Managing interacting species in unassessed fisheries

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Abstract

This paper addresses the management of multispecies fisheries, and suggests the use of restricted fishing policies as an interesting option for unassessed fisheries (as is the case within developing countries). Specifically, we consider a predator-prey system where agents compete to harvest from two interacting fish species. Two management policies are considered: an unrestricted regime where agents can harvest from both species, and a second one where only the predators can be harvested. The performance of both policies is compared from an ecological and an economic point of view. For a sufficiently large number of agents (or for strong biological interaction parameters) the restricted fishing policy is shown to yield both higher long run stock levels and profits. Thus, this contribution suggests that such a policy would require very little monitoring while meeting environmental and economic objectives.

JEL Classification: C72, Q22, Q28
Keywords: Multispecies fisheries, predator-prey system, conservation policy, strategic interactions.

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1 Introduction

Since the works of Gordon [13] or Clark [7], a vast literature has focused on the issues raised by the regulation of fisheries. Most contributions on the analysis of regulatory issues considered species in isolation and suggested mostly market-based approaches like taxes or (transferable) quotas.[1] These approaches have been shown to benefit data-rich fisheries within developed countries (Costello et al. [8]). However, they require strong governance and monitoring, which makes them more difficult to implement for unassessed fisheries in developing countries. Since unmonitored fisheries seem to be more threatened than assessed ones while accounting for over eighty percent of global catch (Costello et al. [9]), the analysis of more broadly appropriate policies has practical significance. Recent contributions have raised the idea of designing new management policies that account for species interactions or diversity. Still, there are few analyses of regulatory tools in situations characterized by economic competition (strategic externalities) and biological interactions. The present paper aims to contribute to this line of research. We will analyse a competitive situation in which two interdependent fish species are harvested, and we will assess the performance of a restricted management policy where agents are allowed to harvest only one species.

The literature on fisheries economics has adopted two main perspectives. First focuses on the analysis of either the socially optimal management policies (Agar [1], Strobele and Wacker [26], Tu and Wilman [27]) or the open access bionomic equilibrium. This type of contribution provides insights on the design of economic instruments in order to achieve socially optimal outcomes. Taxes and transferable quotas are usually suggested on the ground of economic efficiency, even though biological interactions are rarely accounted for in multispecies situations (Asche et al. [5], Costa Duarte [10], Ussif and Sumaila [28]). A growing number of contributions stress the importance of acknowledging species interactions or diversity in designing sustainable management policies (Sterner [25], Akpalu [3], Akpalu and Bitew [2]); some of them suggest the use of instruments requiring weaker governance and monitoring than market based approaches. Examples of such instruments are the introduction of marine protected areas or marine reserves (Schnier [24], White et al [29]), or conservation policies where certain species are harvested on the basis of non use values (Hoekstra and van den Bergh [16]). They are now often supported because they account for the specific interactions existing between species. Moreover, they seem to constitute more appropriate tools for unassessed fisheries.

A second part of the literature focuses on game-theoretic models of fisheries in order to analyse the impact of strategic externalities on the sustainability of

[1] Contributions on multispecies fisheries such as Quirk and Smith [21] or Anderson [4] focused on the difference between open-access harvesting and socially optimal harvesting.

the species. These models often consider the case of a single species only (Levhari and Mirman [19], Plourde and Yeung [20]). A few contributions consider that agents might exploit several stocks simultaneously, and that these stocks might be biologically dependent. One might quote Fischer and Mirman ([11], [12]), Hannesson [14], who analyse a two-country, two-species model and characterize the optimal non-cooperative consumption policies, or Kronbak and Lindroos [18] who characterize the number of exploiters that may be sustained in a non-cooperative equilibrium without driving one stock to extinction. A recent literature has provided game-theoretic treatments of marine protected areas (Sanchirico and Wilen [23], Busch [6]) and highlighted their potential as environmental conservation tools. Since the focus of these contributions is the impact of strategic externalities on the commons problem, regulatory issues are usually not accounted for explicitly.

The aim of this paper is to analyze the problem of regulating an area subject to weak governance and/or monitoring, characterized by biological and strategic interdependencies, and to examine the effectiveness (from the point of view of both environmental and economic objectives) of a simple instrument based on a restricted fishing policy (where the harvest of one species is forbidden).

More specifically, we analyze the following problem. A group of fishermen compete for the harvest of two interacting species, for which biological dependence is characterized as a predator-prey relationship. Two management regimes are available: one where fishermen can exploit the stocks of both species[2], and the second where they can harvest the predator species only. Two main results are shown. First, the restricted fishing policy yields higher long-run stock levels for both species as long as the number of agents (or the value of the biological interaction parameters) is sufficiently large. Second, when this policy is superior from an environmental point of view it actually yields higher profits from fishing. Thus, it would be more palatable to politically-powerful fishermen and self-enforcing (that is, it would require very little monitoring) since it will be in their best interest to adopt it. These results imply that a simple policy based on restricted fishing would enable one to satisfy two extremely important but often opposite criteria: environmental conservation and economic acceptability. Moreover, this policy would be relatively simple to implement for data-poor fisheries as it would require no information about the agents' characteristics and the biological parameters. This is in contrast with taxes or input quotas, where the information gap that exists between the fishery manager and the fishermen, or weak governance and monitoring, would make their use more challenging (Costello et al. [9]).

Such restriction policies are sometimes implemented in practice, but usually fishing activity of one species is restricted in order to promote its environmental

[2]We thus consider a situation of targeted fishing where agents can exert specific efforts for each fish species.

conservation and without explicit consideration of interactions with other species. Our results suggest that this might not be effective when the evolution of this species depends on another one. For biologically dependent fish species the right one must be targeted by the restriction. Moreover, when the policy is designed appropriately, this study provides additional support based on economic arguments. There are cases where this policy is actually self-enforcing.

The rest of the paper is organized as follows. The model is introduced and described in Section 2. Section 3 analyses the unrestricted management regime, and that of the restricted fishing policy is provided in Section 4. The comparison of both policies is provided in Section 5. Section 6 concludes.

2 The model

Consider a situation with $N \geq 2$ agents, each of whom can harvest from two fish species. Let $x(t)$ be the stock of the prey and $y(t)$ denote the stock of the predators species at time t. Both fish stocks increase over time according to their respective growth function and decrease because of harvesting.

We consider a predator-prey relationship where the prey population density is resource-limited and each predator's functional response is linear (type I) [17]. The evolution of both species is characterised by the following equations:

$$\dot{x}(t) = x(t)\left[\alpha - ax(t) - sy(t) - \sum_{i}^{N} \theta E^{x}(t)\right] \tag{1}$$

$$\dot{y}(t) = y(t)\left[s\beta x(t) - \xi - \sum_{i}^{N} \theta E^{y}(t)\right] \tag{2}$$

where a, s and β are positive constants (smaller than one). We use the widely used logistic growth function to model the crowding effects within the prey population. This means that the multiplication of the species, driven by the intrinsic growth rate α, is limited by the available resource (e.g. insufficient nutrients, oxygen deficiency or other biological characteristics...). Parameter a denotes the resource-limited parameter[3]. Furthermore, in the absence of predators and when there is no fishing activity, the maximum stock level is only determined by the environmental parameter[4]. In the presence of predators, the per capita growth rate

[3]Observe that the resource-limited parameter is the equivalent to the standard carrying capacity approach with $a = \frac{\alpha}{K}$. An enrichment of the ecosystem, a higher carrying capacity K, is equivalent with a lower crowding effect a.

[4]Here, the maximum stock level is $x_{max} = \frac{\alpha}{2a}$.

is reduced in proportion s to the biomass of the predators. Coefficient s indicates which share of the prey one predator is consuming per unit of time. Following the Lotka-Volterra model, the rate of prey consumption is assumed to depend linearly on the number of prey[5].

Parameter β denotes the ability of predator to convert food into births. The growth rate of the predators population increases proportionately with their predation rate $s\beta x$. The growth rate of the predators population decreases at a natural death rate, ξ, and according to its harvest level.

Finally, we assume a (widely used) harvest function *a la Schaeffer*, which is characterized by a catch per unit of effort proportional to the abundance of fish species. We further assume a unique catchability coefficient, θ, but we assume that fishermen may choose different effort levels for the two species, $E^j(t)$ $(j = x, y)$ in order to allow for the option to harvest from species selectively. The capture rate of species j is denoted $\theta E^j(t)$.

Beyond the biological interaction between both species, the above evolution rules capture the existing strategic interaction between agents. Within this framework, we assume there are $N \geq 2$ symmetric fishermen who compete for the harvest of two species which stocks evolve according to equations (1) and (2). In the following sections we will analyse two management regimes: one in which agents can harvest from the two species, the other where they can harvest from only one of the two species (specifically, the predators).

3 An unrestricted common-pool resource game

3.1 Characterisation of the equilibrium

In this case agents can harvest from both species. In order to focus on the issues driven by biological interactions, we consider a situation where agents can exert specific fishing efforts for each species, and where the commercial prices of both species are the same. The situation can be described as follows.

Each agent wishes to maximise instantaneous profits from fishing, taking the behavior of the others as given.[6] Thus, as in Ruseski [22], the agents' objective

[5] This is the simplest *functional response* which does not allow for predator satiation but allows us to characterise one important aspect of the prey-predator relationship. There exists two other types of *functional responses* which are non-linear : Type II with a predation rate depending on prey density and Type III, a sigmoidal function. We refer the reader to Yodzis [30] for a detailed discussion.

[6] The reader will keep in mind that we do not consider a situation of open access where the number of agents is such that the zero-profit condition holds. Instead, we focus on a case with

functions can be written in terms of steady state values.[7] In other words, we consider a restricted open access game where the biological stock levels have reached an equilibrium (that is, where $\dot{x} = \dot{y} = 0$). Agent i's maximisation problem can be written as:

$$\Pi_i\left(E_i^x, E_i^y\right) = p\theta[E_i^x \bar{x} + E_i^y \bar{y}] - w(E_i^x + E_i^y),$$

where $\bar{x}$ (respectively, $\bar{y}$) denotes the steady state of the stock of prey (respectively, predators) when both species are being harvested. Thus, $(\bar{x}, \bar{y})$ satisfies $\dot{x} = \dot{y} = 0$.

We now solve for the Nash equilibria of the above game: each agent maximises profits, assuming that the other agents' effort levels are fixed. More specifically, we will focus on the case where neither species is driven to extinction, that is, where $\bar{x}$ and $\bar{y}$ are positive.

As the game is symmetric, we will notice that $E_1^x = \cdots = E_N^x = E^x$ and $E_1^y = \cdots = E_N^y = E^y$; let us first provide some explanation by using the appropriate first order conditions.

The two per capita optimal effort levels result from a trade-off between the marginal benefit and the marginal cost of an extra unit of effort. More specifically, simple inspection of the first order condition related to E^x (condition (see the proof of Proposition 1 in appendix) yields the following intuition. The right hand side is simply the marginal cost from an increase in the effort level dedicated to the prey species. The left hand side is the corresponding marginal benefit, which is the difference between the direct benefit through the extra harvest (given by $\bar{x}$) and the loss resulting from the negative effect on the long run stock level of predators (given by $\frac{s\beta\theta^2 E^y}{s^2\beta}$). This net effect is valued at price p. The same type of logic holds for the optimal effort level dedicated to the predator species. Specifically, we have:

$$p\theta\bar{y} + p\theta E^y \frac{\partial \bar{y}}{\partial E^y} + p\theta E^x \frac{\partial \bar{x}}{\partial E^y} = w. \tag{3}$$

On the left hand side, the first term corresponds to the extra catch, the second one is the effect on the long run stock level of predators, and the last one characterises the effect on the long run stock level of prey (which is positive as $\frac{\partial \bar{x}}{\partial E^Y} = \frac{\theta}{s\beta}$).

Now that the optimal tradeoff has been explained, let us provide the characterisation of the equilibrium:

a fixed number of agents $N \geq 2$ in which fishermen will make positive profits.

[7] The dynamic resource problem is de-emphasised in order to focus on the potential provided by the species' biological dependence. The same type of analysis may be pursued (but at the cost of heavier calculations) in a fully dynamic setting provided that one focuses on the stationary optimal solution.

Proposition 1. *The interior Nash equilibrium exists if and only if the following conditions hold:*

$$s\beta w - p\theta\xi > 0, \tag{4}$$

and either

$$N < \frac{1}{\beta} \text{ and } a(N+1)(s\beta w - p\theta\xi) + (N-\beta)[s^2\beta w - p\theta(s\alpha\beta - a\xi)] > 0, \tag{5}$$

or

$$N > \frac{1}{\beta} \text{ and } a(N+1)(s\beta w - p\theta\xi) + (N-\beta)[s^2\beta w - p\theta(s\alpha\beta - a\xi)] < 0. \tag{6}$$

Provided that these conditions are satisfied, the pair of equilibrium effort levels (E^x, E^y) *is characterised as follows:*

$$E^x = \frac{a(N+1)(s\beta w - p\theta\xi) - (N-\beta)[p\theta(s\alpha\beta - a\xi) - s^2\beta w]}{sp\theta^2(1-N\beta)(N-\beta)} \tag{7}$$

$$E^y = \frac{s\beta w - p\theta\xi}{p\theta^2(N-\beta)} \tag{8}$$

and the corresponding long run stocks of the resources are:

$$\bar{x} = \frac{\xi + N\theta E^y}{s\beta} \;,\; \bar{y} = \frac{s\alpha\beta - a\xi - a\theta N E^y - s\beta N\theta E^x}{s^2\beta} \tag{9}$$

>From the above result, effort levels are shown to have direct (through the harvest level) and indirect effects (due to the biological dependence between the two species). Characterising the impact of changes in the parameter values on the equilibrium size of the fish stocks might be useful to further our understanding of the situation at hand. This is the goal of the next sub-section.

3.2 Comparative statics

We will now provide results of comparative statics with two objectives in mind : *(i)* the investigation of direct effects on the equilibrium stock and on the fishing activities, and *(ii)* identifying how the two different types of externalities (biological and that due to strategic interactions[8]) drive the effects resulting from changes in the parameter values.

[8] An externality occurs when an increase in fishing effort by one fisherman reduces the stock that is available for the others. Biological externalities occur when the harvested species interact with other species that is also harvested.

More specifically, as we are interested in the impact of parameters that might be amended by using public policies, we will focus on the effect of the cost of effort, the market price, and the number of agents. We will provide some results regarding the effect of the prey growth rate and of the crowding effect as well (as it might be possible to get some information on these biological parameters).

First, the effect of changes in the value of the effort cost is driven by its effect on the equilibrium effort levels. Specifically, we have:

$$\frac{\partial \bar{x}}{\partial w} = \frac{N\theta}{s\beta}\frac{\partial E^y}{\partial w} \quad , \quad \frac{\partial \bar{y}}{\partial w} = -\frac{N\theta}{s^2\beta}\left[a\frac{\partial E^y}{\partial w} + s\beta\frac{\partial E^x}{\partial w}\right]$$

The effect on E^y is positive, which implies that the effect on the equilibrium size of the stock of prey is positive too. However, it is positive on E^x if and only if the number of agents is sufficiently small (see the appendix for the formal derivations). As the effect on E^x outweighs the effect on E^y, we conclude that an increase in the cost of effort results in an increase in the equilibrium size of the predator stock if and only if the number of agents is sufficiently small.

Second, an increase in the value of the market price impacts on the equilibrium stock levels through its effect on the equilibrium effort levels. As with the effort cost, we have (focusing on the effect on $\bar{x}$):

$$\frac{\partial \bar{x}}{\partial p} = \frac{N\theta}{s\beta} \cdot \frac{\partial E^y}{\partial p}$$

Since the market price and the cost of effort have opposite effects, we conclude immediately that it impacts negatively on the stock of prey, and positively on the stock of predators if and only if the number of agents is sufficiently small.

Finally, an increase in the number of agents can be decomposed into a direct effect (an extra agent results in an increase of the aggregate catch) and an indirect one (through the effect on the equilibrium effort levels)). Specifically, regarding the effect on the stock of prey, we have:

$$\frac{\partial \bar{x}}{\partial N} = \frac{\theta}{s\beta}\left[E^y + N\frac{\partial E^y}{\partial N}\right]$$

The first term between brackets (on the right hand side of the above equality) measures the direct effect (which is positive) while the second term measures the indirect one. The indirect effect is negative, as an extra agent results in a lower equilibrium effort level. The net effect is negative: the effect on the individual effort level outweighs that on the aggregate catch. The same decomposition applies for the effect on the stock of predators, but the net effect is ambiguous.

Table 1 summarises the above discussion and report the qualitative effects of changes in the parameter values on the equilibrium sizes of the stocks of the two fish species.

Table 1: Comparative Statics - Stocks equilibrium

Parameters	Impact on $\overline{x}$	Impact on $\overline{y}$ — Food conversion rate of predation: *small*	*high*
crowding effect	/	-	+
growth rate of prey	/	+	-
food conversion rate	+	?	?
catchability coef.	-	+	-
effort cost	+	-	+
market price	-	+	-
number of fishermen	-	?	?

As is well known in the literature we notice that parameters characterizing the growth function have no effect on the equilibrium size of the stock of prey (this is known as the "paradox of enrichment", see Hudson [15]). However, this rate and the parameter related to the crowding effect impact on the long run stock of predators. This seems to suggest that a policy designed to exploit the effect of the natural growth rate (by restricting harvest from the prey species) might have an effect on the conservation levels of both species. We are going to introduce such a policy in the next section.

4 A restricted fishing management regime

We now analyse the situation where agents are only allowed to harvest the predators. This is intuitively reasonable from a conservation perspective. If the regulator's goal is to preserve the resources, and it may be possible to restrict access to one of the two species, then a quite natural intuition is that the focus should be put on the prey, since it is the food source of the predators.

For any $i = 1, ..., N$, agent i's problem is then to maximise:

$$\Pi_i^r\left(E_i^y\right) = p\theta E_i^y \overline{y}^r - wE_i^y$$

where $\overline{y}^r$ denotes the long run stock corresponding to the situation where only the predator is being harvested, that is, $\overline{y}^r$ and $\overline{x}^r$ solve the following system:

$$\alpha - a\overline{x}^r - s\overline{y}^r = 0$$

and

$$s\beta\overline{x}^r - \xi - \theta\sum_i^N E_i = 0$$

Such a situation would correspond to a simple policy that could be implemented by a regulator in areas characterised by weak governance and monitoring such as small unassessed fisheries, which are very common in developing countries. The effectiveness of such a policy will be assessed with respect to conservation and economic objectives.

The first step is to characterise the Nash equilibrium.[9] Before providing the result, let us have a look at the optimality condition characterising $\bar{E}$; we have:

$$p\theta\bar{y} + p\theta\bar{E}\frac{\partial\bar{y}}{\partial\bar{E}} = w, \tag{10}$$

where the effect on the long run stock level of predators $\frac{\partial\bar{y}}{\partial\bar{E}}$ is negative. If we compare this condition to the one characterising E^y in the unrestricted regime, we notice that the term depicting the effect on the long run stock level of prey is absent in the case of a restricted fishing policy. Thus, in the unrestricted regime, increasing E^y has potentially opposite effects on the long run stock levels. This absence of conflicting effects enables one to exploit the natural recovery rate of prey by adopting a restricted management regime. To be more explicit, we obtain the following result:

Proposition 2. *The interior Nash equilibrium exists if and only if the following condition holds:*

$$p\theta(s\alpha\beta - a\xi) - s^2\beta w > 0 \tag{11}$$

Provided that this condition is satisfied, the equilibrium effort level $\overline{E}$ is characterised as follows:

$$\overline{E} = \frac{p\theta(s\alpha\beta - a\xi) - s^2\beta w}{a(N+1)p\theta^2} \tag{12}$$

and the corresponding long run stocks of the resources are:

$$\overline{x}^r = \frac{\xi + \theta N\overline{E}}{s\beta} \ , \ \overline{y}^r = \frac{s\alpha\beta - a\xi - aN\theta\overline{E}}{s^2\beta}$$

Globally speaking, results are similar to those obtained above. We observe that the optimal effort level results from a trade-off between the marginal value of the net reproduction of predators and the marginal cost of the predator extra harvest

[9] Again, we focus on the case where neither species is being driven to extinction.

(accounting for the biological interaction with the prey species). Moreover, we can make an additional remark on the optimal trade-off in the two regimes considered.

More specifically, the right hand side of the first order condition characterising $\overline{E}$ is simply the marginal cost from an increase in the effort level dedicated to the predator species, which is the same for both cases. The left hand side is the expression of the marginal benefit. The difference between the restricted and the unrestricted regime is driven by the two following effects. First, there is a "direct" effect: the ability to harvest from one or from the two species yields different effort levels to catch the predators in the two regimes. Second, in the unrestricted regime, the effort level on the prey species affects the marginal benefit related to the other species. Looking at the first order conditions, there is an additional term $s\theta(1-N\beta)E^x$ that will affect the marginal benefit in different ways depending on the number of agents. The second effect can be signed immediately, but the first one is less obvious.

Regarding results of comparative statics, the effects can be analysed more easily in the restricted regime. The effects on the predator population are more obvious than those on the population of prey, since they may result from two opposite effects : *(i)* the effect induced by changes in the predator population and *(ii)* the change in fishing activities. For instance, a higher food conversion rate has a direct positive impact on the optimal effort level, which affects (negatively) the predator population. But this has a positive effect for the predators, since a higher rate makes it easier to sustain a given population level. In this case, the net effect on the predator species is positive, meaning that the biological effect outweighs the increased pressure resulting from the economic activity. On the other hand, the net effect on the prey population is the opposite one.

An increase in the prey natural growth rate has a positive effect on the predator population level. This increase enables agents to adjust their optimal effort level so that the catch level increases, while the impact on the stock levels remains positive.

The effect of changes in the value of the effort cost is driven by its effect on the equilibrium effort level. Specifically, we have:

$$\frac{\partial \bar{x}^r}{\partial w} = \frac{N\theta}{s\beta} \cdot \frac{\partial \bar{E}}{\partial w} \ , \ \frac{\partial \bar{y}^r}{\partial w} = -\frac{aN\theta}{s^2\beta} \cdot \frac{\partial \bar{E}}{\partial w}.$$

The effect on $\bar{E}$ is checked to be negative, which implies that the effect on the equilibrium size of the stock of prey (respectively, predators) is negative (respectively, positive).

Second, since the market price and the cost of effort have opposite effects, we conclude immediately that it impacts positively on the stock of prey, and negatively on the stock of predators. The effect on the second stock is intuitive, while the effect on the first stock highlights that the increased economic pressure put on the predators results in an increase in the stock of prey.

Finally, an increase in the number of agents can be decomposed into a direct effect (an extra agent results in an increase of the aggregate catch, which eases the pressure on the prey) and an indirect one (through the effect on the equilibrium effort level)). Specifically, regarding the effect on the stock of prey, we have:

$$\frac{\partial \bar{x}^r}{\partial N} = \frac{\theta}{s\beta}\left[\bar{E} + N\frac{\partial \bar{E}}{\partial N}\right].$$

The first term between brackets (on the right hand side of the above equality) measures the direct effect (which is positive) while the second term measures the indirect one. The indirect effect is negative, as an extra agent results in a lower equilibrium effort level (and thus a larger population of predators, which affects the size of the stock of prey negatively). The net effect is positive. The same decomposition applies for the effect on the stock of predators, but the conclusion is reversed.

Table 2 summarises the above discussion and reports the qualitative effects of changes in the parameter values on the long-run effort and stock levels.

Table 2: Comparative Statics

Parameters	**Impact on $\overline{x}^r$**	**Impact on $\overline{y}^r$**
crowding effect	-	-
growth rate of prey	+	+
food conversion rate	-	+
catchability coef.	+	-
effort cost	-	+
market price	+	-
number of fishermen	+	-

One may notice that, unlike the unrestricted regime, the parameters characterising the growth function (α and a) impact on the equilibrium stock level of the prey, thus the paradox of enrichment disappears. This is consistent with the main idea underlying a restricted fishing policy, which is to exploit the existing biological interactions and the natural recovery rate of the prey species. By restricting fishing activities, one would like to affect positively the stock levels of both species. While this seems intuitive for the prey species, it is far less obvious that the same intuition holds for the other one. Moreover, the implementation of such a policy may also impact negatively on the profitability of fishing. We will analyse in the next section whether it might meet simultaneously environmental and economic objectives.

5 Comparison of the two policies

We compare both policies (restricted versus unrestricted fishing) from the point of view of environmental conservation and economic acceptability.

5.1 Environmental conservation

A first step is to compare the impact of the policies on the effort level dedicated to the predator species and on its stock level. In order to do this, we will have to impose parameter restrictions to ensure that all appropriate quantities are positive under both management policies. Thus, we assume that conditions (4) and (11) are always satisfied, and that either condition (5) or (6) holds. It can be easily checked that the set satisfying all these conditions is non vacuous. We can now state the following result:

Proposition 3. *Let us assume that conditions (4)-(5)-(11)* <u>or</u> *(4)-(6)-(11) are satisfied. Consider the Nash equilibrium effort levels E^y and $\overline{E}$ corresponding to the unrestricted and restricted fishing game, respectively. Then we have:*

$$E^y > \overline{E} \Leftrightarrow N < \frac{1}{\beta}.$$

We denote by $\overline{y}$ (respectively, $\overline{y}^r$) the equilibrium stock level of the predator species corresponding to the unrestricted (respectively, restricted) fishing policy. We obtain the following comparison:

$$\overline{y} < \overline{y}^r.$$

In other words, the restricted fishing policy always results in a higher long run stock of predators.

The above proposition highlights a very interesting property. A restricted fishing policy where agents can harvest only from the predator species might be useful if the regulator is willing to increase the conservation level of this same species. One might have expected that fishing efforts on predators would necessarily increase as a result of the restriction policy, and that this increase in effort would impact negatively on the corresponding stock. The above proposition highlights that this intuition is not correct. Specifically, a restriction enables to exploit the natural growth rate of the prey in a way that impacts positively on the equilibrium size of the stock of predators, even if the optimal effort level has increased.

More specifically, inspection of the expression of the difference $\overline{y}-\overline{y}^r$ (provided by condition (6) in the appendix) highlights two effects. On one hand, the restricted policy results in an increase in the equilibrium size of the stock of predators through the extra units of prey available to this population (given by $s\beta\theta NE^x$). On the other hand, this policy yields an increase in the catch level (which contributes to a decrease in the equilibrium size of the stock of predators) when the number of agents is sufficiently large (or for sufficiently large values of the food conversion rate). The conclusion is that the direct effect driven by the natural growth rate (more available prey means more food) always dominates.

The impact on the prey species depends on the number of agents and/or on the biological interaction parameter. The comparison can be stated as follows:

Lemma 1. *Under the assumptions of Proposition 3, we have:*

$$\overline{x} > \overline{x}^r \Leftrightarrow N < \frac{1}{\beta},$$

where $\overline{x}$ (respectively, $\overline{x}^r$) denotes the equilibrium stock level of the prey species under the unrestricted (respectively, restricted) fishing policy. Thus, the unrestricted fishing policy results in a higher long run level of the stock of prey if and only if the number of agents is sufficiently small (or conversely, for sufficiently small values of the food conversion rate).

Thus, the comparison depends on a threshold value regarding the number of agents. The expression of this threshold depends in turn on the value of a biological parameter defining the predator-prey interaction, namely the food conversion rate. This is perfectly in line with the logic underlying the use of the restricted fishing policy, where the nature of the interaction is used to provide a regulation instrument.

Again, a simple inspection of the expression of the difference $\overline{x}-\overline{x}^r$ (provided by condition (6) in the appendix) highlights that it results from the difference between the optimal effort levels. This time we may provide an intuition by focusing on the value of the food conversion rate. For low values of this parameter, predators need to harvest a lot from the prey species to sustain a given population level. For such cases, the constraint put on the fishing activity (which enables this species to recover) is not sufficient to offset the activity of the predators. The opposite holds for high values of the conversion rate.

Moreover, there will be an increase in the size of the stock of predators. This implies that there will be more predation, but the above result highlights that more predation does not imply that there will be necessarily a decrease in the size of the stock of prey. In these cases, the use of the policy enables the stock of prey to recover such that the extra harvest resulting from the increase in the number of predators is offset by the natural recovery of the species.

We can summarise the above results as follows. For a sufficiently small number of agents (or for weak biological interaction parameters), a restricted fishing policy implies the existence of a tradeoff between species in terms of conservation goals. While it may help if the regulator is mainly interested in preserving the predator species, the impact on the prey will be negative. The policy enables to increase the long run stock of predators but, due to the low conversion rate, it imposes a high burden on the prey even though its fishing is not allowed. By contrast, for a sufficiently large number of agents, this simple policy enables to satisfy any conservation goal the regulator might have. It enables to obtain higher long run stock levels for *both* species.

The next question is to assess the economic acceptability of such a policy. This will determine whether it would face political opposition from fishermen, who quite often prove to be a powerful interest group. Will agents be willing to accept this restriction or, in other words, can we characterise cases where this policy yields higher profits from fishing? This makes sense, since both effort and stock levels may go up under the restricted fishing policy. This is the aim of the next sub-section.

5.2 Acceptability of the policy

In this section, we would like to assess the feasibility of the policy introduced previously. We will compare the agents' profits from fishing under the unrestricted and restricted fishing policies. The policy will be easy to implement if there are cases where the restricted fishing policy will yield the highest level of profits. Indeed, this would imply that the policy will be self-enforcing. Since the policy has been shown to meet conservation objectives for a sufficiently large number of agents (or for strong biological interaction parameters), we will focus on this case in the next result. We have the following conclusion:

Proposition 4. *Let us consider the case where the restricted fishing policy satisfies conservation objectives, that is, assume that conditions (4)-(6)-(11) are satisfied. Denote Π_N (respectively, Π_N^r) the equilibrium profits from fishing under the unrestricted (respectively, restricted) policy. Then we have:*

$$\Pi_N^r > \Pi_N;$$

In other words, the restricted fishing policy is economically acceptable.

The above proposition enables to derive an interesting conclusion. Indeed, it is shown that the policy that would consist in restricting fishing activities to the predator species is economically feasible (that is, acceptable by the agents as it yields the highest level of profits) when it fulfills a conservation objective as well. Specifically, the resulting long run stock levels of both species would be higher under fishing restrictions. Thus, such a simple policy would be consistent with both economic and conservation objectives.

5.3 Discussion

Before concluding this section, we would like to briefly discuss three features of the above analysis. Firstly, all the results obtained generalise to the case where species have different market prices. In such a setting one can obtain a few additional insights (for instance, a higher price for the predator species might tend to reinforce the economic acceptability of the restricted policy). Still, the setting considered here enables us to present the same main arguments: a simple policy based on fishing restrictions may constitute an interesting option in fisheries characterised by weak governance and monitoring. Moreover, it has the advantage to allow for a reasonably simple picture of the underlying intuition. This is why we decided to present the case of equal prices.

Secondly, as mentioned at the beginning of the analysis, a restricted fishing policy focusing on the recovery of the prey population makes sense since this is the food source of the predators. Indeed, it can be checked that a policy restricting the harvest of predators would result in the present model in extreme equilibrium outcomes: either no harvest (for high marginal costs) or harvest levels leading to the extinction of the predators (for low marginal costs of fishing). Since the main point of the present study is to analyse a policy meeting economic and conservation objectives, we did not present this type of analysis.

Finally, it might be interesting to provide a few words on the case of a sole owner. Assuming $N = 1$ would result in a higher effort level on the predator species than under the restricted fishing policy. This would imply that the long-run stock level of predator would be lower under sole ownership, while the opposite conclusion would hold in the case of the prey species. Thus, compared to the case of a restricted fishing policy, sole ownership (which is yet unlikely in fisheries with weak governance and monitoring) would imply a tradeoff between species as far as environmental conservation is considered.

6 Concluding remarks

When agents compete for the harvest of two dependent species, we highlight the economic and environmental potentials of a simple restricted fishing policy where only the predators are harvested. For a sufficiently large number of agents (or strong biological interaction parameters) this policy satisfies conservation objectives and is economically self-enforcing. Moreover, it does not require knowledge about biological parameters or agents' characteristics, which makes it appropriate for unassessed fisheries where governance and monitoring are weak. The present policy illustrates that it might be useful to adopt management practices based on the ecosystem by exploiting the potential provided by the interaction between species. By designing the policy such that the prey species is allowed to recover, it is possible to satisfy often conflicting objectives. A number of questions remain for future research. For instance, we would like to develop the analysis presented in this paper to allow for other types of biological interactions. We view it as a first step of a large research agenda.

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Appendix

Compatibility of conditions

Let us summarize the three necessary conditions required in the case without restriction :

$$s\beta w - p\theta\xi > 0 \ ; \ a(N+1)(s\beta w - p\theta\xi) - (N-\beta)\left[p\theta(s\alpha\beta - a\xi) - s^2\beta w\right] \lesseqgtr 0$$

and in the restricted case : $p\theta(s\alpha\beta - a\xi) - s^2\beta w > 0$. We thus observe

$$a(N+1)(s\beta w - p\theta\xi) - (N-\beta)\left[p\theta(s\alpha\beta - a\xi) - s^2\beta w\right] \Leftrightarrow \underbrace{a(N+1)s\beta w - ap\theta\xi}_{>0} \lesseqgtr s\beta(N-\beta)\underbrace{(p\theta\alpha - sw)}_{>0} + a\beta p\theta\xi$$

Therefore

$$\begin{aligned} & a(N+1)s\beta w - ap\theta\xi > s\beta(N-\beta)(p\theta\alpha - sw) + a\beta p\theta\xi > 0 \\ \text{or} \quad & a(N+1)s\beta w - ap\theta\xi < s\beta(N-\beta)(p\theta\alpha - sw) + a\beta p\theta\xi < 0 \end{aligned}$$

Proof of proposition 1

Let us focus on the case of agent i; by assumption we focus on the positive levels of stock $\bar{x}$ and $\bar{y}$ that satisfy:

$$\alpha - a\bar{x} - s\bar{y} = \theta \sum_i^N E_i^x, \qquad s\beta\bar{x} - \xi = \theta \sum_i^N E_i^y$$

>From the second equality we deduce that $\bar{x} = \frac{\xi + \theta\sum_i^N E_i^y}{s\beta}$ and then plugging this expression into the first equality, we obtain $\bar{y} = \frac{s\alpha\beta - a\xi - a\theta\sum_i^N E_i^y - s\beta\theta\sum_i^N E_i^x}{s^2\beta}$.

Using these expressions for $\bar{x}$ and $\bar{y}$, we can rewrite the expression of the first agent's profits from fishing as a function of his pair of effort levels:

$$\Pi_i(E_i^x, E_i^y) = p\theta\left[\frac{E_i^x\left(\xi + \theta\sum_i^N E_i^y\right)}{s\beta} + \frac{E_i^y\left(s\alpha\beta - a\xi - a\theta\sum_i^N E_i^y - s\beta\theta\sum_i^N E_i^x\right)}{s^2\beta}\right] - w(E_i^x + E_i^y).$$

The above expression is continuously differentiable and strictly concave as a function of (E_i^x, E_i^y). The first order conditions are then necessary and sufficient. Differentiating respectively with respect to E_i^x and E_i^y, we obtain:

$$\begin{aligned} \frac{\partial\Pi_i}{\partial E_i^x} &= \frac{p\theta}{s^2\beta}\left[s\left(\theta\sum_i^N E_i^y + \xi\right) - s\beta\theta E_i^y\right] - w = 0 \\ \frac{\partial\Pi_i}{\partial E_i^y} &= \frac{p\theta}{s^2\beta}\left[s\theta E_i^x + s\alpha\beta - a\left(\theta\sum_i^N E_i^y + \xi\right) - s\beta\theta\sum_i^N E_i^x - a\theta E_i^y\right] - w = 0 \end{aligned}$$

Now, this symmetric game requires that $E_1^x = \cdots = E_N^x = E^x$ and $E_1^y = \cdots = E_N^y = E^y$ in the above first order conditions; thus, we obtain:

$$\frac{p\theta}{s^2\beta}[s\theta E^y(N-\beta) + s\xi] = \frac{p\theta}{s^2\beta}[s\theta E^x(1-N\beta) + s\alpha\beta - a\xi - a(N+1)\theta E^y].$$

Solving the above system for E^x and E^y, we obtain:

$$E^x = \frac{a(N+1)(s\beta w - p\theta\xi) + (N-\beta)[s^2\beta w - p\theta(s\alpha\beta - a\xi)]}{sp\theta^2(1-N\beta)(N-\beta)} \ , \ E^y = \frac{s\beta w - p\theta\xi}{(N-\beta)p\theta^2}$$

as stated in the Proposition. It remains to notice that condition (4) ensures that E^y is positive, while the same property for E^x requires that either condition (5) or (6) be satisfied. This concludes the proof.

Comparative statics in the unrestricted case

Let us compute the derivative of E^x with respect to the different parameters :

$$
\begin{aligned}
\frac{\partial E^x}{\partial a} &= \frac{(N+1)(s\beta w - p\theta\xi) + (N-\beta)p\theta\xi}{sp\theta^2(1-N\beta)(N-\beta)} \; ; \; \frac{\partial E^x}{\partial w} = \frac{a(N+1)\beta + (N-\beta)s\beta}{p\theta^2(1-N\beta)(N-\beta)} \\
\frac{\partial E^x}{\partial \alpha} &= \frac{-\beta}{\theta(1-N\beta)} \; ; \; \frac{\partial E^x}{\partial p} = -\frac{\beta w\left[a(N+1) + s(N-\beta)\right]}{p\theta^2(1-N\beta)(N-\beta)} \\
\frac{\partial E^x}{\partial N} &= \frac{a\left[N\beta(N+2) - \beta(1+\beta) - 1\right](s\beta w - p\theta\xi) + \beta(N-\beta)^2\left[s\beta^2 w - p\theta(s\alpha\beta - a\xi)\right]}{(1-N\beta)^2(N-\beta)^2} \\
\frac{\partial E^x}{\partial \beta} &= \frac{a[(N+1)sw - p\theta\xi] + s^2 w(N-2\beta) + p\theta s\alpha N}{(1-N\beta)(N-\beta)} + (N^2 - 2N\beta + 1)\frac{a(N+1)(s\beta w - p\theta\xi) + (N-\beta)[s\beta^2 - p\theta(s\alpha\beta - a\xi)]}{(1-N\beta)^2(N-\beta)^2}
\end{aligned}
$$

Similarly, we obtain for E^y :

$$
\begin{aligned}
\frac{\partial E^y}{\partial w} &= \frac{s\beta}{(N-\beta)p\theta^2} \; ; \; \frac{\partial E^y}{\partial \beta} = \frac{swN - p\theta\xi}{p\theta^2(N-\beta)^2} \\
\frac{\partial E^y}{\partial p} &= -\frac{s\beta w}{p^2\theta^2(N-\beta)} \; ; \; \frac{\partial E^y}{\partial N} = -\frac{s\beta w - p\theta\xi}{(N-\beta)^2 p\theta^2}
\end{aligned}
$$

and for $\overline{x}$:

$$
\begin{aligned}
\frac{\partial \overline{x}}{\partial \beta} &= \frac{Nsw - p\theta\xi}{sp\theta(N-\beta)^2} \; ; \; \frac{\partial \overline{x}}{\partial w} = \frac{N}{p\theta(N-\beta)} \\
\frac{\partial \overline{x}}{\partial p} &= -\frac{Nw}{p^2\theta(N-\beta)} \; ; \; \frac{\partial \overline{x}}{\partial N} = -\left(\frac{s\beta w - p\theta\xi}{sp\theta(N-\beta)^2}\right)
\end{aligned}
$$

and for $\overline{y}$:

$$
\begin{aligned}
\frac{\partial \overline{y}}{\partial a} &= -\frac{swN(\beta+1) - (N+1)p\theta\xi}{s^2 p\theta(1-N\beta)(N-\beta)} \; ; \; \frac{\partial \overline{y}}{\partial w} = -\frac{N[a(1+\beta) + s\beta(N-\beta)]}{sp\theta(1-N\beta)(N-\beta)} \\
\frac{\partial \overline{y}}{\partial \alpha} &= \frac{1}{s(1-N\beta)} \; ; \; \frac{\partial \overline{y}}{\partial p} = \frac{Nw[a(1+\beta) + s\beta(N-\beta)]}{sp^2\theta(1-N\beta)(N-\beta)} \; ; \; \frac{\partial \overline{y}}{\partial \theta} = \frac{Nw[a(1+\beta) + s\beta(N-\beta)]}{sp\theta^2(1-N\beta)(N-\beta)} \\
\frac{\partial \overline{y}}{\partial N} &= -\frac{a(1+\beta)(N^2-1)(s\beta w - p\theta\xi) + (N-\beta)^2[s^2\beta w - p\theta(s\alpha\beta - a\xi)]}{s^2 p\theta(1-N\beta)^2(N-\beta)^2}
\end{aligned}
$$

Proof of proposition 2

Let us focus on the case of agent i. We obtain the expressions of $\overline{x}^r$ and $\overline{y}^r$ as functions of E_i by using a similar reasoning than in the proof of the first Proposition. The agents' profits can then be rewritten as: $p\theta E_i\left(\frac{s\alpha\beta - a\xi - a\theta\sum_i^N E_i}{s^2\beta}\right) - wE_i$.
The appropriate first order condition is:

$$
\frac{\partial \Pi_i^r}{\partial E_i} = p\theta\left(\frac{s\alpha\beta - a\xi - a\theta\sum_i^N E_i}{s^2\beta}\right) - E_i\frac{ap\theta^2}{s^2\beta} = w.
$$

This symmetric game requires $E_i = \cdots = E_N = E$, and we obtain: $\frac{p\theta}{s^2\beta}[s\alpha\beta - a\xi - a(N+1)\theta E] = w$, which yields $\overline{E} = \frac{1}{a\theta(N+1)}\left[\frac{p\theta(s\alpha\beta - a\xi) - s^2\beta w}{p\theta}\right]$.
It remains to notice that (11) ensures that $\overline{E}$ is positive. This concludes the proof.

Comparative statics in the restricted case

Let us compute the derivative of $\overline{E}$ with respect to the different parameters :

$$\begin{aligned}
\frac{\partial \overline{E}}{\partial a} &= -\frac{p\theta(s\alpha\beta - a\xi) - s^2\beta w}{a^2 p\theta^2(N+1)} - \frac{\xi}{a\theta(N+1)} \; ; \; \frac{\partial \overline{E}}{\partial \alpha} = \frac{s\beta}{a\theta(N+1)} \; ; \; \frac{\partial \overline{E}}{\partial \beta} = \frac{s(sw - p\theta\alpha}{ap\theta^2(N+1)} \\
\frac{\partial \overline{E}}{\partial w} &= -\frac{s^2\beta}{ap\theta^2\theta(N+1)} \; ; \; \frac{\partial \overline{E}}{\partial p} = \frac{s^2\beta w}{ap^2\theta^2(N+1)} \; ; \; \frac{\partial \overline{E}}{\partial N} = -\frac{p\theta(s\alpha\beta - a\xi) - s^2\beta w}{ap\theta^2(N+1)^2}
\end{aligned}$$

Let us compute the derivative of $\overline{x}^r$ with respect to the different parameters :

$$\begin{aligned}
\frac{\partial \overline{x}^r}{\partial a} &= \frac{-N(p\theta\alpha - sw)}{a^2 p\theta(N+1)} \; ; \; \frac{\partial \overline{x}^r}{\partial \alpha} = \frac{N}{a(N+1)} \; ; \; \frac{\partial \overline{x}^r}{\partial \beta} = \frac{-\xi}{s\beta^2(N+1)} \\
\frac{\partial \overline{x}^r}{\partial w} &= -\frac{sN}{ap\theta(N+1)} \; ; \; \frac{\partial \overline{x}^r}{\partial p} = \frac{Nsw}{ap^2\theta(N+1)} \; ; \; \frac{\partial \overline{x}^r}{\partial N} = \frac{p\theta(s\alpha\beta - a\xi) - s^2\beta w}{ap\theta s\beta(N+1)^2}
\end{aligned}$$

Let us compute the derivative of $\overline{y}^r$ with respect to the different parameters :

$$\begin{aligned}
\frac{\partial \overline{y}^r}{\partial a} &= -\frac{\xi}{s^2\beta(N+1)} \; ; \; \frac{\partial \overline{y}^r}{\partial \alpha} = \frac{1}{s(N+1)} \; ; \; \frac{\partial \overline{y}^r}{\partial \beta} = \frac{a\xi}{s^2\beta^2(N+1)} \\
\frac{\partial \overline{y}^r}{\partial w} &= \frac{N}{p\theta(N+1)} \; ; \; \frac{\partial \overline{y}^r}{\partial p} = -\frac{Nw}{p^2\theta(N+1)} \; ; \; \frac{\partial \overline{y}^r}{\partial N} = \frac{-p\theta(s\alpha\beta - a\xi) + s^2\beta w}{p\theta s^2\beta(N+1)^2} < 0 \; ;
\end{aligned}$$

Proof of proposition 3

Let us first compare the equilibrium effort levels. We obtain:

$$\begin{aligned}
E^y - \overline{E} &= \frac{s\beta w - p\theta\xi}{(N-\beta)p\theta^2} - \frac{1}{a(N+1)}\frac{p\theta(s\alpha\beta - a\xi) - s^2\beta w}{p\theta^2} \\
&= \frac{1}{p\theta^2}\left(\frac{a(N+1)(s\beta w - p\theta\xi) + (N-\beta)[s^2\beta w - p\theta(s\alpha\beta - a\xi)]}{a(N+1)(N-\beta)}\right).
\end{aligned}$$

If conditions (5) hold then the right hand side of the above expression is positive. Conversely, it is negative if condition (6) is satisfied. This concludes the first part of the proof.

Let us now move on to the comparison of the resulting stock levels. We have:

$$\overline{y} = \frac{s\alpha\beta - a\xi - a\theta N E^y - s\beta\theta N E^x}{s^2\beta}, \quad \overline{y}^r = \frac{s\alpha\beta - a\xi - a\theta N\overline{E}}{s^2\beta}.$$

Thus, we obtain:

$$\overline{y} - \overline{y}^r = \frac{a\theta N(\overline{E} - E^y) - s\beta\theta N E^x}{s^2\beta}.$$

Since $\overline{E} - E^y < 0$ when $\beta < \frac{1}{N}$ while E^x is positive, we conclude immediately that the right hand side term of the above equality is negative in this case.

It remains to show that it remains negative when $\beta > \frac{1}{N}$. From the above equality, we deduce that the sign of the difference $\overline{y} - \overline{y}^r$ is equivalent to the sign of $a\theta(\overline{E} - E^y) - s\beta\theta E^x$, which expression is as follows:

$$\begin{aligned}
a(\overline{E} - E^y) - s\beta E^x &= -\frac{1}{p\theta^2}\left(\frac{s^2\beta w - p\theta(s\alpha\beta - a\xi)}{N+1} + \frac{a(s\beta w - p\theta\xi)}{(N-\beta)}\right) \\
&- \frac{\beta}{p\theta^2}\left(\frac{a(N+1)(s\beta w - p\theta\xi)}{(1-N\beta)(N-\beta)} + \frac{s^2\beta w - p\theta(s\alpha\beta - a\xi)}{1-N\beta}\right) \\
&= -(1+\beta)\frac{(N-\beta)\left[s^2\beta w - p\theta(s\alpha\beta - a\xi)\right] + a(N+1)(s\beta w - p\theta\xi)}{p\theta^2(N+1)(1-N\beta)(N-\beta)}
\end{aligned}$$

The first thing one may notice is that, since condition (6) is assumed to hold, we can conclude that the numerator of the above expression is positive. Moreover, as $N > \beta > \frac{1}{N}$ we conclude that $1 - N\beta < 0$ while $N - \beta$ remains positive, which implies that the denominator is negative. We conclude that the sign of this difference is negative, and so $\overline{y} - \overline{y}^r < 0$ remains valid. This concludes the proof.

Proof of lemma 1

From the proof of Propositions 1 and 2, we have $\overline{x} = \frac{\xi + N\theta E^y}{s\beta}$ and $\overline{x}^r = \frac{\xi + N\theta \overline{E}}{s\beta}$. We deduce then

$$\overline{x} - \overline{x}^r = \frac{N\theta(E^y - \overline{E})}{s\beta}.$$

From Proposition 3 we know that $E^y - \overline{E} > 0$ if and only if parameter β is smaller than $\frac{1}{N}$. This concludes the proof.

Proof of proposition 4

Using $E^x = \frac{s^2\beta w - p\theta(s\alpha\beta - a\xi) + ap\theta^2(N+1)E^y}{sp\theta^2(1-N\beta)}$ and the expressions of $\overline{x}$, $\overline{y}$ and $\overline{y}^r$, we compute the following expression:

$$\Pi - \Pi^r \quad = \quad p\theta[E^x\overline{x} + E^y\overline{y}] - w(E^x + E^y) - p\theta\overline{E}\overline{y}^r + w\overline{E}$$

After a few computations:

$$= \frac{1}{s^2\beta p\theta^2(1-N\beta)} \Big\{ aN \left(p\theta^2\right)^2 \left[(N-\beta)(E^y)^2 + (1-N\beta)\overline{E}^2 \right] + p\theta^2 \left[E^y \left[(N-1)(s^2\beta w - p\theta(s\alpha\beta - a\xi)) \right.\right.$$
$$\left.\left. -a(N+1)(s\beta w - p\theta\xi) \right] + (1-N\beta)(s^2\beta w - p\theta(s\alpha\beta - a\xi))\overline{E} \right] + (p\theta\xi - s\beta w)\left[s^2\beta w - p\theta(s\alpha\beta - a\xi) \right] \Big\}$$

Denoting $\Pi - \Pi^r = \frac{T}{s^2\beta p\theta^2(1-N\beta)}$, the expression of T becomes after simplification:

$$\begin{aligned} T \quad &= \quad \frac{1}{a(N+1)^2(N-\beta)} \left\{ -a^2(N+1)^2(s\beta w - p\theta\xi)^2 - (1-N\beta)(N-\beta)[p\theta(s\alpha\beta - a\xi) - s^2\beta w]^2 \right. \\ &- \quad \left. a(N+1)^2(1-\beta)(s\beta w - p\theta\xi)[s^2\beta w - p\theta(s\alpha\beta - a\xi)] \right\} \end{aligned}$$

The expression between brackets can be rewritten as follows:

$$a(N+1)^2(s\beta w - p\theta\xi)\{-a(s\beta w - p\theta\xi) + \frac{N-\beta}{N+1}[p\theta(s\alpha\beta - a\xi) - s^2\beta w]\} + a(N+1)^2(s\beta w - p\theta\xi)\left[(1-\beta) - \frac{N-\beta}{N+1}\right][p\theta(s\alpha\beta - a\xi) - s^2\beta w]$$
$$+(N-\beta)(N\beta - 1)[p\theta(s\alpha\beta - a\xi) - s^2\beta w]^2.$$

After simplification, it becomes

$$a(N+1)^2(s\beta w - p\theta\xi)\{-a(s\beta w - p\theta\xi) + \frac{N-\beta}{N+1}[p\theta(s\alpha\beta - a\xi) - s^2\beta w]\}$$
$$+[p\theta(s\alpha\beta - a\xi) - s^2\beta w]\{a(N+1)(1-N\beta)(s\beta w - p\theta\xi) + (N-\beta)(N\beta - 1)[p\theta(s\alpha\beta - a\xi) - s^2\beta w]\}$$

or finally

$$a(N+1)^2(s\beta w - p\theta\xi)\{-a(s\beta w - p\theta\xi) + \frac{N-\beta}{N+1}[p\theta(s\alpha\beta - a\xi) - s^2\beta w]\}$$
$$+[p\theta(s\alpha\beta - a\xi) - s^2\beta w](N\beta - 1)\{-a(N+1)(s\beta w - p\theta\xi) + (N-\beta)[p\theta(s\alpha\beta - a\xi) - s^2\beta w]\}.$$

Let us now show that the above two terms are positive when conditions (4)-(6)-(11) are valid. Indeed, this implies that

$$-a(N+1)(s\beta w - p\theta\xi) + (N-\beta)[p\theta(s\alpha\beta - a\xi) - s^2\beta w] > 0$$

and that $N\beta - 1 > 0$ as well, which in turn implies immediately that the second term is positive. Since $(s\beta w - p\theta\xi)$ is positive, condition (6) enables to conclude that the first term is positive too. Then we can conclude that T is positive, and thus, since $(1 - N\beta)$ is negative, we have:

$$\Pi - \Pi^r = \frac{T}{s^2\beta p\theta^2(1-N\beta)} < 0,$$

which concludes the proof.

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