Journal of Environmental Economics and Management*, 21, p.201–224.
- [25] Vives X., (1999), Oligopoly pricing, The MIT Press, 425p.
- [26] Zeitouni N. and Dinar A., (1997), Mitigating negative water quality and quality externalities by joint management of adjacent aquifers, *Environmental and Resource Economics*, 9, p.1–20.

## 6 Appendix

### 6.1 Proofs

**Proof of Corollary 1:** Let  $RC$  be the cost ratio function:

$$RC(g) = \frac{MC(h^a, g)}{MD(h^a, r, g)} \quad (56)$$

$$= \frac{1}{\rho} \frac{C_g(h^a, g) - \frac{1}{\delta} \cdot C_h(h^a, g)}{\frac{1}{\rho} K - \frac{1}{\delta} \cdot C_h(h^a, g)}. \quad (57)$$

Notice that  $RC'(g) \leq 0$  if and only if

$$\left(-\frac{1}{\delta} \cdot C_{gh}\right) \left(\frac{1}{\rho} K - C_g\right) + C_{gg} \times \left(\frac{1}{\rho} K - \frac{1}{\delta} \cdot C_h\right) \leq 0. \quad (58)$$

The condition in Proposition 5 can be rewritten as follows:

$$RC(g_G^a) \leq \frac{\mu}{\theta} = RC(g_B^a) \leq RC(0). \quad (59)$$

Assume that  $RC' \geq 0$  for all  $0 \leq g \leq \max\{g_G^a, g_B^a\}$ . Using (59), we have  $RC(\max\{g_G^a, g_B^a\}) \leq RC(0)$  and then  $\max\{g_G^a, g_B^a\} \leq 0$ , i.e.  $n_B = n_G = 0$ . This is impossible in an asymmetric SNE. Hence, there exists  $0 \leq \tilde{g} \leq \max\{g_G^a, g_B^a\}$  such that  $RC'(\tilde{g}) < 0$ . Using (58) and  $C_{gh} < 0$ , we have:

$$\frac{1}{\rho} K - C_g(h^a, \tilde{g}) \leq 0. \quad (60)$$

Using  $C_{gg} \geq 0$ , we find

$$\frac{1}{\rho} K \leq C_g(h^a, g_G^a) \text{ and } \frac{1}{\rho} K \leq C_g(h^a, g_B^a). \quad (61)$$

Since  $C_h < 0$ , we have

$$\frac{1}{\rho} \leq RC(g_G^a) \text{ and } \frac{1}{\rho} \leq RC(g_B^a). \quad (62)$$

Using (59), we conclude that  $\frac{\mu}{\theta} \geq \frac{1}{\rho}$ .  $\square$

**Proof of Proposition 7:** As we are in a system of linear differential equations, local stability implies global stability. In addition, proving stability, we confirm the existence of the different open loop SNE (see Engwerda [5]). For open loop SNE the best stability behavior that we can expect is saddle point stability.

Saddle point stability of a dynamic system means that there exists a saddle manifold of dimension equal to the number of state variables, so that, if the initial conditions of the adjoint variables are appropriately chosen, the solution of the canonical system starts from the stable manifold and converges to the SNE (see Dockner et al. [4] page 256). We analyze the Jacobian matrix given by :

$$\begin{pmatrix} \frac{\partial \dot{h}}{\partial h} & \frac{\partial \dot{h}}{\partial p} \\ \frac{\partial \dot{p}}{\partial h} & \frac{\partial \dot{p}}{\partial p} \end{pmatrix}.$$

We consider each kind of equilibria successively:

First consider the groundwater use (symmetric) equilibria. Using the first order conditions we have

$$\mu(1 - \mu g) - c + h - p = 0,$$

or,

$$g = \frac{\mu - c + h - p}{\mu^2}.$$

The dynamic system becomes

$$\dot{p} = \delta p - g \quad \text{and} \quad \dot{h} = \rho R - Ng,$$

and the characteristic matrix is

$$\begin{bmatrix} -\frac{N}{\mu^2} & \frac{N}{\mu^2} \\ -\frac{1}{\mu^2} & \delta + \frac{1}{\mu^2} \end{bmatrix}$$

The dynamic system is stable, it is a saddle point, because the determinant is negative ( $-N\delta/\mu^2 < 0$ ).

Second, consider the (symmetric) conjunctive use equilibria. From the first order conditions, we have

$$\mu(1 - \mu g - \theta r) - c + h - p = 0, \quad \mu(1 - \mu g - \theta r) - K - \rho p = 0,$$

and then,

$$\frac{p - h + c}{\mu} = \frac{K + \rho p}{\theta}, \tag{63}$$

and deriving w.r.t.  $t$ , we obtain

$$\frac{\mu \rho \dot{p}}{\theta} + \dot{h} - \dot{p} = 0. \tag{64}$$

The dynamic system is then

$$\dot{p} = \delta p - g, \quad \dot{h} = \rho R - \rho N r - Ng.$$

Replacing in (64) we obtain a new equation in  $h, p, g, r$ . Using this equation with  $\mu(1 - \mu g) - c + h - p = 0$ ,

we obtain  $g, r$  in function of  $h, p$ .

The characteristic matrix becomes

$$\begin{bmatrix} -\frac{\rho N(\theta - \mu\delta)}{\mu\theta(\mu\rho N - \mu\delta - N\theta + \theta)} & -\frac{N(\mu\delta - \theta)(\rho\mu^2\delta - \mu\delta\theta + \rho)}{\mu\theta(\mu\rho N - \mu\delta - N\theta + \theta)} \\ -\frac{\rho N}{\mu(\mu\rho N - \mu\delta - N\theta + \theta)} & \frac{N(\rho\mu^2\delta - \mu\delta\theta + \rho)}{\mu(\mu\rho N - \mu\delta - N\theta + \theta)} \end{bmatrix}$$

The determinant is 0 and this implies that the characteristic matrix has an eigenvalue that equals to zero. We can deduce that this zero eigenvalue is due to equation (63), in which  $h$  and  $p$  are linearly independent. Independently of the sign of the other eigenvalue, the only way to define initial conditions for the adjoint variable is to remain in the SNE (if it exists). In other words the SNE is the only stable manifold. The conjunctive use SNE is an equilibrium of our problem if we find an optimal trajectory starting at  $h(0)$  that arrives at this manifold.

Third, consider the asymmetric equilibria such that  $n_G \geq 1$  farmers use groundwater only and  $n_R \geq 1$  farmers use rainwater only. The first order conditions give:

$$g = \frac{\mu - c + h - p}{\mu^2}, \quad r = \frac{\theta - K}{\theta^2}.$$

and the dynamic system becomes:

$$\dot{p} = \delta p - g, \quad \dot{h} = \rho R - \rho n_R \frac{\theta - K}{\theta^2} - n_G.$$

This system is stable (saddle point).

Fourth, consider the other types of asymmetric equilibria, i.e. asymmetric equilibria where some farmers use both rainwater and groundwater. As in the symmetric conjunctive use equilibrium, the first order conditions imply that  $h(t)$  and  $p_B(t)$  for the group that uses both sources are proportional. Hence, the conclusion is the same as in the (symmetric) conjunctive equilibrium.

## 6.2 Computations for Figure 1:

Let us consider the simple functional forms (for production 47 and costs 49) introduced above.

*The groundwater use SNE:*

The SNE conditions (19), (20) and (21) for the groundwater use SNE are given by :

$$g^{GW} = \frac{\rho R}{N}, \quad (65)$$

$$p^{GW} = \frac{\rho R}{\delta N}, \quad (66)$$

$$h^{GW} = c - \mu + \frac{\rho R}{N} \left( \frac{1 + \delta \mu^2}{\delta} \right), \quad (67)$$

$$\lambda_r^{GW} = \frac{\rho R}{N} \left( \frac{\delta \theta \mu + \rho}{\delta} \right) - (\theta - K), \quad (68)$$

Note that,  $\frac{\partial g^{GW}}{\partial \rho} > 0$ ,  $\frac{\partial g^{GW}}{\partial R} > 0$  and the positivity of the Lagrangian multiplier  $\lambda_r^{GW} \geq 0$  directly implies  $R \geq \frac{\delta N(\theta - K)}{\rho(\mu \delta \theta + \rho)} \equiv R_1$ .

*The conjunctive use SNE:* The closed form solution to the conjunctive use SNE can be shown to result in the following SNE values :

$$g^c = \left( \frac{\delta \rho}{\theta \mu \delta \rho - \theta^2 \delta + \rho^2} \right) \left( \frac{N(\theta - K) - \theta^2 R}{N} \right), \quad (69)$$

$$r^c = \frac{R}{N} - \frac{1}{N} \left[ \frac{\delta (N(\theta - K) - \theta^2 R)}{\theta \mu \delta \rho - \theta^2 \delta + \rho^2} \right], \quad (70)$$

$$p^c = \frac{\rho (N(\theta - K) - \theta^2 R)}{N (\theta \mu \delta \rho - \theta^2 \delta + \rho^2)}, \quad (71)$$

$$h^c = c - \mu + \frac{\mu}{N} \left[ g^c \left( \frac{\rho \mu - \theta}{\rho} \right) + \theta R \right] + \frac{g^c}{\delta}. \quad (72)$$

It is straightforward that equation (69) implies two situations that ensure  $g^c > 0$ , depending on the value of  $R$ . In the first situation, the productivity of groundwater is sufficiently small (and/or the productivity of rainwater is sufficiently large) and the recharge is sufficiently large, in the second situation the productivity of groundwater is sufficiently large (and/or the productivity of rainwater is sufficiently small) while the recharge is sufficiently small.

We have to check the various ranges allowing to have positive values in the long run for rainwater collection, the shadow price and the water table,  $\{w_r^c; p^c; h^c\}$ , since all of them depend on the parameters of  $\mu$  and  $R$ . The positivity of  $g^c$  implies two cases : either  $R < \bar{R} \equiv \frac{N(\theta - K)}{\theta^2}$  and  $0 < \frac{\theta^2 \delta - \rho^2}{\delta \theta} < \mu$ , or  $\bar{R} \equiv \frac{N(\theta - K)}{\theta^2} < R$  and  $0 < \mu < \frac{\theta^2 \delta - \rho^2}{\delta \theta}$  (which also implies  $0 < \theta^2 \delta - \rho^2$ ). The positivity of  $r^c$  implies two cases : either  $R_1 \equiv \frac{N \delta (\theta - K)}{\rho(\mu \delta \theta + \rho)} < R$  and  $0 < \frac{\theta^2 \delta - \rho^2}{\delta \theta} < \mu$ , or  $R < R_1 \equiv \frac{N \delta (\theta - K)}{\rho(\mu \delta \theta + \rho)}$  and  $0 < \mu < \frac{\theta^2 \delta - \rho^2}{\delta \theta}$ .



(which also implies  $0 < \theta^2\delta - \rho^2$ ). When  $\frac{\theta^2\delta - \rho^2}{\delta\theta} < \mu$ , we have  $\bar{R} - R_1 = \mu\theta\delta\rho - \theta^2\delta + \rho^2 > 0$ . Therefore, in this case,  $R \in (R_1; \bar{R})$ . When  $\mu < \frac{\theta^2\delta - \rho^2}{\delta\theta}$ , we observe that  $\bar{R} - R_1 = \mu\theta\delta\rho - \theta^2\delta + \rho^2 < 0$ . Therefore, in this case,  $R \in (\bar{R}; R_1)$ .

Finally, if a conjunctive use SNE exists then either:

$$R_1 < R < \bar{R} \text{ and } \frac{1}{\rho} \left( 1 - \frac{1}{\delta} \left( \frac{\rho}{\theta} \right)^2 \right) < \frac{\mu}{\theta}, \quad (73)$$

or,

$$\bar{R} < R < R_1 \text{ and } \frac{\mu}{\theta} < \frac{1}{\rho} \left( 1 - \frac{1}{\delta} \left( \frac{\rho}{\theta} \right)^2 \right). \quad (74)$$

Moreover it is easy to check that when  $R < \bar{R}$ ,  $\frac{\partial g^c}{\partial \rho} < 0$ ,  $\frac{\partial g^{GW}}{\partial R} < 0$ . Let us now consider the different types of *asymmetric SNE* characterization.

“*Full specialization:*”

Conditions (37), (38), (40), and the two Lagrangian multipliers can now be written as follows :

$$r_R^a = \frac{\theta - K}{\theta^2}, \quad (75)$$

$$g_G^a = \frac{\rho}{n_G} \left[ R - n_R \left( \frac{\theta - K}{\theta^2} \right) \right], \quad (76)$$

$$h^a = c - \mu + \left( \frac{1 + \mu^2\delta}{\delta} \right) \frac{\rho}{n_G} \left[ R - n_R \frac{\theta - K}{\theta^2} \right], \quad (77)$$

$$\lambda_{rG}^a = \left( \frac{\rho}{\delta} + \mu\theta \right) \frac{\rho}{n_G} \left[ R - n_R \frac{\theta - K}{\theta^2} \right] - (\theta - K), \quad (78)$$

$$\lambda_{gR}^a = \mu \frac{\theta - K}{\theta} - \left( \frac{1 + \mu^2\delta}{\delta} \right) \frac{\rho}{n_G} \left[ R - n_R \frac{\theta - K}{\theta^2} \right]. \quad (79)$$

As previously some supplementary necessary conditions have to be checked in order to ensure positive values for the two Lagrangian multipliers  $\lambda_{rG}^a$  and  $\lambda_{gR}^a$ . The investigation of these conditions (provided below) outlines that the asymmetric SNE requires that the natural recharge belongs to a critical interval and the productivity of groundwater pumping is sufficiently high.

All the Lagrangian multipliers must be positive :

$$\lambda_{rG}^a = \left( \frac{\rho}{\delta} + \mu\theta \right) \frac{\rho}{n_G} \left[ R - n_R \frac{\theta - K}{\theta^2} \right] - (\theta - K) \geq 0, \quad (80)$$

$$\lambda_{gR}^a = \mu \frac{\theta - K}{\theta} - \left( \frac{1 + \mu^2\delta}{\delta} \right) \frac{\rho}{n_G} \left[ R - n_R \frac{\theta - K}{\theta^2} \right] \geq 0. \quad (81)$$

Condition (81) implies that  $R < \frac{\theta-K}{\theta^2} \left[ N - n_G + \frac{\mu\theta n_G \delta}{\rho(1+\mu^2\delta)} \right] \equiv R_3$  and condition (80) implies that  $R > \frac{\theta-K}{\theta^2} \left[ N - n_G + \frac{\theta^2 \delta n_G}{\rho(\rho+\mu\delta\theta)} \right] \equiv R_2$ . We can easily check that  $R_3 - R_1 = n_G \delta \rho \theta (\mu\rho - \theta) \geq 0$  because  $\mu\rho - \theta \geq 0$ . Finally if a specialized asymmetric SNE exists then we have  $R_2 < R < R_3$ . Note that  $\frac{\partial g_G^a}{\partial \rho} > 0$ ,  $\frac{\partial g_G^a}{\partial R} > 0$ .

“*GW+Conjunctive:*” Consider the asymmetric equilibria such that a subset of  $n_G \geq 1$  farmers use groundwater to irrigate while  $n_B \geq 1$  farmers use a combination of both irrigation strategies (and  $n_R = 0$ ). Conditions (37), (38), (40) are now as follows :

$$\lambda_{rG}^a = \frac{\theta(\mu\rho - \theta) [\mu\rho\theta\delta R - N\delta(\theta - K) + \rho^2 R]}{n_B [\mu^2\delta(\mu\rho\theta\delta - \delta\theta^2 + \rho^2) + \rho^2] + N\theta\delta(\mu\rho - \theta)}, \quad (82)$$

$$r_B^a = \frac{(\mu^2\delta + 1)}{\theta(\mu\rho - \theta)} \lambda_{rG}^a, \quad (83)$$

$$g_B^a = \delta \frac{\rho(\mu^2\delta + 1) [n_B(\theta - K) - R\theta^2] + n_G\theta\delta\mu(\theta - K)}{n_B [\mu^2\delta(\mu\rho\theta\delta - \delta\theta^2 + \rho^2) + \rho^2] + N\theta\delta(\mu\rho - \theta)}, \quad (84)$$

$$g_G^a = \delta \frac{n_B(\theta - K) [\rho + \delta\mu(\rho\mu - \theta)] + \theta\rho R(\mu\rho - \theta)}{n_B [\mu^2\delta(\mu\rho\theta\delta - \delta\theta^2 + \rho^2) + \rho^2] + N\theta\delta(\mu\rho - \theta)}. \quad (85)$$

Positivity of the Lagrangian multiplier  $\lambda_{rG}^a \geq 0$  and  $r_B^a > 0$  imply that  $\mu\rho - \theta > 0$ , which directly implies that the denominator  $n_B [\mu^2\delta(\mu\rho\theta\delta - \delta\theta^2 + \rho^2) + \rho^2] + N\theta\delta(\mu\rho - \theta) > 0$ . Moreover, the positivity of this multiplier implies  $R > R_1$ . Positivity of  $g_B^a$  implies that  $R < R_3$ .

We can easily check that  $R_3 - R_1 = \left( \frac{\theta-K}{\theta^2} \right) \left( \frac{N\delta\theta(\rho\mu-\theta)+n_B(\mu^2\theta\delta^2(\mu\rho-\theta)+\rho^2(1+\delta\mu^2))}{\rho^2(\mu\theta\delta+\rho)(\mu^2\delta+1)} \right) > 0$ . Finally, the necessary conditions for this kind of asymmetric SNE are  $\mu \geq \frac{\theta}{\rho}$  and  $R_1 < R < R_3$ .

“*RW+Conjunctive:*” Now consider the asymmetric equilibria such that a group of  $n_R \geq 1$  farmers uses rainwater only and a group of  $n_B \geq 1$  farmers uses a combination of both irrigation strategies (and  $n_G = 0$ ). Conditions (37), (38), (40) become :

$$\lambda_{gR}^a = \frac{\rho [N(\theta - K) - R\theta^2] (\mu\rho - \theta)}{\theta n_R (\delta\mu\theta\rho - \theta^2\delta + \rho^2)}, \quad (86)$$

$$r_R^a = \frac{\theta - K}{\theta^2}, \quad (87)$$

$$g_B^a = \frac{\delta\rho [N(\theta - K) - R\theta^2]}{n_R (\delta\mu\theta\rho - \theta^2\delta + \rho^2)} = \lambda_{gR}^a \frac{\delta\theta}{\mu\rho - \theta}, \quad (88)$$

$$r_B^a = \frac{\rho [\theta^2 R - n_B(\theta - K)] (\theta\delta\mu + \rho) - \theta^2\delta n_R(\theta - K)}{\theta^2 n_R (\delta\mu\theta\rho - \theta^2\delta + \rho^2)}. \quad (89)$$

Positivity of the Lagrangian multiplier  $\lambda_{gR}^a \geq 0$  implies with  $g_B^a > 0$  that  $\mu\rho - \theta > 0$ , which implies that  $\delta\mu\theta\rho - \theta^2\delta + \rho^2 > 0$ . Condition (86) and  $\lambda_{gR}^a \geq 0$  implies  $\bar{R} > R$ . Positivity of rainwater use,  $r_B^a > 0$ , implies that  $R_2 < R$ . We then verify that  $\bar{R} > R_2 \Leftrightarrow \rho\theta\delta\mu + \rho^2 > \theta^2\delta$ . Finally, the necessary conditions for the existence of this kind of asymmetric equilibria are  $\mu \geq \frac{\theta}{\rho}$  and  $\bar{R} > R > R_2$ .

“RW+GW+Conjunctive:”

Finally, consider the SNE such that  $n_R \geq 1$  farmers use rainwater only,  $n_G \geq 1$  farmers use groundwater only and  $n_B \geq 1$  use both rainwater and groundwater.

$$\lambda_{rG}^a = \frac{(\rho\mu - \theta)}{\theta} \left( \frac{(\theta - K)(-N\theta^2\delta - n_R(\rho\mu\delta\theta + \rho^2 - \delta\theta^2)) + \rho\theta^2 R(\theta\delta\mu + \rho)}{n_B(\theta\delta\mu + \rho)(\rho + \rho\mu^2\delta - \theta\delta\mu) + (n_B + n_G)\theta\delta(\rho\mu - \theta)} \right), \quad (90)$$

$$g_G^a = \frac{\delta}{\theta} \left( \frac{(\theta - K) [\theta(n_B + n_R)(\rho + \mu\delta(\mu\rho - \theta)) - \mu n_R(\delta\mu\theta\rho - \theta^2\delta + \rho^2)] + \rho R\theta^2(\mu\rho - \theta)}{n_B(\theta\delta\mu + \rho)(\rho + \rho\mu^2\delta - \theta\delta\mu) + (n_B + n_G)\theta\delta(\rho\mu - \theta)} \right) \quad (91)$$

$$\lambda_{gR}^a = \frac{(\mu\rho - \theta)}{\theta} \left( \frac{\rho(N(\theta - K) - R\theta^2)(\mu^2\delta + 1) - n_G(\theta - K)[\rho + \mu^2\rho\delta - \theta\delta\mu]}{n_B(\theta\delta\mu + \rho)(\rho + \rho\mu^2\delta - \theta\delta\mu) + (n_B + n_G)\theta\delta(\rho\mu - \theta)} \right), \quad (92)$$

$$r_R^a = \frac{\theta - K}{\theta^2}, \quad (93)$$

$$g_B^a = \frac{\delta\theta}{\rho\mu - \theta} \lambda_{gR}^a, \quad (94)$$

$$r_B^a = \frac{(1 + \mu^2\delta)}{\theta(\rho\mu - \theta)} \lambda_{rG}^a. \quad (95)$$

Condition  $\lambda_{rG}^a \geq 0$  with  $g_B^a > 0$  imply that  $\mu\rho - \theta > 0$ . Condition  $\lambda_{rG}^a \geq 0$  implies that

$$R \geq \left( \frac{\theta - K}{\theta^2} \right) \left[ \frac{N\theta^2\delta(\mu\rho - \theta) + n_R(\mu\rho - \theta)(\delta\theta(\mu\rho - \theta) + \rho^2)}{\rho(\mu\rho - \theta)(\mu\theta\delta + \rho)} \right] \equiv R_4, \quad (96)$$

and condition  $\lambda_g^{a4*} \geq 0$  implies

$$R \leq \left( \frac{\theta - K}{\theta^2} \right) \left[ N - \frac{n_G(\rho + \mu^2\rho\delta - \theta\delta\mu)}{\rho(\mu^2\theta + 1)} \right] \equiv R_5. \quad (97)$$

The positivity of  $g_G^a > 0$  implies

$$R > \left( \frac{\theta - K}{\theta^2} \right) \left[ \frac{-\theta(n_B + n_R)(\rho + \mu^2\delta\rho - \mu\delta\theta) + \mu n_R(\delta\mu\theta\rho - \theta^2\delta + \rho^2)}{\rho(\mu\rho - \theta)} \right] \equiv R_6. \quad (98)$$

We easily check that  $R_5 - R_4 = (n_B + n_G)(1 + \mu^2\delta)(\delta\mu\theta\rho - \theta^2\delta + \rho^2) - n_G(\theta\delta\mu + \rho)(\rho + \rho\mu^2\delta - \theta\delta\mu)$  which is the positive denominator. Similarly for  $R_4 - R_6 > 0$ . Therefore  $R_5 > R_4 > R_6$ .

Finally, the necessary conditions for this kind of asymmetric SNE are  $\mu > \frac{\theta}{\rho}$  and  $R_4 < R < R_5$ .

To construct the graphs of Figure 1, we also use the following result: if  $\mu > \frac{\theta}{\rho}$  we can check that

$$\bar{R} - R_3 = \frac{M(\theta - K)}{\theta^2} \left( \frac{\rho + \mu\delta(\mu\rho - \theta)}{\rho(1 + \mu^2\delta)} \right) > 0, \quad (99)$$

and,

$$R_2 - R_1 = \frac{(N - M)(\theta - K)}{\theta^2} \left( \frac{\rho^2 + \delta\theta(\mu\rho - \theta)}{\rho(\rho + \mu\delta\theta)} \right) > 0. \quad (100)$$

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**Contact :**

Stéphane MUSSARD : [mussard@lameta.univ-montp1.fr](mailto:mussard@lameta.univ-montp1.fr)



