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Dynamic management of water transfer between two interconnected river basins*

Francisco Cabo† Katrin Erdlenbruch‡ Mabel Tidball§

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Abstract

This paper analyzes the dynamic interaction between two regions with interconnected river basins. Precipitation is higher in one river- basin while water productivity is higher in the other. Water transfer increases productivity in the recipient basin, but may cause environmental damage in the donor basin. The recipient faces a trade-off between paying the price of the water transfer, or investing in alternative water supplies to achive a higher usable water capacity. We analyze the design of this transfer using a dynamic modeling approach, and compare solutions with different information structure with the cooperative solution. Contrary to standard games, where decision variables differ among players, we assume that both players take the decisions concerning the water transfer. The equilibrium between supply and demand determines the optimal transfer price and amount. If the problem were set as a static game, the non-cooperative solution would match the cooperative solution. However, in a more realistic dynamic setting, in which the recipient uses a feedback information structure, the cooperative solution will not emerge as the equilibrium solution. The transfer amount is lower than in the case of cooperation, while the investment in usable water capacity is higher. Finally we numerically compare our results to the Tagus-Segura water transfer described in Ballestero (2004).

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1 Introduction

This paper analyzes the interaction between two neighboring regions with differt water inflows and water productivity. If the river- basin in one region receives more rainfall while the water productivity is higher in the other (for example: higher fertility of the irrigated soil, or demand for water for highly productive activities like tourism), then the existence of a transfer infrastructure that enables the transfer of water from the former to the latter would increase overall productivity (see for example Dinar and Wolf 1994).[1] Independently of whether the two river-basins are located in the same or in two adjacent countries, using such an aqueduct to transport the water from the donor to the recipient would help increase efficiency, and can therefore be regarded as a good decision (by a central government) or agreement (between the two parties), from an economic point of view.

Because water inflows in the donor basin are large, after covering the demands from households and industrial activity in this region, there will be a water surplus. As long as the water transfer does not exceed the water surplus, it will not harm the donor's economy. Nevertheless, the water transfer reduces the water level in the donor's river- basin, causing environmental degradation, and hence reducing the welfare of the donor. The water transfer improves productivity in the recipient basin but also increases the environmental constraint in the donor basin (for environmental effects of water transfers, see, for example Kumar (2006)). A transfer payment must thus be set up to compensate the donor for forgone benefits from holding on to the water resource.

The payment of the water transfer can be settled by a central planner (who has to determine how to share the gains from cooperation), or through a bargaining process between the two regions in a water market. Considering that the water transfer is freely agreed between the two regions, the transfer price will be determined in a demand-supply setting in a market (see for example Ballestero (2004)). Here, we consider a bilateral monopoly with a single water seller and a single water buyer, like in Lekakis (1998).

Addressing water scarcity through inter-basin water transfer is just one possibility for the recipient. Water- savings, water recycling, or the production of fresh water by desalination plants are alternative ways to increase the supply of usable water that should be considered

[1] Water trade between two regions is mutually beneficial when one region is characterized by a relatively less binding water constraint and the other with a relatively efficient water-use technology, see [7].

when long-term decisions concerning water supply are taken.[2] Each of these three alternatives (water savings, water recycling, water production) requires investment in infrastructure, which gives the problem a dynamic dimension: an intertemporal trade-off between current water transfer payments and investments to increase the usable water capacity.

The water transfer from the Tagus basin in the center of Spain to the Segura basin in south-eastern Spain is a good illustration. While the Segura basin is an arid area with low precipitation and high evapotranspiration, the Tagus basin is more humid. The productivity of water also differs between the two regions. If irrigated, the fertility of the soil in south-eastern Spain is very high. In addition, the region receives many tourists thus increasing the demand for water. The imbalance between water productivity and rainfall led the Spanish central government to built a 230 km network of canals, aqueducts and tunnels to transfer water from the Tagus to the Segura basin. As a result of the transfer, the water quality in the Tagus basin has deteriorated, leading the regional government in the Tagus basin to complain about the transfer, triggering a national political debate on alternatives like desalination plants. Our model considers these three main features: the use of an existing aqueduct, environmental degradation in the donor basin, and the possibility of investing in the equipment required for alternative sources of water supply.

Other examples of inter-basin water transfer already in operation or in the planning stage can be found in the literature (see, for example, Bhaduri and Barbier (2008)). Transfers can be set up within the same country (e.g. the Snowy River Scheme in Australia, the São Francisco Interlinking Project in Brazil, the Olmos Transfer Project in Peru, the South-North Water Transfer Project in China, or the Archeelos Diversion in Greece), or between countries (e.g. from the Kosi in Nepal to the Ganges in India and Bangladesh, or the Lesotho Highlands Water Project between Lesotho and South Africa, to mention just a few).

Most of the literature refers to water- share by two neighboring countries: the country located upstream and the country located downstream. For an overview of these studies see Ambec and Ehlers (2008), Bhaduri and Barbier (2008), Ambec and Sprumont (2002), or Kilgour and Dinar (1995, 2001). These models seek efficient water sharing agreements that allow the downstream country to compensate the upstream country for using more water. An inter-basin water transfer system shares the price charged by the donor for the water transferred to the

[2] In a case study in south-eastern Spain Bravo et al. (2010) affirm that the best supply oriented policy is a combination of inter-basin water transfer and desalination.

recipient with these models. However, in our model, the environmental problems caused by the water transfer between two river-basins differ from those linked to the management of a single river. With an inter-basin water transfer, the two regions face very different environmental constraints. In particular, the transfer only decreases the water level in the donor river, which faces the highest environmental restriction.[3]

Moreover, in our model, the recipient is able to invest in alternative water supplies, which gives the model a dynamic dimension. The dynamic interaction between donor and recipient is analyzed as a deterministic[4] differential game with an infinite time horizon. Contrary to the standard formulation, with player-specific decision variables, here the two regions decide on the amount of water to be transferred. The donor (resp. the recipient) decides on the supply (resp. the demand) of water transferred as a function of its price. The equilibrium of this bilateral monopoly will determine the price and quantity of the transfer. Another characteristic of our model is the coexistence of a static donor and a dynamic recipient who faces an intertemporal dilemma between paying the price of the water transfer or investing in improving the usable water capacity of his/her region. For such a game, we prove that the supply (as a function of price) and demand (as a function of usable water and price) for water transfer do not differ when we change from open-loop players, who commit to a certain path of actions at the start of the game, to more realistic feedback players who can constantly adjust their behavior to the available information concerning the capacity of usable water. However, a feedback recipient is aware of the negative relation between usable water capacity and the price of the transfer. This gives the recipient market power, he/she invests more in water infrastructure, and hence demands less water from the donor and pays a lower price for it. In either case, with or without commitment, the existence of an aqueduct to transport water from donor to recipient at a market price makes both players better off.

A static analysis would ignore the recipient's dynamic dilemma faced with having to choose between current water transfer payments or investments in water infrastructure. In this setting, the decentralized and the cooperative solutions coincide. But this result is not necessarily valid in a dynamic setting. It is true that the open-loop solution in the non-cooperative dynamic

[3]Here, we ignore the environmental damage that the water transfer could cause to the recipient region. Such effects are reported by Cox (1999), Elvira and Almodóvar (2001), or Gupta and Zaag (2008) and references therein.

[4]To address this problem, we do not focus on inter-annual and intra-annual variations in hydrological resources, which can be modeled with stochastic models (see Ballestero (2004)).

game is also Pareto efficient. However, without an institution to enforce the commitment, a feedback recipient invests more in usable water capacity, demands less water and pays a lower price than is socially optimal. Thus, in a dynamic setting, concluding on Pareto efficiency would be misleading.

For the case study of the Tagus-Segura transfer, Ballestero (2004) determines the quantity and price of the water transfer by simulating the stochastic supply and demand curves in a static approach. After calibrating the model, we compare our results with those of Ballestero. Having the opportunity to invest in usable water capacity reduces the recipient's dependence on water transfer and, hence, less water is transferred at a lower price. This effect is stronger under feedback strategies, when players do not commit to follow open-loop strategies at the beginning of the game.

The paper is organized as follows. In Section 2, we present the welfare functions for the donor and the recipient as well as the dynamics of the capacity of usable water. In Section 3, we solve the differential game under open-loop and feedback information structures. We then compare the results for the decentralized solutions with or without commitment. In Section 4, we prove that the two players are better off when they can bargain on the price and quantity of the water transfer than under the case of no transfer. In Section 5, we describe the cooperative solution. In section 6, we prove that although the static non-cooperative solution is Pareto efficient, this is not the case for the dynamic non-cooperative solution under feedback strategies. In Section 7, we compare our results with those in Ballestero (2004) using numerical simulations. Finally, in Section 8 we present our conclusions.

2 The model

In this section, we describe the dynamic interaction between a region with higher precipitation (donor) and a region with higher water productivity (recipient).

2.1 The Donor

The river- basin in the donor region is characterized by relatively high precipitation rates and relatively low productive uses, which creates the conditions for the existence of a water surplus. An inherent characteristic of water inflows and outflows is their randomness. The intensity and the duration of rainfall events or of snow melt are irregularly distributed over the year. Likewise, water needs for irrigation or household water use vary during the year and even at

times of day. Nevertheless, developed countries have managed to reduce the impact of periods of drought by building dams in the head of the rivers, which help to maintain the water level of the river stable. Nearly all interbasin water transfer projects include such infrastructure (UNESCO (1999)) that allows the river authority to release more water when the demand is higher and inflows are lower (and vice versa). Here we make the extreme assumption that, with no water transfer, the water surplus, or the water level in the river after covering demands, remains constant over time. Hence, we define R as the maximum possible water surplus in the river without the risk of exhausting the reservoir.

The water stored in the dam can be released into the local river, or transferred through the aqueduct to the recipient. Let us denote, $\tau(t)$ this transfer. Henceforth, the water transfer reduces the surplus or the water level in the river. Because the water demand is already covered, the donor's agriculture, industry or households are not directly affected by the transfer. However, the reduction in the water level in the river reduces the quality of the water, (for example the Tagus basin in Spain collects water from the city of Madrid which means it is important to keep minimum water levels high enough to maintain the dilution capacity of the river). In the following, we assume that the associated environmental problem grows more than linearly[5] with the amount of water transferred. The environmental amenities or environmental services based on the water level in the river can be represented by:

$$E(\tau(t)) = c\left(R - \frac{1}{2}\frac{\tau(t)^2}{R}\right), \qquad R, c > 0.$$

Note that the effect of the water surplus is twofold. Firstly, in the absence of water transfer, environmental amenities increase linearly with the water surplus in the river: cR. Secondly, for increments in the water transfer, the marginal reduction in environmental amenities is inversely proportional to this surplus, (while proportional to the share of water transferred): $c\tau(t)/R$. In the following, we assume that environmental amenities and damage are measured in monetary terms.

The donor receives a monetary payment, $p(t)$, from the recipient for each unit of water transferred. The instantaneous welfare function for the donor is then expressed as:

$$F^d(p(t), \tau(t)) = E(\tau(t)) + p(t)\tau(t) = c\left(R - \frac{1}{2}\frac{\tau(t)^2}{R}\right) + p(t)\tau(t). \tag{1}$$

[5]In fact, although not included in the model, an excessively low level might be an irreversible catastrophe (see for example Tsur and Zemel (1995), (2004)).

2.2 The Recipient

If precipitation is low in the river- basin in the recipient region, but water productivity is high, then this region will be willing to pay for the water transferred from the donor. Productive activities in this region (e.g. agriculture or tourism) also depend on other inputs such as labor or capital. Nevertheless, to focus on the effect of available water on production activities, we consider them as fixed inputs. There are two different quantities of water used in the recipient region: a fixed amount available in the recipient's own river- basin, and a variable quantity transferred by the donor, $\tau(t)$. Additionally, the recipient has an alternative way of increasing the volume of available water. He/she could invest in the equipment required to either save water by reducing water use or water leakage from the distribution network, or by increasing recycling (use of gray water), or by producing water (for example by desalination plants, Merett (1997)). We combine these three possibilities in a single variable, $x(t)$, defined as the capacity to produce, recycle or save water using existing equipment (in the following usable water capacity). It is measured in cubic meters. Capacity increases with new investments and decreases with depreciation:

$$\dot{x}(t) = s(t) - \delta x(t), \qquad x(0) = x_0 \geq 0, \tag{2}$$

where $\delta > 0$ is the depreciation rate and $s(t)$ is the investment to replenish and further increase current capacity, i.e. the cubic meters additionally saved, produced or recycled above the current capacity.

In the recipient region, welfare comes from the amount of available water: either water transferred, $\tau(t)$, or the region's usable water capacity, $x(t)$.[6] Welfare increases with the amount of available water at a decreasing rate. In addition, investments in new capacity are increasingly costly, so we consider quadratic investments costs. This can reflect both the existence of increasing transaction costs and the incremental cost of successive projects to produce, save or recycle water. Finally, instantaneous welfare decreases with transfer payments made to the donor. Hence, the welfare function of the recipient is expressed by:

$$\begin{aligned} &F^r(p(t), \tau(t), x(t), s(t)) = Q(\tau(t), x(t)) - p(t)\tau(t) - C(s(t)) \\ &= d\left(\tau(t) + x(t) - \alpha\frac{(\tau(t) + x(t))^2}{2}\right) - p(t)\tau(t) - \beta\frac{s(t)^2}{2}, \end{aligned} \tag{3}$$

with $d, \alpha, \beta > 0$.

[6] Here we disregard the water from the recipient's own river- basin (assumed constant) and focus on these two alternative sources of water.

3 The Nash game

In this section, we present the dynamic interaction between the donor and recipient regions. The amount and the price of the water transfer are determined from the supply and demand decisions taken by the donor and the recipient respectively. The donor determines the supply of water in order to maximize the stream of welfare discounted at a constant rate, ρ, with an infinite time horizon:[7]

$$\max_{\tau} \int_0^{\infty} \left[c\left(R - \frac{\tau^2}{2R}\right) + p\tau \right] e^{-\rho t} dt. \tag{4}$$

Correspondingly, the recipient must decide on the demand for water from the donor and on the investment in usable water capacity, to maximize discounted welfare:

$$\max_{\tau,s} \int_0^{\infty} \left[d\left(\tau + x - \alpha\frac{(\tau+x)^2}{2}\right) - p\tau - \beta\frac{s}{2} \right] e^{-\rho t} dt. \tag{5}$$

The recipient is a farsighted player whose maximization problem is subject to the evolution of usable water capacity in (2). By contrast, the donor behaves as a static or myopic player[8] because the amount and the price of water transfer has no effect on the dynamics of usable water capacity, and because neither the stock nor the investments in usable water capacity influence the donor's welfare.

In contrast to standard differential games, where control variables differ among players, here we consider that both players decide on the amount of water to be transferred from the donor to the recipient. Thus, optimality conditions for problems (4) and (5) give the supply and the demand of water as a function of its price:

$$\tau^S(p) = \frac{pR}{c}, \qquad \tau^D(p,x) = \frac{d-p}{d\alpha} - x. \tag{6}$$

The amount and the price of the water transfer at equilibrium is obtained by equating supply and demand, $\tau^S(p) = \tau^D(p,x)$. This we denote a Nash equilibrium.

$$\left(p^N(x), \tau^N(x)\right) = \left(\frac{cd}{Y+c}(1-\alpha x), \frac{dR}{Y+c}(1-\alpha x)\right), \tag{7}$$

where $Y = Rd\alpha$. From this definition, it immediately follows that the water transfer would only take place if the usable water capacity remains below $1/\alpha$. From (6) it follows that $1/\alpha$ would

[7] Here and henceforth time argument is omitted when no confusion can arise.

[8] Problem (4) could be written as:

$$\max_{\tau} \left[c\left(R - \frac{\tau^2}{2R}\right) + p\tau \right].$$

be the demand for water transfer at a zero price and for zero usable water capacity. Hence, this amount can be understood as the maximum water requirements of the recipient.

The solution to this problem depends on the information structure of the players. We thus distinguish between the open-loop and the feedback or Markov perfect equilibria. An open-loop information structure assumes that the players only have the knowledge concerning the initial state of the game, x_0, and commit to follow an optimal path from this point on. Conversely, in a Markov perfect solution, players adapt their decisions by taking into account the state of the game at each point in time, $x(t)$. In the present paper, the donor is a myopic agent, and hence, his/her optimal decision is always given by $\tau^S(p)$ in (6). Conversely, the farsighted recipient would act differently in the two solution concepts considered.

3.1 Open-loop Nash equilibrium

In this section, we compute the commitment solution for the game described by the optimization problems in (4) and (5), subject to the dynamics of usable water capacity in (2) and the equilibrium condition $\tau^S(p) = \tau^D(p, x)$. The optimal open-loop solution is described by the Nash equilibrium price and quantity in (7), and the optimal investment in usable water capacity is given by:

$$s^{\text{OL}}\left(\lambda_r^{\text{OL}}\right) = \frac{\lambda_r^{\text{OL}}}{\beta}, \tag{8}$$

with λ_r^{OL} the costate variable for the recipient associated with x.

Further, the system dynamics is described by:

$$\dot{x} = \frac{\lambda_r^{\text{OL}}}{\beta} - \delta x, \tag{9}$$

$$\dot{\lambda}_r^{\text{OL}} = (\rho + \delta)\lambda_r^{\text{OL}} - \frac{cd}{c+Y}(1 - \alpha x). \tag{10}$$

The optimal time paths for the usable water capacity and its shadow price are:

$$x^{\text{OL}}(t) = (x_0 - \bar{x}^{\text{OL}})e^{\phi^{\text{OL}}t} + \bar{x}^{\text{OL}}, \tag{11}$$

$$\lambda_r^{\text{OL}}(t) = \beta\left[(\phi^{\text{OL}} + \delta)x^{\text{OL}}(t) - \phi^{\text{OL}}\bar{x}^{\text{OL}}\right], \tag{12}$$

with

$$\bar{x}^{\text{OL}} = \frac{dc}{(c+Y)\Lambda + dc\alpha} < \frac{1}{\alpha}, \tag{13}$$

$$\phi^{\text{OL}} = \frac{1}{2\beta}\left[\rho\beta - \sqrt{\Delta^{\text{OL}}}\right] < -\delta, \quad \text{with} \quad \Delta^{\text{OL}} = (\rho + 2\delta)^2\beta^2 + 4\frac{dc\alpha\beta}{(c+Y)}. \tag{14}$$

and $\Lambda = \beta\delta(\rho + \delta)$.

Here and henceforth, we assume that the initial stock of the usable water capacity, x_0, is lower than its long-run value (for instance, it might initially be zero). Then, since the capacity of usable water at the steady state is lower than $1/\alpha$, we can guarantee that this capacity always remains below $1/\alpha$. Consequently, a positive amount of water is transferred at a positive price.

Given the optimal time paths for the state and the costate variables, the time paths for τ, p and s can immediately be obtained from (7) and (8). Their steady-state values are given by:

$$\bar{p}^{\mathrm{OL}} = \frac{cd\Lambda}{(c+Y)\Lambda + d\alpha c}, \quad \bar{\tau}^{\mathrm{OL}} = \frac{Rd\Lambda}{(c+Y)\Lambda + d\alpha c}, \quad \bar{s}^{\mathrm{OL}} = \frac{\delta dc}{(c+Y)\Lambda + d\alpha c}. \tag{15}$$

3.2 Feedback Nash equilibrium

Open-loop solutions present a problem of credibility. With an OL information structure, the recipient, who is the only farsighted agent, commits to an optimal demand path from the start. Given the supply decided by the static donor, the amount and the price of the water transfer are known as functions of time. Why would he/she stick to the strategy he/she originally agreed to in the light of subsequent information (the state of the system at any time)?

Conversely, when the recipient plays feedback, he/she determines the demand by taking the current usable water capacity into account. Thus the demand for water is not a function of time, but a function of the state of the system. Consequently, the equilibrium price and amount of water transfer are state- dependent. We consider this to be a more realistic solution, while the open-loop solution can be regarded as a benchmark scenario, which requires commitment and is more difficult to achieve in practice.

While the static donor behaves in the same way as in the open-loop scenario, the maximization problem for the farsighted recipient can be written as the Hamilton-Jacobi-Bellman(HJB) equation:

$$\rho V_r^{\mathrm{F}}(x) = \max_{\tau,s} \left\{ d\left[(x+\tau) - \frac{\alpha(x+\tau)^2}{2}\right] - p\tau - \beta\frac{s^2}{2} + (V_r^{\mathrm{F}})'_x(x)(s-\delta x) \right\},$$

where $V_r^{\mathrm{F}}(x)$ is the value function for the recipient. Again, the price and quantity of water transfer at equilibrium are determined from the market-clearing condition $\tau^S(p) = \tau^D(p,x)$.

Proposition 1 *For any differential game between a static and a farsighted player described by:*

$$\max_{\tau}\left[E(\tau) + p\tau\right], \quad \max_{\tau,s}\int_0^\infty \left[Q(\tau,x) - p\tau - C(s)\right]e^{-\rho t}dt,$$

$$s.a.: \quad \dot{x} = f(s,x), \quad \tau^D(p,x) = \tau^S(p),$$

the optimal price and quantity can be written as the same function of the state under the feedback and the open-loop Nash equilibria.

Proof. The optimal demand for water chosen by the static player is characterized by equation $p = -E'(\tau)$ regardless of the information structure considered. Moreover, because the amount of water transfer does not affect the dynamics of the usable water capacity, the supply of water transferred is also the same for the open-loop or the feedback solution, and is given by equation: $p = Q'_\tau(\tau, x)$. Equating supply and demand, the price and the optimal amount of water transfer can be written as the same function of the state. ■

Thus, the price and the optimal quantity in the feedback Nash equilibrium are again given by (7), while the optimal investment in usable water capacity reads:

$$s^{\mathrm{F}}(x) = \frac{(V_r^{\mathrm{F}})'_x(x)}{\beta}, \tag{16}$$

where $(V_r^{\mathrm{F}})'_x(x)$ is the marginal value of additional units of the usable water capacity, x.

Given the linear-quadratic structure of the problem, we make the conjecture of a quadratic value function: $V_r^{\mathrm{F}}(x) = a_r^{\mathrm{F}} x^2/2 + b_r^{\mathrm{F}} x + c_r^{\mathrm{F}}$. Taking into account the optimal values of τ, p and s, the dynamics of the usable water capacity, (2), and solving the associated Ricati system of equations, we obtain the time path of the usable water capacity as:

$$x^{\mathrm{F}}(t) = (x_0 - \bar{x}^{\mathrm{F}})e^{\phi^{\mathrm{F}} t} + \bar{x}^{\mathrm{F}}, \tag{17}$$

with

$$\bar{x}^{\mathrm{OL}} < \bar{x}^{\mathrm{F}} \equiv \frac{cd(c+2Y)}{\Lambda(c+Y)^2 + dc\alpha(c+2Y)} < \frac{1}{\alpha}, \tag{18}$$

$$\phi^{\mathrm{F}} \equiv \frac{1}{2\beta}\left[\rho\beta - \sqrt{\Delta^{\mathrm{F}}}\right] < \phi^{\mathrm{OL}} < -\delta, \quad \text{with} \quad \Delta^{\mathrm{F}} = (\rho+2\delta)^2\beta^2 + 4\frac{dc\alpha\beta(2Y+c)}{(c+Y)^2}. \tag{19}$$

Again, because we assume $x_0 < \bar{x}^{\mathrm{OL}}$ then $x(t) < \bar{x}^{\mathrm{OL}} < \bar{x}^{\mathrm{F}} < 1/\alpha$ and as a consequence $\tau^{\mathrm{F}}(t), p^{\mathrm{F}}(t) > 0, \forall t > 0$. Knowing the time path of the usable water capacity allows the amount and the price of water transfer to be computed, while the coefficients of the value function are:

$$a_r^{\mathrm{F}} = \beta(\delta + \phi^{\mathrm{F}}) = \frac{(\rho+2\delta)\beta - \sqrt{\Delta^{\mathrm{F}}}}{2}, \quad b_r^{\mathrm{F}} = \frac{dc(c+2Y)}{(c+Y)^2(\rho - \phi^{\mathrm{F}})} = \frac{2dc\beta(c+2Y)}{(c+Y)^2\left(\rho\beta + \sqrt{\Delta^{\mathrm{F}}}\right)},$$

$$c_r^{\mathrm{F}} = \frac{(b_r^{\mathrm{F}})^2}{2\beta\rho} + \frac{d}{2\alpha\rho}\frac{Y^2}{(c+Y)^2}.$$

and their steady-state values are given by:

$$\bar{p}^{\mathrm{F}} = \frac{dc\Lambda}{\Lambda(c+Y) + dc\alpha\frac{c+2Y}{c+Y}}, \quad \bar{\tau}^{\mathrm{F}} = \frac{dR\Lambda}{\Lambda(c+Y) + dc\alpha\frac{c+2Y}{c+Y}}, \quad \bar{s}^{\mathrm{F}} = \frac{cd\delta}{\Lambda\frac{(c+Y)^2}{c+2Y} + dc\alpha}. \tag{20}$$

3.3 Comparison between the open-loop and the feedback Nash equilibria

In this section, we compare the optimal solutions under OL and feedback information structures. When the recipient does not commit at the beginning of the game, but determines the demand for water transfer and the investment in usable water capacity as a function of the existing stock of this capacity, this stock attains a higher value, not only in the long run, but also at any point in time. To obtain a larger stock, investments generally need to be higher. This becomes clear in the long run ($\bar{s}^{\mathrm{F}} > \bar{s}^{\mathrm{OL}}$ immediately follows from (15) and (20)), even though we have not proved it at each point in time.

Proposition 2 *If $x_0 < \bar{x}^{OL}$, then $x^{OL}(t) < x^{F}(t) \quad \forall t > 0$.*

Proof. Because $\bar{x}^{\mathrm{OL}} < \bar{x}^{\mathrm{F}}$, then $x_0 < \bar{x}^{\mathrm{OL}}$ ensures $x_0 < \bar{x}^{\mathrm{F}}$. Further, since $\phi^{\mathrm{F}} < \phi^{\mathrm{OL}} < 0$, then $x^{\mathrm{F}}(t)$ converges faster towards a higher steady state value than $x^{\mathrm{OL}}(t)$. Therefore, the proposition follows. ■

Because the usable water capacity in the recipient basin is greater without commitment, the amount of the water transfer and the price paid for it will be lower.

Proposition 3 *If $x_0 < \bar{x}^{OL}$, then $\tau^{OL}(t) > \tau^{F}(t)$, and $p^{OL}(t) > p^{F}(t) \quad \forall t > 0$.*

Proof. From Proposition 2, $x^{\mathrm{OL}}(t) < x^{\mathrm{F}}(t) \quad \forall t > 0$. And the expressions for the amount of water transfer and its price in (7), as functions of x, are valid both under open-loop or feedback information structures. The proposition follows. ■

Hence, with OL information, the recipient invests less and accepts a higher water transfer at a higher price. With feedback information, the recipient invests more, which leads to a lower transfer amount at a lower price. We can thus anticipate that the OL situation generates a higher payoff for the donor and the feedback situation generates a higher payoff for the recipient.

Proposition 4 *The donor's welfare is higher under open-loop than under feedback: $V_d^{OL} > V_d^{F}$.*

Proof. Using (7) we can compute $F^d(p^N(x), \tau^N(x))$. The derivative of this function w.r.t. x is negative:

$$\frac{dF^d(p^N(x), \tau^N(x))}{dx} = \frac{cRd^2\alpha(-1+\alpha x)}{(Y+c)^2} < 0,$$

because $x < 1/\alpha$. Since, by proposition 2, we know that $x^{OL} < x^{F}$, then we conclude $V_d^{OL} > V_d^{F}$. ■

The donor is thus better off playing open- loop.

In the case of feedback, in contrast to the open- loop solution, the recipient does not regard the price as fixed, but is aware of the negative relationship between the price and the usable water capacity, $p'(x) < 0$ (assuming an affine decreasing function $p'(x)$ would be constant). For a Hamiltonian that considers this feedback effect, the shadow price of the usable water capacity would decrease faster and towards a higher long-run value.[9]

A feedback information structure gives the investment in usable water capacity an added-value. We can interpret this result by arguing that the recipient has market power since he/she knows to what extent the price decreases with an increase in usable water capacity. This market power allows him/her to buy water at a lower price. Necessarily, less water is supplied at a lower price. The recipient is thus better off when his/her decisions are linked to the stock of usable water capacity.

We can illustrate this with a numerical example. We ran 10^5 iterations in which the parameters were drawn randomly from a uniform distribution, with $\delta \in [0.001, 0.5]$, $\rho \in [0, 0.5]$, $R, d, \alpha \in [0.001, 1]$ and all other parameters in $[0, 1]$. In all cases, we assumed $x(0) = 0$. In all cases where the transfer was feasible, i.e. smaller than the maximum possible surplus in the water ($\tau < R$), it was better for the recipient to play feedback than open-loop: $V_r^{\mathrm{OL}}(x_0) < V_r^{\mathrm{F}}(x_0)$.

4 No water transfer

In this section, we analyze the optimal investment in usable water capacity either in absence of an aqueduct to transfer the water, or assuming that the players decide not to transfer water. With no water transfer, the donor would not face any optimization problem, while the recipient would choose to invest in usable water capacity to maximize:

$$\max_s \int_0^\infty \left[d\left(x - \alpha \frac{x^2}{2} \right) - \beta \frac{s}{2} \right] e^{-\rho t} dt,$$

subject to (2).

Proposition 5 *With no water transfer, the investment in usable water capacity, its stock and*

[9] Indeed, the feedback Nash solution could be alternatively computed using the Hamiltonian. It would read:

$$H^{\mathrm{F}} = d\left[(x + \tau) - \alpha \frac{(x+\tau)^2}{2} \right] - p(x)\tau - \beta \frac{s^2}{2} + \mu(as - \delta x).$$

its shadow price are given by:[10]

$$s^{NT}(\lambda_r^{NT}) = \frac{\lambda_r^{NT}}{\beta}, \tag{21}$$

$$x^{NT} = (t) = (x_0 - \bar{x}^{NT})e^{\phi^{NT}t} + \bar{x}^{NT}, \tag{22}$$

$$\lambda_r^{NT}(t) = \beta\left[(\phi^{NT} + \delta)x^{NT}(t) - \phi^{NT}\bar{x}^{NT}\right],$$

with

$$\frac{1}{\alpha} > \bar{x}^{NT} \equiv \frac{d}{\Lambda + d\alpha} > \bar{x}^{F} > \bar{x}^{OL},$$
$$\phi^{NT} \equiv \frac{1}{2\beta}\left[\rho\beta - \sqrt{\Delta^{NT}}\right] < \phi^{F} < \phi^{OL} < 0, \qquad \Delta^{NT} = (\rho + 2\delta)^2\beta^2 + 4\alpha\beta d,$$

being λ_r^{NT} the shadow price of the usable water capacity when no transfer is possible.

The usable water capacity is highest when no transfer occurs between the two river-basins. Furthermore, because the speed of convergence is the fastest at this scenario, the stock of usable water capacity is also the highest at any time, t. Solving this problem using the HJB equation, with value function $V_r^{\mathrm{NT}}(x) = a_r^{\mathrm{NT}}x^2/2 + b_r^{\mathrm{NT}}x + c_r^{\mathrm{NT}}$, the coefficients are given by:

$$a_r^{\mathrm{NT}} = \beta(\delta + \phi^{\mathrm{NT}}) = \frac{(\rho + 2\delta)\beta - \sqrt{\Delta^{\mathrm{NT}}}}{2}, \quad b_r^{\mathrm{NT}} = \frac{d}{\rho - \phi^{\mathrm{NT}}} = \frac{2d\beta}{\rho\beta + \sqrt{\Delta^{\mathrm{NT}}}}, \quad c_r^{\mathrm{NT}} = \frac{(b_r^{\mathrm{NT}})^2}{2\beta\rho}.$$

In what follows it is proved that both regions are better off when an aqueduct exists and is used to transfer water from the supplier to the recipient.

Proposition 6 *If $x_0 < \bar{x}^{OL}$ the donor would be better off playing feedback Nash than under the no water transfer scenario, $V_d^{NT}(x) < V_d^{F}(x)$.*

Proof. The donor's instantaneous welfare playing Nash surpasses its welfare without water transfer if and only if:

$$F^d(p^{\mathrm{N}}(x^{\mathrm{F}}(t))), \tau^{\mathrm{N}}(x^{\mathrm{F}}(t))) > F^d(0,0) \equiv cR.$$

That is,

$$-c\frac{\tau^{\mathrm{N}}(x^{\mathrm{F}})^2}{2R} + p^{\mathrm{N}}(x^{\mathrm{F}})\tau^{\mathrm{N}}(x^{\mathrm{F}}) > 0.$$

Under assumption $x_0 < \bar{x}^{\mathrm{OL}}$, it holds that $x^{\mathrm{F}} < 1/\alpha$ and then $\tau^{\mathrm{N}} > 0$. Hence the condition above can be written as:

$$p^{\mathrm{N}}(x^{\mathrm{F}}) > c\frac{\tau^{\mathrm{N}}(x^{\mathrm{F}})}{2R}.$$

[10] Superscript NT refers to the case of no transfer.

That is:

$$\frac{cd}{Y+c}(1-\alpha x^{\mathrm{F}}) > \frac{c}{2R}\frac{dR}{Y+c}(1-\alpha x^{\mathrm{F}}) \Leftrightarrow 1 > \frac{1}{2}.$$

If the instantaneous welfare of the donor is greater at any time, then the aggregate discounted welfare would also be greater. We hence have $F^d(p^{\mathrm{N}}(x^{\mathrm{F}}(t))), \tau^{\mathrm{N}}(x^{\mathrm{F}}(t))) > F^d(0,0) \quad \Rightarrow$ $V_d^{\mathrm{F}}(x) > V_d^{\mathrm{NT}}(x)$. ■

The monetary payment to the donor for the water transfer more than offsets the reduction in the environmental amenities caused by the decrease in the water level in the donor's river. The only requirement is that the usable water capacity in the recipient basin is initially below its long run value.

Proposition 7 *If $x_0 < \bar{x}^{OL}$ the recipient is better off playing feedback Nash than under the no water transfer scenario, $V_r^{NT}(x) < V_r^F(x)$.*

Proof. The instantaneous payoff for the recipient if it behaves optimally in absence of an aqueduct is: $F^r(0,0,x^{\mathrm{NT}}(t), s^{\mathrm{NT}}(t))$.

Let us assume that the recipient continues with the investment $s^{\mathrm{NT}}(t)$, and hence with the usable water capacity $x^{\mathrm{NT}}(t)$, but now an aqueduct allows the transfer of water, for which the price is settled playing à la Nash. Then, the variation in its instantaneous utility with a marginal increment in the amount of water transfer would be:

$$F_\tau^r = d(1-\alpha(x^{\mathrm{NT}}+\tau)) - p(\tau) - p'(\tau)\tau, \quad \text{where} \quad p(\tau) = p^S(\tau) = \frac{c\tau}{R}.$$

And it can be easily derived that:

$$F_\tau^r > 0 \Leftrightarrow \tau < \frac{dR}{2c+Y}(1-\alpha x^{\mathrm{NT}})$$

Assumption $x_0 < \bar{x}^{\mathrm{OL}}$ implies $x^{\mathrm{NT}} < 1/\alpha$, and starting with no water transfer ($\tau = 0$), the above condition always holds for initial increments in τ. The instantaneous welfare increases with a marginal increment in water transfer at any point in time. Consequently, playing the optimal strategy that the recipient would choose if an aqueduct did not exist, $s^{NT}(t)$, and setting $\tau = 0$ is not optimal. The recipient can do better by marginally increasing the amount of water transferred.

$$\exists \hat{\tau} > 0 \mid W(x_0, 0, s^{\mathrm{NT}}(t)) < W(x_0, \hat{\tau}, s^{\mathrm{NT}}(t)),$$

where $W(x_0, \tau(t), s(t)) = \int_0^\infty F^r(\tau(t), p(t), x(t), s(t))e^{-\rho t}dt$, with $\dot{x}(t) = s(t) - \delta x(t)$, $x(0) = x_0$.

When the recipient plays Nash, he/she chooses the demand of water transfer and the investment to maximize $W(x_0, \tau(t), s(t))$ subject to (2) and the equilibrium condition between supply and demand. Therefore, it must hold that:

$$W(x_0, \tau^{\mathrm{F}}(t), s^{\mathrm{F}}(t)) \geq W(x_0, \hat{\tau}, s^{\mathrm{NT}}(t)) > W(x_0, 0, s^{\mathrm{NT}}(t)).$$

Playing Nash gives a higher payoff to the recipient than not to play. ■

Through a numerical analysis identical to the one carried out to compare the value functions under open-loop and feedback strategies, for 10^5 iterations, we can confirm that in all cases where the transfer was feasible, it was better for the recipient to play rather than to stick to the no water transfer strategy, $V_r^{\mathrm{NT}}(x_0) < V_r^{\mathrm{F}}(x_0)$. In fact, we found that the value function for the recipient and for the leader without water transfers was smaller than their value function either under open-loop or feedback information structures: $V_i^{\mathrm{NT}}(x_0) < \min\{V_i^{\mathrm{OL}}(x_0), V_i^{\mathrm{F}}(x_0)\},\ i \in \{r, d\}$.

5 Cooperation

In the particular case in which the donor and the recipient are located in the same country, a central government might decide on the amount of water to be transferred and the investment in usable water capacity in the donor region. The maximization problem in the cooperative case reads:

$$\max_{\tau, s} \int_0^\infty e^{-\rho t} \left[c\left(R - \frac{\tau^2}{2R}\right) + d\left[(x+\tau) - \alpha\frac{(x+\tau)^2}{2}\right] - \beta\frac{s^2}{2}\right] dt,$$

$$\text{s.a.: } \dot{x} = s - \delta x, \quad x(0) = x_0.$$

The amount of water transferred and the investment in usable water capacity in the cooperative equilibrium are:

$$\tau^{\mathrm{C}}(x) = \frac{dR}{c+Y}(1 - \alpha x), \quad s^{\mathrm{C}}(\lambda^{\mathrm{C}}) = \frac{\lambda^{\mathrm{C}}}{\beta},$$

where λ^{C} is the shadow price of x for the central planner. And the system dynamics is described by the system of differential equations:

$$\dot{x} = \frac{\lambda^{\mathrm{C}}}{\beta} - \delta x,$$
$$\dot{\lambda}^{\mathrm{C}} = (\rho + \delta)\lambda^{\mathrm{C}} - \frac{cd}{c+Y}(1 - \alpha x).$$

The cooperative equilibrium for τ^{C} and s^{C} coincide with the expressions in (7) and (8) under open- loop. Likewise, the dynamics of the state and the costate variables are identical to their

dynamics in (9) and (10) under open- loop. Consequently, $\lambda^{\mathrm{C}}(t) = \lambda^{\mathrm{OL}}(t)$, $x^{\mathrm{C}}(t) = x^{\mathrm{OL}}(t)$, $\tau^{\mathrm{C}}(t) = \tau^{\mathrm{OL}}(t)$ and $s^{\mathrm{C}}(t) = s^{\mathrm{OL}}(t)$, $\forall t \geq 0$. The investment in, the stock and the shadow price of the usable water capacity, as well as the amount of water transfer, are identical under a central authority or assuming decentralized players following open-loop strategies. The cooperative solution coincides with the Nash open-loop solution, except that there is no monetary payment for the water transfer under cooperation. The commitment Nash solution is then Pareto efficient.

There are two main characteristics of the game that lead to the equivalence between open-loop Nash and cooperation:

- The optimal decision on water transfer does not influence the dynamics of the state of the system (i.e. the usable water capacity). There is neither a direct effect nor an indirect effect through s (i.e. $\dot{x}$ and s^{OL} are independent of τ).
- The stock or the investments in usable water capacity do not directly affect the donor's welfare ($F^d(p,\tau)$ is independent of x and s).

Proposition 8 *For any differential game between a static and a farsighted player described by:*

$$\max_{\tau} \left[E(\tau) + p\tau\right], \quad \max_{\tau,s} \int_0^{\infty} \left[Q(\tau,x) - p\tau - C(s)\right] e^{-\rho t} dt,$$

$$s.a.: \quad \dot{x} = f(s,x), \quad \tau^D(p,x) = \tau^S(p),$$

the non-cooperative open-loop solution coincides with the solution of the cooperative game:

$$\max_{\tau,s} \int_0^{\infty} \left[E(\tau) + Q(\tau,x) - p\tau - C(s)\right] e^{-\rho t} dt,$$

$$s.a.: \quad \dot{x} = f(s,x).$$

Proof. Under open-loop Nash, water transfer (market clearance) and investments are characterized by:

$$p = -E'(\tau) = p = Q'_{\tau}(\tau,x), \quad s = \frac{\lambda_r^{\mathrm{OL}}}{\beta}.$$

Correspondingly, water transfer and investments under cooperation come from the equations:

$$E'(\tau) + Q'_{\tau}(\tau,x) = 0, \quad s = \frac{\lambda_r^{\mathrm{C}}}{\beta}.$$

The amount of water transfer and the investments are therefore described by the same functions of x under the two scenarios.

The system dynamics in the open-loop Nash game also matches the cooperative dynamics:

$$\dot{\lambda}_r^i = \rho\lambda_r^i - \left[Q'_x(\tau,x) + \lambda_r^i f'_x(\lambda_r^i,x)\right], \quad \dot{x}^i = f(\lambda_r^i,x), \quad i \in \{\mathrm{OL},\mathrm{C}\}$$

therefore, $\lambda_r^{\mathrm{OL}}(t) = \lambda_r^{\mathrm{C}}(t)$ and $x^{\mathrm{OL}}(t) = x^{\mathrm{C}}(t)$. Consequently, $s^{\mathrm{OL}}(t) = s^{\mathrm{C}}(t)$ and $\tau^{\mathrm{OL}}(t) = \tau^{\mathrm{C}}(t)$.

■

Open-loop solutions oblige players to follow the optimal strategy decided at the beginning of the game. These types of equilibria are not credible because players might have an incentive to deviate from the committed strategy at any point in time (especially in the case of an infinite time horizon). Unless this commitment could be enforced, rational players would adjust strategies at any time, knowing the state of the game. The recipient would play feedback strategies, linking the demand for water transfer and the investments in the usable water capacity to the actual level of this stock. Its valuation of the capacity of usable water being higher, this would lead to greater investments and lower water transfer. Consequently, without a mechanism to enforce commitment, the non-cooperative solution deviates from Pareto efficiency with over-investment in capacity and a shortfall in water transfer.

6 Static versus dynamic game

In the model presented so far, the recipient faces a trade-off. He/she can buy water from the donor at a market price or can invest in the creation and maintenance of equipment to save, produce or recycle water, the stock of which we call the usable water capacity. As we have seen, usable water capacity is strategic for the recipient because it allows him/her to influence the market price in the transfer market. Until now, we have assumed that building up this water capacity requires time. In this section, we make the simplifying assumption that usable water capacity immediately adjusts to the optimal level, once the investment decision is taken. We also consider that this capacity, once created, never decreases, i.e. that the depreciation rate is zero. These are two unrealistic assumptions but they allow us to transform our dynamic model into a static model so as to compare the results. If both models give the same insights, one might be tempted to rely on the more simple static model, rather than the more complicated dynamic one.

The payoff functions for the two players in the static setting read:

$$F^d(p,\tau) = c\left(R - \frac{1}{2}\frac{\tau^2}{R}\right) + p\tau, \tag{23}$$

$$F^r(p,\tau,s) = d\left[x + \tau - \frac{\alpha}{2}(x+\tau)^2\right] - p\tau - \frac{1}{2}\beta s^2, \quad x = x_0 + s.$$

where s is the investment decision and x the usable water capacity. As before, let us assume initial investments are zero, i.e. $x_0 = 0$, which implies $x = s$ and the recipient's payoff function

can be written as:

$$F^r(p,\tau,s) = d\left[s+\tau-\frac{\alpha}{2}(s+\tau)^2\right] - p\tau - \frac{1}{2}\beta s^2. \tag{24}$$

The Nash equilibrium for this static game is obtained by maximizing $F^d(p,\tau)$ w.r.t. τ, $F^r(p,\tau,s)$ w.r.t. τ and s, and taking into account the market clearing condition to determine the price of the water transfer, $\tau^S(p) = \tau^D(p,s)$. The optimal solution is given by:

$$\tau^{\mathrm{NS}} = \frac{\beta dR}{\alpha dc + \beta(c+Y)}, \quad p^{\mathrm{NS}} = \frac{\beta dc}{\alpha dc + \beta(c+Y)}, \quad s^{\mathrm{NS}} = \frac{dc}{\alpha dc + \beta(c+Y)}. \tag{25}$$

It can be easily seen that the payoffs for the two players at this equilibrium are greater than under the assumption of no water transfer between the regions.

Proposition 9 *$F^d(p^{NS},\tau^{NS}) > F^d(0,0)$ and $F^r(p^{NS},\tau^{NS},s^{NS}) > F^r(0,0,s^{NT})$.*

Proof. We can compute $s^{\mathrm{NT}} = \frac{d}{\beta+d\alpha}$ and it can easily be seen that

$$F^d(p^{\mathrm{NS}},\tau^{\mathrm{NS}}) - F^d(0,0) = \frac{cR\beta^2d^2}{2(\beta Rd\alpha + \beta c + d\alpha c)^2} > 0.$$

$$F^r(p^{\mathrm{NS}},\tau^{\mathrm{NS}},s^{\mathrm{NS}}) - F^r(0,0,s^{\mathrm{NT}}) = \frac{\alpha R^2\beta^3d^3}{2(\beta+d\alpha)(\beta Rd\alpha + \beta c + d\alpha c)^2} > 0.$$

■

Hence, under the above conditions, the water transfer will take place. If the two regions decide to cooperate, they will fix the amount of water transfer, τ, and the alternative source of water, s, in order to maximize their joint welfare:

$$\max_{\tau,s}\left\{F^d(p,\tau) + F^r(p,\tau,s)\right\}.$$

The cooperative solution $\tau^{\mathrm{CS}}, s^{\mathrm{CS}}$ disregards the price of the water transfer. Apart from that, the transfer and the alternative source of water match their non-cooperative values:

$$\tau^{\mathrm{CS}} = \frac{\beta dR}{\alpha dc + \beta(c+Y)}, \quad p^{\mathrm{CS}} = \frac{\beta dc}{\alpha dc + \beta(c+Y)}, \quad s^{\mathrm{CS}} = \frac{dc}{\alpha dc + \beta(c+Y)}. \tag{26}$$

Proposition 10 *$\tau^{NS} = \tau^{CS}$ and $s^{NS} = s^{CS}$.*

Proof. The proof is obvious, comparing equations (25) and (26). ■

Remark 11 *In the static game, the Nash equilibrium is Pareto efficient. In the dynamic game, the lack of an enforcement mechanism leads the recipient to deviate from the open-loop solution which coincided with the cooperative solution. The Nash feedback dynamic equilibrium is not Pareto efficient.*

If we treated the interaction between the donor and the recipient as a static game, the non-cooperative solution would be Pareto efficient. Conversely, if we consider the more realistic dynamic setting with a cumulative usable water capacity that depreciates at a given rate, we have seen that the non-cooperative solution will be the feedback solution, because the donor has a higher payoff function and because commitment is generally not easy to impose. The feedback solution is not equal to the cooperative solution and hence the non-cooperative solution in the dynamic game will not be Pareto efficient.

Therefore, analyzing this problem in a static setting has to major caveats: first, the model fails to accurately reflect reality, in particular the dynamic nature of investments and their depreciation. Second, the model fails to capture the most salient feature of the solution: the possibility to use a feedback strategy in which the recipient takes into account the impact of his/her investments on the price path of the water transfer. Hence, we may mistakenly conclude that the non-cooperative solution is Pareto efficient when it is not. The market equilibrium established in the dynamic game will result in a lower transfer and a higher investment than in the case of cooperation (and with a lower price than in the open-loop case). We illustrate this point in the numerical example that follows.

7 Numerical illustration: the Tagus-Segura transfer

We can link our results to the data of the Tagus-Segura water transfer described in Ballestero (2004), in particular concerning the information on total supply and demand, the transfer amount and the transfer price.

Ballestero reports the following mean amounts for the period 1979-1996 (page 84 Table IV). Surplus[11] in the Tagus donor basin: 593.67 million m^3. Agricultural water needs in the Lorca recipient region: 74 million m^3 (of which some can be covered by available groundwater). He also computes the market clearing transfer amount, τ^B and the associated price, p^B in his static model as: $\tau^B = 58$ million m^3 and $p^B = 0.46$ dollar US/m^3.

We define R as the maximum possible water surplus in the river, $R = 593.67$ million m^3. We also define $1/\alpha$ as the maximum water requirements in the recipient basin. We can therefore set $\alpha = 1/74 = 0.0135$.

The main difference between our model and Ballesteros is the possibility to invest in alternative water supplies. Thus our results will resemble his if we solve the game considering

[11] Surplus is stored water (833.67 million m^3) minus annual water needs in the Tagus basin (240 million m^3).

the welfare function for the donor as in (1), while the instantaneous welfare for the recipient is described by:

$$F^r(p,\tau) = d\left[\tau - \frac{\alpha}{2}(\tau)^2\right] - p\tau.$$

We calibrate our model in such a way that the transfer and the prices that solve this static game match the values found by Ballestero. This gives the parameter values $c = 4.7084$, $d = 2.1275$. We next assume the discount rate $\rho = 0.001$ and the equipment depreciation rate $\delta = 0.1$. Initial investments are set to zero: $x(0) = 0$.

Given these parameter values, we can solve either the static game, or the dynamic game, considering open-loop or feedback information structures, depending on the value of β (which measures the cost of investment). As shown in Table 1, it is immediately clear that at zero investment cost, the recipient basin would satisfy all its water needs by producing, recycling, or saving free water. The water capacity is immediately reached at no cost and as a consequence, no water is transferred. Another extreme case would be defined by infinite investment costs. With no alternative sources of water, the recipient can only rely on the water transfer. This is the case studied by Ballestero. When the investment in water capacity is free, or when it is not feasible, the dynamic game becomes static. As a consequence, the static equilibrium coincides with both dynamic solutions (open-loop or feedback). For intermediate values of investment costs, we have a positive stock of equipment that enables less water to be transferred at a lower price than in Ballestero's case. Moreover, static, open-loop and feedback solutions differ. In the static case the stock of usable water is at its lowest and hence transfers and prices are highest. By contrast, in the dynamic case with no commitment, the investment is highest and hence prices and transfers are lowest. The long-run solution under open-loop information strategies is an intermediate case. We can also confirm that having the opportunity to invest reduces the recipient's dependence on the donor: in the cooperative case (which coincides with the open-loop case) roughly one half of the water needs are supplied by investments in capacity. If, in addition, the recipient exerts his/her market power, his/her dependence on the donor is further reduced with two-thirds of the water needs supplied by investments in capacity. It is also important to note that, in the long run, the total available water in the recipient, $\bar{\tau} + \bar{x}$, is highest in the feedback case, but is still less than his/her water needs. On the other hand, the available water reaches its lowest values in the static case, although still higher than in the case with infinite investment costs.

Figure 1 shows the difference between open-loop and feedback steady-state solutions as

Table 1: Steady state values for different values of β.

Variable	$\beta = 0$	$\beta = \hat{\beta}$			$\beta \to \infty$
	S=OL=F	Static	Open-loop	Feedback	S=OL=F
$\bar{\tau}$	0	57.5647	33.1668	24.8332	58
$\bar{p}$	0	0.4565	0.2630	0.1970	0.46
$\bar{x}$	74	0.5554	31.6837	42.3163	0
$\bar{\tau} + \bar{x}$	74	58.1201	64.8505	67.1495	58

$\hat{\beta}$ s/t difference between OL and F solutions are greatest Ballestero

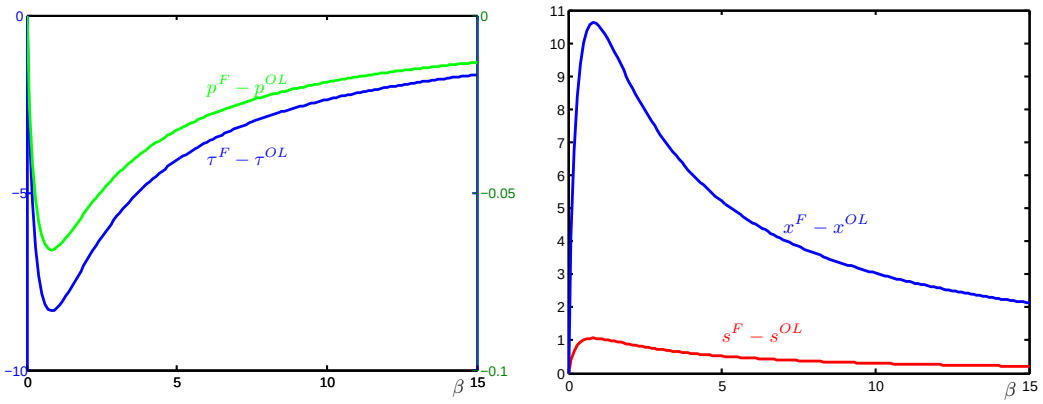


Figure 1: Feedback versus OL steady-state values as a function of β

functions of β: transfer levels and prices in the figure on the left and investment stock and flows in the figure on the right. The difference is maximum for $\beta = \hat{\beta} = 0.8220$. Let us then analyze the time paths of the different solutions. The optimal transfer amounts are shown on the left in figure 2. The dynamic transfer initially starts above the static transfer but immediately falls below it, tending to the steady state values described in table 1 (greater under cooperation). The right side of figure 2 depicts the optimal path of water capacity. In the feedback case, optimal investments are greater, leading the stock to higher long-run levels than under cooperation.

Figure 3 shows the gains for the donor playing open- loop rather than feedback (the top curve) and the gains for the recipient playing feedback rather than open- loop (the bottom curve). First, we confirm that the donor is better off playing open-loop and the recipient is better-off playing feedback. Moreover, cooperation means higher gains for the recipient than losses for the donor. The difference between the curves represents the total gain to be had from

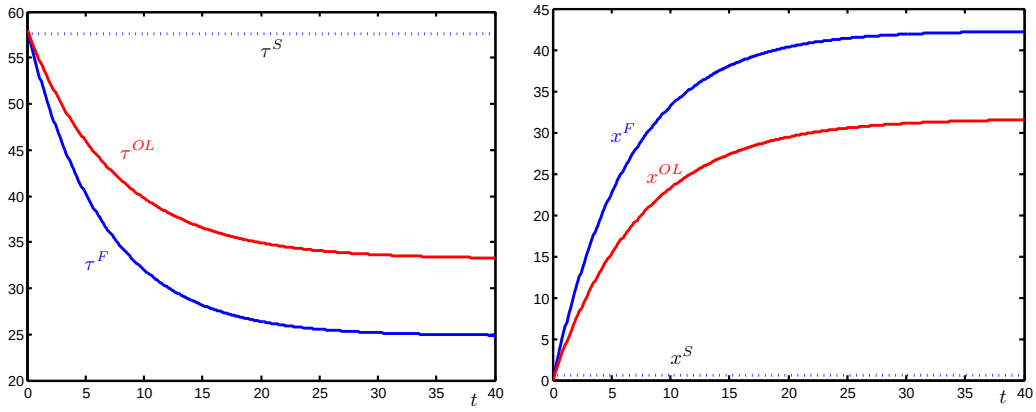


Figure 2: Time paths for transfer and water capacity

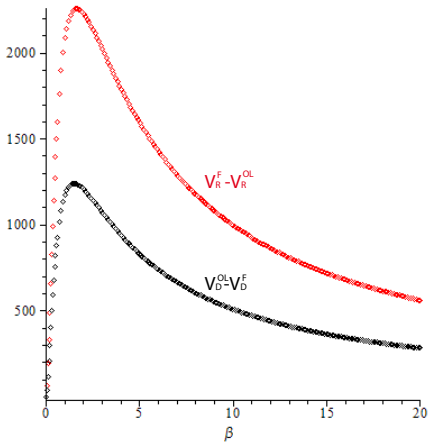


Figure 3: Value function: top curve feedback, bottom curve open- loop

cooperation, as an inverted u-shaped function of β. Total gains from cooperation are highest for intermediate values of investment costs.

8 Conclusions

In this paper we analyze water transfer between two river- basins connected by an aqueduct. The transfer of the water surplus might not affect the donor's economy, but it deteriorates the environment in the donor's region. The recipient's region, which benefits from the transfer, has to pay the donor for forgone benefits from holding on to the water resource. The recipient faces a dynamic dilemma. He/she can pay the price of the water transfer or can invest in alternative water supplies to increase the region's usable water capacity. The donor is a myopic agent whose supply decisions disregard this accumulation process. Our main objective is the optimal

determination of price and quantity of the water transfer considering a bilateral monopoly with a single supplier and a single demander.

The dynamic interaction between the two regions is analyzed as a differential game with two particularities. Firstly, the two players do not determine the optimal value of different variables, on the contrary, they both decide on the amount of water to be transferred. The donor fixes the supply and the recipient fixes the demand, both as a function of the price. The market clearance condition will determine the price and the quantity at equilibrium. The second characteristic is the coexistence of a farsighted buyer with a myopic seller. We a have obtained two main findings that are valid for a rather general differential game with one myopic and one farsighted player taking supply and demand decisions in a bilateral monopoly market. The first result states that supply and demand decisions are the same regardless of whether the farsighted player commits to an optimal path from the start of the game, or adjusts his/her decisions by taking the state of the system into account over time. Thus, optimal price and quantity are the same function of the state under the feedback or the open-loop Nash equilibria. The second and main result of the paper shows that if the interaction between the players is treated as a static game, the non-cooperative solution conincides with the cooperative solution and is then Pareto efficient. Conversely, in a dynamic setting with no mechanism to enforce commitment, the non-cooperative solution is characterized by feedback strategies. The feedback Nash equilibrium does not coincide with the cooperative solution and is therefore not Pareto efficient. Only if the players have no information on the state of the system or their commitment is somehow exogenously enforced, would it be adequate to consider open-loop information structures. And only in that case, the laisser-faire policy would be Pareto efficient.

In the game described in this paper, a feedback recipient who does not commit from the start of the game, is aware of the fact that a greater usable water capacity would reduce the price of the water transfer. Consequently, the recipient invests more in the infrastructure to produce, save, or recycle water. And hence, the increase in usable water capacity is higher under feedback than under open-loop information structures. This implies that less water is transferred at a lower price. Due to this market power, we prove (analytically) that the recipient is better off, while (numerically) the donor is worse off. Since the open-loop solution is Pareto efficient, the aggregate welfare would be higher with a commitment. We further prove that when the price and quantity of the water transfer are optimally chosen, either with or without commitment, both players are better off than in the case of no transfer.

Finally, wecompare the price and quantity of water transfer in the two non-cooperative solutions (open-loop and feedback), and with the supply-demand scheme described in Ballestero (2004), who takes uncertainty into account, but not the recipient's opportunity to invest in the usable water capacity. After calibrating the model, we observe that the opportunity to invest in alternative water supplies reduces the recipient's dependence on water transfer. Less water is transferred at a lower price, and the price decreases over time. In addition, the investment in alternative water supplies is higher under feedback strategies.

In this paper, we analyze the interaction between the players assuming that the aqueduct already exists. Either a central government decided to built it or the two parties agreed to build it. Once the transfer is physically feasible, our analysis focused on the optimal price and quantity. An immediate natural extension would be to analyze how to share the fixed cost of building the aqueduct. A second interesting extension, especially when the two river- basins are located in different countries, would be to study the model considering hierarchical modes of play. Depending on the circumstances, either the donor or the recipient could be regarded as the Stackelberg leader and his/her opponent as the follower.

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