Usefulness of sensitivity analysis for approximate bayesian computation

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Usefullness of Sensitivity Analysis for Approximate Bayesian Computation

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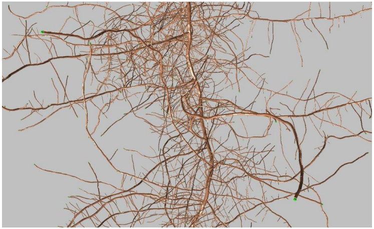

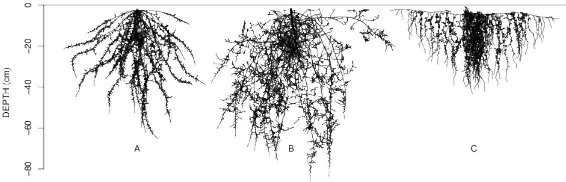

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Overview

1. Review of ABC concepts
2. The root system model
3. Sensitivity Analysis for statistics
4. Sensitivity Analysis for MSE criterion
5. Conclusion and discussion

1. ABC concepts

- Approximate Bayesian Computing (ABC) is a free likelihood method to estimate model parameters
- Definition of statistics (or descriptors)
- Fast computing model

Notations:

Observed data D and simulated data $D^\star$

θ is the vector of parameters with Prior $\pi(.)$

s(.): function that computes a set of statistics (descriptors)

$S = s(D)$ vector of statistics for data D

$S^\star = s(D^\star)$ vector of statistics for data $D^\star$

1. ABC: a free likelihood method

Algorithm (Accept/Reject)

0: Suppose we have observed data D and $S = s(D)$

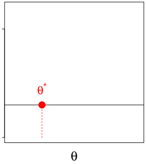
1: Generate $\theta^\star$ from $\pi(.)$

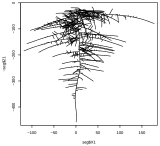
2: Generate $D^\star$ from $f(.|\theta^\star)$

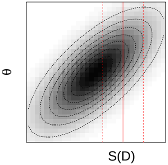
3: Compute statistics $S^\star$ for $D^\star$

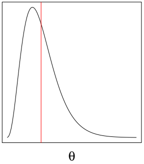
4: Accept $\theta^\star$ if $d_W(S, S^\star) \leq \epsilon$ and return to (1)

Prior	Simulation	Joint Ditribution	Posterior
$\pi(\theta)$	$D^\star$	$(\theta^\star, S^\star)$	$\pi(\theta \| d_W(S, S^\star) \leq \epsilon)$









1. ABC: a free likelihood method

This algorithm gives an approximation of $\pi(\theta|D)$.

Two important points for the approximation:

- The threshold ϵ:

  smaller $\epsilon \rightarrow$ *better* approximation

- $D^\star$ is summarised by the statistics $S^\star$:

  better statistics $\rightarrow$ *better* approximation

2. The root system model

Complexity of plant root system:

Functionning is linked to the dynamics of the architecture. Water and nutriment uptake depend on the root surface..

Plant root system modelling:

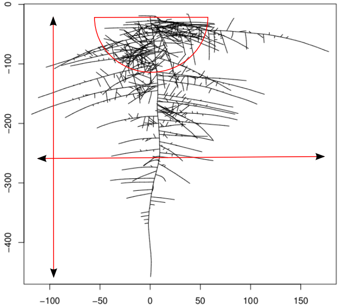
Integration of knowledge and test of new hypotheses
Summarize data into a low number of key values

The stochastic model:

Number of parameters: 14
Output of the model: image of root system

3. The root system model

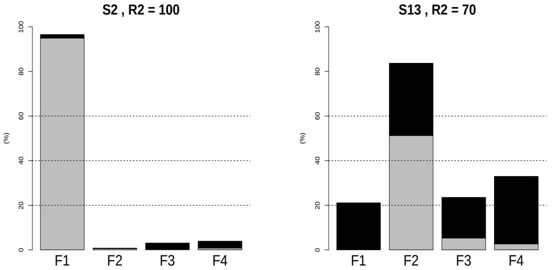
- **4 parameters** over 14 are estimated with images.
- **15 statistics** are computed: size and shape of the root system, density of pixels in different areas, ...



3. Sensitivity analysis of statistics

- Can parameters be estimated with the statistics ?
- Anova: 4 factors with 5 levels, interaction of order 3

Gray: Principal
Black: Interaction



- About 8-10 statistics over the 15 seem to be sufficient to estimate parameters

4. Sensitivity analysis of MSE

- Find the best weights W of d_W to minimize MSE criterion ?
- Point estimate: $\hat{\theta} = Mean\{\theta^\star : d_W(S, S^\star) \leq \epsilon\}$ with

$$d_W^2(S, S^\star) = \sum_{i=1}^{N_S=15} w_i(S_i - S_i^\star)^2 \text{ and } w_i > 0, \sum_{i=1}^{N_S} w_i = 1.$$

- Criterion to evaluate point estimate $\hat{\theta}$:

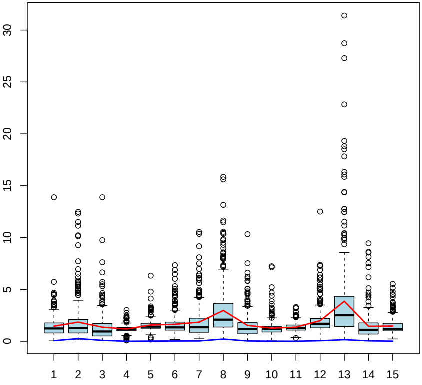
$$MSE_\theta(W) = \sum_{k=1}^{N_\theta=4} \frac{(\hat{\theta}^{(k)} - \theta^{(k)})^2}{\sigma^2_{\theta^{(k)}}}$$

4. Sensitivity analysis of MSE

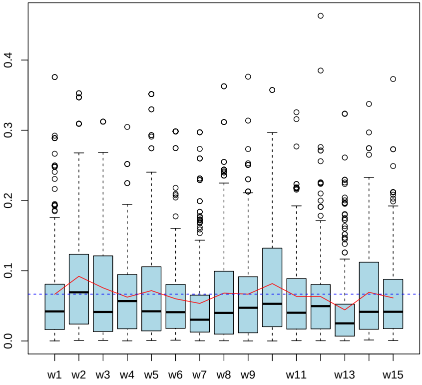
- Generate uniformly a R-sample of weights $W^r, r = 1, ..., R$ with $W^r = (w_1^r, ..., w_{N_S}^r)$ and $\sum_{i=1}^{N_S} w_i^r = 1$
- Generate a N-sample $\theta_l, l = 1, ..., N$ from $\pi(\theta)$.
- For each $\theta_l, l = 1, ..., N$
  - Compute $MSE_{\theta_l}(W^r)$, r=1,...,R
  - Fit a canonical polynomial of degree 2:
    $MSE_{\theta_l}(W) = P_l(W) + e,\ l = 1, ..., N$
    $$\text{with } P_l(W) = \sum_{i=1}^{N_S} \delta_{ii} w_i^2 + \sum_{i=1}^{N_S} \sum_{i<j}^{N_S} \delta_{ij} w_i w_j$$
  - Sensitivity indices by comparing nested polynomials models.

4. Sensitivity analysis of MSE

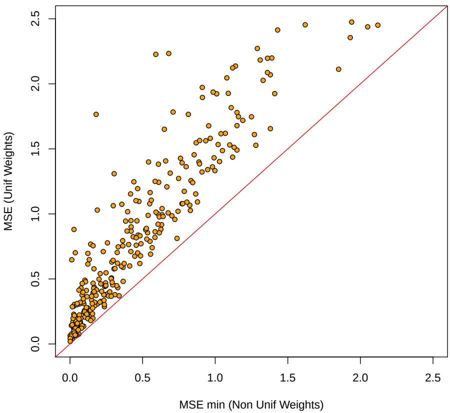
Sensitivity indices

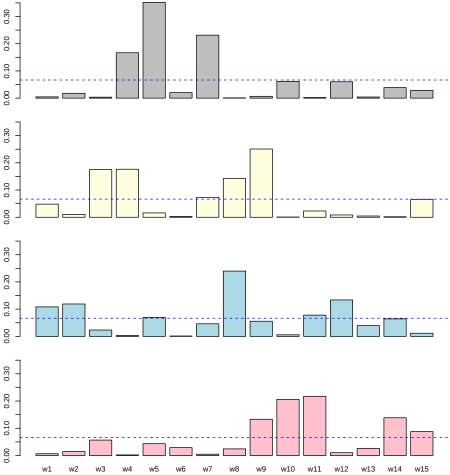


Minimum weights



4. Sensitivity analysis of MSE





5. Conclusion

Conclusion

- Difficult to find an optimal distance (for all θ)
- Interaction between weights associated to statistics
- ABC with three steps:
  1. Pilot ABC ($\rightarrow$ first approximation $\tilde{\theta}$)
  2. Determine optimal weights associated to $\tilde{\theta}$
  3. ABC with the optimal weights ($\rightarrow$ second approximation $\hat{\theta}$)

Future work

- Optimal weights determined by global optimum of P_W
- Study based on the expectations of the statistics (rather one observation)

References

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Factor F3 and F4: