Heterogeneous reactions to heterogeneity in returns from public goods

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1. Introduction

In this paper we address the issue of technology adoption with firms operating in a competitive market and when some environmental regulation is implemented with emission fees or tradable pollution permits. We are not interested in the initial steps of the process, *i.e.* R&D activities and innovation. Instead, we focus on the diffusion of the new technology among firms. In this purpose we model the output market and consider how the regulation of a polluting input shapes the adoption of a new and clean technology.

There exist many papers in the literature interested in the influence of environmental regulation on the adoption of a clean technology, (*e.g.* Milliman and Prince, 1989; Jung et al., 1996; Requate and Unold, 2003). They rely on "partial - partial" equilibrium analysis in the sense where they only consider the market for the polluting good, and not the interaction with the output market. In these papers, the firms engage in pollution abatement but the interplay with the output market equilibrium is not made explicit.

One may argue that the output market is subsumed in the production functions of firms, in particular when a dual approach is taken (minimization of cost functions). In this case the abatement cost encompasses firms' profit losses in the output market. This interpretation is important given that, in many cases, pollution reduction is mainly awaited from changes in the production process. Still, there are two problems with such an interpretation.

Firstly, it is usually assumed in the literature that abatement costs (and thus marginal abatement costs, MACs) are reduced after the adoption of a clean technology. This assumption has been seriously questioned in a series of recent papers. Brechet and Jouvet (2008) notice that the assumption does not hold if the polluting good is an input to the production process. In such a case, they show that the MAC functions of a dirty and a clean firm crosses at least once. In other words, the marginal abatement cost may well increase or decrease after innovation. Amir et al. (2008) analyze under which conditions this result holds by considering a typology of different kinds of innovations. This point was also made by Baker et al. (2008) and by Bauman et al. (2008). Both articles provide some empirical evidences that MACs can actually increase after innovation.

The recent literature has shown that, when a new technology reduces the amount of pollutant input required to produce one unit of output, then the MAC does not necessarily decrease after innovation because of the firm's capacity to increase its market share. This means that the environmental regulation faces conflicting market mechanisms. The fact that the MACs can increase after the adoption of a less polluting technology provides a rationale to the Jevons' paradox because, in such a case, a firm with a new technology can pollute more than firms with the standard technology. This property results from the equilibrium effects on the output market. The increase in output due to the adoption of the new technology can offset the reduction in the emission intensity. This is the very effect stressed by Jevons (1866) in his study about the coal question in the United Kingdom by the end of the XIXth century.

Secondly, the fact that the polluting firms compete in the output market raises another previously unnoticed implication. It has to do with the interplay between firms' abatement costs functions in equilibrium. As noticed just above, the abatement cost of a firm depends on the output market outcome. So it is shaped by the decisions of all other firms through market equilibrium. A firm's incentive to adopt the new technology is influenced by the behavior of all its competitors on the output market. When the regulation is done with tradable permits, then the incentive also depends on the equilibrium on the market for permits, and the interactions between these two markets in equilibrium are also key.

Our main results can be summarized as follows. We first show that there exist situations where only a subset of firms adopts the clean technology. At the equilibrium firms end-up heterogeneous even though they are identical ex-ante. Then, we show that the proportion of clean firms is not monotonic with respect to the tax level or the emission cap. This proportion first increases, and then decreases with the tax level. So, setting the tax too high can discourage innovation. Similarly, strengthening the emission cap can lead to less innovation because the output level becomes too small to push out the innovation. This property has impacts on the output market. It may be the case that the output price decreases with the tax, because of the endogenous adoption of the clean technology. When the tax increases, a larger number of firms adopt the clean technology, and thus increase their output level, and the overall effect can be positive on the aggregate output level.

As far as pollution is concerned, we show that aggregate pollution level can be higher with clean firms in the economy than with dirty firms only. This comes from the fact that a clean firm has a higher activity level than a dirty one, and this may well offset the reduction in pollution intensity. The result is obtained under an emission tax, but a corresponding result holds under tradable permits: the equilibrium permit price can be larger with clean firms than without.

Then, we compare the properties of the policy instruments in terms of pollution and incentive for adoption, *i.e.* their influence on the proportion of adopting firms in equilibrium. We take the usual assumption of a myopic regulator that does not anticipate firms' reaction in terms of technology adoption. Concerning emissions, we show that two situations can arise. In one case, the tax is better than permits for the environment, more precisely, there are too few emissions with a tax and too many with permits compared to the optimal policy. In the other case, the ranking is reversed, permits are favorable to the environment. We discuss the consequences on the welfare comparison of instruments.

Finally, concerning the ranking of instruments with respect to the incentive to adopt the clean technology, three distinct cases must be distinguished. First, for low tax rates or high emission caps, similar results as Requate and Unold (2003) are found: a smaller number of firms innovate with permits than with the tax because the pollution price is lower with permits. In this case the pollution price decreases when one more firm adopts the clean technology, and this price decrease further reduces the incentive to innovate. Second, for high taxes rates or stringent emission caps, similar results are found but with a reversed causality: a smaller number of firms innovate with permits than with the tax, but the price of pollution is higher with permits. Third, for intermediary values of the instruments, new results come out: there is a stronger technology adoption with permits than with the tax because, with permits, technology adoption reduces the pollution price, which fosters innovation.

The paper is organized as follows. In Section 2 we present the model, we define equilibrium in the output market and the technology adoption equilibrium. In Section 3, monotonicity properties of the policy instruments are scrutinized, *i.e.* how the proportion of innovative firms varies with the policy instrument. This will allow us to compare the instruments in terms of welfare and incentive to innovate in Section 4. Section 5 is the conclusion.

2. The model

2.1. Firms and technology

We consider an economy composed of a continuum of ex-ante symmetric firms with a total mass normalized to 1. Firms produce a homogeneous good an sell it at a market price p and face an inverse demand function given by

$$p(Y) = a - bY \tag{1}$$

where $a, b > 0$ and where Y is the aggregate output level. The gross consumer surplus is thus given by $S(Y) = (a - 0.5bY)Y$.

Two production technologies are available. They differ by their output polluting emission rate, $u \in \{1, \alpha\}$. There exists a *standard* technology with an emission rate normalized to 1, and a *clean* technology with an emission rate $\alpha \in (0, 1)$. The aggregate amount of pollution is denoted by E. The production cost $c(y)$ is common to both technologies,

$$c(y) = \frac{\gamma}{2} y^2. \tag{2}$$

The proportion of clean firms in the economy is denoted by $\beta \in [0, 1]$. In the short term, each firm is either clean or standard depending its technology, so β is fixed. We shall compare the properties of an emission tax, denoted by t, and a market for pollution permits (a cap-and-trade system) where the permits price will be denoted by σ. In the short term, with a polluting tax, the profit of a firm with technology $u = 1, \alpha$ is given by:

$$\pi_u(p, y, t) = py - c(y) - tuy. \tag{3}$$

All clean (resp. standard) firms produce the same output level y_α (resp. y_1). Thus, aggregate output and polluting emissions in the economy read as:

$$Y = \beta y_\alpha + (1 - \beta)\, y_1 \tag{4}$$
$$E = \beta \alpha y_\alpha + (1 - \beta)\, y_1 \tag{5}$$

To guarantee positive profits we impose the following condition:

$$\alpha > b/(b + \gamma). \tag{C1}$$

When this condition holds the cost advantage of the clean firms is not sufficient to drive standard firms out of the market. Furthermore, even if some of our results can be obtained without this condition, the existence of the Jevon's paradox (which is central to our analysis) requires to impose a lower bound on the emission rate of the clean technology. This condition is further discussed in Appendix A2.

In the long term each firm decides whether to adopt the clean technology, with a fixed adoption cost f, or not.

In the following we first describe the short term market equilibrium when β is fixed before considering the decision to adopt the clean technology, and the long term equilibrium with an endogenous β. These two steps are presented under the case of an emission tax t. The case with tradable pollution permits will be considered below.

2.2. Output market equilibrium

In the short term β is given and if t is not too large, both types of firms have a positive output level. For a given output price p the output level of a firm of type $u = 1, \alpha$, $y_u(p,t)$, equalizes the output price to its marginal cost :

$$p = c'(y_u) + tu. \tag{6}$$

Total supply of the whole mass of producers $Y(p, \beta, t)$ is thus:

$$Y(p,t,\beta) = \beta y_\alpha(p,t) + (1-\beta)y_1(p,t). \tag{7}$$

At the equilibrium, the price $p(\beta,t)$ clears the market:

$$D(p(\beta,t)) = Y(p(\beta,t),t,\beta)).$$

Because of its lower emissions rate, a clean firm has a lower marginal production cost and has a higher output level than a standard firm. Thus, the adoption of the clean technology by one firm (*i.e.* a marginal increase of β) rises the aggregate output level, pushes the output price down, and thus reduces the output level for all other firms in the economy. The expressions in the linear case are provided in appendix (Appendix A.1).

In equilibrium the aggregate emissions level is:

$$E(\beta,t) = \beta\alpha y_\alpha(p(\beta,t),t) + (1-\beta)y_1(p(\beta,t),t). \tag{8}$$

Comparing the emissions level of a clean and a standard firm is not straightforward. Even though the clean firm has a lower emission rate, it may well pollute more because of a higher output level. This can be seen from the first order condition (6), the production of a firm $u = 1, \alpha$ is $y_u = (p - tu)/\gamma$; so, emissions of a clean firm αy_α are larger than that of a standard firm y_1 if and only if $t(1+\alpha) \geq p$. At the global level, the fact that a clean firm pollutes more than a standard one does not necessarily imply that there are more emissions with clean firms in the economy ($\beta > 0$) than without ($\beta = 0$). This is because, when there are clean firms in the economy, standard firms have a lower output and pollution levels. This leads us to the following Lemma.

Lemma 1. *A clean firm pollutes more than a standard firm if and only if*

$$t > \frac{\gamma a}{\alpha(b+\gamma) + \gamma + b\beta\,(1-\alpha)}. \tag{9}$$

Aggregate emissions are higher with clean firms in the economy ($\beta > 0$) than without ($\beta = 0$) if and only if

$$t > \frac{\gamma a}{\alpha(b+\gamma) + \gamma + b\beta\,(1-\alpha) - b}. \tag{10}$$

Proof. See appendix Appendix A.2. ■

In comparison with the usual framework that can be found in environmental economics about technology adoption, introducing the output market implies that a "clean" firm (a firm that adopts the clean technology) does not necessarily pollute less than a standard firm. For a given output price p, a clean firm produces more than a standard one, and the difference increases with the tax. If the tax level is high enough, the difference in output levels offsets the gain in emission intensity. This is an illustration of the Jevons paradox (Jevons, 1866). This effect is a cornerstone for our analysis and can be reformulated in terms of Marginal Abatement Costs (MACs).[1] For a given output price, the MAC of a clean firm is not necessarily lower than the MAC of a standard firm. It is lower when the tax level is low, but its is higher when the tax level is high. With our specifications, the MAC functions cross once, but with more general specifications they can cross more (see Brechet and Jouvet, 2008; Amir et al., 2008).

Beside the fact that MACs cross when output production is introduced, an additional effect occurs due to the fact that firms compete in the output market. Actually, the adoption of the clean technology by a firm (*i.e.* an increase of β) modifies not only its own MAC, but also the one of all the other firms in the economy via the changing in the output price. The MAC function of a firm does not only depend on its own technology, but also on the technology of all other firms and on market outcome. The larger the proportion of clean firms in the economy, the lower the output price and the lower the MAC of each firm. This property provides a rationale for the second part of Lemma 1. Even if each clean firm pollute more than standard firms, the aggregate pollution level may well be smaller in the presence of clean firms than without because introducing clean firms reduces the MACs, and therefore the emissions, of all standard firms.

Alternatively, one can also consider the effect of the proportion of clean firms in the economy on aggregate emissions. As the number of clean firms increases, the aggregate emission level first decreases and then increases. It can possibly be higher than its initial level (*i.e.* without pollution tax) if the share of clean firms is large enough.

[1] The pollution abatement cost of a firm is the difference in profits, $C_u(p,\nu) = \pi^0 - \pi_u(p,(e^0-\nu)/u,0)$ where π^0 and e^0 are the profit and the quantity of emissions of a firm without environmental policy and ν the abatement.

2.3. Technology adoption equilibrium

The incentive for a firm to adopt the clean technology is given by the difference in profit with and without the clean technology. In this section we show that there exists a unique equilibrium share of clean firms that could be interior, *i.e.* strictly between 0 and 1 (proposition 1). *Ex-ante* symmetric and price-taking firms can become asymmetric in equilibrium. This is because, as shown in the above section, the equilibrium output price is related to the share of clean firms in the economy. Thus, the larger the number of clean firms, the lower the incentive to adopt the clean technology and, at an interior equilibrium, the incentive is nil.

For an output price p, a tax t, and a fixed adoption cost f, the incentive to adopt the clean technology writes:

$$\psi(p,t) = \pi_\alpha(p, y_\alpha, t) - \pi_1(p, y_1, t) - f. \tag{11}$$

In equilibrium the price is endogenously determined by the short term market equilibrium. Three situations can arise in which none, all, or some firms adopt the clean technology at the long-run equilibrium. A Technology Adoption Equilibrium is thus defined as follows.

Definition 1. *A share $\beta \in [0,1]$ is a Technology Adoption Equilibrium (TAE) if and only if*

(i) $\beta = 0$ *and* $\psi(p(0,t),t) \leq 0$*;*

(ii) $\beta = 1$ *and* $\psi(p(1,t),t) \geq 0$*;*

(iii) $\beta \in (0,1)$ *and*

$$\psi(p(\beta,t),t) = 0. \tag{12}$$

The existence and uniqueness of a Technology Adoption Equilibrium is stated in the following proposition.

Proposition 1. *For a given emission t, there exists a unique TAE, denoted by $\beta^t(t)$.*

Proof. We prove the uniqueness of the equilibrium by establishing that at least one and only one of the three situations listed above can arise. The incentive to adopt the clean technology is increasing in the output price:

$$\frac{\partial \psi}{\partial p} = y_\alpha - y_1 > 0;$$

thus, the function $\beta \to \psi(p(\beta,t),t)$ defined on the set $[0,1]$ is decreasing so it is either always negative (and $\beta^t = 0$ is the unique TAE), or always positive (and $\beta^t = 1$ is the unique TAE), or there exists a unique root of (12) which is a TAE $\beta(t)$. [2] ■

[2]We restrict the analysis to fixed costs levels f such that, $\forall t$, $\beta(t) < 1$ and $\exists t$ with $\beta > 0$. See the appendix for details.

Naturally, for a given tax level, the proportion of clean firms in the economy is decreasing with f : it goes from full adoption (all firms are clean) to partial adoption, and eventually to no adoption at all as f increases. From now onwards we shall assume f low enough so that there exists a tax level for which some firms adopt the clean technology. We shall also assume that f is high enough so that, for any positive tax level there is no full adoption in the economy (that is, $\beta < 1$). The latter assumption is made for the sake of exposition and to avoid the fastidious enumerations of different cases. So we assume that

$$\underline{f} < f < \bar{f}. \tag{C2}$$

The expressions for $\underline{f}$ and $\bar{f}$ are given in appendix B.2.

3. Monotonicity properties

The question we address in this section is the following. Does the proportion of clean firms in the economy increases when the environmental policy becomes more stringent? While the existing literature would answer yes, we will show that it is not necessarily the case. We will first consider the case of an emission tax, and then the case of tradable pollution permits. The comparison between those two instruments is relegated to the next section.

3.1. The case of an emission tax

When an emission tax is considered the share of clean firms in the TAE displays some intuitive monotonicity properties. It decreases with the fixed cost and the emission intensity of the clean technology, and it increases with the market size ($1/b$). By contrast, the effect of the tax level is not straightforward, as it is not monotonic. The direct (*ex-ante*) impact of the tax level on the incentive is given by:

$$\frac{\partial \psi}{\partial t} = \frac{\partial \pi_\alpha}{\partial t} - \frac{\partial \pi_1}{\partial t} = y_1 - \alpha y_\alpha. \tag{13}$$

For a given output price, the impact of a change in the tax level on a firm's profit depends on its emissions level (by the envelop theorem). Thus, when the emission level of clean firms is lower than the emission level of a standard firm, then the incentive to adopt the clean technology increases with the tax. This effect is reversed in the opposite case. Consequently, for a given output price the incentive function ψ is bell-shaped with respect to the tax t. A firm has a positive incentive to adopt the clean technology only for intermediary values of the tax. When considering the impact of the tax level on the equilibrium share of clean firms in the economy, an additional indirect effect appears (related to the output price), at an interior equilibria:

$$\left[-\frac{\partial \psi}{\partial p}\frac{\partial p}{\partial \beta}\right]\frac{\partial \beta(t)}{\partial t} = \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial p}\frac{\partial p}{\partial t} = (y_1 - \alpha y_\alpha) + (y_\alpha - y_1)\frac{\partial p}{\partial t} \tag{14}$$

The term depicts the direct effect of the tax on the incentive to adopt the clean technology. The second term is the indirect effect. This effect is positive because the output price increases with the tax and because the incentive increases with the output price (due to the fact that a clean firm has a higher output level than a standard one). Thus, the indirect effect mitigates the direct effect. It never cancels it out, and for sufficiently high tax levels the share of clean firms decreases with t.

Proposition 2. *There exists a critical tax level $\hat{t}$ such that the share of clean firms in the economy is first increasing, and then decreasing with the tax level. $\hat{t}$ is given by:*

$$\hat{t} = 2f\frac{b+\gamma}{a(1-\alpha)} \tag{15}$$

The adoption of the clean technology by the firms yields positive consequences for consumers, because it increases the output level. However, an unexpected consequence of the adoption of the clean technology is that the output price is not necessarily increasing with the tax. For intermediary values of the tax –when firms begin adopting the clean technology– the output price decreases with the tax because a larger number of firms become clean. In the same time, the increase in output due to a higher share of clean firms offsets the direct effect due to a higher tax. After a tax increase, two opposite effects play on the output. On the one hand, there is a direct negative effect due to the increase of firms' marginal costs. On the other hand, there is a positive indirect effect due to the diffusion of the clean technology. It is possible to determine under which conditions one effect dominates the other by considering the difference in profits.

At an interior TAE, equation (12) is satisfied, which means that the difference in profit between a clean and a standard firm is f. Substituting expression (11) for ψ and differentiating (12) with respect to t provides:

$$(y_1 - \alpha y_\alpha) + (y_\alpha - y_1)\frac{dp}{dt} = 0.$$

The first term in LHS of the equation shows how the difference in profits is directly impacted by a change in the tax level. The second term shows how it is altered by the indirect effect of the tax on the output price. The difference in the output levels is positive. This means that an increase in the output price is more beneficial to a clean firm than to a standard one. Therefore, the sign of the price change is the sign of the difference in emissions between a clean and a standard firm. In the case of partial adoption and when clean firms pollute less than standard ones, then a marginal increase in the tax level will decrease the output price. In such a case the positive indirect effect (playing through the adoption) dominates the direct negative effect (related to the tax). This can only happen for a range of taxes where clean firms pollute less than standard ones, while producing more.

Let us now analyze the effect of the tax on the aggregate amount of pollution in the economy. As mentioned before, for a given proportion of clean firms in the economy, the total amount of pollution may can be larger than in an economy without any clean firms, provided the tax is high enough. Alternatively, for a given tax level, the total amount of pollution can be larger with the clean technology if the proportion of clean firms is sufficiently large in the economy. When the proportion of clean firms becomes endogenous (in a TAE), we already know that this proportion will eventually decrease with the tax (for high tax levels). Thus it is unclear whether global emissions could be higher or lower with the clean technology in the long-term. Proposition 3 answers this question.

Proposition 3. *At the Technology Adoption Equilibrium, aggregate pollution is larger than without the clean technology if $t \geq \tilde{t}$. It is lower otherwise. We have:*

$$\tilde{t} = \left[\frac{2\gamma f}{1-\alpha} \frac{b+\gamma}{(1+\alpha)\gamma - (1-\alpha)b} \right]^{1/2}. \tag{16}$$

Furthermore,

$$\tilde{t} > \hat{t}. \tag{17}$$

Proof. See appendix C1. ■

The rationale behind this result was explained in detailed above in the paper. It is important here to stress that the aim of the policy is not to foster innovation *per se*, but to reduce the global pollution level in the economy. We show here that the adoption of a clean technology will not necessarily be effective in curbing global pollution. Figure 1 displays the aggregate pollution level in the economy with respect to the tax level. By construction, this relation is linear without innovation. It becomes non linear in the TAE and pollution is higher (resp. lower) if the tax is above (resp. below) the critical tax level $\tilde{t}$.

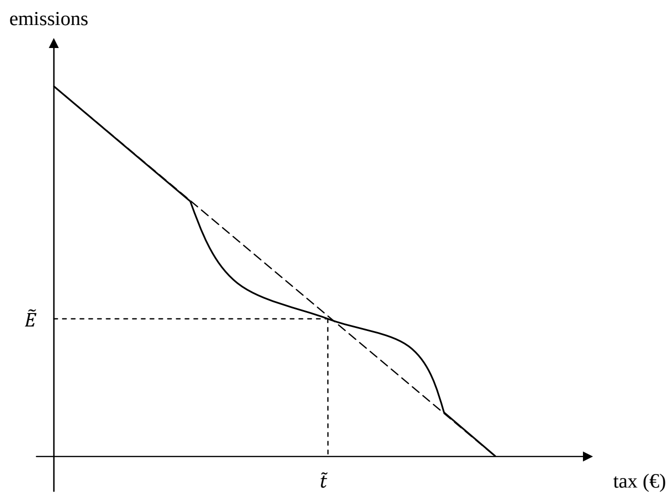


Figure 1: Aggregate emissions with respect to the tax without the clean technology (dashed line) and at the TAE (plain line)

3.2. The case of tradable pollution permits

The results established for a tax have counterparts in the case of tradable emission permits (TEPs). Let us denote by $\bar{E}$ the total amount of emission permits issued by the regulator, and by σ their price. For any permit price σ the long-term demand for permits is $E(\beta(\sigma), \sigma)$, which is decreasing in σ. The equilibrium price $\sigma(\bar{E})$ that clears the market for permits is such that:

$$\bar{E} = E(\beta(\sigma(\bar{E})), \sigma(\bar{E})). \tag{18}$$

The proportion of clean firms in equilibrium when $\bar{E}$ permits are issued is $\beta(\sigma(\bar{E}))$. We have just shown that, with a tax, the global emission level is not necessarily lower with the clean technology than without. This result translates into the result that, with TEPs, the equilibrium permit price is not necessarily lower with the clean technology than without it. This comes from the fact that the demand for permits of clean firms may well be larger than the demand of standard firms (see Lemma 1). In the short-term, for a given β, there exists a threshold in the emission cap such that, for any lower cap the permit price is higher after adoption. In the long-term, a twin proposition to Proposition 3 can thus be provided.

Proposition 4. *At the Technology Adoption Equilibrium, the permit price is higher than without clean technology if $\bar{E} < \tilde{E}$. It is lower otherwise. We have:*

$$\tilde{E} = E(\beta(\tilde{t}), \tilde{t}). \tag{19}$$

Proof. Let us denote *by*σ^0 the permit price without the clean technology. It solves $\bar{E} = E(0, \sigma^0)$. If $\bar{E} < \tilde{E}$ then $\sigma(E) > \sigma(\tilde{E}) = \tilde{t}$, so, by Proposition 2,

$\bar{E} \geq E(0, \sigma)$. Then, $E(0, \sigma) \leq E(0, \sigma^0)$ implies $\sigma \geq \sigma^0$. If $\bar{E} \geq \tilde{E}$ a similar reasoning leads to $\sigma \leq \sigma^0$. ■

Like in the case of a tax on pollution, the proportion of clean firms in the economy is not monotonic with respect to the emission cap $\bar{E}$. It is also bell-shaped. Let us denote by $\hat{E}$ the threshold cap:

$$\hat{E} = E(\beta(\hat{t}), \hat{t}). \tag{20}$$

Finally, as far as the output market equilibrium is concerned, the output price will decrease when the emissions cap is strengthened if the proportion of clean firms is interior (*i.e.* $\beta(\sigma) \in (0, 1)$) and if the clean firms pollute less than the standard ones.

We are now ready to compare the two instruments.

4. Instruments comparison

In this section we shall first analyze the influence of the clean technology on the socially optimal emissions level. In that purpose it is convenient to define two zones according to the effect of the adoption of the clean technology on the marginal abatement cost. Then we shall compare tax and permits in terms of social welfare. Because the equivalence between tax and permits holds in our framework, a regulator that would anticipate the level of technology adoption would reach a similar outcome whatever the policy instrument. More appealing, we shall also consider the case of a myopic regulator that designs its policy instruments while ignoring that firms can adopt a clean technology. Finally, we shall analyze how the choice of instruments influences the proportion of clean firms in the economy and rank the instruments with respect of socially optimal proportion of clean firms.

4.1. Welfare analysis

Let us introduce a damage function representing the social cost of pollution, $D(E)$. This function is strictly increasing and convex in the aggregate pollution level. The economic benefits of economic activity (and so, of the related pollution) are denoted by $B(E)$ and defined as follows:

$$B(E) = \max_{y_1, y_\alpha, \beta} S\left(\beta y_\alpha + (1-\beta) y_1\right) - \left[\beta c(y_\alpha) + (1-\beta) c(y_1)\right] - \beta f. \tag{21}$$

Social welfare is thus given by:

$$W = B(E) - D(E). \tag{22}$$

We denote by E^* the socially optimal level of emissions. It is such that $B'(E^*) = D'(E^*)$. In our framework the economic benefits $B(E)$ can be decentralized by a tax or emission permits market because there is no market failure related to technology adoption. For any amount of emission E the marginal benefit pollution is given by the permit price:

$$B'(E) = \sigma(E) = p - c'(y_1) = [p - c'(y_\alpha)]/\alpha.$$

Thus, the marginal benefit of pollution is given by the inverse price function of permits displayed in Figure 2. This *macro-MAC* captures the relationship between prices and quantities at the Technology Adoption Equilibrium. Figure 2 shows that, in the linear example, even though individual firms have linear MAC functions the macro-MAC function is not linear at all. It is concave-convex.

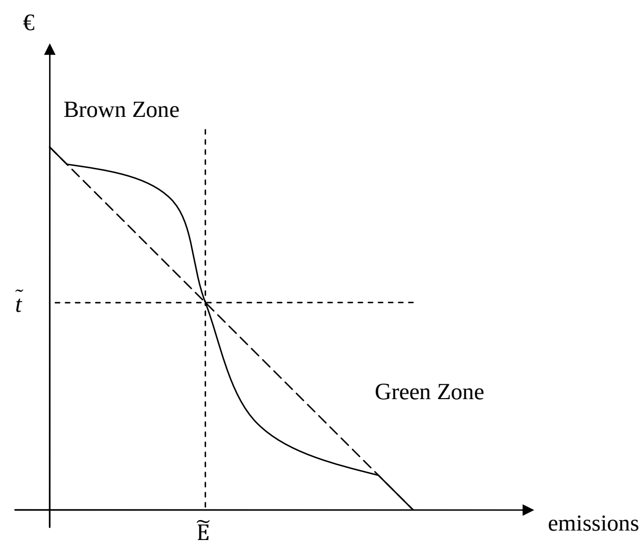


Figure 2: Equilibrium Marginal Abatement Cost function with endogenous technology adoption

Figure 2 allows us to define two zones by comparing the macro-MAC before and after adoption of the clean technology.

Definition 2. *The Green zone is the set of emission levels E such that the macro-MAC is lower after adoption. The Brown zone is the set of emission levels E such that the macro-MAC is higher after adoption.*

In the Green zone, the macro-MAC is lowered after the adoption of the clean technology while it is increased in the Brown zone. The threshold between the two zones is $\tilde{E}$, if one talks in terms of quantities, or $\tilde{t}$ if one talks in terms of prices. In the Green zone, for an emission level larger than $\tilde{E}$, the clean technology has the intuitive effect on the aggregate marginal abatement cost: it reduces it and the level of the emission constraint is lower with the clean technology. On the contrary, in the Brown zone, for a stringent environmental policy $E < \tilde{E}$, the clean technology rises the macro-MAC. The emissions constraint has a higher shadow price when the clean technology is available because it allows to produce more.

The socially optimal pollution level is given by the intersection between the MAC and the marginal environmental damage. The fact that the macro-MAC is not lowered everywhere by the clean technology implies that the optimal level of emissions is either reduced or increased by the clean technology. At the optimum, if the marginal benefits and damages cross in the Green zone (i.e. for relatively high emission levels) then the socially optimal emissions level is reduced with clean technology adoption. In the Brown zone (i.e. for relatively low pollution levels), it is increased. This result is summarized in the following proposition.

Proposition 5. *The so-called clean technology reduces the socially optimal pollution level if and only if the optimal pollution level wo adoption is high ($E > \tilde{E}$).*

This proposition shows that the interactions between pollution, clean technology adoption, and welfare are far from being straightforward. The purpose of the following subsections is to better understand these interactions as well as to draw policy implications.

4.2. Which policy instrument to promote environmental quality?

We shall first compare the policy instruments in their capacity to promote a clean environment. Actually, as it can be understood from the previous section, just promoting the adoption of a clean technology is not an aim that can be socially justified by itself. The aim may be to improve environmental quality so that social welfare is maximized, but again this is not that simple, as shown in the previous Proposition. We shall consider that the regulator is unable to anticipate private firms' reaction in terms of technology adoption as a response to its policy. Thus, the regulator has the choice between setting either an emission tax t or a cap-and-trade system with a cap on global emissions $\bar{E}$.[3] It chooses the level of the instruments by solving the following problem:

$$\max S(y_1) - \frac{1}{2}\gamma y_1^2 - D(y_1). \tag{23}$$

If tradable permits are used, then the regulator will set $\bar{E}$ such that $p(\bar{E}) - \gamma\bar{E} = D'(\bar{E})$. If an emission tax is used, the regulator will set a tax level equal to the marginal damage, $t = D'(\bar{E})$, which also satisfies $E(0,t) = \bar{E}$. Because the regulator does not anticipate technology adoption, both choices are suboptimal. But more important is that the two instruments will yield contrasting economic and environmental outcomes depending on whether the optimum is situated in the Green or in the Brown zone. This is summarized in the following proposition.

Proposition 6. *If the social optimum $E^\star$ is in the Green zone (case 1):*

[3]Because we are in perfect competition in all markets, the way permits are endowed among firms is not an issue.

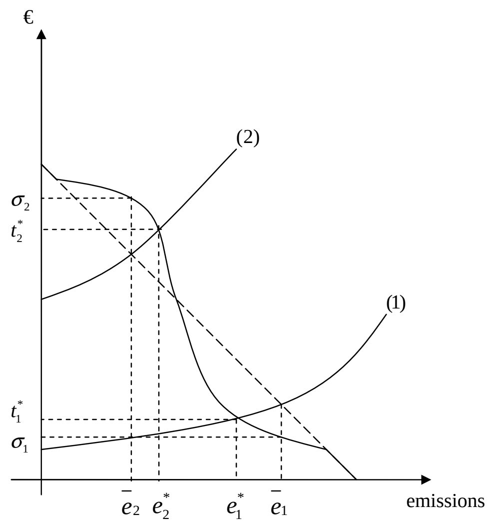
- *With tradable permits, the regulator issues too many permits than the optimal cap and the equilibrium permits price is lower than the optimal one;*
- *With a tax, the regulator sets a tax level which is higher than the optimal one and emissions are smaller than at the optimum.*

If the social optimum $E^\star$ is in the Brown zone (case 2):

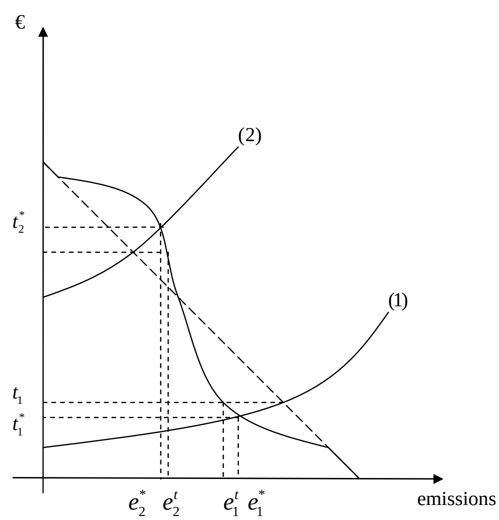
- *With tradable permits, the regulator issues too few permits than the optimal cap and the equilibrium permits price is higher than the optimal one;*
- *With a tax, the regulator sets a tax level which is lower than the optimal one and emissions are larger than at the optimum;*

Figure 3 provides a comprehensive and convenient overview of all possible outcomes as they are stated in the Proposition. Case (1) refers to the case where the optimum is located the Green zone and Case (2) refers to the case where it is located in the Brown zone. Left panel, Fig. 3(a), is for TEPs and right panel, Fig. 3(b), is for a tax.

In the Green zone the comparison is rather intuitive: the regulator that does not anticipate the adoption process overestimates the marginal benefits of polluting emission and sets a tax or an emissions cap larger than optimal ones. From an environmental point of view, in this zone, with a myopic regulator, the tax is good for the environment. In the Brown zone, the situation is the other way round. The regulator underestimates the marginal benefits and sets a tax or an emission cap lower than the optimal ones. In that case, which arises if the environmental marginal damage is large, permits are more favorable to the environment than a tax.



(a) Tradable emission permits



(b) Emission tax

Figure 3: Comparison of myopic emissions quantities and price to the optimal ones, in the Green zone (marginal damage 1) and in the Brown zone (marginal damage 2), with permits (a) and tax (b).

In Proposition 4 we compare the price and the amount of polluting emissions obtained with the two instruments with respect to the optimal ones. One could also compare the two instruments in terms of welfare. As can be seen from

Figure 3 this comparison would depend on the shape of the marginal damage curve. Let us take an example with environmental damages given by:

$$D(E) = t_1 E + \frac{\lambda}{2}(E - \bar{E}_1)^2,$$

with $\bar{E}_1 = E(0, t_1)$. With such a damage function, t_1 and $\bar{E}_1$ are the levels of the instruments that are choosen by the myopic regulator for all λ, because $t_1 = D'(\bar{E}_1), \forall \lambda$. The parameter λ is the slope of the marginal environmental damage; graphically, a change in λ corresponds to a rotation of the marginal environmental damage curve around its intersection with MAC without adoption. It can easily be checked graphically that, the steeper the marginal environmental damage the more favorable to permits the comparison is. This result holds in both zones.

The result that tax and permits yield contrasting outcomes flows from the myopia of the regulator who does not anticipate the adoption process. Even if such an assumption is standard in the literature, another frequent approach would consist in introducing uncertainty or asymmetric information between the regulator and firms. In a seminal paper Weitzman (1974) demonstrated that in such a case a tax is preferred to permits if and only if the slope of the marginal environmental damage is lower than the slope of the marginal economic benefit (*i.e.* the slope of the MAC). Our framework can be used to analyze how the adoption process would modify the comparison of instruments *à la* Weitzman. Without adoption, the slope of the marginal benefit is $b+\gamma$. From Figure (3) one can see that the slope of the marginal benefit with adoption is larger than $b+\gamma$ for an intermediary range of emission levels, and it is lower for very high or very low emission levels. In the former case, the appeal of the tax would be reinforced with adoption, while in the latter case it is the appeal of tradable permits that would be fostered. This result must be contrasted with a recent study of Weber and Neuhoff (2010), where the authors proceed to a comparison of instrument *à la* Weitzman. They show that the comparison is more favorable to tradable permits when innovation is taken into account. Still, in their framework the equilibrium in the output market is not considered and innovation reduces both abatement cost and marginal abatement cost.

4.3. Which policy instrument to promote technology adoption?

Let us now analyze how the choice of instruments affects the share of clean firm at the adoption equilibrium. As previously, we consider that the regulator does not anticipate the adoption process and sets a tax t or issues $\bar{E} = E(0, t)$ permits. We compare the adoption level under each instrument, and the optimal level. The optimal tax level, emission level, and the optimal proportion of clean firms are respectively denoted by t^*, E^* and β^*. The corresponding variables under an emission tax (resp. tradable permits) are denoted by t, E^{tax} and β^{tax} (resp. σ, $\bar{E}$ and β^{tep}).

The results are summarized in the following proposition.

Proposition 7. *If the optimal policy is located in the Brown zone, then the following ranking holds:*

$$\beta^{tep} \leq \beta^* \leq \beta^{tax}.$$

If the optimal policy is located in the Green zone, then the three following cases can arise:

- *for intermediary emission levels, the ranking of instruments is reversed:*

$$\beta^{tax} < \beta^{tep};$$

- *for intermediary emission levels, both instruments lead to too few adopting firms:*

$$\beta^* > \max\{\beta^{tax}, \beta^{tep}\};$$

- *for large emission levels, the same ranking as in the Brown zone holds:*

$$\beta^{tax} < \beta^{tep}.$$

To help understand these results, the comparison can conveniently be carried out by using Figure 4. This Figure depicts how the proportion of clean firms changes with respect to the tax level.

Compared to previous results on technology adoption and instruments choice, here we obtain more contrasted results. In the Brown zone and in part of the Green zone the same ranking holds: there is over-adoption (*w.r.t.* the optimum) with a tax and under-adoption with permits. This ranking is the same as the one established by Requate and Unold (2003). In an intermediary area which is located in the Green zone, several rankings can occur. To better understand the mechanisms at stake it is worth considering how the price of emissions (the tax level or the permits price) differs depending on the instrument.

Let us first explain how the intuitive comparison obtained in the Brown zone is in fact the outcome of a counterintuitive process. There are two mechanisms at stake: (i) the effect of the adoption of the clean technology on the permits price, and (ii) the effect of the emission price (the tax or the permit price) on the incentive to adopt the clean technology. In the literature on clean technology adoption it is shown that, with tradable permits, (i) adoption of a clean technology by a firm decreases its net demand for permits, and consequently their equilibrium price, and that (ii) a lower emission price reduces the incentive to adopt the clean technology by other firms. These two mechanisms are related to the assumption made in the literature that the marginal abatement cost decreases after adoption. In our framework, such a result corresponds to the case of a low tax ($t < t_1$) and a large emission cap, and it occurs in the Green zone for high emission levels. In this case, under permits there are fewer firms that adopt the clean technology and the permit price is lower than the corresponding tax. Actually, in the Brown zone the two steps of this process are reversed: (i) an increase of the number of clean firms increases the demand for permits and thus their equilibrium price, and (ii) the increase in the permit price shrinks the incentive to adopt the clean technology. Therefore, under a regulation with permits, there are fewer firms that adopt the clean technology and the permit price is larger than the corresponding tax. The fact that technology adoption pushes the permits price up does not imply that there is more adoption with permits than with a tax because, in this case, a high permit price does not increase firms' incentive to adopt the clean technology.

In the Green zone, for intermediary values of the tax and the emission cap, the ranking is different because the second step is different (not the first one). This is so because the tax level at which adoption is maximal $\hat{t}$ (cf. Proposition 2) is lower than $\tilde{t}$ (cf. Proposition 3). The situation is illustrated in Figure 4. When the proportion of clean firms is decreasing with respect to the tax (*i.e.* increasing with respect to the emission cap), under permits the permit price is lower than the corresponding tax, but adoption is stronger because a low permit price favors adoption. The optimum is again located between the two instruments in terms of adoption rate. One can see from Figure 4 that, for larger emission levels, both instruments yield too low a level of adoption, even though the comparison between themslves could be in either way. The multiplicity of situations occurs because the proportion of clean firms is not monotonous in the Green zone. There exists a range of emission prices where an increase of this price (or an increase in the stringency of the environmental policy) leads to a decrease in the proportion of adopting firms while, in the meantime, emissions are lower at the TAE than without adoption. Let us note that the proportion of clean firms declines with respect to the emissions price only if a clean firm pollutes more than a standard one. So, in this range of prices, clean firms pollute more than standard ones, but the aggregate amount of emissions is nevertheless smaller with the clean technology than without.

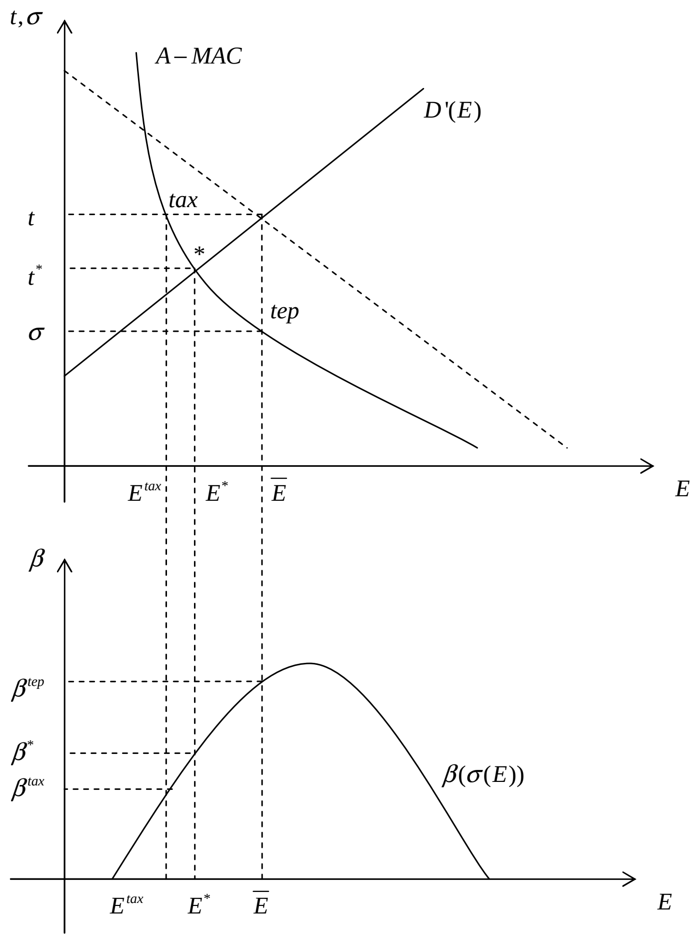


Figure 4: Comparison of tax, tradable permits, and the optimum for the intermediary situation in the Green zone.

In the previous discussion we have stressed that the comparison of adoption rates (Proposition 5) is best understood if linked to the comparison of the quantity and price of emissions (Proposition 4). All these comparisons are gathered and displayed in a more rigorous way in Table 1. Table 1 must be read from the left to the right : this direction corresponds to a less stringent policy (a lower emission tax or a larger emission cap). Hence, the reading begins in the Brown zone and goes gradually to the Green zone. This also corresponds to a move along the MAC in Figure 2 from North-West to South-West. The table can be built up by decreasing the marginal damage function in Figure 4.

Brown Zone	Green Zone			
$\bar{E} < E^* < E^{tax}$ $\sigma > t^* > t$	$\bar{E} > E^* > E^{tax}$ $\sigma < t^* < t$			
$\beta^{tep} < \beta^* < \beta^{tax}$	$\beta^{tax} < \beta^{tep}$		$\beta^{tep} < \beta^{tax}$	
	$\beta^{tax} < \beta^* < \beta^{tep}$	$\beta^{tax} < \beta^{tep} < \beta^*$	$\beta^{tep} < \beta^{tax} < \beta^*$	$\beta^{tep} < \beta^* < \beta^{tax}$

Table 1: An overview of the comparison of instruments (tax and tradable permits), and the optimum

A last comment that deserves some attention is the possibility to regulate the adoption process directly. Once the regulator has mistakenly set an instrument, she could still try to regulate the adoption process, for instance by subsidizing adoption. There exists an important difference between instruments regarding this possibility. Although permits often lead to a suboptimal proportion of clean firms, it is worth stressing that *given*the amount of emissions, the share of clean firms is optimal. This second point is related to the fact that there is no market failure related to adoption in our framework. The only market failure is the environmental externality. If tradable permits are used, the environmental damage is given by the emission cap and this is the only relevant regulatory variable that matters. It is not worth further regulating the output market. The situation is different with a tax because the tax does not directly determine the environmental damage. If the tax is suboptimal then the equilibrium proportion of clean firms is suboptimal, and then it is possible to improve the situation by regulating the adoption process itself.

5. Conclusion

In this paper we compare the properties of a pollution tax and tradable permits when pollutant firms can decide to adopt a cleaner technology. In contrast to the previous literature we explicitly model the output market. As a consequence, the marginal pollution abatement cost of a firm does not necessarily decrease with the adoption of the clean technology and, in equilibrium, it is linked to the decisions of all other firms in the economy.

Having established the existence and uniqueness of an technology adoption equilibrium, we show that the share of innovating firms is not monotonic with respect to the stringency of the environmental policy. For instance, the share of innovating firms first increases, and then decreases with the tax level. As far as pollution is concerned, we show that global emissions can be higher with clean firms than only with dirty firms. This results from the fact that a clean firm has a higher activity level than a dirty firm and that this may well offset the reduction in pollution intensity.

We also compare the policy instruments to the optimal policy and we analyze their properties in terms of incentive for adoption. We take the common assumption of a myopic regulator that does not anticipate firms' reaction in terms of technology adoption. In terms of emissions we show that, depending on the marginal environmental damage, the policy instruments rank differently on the emission level. When the marginal environmental damage is low, using tradable permits leads to a too soft environmental policy, while the tax leads to a too stringent policy. When the marginal damage is high, the reverse holds. In terms of technology adoption, we show that the ranking of policy instruments also depends on the stringency of the policy. In extreme cases, i.e. for a very low or a very high pollution price, adoption is too low with tradable permits and too high with the tax. And for intermediary cases, the ranking may well be reversed.

One contribution of the paper is to scrutinize all the potential market outcomes that may arise when technology adoption and environmental policy interact. Another important contribution is about policy implications. These can be summarized as follows.

First, it is wrong to claim that innovation in a clean technology is always good for the environment. This flows from market equilibrium effects when the output market is taken into account. In other words, a pure technological ex-ante analysis is unable to foresee the global impact of a technology in the economy. Second, in most cases a 'pro-innovation regulator' should better implement an emission tax, but this may well increase the global pollution level. Third, in terms of environmental quality, emissions are lower with a tax than with permits only if the marginal environmental damage is low, otherwise they are increased with the tax. Fourth, before choosing the optimal policy instrument a 'pro-welfare regulator' in a situation of uncertainty à la Weitzman (1974) should carefully assess how the marginal abatement curve at the sectoral level is shaped by technological innovation. In particular, it is not true that the long term slope of the marginal abatement costs is always lower than the short-term one. Fifth, an interesting empirical question would be to evaluate the range of the zone of indeterminacy (between the Green and the Brown zones). Even though it looks rather narrow in our linear example, it may well be larger for more realistic production and damages functions. If it were the case, then it may become tricky to implement efficient environmental policies. This is an important avenue for future empirical research.

Acknowledgment

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Appendix A. Short-term equilibrium

We first provide the expressions for output and prices before considering emissions.

Appendix A.1. Production

Let $\beta \in [0,1]$. With p fixed, quantities are $y_u = (p - tu)/\gamma$ for $u = \alpha, 1$, so,

$$y_\alpha = y_1 + t(1-\alpha)/\gamma, \tag{A.1}$$

and the aggregate supply $Y = \beta y_\alpha + (1-\beta)y_1$ is

$$\begin{aligned} Y &= \beta\left(y_1 + t(1-\alpha)/\gamma\right) + (1-\beta)\, y_1 \\ &= y_1 + \beta t \frac{1-\alpha}{\gamma} = y_\alpha - (1-\beta)\, t \frac{1-\alpha}{\gamma}. \end{aligned} \tag{A.2}$$

At the equilibrium the price is $a - bY$ so

$$\begin{aligned} a - bY = \gamma y_\alpha + t\alpha &= \gamma \left[Y + (1-\beta)\, t \frac{1-\alpha}{\gamma} \right] + t\alpha \\ &= \gamma Y + t\left[1 - \beta\,(1-\alpha)\right] \end{aligned}$$

hence,

$$Y(p(\beta,t),t,\beta) = \frac{a - t\left[1-\beta(1-\alpha)\right]}{b+\gamma}. \tag{A.3}$$

and the price is:

$$p(\beta,t) = \frac{\gamma a + bt\left[1-\beta(1-\alpha)\right]}{b+\gamma}. \tag{A.4}$$

Firms' output levels are:

$$y_\alpha = \frac{a - t\alpha}{b+\gamma} + t\,(1-\beta)\,\frac{b(1-\alpha)}{\gamma(b+\gamma)} \tag{A.5}$$

and

$$y_1 = \frac{a-t}{b+\gamma} - t\beta\frac{b(1-\alpha)}{\gamma(b+\gamma)} \tag{A.6}$$

Note that the expressions for production (A.5) and (A.6) are valid if t is not too large, i.e.

$$t \leq \gamma a/[(\gamma + \beta b(1-\alpha)]. \tag{A.7}$$

If the tax is larger than this threshold then standard firms do not produce at all and only clean ones do. For even larger tax levels, $t > a/\alpha$, no firm produces.

Appendix A.2. Emissions, proof of lemma 1

Emissions of a clean and a standard firm are respectively: αy_α and y_1. With the first-order conditions:

$$\alpha y_\alpha - y_1 = \frac{1}{\gamma}\left[t(1-\alpha^2) - p(1-\alpha)\right] = \frac{(1-\alpha)}{\gamma}\left[t(1+\alpha) - p\right], \tag{A.8}$$

substituting p by (A.4) gives:

$$\alpha y_\alpha - y_1 = \frac{(1-\alpha)}{\gamma(b+\gamma)}\left\{t\left[(1+\alpha)\gamma + \alpha b + b\beta(1-\alpha)\right] - \gamma a\right\}. \tag{A.9}$$

It is positive if inequality (9) is satisfied.

Next, total emissions are given by (8) and the comparison of emissions with and without the clean technology corresponds to $E(\beta,t)$ and $E(0,t)$. With a slight abuse of notation ($y_u(\beta)$ is used for $y_u(p(\beta,t),t)$) the difference $E(\beta,t) - E(0,t)$ is:

$$\beta\alpha y_\alpha(\beta) + (1-\beta)y_1(\beta) - y_1(0) = \beta\left(\alpha y_\alpha(\beta) - y_1(\beta)\right) - \left(y_1(0) - y_1(\beta)\right),$$

the first term is given by (A.8), and the second one is $t\beta b(1-\alpha)/\gamma(b+\gamma)$ thus, $E(\beta,t)$ is higher than $E(0,t)$ if and only if

$$\gamma a < t\left[(1+\alpha)\gamma - (1-\alpha)(1-\beta)b\right].$$

which is equivalent to (10) (the second factor of the RHS is positive thanks to C1).

To derive (9) and (10), we assume that both types of firms have a positive output, i.e. that (A.7) holds. We should verify under which conditions these inequalities are not contradictory. First, the right hand side of (9) is lower than the right hand side of (A.7); so, for any parameters value there are taxes such that a clean firm produce more than a standard one while both type of firms produce. Then, (10) and (A.7) are simultaneously satisfied if and only if $\alpha\gamma \leq (1-\alpha)b$ which corresponds to $C1$. It means that α should not be too small to be sure that a partially clean industry emits more than a fully standard one while standard firms still produce. If this inequality is not satisfied it is still possible to find a threshold tax such that there are more emissions with $\beta > 0$ than with $\beta = 0$, but with only clean firms producing. To get convinced it is sufficient to think of the situation at $t = a$ where standard firms do not produce but clean firms do.

Appendix B. Technology Adoption Equilibrium

Appendix B.1. Expression of $\beta(t)$

The incentive (11) is:

$$\begin{aligned}\psi(p,t) &= \frac{1}{2\gamma}\left[(p-t\alpha)^2 - (p-t)^2\right] - f \\ &= \frac{1}{2\gamma}t(1-\alpha)\left[2p - t(1+\alpha)\right] - f. \end{aligned} \quad \text{(B.1)}$$

If $\beta(t) \in (0,1)$ it solves $\psi(p(\beta,t),t) = 0$, replacing p by the expression (A.4) gives an equation linear in β, and solving this equation with respect to β gives a function $\phi(t)$:

$$\phi(t) = \frac{1}{1-\alpha}\left\{1 + \frac{\gamma a}{bt} - \frac{b+\gamma}{b}\left[\frac{\gamma f}{(1-\alpha)t^2} + \frac{1+\alpha}{2}\right]\right\}, \quad \text{(B.2)}$$

such that the equilibrium share of clean firms is:

$$\beta(t) = \begin{cases} 0 \text{ if } \phi(t) < 0, \\ 1 \text{ if } \phi(t) > 1, \\ \phi(t) \text{ otherwise.} \end{cases} \quad \text{(B.3)}$$

Appendix B.2. Conditions

We establish the conditions on f to be sure that, for all t, β is strictly smaller than 1, and that there is at least one t such that $\beta > 0$.

- Whether $\beta(t) = 1$ can occur or not depends upon f. It is the case if there is some t such that $\psi(p(1,t),t) \geq 0$. Putting (A.4) for $\beta = 1$, into (B.1), the inequality $\psi(p(1,t),t) > 0$ is equivalent to

$$t\frac{1-\alpha}{b+\gamma}\left[2\gamma a - [(1+\alpha)\gamma + (1-\alpha)b]t\right] - 2\gamma f > 0, \tag{B.4}$$

  and there is t that satisfies this inequality if and only if the discriminant is positive,

$$(1-\alpha)\frac{\gamma a^2}{b+\gamma} \geq 2f\left[(1+\alpha)\gamma + (1-\alpha)b\right]. \tag{B.5}$$

  Thus, we assume that f is sufficiently large so that there is never full adoption:

$$\underline{f} \equiv \frac{1}{2}\frac{1-\alpha}{b+\gamma}\frac{\gamma a^2}{(1+\alpha)\gamma + (1-\alpha)b} \tag{B.6}$$

- Similarly, there is some adoption for some tax if there is a t such that $\psi(p(0,t),t) > 0$. This is the case if and only if

$$-[(1+\alpha)\gamma - (1-\alpha)b]t^2 + 2\gamma a t - 2\gamma f(b+\gamma)(1-\alpha) > 0$$

  The factor of t^2 is negative because $(1+\alpha)\gamma > (1-\alpha)b$ from C1. The left hand side is strictly positive for some $t < a$ if and only if the discriminant is positive, that is,

$$\bar{f} \equiv \frac{1}{2}\frac{1-\alpha}{b+\gamma}\frac{\gamma a^2}{(1+\alpha)\gamma - (1-\alpha)b}; \tag{B.7}$$

  and, in that case, the smaller root is smaller than a (using C1).

- One can impose another condition on f to ensure that at $t = a$ there is no adoption $\beta(a) = 0$–this is the same condition that ensures that the second root of the left-hand side of (B.4) is smaller than a. f should be sufficiently large, i.e. $f \geq a^2(1-\alpha)^2/2\gamma$–this condition and B.7 are not contradictory i.e. $a^2(1-\alpha)^2/2\gamma < \bar{f}$.

- Finally, to ensure that at the TAE standard firms produce, the inequality (A.7) should be satisfied, from (B.2), $y_1(\beta(t),t) > 0$ if and only if

$$t < (2\gamma f)^{1/2}/(1-\alpha). \tag{B.8}$$

Appendix C. Proofs of Propositions 2 and 3

Appendix C.1. Proof of Proposition 2

By differentiation of $\phi(t)$:

$$\frac{\partial \phi}{\partial t} = -\frac{\gamma a}{bt^2} + 2\frac{b+\gamma}{b}\frac{\gamma f}{(1-\alpha)t^3}$$

therefore, the share of clean firm decreases if and only if

$$t \geq 2f\frac{b+\gamma}{a(1-\alpha)} = \hat{t}. \tag{C.1}$$

It should be verified that at this tax, standard firms produce, i.e. (B.8) is satisfied. From (B.8) and (C.1), it is the case if and only if $f < 0.5a^2/(b+\gamma)^2$. And, thanks to C1, $\bar{f} < 0.5a^2(b+\gamma)^2$.

Let us show that $\tilde{t}$ is lower than $\hat{t}$,

$$\hat{t}^2 = 4f^2\frac{b+\gamma)^2}{a^2(1-\alpha)^2} < \frac{\gamma f}{1-\alpha}\frac{2(b+\gamma)}{(1+\alpha)\gamma-(1-\alpha)b} = \tilde{t}^2$$

because $f < \bar{f}$ and from the expression (B.7) of $\bar{f}$.

Appendix C.2. Proof of Proposition 3

Total emissions are higher with the clean technology than without it if and only if condition (10) is satisfied at $\beta(t)$. Plugging the expression (B.2) into (10) gives the condition

$$(b+\gamma)\left\{\frac{\gamma f}{(1-\alpha)t} - t\left[\frac{1+\alpha}{2} - \frac{b}{b+\gamma}\right]\right\} < 0.$$

This condition is satisfied if $t < \tilde{t}$ ($(1+\alpha)/2 > b/(b+\gamma)$ with C1), where $\tilde{t}$ is given by (16). And, at $\tilde{t}$ the condition (B.8) is satisfied thanks to C1.

Appendix D. Comparison of instruments

Appendix D.1. Proof of Proposition 4

First, Let us consider that the social optimum $E^\star$ is in the Green Zone.

- with TEPs, the regulator sets a cap $\bar{E}$ such that $D'(\bar{E}) = p(\bar{E}) - \gamma\bar{E}$. By definition of the Green Zone, $p(\bar{E}) - \gamma\bar{E} < \sigma(\bar{E})$, and by concavity of the welfare function $\bar{E} \geq E^*$, and, $\sigma(\bar{E}) \leq \sigma(E^*)$ (σ is decreasing).

- With a tax, the regulator sets $t = p(\bar{E}) - \gamma\bar{E}$. Given than we are in the Green Zone $t \geq \sigma(\bar{E})$ so $E(\beta(t), t) \leq \bar{E}$. Hence $t > D'(E(\beta(t), t))$ and the tax is larger than the optimal one (by concavity of W), and, $E(\beta(t), t)$ is larger than E^*.

Second, if the social optimum is in the Brown Zone, the same token leads to the proposition.

Appendix D.2. Proof of Proposition 5

We prove the content of Table 1 concerning β, and proposition 5 follows.

In the Brown Zone, $t > \tilde{t}$, from Proposition 4, the permit price σ is larger than the optimal tax which is larger than t. And $\tilde{t} > \hat{t}$ so $\beta(t)$ is decreasing and

$$\beta^{tax} = \beta(t) \geq \beta^* \geq \beta(\sigma) = \beta^{tep}.$$

In the Green Zone things are more complicated because $\beta(t)$ is not monotonous, for $\hat{t} \leq \tilde{t}$. From Proposition 4, in the Green Zone, $t \geq t^* \geq \sigma$, with three equalities at $t = \tilde{t}$ and at t such that $\beta(t) = 0$ and strict inequalities if $\beta^* > 0$ and $t < \tilde{t}$.

The intuition of the proof can be illustrated graphically: in the set $(0, \tilde{t})$, the adoption function $\beta(t)$ is bell-shaped (cf Proposition 2), when t increases the three prices t,t^* and σ increase, and the corresponding adoption shares go over the bell one after the other.

Formally, we parametrized everything in t, which corresponds to a parametrization of the damage function, an increase in t is related to an increase of the marginal damage function. Therefore, for any tax t, the corresponding optimal price is a function of t, $t^* = f(t)$ a strictly increasing function with $f(t) \leq t$ (for $t \leq \tilde{t}$). We can then define the three adoption functions as functions of t:

$$\beta^{tax}(t) = \beta(t);\ \beta^*(t) = \beta(f(t));\ \beta^{tep}(t) = \beta(\sigma(\bar{E})).$$

All three functions are bell-shaped, first increasing then decreasing, but slightly shifted. Let us denote by t_1, t_2 and t_3 the taxes that equalize these functions one with another:

$$\beta^{tax}(t_1) = \beta^*(t_1);\ \beta^{tax}(t_2) = \beta^{tep}(t_2);\ \beta^*(t_3) = \beta^{tep}(t_3).$$

The situation is depicted Figure D.5.

Note that β^{tax} and β^{tep} do not depend on the shape of the damage function, but they depend on the location of β^* between the two. So the comparison made in Table 1 is independent of the shape of the damage function; it depends on the thresholds t_1 and t_3.

Then, the comparisons between the βs can be carried out by pairs. For β^{tax} and β^*, the comparison is the following: if $t \leq t_1$ (resp. $t \geq t_1$) then $\beta^{tax} \geq \beta^*$ (resp. $\beta^{tax} \leq \beta^*$). It can be proved by considering the function:

$$h(t) = \beta^{tax}(t) - \beta^*(t),$$

which is positive for $t \geq \hat{t}$ (because β is increasing and $t^* < t$), it is negative at t such that $f(t)(= t^*) = \hat{t}$, it is decreasing in between, and null at t_1, so it is positive (resp. negative) for all t smaller (resp. larger) than t_1.

Similarly, if $t \leq t_2$ (resp. $t \geq t_2$) then $\beta^{tax} \geq \beta^{tep}$ (resp. $\beta^{tax} \leq \beta^{tep}$). And, if $t \leq t_3$ (resp. $t \geq t_3$) then $\beta^{tep} \geq \beta^*$ (resp. $\beta^{tep} \leq \beta^*$).

Finally, it must be showed that $t_1 \leq t_2 \leq t_3$. For the first inequality, at t_1, β^{tax} is decreasing while β^* is increasing. So, for $t \leq f(t_1)$, $\beta(t)$ is increasing and $\sigma \leq f(t_1)$ so $\beta^{tep} \leq \beta^* \leq \beta^{tax}$. Therefore, $t_2 \geq t_1$. The second inequality is proved similarly. Table 1 directly follows.

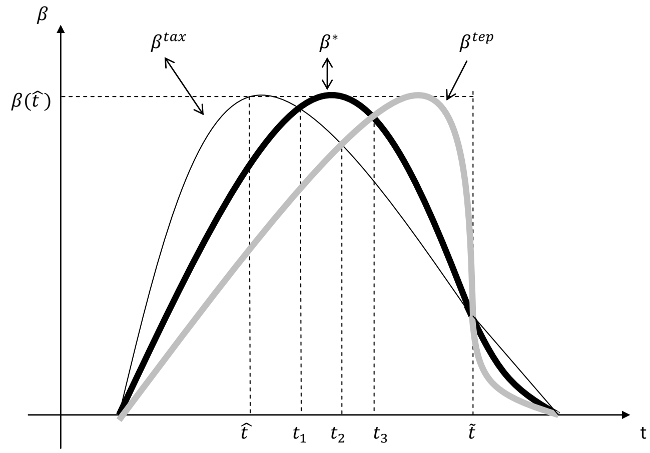

Figure D.5: The adoption with a tax, at the optimum (thick line) and with TEPs (grey line), as function of the tax.