Quantity-quality management of groundwater resource

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Abstract

We consider a problem of groundwater management in which a group of farmers overexploits a groundwater stock and causes excessive pollution. A Water Agency wishes to regulate the farmer's activity, in order to reach a minimum quantity and quality level but it is subject to a budget constraint and cannot credibly commit to time-dependent optimal policies. We construct a Stackelberg game to determine a set of constant policies that brings the groundwater resource back to the desired state. We define a set of conditions for which constant policies exist and compute the amount of these instruments in an example.

JEL classification: H23, Q15, Q25.

Key words: groundwater, quantity-quality management, Stackelberg game

1 Introduction

The problem of groundwater management is a typical common pool resource problem where several users have to share a same resource stock. However, water resource management has to be considered along two dimensions, quantity and quality. Optimal public policies have to tackle both the externalities related to quantity and to quality. In this paper, we consider an endogeneous pollution externality from agricultural production and discuss optimal quantity-quality regulation by a Water Agency with restricted regulatory power.

Many articles have focused on the need of public intervention to regulate private exploitation of groundwater. In a simple quanitity management model with stock and pumping cost externalities,[1] Gisser and Sanchez 1980 [4] argued that the difference between the competetive and the optimal outcome is too small to justify policy intervention (see Koundouri 2004 [5] for a survey.). The consideration of more complicated resource problems

[1]The stock externality arises because the extraction of each resource user is constraint by the total groundwater stock ; the pumping cost externality arises because the cost of pumping groundwater depends on the level of the groundwater table, see Provencher and Burt 1993 [8].

and other externalities has shown that public intervention can be necessary, for example when several resources are linked with each other (Zeitouni and Dinar 1997 [15]), when groundwater has a buffer value against surface water scarcity (Provencher and Burt 1993 [8][2]), or when quality is taken into account (Roseta-Palma 2003 [10]).

Concerning water quality, a great deal of attention has been given to the issue of saltwater intrusion in coastal aquifers (see for example Cummings 1971 [1], Zeitouni and Dinar 1997 [15], Dinar and Xepapadeas 1998 [2], Tsur and Zemel 2004 [11], Moreaux and Reynaud 2006 [7]). With the intensification of agricultural production, inland resources are more and more threatened by quality degradation, via nitrate infiltration. Because groundwater resources are often used for drinking water, the issue is of importance also outside the agricultural sector. It is for example addressed by several European Policies, such as the Water Framework Directive (Directive 2000/60/EC), which fixes the objective of "good water quality" in 2015, the Directive on the protection of groundwater against pollution and deterioration (Directive 2006/118/EC) or the Nitrates Directive (Directive 91/676/EEC) , which specifically tackles pollution from agricultural production.

A large literature exists on the issues of nitrate pollution and non-point source pollution resulting from agricultural activity, including dynamic models (for example Yadav 1997 [14], Xepapadeas 1992 [12]). However, as Koundouri [5] states, these models "generally avoid the relationship between contamination and water-use decisions. The assessment of how much groundwater should be pumped is absent from these models". The first work that brings together these aspects in a general dynamic setting is Roseta-Palma (2002 [9] and 2003 [10]). She considers the impact of contaminant discharges on groundwater quality and in particular two special effects: the stock dilution effect which describes the beneficial impact of water volume on water quality and the contaminating vector effect in which contaminants infiltrate more easily into the soil when carried with irrigation water. Roseta-Palma 2003 shows that public regulation should address both quantity and quality to be optimal. She also confirms numerically that policy intervention is justified even if gains from quantity regulation are small, as in Gisser and Sanchez [4], because of the importance to meet quality standards.

However, Roseta-Palma (2003) and most other articles consider only one optimal tool for policy intervention: dynamic taxation.[3] Although a dynamic tax has a conceptual appeal, it is quite irresalistic in real-life contexts. Indeed, it requires that the regulator chooses an optimal policy that changes continuously, depending on the individual actions taken. Roesta-Palma points at some implementation problems but focuses on those linked to informational constraints on individual production and pollution functions. In this pa-

[2]Reducing groundwater stocks then generates the so-called risk-externality, see Provencher and Burt 1993 [8].

[3]As argued by Provencher and Burt [8], permit allocation does solve neither the risk externality nor the cost externality.

per, we study the case where the water regulator imposes constant policies over a reasonable time period, for example a year. This is what we observe in the field: many taxation and subsidy rates are revised every year, or set for a couple of years.

In the following, we analyse the case of a group of irrigating farmers which use the same groundwater resource. Fertilizer used by the farmers leaches into the groundwater and causes nitrate pollution, mitigated by the stock dilution effect and the natural decay rate of the contaminant. However, the individual farmer does not observe this pollution. The regulator, a Water Agency, aims at preserving a given quantity level to provide water for a nearby town and wishes to maintain drinking water quality. The Water Agency can levy taxes (on withdrawal and pollution), give subsidies or invest to ameliorate the contaminant decay rate, for example with green manure. However, she is subject to a budget constraint and can credibly commit only to constant policies. We therefore construct an open-loop dynamic Stackelberg game, similar to Krawczyck and Zaccour [6], to model farmers' optimal decisions in the face of these constant incentive policies. In the example we make, we use a linear state open-loop game for which the equilibrium is known to be subgame perfect and equivalent with the feedback Stackelberg equilibrium (see for example Xepapadeas 1995 [13] for a general feedback Stackelberg model).

We find that, under given conditions, there is indeed a set of constant optimal policies which fulfills all the constraints the Water Agency has to respect. Maybe surprisingly, in our simple example, the optimal poliy-mix consists in two input-subsidies (on water withdrawals and fertlizer use).

The paper is structured as follows. In section (2) we present the problem, a simplified agro-economic model including a groundwater resource. In section (3), we present the Stackelberg game and characterize its solution. In section (4) we consider a numerical example and compute the optimal taxation and investment policy in this context. In the last section, we conclude and give some perspectives for future research.

2 The problem

2.1 Farmers

Consider a group of farmers $i = 1, ...N$, situated above the same groundwater resource, $G(t)$, with t continuous time. Agricultural production, $y_i(\cdot)$ depends on two inputs: fertilizer, $\gamma_i(t)$, and irrigation water, $g_i(t)$, which each farmer pumps in the groundwater resource. Let ρ be the discount rate, and T the considered time horizon. Before tax and subsidy, the i's agent pay-off function is given by:

$$B_i = \int_0^T e^{-\rho t}[p_i y_i(g_i(t), \gamma_i(t)) - c_{g,i}(G(t), g_i(t)) - c_{\gamma,i}(\gamma_i(t))]dt \tag{1}$$

where p_i the price of the agricultural production, $c_g(\cdot)$ are pumping and distribution costs of irrigation water and $c_\gamma(\cdot)$ are costs of fertilizer use. Prices are assumed to be constant and farmers are price-takers. There is no conceptual difficulty in extending our model to an oligopolistic setting where the farmers compete with an homogenous product à la Cournot. Also, we may consider the case of organic producers, where prices may increase with, e.g. the water quality, the level of used fertilizer. Note, however, that the computation of equilibrium in such circumstances becomes more tedious.

We assume that agricultural production is increasing with inputs but at decreasing returns to scale; irrigation water and fertilizers are complementary goods:

$$\frac{\partial y_i}{\partial g_i} \geq 0, \frac{\partial y_i}{\partial \gamma_i} \geq 0, \frac{\partial^2 y_i}{\partial g_i^2} \leq 0, \frac{\partial^2 y_i}{\partial \gamma_i^2} \leq 0, \frac{\partial^2 y_i}{\partial g_i \partial \gamma_i} \geq 0.$$

We further assume that costs are increasing with both inputs but decreasing with the groundwater stock (the higher the water table, the lower the pumping costs).

$$\frac{\partial c_{g,i}}{\partial g_i} \geq 0, \frac{\partial c_{\gamma,i}}{\partial \gamma_i} \geq 0, \frac{\partial c_{g,i}}{\partial G} \leq 0.$$

Farmers are subject to public policies of the Water Agency: a tax,τ, on the use of polluting fertilizer and a tax, ϕ on individual water withdrawals.[4] Considering these public policies, the i's agent profits are thus given by:

$$\pi_i = \int_0^T e^{-\rho t}[p_i y_i(g_i(t), \gamma_i(t)) - c_{g,i}(G(t), g_i(t)) - c_{\gamma,i}(\gamma_i(t)) - \tau\gamma_i(t) - \phi g_i(t)]dt. \quad (2)$$

2.2 Water quantity and water quality

The groundwater stock, $G(t)$, evolves according to the following equation of motion:

$$\dot{G} = -\sum_i g_i(t) + r, \quad G_i(0) = G_0 \quad \text{given.} \quad (3)$$

The water volume increases with the mean recharge rate, r, and decreases with total water withdrawals, $\sum_i g_i(t)$. G_0 is the initial water volume.

The quality of groundwater, $Q(t)$, depends on total fertilizer use, the regenerative capacity of the resource and the environment and the total water volume[5]

$$\dot{Q} = -(\delta + \theta)\sum_i \gamma_i(t) + uG(t), \quad Q_i(0) = Q_0 \quad \text{given.} \quad (4)$$

[4]We do not impose any sign on these instruments. If after optimisation they are negative, subsidies should be set up rather than taxes.

[5]It would be more realistic to have an evolution of the form

$$\dot{Q} = -\frac{(\delta + \theta)\sum_i \gamma_i(t)}{G(t)}, \quad Q_i(0) = Q_0 \quad \text{given.}$$

However, solving for this would be of formidable difficulty.We suppose that having the above formulation provides a good approximation of quality motion.

δ is a parameter measuring the natural pollution decay rate and θ a decay rate controllable by the Water Agency. The cost of this effort is given by $c_{theta}(theta)$, an increasing function, satisfying $c_{theta}(0) = 0$. To fix ideas, the Water Agency may for example favour the use of plants containing nitrogen-fixing symbiotic bacteria[6]. Q_0 is the initial quality of the groundwater stock, which is observable by the Water Agency. Water quality thus deteriorates because of fertilizer use, but at a rate which depends on the natural decay of pollutants and the nitrate fixing capacity of additional plants. The water volume available induces a dilution effect wich mitiages overall pollution.

2.3 Water Agency

The Water Agency is concerned with both, water quantity and water quality. The Agency wishes to reach a given quantity or a given quality level at time T.

$$Q(T) \geq \alpha_Q Q_0 \tag{5}$$

$$G(T) \geq \alpha_G G_0. \tag{6}$$

where α_Q and α_G are given non-negative parameters. For example, some water should be safeguarded for urban or industrial uses. In addition, a minimum quality level could be necessary to use this water outside the agricultural sector, e.g. for drinking. The Agency may levy taxes, τ and ϕ, and limit pollution, θ, which comes at a cost $c_\theta(\theta)$. As stated before, for the sake of realism, we suppose that the tax and subsidy rates, as well as the cleaning effort, are constant over the considered time period, from 0 to T[7]. The Water Agency is subject to a budget constraint. The budget at time T should be in equilibrium, $Y(T) = 0$, given Y_0 the initial budget:

$$0 = Y(0) + \int_0^T e^{-\rho t}[-c_\theta(\theta) + \tau \sum_i \gamma_i(t) + \phi \sum_i g_i(t)]dt. \tag{7}$$

The above isoperimetric constraint can be rewritten in the form of a state equation, that is,

$$\dot{Y} = e^{-\rho t}[-c_\theta(\theta) + \tau \sum_i \gamma_i(t) + \phi \sum_i g_i(t)] \quad \text{with} \quad Y(0) = Y_0 \quad \text{and} \quad Y(T) = 0. \tag{8}$$

[6]This is the concept of green manure: white mustard (Sinapis alba), vetches (Vicia), phacelia or rapeseed (Brassica napus) for example are able to fix nitrogen in the field. They are set up after the main harvest, in automn and destroyed in winter. French farmers for example have been eligible to a damage payment, the *Indemnité compensatoire de couverture des sols* (Code de l'environnement LII1.1.3.3) for the introduction of these nitrogen fixing plants. This subsidy amounted to 60 euros/ha in 2003 and 30 euros/ha in 2006.

[7]We can seasily let the cleaning effort vary over time.

3 A Stackelberg game

3.1 The game model

3.1.1 The follower's problem

Each farmer choses the amount of inputs, $g_i(t)$ and $\gamma_i(t)$, that maximises profits, $\pi_i(G_0)$, given the constraints he observes:

$$(\hat{g}_i, \hat{\gamma}_i) = arg \quad max_{g_i(t),\gamma_i(t)} \quad (2) \quad \text{subject to} \quad (3). \tag{9}$$

Farmers are partially myopic: they do consider the impact of their decisions on water quantity but do not consider the impacts on water quality. Indeed, the height of the water table (and therefore the water stock) is supposed to be more easily observable to the farmer than the water quality.[8] In addition, total water quantity directly affects the farmer's pumping costs, $c_{g,i}(G(t), g_i(t))$. Farmers take into account water quality only indirectly through the taxes they have to pay, if $\tau > 0$. Likewise, water quantity is considered indirectly, through the taxes they have to pay, if $\phi > 0$.

3.1.2 The leader's problem

The Water Agency choses a set of constant policies ($\hat{\tau}$,$\hat{\phi}$,$\hat{\theta}$) that allows to reach the quantity and quality targets $G(T) \geq \alpha_G G_0$ and $Q(T) \geq \alpha_Q Q_0$,[9]. She considers the dynamics of quantity and quality, the budget constraint and the farmer's reaction to the public policies:

$$\text{Choose} \quad (\hat{\tau}, \hat{\phi}, \hat{\theta}) \quad \text{subject to} \quad (3), (4), (5), (6), (7) \text{ and } (9). \tag{10}$$

A solution to (10) defines an open loop Stackelberg equilibrium. The leader announces a set of public policies. The follower takes them into account in his optimisation process. In our case, the follower considers only: $\hat{\tau}$ and $\hat{\phi}$, but ignores $\hat{\theta}$ which only affects water quality. The leader then computes the optimal value of $\hat{\tau}$, $\hat{\phi}$, and $\hat{\theta}$ given the reaction of the follower and her own constraints. It is well known that open-loop Stackelberg equilibrium is in general time inconsistent. That is, if given the choice, then the leader may reoptimize at an intermediate date and change her decisions for the remaining time period. In our case however, it seems reasonable to assume that the Water Agency announces and commits to her public policy, probably by legislating, over the short time horizon we consider, for example a year. Any changes can be implemented in the following period.

[8] We could also consider the case where farmers are completely myopic, i.e. do not consider the dynamics of water quality and quantity.

[9] Note that this is a problem of cost effectiveness and not of optimizing total surplus.

3.2 The solution of the game

3.2.1 The follower's reaction

The i's follower's current-value Hamiltonian is:

$$H_i = p_i y_i(g_i(t), \gamma_i(t)) - c_{g,i}(G(t), g_i(t)) - c_{\gamma,i}(\gamma_i(t)) - \tau\gamma_i(t) - \phi g_i(t) \\ +\lambda_i(t)[-\sum_i g_i(t) + r] \tag{11}$$

where λ_i is the current-value adjoint state variable for each follower. Assuming an interior solution, the necessary conditions are:

$$\frac{\partial H_i}{\partial g_i} = 0 \quad \Rightarrow \quad p_i \frac{\partial y_i}{\partial g_i} - \frac{\partial c_{g,i}}{\partial g_i} - \phi - \lambda_i = 0 \tag{12}$$

$$\frac{\partial H_i}{\partial \gamma_i} = 0 \quad \Rightarrow \quad p_i \frac{\partial y_i}{\partial \gamma_i} - \frac{\partial c_{\gamma,i}}{\partial \gamma_i} - \tau = 0 \tag{13}$$

$$\dot{\lambda}_i = \rho\lambda_i - \frac{\partial H_i}{\partial G} \quad \Rightarrow \quad \dot{\lambda}_i = \rho\lambda_i + \frac{\partial c_{g,i}}{\partial G} \tag{14}$$

We also have the transversality condition

$$\lambda_i(T) = 0 \tag{15}$$

and the equation of motion for the resource stock:

$$\dot{G} = -\sum_i g_i(t) + r, \quad G(0) = G_0. \tag{16}$$

Equations (12) and (13) are the usual optimality conditions that state that, at the optimum, marginal revenus from production equal marginal costs. In equation (12), marginal revenues are due to the use of one additional unit of water. Marginal costs are given by marginal costs of pumping and distributing irrigation water, by the taxes paid per unit of water pumped and by the marginal shadow price of using water today, instead of tomorrow. In equation (13), marginal revenues due to the use of one additional unit of fertilizer are equal to marginal costs of buying fertilizers and the taxes paid per unit of fertilizer. Finally, equation (14) describes how the shadow price evolves, taking into account the stock effect on costs and on subsidies. The optimal reaction of the i's follower is of the form:

$$\tilde{g_i(t)} = f_g(\gamma_i(t), G(t), \lambda_i(t), \tau, \phi) \tag{17}$$

$$\tilde{\gamma_i(t)} = f_\gamma(g_i(t), G(t), \lambda_i(t), \tau, \phi). \tag{18}$$

The optimal reaction can be plotted into the leader's problem to solve the Stackelberg game. However, it is not always possible to compute this optimal response analytically. We therefore propose in the following another more general way to sovle the problem.

3.2.2 The leader's decision

The leader's current value Hamiltonian is given by:

$$\begin{aligned} H_l &= \kappa(t)\dot{Q} + \mu(t)\dot{G} + \nu(t)\dot{Y} \\ &+ \sum_i \omega_i(t)\frac{\partial H_i}{\partial g_i(t)} + \sum_i \xi_i(t)\frac{\partial H_i}{\partial \gamma_i(t)} + \sum_i \zeta_i(t)\left(\rho\lambda_i - \frac{\partial H_i}{\partial G(t)}\right) \end{aligned} \quad (19)$$

We have the following end-time conditions:

$$Q(T) \geq \alpha_Q Q_0, \quad G(T) \geq \alpha_G G_0, \quad Y(T) = 0. \quad (20)$$

The "usual" necessary conditions are:

$$\dot{\kappa} = \rho\kappa - \frac{\partial H_l}{\partial Q} \Rightarrow \dot{\kappa} = \rho\kappa \quad (21)$$

$$\dot{\mu} = \rho\mu - \frac{\partial H_l}{\partial G} \Rightarrow \dot{\mu} = \rho\mu + \kappa u - \nu[\frac{\partial \dot{Y}(G,...)}{\partial G}] + \sum_i \omega_i[\frac{\partial^2 c_{g,i}}{\partial g_i \partial G}] + \sum_i \zeta_i[\frac{\partial^2 c_{g,i}}{\partial G^2}] \quad (22)$$

$$\dot{\nu} = \rho\nu - \frac{\partial H_l}{\partial Y} \Rightarrow \dot{\nu} = \rho\nu \quad (23)$$

Equations (21) and (23) tell us that the current-value shadow price of the budget and of water quality are constants. Equation (22) indicates that the evolution of the current-value shadow price for water quantity depends on the impact of water quantity on the followers' cost functions and on water quality.

Following Dockner et al. [3], there are also a series of "special conditions":

$$\int_0^T \frac{\partial H_l}{\partial \tau}dt = 0 \Rightarrow \int_0^T [\nu(t)\frac{\partial \dot{Y}}{\partial \tau} - \sum_i \xi_i(t)]dt = 0 \quad (24)$$

$$\int_0^T \frac{\partial H_l}{\partial \phi}dt = 0 \Rightarrow \int_0^T [\nu(t)\frac{\partial \dot{Y}}{\partial \phi} - \sum_i \omega_i(t)]dt = 0 \quad (25)$$

$$\int_0^T \frac{\partial H_l}{\partial \theta}dt = 0 \Rightarrow \int_0^T [\kappa(t)\frac{\partial \dot{Q}}{\partial \theta} + \nu(t)\frac{\partial \dot{Y}}{\partial \theta}]dt = 0. \quad (26)$$

Equations (24) and (25) state that the impact of the tax policy (τ and ϕ respectively) on the evolution of the budget should be balanced with the value that this constraint imposes on the follower (the sum of the state ajoint variables), over the considered time period. Equation (26) says that the impact of the subsidy (θ) on the evolution of the budget, in value terms, should be balanced with its impact on water quality, over the considered time period.

4 A simple example

4.1 Assumptions

We illustrate in this section the type of insight that can be obtained using our model. To keep things as simple as possible, we assume two identical players. The agricultural production function is linear in inputs and the production cost functions are linear-quadratic with respect to inputs.

$$y_i = A g_i \gamma_i, \tag{27}$$

$$c_{i,g} = Z - CG(t) + E g_i + \frac{M g_i^2}{2}, \tag{28}$$

$$c_{i,\gamma} = L\gamma_i + \frac{K\gamma_i^2}{2}. \tag{29}$$

The investment cost function is supposed to be linear:

$$c_\theta = D\theta. \tag{30}$$

We also need to verify: $g_i(t) \geq 0$ and $G(t) \geq 0$.
Further, we suppose that the planning horizon is sufficiently short, e.g. T corresponds to a fiscal year, or 12 months, and hence we set $\rho = 0$. Other parameter values are:

$$p_i = 6, A_i = 0.8, M, K, Z = 1, C = 0.02, E = 0.2, L = 2, D = 1000, r = 0.05, u = -0.0001, \delta = 0.8$$

and for the stocks:

$$\begin{aligned} G0 = 100, \quad \alpha_G = 0.95, \quad & \text{hence with a binding constraint:} \quad G12 = 95, \\ Q0 = 7, \quad \alpha_Q \approx 0,79 \quad & \text{such that, with a binding constraint:} \quad Q12 = 5.5, \\ & Y0 = 1, \quad \text{and} \quad Y12 = 0. \end{aligned}$$

4.2 Results

Figure (1) represents water quantity and quality as they are chosen by the follower, without any policy intervention by the Water Agency. In our case, the follower depletes the quantity to a level of $G(T) = 90$ and drives quality down to $Q(T) = 4.79$. Assume that water then is polluted.

By the end of the year, the Water Agency wishes to reach a water level of $G(T) = 95$ and wishes to have a better water quality, let's say $Q(T) = 5.5$. After following the Stackelberg game, we can define the optimal instruments. Quite surprisingly, the optimal policy-mix consists in input subsidies, rather than taxes. The Water Agency should set a constant subsidy on water withdrawals $\hat{\phi} = -0.15$, a constant subsidy on fertilizer use, $\hat{\tau} = -0.96$. The intervention on green manure is zero[10].

[10]This is not surprising as in our example, the cost of intervention is set to be extremely high.

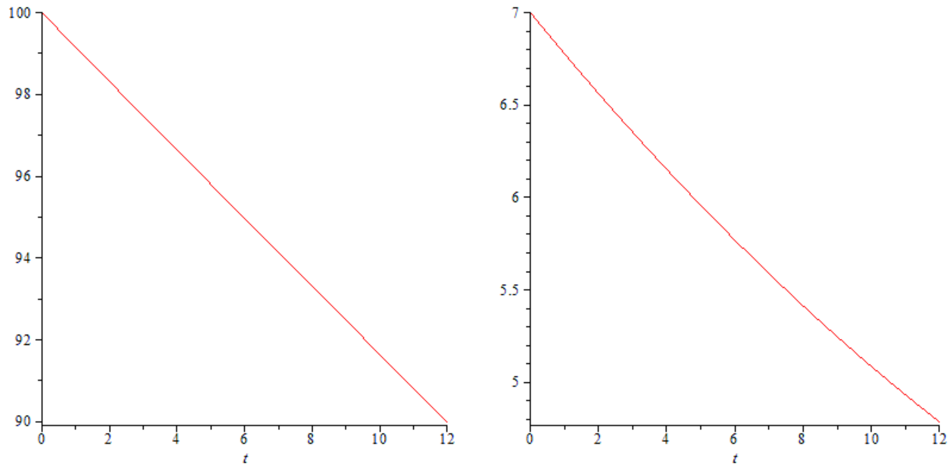

Figure 1: Water quantity, $G(t)$ and Water quality $Q(t)$ chosen by the follower, without policy intervention.

To explain the fact that optimal policies are input subsidies, rather than taxes, we may analyse the form of the optimal input variables $\tilde{g}(t)$ and $\tilde{\gamma}(t)$. We have:

$$\tilde{\gamma}_i(t) = \frac{p_i A_i - L - \tau}{K}. \tag{31}$$

Optimal fertilizer use depends positively on output prices, p_i and production efficiency, A_i and negatively on fertlizer costs, L and K, and fertilizer taxes, τ. To reduce fertilizer use, τ should be positive, all other variables being equal. Yet, this is not the case. In addition, water input depends positively on τ. It is given by the following equation:

$$\tilde{g}_i(t) = \frac{p_i A_i (L + \tau) + \lambda_i(t) K + EK + \phi K}{p_i^2 A_i^2 - MK}, \tag{32}$$

with

$$\tilde{\lambda}_i(t) = -Ct + \lambda_0.$$

Positive τ would increase water withdrawals, all other variables being equal. In our example, the optimal value of τ should allow $\tilde{g}(t)$ to decrease. But decreasing water consumption also decreases production costs and the value of production, the other variable in $\tilde{g}(t)$, etc. The choice of the optimal policy instrument is hence not straightforward.

Figures (2) and (3) show that after implementing the above optimal constant policy instruments, the quantity and quality targets are met. The follower now depletes less the groundwater stock (red line below black line in figure (2)) and he pollutes less, that is

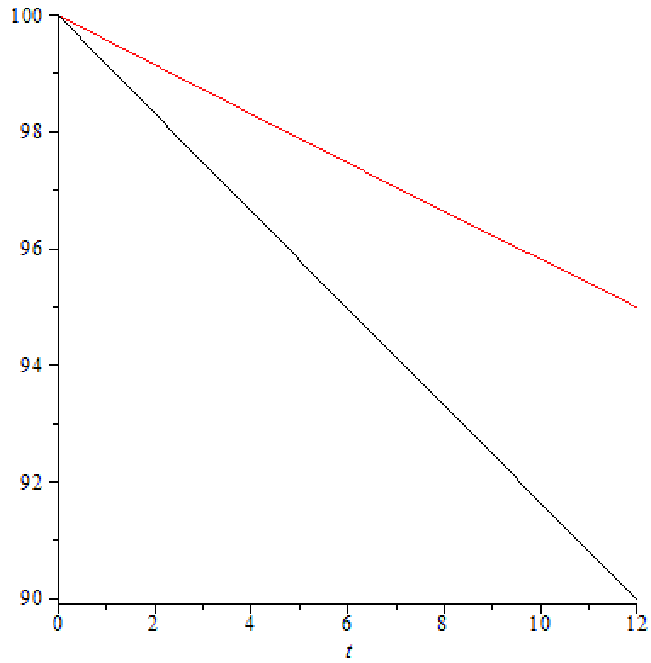

Figure 2: Optimal water stock, $G(t)$, before (black, below) and after (red, above) policy intervention

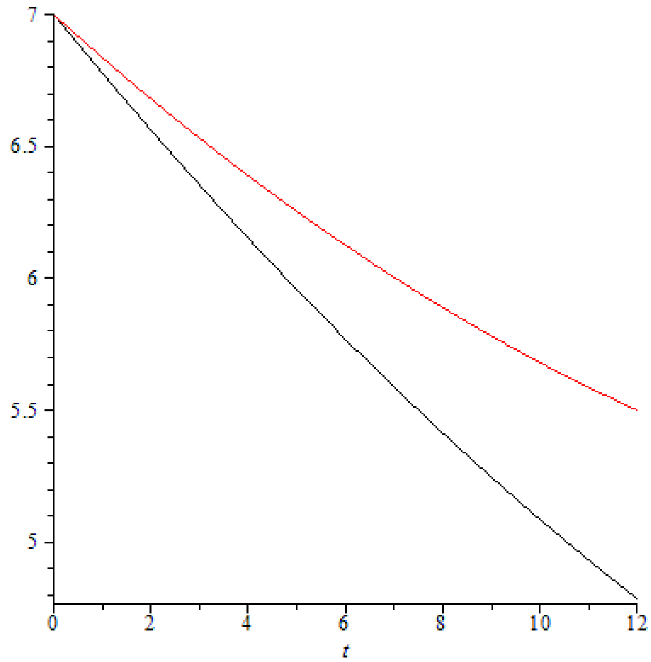

Figure 3: Optimal water quality, $Q(t)$, before (black, below) and after (red, above) policy intervention

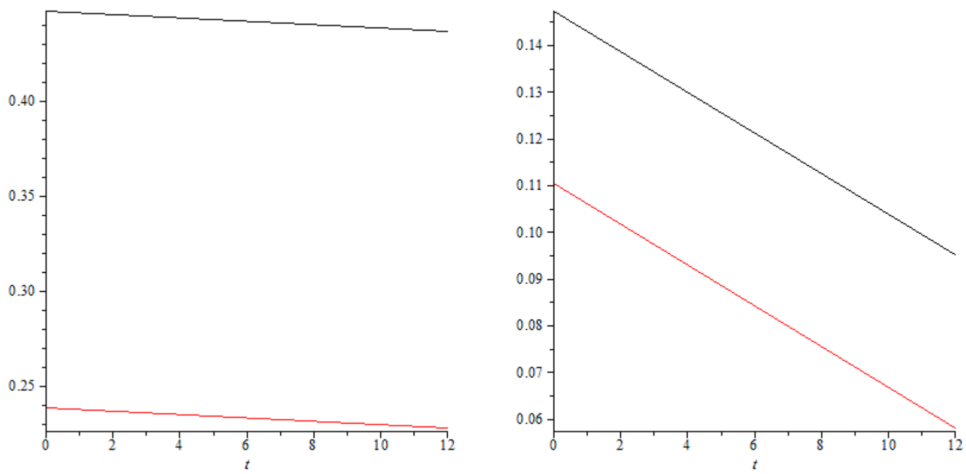

Figure 4: Optimal input use before policy intervention (black, above) and after policy intervention (red, below). Left-hand side: water use, $g(t)$. Right-hand side: fertilizer use, $\gamma(t)$.

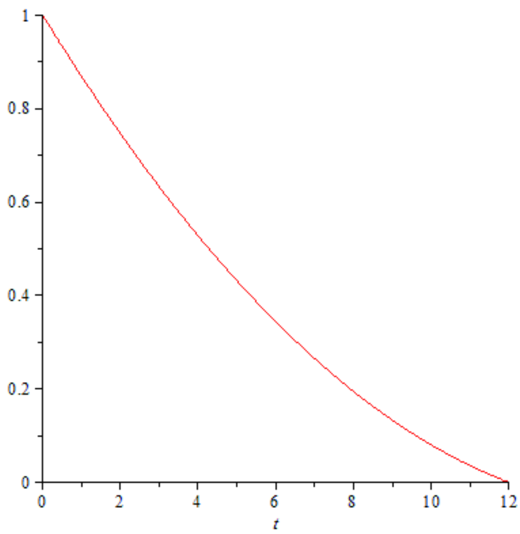

Figure 5: Evolution of optimal budget,$Y(t)$ initial and end values are given.

quality is always higher (red line above black line in figure (3)).

In line with previous results, figure (4) shows that the follower uses less inputs over time, after the policy intervention. The left-hand side shows the optimal evolution of water inputs, the right-hand side the optimal evolution of fertilizer inputs. Before policy intervention, water input use started at a level of $g = 0.45$, after policy intervention, it starts at a level of $g = 0.15$. Likewise, fertilizer use started at a level of 0.45 before policy intervention, and $g = 0.24$ after policy intervention. Input-use over time is decreasing.

We can also compute total gains for the followers, over the fiscal year (that is from $t = 0$ to $T = 12$). In absence of the optimal policy instruments, both followers earn: $\pi_i = 19.21$. After implementation of the optimal policy instruments, the followers earn $\pi_i = 16.37$. The subsidy is thus not sufficient to compensate the forgone production earnings, as fertilizer and water use have to be reduced. Finally, we can confirm that the Water Agency's budget constraint holds: it starts in $Y0 = 1$ and ends in $Y(T) = 0$ (see figure (5)). The budget is in equilibrium.

5 Concluding remarks

We have constructed a model of groundwater management in which a group of farmers overexploits a groundwater stock and causes excessive pollution, by using too much irrigation water and fertilizer. We have shown that there exists a set of constant policies which the regulator can impose, in order to bring the water resource back to a given quantiy and quality level. To find the optimal policy-mix, we have constructed a linear-state open-loop Stackelberg game, which is equivalent to a feedback Stackelberg game. We have shown that, in addition to the usual first order conditions, we need some special conditions to account for the realism that the Water Agency can only impose constant policies.

In further work, it would be interesting to compare our solution to a social optimal solution in which taxation is dynamic. We could in particular explain the link between the constant optimal policies and the dynamic ones.

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