Performance of bankruptcy rules in CPR allocation when resource diversication is available

Mabel Tidball, Marianne Lefebvre

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Abstract

Common pool resource (CPR) users face the risk of resource shortage if the sum of the claims they have over the common pool resource is incompatible with the actual resource size. However, in many situations, agents can diversify their resources and substitute an alternative safe resource but costly to a free but risky CPR. This paper adresses the original question of the interaction between the sharing rule of the CPR (required to allocate the available resource in case of shortage) and the resource diversification choices of agents. It also bridges the gap between the CPR and the bankruptcy litterature by measuring the performance of traditionnal bankruptcy sharing rules in CPR management. We find the optimal sharing rule and the optimal diversification choices under different assumptions concerning the group of CPR users: heterogeneity in valuation of the resource (under risk neutrality) and heteorogeneity in risk tolerance (assuming equal valuation). The first-best sharing rule, leading to optimal diversification level and greatest efficiency, is a rule which is independant of users' claims to the CPR.*Preliminary draft. Please do not quote.*

Introduction

Common pool resources are most often managed through the –formal or informal - allocation of access rights or use licenses to each member of the group entitled to claim or request a share of the resource. Optimal allocation rules when the total availability of the resource is known with certainty impose that the marginal value of the resource to each user be equal. However, in many cases, CPR are natural resources and quantities available can vary stochastically due for example to climatic conditions. When the size of the CPR is uncertain, users are exposed to the risk of shortage. There is a need to establish a sharing rule, ie a rationing scheme defining the share of the total resource allocated to each user when the sum of claims is greater than availability. Users therefore face the risk to have access to a lower quantity of the CPR than they initially planned. When possible, they may want to diversify their risk, for example by reducing their dependence on the free but risky CPR and investing into a safe but costly alternative resource, playing the role of a self-insurance mechanism [4]. As the sharing rule determines the resource allocated to an agent in case of shortage, it may affect the diversification or self-insurance decisions of the agent. As a result, when designing management rules for the CPR, one must also include the joint question of the optimal investment in self-insurance at the society level and the design of the sharing rule.

CPR shortage is equivalent to a "bankruptcy" problem, occuring when there are overlapping claims over a resource. The canonical example of a bankruptcy is that of a bankrupt firm that is to be liquidated; namely, a situation in which the creditors 'entitlements exceed the worth of the firm. Another familiar example refers to the division of an estate among several heirs when the estate falls short of the deceased commitments. Water allocation and river sharing problems have also been shown to be equivalent to bankruptcy problems [1]. The bankruptcy literature provides well-behaved and acceptable sharing rules that are usefull to consider for CPR management [2, 6]. Surprinsingly, the CPR and the bankrupcty literature are rarely mobilized together.

Such situation of CPR shortage arises frequently in irrigated agriculture, when several farmers use the same water resource for irrigation. In France for example, farmers make decisions on their cropping patterns and production plans before knowing with certainty the availability of irrigation water for the coming season and without knowing either the water needs of other farmers. Each farmer makes a formal request for water use licenses to match his irrigation needs. Volumetric licenses are granted on an annual basis by the authority in charge of water management. These licenses are equivalent to an authorization to pump a given volume into the river, managed as a CPR whose size is uncertain. In times of shortage, when water flows are too low to fulfil licenses, water is shared through a system of temporary restriction roasters, managed and controlled by public authorities. But French farmers can also partially secure their access to irrigation water by investing into on farm storage[1], pumping equipment for groundwater, contracts with water companies guaranteeing pressurized water from dams etc. Such opportunities allow farmers to mitigate the risk of a shortage as they reduce the financial losses by increasing water quantities available in case of drought. When doing so, they agree to forego part of their claims on the uncertain river water. The deal with public authorities is the following: their investments in substitute resources, such as reservoirs filled in winter, are subsidized but they must hand back a share of their pumping licenses corresponding to the safe volume obtained. Clearly, by relying more on alternative resources, they relieve the collective pressure on the river and reduce the probability that total demand exceeds total availability, as well as the severity of shortage when it occurs. However, from a social welfare perspective, there is a need to find the right balance between economic losses due to the risk of water shortage and investment costs in alternative safe resources. This optimal "risk-taking level" will also depend both on the marginal value of water for each farmer but also on their preference relative to risks of water shortage. Water managers[2], in charge of the allocation of irrigation volumes at the scale of river basins, must address this crucial question when designing the rules for sharing water between farmers in times of drought.

We develop a model that allows us to study the interactions between the sharing rules of the common-pool water resource and farmers' water resource diversification strategies. In a more general setting, this paper addresses three issues which are only very partially developed by the existing literature on CPR: i) how does the sharing rule affect the CPR users' diversification or self-insurance strategies (their overall risk-taking level), ii) how does the sharing rule allocate risk across users? iii) can we identify a sharing rule

[1]French farmers show a growing interest in individual reservoirs in order to diversify their water resources. These reservoirs are called "*réserves de substitution*" or "*retenues collinaires*" in France. Some of these reservoirs are not individual but shared between a small number of neighbours. The reservoirs are filled during winter, when water is relatively abundant and when the reservoirs' filling activity does not compete with irrigation. This resource is perceived as safe by the farmers because the quantities are known in advance (farmers can observe the quantity stored at the end of winter) and administrative restrictions do not apply to this resource. Reservoir building is assumed to have no effect on the probability of water shortage occurrence [5].

[2]Called « organismes uniques » in France

leading to a Pareto optimum? Our paper thus contributes to the risk sharing literature by adressing the issue of optimal risk sharing and risk-taking simultaneously [3, 13]. It also contributes to the empirical literature about the specific issue of water sharing rules in times of shortage. Under the model assumptions on agents' preferences and resource valuations, we determine the socially optimal sharing rule and the optimal request to the CPR for each agent (the optimal investment in the alternative resource is the dual decision) under different assumption regarding agents' risk preferences and valuation of the resource. Then we compare the performance of different sharing rules, which are empirically relevant and described in the bankruptcy literature. The objective of the resource manager is to design a sharing rule such that the sum of private decisions of CPR users corresponds to the social optimum.

The paper is organized as follows. The first section outlines the main assumptions of the model. In section 2 we consider the case of risk neutral agents with heterogeneous resource valuations. In section 3 we include agents' risk preferences through a mean-variance model. Section 4 concludes and draws recommendations for resource management under uncertain resource size when a substitute resource is available.

1 The model

The model is intended to provide answers to two questions:

(Q1) What is the optimal sharing of the resource when the total request exceeds the CPR size and the optimal investment in an alternative and substitute resource?

(Q2) How well different sharing rules described in the bankruptcy literature perform compared to the optimal solution?

The first questions is related to the solution that would be chosen by a benevolent regulator, who can decide both on the sharing rule and on each individual's portfolio. The second question considers the more realistic issue where the regulator can only choose the sharing rule. He is therefore constrained by the agents' choice of their portfolio, i.e. a portofolio that maximizes their objective function. Different sharing rules are described in the bankrucptcy literature and have been extensively used as policy tools, including in the French water management context. The answers to the first question will provide a benchmark with respect to which we can evaluate the performance of various sharing rule that a regulator is already implementing or might want to implement in the future. The answer to the last question can help a practical implementation of a scarcity sharing rule, for example in the context of the water law reform that France is presently implementing.

1.1 Assumptions

We consider n identical agents who choose the quantity (H) of a single input (water) to maximize their profit. The unique profit maximizing quantity of input is equal for all agents and noted $\bar{H}$. $\bar{H}$ can be interpreted as the historical allocation of pumping rights into the river, satisfying the historical needs of water input. Since pumping rights have been largely over allocated, one knows that no more water will be made available to the irrigants: $\bar{H}$ is not only the profit maximizing quantity of resource but also the maximum quantity of water available to each water user.

Each agent can obtain the optimal quantity of input $\bar{H}$ by combining two possible sources: the CPR resource and a private rescource. While drawing on the private resource is secure, the claims on the CPR are not necessarily satisfied since it involves a risk of shortage. On the other hand, requests for the common pool are costless, while drawing on the secure resource entails a costs of $c > 0$ per unit. We assume that the two resources are perfect substitutes, allowing for any combination of amounts.The decision we focus on is the CPR quantity requested by an agent to the CPR: we denote R_i agent's i request. The individual request to the CPR is constrained by $0 \leq R_i \leq \bar{H}$: it cannot exceed the total resource needs $\bar{H}$. $\bar{H} - R_i$ is the complementary quantity requested from the secure resource.

Agents are exposed to a systemic risk on the size x of the CPR. To obtain explicit results, we assume that x follows a uniform distribution on the interval $[a; b]$ $(a \geq 0)$: $F(x) = \frac{x-a}{b-a}$ and $f(x) = \frac{1}{b-a}$[3]. The quantity received by agent i from the CPR Q_i depends on the size of the resource x and the total CPR quantity requested $R = \sum R_i$. If the size of the resource is larger than total request R, agent i gets R_i from the CPR. In the opposite case, a rationing scheme is implemented. The sharing rule determines the quantity $\theta_i(x)$[4]that agent i gets in case of restriction. As a result, $Q_i = R_i \; if \; R < x$ and $Q_i = \theta_i(x) \; if \; R > x$.

[3]Nevertheless, all the optimization results can be obtained in implicit form with any other distribution of x.

[4]The quantity received can also be a function of claims: $\theta_i(x, R_i, R_{-i})$

There are many possible ways to define the relation between θ_i and x. However two obvious restrictions are necessary : (C1) $\sum \theta_i = x$ and (C2) $0 \leq \theta_i \leq R_i$. According to restriction (C1) the sharing rule must exhaust the total quantity available x. It means that any rule should be defined such that the available resource x is always fully allocated when $R > x$. This is of course a condition for optimality as a share of the resource not allocated will not contribute to social welfare[5]. Restriction (C2) states that no agent can get a negative quantity nor get a share of the CPR that exceeds her request ($0 \leq \theta_i \leq R_i$). The sharing rules should thus always be specified to ensure this condition. Section 1.2 presents the different sharing rules mentionned in the bankruptcy literature we will consider.

The profit of agent Π_i is equal to the value of the units she gets from both resources minus the cost of the units obtained from the secure resource. For simplicity, we assume that the value of each unit of resource is constant and equal to υ_i. In section 2, we allow for heterogeneous valuations of the resource under the assumption of risk neutrality whereas in section 3, all agents have the same valuation of the resource $\upsilon = 1$.We consider that the valuation of the resource is always strictly higher than the cost of the alternative resource ($\upsilon_i > c$).

$$\Pi_i = \upsilon_i \left[(\bar{H} - R_i) + Q_i(x, R_i, R_{-i})\right] - c(\bar{H} - R_i) \tag{1}$$

1.2 Bankruptcy sharing rules

The bankruptcy literature provides three principal sharing rules[6]. In any bankruptcy situation, individuals first define their claims on a resource before knowing the size of the resource. The request to the CPR R_i of an agent is her “claim”. A “sharing rule” is a mechanism for solving a bankrupcty problem used if the sum of all claims exceeds the amount of available resource. The amont of available resource is called “estate”, the CPR users that have claims on the resource are the “creditors” and the share of the estate allocated to an agent is the “award”[2, 6].

[5] In the field, when water is not fully allocated to irrigators, the water remains into the river and contributes to “environmental flows”. This is of course not a pure loss for the society. In the model, we consider that x is the quantity of resource that has been allocated to agricultural users such that they can use all of it without compromising the needs of other users (like the environment).

[6] This problem has been adressed by Jewish scholars at least since the era of the collation of the Talmud [2, 10, 12]. Such rules have also been observed in different contexts for water management.

The best-known rule is the proportional rule, which recommends awards to be proportional to claims. In the case of shareholders of a firm, the idea behind the proportional rule is natural: each share is awarded equally. The idea of equality underlies another well-known rule: the constrained equal-awards rule. It makes awards as equal as possible to all creditors, subject to the condition that no creditor receives more than her claim. A dual formulation of equality, focusing on the losses creditors incur, as opposed to what they receive, underlies the constrained equal-losses rule. It proposes losses as equal as possible for all creditors, subject to the condition that no creditor ends up with a negative award. We slightly modify these rules such that they fullfill conditions (C1) and (C2) defined as the minimal requirements of an optimal rule: the rule should allocate all the estate and the award should be positive or nulle and lower than the claim.

(i) The *CA (constrained awards) rule* divides the estate among the agents independently from their claims. In case of shortage, an agent who requested R_i receives $\theta_i = \Omega_i x$ with $\sum_{i=1}^{n} \Omega_i = 1$. Under equality requirement, the Constrained Equal Awards rule (CEA) is such that $\Omega_i = \frac{1}{n}$. In the case of two agents, and in order to satisfy constraint (C1) and (C2), we rewrite the CA rule as follows: $\theta_i^{CA} = min\left[\Omega_i x; R_i\right]$ and $\theta_j^{CA} = min\left[x - \theta_i; R_j\right]$. It garantees that no agent receives more than her claim and all the resource x is allocated when $x < R$. For example, if agent i's claim is lower than $\Omega_i x$, the other agent will be allocated all the resource left $x - R_i$. This is illustrated in figure 1.

(ii) The *proportional rule* divides the amount of resource available proportionally to the agents' claims. In case of shortage an agent who requested R_i receives $\theta_i^P = x\frac{R_i}{R}$. This rule automatically satisfies constraint (C1) and (C2).

(iii) The *CL (constrained losses) rule* allocates the missing amount of resource. It divides equally the difference between the aggregate claim and the total amount available, provided no agent ends up with a negative transfer. In case of shortage an agent who requested R_i receives $\theta_i = R_i - \beta_i \left(R - x\right)$ with $\sum_{i=1}^{n} \beta_i = 1$. Under equality requirement, the constrained equal losses rule (CEL) is such that $\beta_i = \frac{1}{n}$. For two agents, in order to satisfy constraint (C1) and (C2), we rewrite the CA rule as follows: $\theta_i^{CL} = min\left[max\left[R_i - \beta_i\left(R - x\right); 0\right]; R_i\right]$ and $\theta_j^{CL} = min\left[x - \theta_i; R_j\right]$. It garantees that all the resource x is allocated when $x < R$ and the award is not negative.

We study the performance of these three rules in sharing the risk of resource shortage and induce agents to take the optimal diversification choices. The answer to these questions will be given under different assumptions concerning agents preferences. In section 2, we first solve the model assuming risk neutrality of the agents in order to isolate the effect of heterogeneous valuations of the resource. In section 3, we introduce a mean-variance objective function to observe the effect of heterogeneity in risk tolerance.

2 Risk neutral agents

A risk neutral agent chooses her portfolio of resource $\{R_i, \bar{H} - R_i\}$ such as to maximize her expected profit defined as follows.

$$E[\Pi]_i = (\upsilon_i - c)(\bar{H} - R_i) + \frac{\upsilon_i}{b-a}\left[\int_R^b R_i \, dx + \int_a^R \theta_i(x) \, dx\right] \tag{2}$$

For the regulator, as underlined in the introduction, the problem is twofold: the optimal sharing of the resource is jointly determined with the optimal request to the CPR. Formally, the regulator chooses both the vector of requests $\{R_i^*\}$ and the sharing rule $\{\theta_i^*\}$ in order to maximize the weighted sum of the expected profits of all group members, with λ_i the weight of agent i in the social welfare function. We consider a vast array of rules $\theta_i(x, R_i, R_{-i})$ where the individual quantities allocated to agent i in case of shortage may depend on the amount of resource available x and the vector of requests (R_i, R_{-i}). For simplicity of computation we focus on the case of two agents (i, j). The results can easily be extented to the case of n agents. The program of the regulator writes as follows:

$$\begin{aligned} \underset{\{R_i\},\{\theta_i(x,R_i,R_{-i})\}}{max\ SW} &= \sum_{i=1}^{2}\lambda_i\left[(\upsilon_i - c)(\bar{H} - R_i) + \frac{\upsilon_i}{b-a}\left[\int_R^b R_i \, dx + \int_a^R \theta_i(x, R_i, R_{-i}) \, dx\right]\right] \quad (3)\\ /c \quad (1) &\quad \sum \theta_i = x \\ (2) &\quad 0 \leq \theta_i \leq R_i \ \forall i \end{aligned}$$

We first solve the full optimization program of the regulator (optimal request and optimal sharing). Then, we compare the performance of the different bankruptcy sharing rules mentionned in section 1 with respect to the optimum.

2.1 Optimum

Proposition 1: When the social value of the resource used by i is greater than the social value of the resource used by j $(\lambda_i \upsilon_i > \lambda_j \upsilon_j)$[7]**, the optimal sharing rule is** :

$$\theta_i^*(x) = min\,[x\,;\ R_i] \tag{4}$$
$$\theta_j^*(x) = x - \theta_i^*(x)$$

The optimal requests to the CPR are:

$$R_i^* = max\left[min\left[a + c(b-a)\frac{\lambda_i - \lambda_j}{\lambda_i\upsilon_i - \lambda_j\upsilon_j}; \bar{H}\right]; 0\right]$$
$$R_j^* = max\left[min\left[\frac{c}{\upsilon_j}(b-a)\frac{\lambda_i\,(\upsilon_i - \upsilon_j)}{\lambda_i\upsilon_i - \lambda_j\upsilon_j}; \bar{H}\right]; 0\right]$$
$$R^* = min\left[a + \frac{c}{\upsilon_j}(b-a); 2\bar{H}\right] \tag{5}$$

Proof:

Under the constraint $\theta_i + \theta_j = x$, the social welfare function simplifies in:

$$SW = \lambda_i(\upsilon_i - c)(\bar{H} - R_i) + \lambda_j(\upsilon_j - c)(\bar{H} - R_j)$$
$$+ \frac{1}{b-a}\int_R^b [\lambda_i\upsilon_i R_i + \lambda_j\upsilon_j R_j]\ dx + \frac{1}{b-a}\int_a^R [(\lambda_i\upsilon_i - \lambda_j\upsilon_j)\theta_i(x, R_i, R_{-i}) + \lambda_j\upsilon_j x]\ dx$$

For $\lambda_i\upsilon_i - \lambda_j\upsilon_j > 0$, the social welfare is increasing in $\theta_i(x, R_i, R_{-i})$. Thus, $\theta_i(x, R_i, R_{-i})$ should be maximum.

From constraint (C2) and the fact that θ_i is used as a sharing rule only if $R_i + R_j > x$, we get the constraint (C3): $0 \leq x - R_j \leq \theta_i \leq R_i$. As we only have θ_i into the optimization problem, the constraint $0 \leq \theta_j \leq R_j$ disappears, and we need to consider constraint (C3).

[7] $\lambda_i\upsilon_i$ captures the social value of the resource when used by agent i: it equals to the private value υ_i multiplied by the weight of agent i λ_i in the social welfare function.

Thus, $\theta_i(x, R_i, R_{-i})$ should be maximum under constraint (C3).

$$\theta_i^*(x) = max\left[min\left[x; R_i\right]; max\left[x - R_j; 0\right]\right]$$

This optimal sharing rule defined in (4) verifies constraint (C2) and (C3) as illustrated in figure 1. For any $\{R_i^*\}$, $\theta_i^*(x) = min\left[x\, ;\, R_i\right] \leq x - R_j$. As a result, we can simplify the optimal sharing rule and write it as (4).

We verify that $\theta_j^*(x)$ verifies constraint (C2). Indeed, if $\theta_i^*(x) = min\left[x\, ;\, R_i\right] = x$, $\theta_j^*(x) = 0$ verifies constraint (C2). If $\theta_i^*(x) = min\left[x\, ;\, R_i\right] = R_i$, $\theta_j^*(x) = x - R_i$. It also verifies constraint (C2) as $x < R$ implies $x - R_i < R_j$ and $x > R_i$ implies $x - R_i > 0$.

Plugging the optimal rule into the social welfare function, the social welfare function writes as:

$$\begin{aligned} SW = \lambda_i & \left[(\upsilon_i - c)(\bar{H} - R_i) + \frac{\upsilon_i}{b-a}\left[\int_R^b R_i\, dx + \int_{R_i}^R R_i\, dx + \int_a^{R_i} x\, dx\right]\right] \\ + \lambda_j & \left[(\upsilon_j - c)(\bar{H} - R_j) + \frac{\upsilon_j}{b-a}\left[\int_R^b R_j\, dx + \int_{R_i}^R (x - R_i)\, dx + \int_a^{R_i} 0\, dx\right]\right] \end{aligned} \tag{6}$$

The FOC in (Ri,Rj) are:

$$\begin{aligned} \frac{\partial SW}{\partial R_i} &= \frac{\lambda_i c(b-a) + \lambda_i \upsilon_i (a - R_i) - \lambda_j \upsilon_j R_j}{b-a} = 0 \\ \frac{\partial SW}{\partial R_j} &= \frac{\lambda_j c(b-a) + \lambda_j \upsilon_j (a - R_i - R_j)}{b-a} = 0 \end{aligned}$$

We verify that R_i^* and R_j^* are maxima

$$\begin{aligned} \frac{\partial^2 SW}{\partial R_i^2}(R_i^*, R_j^*) &= -\tfrac{\lambda_i \upsilon_i}{b-a} && < 0 \\ \frac{\partial^2 SW}{\partial R_j^2}(R_i^*, R_j^*) &= -\tfrac{\lambda_j \upsilon_j}{b-a} && < 0 \\ \left(\frac{\partial^2 SW}{\partial R_i^2}\frac{\partial^2 SW}{\partial R_j^2} - \left(\frac{\partial^2 SW}{\partial R_i \partial R_j}\right)^2\right)(R_i^*, R_j^*) &= -\tfrac{\lambda_j \upsilon_j}{(b-a)^2}(\lambda_i \upsilon_i - \lambda_j \upsilon_j) && > 0 \end{aligned}$$

■

The optimal sharing rule is such that the agent with the higher social value of the resource gets all the resource, provided the constraint (C1) and (C2) are verified.

The optimal total request to the CPR is such that requesting more than R^* would be sub-optimal because the probability of restriction would be too high; requesting less than R^* would be sub-optimal because the alternative resource is costly. This optimal total request to the CPR is at least equal to the minimum quantity of CPR available (the lower bound of the interval a). It is increasing with the cost of the alternative resource c adjusted to the value of the resource for the lower value user υ_j. The lower-value user is determinant as the alternative resource would appear to him to be more costly. A decrease in the average quantity available (a decrease in a and/or b) leads to a lower optimal request to the CPR. Note that the optimal request is independant of the total demand $\bar{H}$, as long as $R_i^* < \bar{H}$. The optimal total request is also independant of the weights λ. The weights only determine how this optimal total request is shared between both agents. Whether the agent with the higher social value for the resource will request more or less than the other depends on their relative valuations.

Remark 1: When there is only one agent, the question of the optimal sharing of the resource is not relevant. The regulator maximizes:

$$SW = (\upsilon_i - c)(\bar{H} - R_i) + \frac{\upsilon_i}{b-a}\left[\int_{R_i}^{b} R_i \, dx + \int_{a}^{R_i} x \, dx\right]$$

We find that the optimal request is also equal to (5). This suggests that the sharing problem is independant from the question of the optimal request at optimum.

Remark 2: Intuitively, the sharing problem is an issue only if agents are heterogeneous. If agents are perfectly identical (equal valuation and equal weight), any sharing rule is optimal. The optimal total request to the CPR remains equal to (5).

Proof:

For $\upsilon_i = \upsilon_j = \upsilon$ and $\lambda_i = \lambda_j = \frac{1}{2}$, the social welfare function simplifies in:

$$SW = (\upsilon - c)(2\bar{H} - R) + \frac{\upsilon}{b-a}\left[\int_{R}^{b} R \, dx + \int_{a}^{R} x \, dx\right] \tag{7}$$

The social welfare function does not depend on $\{\theta_i\}$, so any sharing rule such that $\theta_i + \theta_j = x$, $0 \leq \theta_i \leq R_i$ is optimal.

The social welfare is only function of R in 7. The optimal request $R^*$is such that:

$$\frac{\partial SW}{\partial R} = c - \upsilon + \upsilon \frac{b-R}{b-a} = 0$$

The optimal total request is:

$$R^* = min\left[\frac{c}{\upsilon}(b-a) + a; 2\bar{H}\right].$$

We verify that R^* is a maximum $\frac{\partial^2 SW}{\partial R^2}(R^*) = -\frac{\upsilon}{b-a} < 0$

■

2.2 Performance of the sharing rules

Let us now assume that the regulator cannot decide the agent's diversification choices, but can only define the quantity of the resource allocated to each agent in case of shortage through the sharing rule. We are therefore interested in defining a sharing rule $\{\theta_i^*\}$ such that agents' diversification choices and requests to the CPR are optimal. Formally, the agents and the regulator play a Stackelberg game where the regulator is the leader and the agents the followers. The timing of the game is as follows:

step 1: The regulator announces the sharing rule.

step 2: Each agent chooses R_i such as to maximize her expected profit. Because the sharing rule introduces strategic interactions between agents, we calculate the best response of each agent and check that they constitute a Nash equilibrium.

Knowing the best response of the agents, the regulator should choose the sharing rule announced in step 1 such that the decentralized requests of agents chosen in step 2 coincide with the socially optimal request determined in (5) . This will ensure that the rule maximizes social welfare.

Proposition 2: The CA rule where the regulator chooses Ω_i favoring the agent with the highest social value for the resource ($\Omega_i = 1$ and $\Omega_j = 0$) maximizes social welfare (for $\lambda_i \upsilon_i > \lambda_j \upsilon_j$).

Indeed, the sharing rule $\theta_i^*(x) = min\left[x\, ;\, R_i\right]$ and $\theta_j^*(x) = x - \theta_i^*(x)$ is equivalent to the CA rule with such parameters. It is the only rule such that the sum of private request decisions corresponds to the optimal request to the CPR. As a result, this rule maximizes social welfare.

Proof:

We plug the different sharing rules defined in section 1 into the objective function of the agents and compute the nash equilibrium.

CA rule: $\theta_i = min\left[\Omega_i x; R_i\right]$ **and** $\theta_j = min\left[x - \theta_i; R_j\right]$

There are three possible cases to consider in order to satisfy constraint (C1) and (C2).

case 1: If $\frac{R_i}{\Omega_i} > R$ and $\frac{R_j}{(1-\Omega_i)} > R$:

$E\left[\Pi\right]_i = (\upsilon_i - c)(\bar{H} - R_i) + \frac{\upsilon_i}{b-a}\left[\int_R^b R_i\, dx + \int_a^R \Omega_i x\, dx\right]$

$E\left[\Pi\right]_j = (\upsilon_j - c)(\bar{H} - R_j) + \frac{\upsilon_j}{b-a}\left[\int_R^b R_j\, dx + \int_a^R (1-\Omega_i) x\, dx\right]$

case 2: If $\frac{R_i}{\Omega_i} < R$ and $\frac{R_i}{\Omega_i} < \frac{R_j}{(1-\Omega_i)}$:

$E\left[\Pi\right]_i = (\upsilon_i - c)(\bar{H} - R_i) + \frac{\upsilon_i}{b-a}\left[\int_R^b R_i\, dx + \int_{R_i/\Omega_i}^R R_i\, dx + \int_a^{R_i/\Omega_i} \Omega_i x\, dx\right]$

$E\left[\Pi\right]_i = (\upsilon_j - c)(\bar{H} - R_j) + \frac{\upsilon_j}{b-a}\left[\int_R^b R_i\, dx + \int_{R_i/\Omega_i}^R (x - R_i)\, dx + \int_a^{R_i/\Omega_i} (1-\Omega_i) x\, dx\right]$

case 3: If $\frac{R_j}{(1-\Omega_i)} < R$ and $\frac{R_i}{\Omega_i} > \frac{R_j}{(1-\Omega_i)}$:

$E\left[\Pi\right]_i = (\upsilon_i - c)(\bar{H} - R_i) + \frac{\upsilon_i}{b-a}\left[\int_R^b R_i\, dx + \int_{R_j/(1-\Omega_i)}^R x - R_j\, dx + \int_a^{R_j/(1-\Omega_i)} \Omega_i x\, dx\right]$

$E\left[\Pi\right]_i = (\upsilon_j - c)(\bar{H} - R_j) + \frac{\upsilon_j}{b-a}\left[\int_R^b R_i\, dx + \int_{R_j/(1-\Omega_i)}^R R_j\, dx + \int_a^{R_j/(1-\Omega_i)} (1-\Omega_i) x\, dx\right]$

We verify ex-post that the Nash equilibrium solution verifies the condition of case 2.

In that case, the Nash equilibrium requests are:

$$R_i = max\left[min\left[\Omega_i\left(a + \frac{c(b-a)}{\upsilon_i}\right); \bar{H}\right]; 0\right]$$
$$R_j = max\left[min\left[(1-\Omega_i)a + \frac{c(b-a)}{\upsilon_j} - \frac{c(b-a)}{\upsilon_i}\Omega_i; \bar{H}\right]; 0\right]$$

This corresponds to the optimal requests defined in (5) for $\lambda_i = 1$, $\lambda_j = 0$, $\Omega_i = 1$ and $\Omega_j = 0$. The total equilibrium request is : $max\left[min\left[a + \frac{c}{\upsilon_j}(b-a); 2\bar{H}\right]; 0\right]$, which corresponds exactly to the optimal total request. In other words, any CA rule ($\forall\Omega_i$) decentralizes any optimum ($\forall\lambda_i$) at the group level.

Proportional rule: $\theta_i = \frac{R_i}{R_i+R_j}x$

The objective function of the agents are:

$E\left[\Pi\right]_i = (\upsilon_i - c)(\bar{H} - R_i) + \frac{\upsilon_i}{b-a}\left[\int_R^b R_i\, dx + \int_a^R \frac{R_i}{R_i+R_j}x\, dx\right]$

$E\left[\Pi\right]_j = (\upsilon_j - c)(\bar{H} - R_j) + \frac{\upsilon_j}{b-a}\left[\int_R^b R_j\, dx + \int_a^R \frac{R_j}{R_i+R_j}x\, dx\right]$

As we have no explicit solution for the nash equilibrium requests Ri and Rj, we can't compare directly the nash equilibrium requests with the optimal requests. We verify that the FOC conditions do not cancel out for Ri=Ri* and Rj=Rj*.

CL: rule $\theta_i = min\left[R_i - \beta_i(R-x); 0\right]$ **and** $\theta_j = min\left[x - \theta_i; R_j\right]$

There are three possible cases to consider in order to satisfy constraint (C1) and (C2).

case 1: If $R_i - \beta_i(R-x)$ and $R_j - (1-\beta_i)(R-x)$ do not cancel out for $x\epsilon\left[a;b\right]$

$E\left[\Pi\right]_i = (\upsilon_i - c)(\bar{H} - R_i) + \frac{\upsilon_i}{b-a}\left[\int_R^b R_i\, dx + \int_a^R R_i - \beta_i(R-x)\, dx\right]$

$E\left[\Pi\right]_j = (\upsilon_j - c)(\bar{H} - R_j) + \frac{\upsilon_j}{b-a}\left[\int_R^b R_j\, dx + \int_a^R R_j - (1-\beta_i)(R-x)\, dx\right]$

case 2: If $R_i - \beta_i(R-x) = 0$ for $x\epsilon\left[a;b\right]$ and $R_j - (1-\beta_i)(R-x)$ do not cancel out for $x\epsilon\left[a;b\right]$

$E\left[\Pi\right]_i = (\upsilon_i - c)(\bar{H} - R_i) + \frac{\upsilon_i}{b-a}\left[\int_R^b R_i\, dx + \int_{R-R_i/\beta_i}^R R_i - \beta_i(R-x)_i\, dx + \int_a^{R-R_i/\beta_i} 0\, dx\right]$

$E\left[\Pi\right]_i = (\upsilon_j - c)(\bar{H} - R_j) + \frac{\upsilon_j}{b-a}\left[\int_R^b R_i\, dx + \int_{R-R_i/\beta_i}^R R_j - (1-\beta_i)(R-x)\, dx + \int_a^{R-R_i/\beta_i} x\, dx\right]$

case 3: If $R_j - (1-\beta_i)(R-x) = 0$ for $x\epsilon\left[a;b\right]$ and $R_i - \beta_i(R-x)$ do not cancel out for $x\epsilon\left[a;b\right]$

$E\left[\Pi\right]_i = (\upsilon_i - c)(\bar{H} - R_i) + \frac{\upsilon_i}{b-a}\left[\int_R^b R_i\, dx + \int_{R-R_j/(1-\beta_i)}^R R_i - \beta_i(R-x)_i\, dx + \int_a^{R-R_j/(1-\beta_i)} x\, dx\right]$

$E\left[\Pi\right]_i = (\upsilon_j - c)(\bar{H} - R_j) + \frac{\upsilon_j}{b-a}\left[\int_R^b R_i\, dx + \int_{R-R_j/(1-\beta_i)}^R R_j - (1-\beta_i)(R-x)\, dx + \int_a^{R-R_j/(1-\beta_i)} 0\, dx\right]$

We verify that the FOC conditions do not cancel out for Ri=Ri* and Rj=Rj*.

For the special case where $\upsilon_i = \upsilon_j$, we show that the FOC evaluated at Ri* and Rj* are nulle in case 3. We can verify that RiNash=Ri* and RjNash=Rj* are such that $R_j - (1-\beta_i)(R-x) = 0$ for $x\epsilon\left[a;b\right]$ and $R_i - \beta_i(R-x)$ do not cancel out for $x\epsilon\left[a;b\right]$. The conditions to be in case 3 are thus fullfiled. The CEL rule, $\forall\beta_i$, can lead to the optimal requests to the CPR when valuations are equal.

■

The CA depends on request only through the constraint (C2). This rule does thus not create any strategic interaction despite the presence of R_i in the definition of the rule.

The sum of the private decisions of the agents corresponds to the optimal request to the CPR. As a result, social welfare is maximized with the CA rule defined in proposition 2. On the contrary, the proportional and CL rules depend directly on request as the quantity of resource receives is a proportion of the request. These later rules fail to decentralize the optimum as they introduce strategic interactions between agents. They give incentives to the agent to request more from the CPR than optimum. The social welfare is lower with the proportional and CL rules than with the CA rule. In order to decentralize the optimum, the sharing rule should thus not depend on directly on requests.

In this second section, we have answered our three questions assuming agents are risk neutral and heterogeneous in their valuation of the resource:

(Q1) We determined the optimal request to the CPR. The optimal sharing rule is such that the agent i with the higher value for the resource ($\lambda_i \upsilon_i > \lambda_j \upsilon_j$) gets all the available resource, up to her request (the other agent will be allocated the resource left).

(Q2) The CA rule with $(\Omega_i; \Omega_j) = (1; 0)$ maximizes social welfare when the agents decides privately their request to the CPR knowing the sharing rule.

The next section aims at extending these results to another source of heteorogeneity: agents' risk tolerance.

3 Mean-Variance agents

In this section we extend the model to account for risk-aversion and heterogeneous risk preferences, by relying on the two-moment decision model first introduced by Markowitz[8]. In the two-moment decision model, agents rank choice possibilities according to their mean and their variance[8]. For instance, a risk-averse agent will accept an increase in the variance of his return only if he receives a compensation in terms of a higher mean return. The two-moment decision model is compatible with any von-Neumann Morgenstern utility function as long as the class of available choice options is

[8]In a mean-variance model, the variance is considered a good measure of the degree of riskiness. Of course, in many cases, the cost of risk will also depend upon the other moments of the distribution of wealth risk such as skewness (third moment) and kurtosis (fourth moment). We can assume that even if higher moments are potentially important for the irrigator, such a precise information on the distribution of the water volumes available is rarely available. As a result, the agent won't be able to take into account these dimensions when taking a decision under uncertainty.

restricted to distributions that differ from each other only by location and scale (Meyer condition) [11, 9]. However, for tractability and computation of explicit solutions, we shall adopt the standard linear specification $V(\mu,\sigma) = \alpha\mu - \beta\sigma^2$, where μ and σ^2 correspond to the mean and the variance respectively. In this specification β measures the agent's risk-aversion (or marginal utility for risk as measured by σ^2) and α measures his marginal utility for expected returns. The marginal rate of substitution $\frac{\alpha}{\beta}$ is therefore a constant with this specification.

We use a mean-variance objective function $\Phi\left[\Pi_i\right]$ defined in (8) : the first term is the expected profit and the second term capture the effect of the variance on the expected utility[9]. The individual parameter T_i measures the agent's risk tolerance, defined as the inverse of risk-aversion. The marginal utility of expected return is assumed equal to 1. As a result, T_i is also the constant marginal rate of subsitution between the mean and the variance.The higher T_i, the lower is the weight of the variance in the agent objective function and the lower is the cost of risk.

$$\Phi\left[\Pi_i\right] = E\left[\Pi_i\right] - \frac{1}{2T_i} Var\left[\Pi_i\right] \tag{8}$$

$$E\left[\Pi_i\right] = (1-c)(\bar{H} - R_i) + \frac{1}{b-a}\left[\int_R^b R_i\, dx + \int_a^R \theta_i(x)\, dx\right]$$

$$\begin{aligned} Var\left[\Pi_i\right] = {} & \frac{1}{b-a}\left[R_i^2(b-R) + \int_a^R \theta_i^2(x, R_i, R_j)\, dx\right] \\ & - \frac{1}{(b-a)^2}\left[R_i^2(b-R)^2 + 2R_i(b-R)\int_a^R \theta_i(x, R_i, R_j)\, dx + \left[\int_a^R \theta_i(x, R_i, R_j)\, dx\right]^2\right] \end{aligned}$$

The above specification has several advantages : firstly, risk-attitudes are captured by a single coefficient (T_i); secondly it takes into account the three components of the cost of risk: (i) the risk premium that agents incur due to the variability of the resource received from the CPR; (ii) the cost $c(\bar{H} - R_i)$ of relying on the secure resource; (iii) the opportunity cost of self insurance due to the substitution between the CPR and the

[9]Note that the risk born by an agent is endogeneous and different from the exogeneous systemic risk on the CPR resource size (given by the distribution of x). The risk born by agent i is given by the distribution of the profit function defined in (1). We approximate the risk by the variance of profits in this model.

alternative resource: requiring a unit from the secure resource decreases the share of the free CPR that one can receive. Last but not least, we get some computable analytical results for this model whereas a more general model is more difficult to solve[10] .

To keep things tractable, we assume equal valuation of the resource when agents are risk averse ($v_i = v_j = 1$). As a result, there is only one source of heterogeneity (the risk tolerance). Under risk neutrality, we have shown that the optimal sharing of the CPR in case of shortage is such that the agent with the higher value for the resource gets all of it. We can now study how heterogeneity in risk tolerance modifies the optimal risk sharing and risk taking problem. Does the optimal sharing rule depends on risk tolerance ? Are the optimal diversification choices modified ?

3.1 Optimum

As in the risk neutral case, the regulator chooses both the vector $\{R_i^*\}$ and the sharing rule $\{\theta_i^*\}$ in order to maximize social welfare. The social welfare is defined as a weighted sum of the objective function of the agents with λ_i the weight of agent i in the social welfare function. We solve the model for equal weights, for two agents.

The social welfare function writes as (9). The same constraints hold under risk aversion: (C1) $\theta_i + \theta_j = x$ and (C2) $0 \leq \theta_i \leq R_i$.

$$SW = \sum_{i=1}^{2} \left(E\left[\Pi_i\right] - \frac{1}{2T_i} Var\left[\Pi_i\right] \right) \tag{9}$$

Proposition 3: The optimal sharing rule is function of individual risk tolerance (for sharing rules of the form presented in 1.2).

$$\theta_i^{**} = \frac{T_i}{T_i + T_j} x \tag{10}$$

[10]"Many writers have made valuable contributions the problem of optimal risk decisions by emphasizing to analyses of means and variances. These writers have realized that the results can be only approximate, but have also realized that approximate but computable results are better than none" (Samuelson 1979). As mentionned by Liu (2004) [7], "the popularity of the mean–variance analysis is possibly not because of its precision of approximating the expected utility theory but because of its simplicity and the power of its implications".

The optimal individual requests are increasing with risk tolerance.

$$\begin{aligned} R_i^* &= min\left[\frac{T_i}{T_i+T_j}Z^* \ ; \ \bar{H}\right] \\ R_j^* &= min\left[\frac{T_j}{T_i+T_j}Z^* \ ; \ \bar{H}\right] \end{aligned} \tag{11}$$

with Z^* unique solution of $g(Z)$ in $[0;b]$ and

$$g(Z) = Z^3 - Z^2(b+2a) + Z\left(2(T_i+T_j)(a-b)+2ba+a^2\right) - a^2b + 2(T_i+T_j)(b-a)(a+c(b-a)) \tag{12}$$

Proof:

A general answer to the first question (optimum) cannot be given without specifying a functionnal form for the sharing rule. We consider three functionnal forms, inspired by the literature on bankruptcy and presented in 1.2. We first consider the array of rules $\theta_i(x)$ independant of the requests $\{R_i\}$ of the form $\theta_i = \Omega_i x$. We then consider rules that depend directly on $\{R_i\}$ in two different ways: $\theta_i(x, R_i, R_j) = \frac{R_i}{R}x$ and $\theta_i(x, R_i, R_j) = R_i - \beta_i(R - x)$. We find the optimal rules among theses classes of rules by optimizing on $\{\Omega_i\}$ and $\{\beta_i\}$. We verify ex-post that the optimal rules satisfies constraint (C1) and (C2).

We show that the optimal rules of the three form are equal for optimal request $(\theta_i(x, R_i^*, R_j^*))$. It is straightforward that the social welfare is equal for the three classes of rules considered here.

The optimal rule is of the form $\theta_i = \Omega_i x$

We solve the problem under the constraint $\theta_i = \Omega_i x$. The regulator chooses $\{\Omega_i^*\}$ and $\{R_i^*\}$ such as to maximize social welfare defined in (9) under the constraint (C2) and (C3).

The FOC of the problem are $\frac{\partial SW}{\partial R_i} = 0$ and $\frac{\partial SW}{\partial \Omega_i} = 0$ for i=i,j.

To solve the FOC we proceed as follows: From $\frac{\partial SW}{\partial \Omega_i} = 0$, we obtain Ω_i^{int} as a function of R_i and R_j. Replacing Ω_i^{int} in $\frac{\partial SW}{\partial R_i} - \frac{\partial SW}{\partial R_j} = 0$, we obtain three possible solutions: $R_i + R_j = a$, $R_i + R_j = b$, $R_i = \frac{T_i}{T_j}R_j$.

We can easily verify that $R_i + R_j = a$ does not verify the FOC $\frac{\partial SW}{\partial R_i} = 0$.

Replacing $R_i = \frac{T_i}{T_j} R_j$ in $\frac{\partial SW}{\partial R_i} = 0$, we obtain that the solution R_i that maximizes social welfare is given by (12). The factor depends on risk tolerance relative to the pair total risk tolerance $\left(\frac{T_i}{T_i+T_j}\right)$.

We can verify that the social welfare for this solution $SW \mid_{R_i = \frac{T_i}{T_j} R_j}$ is higher than $SW \mid_{R_i + R_j = b}$.

We then replace $\{R_i^*\}$ in Ω_i^{int} to find Ω_i^* defined in (10).

Furthermore, we verify ex-post that the constraint (C2) is verified: $\Omega_i^* x < R_i^*$ for all $x < R^*$ and for i and j. Indeed verifying if constraint (C2) for i and j is equivalent to verify constraint (C3) $x - R_j^* \leq \theta_i^* \leq R_i^*$.This is illustrated in figure 2.

The optimal rule is of the form $\theta_i(x, R_i, R_j) = \frac{R_i}{R} x$

We solve the same problem under the constraint $\theta_i = \frac{R_i}{R} x$. As above, the regulator chooses $\{R_i^*\}$ such as to maximize social welfare defined in (9). There is no parameter to optimize because there is a unique rule of the form $\theta_i(x, R_i, R_j) = \frac{R_i}{R} x$ such that $\theta_i + \theta_j = x$ and $0 \leq \theta_i \leq R_i$.

The FOC of the problem are $\frac{\partial SW}{\partial R_i} = 0$ for i=i,j. Solving $\frac{\partial SW}{\partial R_i} - \frac{\partial SW}{\partial R_j} = 0$, we obtain three possible solutions: $R_i + R_j = a$, $R_i + R_j = \frac{4}{3} b - \frac{1}{3} a$, $R_i = \frac{T_i}{T_j} R_j$.

We can easily verify that $R_i + R_j = a$ and $R_i + R_j = \frac{4}{3} b - \frac{1}{3} a$ does not verify the FOC $\frac{\partial SW}{\partial R_i} = 0$

Replacing $R_i = \frac{T_i}{T_j} R_j$ in $\frac{\partial SW}{\partial R_i} = 0$, we obtain that the solution R_i that maximizes social welfare is given by (12).

As a result, the solution $\{R_i^*\}$is the same under the two functional form described in 2.1.1 and 2.2.2.

If we replace $\{R_i^*\}$ in $\theta_i(x, R_i, R_j) = \frac{R_i}{R} x$, we find $\theta_i^{**} = \frac{T_i}{T_i + T_j} x$.

The optimal rule is of the form $\theta_i(x, R_i, R_j) = min\left[max\left[R_i - \beta_i\left(R - x\right); 0\right]; R_i\right]$

We solve the problem under the constraint $\theta_i = R_i - \beta_i\left(R - x\right)$, $\sum \beta_i = 1$.

The regulator chooses $\{\beta_i^*\}$ and $\{R_i^*\}$ such as to maximize social welfare defined in (9).

The FOC of the problem are $\frac{\partial SW}{\partial R_i} = 0$ and $\frac{\partial SW}{\partial \beta_i} = 0$ for i=i,j. From $\frac{\partial SW}{\partial \beta_i} = 0$, we obtain $\beta_i^* = \frac{T_i}{T_i + T_j}$.

Replacing β_i^* in $\frac{\partial SW}{\partial R_i} = 0$ we obtain $R^* = Z$ given by (12). For this rule individual requests are not determined. One can choose for example the repartition defined in (11).

We verify ex-post that condition (C1) and (C2) are verified: $0 < R_i^* - \beta_i^* (R^* - x) < R_i^*$ and $0 < R_j^* - (1 - \beta_i^*)(R^* - x) < R_j^*$ for $x \epsilon [a; R^*]$. As a result, $min\left[max\left[R_i^* - \beta_i^*(R^* - x); 0\right]; R_i^*\right] = R_i^* - \beta_i^*(R^* - x)$.

If we replace $\{R_i^*\}$ in $\theta_i^*(x, R_i, R_j) = R_i - \beta_i^*(R - x)$, we find $\theta_i^{**} = \frac{T_i}{T_i + T_j} x$.

■

Remark : We can verify that the total optimal request to the CPR (11) tend to the risk neutrality solutions when risk tolerances tends to infinite.

Proof:

$R_i^* = \frac{T_i}{T_i+T_j} Z^*$ where Z^* is given by the solution of $g(Z)$. Note that $Z^* = R_i^* + R_j^*$. The roots of this polynom in Z tend to the roots of the limit polynom $g^\infty(Z)$ given by

$$g^\infty(Z) = Z\left(2(T_i + T_j)(a - b)\right) + 2(T_i + T_j)(b - a)(a + c(b - a)).$$

The solution to $g^\infty(Z) = 0$ is equal to: $Z = a + c(b - a)$

Thus, $\lim_{T_i, T_j \to \infty} Z = a + c(b - a)$ and $\lim_{T_i, T_j \to \infty} R^* = \frac{T_i}{T_i+T_j}(a + c(b - a)) + \frac{T_j}{T_i+T_j}(a + c(b - a)) = a + c(b - a)$. This is the optimal total request under risk neutrality with $\upsilon_i = \upsilon_j = 1$.

■

3.2 Performance of the optimal sharing rules

As in the risk neutral case, we verify which rules the regulator can use in order to maximize social welfare. The best rule is such that the decentralized nash equilibrium requests of agents chosen coincide with the socially optimal request determined in(11)
.

Proposition 4: The CA sharing rule where resource is shared according to risk tolerance ($\theta_i^{**} = \frac{T_i}{T_i+T_j} x$) maximizes social welfare.

The only rule such that the sum of private request decisions corresponds to the optimal request depends on risk tolerance. As a result, this rule maximizes social welfare. Any sharing rule that depends on request to the CPR leads to suboptimal request to the CPR because it creates stategic interactions. This result is simular to proposition 2.

Proof:

We plug the different sharing rules defined in section 1 into the objective function of the agents and compute the nash equilibrium.

CA rule: $\theta_i = min\,[\Omega_i x; R_i]$ **and** $\theta_j = min\,[x - \theta_i; R_j]$

As for proposition 2, we consider three possible cases in order to satisfy constraint (C1) and (C2) (see proof of proposition 2 for details of the different cases). We find that the FOC for Nash equilibrium evaluated at the optimal solutions Ri* and Rj* cancel out for $\Omega_i = \Omega_i^*$. In other words, the CA rule with $\Omega_i = \Omega_i^*$ is the only rule that decentralizes the optimum in the class of CA rules. We verify ex-post that the Nash equilibrium solution verifies the condition $R = \frac{R_i}{\Omega_i} = \frac{R_j}{1-\Omega_i}$ as Rnash=R*, Rinash=Ri* and Rjnash=Rj*. As a result, the first case only is relevant.

Proportional rule: $\theta_i = \frac{R_i}{R_i+R_j}x$

As we have no explicit solution for the nash equilibrium requests Ri and Rj, we can't compare directly the nash equilibrium requests with the optimal requests. We verify that the FOC conditions do not cancel out for Ri=Ri* and Rj=Rj*.

CL: rule $\theta_i = min\,[R_i - \beta_i(R - x); 0]$ **and** $\theta_j = min\,[x - \theta_i; R_j]$

As for proposition 2, there are three possible cases to consider in order to satisfy constraint (C1) and (C2) (see proof of proposition 2 for the details of the cases). We verify numerically that the FOC conditions do not cancel out for Ri=Ri* and Rj=Rj* in the three cases.

■

In this third section, we have answered our three questions assuming agents have mean-variance preferences and have the same social value for the resource ($\lambda_i \upsilon_i = \lambda_j \upsilon_j = 1$):

(Q1) We determined the optimal request to the CPR. The optimal sharing rule depends on individual risk tolerance relatively to total risk tolerance.

(Q2) The CA rule with $\Omega_i = \frac{T_i}{T_i+T_j}$ maximizes social welfare when the agents decides privately their request to the CPR knowing the sharing rule.

4 Conclusion and policy recommendations

In this paper, we have studied the relative efficiency of different sharing rules in order to minimize the social cost of a CPR shortage, when vulnerability to the shortage or bankruptcy risk is endogeneous: the larger their requests to the CPR, the higher the probability of shortage and the larger the extent of the shortage. The innovation of the paper is to determine simultaneously the optimal sharing of the shortage risk and the optimal diversification choices of the CPR users. The social cost of resource shortage is minimized if the risk is shared optimally between agents and if agents make collectively optimal diversification choices. We determine the optimal request to the common-pool resource and the dual decision: the optimal investment in the secure resource. Investing too much in the safe resource is suboptimal as it has a cost. On the opposite, investing two few is also suboptimal as the occurence and severity of shortage become too high. When agents are only heterogeneous in their valuation of the resource (they are all risk neutral), the optimal rule is such that the agent with the higher value for the resource gets all the resource available in case of shortage. In the mean-variance model with agents heterogeneous in their risk tolerance, the optimal sharing rule depends on individual risk tolerance relative to the total risk tolerance.

We have compared the performance of different bankruptcy rules in decentralizing the optimal diversification choices: the constrained awards rule, the proportional rule and the constrained losses rule. We have shown that the only rule that decentralize the optimum does not depend directly on the requests to the CPR. Any rule sharing the resource according to the individual requests introduces strategic interactions between agents and fail to incite the agents to diversify optimally their resource portfolio. The constrained awards rule is the rule to be favored by the water manager. The parameter of the rule have to be adjusted according to valuation of the resource or risk tolerance.

The practical conclusion of this work for water management is that a water allocation rule in case of restrictions defined as a proportion of river pumping rights or licences is not efficient when access to an alternative resource is possible. However, this rule is much in used in many countries. For example in some regions in Spain, volumes distributed are proportions of the subscribed quotas. When the sharing rule impacts the diversification choices of the water-users, the regulator should rather defined the restriction rule independantly from the claims (in proportion of valuation of the resource or risk tolerance). Of course, the claims of the users in the risky water resource is an indication of their risk tolerance. A more risk tolerant agent will choose to rely more on river flows and will not invest as much in an individual reservoir. Nevertheless, the water manager should announce clearly that the restrictions will not be organized according to the irrigators' claims on the resource.

We leave for future research the generalization of these results in the expected utility framework. This could enable to extend the optimal risk-sharing results of Borch (1962) and Wilson (1968) to the situation where risk taking and risk sharing should be jointly determined. Further work could also study how to design a contract such that agents reveal thruthfully their risk tolerance to the regulator. Such information is indeed required to implement the optimal rule under risk aversion.

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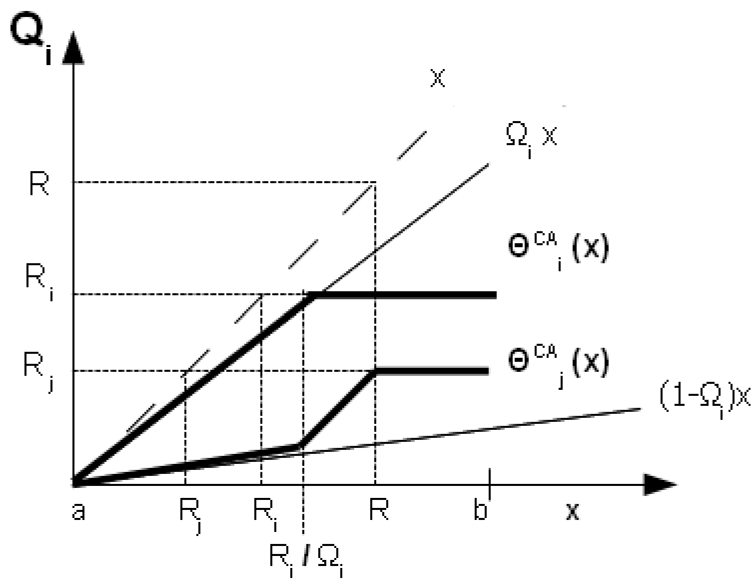

Figure 1: CA rule when $\frac{R_i}{\Omega_i} < R$(case 3)

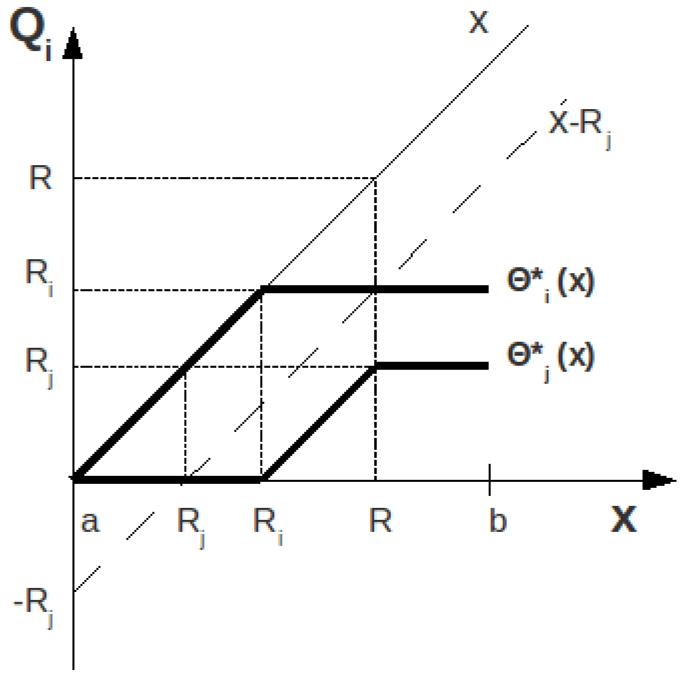

Figure 2: Optimal sharing rule under constraint (C2) and (C3) under risk neutrality

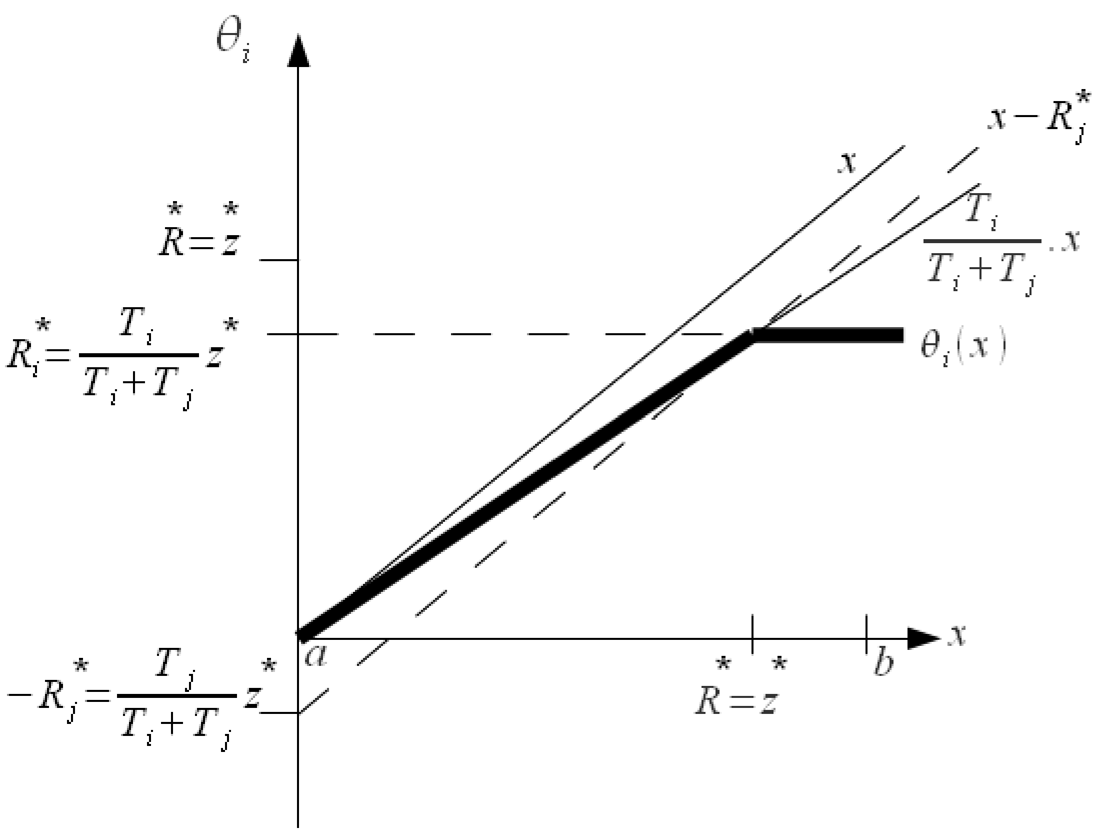

Figure 3: Optimal sharing rule of the CA form verifies constraint (C2) and (C3) under risk aversion