A water agency faced with quantity-quality management of a groundwater resource

Katrin Erdlenbruch, Mabel Tidball, Georges Zaccour

LAMETA
Laboratoire Montpelliérain d'Economie Théorique et Appliquée

UMR
Unité Mixte de Recherche

DOCUMENT de RECHERCHE

« A Water Agency faced with Quantity-quality Management of a Groundwater Resource »

Katrin ERDLENBRUCH
Mabel TIDBALL
Georges ZACCOUR

DR n°2012-09

Unite de Formation et de Recherche d'Economie
Avenue Raymond DUGRAND C.S. 79606
3 4 9 6 0 MONTPELLIER Cedex 2

E-mail : lameta@lameta.univ-montp1.fr
web : www.lameta.univ-montp1.fr

Université Montpellier 1 · agro Montpellier Ecole Nationale Supérieure Agronomique · CNRS Centre National de la Recherche Scientifique · INRA

A Water Agency faced with quantity-quality management of a groundwater resource

Katrin Erdlenbruch*
Irstea, Montpellier, France

Mabel Tidball
INRA, Montpellier, France†

Georges Zaccour
Chair in Game Theory and Management, GERAD, HEC Montréal, Canada

Version: 27/03/2012

Abstract

We consider a problem of groundwater management in which a group of farmers overexploits a groundwater stock and causes excessive pollution. A Water Agency wishes to regulate the farmer's activity, in order to reach a minimum quantity and quality level but it is subject to a budget constraint and cannot credibly commit to time-dependent optimal policies. We construct a Stackelberg game to determine a set of constant policies that brings the groundwater resource back to the desired state. We define a set of conditions for which constant policies exist and compute the amount of these instruments in an example.

JEL classification: H23, Q15, Q25.

Key words: groundwater, quantity-quality management, Stackelberg game, constant policies

1 Introduction

The management of groundwater is a typical common-pool renewable resource problem where several users have to share a same resource stock, with, however, one additional important feature, namely, the quality of the stock. Therefore, any attempt to regulate the use of water, has to tackle the externalities related to both quantity and quality. In this paper, we consider an endogenous pollution externality from agricultural production and discuss optimal quantity-quality regulation by a water agency with restricted regulatory power.

A significant literature has so far analyzed the need for public intervention to regulate private exploitation of groundwater. Using a simple quantity model with stock and pumping cost externalities,[1] Gisser and Sanchez [5] argued that the difference between the competitive and the optimal outcome is too small to justify policy intervention (see Koundouri [6] for a survey). However, the consideration of more complicated resource problems and other externalities has shown that public intervention can be necessary, e.g., when several resources are linked to each other (Zeitouni and Dinar [17]), when groundwater has a buffer value against surface water scarcity

*Corresponding author: katrin.erdlenbruch@irstea.fr
†K. Erdlenbruch and M. Tidball acknowledge financial support from the French National Research Agency through grant ANR-08-JCJC-0074-01.
[1]The stock externality arises because the extraction of each resource user is constrainted by the total groundwater stock; the pumping cost externality arises because the cost of pumping groundwater depends on the level of the groundwater table, see, e.g., Provencher and Burt [10].

(Provencher and Burt [10][2]), or when quality is taken into account (Roseta-Palma [12]).

Concerning water quality, a focal point was the issue of saltwater intrusion in coastal aquifers (see, e.g., Cummings [2], Zeitouni and Dinar [17], Dinar and Xepapadeas [3], Tsur and Zemel [13], Moreaux and Reynaud [9]). With the intensification of agricultural production, inland resources are increasingly threatened by quality degradation, via nitrate infiltration. Because groundwater resources are often used for drinking water, the issue is of importance also outside the agricultural sector. For instance, quality is addressed by several European policies, such as the Water Framework Directive (Directive 2000/60/EC), which fixes the objective of "good water quality" in 2015, the Directive on the protection of groundwater against pollution and deterioration (Directive 2006/118/EC) or the Nitrates Directive (Directive 91/676/EEC), which specifically tackles pollution from agricultural production.

A large literature exists on the issues of nitrate pollution and non-point source pollution resulting from agricultural activity, including dynamic models (e.g., Yadav [16], Xepapadeas [14]). Yet, as Koundouri [6] states, these models "generally avoid the relationship between contamination and water-use decisions. The assessment of how much groundwater should be pumped is absent from these models". The first work that brings together these aspects in a general dynamic setting is Roseta-Palma ([11] and [12]). She considers the impact of contaminant discharges on groundwater quality and in particular two special effects: the stock dilution effect which describes the beneficial impact of water volume on water quality, and the contaminating vector effect in which contaminants infiltrate more easily into the soil when carried with irrigation water. Roseta-Palma shows that public regulation should address both quantity and quality to be optimal. She also numerically confirms that policy intervention is justified even if gains from quantity regulation are small, as in Gisser and Sanchez [5], because of the importance to meet quality standards.

However, Roseta-Palma (2003) and most other articles consider dynamic taxation as the only tool for policy intervention.[3] Although a dynamic tax has a conceptual appeal, it is quite unrealistic in many real-life contexts. Indeed, it requires that the regulator chooses an optimal policy that changes continuously, depending on the individual actions taken. Roseta-Palma points at some implementation problems but focuses on those linked to informational constraints on individual production and pollution functions. In this paper, we study the case where the water regulator imposes constant policies over a reasonable time period, for example a year, which corresponds to the length of a fiscal exercise.

We consider a group of irrigating farmers using the same groundwater resource. Fertilizer used by the farmers leaches into the groundwater and causes nitrate pollution, mitigated by the stock dilution effect and the natural decay rate of the contaminant. We assume that the farmers are atomistic players who optimize their individual payoffs without taking into account the impact of their decisions, i.e., water withdrawal and use of fertilizers, on the stock of water and its quality. In order to insure a sustainable use of the resource, a water agency is in charge of regulating the quantity and quality of the groundwater. Regulation takes the form of tax (or subsidy) on water withdrawal and pollution (i.e., use of fertilizers). We shall consider and contrast the results of the following three scenarios:

Laissez-faire scenario: As the name suggests, in this case the use of water is not regulated. This scenario is seen as a benchmark.

Regulation with budget constraint: In this scenario, the water agency is endowed with

[2]Reducing groundwater stocks then generates the so-called risk-externality, see Provencher and Burt 1993 [10].

[3]As argued by Provencher and Burt [10], permit allocation does solve neither the risk externality nor the cost externality.

a budget at the start of the planning horizon and must balance its book at the end of the fiscal exercise.

Regulation with no budget constraint: In this scenario, the water agency does not dispose of a budget and its problem is simply to find the optimal tax or subsidy of water withdrawal and feryilizer use. This allows us to better understand the results including the budget constraint.

In all scenarios, we retain a mode of play à la Stackelberg where the role of leader is assumed by the water agency and the farmers are the followers. As the considered planning horizon is short (a fiscal year), we suppose that the water agency seeks constant tax or subsidy policies. We believe that constant policies are more realistic, from an implementability point of view, than time-varying ones. Indeed, it would be peculiar to require that the water agency produces a new tax rate at each instant of time throughout the duration of a fiscal exercise.

Therefore, we construct an open-loop dynamic Stackelberg game to model farmers' optimal decisions in the face of these constant incentive policies.[4]

We find that, under given conditions, there is indeed a set of constant optimal policies which fulfills all the constraints the Water Agency has to respect. In our simple example, the optimal policy-mix consists in an input-tax on water withdrawals and and input subsidy on fertilizer use.

The paper is structured as follows. In section 2 we present the problem, a simplified agro-economic model including a groundwater resource. In section 3, we present the Stackelberg game and characterize its solution. In section 4 we consider two examples and compute the optimal taxation policy in this context. Finally, in the last section, we conclude and give some perspectives for future research.

2 The Model

2.1 The Farmers

Consider a group of N farmers growing a single agricultural product and located above a same groundwater resource. Time t is continuous and the planning period given by the interval $[0, T]$. The agricultural production $y_i(t)$, of farmer $i = 1, ...N$, at time $t \in [0, T]$, depends on two inputs, namely, the quantity of fertilizer spread on cultivated land, $f_i(t)$, and the volume of irrigation water, $w_i(t)$, that each farmer pumps in the groundwater resource. We assume that the production function $y_i(w_i(t), f_i(t))$ is increasing in both inputs, but at decreasing returns to scale, that is,

$$\frac{\partial y_i}{\partial w_i} \geq 0, \frac{\partial y_i}{\partial f_i} \geq 0, \frac{\partial^2 y_i}{\partial w_i^2} \leq 0, \frac{\partial^2 y_i}{\partial f_i^2} \leq 0, \frac{\partial^2 y_i}{\partial w_i \partial f_i} \geq 0. \tag{1}$$

The positive sign of the last derivative means that irrigation water and fertilizers are complementary inputs.

Soil fertilization and water pumping are costly. The fertilization cost $c_f(\cdot)$, which includes the purchasing cost of fertilizers and their land application, depends on the quantity of fertilizer used. We assume that this cost is convex and increasing, i.e.,

$$\frac{\partial c_f}{\partial f_i} \geq 0, \quad \frac{\partial^2 c_f}{\partial f_i^2} \geq 0. \tag{2}$$

The cost of pumping and distributing water $c_w(\cdot)$ depends on the volume withdrawn, and on the depth of the aquifer, denoted $D(t)$. We assume that $c_w(\cdot)$ is jointly convex in its arguments

[4]For a general feedback Stackelberg model see for example Xepapadeas 1995 [15]).

and satisfies the following assumptions:

$$\frac{\partial c_w}{\partial w_i} \geq 0, \quad \frac{\partial^2 c_w}{\partial w_i^2} \geq 0, \quad \frac{\partial c_w}{\partial D} \geq 0, \quad \frac{\partial^2 c_w}{\partial D^2} \geq 0, \quad \frac{\partial^2 c_w}{\partial w_i \partial D} \geq 0. \tag{3}$$

The first derivative states that the cost of pumping water is increasing in the input w_i. The second derivative implies that the marginal cost of pumping is increasing. This can be justified by the fact that the consumption of energy increases non-linearly in the volume of pumped water. The sign of the third derivative captures the idea that the larger D (meaning that water must be lifted a longer distance to surface), the higher the cost of pumping water. Increasing marginal returns are assumed through $\frac{\partial^2 c_{wi}}{\partial D^2} \geq 0$. Finally, the non negativeness of the last derivative means that the marginal cost of pumping water might be increasing in D.

The farmers are price-takers and the price p_i of the agricultural product is constant throughout the short duration of the planning horizon. Indeed, the period considered is defined as a season or a fiscal year. Further, the farmers are subject to public policies of the water agency, namely, they pay a tax τ on the use of polluting fertilizer, and a tax ϕ on individual water withdrawals. Consequently, the i's agent profit reads as follows:

$$\pi_i = \int_0^T \left(p_i y_i(w_i(t), f_i(t)) - c_w(D(t), w_i(t)) - c_f(f_i(t)) - \tau f_i(t) - \phi w_i(t) \right) dt. \tag{4}$$

We make the following two remarks:

1. There is no conceptual difficulty in extending the model to an oligopolistic setting where the farmers compete with an homogenous product à la Cournot. Actually, we can also consider a differentiated product (e.g., organic and regular), where the price does not only depend on the quantity put on the market, but also on the quality of irrigation water and the quantity of fertilizer used in farmig. Obviously, the more sophisticated the model, the more complex the computation of the equilibrium policies.

2. Given the short-term planning horizon, we do not discount farmer i's profit. Including a discount factor does not pose any particular difficulty.

2.2 The Dynamics

The depth of the aquifer depends on withdrawals by farmers and on natural recharge. Denote by $r(t)$ the mean recharge rate of the groundwater stock. The evolution of D is described by the differential equation

$$\dot{D}(t) = g\left(\sum_i w_i(t), r(t)\right), \quad D(0) = D_0 \quad \text{given}, \tag{5}$$

where D_0 is a measurement of the initial water distance, with

$$\frac{\partial g}{\partial w_i} > 0, \quad \frac{\partial g}{\partial r} < 0.$$

The quality of the groundwater deteriorates with the quantity of fertilizer used by each farmer. Further, the larger the volume of the stock of water, the higher the dilution (mitigation) capacity, and the better the quality. As there is a monotone relationship between the volume of water and the depth of the aquifer, we can model the evolution of quality as function of D and of the water withdrawals. More specifically, the evolution of water quality is modeled by the following differential equation:

$$\dot{Q}(t) = h\left(\sum_i f_i(t), D(t)\right), \quad Q(0) = Q_0 \quad \text{given}, \tag{6}$$

where Q_0 is a measurement of the initial water quality, with

$$\frac{\partial h}{\partial f_i} < 0, \qquad \frac{\partial h}{\partial D} < 0.$$

We do not make for the moment any additional assumption on h, but simply note that this function is not necessarily linear.

Remark: For now, our modeling of the water quality evolution does not account for any abatement activity that the farmers and/or the water agency may undertake to improve the quality of water. Indeed, it is technically possible to influence that quality by, e.g., favouring the use of plants containing nitrogen-fixing symbiotic bacteria. This is the concept of green manure: for instance, white mustard (*Sinapis alba*), vetches (*Vicia*), phacelia or rapeseed (*Brassica napus*) are able to fix nitrogen in the field. They are set up after the main harvest, in automn and destroyed in winter. In some European countries, farmers were eligible to a damage payment for the introduction of these nitrogen fixing plants.[5]

2.3 The Water Agency

Whereas the definition of an objective function for a farmer is easy, the task of doing so for the water agency is not that straightforward. Ideally, one would like to define a welfare function to assess the value to society of any particular governmental policy. However, writing down such a function is a highly complex problem from a theoretical, as well as from a practical point of view. In this paper, we adopt a pragmatic approach and assume that the water agency uses its public policy to approach as close as possible pre-determined levels of quality and quantity of water at time T. These levels correspond to a quality norm and a minimum amount of water, which should be preserved for future periods:

$$Q_b(T) = Q_b, \tag{7}$$

$$D_b(T) = D_b, \tag{8}$$

More precisely, the water agency wishes to minimize the distance between current and desired quality and quantity levels, that is,

$$\theta = \left[\alpha(Q(T) - Q_b)^2 + (1-\alpha)(D(T) - D_b)^2\right], \tag{9}$$

where α and $(1-\alpha)$ are positive weights that measure the importance of the quality and quantity goal, respectively. Such objective seems to be in line with the philosophy of public-policy makers who would like to see a clear statement of what a government program is aimed at.

To achieve its goals, the water agency can levy (constant) taxes on fertilizer and water use. The agency is endowed with some financial resources at the initial instant of time, and is required to balance its books at the end of the planning horizon. The equilibrium-budget constraint at T reads as follows:

$$0 = b_0 + \int_0^T [\tau \sum_i f_i(t) + \phi \sum_i w_i(t)] dt, \tag{10}$$

where b_0 is the available budget at time 0. The above budget equation is an isoperimetric constraint that can be rewritten in the form of a state equation as follows:

$$\dot{Y}(t) = [\tau \sum_i f_i(t) + \phi \sum_i w_i(t)] \quad \text{with} \quad Y(0) = b_0 \quad \text{and} \quad Y(T) = 0, \tag{11}$$

where $Y(t)$ represents the funds available at time $t \in [0, T]$.

We make the following clarification remarks:

[5] In France for example, the *Indemnité compensatoire de couverture des sols* (Code de l'environnement LII1.1.3.3) amounted to 60 euros/ha in 2003.

1. The tax rates τ and ϕ do not vary with time and/or the state of the system (quality of the growndwater and depth of the aquifer) during the planning interval $[0, T]$. Although state or time dependent tax rates may be conceptually attractive, they are difficult to implement in reality. Indeed, it will be very difficult for public agencies to explain a policy that continuously changes over time, and farmers will hardly accept such a mechanism. We believe that our assumption simply reflects actual practice where tax rates (as well as other public service prices) are set constant by governmental agencies for the whole duration of the fiscal exercise, typically a year. Such constant tax and subsidy policies were used by Krawczyk and Zaccour [7] in a dynamic game where a local government aims at controlling pollution emissions by decentralized agents.

2. We do not impose any sign on the instruments τ and ϕ. If optimization leads to negative values, then subsidies should be set up rather than taxes. The sign of τ and ϕ will of course depend on the objective of the water agency.

3. We assumed that the water agency must balance its budget at T. If the agency is allowed to realize a surplus, then the budget constraint, equation (10) becomes an inequality, i.e.,

$$0 \leq b_0 - \int_0^T [\tau \sum_i f_i(t) + \phi \sum_i w_i(t)] dt.$$

To keep it simple, we shall consider the case where the budget constraint is binding and, as stated in the introduction, we shall contrast the results of this scenario to the case where there is no budget constraints and the one with no regulation.

3 A Stackelberg Game

In the previous section, we defined a finite-horizon differential game, with $N + 1$ players (N farmers and a regulator). The model involves three state variables, namely, the quantity D and quality Q of water and the water agency's budget, Y. The control variables of a farmer are the water withdrawal w_i and the quantity of fertilizer f_i. The water agency chooses the tax rates τ and ϕ, which can assume any sign.

The game is played à la Stackelberg. The water agency takes the leader's role and announces its strategy before the farmers make their decisions. Given the leader's announcement of the tax policy (τ, ϕ), the farmers, acting as followers, play a Nash game and choose w_i and f_i. We suppose that the farmers employ open-loop strategies, that is, at the initial instant of time, each player decides upon a strategy which depends only on time. It is well known that open-loop Stackelberg equilibria are in general time inconsistent.[6] This means that given the opportunity to revise his strategy at an intermediate instant of time, the leader would like to choose another strategy than the one he selected at the initial instant of time. Therefore, an open-loop Stackelberg equilibrium only makes sense if the leader can credibly precommit to his strategy. In the present game it seems plausible to assume precommitment on the part of the water agency: in practice a tax scheme is determined and announced from the outset and when the regulator's decision is irrevocable, the announcement will be credible.

3.1 The Followers' Reaction Functions

To solve for Stackelberg equilibrium, we first determine the reaction functions of the followers and next solve the (optimal-control) problem of the leader. Each farmer chooses the levels of inputs, $w_i(t)$ and $f_i(t)$, that maximize profits, given by equation (4). Note that the water

[6] See, e.g., Martín-Herrán et al. (2005) and Buratto and Zaccour (2009) for examples where open-loop Stackelberg equilibria are time consistent.

quality does not appear in the payoff function of a farmer, and hence it is irrelevant for this agent. Further, we suppose that the budget constraint and the evolution of the water distance are private information detained by the water agency, i.e., the farmers do not observe these state equations. We shall omit from now on the time argument when no ambiguity may arise.

Assuming an interior solution, the first-order equilibrium conditions are:

$$\frac{\partial H_i}{\partial w_i} = p_i \frac{\partial y_i(w_i, f_i)}{\partial w_i} - \frac{\partial c_w(D, w_i)}{\partial w_i} - \phi = 0, \tag{12}$$

$$\frac{\partial H_i}{\partial f_i} = p_i \frac{\partial y_i(w_i, f_i)}{\partial f_i} - c_f'(f_i) - \tau = 0. \tag{13}$$

Equations (12)-(13) are the usual optimality conditions stating that, at the optimum, marginal revenue from production equal marginal costs. In equation (12), marginal revenues are due to the use of one additional unit of water. Marginal costs are given by marginal costs of pumping and distributing irrigation water and by the taxes paid per unit of water pumped In equation (13), marginal revenues due to the use of one additional unit of fertilizer are equal to marginal cost of buying fertilizers and the tax paid per unit of fertilizer.

Using (12)-(13), we can express w_i and f_i as functions of the state variable, D and the instruments of the water agency. Denote by $\tilde{f}_i\left(D, \tau, \phi\right)$ and $\tilde{w}_i\left(D, \tau, \phi\right)$ these reaction functions.

3.2 The Leader's Problem

The leader solves an optimal-control problem which is not standard because the water agency is looking for a constant tax policy throughout the planning horizon. The water agency choses this policy so as to minimize the distance between observed and desired quantity and quality levels at the end of the planning horizon, T, taking into account the followers reactions and the evolution of all the state variables (see equation (10).

Substituting for $\tilde{f}_i\left(D, \tau, \phi\right)$ and $\tilde{w}_i\left(D, \tau, \phi\right)$ in the water agency's budget, quantity and quality equations leads to

$$\dot{Y}\left(t\right) = \tau \sum_i \tilde{f}_i\left(D, \tau, \phi\right) + \phi \sum_i \tilde{w}_i\left(D, \tau, \phi\right), \quad Y(0) = b_0, \quad Y(T) = 0, \tag{14}$$

$$\dot{D} = g\left(\sum_i \tilde{w}_i\left(D, \tau, \phi\right), r\right), \quad D(0) = D_0, \tag{15}$$

$$\dot{Q}\left(t\right) = h\left(\sum_i \tilde{f}_i\left(D, \tau, \phi\right), D(t)\right), \quad Q_i(0) = Q_0 \quad \text{given.} \tag{16}$$

The leader's Hamiltonian reads as follows:

$$H_L\left(D\left(t\right), \mu^D\left(t\right), Q\left(t\right), \mu^Q\left(t\right), Y\left(t\right), \mu^Y\left(t\right), \tau, \phi\right) = \mu^D\left(t\right) g\left(\sum_i \tilde{w}_i\left(D\left(t\right), \lambda_i\left(t\right), \tau, \phi\right), r\right)$$

$$+\mu^Q\left(t\right) h\left(\sum_i \tilde{f}_i\left(D, \lambda_i, \tau, \phi\right), D(t)\right) + \mu^Y\left(t\right)\left(\tau \sum_i \tilde{f}_i\left(D, \lambda_i, \tau, \phi\right) + \phi \sum_i \tilde{w}_i\left(D, \lambda_i, \tau, \phi\right)\right),$$

where the $\mu^D\left(t\right), \mu^Q\left(t\right)$ and $\mu^Y\left(t\right)$ are adjoint variables appended to the state variables $D\left(t\right), Q\left(t\right)$ and $Y\left(t\right)$.

Assuming an interior solution, along with the four state equations in (15)-(16), the first-order optimality conditions are as follows:[7]

$$\begin{aligned}
\dot{\mu}^D &= -\frac{\partial H_L}{\partial D}, \quad \mu^D(T) = 2(1-\alpha)(D(T) - D_b), && (17)\\
\dot{\mu}^Q &= -\frac{\partial H_L}{\partial Q}, \quad \mu^Q(T) = 2\alpha(Q(T) - Q_b), && (18)\\
\dot{\mu}^Y &= -\frac{\partial H_L}{\partial Y}, && (19)\\
\int_0^T \frac{\partial H_L}{\partial \tau} dt &= 0, && (20)\\
\int_0^T \frac{\partial H_L}{\partial \phi} dt &= 0. && (21)
\end{aligned}$$

Recall that the optimality conditions in (20) and (21) take the form of an integral because of our restriction of the leader's tax policies to constant ones. Further, as the values of state variable $Y(t)$ are given at 0 and T, the adjoint variable μ^Y is free. Finally, we note that the leader's optimality conditions include 8 equations and same number of unknowns.

4 Illustration

We illustrate in this section the type of insight that can be obtained using our model. To keep things as simple as possible, we assume that the n farmers are identical. Given our settings of price-taking farmers located on the same growndwater, this assumption is not severe.

4.1 Production functions and dynamics

We adopt the following production function:

$$y_i = Aw_i f_i + Bw_i + Ef_i - K\frac{1}{2}f_i^2 - \frac{1}{2}Mw_i^2 + G,$$

where A, B, E, K, M and G are non-negative parameters. Some restrictions on these parameters will be required to satisfy the conditions in (1), namely:

$$\begin{aligned}
\frac{\partial y_i}{\partial w_i} &= Af_i + B - Mw_i \geq 0, \quad \frac{\partial^2 y_i}{\partial w_i^2} = -M \leq 0,\\
\frac{\partial^2 y_i}{\partial w_i \partial f_i} &= A > 0, \quad \frac{\partial y_i}{\partial f_i} = Aw_i + E - Kf_i \geq 0, \quad \frac{\partial^2 y_i}{\partial f_i^2} = -K < 0.
\end{aligned}$$

The revenue function of farmer i is given by py_i. Using the above derivatives, it is easy to verify that for the revenue function to be concave, it is necessary to have the determinant of the Hessian matrix non-negative, i.e.,

$$p^2A^2 - p^2KM \leq 0. \tag{22}$$

In some of the following examples, we will use the simplifying assumption $A = 0$. In that case, the determinant of the Hessian matrix is negative, i.e.,

$$-p^2KM \leq 0. \tag{23}$$

The fertilizer aquisition cost is given by

$$c_f(f_i) = Lf_i,$$

[7] As there is no particular need for it, we do not write these conditions in full. We will detail them in the example

where L is a positive parameter. The irrigation cost is specified as

$$c_w(w_i) = (Z + CD)w_i,$$

where Z and C are positive parameters. The term Zw_i represents the cost of distributing water, and CDw_i is the water-pumping cost that depends on the distance between the topsoil and the watertable. Clearly, the above cost functions satisfy the requirements in (3) and (2).

The dynamics of the depth and quality of the growndwater are modeled as follows:

$$\begin{aligned}\dot{D} &= \sum_i w_i - r, \quad D(0) = D_0, \\ \dot{Q} &= \left(-\delta \sum_i f_i\right) D, \quad Q(0) = Q_0,\end{aligned}$$

where δ is a non-negative parameter.Note that quality is non-increasing, for all t.

4.2 Analytical results without complementarity

Suppose first there is no cross-effect between inputs, i.e. $A = 0$.

4.2.1 Optimal input and policy choice

Given our functional specifications, farmer i's Hamiltonian now reads:

$$H_i = p\left(Bw_i + Ef_i - \frac{1}{2}Kf_i^2 - \frac{1}{2}Mw_i^2 + G\right) - (Z + CD)w_i - Lf_i - \tau f_i - \phi w_i.$$

Assuming an interior solution, the first-order equilibrium conditions of farmer $i, i = 1, \ldots, N$, are given by:

$$\frac{\partial H_i}{\partial w_i} = 0 \Leftrightarrow \tilde{w}_i(\tau, \phi) = \frac{pB - CD - Z - \phi}{pM}, \tag{24}$$

$$\frac{\partial H_i}{\partial f_i} = 0 \Leftrightarrow \tilde{f}_i(\tau, \phi) = \frac{pE - L - \tau}{pK}. \tag{25}$$

Note that the solution is fully symmetric, i.e. $\tilde{w}_i(\tau, \phi) = \tilde{w}(\tau, \phi)$ and $\tilde{f}_i(\tau, \phi) = \tilde{f}(\tau, \phi)$, for all $i = 1, \ldots N$.

Proposition 1 *Farmers auto-regulate their water use when the water distance increases. Further, farmers use less water (fertilizer) when the water (fertilizer) input is taxed.*

Proof. From equations (24) and (25), it is obvious that:

$$\begin{aligned}\frac{\partial \tilde{w}}{\partial D} &< 0, \ \frac{\partial \tilde{f}}{\partial D} = 0, \\ \frac{\partial \tilde{w}}{\partial \tau} &= 0, \ \frac{\partial \tilde{w}}{\partial \phi} < 0, \ \frac{\partial \tilde{f}}{\partial \tau} < 0, \ \frac{\partial \tilde{f}}{\partial \phi} = 0.\end{aligned}$$

The larger the water distance, which is synonymous to a higher pumping cost, the lower the farmer's water use. The higher the water (fertilizer) tax, the lower the optimal irrigation (fertilizer) use.□

After substituting for $\tilde{w}(\tau, \phi)$ and $\tilde{f}(\tau, \phi)$ in the state equations $\dot{D}, \dot{Q}$ and $\dot{Y}$, we obtain the following Hamiltonian for the leader:

$$\begin{aligned} H_L &= \mu^D N(\tilde{w}_i - r) - \mu^Q \delta N \tilde{f}_i D + \mu^Y \left[\tau N \tilde{f}_i + N\phi\tilde{w}_i\right] \\ &= N\left(\mu^D + \mu^Y\phi\right)\left[\frac{pB - CD - Z - \phi}{pM}\right] + N\left(-\mu^Q\delta D + \mu^Y\tau\right)\left[\frac{pE - L - \tau}{pK}\right] - rN\mu^D, \end{aligned}$$

where $\mu^D(t), \mu^Q(t)$ and $\mu^Y(t)$ are adjoint variables appended to the state variables $D(t), Q(t)$ and $Y(t)$. The first-order optimality conditions and the solution procedure are given in the Appendix (A.1.1). Solving, we get:

$$D(t,\phi) = e^{-\rho t} D_0 + \Theta(\phi)(1 - e^{-\rho t}), \tag{26}$$

$$Q(t,\phi,\tau) = Q_0 - \Lambda(\tau)\int_0^t D(s)ds, \tag{27}$$

$$Y(t,\phi,\tau) = b_0 \frac{N}{pKM}\left[(pE - L - \tau)\tau M + (pB - Z - \phi)\phi K\right] t - \phi\rho\int_0^t D(s)ds, \tag{28}$$

where

$$\Theta(\phi) = \left[\frac{N(pB - Z - \phi) - rpM}{NC}\right], \quad \Lambda(\tau) = \frac{\delta N(pE - L - \tau)}{pK}, \quad \rho = \frac{NC}{pM}$$

and

$$\int_0^t D(s)ds = \frac{D_0 - D(t)}{\rho} + \Theta(\phi)t.$$

We also have to consider the special conditions as we can see in the Appendix, equations (40) and (41) . We therefore end up with a system of 3 equations (47)-(49) for the three unknowns τ, ϕ and μ^Y which we can solve. We can then insert the results in the system dynamics and the reaction functions of the followers.

Proposition 2 *The use of optimal input taxes leads to a better water quality over time. The use of the optimal water tax decreases the water-table distance over time, i.e. leads to a greater groundwater volume.*

Proof. Proof in the Appendix, (A.1.2). □

4.2.2 No budget constraint case

If we assume away the budget constraint, the first-order optimality conditions of the leader become:

$$\dot{\mu}^D = \frac{N}{pKM}\left[KC\mu^D + \mu^Q\delta M(L + \tau - pE)\right], \quad \mu^D(T) = 2(1-\alpha)(D(T) - D_b), \tag{29}$$

$$\int_0^T \frac{\partial H_L}{\partial \tau} dt = 0 \Leftrightarrow -\delta\mu^Q \int_0^T D dt = 0, \tag{30}$$

$$\int_0^T \frac{\partial H_L}{\partial \phi} dt = 0 \Leftrightarrow \int_0^T \mu^D dt = 0. \tag{31}$$

with $\dot{D}$, $\dot{Q}$ and $\dot{\mu^Q}$ as before, see Appendix, equations (34), (38) and (39).
Solving the differential equations of quantity and quality yields, as before,

$$\begin{aligned} D(t,\phi) &= e^{-\rho t} D(0) + \Theta(\phi)(1 - e^{-\rho t}), \\ Q(t,\phi,\tau) &= Q_0 - \Lambda(\tau)\left[\frac{(D_0 - \Theta(\phi))}{\rho}(1 - e^{-\rho t}) + \Theta(\phi)t\right]. \end{aligned}$$

Proposition 3 *Without budget constraint, the desired quantity and quality level will be reached.*

Proof. If we assume $D(t) > 0$ for all t, then condtion (30) implies

$$\mu^Q = 0 \iff Q(T) = Q_b.$$

Substituting for $Q(T) = Q_b$ in (29) and solving leads to

$$\mu^D(t) = \frac{2(D(T) - D_b)(1-\alpha)e^{\rho t}}{e^{\rho T}}.$$

Now, condition (31) implies

$$\mu^D = 0 \iff D(T) = D_b.$$

□

The taxes ϕ^* and τ^* that verify $D(T) = D_b$ and $Q(T) = Q_b$ are then given by

$$\phi^* = \left(\frac{e^{-\rho T} D_0 - D_b}{1 - e^{-\rho T}} - \frac{r}{\rho}\right) C - Z + pB,$$

$$\tau^* = pE - L - \frac{Q_b - Q_0}{V}\frac{pK}{\delta N},$$

where

$$V = \frac{D_b - D_0}{\rho} + \frac{e^{-\rho T} D_0 - D_b}{1 - e^{-\rho T}} T < 0,$$

see Appendix, (A.1.3). The signs of ϕ^* and τ^* cannot be ascertained unambiguously as they depend on the values of the parameters.

Finally, supposing that $Q_b < Q_0$ and $D_b > D_0$, optimal levels of inputs are given by:

$$\begin{aligned} w(t)^* &= \frac{1}{N}\left(r + \rho e^{-\rho t}\left(\frac{D_b - D_0}{1 - e^{-\rho T}}\right)\right) > 0, \\ f^* &= \frac{Q_b - Q_0}{V\delta N} > 0 \end{aligned}$$

and the optimal evolution of water quantity and water quality is given by:

$$\begin{aligned} D(t, \phi^*) &= e^{-\rho t} D_0 - \frac{e^{-\rho T} D_0 - D_b}{1 - e^{-\rho T}}(1 - e^{-\rho t}), \\ Q(t, \phi^*, \tau^*) &= Q0 + \Lambda(\tau^*)\frac{D0 - \Theta(\phi^*)}{\rho}(e^{-\rho t} - 1) - \Theta(\phi^*) t, \end{aligned}$$

where

$$\Lambda(\tau^*) = \frac{Q_b - Q_0}{V}, \quad \Theta(\phi^*) = -\frac{e^{-\rho T} D_0 - D_b}{1 - e^{-\rho T}}.$$

Remark 1 *If the regulator aims at an end-of-period quantity that is equal to initial-date quantity, i.e., $D_0 = D_b$, then the solution becomes*

$$\begin{aligned} \phi^* &= \left[pB - \left(D_0 + \frac{r}{\rho}\right) C - Z\right], \\ \tau^* &= pE - L + \frac{Q_b - Q_0}{D_0 T \delta N} pK, \\ w^* &= \frac{r}{N} > 0, \qquad f^* = \frac{Q_0 - Q_b}{D_0 T \delta N} > 0, \\ D^* &= D_0, \qquad Q(t)^* = Q_0 - \frac{Q_0 - Q_b}{D_0 T} t. \end{aligned}$$

We can now compute the budget corresponding to these policies, i.e.,

$$\begin{aligned}\dot{Y}(t) &= \left(\frac{Q_0 - Q_b}{D_0 T\delta}\right)\left[pE - L + \frac{Q_0 - Q_b}{D_0 T\delta N}pK\right] + r\left[pB - \left(D_0 + \frac{r}{\rho}\right)C - Z\right] \\ Y(T) &= \left(\frac{Q_0 - Q_b}{D_0 T\delta}\right)\left[pE - L + \frac{Q_0 - Q_b}{D_0 T\delta N}pK\right] + r\left[pB - \left(D_0 + \frac{r}{\rho}\right)C - Z\right]T + b_0\end{aligned}$$

Remark 2 *If the regulator aims at an end-of-period quality that is equal to initial-date quality, i.e., $Q_0 = Q_b$, then we have the special case where $f^* = 0$.*

4.2.3 No regulation case

In the absence of any regulation, the equilibrium conditions for the farmers in (24)-(25) yield the following water withdrawal and fertilizer levels:

$$\tilde{w} = \frac{pB - CD - Z}{pM}, \tag{32}$$

$$\tilde{f} = \frac{pE - L}{pK}. \tag{33}$$

Consequently, the quantity and quality trjectories are given by

$$\begin{aligned}D(t)^* &= e^{-\rho t}D(0) + \Theta(1 - e^{-\rho t}), \\ Q(t)^* &= Q_0 - \Lambda\left[\frac{(D_0 - \Theta)}{\rho}(1 - e^{-\rho t}) + \Theta t\right],\end{aligned}$$

where

$$\begin{aligned}\Theta &= \left[\frac{N(pB - Z) - rpM}{NC}\right], \\ \Lambda &= \frac{\delta N(pE - L)}{pK}, \qquad \rho = \frac{NC}{pM}.\end{aligned}$$

4.3 Analytical results with complementarity

Suppose now a cross-effect between inputs, i.e. $A \neq 0$.

4.3.1 Optimal input and policy choice

Given our functional specifications, farmer i's Hamiltonian now reads:

$$H_i = p\left(Aw_i f_i + Bw_i + Ef_i - \frac{1}{2}Kf_i^2 - \frac{1}{2}Mw_i^2 + G\right) - (Z + CD)w_i - Lf_i - \tau f_i - \phi w_i.$$

The corresponding reaction functions now depend on the water distance, $D(t)$ and on both input taxes:

$$\begin{aligned}\tilde{w}_i(\tau, \phi) &= \frac{A(L + \tau - pE) + K(CD + Z - pB + \phi)}{p(A^2 - KM)}, \\ \tilde{f}_i(\tau, \phi) &= \frac{M(L + \tau - pE) + A(CD + Z + \phi - pB)}{p(A^2 - KM)}.\end{aligned}$$

Note that $\frac{\partial \tilde{w}_i}{\partial D} < 0$ and $\frac{\partial \tilde{f}_i}{\partial D} < 0$ as $pA^2 - pKM < 0$ and $C > 0$. In addition, we have $\frac{\partial \tilde{w}_i}{\partial \phi} < 0$, $\frac{\partial \tilde{w}_i}{\partial \tau} < 0$ and $\frac{\partial \tilde{f}_i}{\partial \phi} < 0$, $\frac{\partial \tilde{f}_i}{\partial \tau} < 0$.

Proposition 4 *Farmers auto-regulate both their water use and their fertilizer use when the water distance increases. Further, farmers use less water (fertilizer) input when either water or fertilizer inputs are taxed.*

Proof. See Appendix(A.2.1) □

Proposition 5 *The use of optimal input taxes leads to both better water quality and a smaller water distance, whatever the type of input taxes used, a water tax or a fertilizer tax.*

Proof. See Appendix(A.2.2) □

4.3.2 No budget constraint case

Proposition 6 *Without budget constraint the desired quantity and quality level will again be reached.*

Proof. See appendix (A.2.3). □

Remark 3 *The impact of the complementarity parameter A on the optimal input choice and on optimal water quantity and quality is ambiguous.*

4.4 Some numerical examples

We can finally give some numerical examples. Remember that we have a short time horizon of one fiscal year and hence $T = 1$. Consider a case where $A = 1$. Next suppose that ten farmers exploit the same groundwater resource. Parameter values are given by:

$$A = 1, B = 4, C = 1, E = 4, G = 1, K = 2, L = 0.1, M = 2,$$

$$N = 10, p = 100, r = 0.7, Z = 0.1, \delta = 0.01,$$

and

$$D_0 = 10, D_b = 20, Q_0 = 12, Q_b = 2, Y_0 = 2500, \alpha = 0.5.$$

Note that the impact of fertilizer and water inputs on production is symetric, i.e. $B = E = 4$ and $K = M = 2$. Likewise, the Water Agency attaches the same importance to quantity and quality management, $\alpha = 0.5$. In addition, the distance between the initial and the desired water quantity and quality levels is equal: $D_b - D_0 = Q_0 - Q_b = 10$. Figure 1 depicts the optimal evolution of the water-table distance and the water quality. In the laisser-faire case (dashed lines), both the water table distance and the water quality are depleted beyond the desired levels. When optimal tax policies are implemented (solid lines), the quantity and quality degradations are tempered. We have:

$$\phi = 520.84, \tau = -232.27,$$

and after policy intervention:

$$D(T) = 21.26, Q(T) = 6.10, w(T) = 1.16, f(T) = 3.74.$$

The optimal policy consists in taxing water inputs and subsidizing fertilizer inputs. This is not surprising for two reasons: first, the budget constraint leads to a tax-subsidy policy mix (as long as $b_0 > 0$). Second, water withdrawals have an impact on water quantity and on water quality (through the dilution effect) while fertilizer use only has an impact on water quality. In this symetric example, it is thus more interesting to tax water and to subsidize fertilizer. Note that optimal input use is smaller than before when optimal policies are in place.

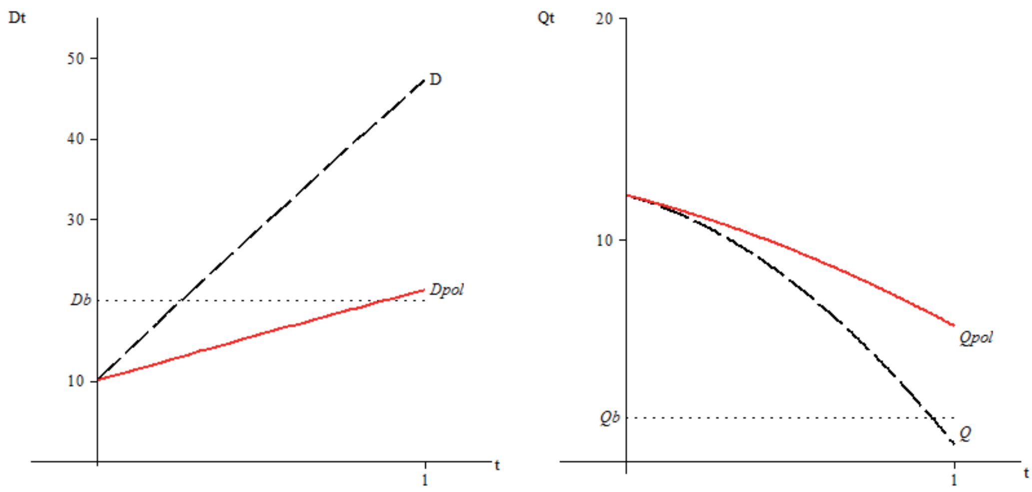

Figure 1: Optimal evolution of water-table distance, $D(t)$, and water quality, $Q(t)$, with policy (solid line) and without policy (dashed line).

Next, look at the impact of α on the optimal policy, as shown in figure 2. When $\alpha = 1$, the Water Agency does not attach any importance on the water-table distance. All effort goes into the regulation of water quality. The optimal policy is such that the quality requirement is met, but the water distance is degraded. The smaller α, the greater the importance that it attached on the level of the water-table and the flatter the evolution of the water-table distance.

We can also test the impact of the desired levels of the water-table and the water quality on the optimal policy. We compare the following cases: D_b varying from 12 to 26 while $Q_0 - Q_b = 10$ and Q_b varying from 4 to 18 while $D_b - D0 = 10$. Results[8] are shown in figure 3.

The greater the constraint (for Q_b big and D_b small), the higher the water-input tax, ϕ, and the lower the fertilizer-input subsidy, τ (remember the budget constraint, which implies that $Y(T) = 0$). This in turn means lower gains for farmers (for binding constraints). Likewise, the greater the constraints, the smaller the final water-table distance, the greater the final water quality and the smaller final water and fertilizer input. Overall, we can see that the sensitivity of the results is greater for D_b than for Q_b. This is again due to the dilution effect.

Finally, note that we can also generate a case in which the water input is subsidized and the fertilizer input is taxed. When the constraint on the water-table is not strong (for example $D_b = 36$ instead of $D_b = 20$) compared to the quality constraint (for example $Q_b = 10$ instead of $Q_b = 2$) and the water productivity small (for example $B = 0.1$ instead of $B = 4$). This is shown in figure 4. Now we have:

$$\phi = -309.04, \tau = 265.18,$$

and considering some parameter values at final time with policy intervention:

$$D(T) = 33.59, Q(T) = 7.88, w(T) = 2.35, f(T) = 1.85.$$

[8] Note that changes in D_b are built on the baseline-case, with $D_0 = 10$. For changes in Q_b we considered an initial quality level of $Q_0 = 22$ in order to be able to cover the same range of variation for Q_b and D_b.

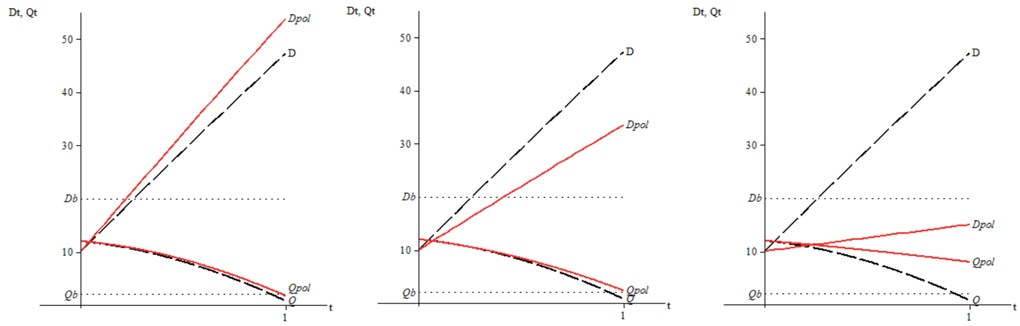

Figure 2: Optimal evolution of water-table distance and water quality for $\alpha = 1, \alpha = 0.7, \alpha = 0.3$.

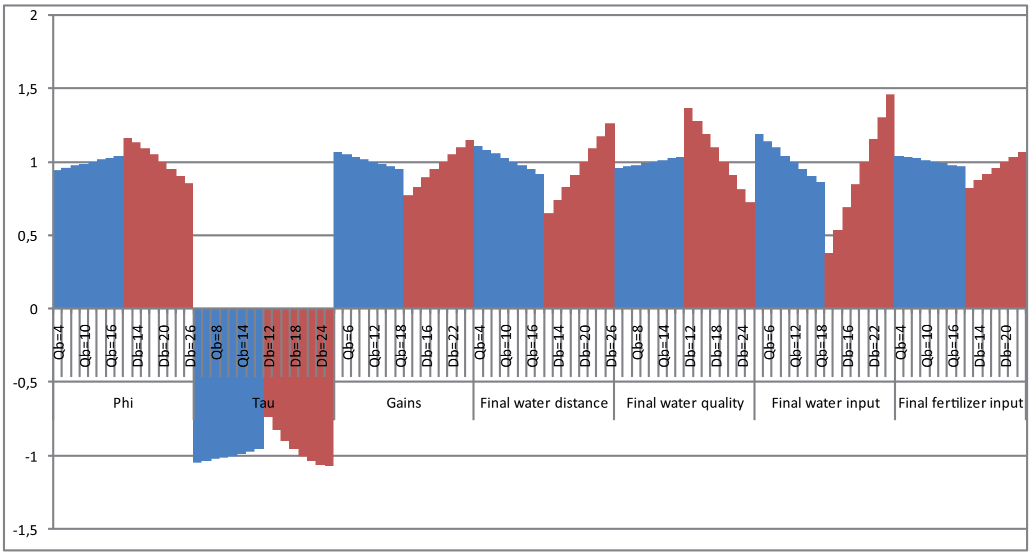

Figure 3: Variation of optimal policy as a function of desired levels of water quality, Q_b (blue) and water quantity, D_b (red).

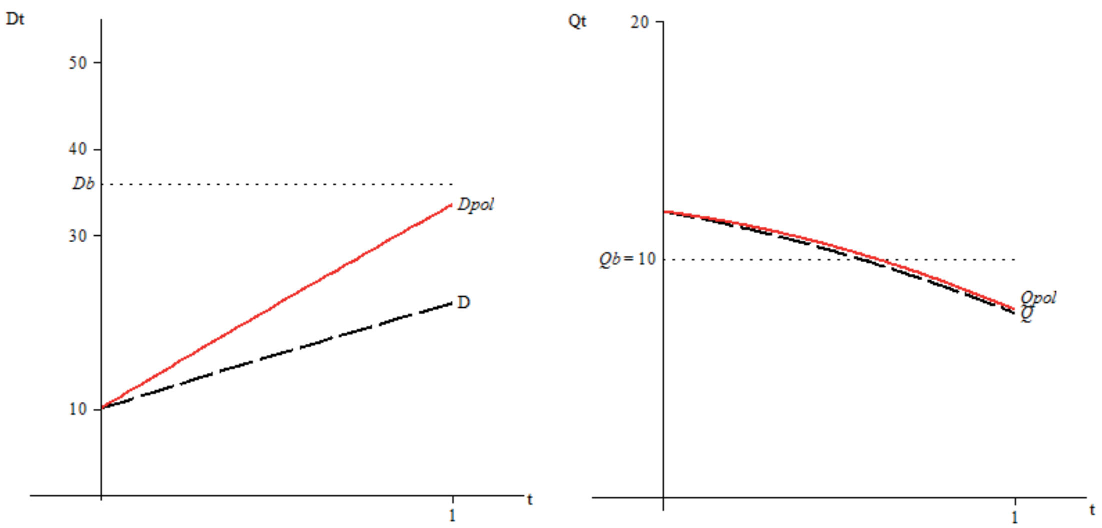

Figure 4: Optimal evolution of water-table distance, $D(t)$, and water quality, $Q(t)$, where the water-input is subsidized and fertilizer input taxed.

5 Some concluding remarks and perspectives

We have constructed a model of groundwater management in which a group of farmers overexploits a groundwater stock and causes excessive pollution, by using too much irrigation water and fertilizer. We have shown that there exists a set of constant policies which the regulator can impose, in order to bring the water resource close to a given quantity and quality level. To find the optimal policy-mix, we have constructed a linear-state open-loop Stackelberg game, which is equivalent to a feedback Stackelberg game. We have shown that, in addition to the usual first order conditions, we need some special conditions to account for the realism that the Water Agency can only impose constant policies.

In further work, it would be interesting i) to compare the constant policy solution to a solution where policies are time-dependent and ii) to compare our solution to a social optimal solution where the leader optimizes joint welfare but implements constant policies.

A Appendix

A.1 Case $A = 0$

A.1.1 First-order optimality conditions and solutions with budget constraint

$$
\begin{aligned}
\dot{D} &= \frac{N\left(pB - CD - Z - \phi\right)}{pM} - r, \quad D(0) = D_0, && (34)\\
\dot{\mu}^D &= \frac{N}{pKM}\left[KC\left(\mu^D + \mu^Y\phi\right) + \mu^Q\delta M\left(L + \tau - pE\right)\right], \mu^D\left(T\right) = 2(1-\alpha)(D(T) - D_b), && (35)\\
\dot{Y}\left(t\right) &= \left(\frac{N}{pKM}\right)\left[\left(pE - L - \tau\right)\tau M + \left(pB - CD - Z - \phi\right)\phi K\right], Y(0) = b_0, Y(T) = 0, && (36)\\
\dot{\mu}^Y &= 0 \Rightarrow \mu^Y(t) = constant = \mu^Y, && (37)\\
\dot{Q} &= \frac{\delta ND\left(L + \tau - pE\right)}{pK}, \quad Q(0) = Q_0, && (38)\\
\dot{\mu}^Q &= 0, \quad \mu^Q\left(T\right) = 2\alpha(Q\left(T\right) - Q_b) \Rightarrow \mu^Q\left(t\right) = 2\alpha(Q\left(T\right) - Q_b), && (39)
\end{aligned}
$$

$$
\begin{aligned}
\int_0^T \frac{\partial H_L}{\partial \tau} dt &= 0 \Leftrightarrow \int_0^T \left\{\mu^Y\left(L + 2\tau - pE\right) - \mu^Q\delta D\right\} dt = 0, && (40)\\
\int_0^T \frac{\partial H_L}{\partial \phi} dt &= 0 \Leftrightarrow \int_0^T \left\{\mu^Y\left(CD + Z + 2\phi - pB\right) + \mu^D\right\} dt = 0. && (41)
\end{aligned}
$$

Denote by

$$
\begin{aligned}
\Theta\left(\phi\right) &= \left[\frac{N\left(pB - Z - \phi\right) - rpM}{NC}\right],\\
\Lambda\left(\tau\right) &= \frac{\delta N\left(pE - L - \tau\right)}{pK} > 0, \qquad \rho = \frac{NC}{pM} > 0.
\end{aligned}
$$

Solving, we get:

$$
\begin{aligned}
D(t,\phi) &= e^{-\rho t}D_0 + \Theta(\phi)(1 - e^{-\rho t}), && (42)\\
Q\left(t,\phi,\tau\right) &= Q_0 - \Lambda(\tau)\int_0^t D(s)ds = Q_0 - \Lambda(\tau)\left[\frac{(D_0 - \Theta(\phi))}{\rho}(1 - e^{-\rho t}) + \Theta(\phi)t\right], && (43)\\
Y\left(t,\phi,\tau\right) &= b_0\frac{N}{pKM}\left[(pE - L - \tau)\tau M + (pB - Z - \phi)\phi K\right]t - \phi\rho\int_0^t D(s)ds, && (44)
\end{aligned}
$$

$$
\begin{aligned}
0 &= \mu^Y\left(L + 2\tau - pE\right)T + 2\alpha(Q\left(T\right) - Q_b)\delta\int_0^T D(s)ds, && (45)\\
0 &= \frac{\mu^Y C}{\rho}\left[\left(1 - e^{-\rho T}\right)D_0 + \Theta(\phi)(\rho T + e^{-\rho T} - 1)\right] + \left(Z + 2\phi - pB\right)\mu^Y T + \int_0^T \mu^D dt, && (46)
\end{aligned}
$$

where

$$
\int_0^t D(s)ds = \frac{D_0 - D(t)}{\rho} + \Theta(\phi)t,
$$

and

$$
\mu^D(t,\phi,\tau) = C_1 e^{\rho t} - \frac{\mu^Q(t)\delta M(pE - L - \tau)}{NCK} - \mu^Y\phi, \quad C_1 \quad \text{s.t.} \quad \mu^D(T,\phi,\tau) = 2(1-\alpha)(D(T,\phi,\tau) - D_b).
$$

Note that $Q(T)$ involves a multiplicative term $\tau\phi$ through the product of $\Lambda(\tau)$ and $\Theta(\phi)$. Therefore, after substitution for $Q(T)$ in equations (45) and (46) above, we end up with two equations of the form

$$\begin{aligned} 0 &= a_0 + a_1\tau\mu^Y + a_2\tau\phi + a_3\tau\phi^2 + a_4\phi + a_5\tau + a_6\phi^2, \quad (47) \\ 0 &= c_0 + c_1\mu^Y + c_2\tau + c_3\phi + c_4\phi\mu^Y + c_5\tau\phi + c_6\tau^2\phi + c_7\tau^2, \quad (48) \end{aligned}$$

where $a_0, \ldots, a_6$, and $c_0, \ldots, c_7$ are constants. The budget constraint at T in equation (44) reads as follows:

$$Y(T, \phi, \tau) = 0 = d_0 + d_1\tau + d_2\tau^2 + d_3\phi + d_4\phi^2, \quad (49)$$

where $d_0, \ldots, d_4$ are constants. We have all the information to solve a system of 3 equations (47)-(49) for the three unknowns τ, ϕ and μ^Y.

A.1.2 Proof of $\frac{\partial D}{\partial \phi} < 0$, $\frac{\partial Q}{\partial \phi} > 0$, $\frac{\partial Q}{\partial \tau} > 0$

Proof. First of all, we have:

$$sign(\frac{\partial D(t,\phi)}{\partial \phi}) = sign(\frac{\partial \Theta(\phi)}{\partial \phi}) = sign(-1) < 0.$$

Next, remember

$$Q(t, \phi, \tau) = Q_0 - \delta N \int_0^t \tilde{f(t)} D(t) dt.$$

We thus have:

$$\frac{\partial Q}{\partial \phi} = -\delta N \int_0^t \frac{d}{\partial \phi}\left[\tilde{f(s)} D(s)\right] ds = -\delta N \int_0^t \left[\frac{\partial \tilde{f(s)}}{\partial \phi} D(s) + \tilde{f}\frac{\partial D}{\partial \phi}\right] ds > 0$$

as

$$\frac{\partial \tilde{f}}{\partial \phi} = 0, D(s) > 0, \tilde{f} > 0, \frac{\partial D}{\partial \phi} < 0.$$

Likewise:

$$\frac{\partial Q}{\partial \tau} = -\delta N \int_0^t \frac{d}{\partial \tau}\left[\tilde{f(s)} D(s)\right] ds = -\delta N \int_0^t \left[\frac{\partial \tilde{f(s)}}{\partial \tau} D(s) + \tilde{f}\frac{\partial D}{\partial \tau}\right] ds > 0$$

as

$$\frac{\partial \tilde{f}}{\partial \tau} < 0, D(s) > 0, \tilde{f} > 0, \frac{\partial D}{\partial \tau} = 0.$$

□

A.1.3 Case $A = 0$ without budget constraint

We have $V < 0$ for all $D_b \geq D_0$ and $T \geq 0$. In fact

$$\begin{aligned} V(D_b = D_0) &= -D_0 T < 0, \\ \frac{\partial V}{\partial D_b} &= \frac{1 - e^{-\rho T} - \rho T}{\rho\left(1 - e^{-\rho T}\right)} < 0 \quad \forall T. \end{aligned}$$

V is decreasing with D_b and hence always negative. Note that $V < 0$ implies that $f^* > 0$, as $Q_b < Q_0$.

A.2 Case $A \neq 0$

A.2.1 Proof of $\frac{\partial \tilde{f}}{\partial \phi} < 0$, $\frac{\partial \tilde{f}}{\partial \tau} < 0$, $\frac{\partial \tilde{w}}{\partial \phi} < 0, \frac{\partial \tilde{w}}{\partial \tau} < 0$,

Solving $\dot{D} = N\tilde{w} - r$, we get:

$$D(t) = e^{-\rho t} D_0 + \Theta(\tau, \phi)(1 - e^{-\rho t}),$$

where

$$\rho = \frac{-NKC}{p(A^2 - MK)} > 0,$$

$$\Theta(\tau, \phi) = -\frac{A(L + \tau - pE) + K(Z - pB + \phi)}{KC} - \frac{r}{\rho}.$$

We therefore have:

$$\begin{aligned} \frac{\partial D}{\partial \tau} &= -\frac{A}{KC}\left[1 - e^{-\rho t}\right] < 0, \\ \frac{\partial D}{\partial \phi} &= -\frac{1}{C}\left[1 - e^{-\rho t}\right] < 0, \end{aligned}$$

and:

$$\begin{aligned} \frac{\partial \tilde{f}}{\partial \phi} &= -\frac{1}{p(A^2 - MK)} A \left[-1 - C\frac{\partial D}{\partial \phi}\right] = -\frac{1}{p(A^2 - MK)} A \left[-e^{-\rho t}\right] < 0 \\ \frac{\partial \tilde{f}}{\partial \tau} &= -\frac{1}{p(A^2 - MK)} \left[-AC\frac{\partial D}{\partial \tau} - M\right] = -\frac{1}{p(A^2 - MK)}(\frac{A^2}{K} - M) < 0 \\ \frac{\partial \tilde{w}}{\partial \phi} &= -\frac{1}{p(A^2 - MK)} \left[-K - KC\frac{\partial D}{\partial \phi}\right] = -\frac{1}{p(A^2 - MK)} K \left[-e^{-\rho t}\right] < 0 \\ \frac{\partial \tilde{w}}{\partial \tau} &= -\frac{1}{p(A^2 - MK)} \left[-A - KC\frac{\partial D}{\partial \tau}\right] = -\frac{1}{p(A^2 - MK)} A \left[-e^{-\rho t}\right] < 0 \end{aligned}$$

as

$$-\frac{1}{p(A^2 - MK)} > 0.$$

A.2.2 Proof of $\frac{\partial D}{\partial \phi} < 0$, $\frac{\partial D}{\partial \tau} < 0$, $\frac{\partial Q}{\partial \phi} > 0, \frac{\partial Q}{\partial \tau} > 0$

Proof. Remember from proof (A.2.1) that

$$\begin{aligned} \frac{\partial D}{\partial \tau} &= -\frac{A}{KC}\left[1 - e^{-\rho t}\right] < 0, \\ \frac{\partial D}{\partial \phi} &= -\frac{1}{C}\left[1 - e^{-\rho t}\right] < 0. \end{aligned}$$

The proof follows the same reasoning as in proof (A.1.2). We have:

$$\frac{\partial Q}{\partial \phi} > 0$$

as

$$\frac{\partial \tilde{f}}{\partial \phi} < 0, D(s) > 0, \tilde{f} > 0, \frac{\partial D}{\partial \phi} < 0.$$

and

$$\frac{\partial Q}{\partial \tau} > 0$$

as

$$\frac{\partial \tilde{f}}{\partial \tau} < 0, D(s) > 0, \tilde{f} > 0, \frac{\partial D}{\partial \tau} < 0.$$

□

A.2.3 Case A>0 witouth budget constraint. Proof of $D(T) = Db$ and $Q(T) = Qb$

Proof. We now have:

$$H_L = \mu^D N(\tilde{w}_i - r) - \mu^Q \delta \tilde{f}_i D$$

$$\int_0^T \frac{\partial H_L}{\partial \tau} dt = 0 \Leftrightarrow \int_0^T \left[\mu^D NA - \mu^Q \delta MD\right] dt = 0, \tag{50}$$

$$\int_0^T \frac{\partial H_L}{\partial \phi} dt = 0 \Leftrightarrow \int_0^T \left[\mu^D NK - \mu^Q \delta AD\right] dt = 0. \tag{51}$$

From (50) and (51), we have:

$$\int_0^T \left[\mu^D - \mu^Q \frac{\delta M}{NA} D\right] - \left[\mu^D - \mu^Q \frac{\delta A}{NK} D\right] dt = 0 \Rightarrow \frac{\delta}{N} \int_0^T \mu^Q D \left[A^2 - MK\right] = 0.$$

As $D > 0$ and $[A^2 - MK] \neq 0$, we have: $\mu^Q = 0 \Leftrightarrow Q(T) = Qb$.

In addition, we have:

$$\dot{\mu^D} = -\frac{\partial H_L}{\partial D} = \rho \mu^D + \mu^Q \frac{\delta ACD}{p(A^2 - MK)} - \mu^Q \frac{\delta[M(pE - L - \tau) + A(pB - CD - Z - \phi)]}{p(A^2 - MK)}$$

with:

$$\mu^D(T) = 2(1 - \alpha)(D(T) - D_b) \quad \text{and} \quad \mu^Q = 0.$$

Hence:

$$\dot{\mu^D} = -\frac{NKC}{p(A^2 - MK)} \mu^D, \quad \mu^D(T) = 2(1 - \alpha)(D(T) - D_b).$$

Therefore: $\mu^D = 0 \Rightarrow D(T) = D_b$. $\square$

References

[1] A. Buratto, Zaccour, G., 2009, Coordination of Advertising Strategies in a Fashion Licensing Contract, Journal of Optimization Theory and Applications, 142, 31-53.

[2] Cummings, R. G., 1971, Optimum Exploitation of Groundwater Reserves with Saltwater Intrusion, Water Resources Research 7 (6), 1415-1424.

[3] Dinar, A., Xepapadeas, A., 1998, Regulating water quantity and quality in irrigated agriculture, Journal of Environmental Management 54, 273-289.

[4] Dockner,E.J., Jorgensen, S., Ngo Van Long, Sorger, G., 2000, Differential Games in Economics and Management Science, Cambridge University Press, 350p.

[5] Gisser, M., Sanchez, D.A., 1980, Competition versus optimal control in groundwater pumping, Water Resources Research 31, 638-642.

[6] Koundouri, P., 2004, Current Issues in the economics of groundwater resource management, Journal of Economic Surveys 18 (5), 703-740.

[7] Krawczyck, J.B., Zaccour, G., 1996, Pollution management through levies and subsidies, Modelling and Control of National and Regional Economies, eds. LJ. Vlacici et al.; an Imprint of Elsevier Science, Pergamon, 1996, 241-246.

[8] G. Martín-Herrán, Taboubi, S., Zaccour, G., 2005, A Time-Consistent Open-Loop Stackelberg Equilibrium of Shelf-space Allocation, Automatica 41(6), 971-982.

[9] Moreaux, M., Reynaud, A., 2006, Urban freshwater needs and spatial cost externalities for coastal aquifers: A theoretical approach, Regional Science and Urban Economics 36, 163-186.

[10] Provencher, B., Burt, O., 1993, The Externalities Associated with the Common Property Exploitation of Groundwater, Journal of Environmental Economics and Management 24, 139-158.

[11] Roseta-Palma, C., 2002, Groundwater Management When Water Quality Is Endogenous, Journal of Environmental Economics and Management 44, 93-105.

[12] Roseta-Palma, C., 2003, Joint Quantity/Quality Management of Groundwater, Environmental and Resource Economics 26, 89-106.

[13] Tsur, Y., Zemel, A., 2004, Endangered aquifers: Groundwater management under threats of catastrophic events, Water Resources Research 40, 1-10.

[14] Xepapadeas, A., 1992, Environmental Policy Design and Dynamic Nonpoint-Source Pollution, Journal of Environmental Economics and Management 23, 22-39.

[15] Xepapadeas, A., 1995, Managing the International Commons: Resource Use and Pollution Control, Environmental and Resource Economics 5, 375-391.

[16] Yadav, S. N., 1997, Dynamic Optimization of Nitrogen Use When Groundwater Contamination Is Internalized at the Standard in the Long Run, American Journal of Agricultural Economics 79, 931-945.

[17] Zeitouni, N., Dinar, A., 1997, Mitigating Negative Water Quality and Quality Externalities by Joint Management of Adjacent Aquifers, Environmental and Resource Economics 9, 1-20.

Documents de Recherche parus en 2012[1]

DR n°2012 - 01 : Abdoul Salam DIALLO, Véronique MEURIOT, Michel TERRAZA
« Analyse d'une nouvelle émergence de l'instabilité des prix des matières premières agricoles »

DR n°2012 - 02 : Emmanuel DUGUET, Christine Le CLAINCHE
« Chronic Illnesses and Injuries: An Evaluation of their Impact on Occupation and Revenues »

DR n°2012 - 03 : Ngo Van LONG, Antoine SOUBEYRAN, Raphael SOUBEYRAN
« Knowledge Accumulation within an Organization »

DR n°2012 - 04 : Véronique MEURIOT
« Une analyse comparative de la transmission des prix pour l'orientation des politiques publiques : le cas du riz au Sénégal et au Mali »

DR n°2012 - 05 : Daniel SERRA
« Un aperçu historique de l'économie expérimentale : des origines aux évolutions récentes »

DR n°2012 - 06 : Mohamed CHIKHI, Anne PEGUIN-FEISSOLLE, Michel TERRAZA
« Modélisation SEMIFARMA-HYGARCH de la persistance du cours du Dow Jones »

DR n°2012 - 07 : Charles FIGUIERES, Fabien PRIEUR, Mabel TIDBALL
« Public Infrastructure, non Cooperative Investments and Endogenous Growth »

DR n°2012 - 08 : Emmanuel DUGUET, Christine LE CLAINCHE
« The Impact of Health Events on Individual Labor Market Histories : the Message from Difference in Differences with Exact Matching »

DR n°2012 - 09 : Katrin ERDLENBRUCH, Mabel TIDBALL, Georges ZACCOUR
« A Water Agency faced with Quantity-quality Management of a Groundwater Resource »

[1] La liste intégrale des Documents de Travail du LAMETA parus depuis 1997 est disponible sur le site internet : http://www.lameta.univ-montp1.fr

Contact :

Stéphane MUSSARD : mussard@lameta.univ-montp1.fr