The double spatial lag model

Ghislain Geniaux

The Spatial^2 Lag Model (S2LM)

[1] G. GENIAUX

[1]INRA SAD Ecodeveloppement Avignon (UR 767)

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Plan

Introduction

Several methodological contributions proposed estimators adapted to situations where some economic behaviors can change depending on their location.

Two main approaches in spatial regression:

- spatial regimes ($\longrightarrow$ stratification / segmentation)
- spatially varying coefficients model.

Introduction

The first approaches in this field consisted in parametric models that incorporate the spatial coordinates of observations by different ways, such as the "Trend Surface Analysis" method (Ripley, 2005) and its generalization proposed by Casetti (1997, 1972) under the name of "Variable Expansion" method.

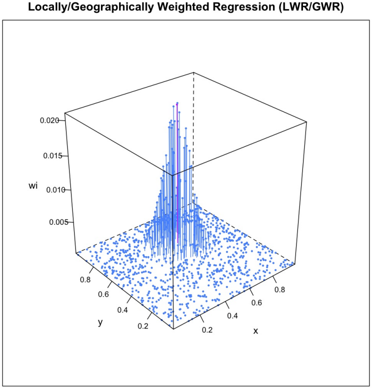
Introduction

Since, the main contributions considered semi-parametric methods like the LWR (McMillen, 1996) and GWR (Brunsdon et al., 1996) which provide a methodological framework to estimate local values of the parameters using local regressions.

Introduction

The idea of LWR/GWR is to estimate as many regressions as focal points i' (data coordinates or other sets of coordinates) by weighting the local regressions with respect to the observations' distance to these focal points. Let's note M the weight matrix with:

$$m_{i'i} = exp(-d^2_{i'i}/h^2)$$



Introduction

By defining $M_{i'}$ a matrix $n \times n$ such that $M_{i'} = diag[m_{i'1}, ..., m_{i'n}]$ with zero for all others elements, the vector of local coefficients (for a focal point) is as follows:

$$\hat{\beta}(u_{i'}, v_{i'}) = (X' M_{i'} X)^{-1} X' M_{i'} Y \tag{1}$$

A notable contribution to take into account spatial autocorrelation in a local regression framework has been made by Pace and LeSage (2004). They propose a Spatial Autoregressive Local Estimation (SALE) which is based on a recursive approach for maximum likelihood estimation of SLM, implying estimates on subsamples related to a neighboring of each observation.

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A MGWR (Fotheringham et al., 1999) model can be expressed as:

$$y_i = \sum_{j=1}^{q} \beta_j x_{ij} + \sum_{j=q+1}^{p} \beta_j(u_{i'}, v_{i'}) x_{ij} + \epsilon_i \quad i', i = 1, 2, .., n \qquad (2)$$

Mei (2004) and Geniaux et al. (2011) propose the use of a two steps estimator for mixed GWR that is easy to implement.

Aim of the paper

We consider a regression model that includes a spatial lag of the endogenous variable and that allows the spatial parameter to vary among locations: $Y_i = \lambda(u_i, v_i)WY_i + X_i\beta + \epsilon_i$

We propose for estimating this model to use a two steps estimator based on a linearized GMM and a mixed GWR.

S2LM

$$Y_i = \lambda(u_i, v_i; M)WY_i + X_i\beta + \epsilon_i \tag{3}$$

$$with \quad w_{ij} = \frac{e^{(-d_{ij}/h_1)^2}}{\sum_j e^{(-d_{ij}/h_1)^2}} \quad \left(resp. \quad m_{ij} = e^{(-d_{ij}/h_2)^2}\right)$$

$\lambda(u_i, v_i)$ could vary du to :

1. True local variations of i's behavior
2. Bad specification of W
3. Bad model specification (SEM, SARAR, non linearities, ...)

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GMM estimator

First consider λ constant. Then a GMM estimator of $\theta = (\lambda, \beta)$ could be used with:

$$\theta = \underset{\theta \in \Theta}{\text{argmin}} \; Q(\theta)$$

Where:

- $Q(\theta) = S_N{}'(\theta)\Sigma_N S_N(\theta)$
- $S_N(\theta) = \frac{1}{N} Z' \tilde{u(\theta)}$
- $Z \in \left[X, WX, W^2X, W^3X\right]$
- Σ_N some positive definite matrix
- $u(\theta)$ the vector of residuals of the regression (3) considering λ constant

Linearized GMM estimator

With a convergent $\hat{\theta}$, we can use a LGMM estimator (KM2008):

$$u(\hat{\theta}_{IV}) = P_Z Y - \hat{\lambda}_{IV} P_Z WY - P_Z X \hat{\beta}_{IV} \tag{4}$$

The gradients of the GMM are quite simple:

$$\begin{aligned} G_{\beta_i} &= P_Z X \quad (5) \\ G_{\lambda_i} &= P_Z WY \quad (6) \end{aligned}$$

To linearize the error term $u_i(\beta, \lambda)$, we use a taylor approximation.

$$\tilde{u}_i(\beta, \lambda) - \tilde{u}_i(\hat{\beta}_{IV}, \hat{\lambda}_{IV}) = G(\theta(u_i, v_i) - \theta_0) \tag{7}$$

where $G(.)$ is a gradient term: $\tilde{u}_i(\beta, \lambda) = G(\theta(u_i, v_i) - \theta_0) + \tilde{u}_i(\hat{\beta}_{IV}, \hat{\lambda}_{IV})$

S2LM stimator

1. In the first step, estimate β_{OLS} by OLS and keep the residuals vector $u(\beta_{OLS}, \lambda = 0)$. Estimate the gradient terms :

$$G_{\beta_i}(\hat{\beta}_{IV}, \hat{\lambda}_{IV}) = X$$
$$G_{\lambda_i}(\hat{\beta}_{IV}, \hat{\lambda}_{IV}) = WY$$

2. In the second step, Regress G_β and G_λ on Z. The predicted values are $\hat{G}_\beta$ and $\hat{G}_\lambda$. Finally, use a Mixed GWR to estimate:

$$u_{(\beta_{\mathbf{OLS}}, \lambda=0)} + G'_\beta \hat{\beta}_{OLS} = \beta \hat{G}_\beta + \lambda(u_i, v_i; M)\hat{G}_\lambda + \epsilon_i$$

MC Settings

We focus our experiments on the ability of the proposed estimator to identify the spatial pattern of $\lambda(u_i, v_i, M)$ when W is known.

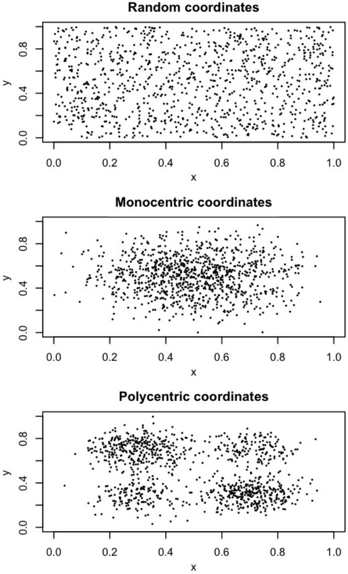
M is considered unkonwn.

MC Settings

An experiment is defined by a quadruplet $(l, h, lambda_c, lambda_v)$

MC Settings: locations l

$l \in (random, monocentric, polycentric)$.



MC Settings: Weighting scheme *wt*

The weighting scheme type for W is an exponential distance kernel with $w_{ij} = \frac{e^{-(d_{ij}/h_1)^k}}{\sum_j w_{ij}}$ and we choose a set of bandwidth h equal to the distance of neighboring that contains 4, 10 or 20 mean number of neighbors (mnb), so we have $h_1 \in (h_1^4, h_1^{10}, h_1^{20})$.

MC Settings: spatial pattern of $\lambda(u_i, v_i)$

Three different spatial patterns for $\lambda(u_i, v_i)$:

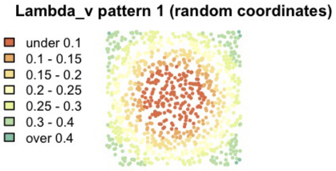
$\lambda_c = \sum_i \lambda(u_i, v_i)/n \in (0, 0.2, 0.4, 0.6)$.

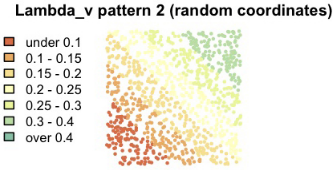
Pattern 1: $\lambda(u_i, v_i) =$

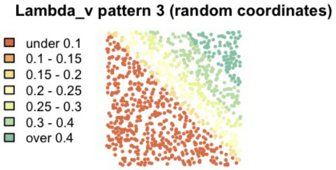
$\lambda_c + f(\sqrt{(u_i - 0.5)^2 + (v_i - 0.5)^2})$

Pattern 2: $\lambda(u_i, v_i) = \lambda_c + f(u_i + v_i)$

Pattern 3: $\lambda(u_i, v_i) = \lambda_c + f_{u_i+v_i<1}(u_i + v_i)$







MC settings

- $\beta = (1, 0.5)$
- $X_1 = 1$, $X_2 \sim N(0, 3)$
- Coordinates of data (u_i, v_i) and X are drawn once and only once for each type I.
- 1000 replications of ϵ using normal random ($\epsilon \sim N(0, 1)$) to generate the vector of y

Random locations Case.

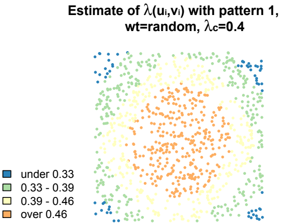
$n = 1000$, $r = 1000$, $\beta = (1, 0.5)$, $l = random$, W known

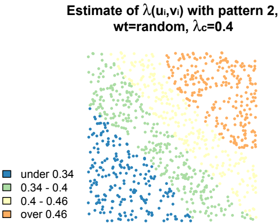
Exp. parameters			OLS		IV				S2LM				
λ_c	$mnb(h)$	$pattern$	β_0	β_1	β_0	β_1	λ	$RMSE_\lambda$	β_0	β_1	$\bar{\lambda}(u_i, v_i)$	$RMSE_\lambda$	$h_2^\star$
0	4	1	0.9989	0.4997	0.9928	0.4995	0.0032	0.0014	0.9937	0.4994	0.0027	0.0464	0.845
0	10	1	0.9989	0.4997	0.9896	0.4995	0.0047	0.0021	0.9914	0.4994	0.0037	0.0685	0.8418
0	20	1	0.9989	0.4997	0.9866	0.4995	0.0061	0.0029	0.9907	0.4995	0.004	0.0955	0.8354
0.2	4	1	1.5269	0.5011	1.0949	0.4986	0.1702	0.0995	1.0085	0.4999	0.2103	0.0646	0.1846
0.2	4	2	1.529	0.5048	0.9537	0.5007	0.2264	0.0956	1.0028	0.4997	0.2064	0.0514	0.3046
0.2	4	3	1.5384	0.5047	0.9617	0.5005	0.2262	0.1101	1.0136	0.4997	0.2049	0.0685	0.247
0.2	10	1	1.5315	0.4993	1.1433	0.4981	0.1519	0.116	1.0235	0.4995	0.2048	0.0834	0.1842
0.2	10	2	1.5317	0.503	0.9339	0.5004	0.2339	0.1063	0.9935	0.4996	0.2098	0.0695	0.3131
0.2	10	3	1.5405	0.5027	0.9662	0.5003	0.224	0.1181	1.0129	0.4995	0.2048	0.084	0.2551
0.2	20	1	1.5341	0.4984	1.2338	0.4978	0.1168	0.1486	1.0747	0.4992	0.1845	0.1116	0.1807
0.2	20	2	1.534	0.502	0.8334	0.5001	0.2727	0.1309	0.9579	0.4996	0.2234	0.0962	0.3263
0.2	20	3	1.5422	0.5016	0.9243	0.5	0.2398	0.1337	1.0149	0.4994	0.2036	0.1062	0.2645

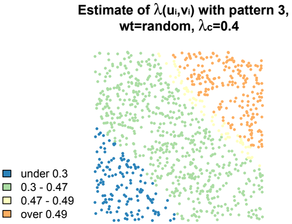
Random locations Case.

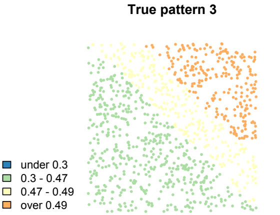
$n = 1000$, $r = 1000$, $\beta = (1, 0.5)$, $l = random$, W known

Exp. parameters			OLS		IV				S2LM				
λ_c	$mnb(h)$	$pattern$	β_0	β_1	β_0	β_1	λ	$RMSE_\lambda$	β_0	β_1	$\bar{\lambda}(u_i, v_i)$	$RMSE_\lambda$	$h_2^\star$
0.4	4	1	2.3905	0.5102	1.1574	0.4986	0.3644	0.0984	0.9853	0.5002	0.4274	0.0571	0.2327
0.4	4	2	2.3922	0.5171	0.935	0.5005	0.4305	0.0935	1.0158	0.4996	0.4051	0.0399	0.421
0.4	4	3	2.414	0.517	0.9485	0.5004	0.4302	0.1082	1.0325	0.4996	0.4046	0.0621	0.4005
0.4	10	1	2.4081	0.5029	1.2329	0.4978	0.343	0.1147	1.0122	0.4998	0.419	0.0704	0.2263
0.4	10	2	2.4064	0.5095	0.9058	0.5003	0.4385	0.1017	0.9995	0.4995	0.4095	0.0549	0.4265
0.4	10	3	2.4266	0.5091	0.9578	0.5004	0.4268	0.1131	1.0475	0.4993	0.3995	0.0726	0.3973
0.4	20	1	2.4176	0.4993	1.3502	0.4973	0.309	0.1475	1.1179	0.4992	0.3865	0.0985	0.2096
0.4	20	2	2.4149	0.5059	0.7304	0.4999	0.4888	0.1313	0.9335	0.4994	0.4286	0.0764	0.4386
0.4	20	3	2.4334	0.5053	0.8716	0.5	0.451	0.128	1.0777	0.4993	0.3903	0.0878	0.3884
0.6	4	1	4.2053	0.5306	1.3348	0.4986	0.5536	0.1	0.9501	0.5004	0.6583	0.0674	0.327
0.6	4	2	4.1976	0.5464	0.9016	0.5001	0.6366	0.0924	1.0608	0.4996	0.6027	0.0305	0.5974
0.6	4	3	4.2696	0.5465	0.9246	0.5002	0.6373	0.1075	1.1007	0.4996	0.6029	0.0577	0.6096
0.6	10	1	4.25	0.5097	1.4455	0.4969	0.5328	0.1159	1.0219	0.5003	0.642	0.0652	0.313
0.6	10	2	4.2367	0.5248	0.8401	0.5002	0.6474	0.0995	1.0356	0.4993	0.6071	0.0413	0.5997
0.6	10	3	4.3024	0.524	0.935	0.5005	0.6341	0.1096	1.1808	0.4994	0.5865	0.0664	0.592
0.6	20	1	4.2671	0.5001	1.4042	0.4957	0.5395	0.135	1.1644	0.4992	0.6093	0.0816	0.2866
0.6	20	2	4.2557	0.515	0.4802	0.4993	0.7146	0.143	0.9626	0.4991	0.6212	0.0561	0.6072
0.6	20	3	4.3148	0.514	0.7171	0.4999	0.6737	0.1293	1.3701	0.4994	0.5503	0.0897	0.5597


Estimate of $\lambda(u_i,v_i)$ with pattern 1, wt=random, λ_c=0.4
under 0.33
0.33 - 0.39
0.39 - 0.46
over 0.46


Estimate of $\lambda(u_i,v_i)$ with pattern 2, wt=random, λ_c=0.4
under 0.34
0.34 - 0.4
0.4 - 0.46
over 0.46


Estimate of $\lambda(u_i,v_i)$ with pattern 3, wt=random, λ_c=0.4
under 0.3
0.3 - 0.47
0.47 - 0.49
over 0.49


True pattern 3
under 0.3
0.3 - 0.47
0.47 - 0.49
over 0.49

Application example

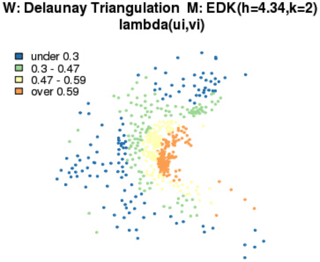
Housing prices in Boston database (Harrison and Rubinfeld, 1978) contains an aggregated house price index (median) for 506 census tracts and a set of 14 covariates, augmented with census coordinates projected to UTM Zone 19. Anselin (2003) proposes this SLM:

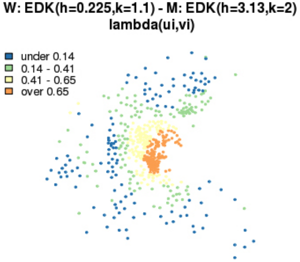
$$\begin{aligned} log(MEDV) =& \lambda W \; log(MEDV) + \beta_0 + \beta_1 CRIM + 2 + \beta_3 INDUS + \beta_4 CHAS \\ & + \beta_5 NOX^2 + \beta_6 RM^2 + \beta_7 AGE + \beta_8 log(DIS) + \beta_9 log(RAD) \\ & + \beta_{10} TAX + \beta_{11} PTRATIO + \beta_{12} B + \beta_{13} log(LSTAT) + \epsilon \end{aligned}$$

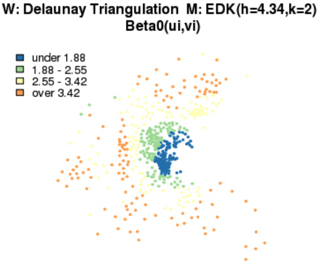
$$
\begin{aligned}
log(MEDV) = & \beta_0(u_i, v_i) + \lambda(u_i, v_i)W\ log(MEDV) + \beta_1 CRIM + 2 \\
& + \beta_3 INDUS + \beta_4 CHAS + \beta_5 NOX^2 + \beta_6 RM^2 + \beta_7 AGE \\
& + \beta_8 log(DIS) + \beta_9 log(RAD) + \beta_{10} TAX \\
& + \beta_{11} PTRATIO + \beta_{12} B + \beta_{13} log(LSTAT) + \epsilon
\end{aligned} \tag{8}
$$

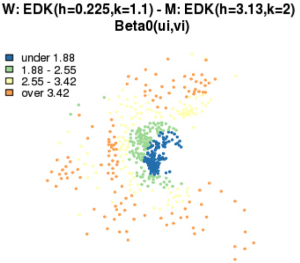
Choosing (h_1, k_1) and (h_2)

	W	*EDK*	*DTRI*
First step	$h_1^\star / k^\star$	0.225/1.1	NA
	SSR_{IV}	14.418	**13.276**
Second step	$h_2^\star$	3.13	4.34
	$SSR_{\boldsymbol{s2LM}}$	10.301	**9.606**
Third step	$SMPA_{\boldsymbol{s2LM}}(n = 1)$	0.0265	**0.0251**
	$SMPA_{IV}(n = 1)$	0.0289	0.0263
Third step	$SMPA_{\boldsymbol{s2LM}}(n = 10)$	0.0321	**0.0324**
	$SMPA_{IV}(n = 10)$	0.0349	0.0343
Third step	$SMPA_{\boldsymbol{s2LM}}(n = 50)$	**0.0327**	0.0333
	$SMPA_{IV}(n = 50)$	0.0352	0.0351


W: Delaunay Triangulation M: EDK(h=4.34,k=2)
lambda(ui,vi)
under 0.3
0.3 - 0.47
0.47 - 0.59
over 0.59


W: EDK(h=0.225,k=1.1) - M: EDK(h=3.13,k=2)
lambda(ui,vi)
under 0.14
0.14 - 0.41
0.41 - 0.65
over 0.65


W: Delaunay Triangulation M: EDK(h=4.34,k=2)
Beta0(ui,vi)
under 1.88
1.88 - 2.55
2.55 - 3.42
over 3.42


W: EDK(h=0.225,k=1.1) - M: EDK(h=3.13,k=2)
Beta0(ui,vi)
under 1.88
1.88 - 2.55
2.55 - 3.42
over 3.42

	OLS	IV	S2LM
W		DTRI	DTRI
M			EDK(4.34,2)
	coef *(p-value)*	coef *(p-value)*	coef *(p-value)*
CRIM	-0.0119 *(< 2e-16)*	-0.00739 *(1.39e-09)*	-0.0058 *(7.50e-09)*
ZN	8.02e-05 *(0.874)*	0.000374 *(0.413)*	0.00112 *(0.03260)*
INDUS	0.00024 *(0.919)*	0.00133 *(0.535)*	0.00181 *(0.2360)*
CHAS1	0.0914 *(0.00613)*	0.0116 *(0.706)*	0.00432 *(0.8926)*
I(NOX^2)	-0.638 *(2.88e-08)*	-0.289 *(0.00713)*	-0.33471 *(0.0008)*
I(RM^2)	0.00633 *(1.89e-06)*	0.00675 *(2.04e-08)*	0.00808 *(5.02e-14)*
AGE	9.07e-05 *(0.863)*	-0.000238 *(0.616)*	-0.00154 *(0.00256)*
log(DIS)	-0.191 *(1.78e-08)*	-0.155 *(4.45e-07)*	-0.19951 *(5.66e-05)*
log(RAD)	0.0957 *(7.91e-07)*	0.0771 *(1.08e-05)*	0.09359 *(5.26e-06)*
TAX	-0.00042 *(0.000664)*	-0.000373 *(0.000817)*	-0.00042 *(0.0002)*
PTRATIO	-0.0311 *(1.14e-09)*	-0.014 *(0.00357)*	-0.01362 *(0.0027)*
B	0.000364 *(0.00046)*	0.000287 *(0.00221)*	0.00032 *(0.0003)*
log(LSTAT)	-0.371 *(<2e-16)*	-0.236 *(<2e-16)*	-0.21437 *(<2e-16)*
$\beta_0(u_i, v_i)$	4.56 *(6.24e-111)*	2.38 *(2.61e-20)*	Min.: 0.7284 1st Qu.: 2.0446 Median: 2.3850 Mean: 2.5532 3rd Qu.: 2.9209 Max.: 6.5365
$\lambda(u_i, v_i)$		0.4643 *(4.33e-02)*	Min.: -0.8631 1st Qu.: 0.2982 Median: 0.4703 Mean: 0.4189 3rd Qu.: 0.5857 Max.: 1.0082
n=506	DF=492		
SSR	16.378	13.275	9.606
R2	0.9965	0.9972	0.9979
SMPA (m=1)	0.0351	0.0263	0.0251
SMPA (m=10)	0.0347	0.0343	0.0324
SMPA (m=50)	0.0348	0.0351	0.0333

Conclusion

Our proposition has several advantages, compared to a simple extension of the GWR that takes into account spatial autocorrelation.

- First, the mixed GWR framework with a limited number of spatial varying parameters allows to avoid numerous problems relative to the GWR framework.
- Second, it is usable for large datasets
- Third, our estimator seems very promising by allowing a substantial gain in predictive accuracy on different datasets tested.

Conclusion

But it remains a lot of works to do:

1. To rerun the MC experiments with adaptive bandwidths.
2. To ameliorate the criteria to choose the bandwidth of M.
3. To test a wider range of spatial patterns of spatial lag parameter in order to draw conclusions on the accuracy of θ estimates.
4. To run Monte Carlo experiments with unobserved spatial heterogeneity
5. Finding a way to deal with the presence of spatial discontinuities of $\lambda(u_i, v_i)$
6. Extension to Logit/Probit model.

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