Seasonality and the evolutionary divergence of plant parasites

Frédéric Marie Hamelin, Magda M. Castel, Sylvain Poggi, Didier Andrivon, Ludovic Mailleret

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8^{th} European Conference on Mathematical and Theoretical Biology

Kraków, 28 June - 2 July 2011

Biotrophic plant parasites

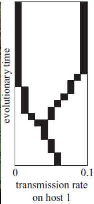
- **feed, grow and reproduce on their living host plant**
- cause massive damage to staple food crops


- **ubiquitous coexistence of related plant parasite species**[1]

[1] Brasier, 1987

Temporal heterogeneity in host availability

- Spatial host heterogeneity promotes evolutionary divergence[2]

- **Can seasonality promote evolutionary divergence as well?**


[2]Gudelj *et al.*, 2004

Biotrophic parasites' life cycle: Potato Late Blight

During early spring

- **Primary infection** phase: seedlings' infection by **inoculum from previous seasons**

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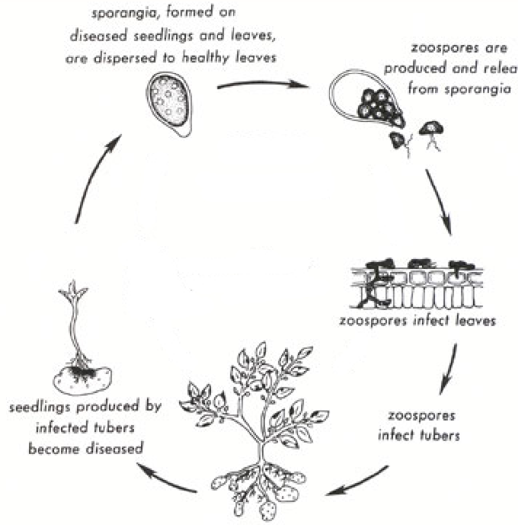
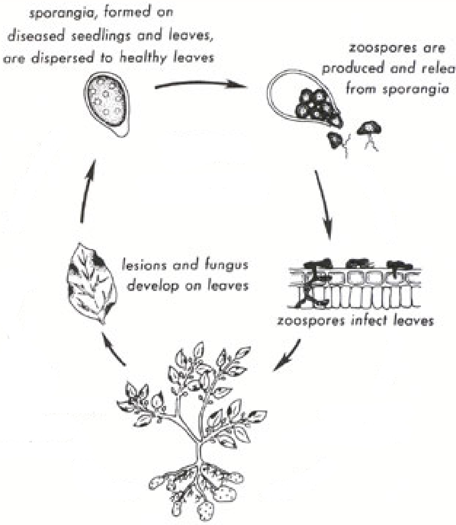


Biotrophic parasites' life cycle: Potato Late Blight

During the **season**

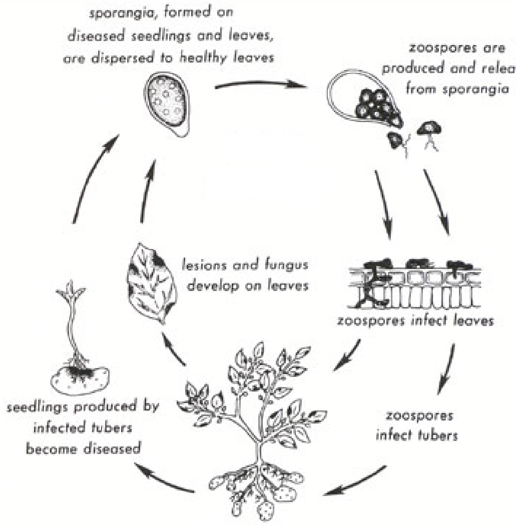
- **Secondary infection** phase: the parasite spreads from host to host through **inoculum from the current season**



Biotrophic parasites' life cycle: Potato Late Blight

Two **complementary** transmission routes:

- **between** season transmission
- **within** season contagion



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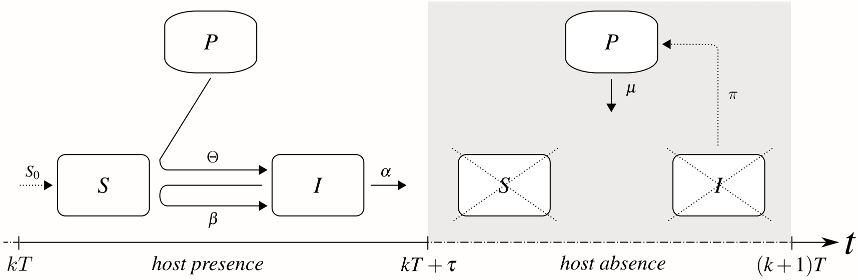
Model Basic Assumptions

(1) **Different** important **time windows** in such epidemic systems:



(2) Fast primary infection.
This requires a mixed **continous/discrete** modelling framework

The ecological model in compact form[3]

With the **continous part**

$$\begin{cases} \dot{S} = -\sum_i \beta_i S I_i, \\ \dot{I}_i = \beta_i S I_i - \alpha I_i. \end{cases}$$

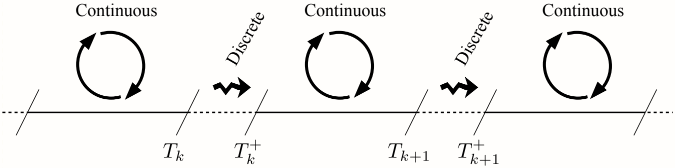
Susceptible/healthy hosts

Infected/infectious hosts, r or m

And the **discrete part**

$$\begin{cases} S((n+1)T^+) & = & S_0 \exp\left(-\sum_i F_i((n+1)T)\right), \\ I_i((n+1)T^+) & = & S_0\left[1-\exp\left(-\sum_i F_i((n+1)T)\right)\right] \times \left(\frac{F_i((n+1)T)}{\sum_i F_i((n+1)T)}\right), \end{cases}$$

with $F_i((n+1)T) = \pi \mathrm{e}^{-\mu_i(T-\tau)} \frac{\theta}{\delta} I_i(nT+\tau)$.



[3] Mailleret et al., 2011

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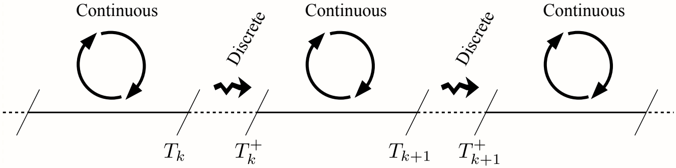
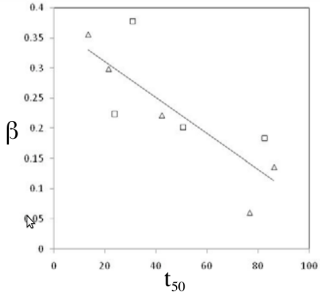


[3] Mailleret et al., 2011

Evolutionary trade-off

Experimental evidence[4] of a **negative relationship** between

- **within season transmission** ability
- **season-to-season survival** ability



Gosme *et al.*, 2009, take-all of wheat

higher infection rate ⇔ **lower season-to-season survival**

To **capture** this, let $\mu = f(\beta)$, **with** $f' > 0$.

[4] Abang et al. 2006, Carson 1998.

Evolutionary invasion analysis

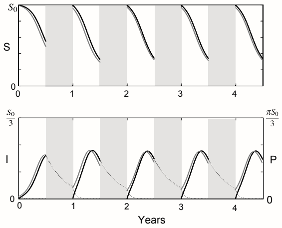
Adaptive Dynamics, a framework to address phenotypical **evolution**

- consider a **resident** population at ecological "equilibrium",
- **challenge** it with a small **mutant** sub-population

Assuming the resident is at a T-periodic equilibrium $(S_r^\circ(\cdot), I_r^\circ(\cdot))$,

let

$$\bar{S}_r = \frac{1}{\tau}\int_0^\tau S_r^\circ(t)\mathrm{d}t\,.$$



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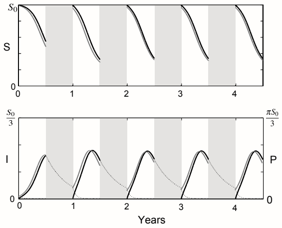
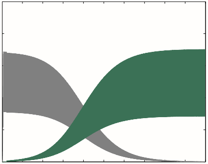


Invasion fitness

The mutant invasion criterion is define by the **invasion fitness**:

$$s(\beta_r, \beta_m) = (\beta_m - \beta_r)\bar{S}^{\circ}(\beta_r)\tau - (f(\beta_m) - f(\beta_r))(T - \tau)$$



- The small mutant can invade provided

$$s(\beta_r, \beta_m) > 0.$$

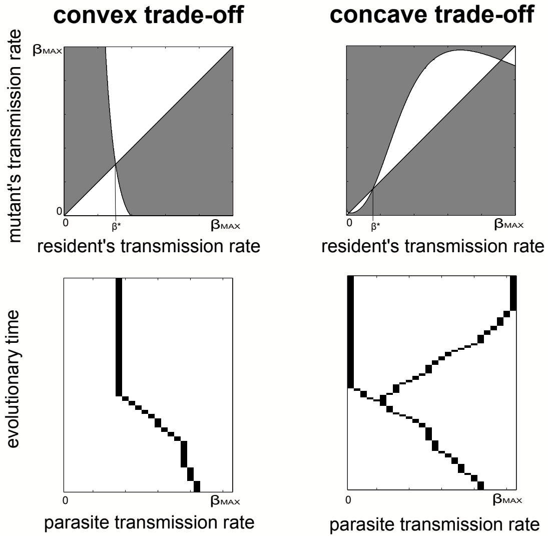
We are interested in **singular traits** $\beta^\star$ s.t.

$$\mathrm{D}_2 s(\beta^\star, \beta^\star) = 0\,.$$

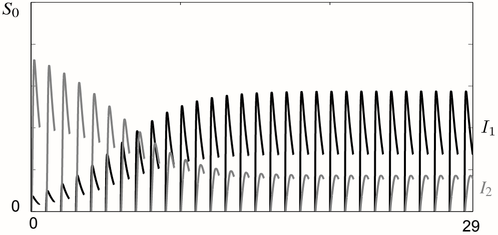
The necessary condition for a branching point reads

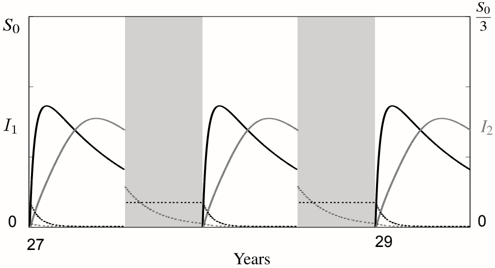
$$\mathrm{D}_{22} s(\beta^\star, \beta^\star) = -f''(\beta^\star)(T - \tau) > 0\,.$$

Evolutionary dynamics



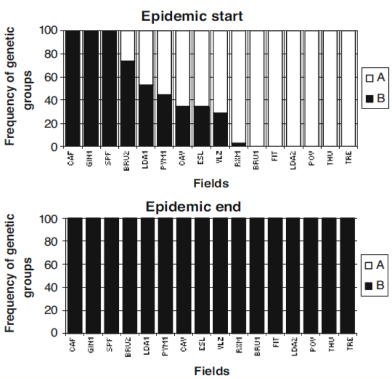
Ecological dynamics at the dimorphic evolutionary endpoint





Conclusion

- (evolution can promote) ecological niche differentiation through time partitioning[5]



[5]from Montarry et al., 2007

Thank you for your attention!

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Annexes

Full model with two strains

$$
\left\{
\begin{array}{ll}
\dot{P}_i = -\Delta P_i\,, & \text{Primary inoculum, type } r \text{ or } m \\
\dot{S} = -\sum_i \Theta P_i S - \sum_i \beta_i S I_i\,, & \text{Susceptible/healthy hosts} \\
\dot{I}_i = \Theta P_i S + \beta_i S I_i - \alpha I_i\,. & \text{Infected/infectious hosts, } r \text{ or } m \\
\hline
P_i(\tau^+) = P_i(\tau) + \pi I_i(\tau)\,, & \\
S(\tau^+) = 0\,, & \text{Transition from growing season} \\
I_i(\tau^+) = 0\,. & \text{to winter season. } t = \tau. \\
\hline
\dot{P}_i = -\mu_i P_i\,. & \text{Overwintering. } t \in (\tau, T). \\
\hline
P_i(T^+) = P_i(T)\,, & \\
S(T^+) = S_0\,, & \text{Beginning of a new cycle.} \\
I_i(T^+) = 0\,. & t = T.
\end{array}
\right.
$$

Annexes

Making primary infections fast

- Let $\delta = \varepsilon\Delta$, $\theta = \varepsilon\Theta$, with $0 < \varepsilon \ll 1$[6].
- The **within-season** model writes, in a **slow-fast form**,

$$\begin{cases} \varepsilon\dot{P}_i = -\delta \mathbf{P_i}, & \text{Primary inoculum} \\ \varepsilon\dot{S} = -\sum_{\mathbf{i}} \theta \mathbf{P_i S} - \sum_i \varepsilon\beta_i S I_i, & \text{Susceptible/healthy plants} \\ \varepsilon\dot{I}_i = \theta \mathbf{P_i S} + \varepsilon\beta_i S I_i - \varepsilon\alpha I_i. & \text{Infected/Infectious Plants} \end{cases}$$

- And **neglecting terms** in $O(\varepsilon)$

$$\begin{cases} P \to 0\,, \\ S \to S_0 \exp\left(-\sum_i \frac{\theta}{\delta} P_{i,0}\right)\,, \\ I \to S_0 \left[1 - \exp\left(-\sum_{\mathbf{i}} \frac{\theta}{\delta} P_{i,0}\right)\right] \left(\frac{\frac{\theta}{\delta} P_{i,0}}{\sum_{\mathbf{i}} \frac{\theta}{\delta} P_{i,0}}\right). \end{cases}$$

where $P_{i,0} = P_i((n+1)T^+) = \pi e^{-\mu_i(T-\tau)} I_i(nT+\tau)$.

[6] Madden and van den Bosch (2002)

Annexes

Making primary infections fast

- Defining fast time $t' = t/\varepsilon$,
- The **within-season** model writes, in a **slow-fast form**,

$$\begin{cases} \dot{P}_i' = -\delta \mathbf{P_i}, & \text{Primary inoculum} \\ \dot{S}' = -\sum_{\mathbf{i}} \theta \mathbf{P_i S} - \sum_i \varepsilon \beta_i S I_i, & \text{Susceptible/healthy plants} \\ \dot{I}_i' = \theta \mathbf{P_i S} + \varepsilon \beta_i S I_i - \varepsilon \alpha I_i. & \text{Infected/Infectious Plants} \end{cases}$$

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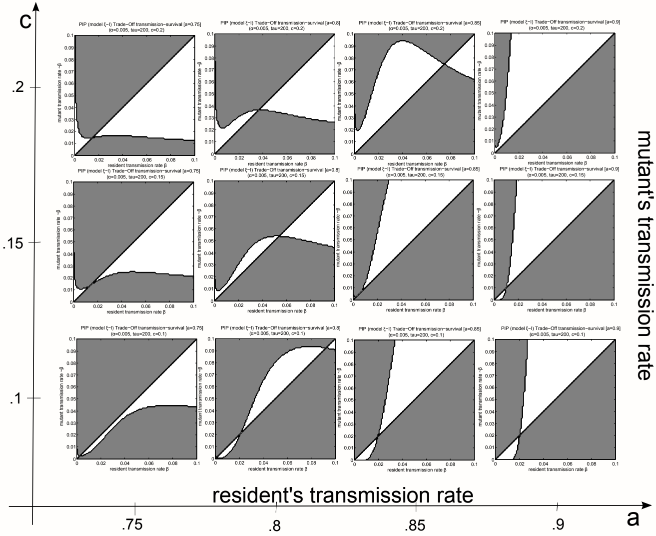
Annexes

PIP robustess around the branching point with a respect to the trade-off



Annexes

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