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# Approximation of epidemic models by diffusion processes and statistical applications

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## Abstract

Amongst various mathematical frameworks, multidimensional continuous-time Markov jump processes  $(Z_t)$  on  $N^d$  form a natural set-up for modeling SIR-like epidemics. We first extend the results of Ethier and Kurtz (2005) on the approximation of density dependent processes by diffusion and Gaussian processes to general density time-dependent Markov processes on  $N^d$ . The normalization by the population size  $N$  of  $(Z_t)$  leads, as  $N \rightarrow \infty$ , to the solution of an ODE system. Recentering  $(Z_t)$  yields a multidimensional Gaussian process. Another approximation leads to a time-dependent diffusion process with small diffusion coefficient ( $\frac{1}{\sqrt{N}}$ ), close to the previous Gaussian process. For inference, we extend the results of Guy *et al.* (2012) on discretely observed diffusion processes with small diffusion coefficient to time-dependent diffusions. Consistent and asymptotically Gaussian estimates are obtained by means of a contrast process in two different asymptotics: fixed number  $n$  of observations (or equivalently  $\Delta$ ) and  $N \rightarrow \infty$ ;  $n \rightarrow \infty$  and  $N \rightarrow \infty$  simultaneously. In the context of epidemics,  $n$  is usually fixed. Adding a correction term, we obtain better estimates non asymptotically (Guy *et al.* 2013). Finally, we simulated two Markov jump processes modeling *SIR*, *SIRS* epidemics in two different epidemic contexts (single and recurrent outbreaks) with different characteristics (variation of parameters  $R_0$ ,  $N$ ,  $n$ ). We obtain that the estimators have good asymptotic properties and behave well for realistic numbers of observations and population sizes. Contrary to the majority of current inference techniques for partially observed processes which demand computer intensive simulations, our method, being mostly analytical, only requires the classical optimization steps.

Ethier SN, Kurtz TG. (2005) Markov processes: characterization and convergence. *Wiley Series in Probability and Statistics*, Wiley.

Guy R., Laredo C., Vergu E. (2012) Parametric inference for discretely observed multidimensionnal diffusions with small diffusion coefficient. *ArXiv: 1206.0916*

Guy R., Laredo C., Vergu E. (2013) Statistical inference for epidemic models approximated by diffusion processes. *To be submitted*