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Partial enclosure of the commons

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Abstract

We examine the efficiency, distributional, and environmental consequences of assigning spatial property rights to *part* of a spatially-connected natural resource, a situation which we refer to as *partial enclosure of the commons*. The model reflects on a large class of institutions and natural resources for which complete enclosure by a sole owner may be desirable, but is often institutionally impractical. When a sole owner is granted ownership of only a fraction of the spatial domain of the resource and the remainder of the resource is competed for by an open access fringe, interesting spatial externalities arise. We obtain sharp analytical results regarding partial enclosure of the commons including: (1) While second best, it always improves welfare relative to no property rights, (2) all resource users are made better off, (3) positive rents arise in the open access area, and the resource will maintain higher abundance. Under spatial heterogeneity, we also characterize spatial regions that are ideal candidates for partial enclosure - typically, society should seek to enclose those patches with high ecological productivity and high self-retention, but whether high economic parameters promote or relegate a patch may depend on one's objective. These results help inform a burgeoning trend around the world to partially enclose the commons.

JEL Classifications: C61, H23, H73, Q2

Key words: Incomplete property rights, natural resources, common property

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1 Introduction & Background

The economics profession has established incontrovertibly that open access or common pool management of natural resources often leads to economic inefficiencies, and possibly ecological disaster ([15], [4], [26], [23]). A large body of evidence highlights mismanagement of fisheries, pastures, forests, groundwater, pollution sinks [34], antibiotic resistance ([2], [14]), among other tragedies of the commons. A massive literature has emerged that proposes economic instruments as solutions, including taxes ([3], [31], [36]), effort restrictions ([1], [28]), fully-delineated property rights ([11], [18], [7]), tradeable permits ([24], [12], [35], [29]), and spatial zoning with taxation ([32]) or with unitization ([20], [17]). Under certain conditions, each of these instruments has benefits and may even 'solve' the tragedy of the commons, and provide first-best outcomes.[1] Yet issues of wealth redistribution, heterogeneity ([19]), or high political and economic costs ([16]) may impede the performance of such instruments and may explain why we rarely observe these instruments being implemented as economic models would suggest. Instead, we tend to observe hybrids where only part of the resource is subsumed within a market structure. Indeed the failure of many natural resource management institutions has been explained by the potential mismatch between the spatial scale of management and that at which ecological processes operate. For instance, Scott [33] states that "the property must be allocated on a scale sufficient to insure that one management has complete control of the asset". Yet this is rarely the case for resources such as water, hunting game, fisheries, oil, and forests. A much more common institutional regime is to assign property rights to a fraction of the natural resource, often leaving the remainder of the resource to be competed for by an open access fringe.

We refer to this situation as 'partial enclosure of the commons'. While the owner of the enclosed area may behave somewhat like a sole owner, the mobility of the resource induces a spatial externality, so the open access fringe may affect his behavior. Despite notable advances on the use of economic policies to internalize various externalities ([21]), the use of partial enclosure remains an unresolved issue. We aim at filling this gap by analyzing the efficiency, distributional and environmental consequences of its application.

We illustrate the ubiquity of partial enclosure by providing some examples. Even if rights are fully delineated in one state's jurisdiction, fish often traverse jurisdictions and are subject to harvest in adjacent open access areas (e.g. outside a nation's exclusive economic zone, on the 'high seas'). But even for those species that are reasonably sessile like reef fish, they are often exploited by different

[1] The OECD report [25] provides a survey of many market-like instruments used to solve these problems.

fleets (e.g. commercial vs. recreational), typically where one fleet has well-defined property rights and the other acts in an open access fashion. Partial enclosure of the commons is even more commonplace for groundwater and oil reserves. Like fish, those resources are mobile (extraction in one location induces a flux). Rights to groundwater and oil stocks are often related to spatial property rights at the surface.[2] The spatial delineation of these rights almost never accords with the spatial domain of the underlying resource. Game, such as deer and waterfowl, have characteristics similar to fish - they migrate and are only partially enclosed on private lands. Even some forest resources share these characteristics. Communities are often granted exclusive ownership over a tract of forest land, where the remainder of the forest is open to others. To the extent that actions outside the tract influence it (e.g. excessive harvest outside may reduce seed dispersal inside), or actions inside affect the open access area, the 'partial enclosure' prerequisites hold. This institutional arrangement is thus characterized by spatial externalities (external effects on adjacent areas) diffusing the institutional effect of partial enclosure by a single owner due to the mobility of the resource. The literature has not assessed the effect of such asymmetric property rights regimes on a spatially connected resource. We are specifically interested in the effects of spatial externalities and heterogeneity on welfare, and in the characterization of the optimal siting of partial property rights for renewable resources.

To address these questions we develop and study an analytical model of partial enclosure of the commons. The model is simple enough to maintain analytical tractability, but contains all of the components essential to describe this ubiquitous institutional arrangement. It is meant to be generically applicable to a wide range of natural resources with certain characteristics. The dynamics of a natural resource are both temporal and spatial. Across time, the natural resource can grow (or shrink) depending on the level of extraction and the degree of regeneration which may, itself, depend on the level of resource stock. The resource also moves across space. We model space as a set of mutually-exclusive and exhaustive patches. In biological sciences this is referred to as a 'metapopulation' - we will keep track of natural resource stock in each patch. Any given patch may be unregulated (i.e. open access - a situation in which current economic returns govern entry, exit, and extraction) or may be managed by an owner who maximizes her private benefits. Owing to spatial movement, behavior in the open access region has important consequences for the sole owner, and vice versa. The ensuing spatial

[2] The "rule of capture" of groundwater or of any mineral resources is historically based on the concept that each landowner has complete ownership of resources under his land, and has an unlimited right to use them. This *Absolute Ownership Doctrine* has led to overexploitation issues in areas where the number of users has grown so that the use of the resource, even if it is limited by land ownership, gets close to that of an open access outcome. It is now commonly rejected because of the existing diffusion/dispersal process of the resource.

and temporal externalities represent a potentially damaging market failure that induce a kind of dynamic spatial game across patches with different characteristics. We solve this problem and explore its consequences.

As a starting point, we note that partial enclosure of the commons will not fix all externalities, and will thus be a second best alternative to sole ownership of the entire resource domain.[3] Despite its ubiquity, the literature on *partial* property rights is sparse. Under what conditions will assigning rights in this way achieve economic, distributional, or ecological improvements over the pure open access case? And if we are to proceed with partial enclosure of the commons, what guiding principles can be generated to design these institutions? The remainder of this paper is devoted to addressing these, and related questions.

To the best of our knowledge, only a single existing paper tackles the issue of partial enclosure of natural resources ([13]).[4] It focuses on uncertainty, instrument, and the congestion problem resulting from the enclosure of some resource pools on other open-access resource pools. By contrast, we investigate whether partial enclosure may increase (aggregate or patch-specific) resource stock levels and/or aggregate economic value (or individual profits). We highlight the influence of spatial externalities and environmental heterogeneity on the optimal assignment of partial property rights. The importance of spatial effects has been documented in the literature on learning externalities and agglomeration economies: it is emphasized how investment decisions made by one agent may influence others who learn from his experience ([22], [27]). In our setting, it is the physical diffusion or dispersal of the resource across space that gives rise to interesting spatial externalities. Given biological heterogeneity, this diffusion effect may have different impacts from one region to another, emphasizing the importance of careful selection of the region in which property rights will be assigned.

2 Model & Results

A natural resource stock (denoted by x) is distributed heterogeneously across a discrete spatial domain consisting of N patches. Patches may be heterogeneous in

[3]Taxes are a possible alternative, but a tax in only one region (analogous to partial enclosure) would be second best. Indeed, it is proven in [32] that a first best outcome would require the use of spatially differentiated taxes (one for each region).

[4]Colombo and Labrecciosa [6] analyze the oligopolistic exploitation of a productive asset under private and common property. They assume that, under private exploitation, the resource is parceled out. Each firm owns and manages the assigned parcel over the entire planning horizon. Thus, fully delineated property rights exist over the entire domain of the resource. As such, they abstract from situations where the resource is fully mobile, and do not analyze (as we do) the impact of spatial externalities and biological heterogeneity on the assignment of partial property rights.

size, shape, economic, and environmental characteristics, and resource extraction can potentially occur in each patch. The only requirement for the delineation of patches is that patches must be homogeneous intra-patch; all ecological and economic variables are constant within each patch. The resource is mobile and can migrate from patch to patch. In particular, denote by $D_{ij} \geq 0$ the (constant) fraction of the resource stock in patch i that migrates to patch j in a single time period. Time is treated in discrete steps. The resource may also grow, and the growth conditions may be patch-specific. Assimilating all of this information, the equation of motion, in the absence of harvest, is given as follows:

$$x_{it+1} = \sum_{j=1}^{N} D_{ji} g(x_{jt}, \alpha_j) \tag{1}$$

Here resource production in patch i is given by $g(x_i, \alpha_i)$ which is extremely general; we follow the literature and require that $\frac{\partial g(x,\alpha)}{\partial x} > 0$, $\frac{\partial g(x,\alpha)}{\partial \alpha} > 0$, $\frac{\partial^2 g(x,\alpha)}{\partial x^2} < 0$, and $\frac{\partial^2 g(x,\alpha)}{\partial x \partial \alpha} > 0$. We also assume that extinction is absorbing ($g(0; \alpha) = 0$) and that the growth rate is finite ($\frac{\partial g(x,\alpha)}{\partial x}|_{x=0} < \infty$). The patch-specific parameter α_i affects resource growth and has many possible interpretations including intrinsic rate of growth, carrying capacity, patch size, etc. All standard biological production functions are special cases of $g(x, \alpha)$. The resource stock that is produced in patch i then disperses across the spatial domain: some fraction stays within patch i (D_{ii}) and some flows to other patches (D_{ij}). Indeed, some may flow out of the system entirely, so the dispersal fractions need not sum to one: $\sum_j D_{ij} \leq 1$.

Because this is a discrete-time model, we must specify the timing of harvest. For patch-i harvest h_{it}, the *residual stock*[5] left for reproduction is given by $e_{it} \equiv x_{it} - h_{it}$. Including harvest, the patch-i equation of motion becomes:

$$x_{it+1} = \sum_{j=1}^{N} D_{ji} g(e_{jt}, \alpha_j). \tag{2}$$

The timing is thus: the present period stock (x_{it}) is observed and then harvested (h_{it}) giving residual stock (e_{it}), which then grows ($g(e_{it}, \alpha_i)$), and disperses across the system (D_{ij}). By the identity $e_{it} \equiv x_{it} - h_{it}$, there is a duality between choosing *harvest* and choosing *residual stock* as the decision variable. We use residual stock because it turns out to bypass many technical issues that arise when one uses harvest as the decision variable. By adopting residual stock as the control, we are able to fully characterize the optimal policy; one can then back out the optimal harvest. This in turn will enable us to provide a precise assessment of partial enclosure as an institutional arrangement.[6]

[5] In the fisheries literature, *residual stock* has been coined *escapement*.

[6] This mathematical convenience was pointed out in [30] and has been adopted by several subsequent contributions ([10], [9] among others).

Patch-i harvesters earn price p_i per unit harvest and marginal harvest cost is a non-increasing function of resource stock in patch i. If h_{it} is harvested in patch i at period t, then the residual stock level is e_{it}, and profit is given by:

$$\Pi_{it} = p_i(x_{it} - e_{it}) - \int_{e_{it}}^{x_{it}} c_i(s)ds \tag{3}$$

where $c_i'(s) \leq 0$ (higher resource stock reduces per-unit harvest cost).

2.1 Open Access Benchmark

In the absence of property rights, resource users are able to costlessly access all patches to seek short-run profit. As such, extraction effort will gravitate in any period to the patches with the highest marginal profit. Indeed, we assume that effort will enter patch i until marginal profit is zero, i.e. until $p_i = c_i(\hat{e}_{it})$ were $\hat{e}_{it}$ denotes the residual stock in patch i when all patches are under open access. As long as costs are sufficiently high in at least one patch ($p_j < c_j(0)$) and there is some self-retention ($D_{jj}>0$), then the stock will never be completely exhausted, even under open access. Rather, in each patch the stock will be extracted down to a level where it becomes unprofitable to extract further. In patches for which this level is positive, stock will grow and redistribute spatially according to Equation 2. Thus, under pure open access of this spatial resource, we have the following benchmark results:

Proposition 1. *Open access residual stock level in patch i satisfies $p_i = c_i(\hat{e}_{it})$ when $c_i(0) > p_i$ and $\hat{e}_{it} = 0$ otherwise.*

2.2 Partial Assignment of Spatial Property Rights

The main purpose of this paper is to examine the consequences of partial enclosure of the commons. Within the model developed here, we implement that concept by assigning exclusive property rights over a single patch to a single owner while the other $N-1$ patches remain open access. This induces a potentially complicated dynamic spatial game between the owner of the enclosed patch and the adjacent open access fringe areas, which are connected to the enclosed patch through the system dynamics. More specifically, assuming that patch i is the *enclosed* patch, the optimal policy function $e_{it} = \phi_i(t, x_{1t}, x_{2t}, ...)$ is potentially time and state dependent. The enclosed patch owner's economic objective is to maximize the expected net present value of harvest, expressed in terms of residual stock level, from patch i over an infinite horizon:

$$\max_{\{e_{it}\}} \sum_{t=1}^{\infty} \delta^t \left[p_i(x_{it} - e_{it}) - \int_{e_{it}}^{x_{it}} c_i(s)ds \right]. \tag{4}$$

Henceforth, all of the analysis will rely on solving the discrete-time difference game that is induced by the patchiness of spatial ownership. Because we focus on the residual stock as the control variable in each spatial patch, and owing to some useful characteristics of the economic environment, this challenge has a straightforward solution. We can immediately write down an implicit expression defining the residual stock level in every patch. These levels (in all patches) are summarized as follows:

Lemma 1. *When patch k is enclosed, and the other $N-1$ patches remain open access, the equilibrium residual stock levels are:*

$$\begin{aligned} &p_i = c_i\left(e_{it}\right) \; for \, i \neq k \\ &p_k - c_k\left(\bar{e}_{kt}\right) = \delta D_{kk}\left[p_k - c_k\left(\bar{x}_{kt+1}\right)\right] g_e\left(\bar{e}_{kt}, \alpha_k\right), \end{aligned}$$

Here, e_{it} denotes the residual stock in patch i when another patch is enclosed, and $\bar{e}_{kt}$ (respectively $\bar{x}_{kt}$) denotes the residual stock (respectively the stock level) when k is enclosed (all other patches being under open access). All proofs are provided in the appendix. Lemma 1 shows that the residual stock level takes just two possible forms. In the open access patches, *harvest* will respond to behavior in the other patches, but *residual stock* will not. Each open access patch is harvested, in each period, down to the open access residual stock level - i.e. where harvesting the next unit of resource stock would entail an economic loss. In the *enclosed* patch, the owner acknowledges the behavior of the (connected) open access patches and realizes that she will not be the residual claimant of any conservative harvesting behavior. Thus, she behaves as if the entire resource that disperses out of her patch will be lost. This is why the only dispersal term to enter the optimal residual stock term is D_{kk} the fraction of the resource that remains in the enclosed patch.

As an extreme point of comparison, we can also consider the problem in which sole ownership is assigned over the entire spatial domain. While this problem has been studied elsewhere, we provide the sole owner's first order conditions in this case as a point of comparison with that described in Lemma 1. Following [9] the complete sole owner chooses residual stock in patch j as follows:

$$p_j - c_j\left(e^*_{jt}\right) = \sum_{l=1}^{N} \delta D_{jl}\left[p_l - c_l\left(x_{lt+1}\right)\right] g_e\left(e^*_{jt}, \alpha_j\right). \tag{5}$$

Equation 5 leads to a very different residual stock than is implied by Lemma 1. Indeed, simple inspection of the right hand side of the Equation 5 highlights that the sole owner would account for the effect on all patches (D_{jl} $\forall l$) not just the fraction of the resource that stays in patch k (D_{kk}).

Now, relying on Lemma 1 and following [9], it turns out that the residual stock level in every patch is constant; this is summarized by the following:

Lemma 2. *The equilibrium harvest strategy for all patches under* partial enclosure*, is for all patches to harvest down to a pre-determined residual stock level that is time and state independent.*

Lemma 2 is extraordinarily useful. Normally, we would expect the optimal residual stock level to be a (possibly time varying) feedback control rule that mapped the state (all possible combinations of resource stock levels in all N patches) into the residual stock. Then finding the equilibrium across all patches would require solving a complicated N-dimensional system. Indeed, if we had specified *harvest* as the control variable, this would be the case. But since the marginal profit and marginal growth conditions depend only on the residual stock level, and not on resource stock, then Lemma 2 obtains, which dramatically simplifies the dynamic game. However, since biological growth, dispersal, and economic returns can vary across patches, the optimal choice will, in general, vary across space too.

2.3 Welfare & Distribution

Accounting for the behavior characterized in Section 2.2, we are initially interested in the consequences of partial enclosure on resource stocks. Because partial enclosure is second best, it is possible that perverse outcomes arise. Is it possible, for example, for partial enclosure, combined with spatial connectivity, to lead to lower natural resource stocks than under pure open access? We can unambiguously answer this question: it turns out that enclosing any patch will always increase resource stock. This is formalized below:

Proposition 2. *Partial enclosure of* any *patch (weakly) increases resource stock in all patches and strictly increases stock in at least one patch.*

Proposition 2 accords with economic intuition. When moving from a system of pure open access to a system in which some fraction of the resource is enclosed by a single owner, that owner will find it optimal to maintain a larger resource stock in her own patch. The adjacent open access patches clearly benefit from this behavior - to the extent that some of this increased resource spills over into adjacent patches, they are residual claimants of this behavior. Thus, the stock rises (weakly) everywhere.

Partial enclosure also has important consequences for profit as is formalized below:

Proposition 3. *Partial enclosure of* any *patch provides a strict Pareto-Improvement (i.e. increases profit in at least one patch without decreasing profit in any patch).*

A powerful, and somewhat counterintuitive corollary immediately emerges. Because the enclosed patch owner has the incentive to raise resource stock in every period, and because some of that resource spills over to the open access sector every period, the presence of *partial enclosure* guarantees positive profits in equilibrium for the open access sector, as is formalized below:

Corollary 1. *Some open access patches retain positive equilibrium profit under partial enclosure, even when extinction is optimal by the open access fringe.*

While Corollary 1 holds for any cost function, it is most striking when $c'(\cdot) = 0$ in which case all open access patches drive the stock extinct. Enclosure of any single patch k induces that patch owner to increase residual stock which bestows a positive externality on all patches j for which $D_{kj} \neq 0$. The result focuses on the positive rents accruing to open access patches when moving from a fully open access to a partial sole ownership management structure.

We have shown that even if exclusive ownership can be implemented only over a small part of the spatial domain, some positive effect is expected on resource stocks and agents' profits. These results arise from behavioral adaptations: when part of the spatial domain is enclosed, behavioral changes (by the enclosed patch owner and the open access sector) ensue.

Before concluding this section, we examine the role of key biological, environmental, and economic parameters on these behavioral changes. When patch i is enclosed, increasing self-retention (D_{ii}) or biological growth (α_i) causes the enclosed patch owner to increase her optimal residual stock. In contrast, increasing price (p_i) causes her to decrease residual stock. Increasing D_{ii} or α_i increases the rate of return to owner i from a larger residual stock, thus she favors a larger stock. The effect of price is more subtle: as it increases, the benefit of increased harvest (and thus lower residual stock) turns out to outweigh the future benefit of higher residual stock. These results are summarized below and we will make extensive use of them in the remainder of the analysis[7]:

Lemma 3. *When patch i is enclosed:*

$$\frac{d\bar{e}_i}{dD_{ii}} \geq 0, \quad \frac{d\bar{e}_i}{d\alpha_i} \geq 0, \quad \frac{d\bar{e}_i}{dp_i} \leq 0.$$

These results will prove useful in the optimal siting of partial enclosure of the commons.

[7]Due to Lemma 2, we might omit subscript t in the expressions of residual stocks and stock levels.

2.4 Siting Partial Enclosure

Thus far we have focused on the welfare effects of *partial enclosure of the commons*. In many real-world contexts a government, NGO, or private agent has the opportunity to enclose part of the commons. For example, current initiatives by development banks (such as the World Bank), NGOs (such as The Nature Conservancy or Rare), and countless local communities, often in developing countries with little or no existing formal governance, involve siting decisions where decision-makers must determine which areas to enclose. We have shown that even haphazard siting decisions will improve welfare and conservation. But there may be considerable differences across sites: enclosing the "right" patch may lead to substantially larger welfare gains, or may give rise to important distributional or conservation effects, than enclosing the "wrong" patch. This section is devoted to analyzing the characteristics of patches that are good candidates for enclosure. We use as a starting point the case when a single patch is enclosed and the remaining $N-1$ patches are open access.[8]

Which patch to enclose may depend on one's objective. For example, many resource conservation groups may advocate partial enclosure, say of a fishery or a forest area, to protect the natural resource itself, often arguing that protecting the resource stock is a first step toward enhancing local, or aggregate, profits. If the objective is to site the enclosure to maximize equilibrium resource stock, we can use the model to derive conditions which define the optimal candidate for enclosure. Indeed, inspecting the expression characterizing the evolution of stock levels, we can immediately characterize the optimal enclosure. It turns out that the difference $g(\bar{e_i}, \alpha_i) - g(e_i, \alpha_i)$ determines it (if the goal is to maximize aggregate resource stock): The decision-maker should enclose the patch with highest such difference (see Appendix for proof). Of course, calculating this difference requires calculating $\bar{e_i}$ and e_i which, in turn, are implicitly defined by economic returns and biological parameters. Thus, even though this condition has a straightforward interpretation, it is difficult to use it in order to assess the effect of key model parameters on optimal enclosure siting. Furthermore, there is no guarantee that maximizing resource stock will coincide with maximizing profits.

Thus, we will seek to determine the role of key model parameters on the optimal siting of partial enclosure by adopting the following strategy: We define a benchmark situation in which all economic and biological parameters are held equal across patches, thus enclosure does not favor any particular patch. Moving from this situation, we then increase the value of a single parameter in a particular

[8] While it may seem restrictive to only allow $1/N^{th}$ of the patches to be enclosed, recall that the patches can be different sizes, so we can consider enclosing any arbitrary percentage of the resource domain.

patch, and assess the impact of the change on economic and environmental objectives. Enclosing any given patch will give rise to dynamics throughout the system, which, in principle, affect all patches (including the enclosed patch), so the entire system's response to enclosure must be accounted for. This approach allows us to derive concrete conclusions about optimal enclosure siting while isolating the effects of any given parameter. We derive guiding principles for optimal enclosure under each possible objective.

In the benchmark case, $D_{ii} = D$, $D_{jk} = Q$, $\alpha_i = \alpha$, $p_i = p$, and $c_i(\cdot) = c(\cdot)\ \forall\ i$ and for $j \neq k$. Starting from this situation, patches are indistinguishable so there is no preference for enclosing any particular patch. Without loss of generality, we assume that a single parameter (either D_{11}, α_1, or p_1) increases in patch 1 and we calculate the comparative static effects given all possible enclosures. Because parameters are identical across all other patches j ($j \neq 1$), we need only calculate the effects of changing parameters in patch 1 when patch 1 is enclosed and when some other patch (we choose patch 2) is enclosed. Thus, under an increase in a single parameter in patch 1, we examine the following cases: (1) Patch 1 is enclosed and all $N-1$ other patches remain open access, and (2) Patch 2 is enclosed and all $N-1$ other patches remain open access.

This procedure allows us to determine, ceteris paribus, whether an increase in self retention (D), biological growth (α), or price (p) will favor, or relegate, a given patch for enclosure. We analyze optimal enclosure under four possible objectives: (1) maximize resource stock in the enclosed patch, (2) maximize aggregate stock, (3) maximize profit in the enclosed patch, and (4) maximize aggregate profit. For each objective, we compute the difference between the payoff when enclosing patch 1 and the payoff when enclosing patch 2. If that difference is positive, then a higher value of the parameter in patch 1 promotes the patch for enclosure (because enclosing patch 1 is preferred to enclosing patch 2). If the difference is negative, then a higher value of the parameter relegates the patch for enclosure (because enclosing patch 2 is preferred to enclosing patch 1).

Aside from the determination of optimal siting of an enclosure, we are interested in the conditions under which these four objectives are consistent or contradictory: If enclosure is sited to maximize local benefits, will this also maximize benefits system-wide? And if enclosure is sited to maximize resource stock, will this also maximize profit?

While our model permits any non-increasing $c(\cdot)$, we begin with the case of linear harvest cost, so $c'(\cdot) = 0$.[9] In that case, the open access patches always harvest the entire local resource stock in each period. Though by Proposition 2 the resource stock is positive in equilibrium - this is because the enclosed patch acts as a donor for all connected patches; provided that $Q > 0$, it supplies all patches

[9]In Section 2.5 we discuss how results may change when $c'(\cdot) < 0$.

with a surplus stock from which to harvest each period. While this case is not trivial to examine, the dynamics are somewhat muted because changes in system parameters do not change the optimal residual stock in the open access patches. Furthermore, the first order condition for the enclosed patch no longer relies on price or cost, which simplifies the comparative statics of behavioral changes.

2.4.1 Self-retention, D_{11}

We begin by assessing the effects of increasing the self-retention parameter D_{11} in patch 1 on the optimal siting of partial enclosure. Higher self-retention allows patch 1 to retain a greater fraction of its growth. If patch 1 is open access (so patch 2 is enclosed), then there is no behavioral shift in patch 1 from the increase in D_{11} - all residual stocks are unchanged. But if patch 1 is enclosed, that owner will increase her residual stock to take advantage of higher retention (see Lemma 3). Proving that profits are larger when patch 1 is enclosed (compared to enclosing some other patch) relies on an envelope theorem: to analyze the total derivative of profit in patch 1 we can ignore the behavioral shift, and can instead focus only on the direct influence of D_{11} on profit. This effect is clearly positive since all non-enclosed patches have zero residual stock. Taking all of the dynamics into account, we find that if costs are linear, then across all four objectives, a higher value of D promotes a patch for enclosure. This result is formalized below:

Proposition 4. *Provided $c'(\cdot) = 0$, a higher value of self-retention (D_{11}) in patch 1 has the following effects:*

1. *(Enclosed Patch Stock): Patch 1 is the best candidate for enclosure.*
2. *(Aggregate Stock): Patch 1 is the best candidate for enclosure.*
3. *(Enclosed Patch Profit): Patch 1 is the best candidate for enclosure.*
4. *(Aggregate Profit): Patch 1 is the best candidate for enclosure.*

Proposition 4 shows that ceteris paribus, across all four objectives, the optimal patch to enclose is the patch with higher self-retention. Thus there is strong consistency between optimal enclosure siting for individual benefit and optimal enclosure siting for aggregate benefit. There is also strong consistency between conservation objectives (i.e. maximizing resource stock) and economic objectives (maximizing profit).

2.4.2 Biological growth, α_1

Our resource growth model from Section 2 is quite general and permits a wide range of interpretations for the parameter α_i. Regardless of the interpretation, we can think of higher α_1 as representing *improved* growth conditions in patch 1. Here we examine the role of α_1 on optimal enclosure siting: will improved growth conditions in a patch promote it as a candidate for enclosure? If patch 1 is open access (so patch 2 is enclosed), there will be no adjustment in residual stock in patch 1 because $p_1 = c_1(e_1)$. On the other hand, if patch 1 is enclosed, then the increase in α_1 leads to an unambiguous increase in residual stock in 1 (see Lemma 3). This also has positive externalities on adjacent patches. Again, in the case of linear cost we will prove that across all four objectives, a higher value of α_1 promotes patch 1 for enclosure. The result is formalized as follows:

Proposition 5. *Provided $c'(\cdot) = 0$, a higher value of biological growth (α_1) in patch 1 has the following effects:*

1. *(Enclosed Patch Stock): Patch 1 is the <u>best</u> candidate for enclosure.*
2. *(Aggregate Stock): Patch 1 is the <u>best</u> candidate for enclosure.*
3. *(Enclosed Patch Profit): Patch 1 is the <u>best</u> candidate for enclosure.*
4. *(Aggregate Profit): Patch 1 is the <u>best</u> candidate for enclosure.*

Proposition 5 reveals that α has the same effects as D on optimal siting of the enclosure: Higher biological growth always promotes a patch as an ideal candidate for partial enclosure regardless of one's objective.

2.4.3 Market price, p_1

Finally we consider the effect of an increase in p_1 on optimal enclosure siting. A price increase in a single patch can have complex and far-reaching effects on the stock and profit because both the enclosed patch owner and the open access patches may change residual stock in response to the price increase. However, when $c'(\cdot) = 0$ (linear cost), neither the open access patch nor the enclosed patch will change residual stock following a price rise. Thus, residual stock is unaffected by the decision about which patch to enclose. However, profit is affected by the price rise, and thus the effects of higher price will depend on which patch is enclosed. Because resource stocks are unaffected, it is clear that to maximize enclosed patch profit, one should enclose the patch with the elevated price. But to maximize system-wide profit, it will depend on the spatial externality. When the patches are weakly connected (so Q is small), then the patch with high price should be enclosed. But when the patches are tightly connected, the patch with the elevated price should be left open access. These results are summarized as below.

Proposition 6. *Provided $c'(\cdot) = 0$, a higher price (p_1) in patch 1 has the following effects:*

1. *(Enclosed Patch Stock): All patches are equally desirable for enclosure.*
2. *(Aggregate Stock): All patches are equally desirable for enclosure.*
3. *(Enclosed Patch Profit): Patch 1 is the best candidate for enclosure.*
4. *(Aggregate Profit): Patch 1 is the worst candidate for enclosure if $D \leq Q$ or δ is small. Patch 1 is the best candidate for enclosure if Q is small and δ is large.*

Proposition 6 reveals an interesting tension between the biophysical parameters and the economic parameters. While we found that higher biophysical parameters always promoted a patch for enclosure, we find nearly the opposite for the key economic parameter, at least when harvest costs are linear.

Taken together, these results suggest that if the objective is to maximize local (i.e. enclosed patch) benefits from enclosure, it is typically optimal to enclose the patch with a high level of self-retention, a high biological parameter, or a high price. But if the goal is to site the enclosure to improve the system overall (whether system-wide profit or system-wide stock), the best candidate for enclosure may be a patch with high self-retention, high biological growth, or *low* price. This reveals an interesting tension between local and system-wide benefits. If the enclosure is to be sited by an agent who derives only local benefits, the enclosure may in fact *minimize* system-wide benefit, though this result can only occur if there is heterogeneity in economic returns across space. Conversely, if the enclosure is sited by a social planner who seeks to maximize aggregate benefits, payoffs in the local enclosure may suffer. Table 1 summarizes the results of Propositions 4-6, where a "+" indicates that the patch with elevated parameter is the best candidate for enclosure and a "-" indicates that the patch with the elevated parameter is the worst candidate for enclosure.

Table 1: Summary of results for linear cost ($c'(\cdot) = 0$)

Objective	D	α	p
Enclosed Stock	$+$	$+$	0
Aggregate Stock	$+$	$+$	0
Enclosed Profit	$+$	$+$	$+$
Aggregate Profit	$+$	$+$	$+/-$

2.5 Extension to non-linear cost

The siting results above have been proven only in the case of linear cost ($c'(\cdot) = 0$). While this is a common assumption in natural resource models, it fails to capture the possibility of a stock effect, where harvest costs rise as scarcity sets in. In that case $c'(\cdot) < 0$ which has important economic and behavioral implications. First, this assumption implies that even in the open access patches, stocks will not be fully exhausted. As the resource becomes scarce, the costs rise to such a degree that, even under open access, the marginal profit eventually hits zero and harvesting ceases. Second, in the enclosed patch the optimal residual stock level will depend on both price and residual stock from the open access fringe. These facts link the system together in a more nuanced way than when costs are linear, rendering the spatial externalities more complex. Thus, one might predict that there is an enhanced role for connectivity (both self-retention, D and migration Q) to drive results. Indeed, when $c'(\cdot) < 0$ we find that the result often hinges on connectivity. Table 2 summarizes our results for the case of $c'(\cdot) < 0$.

Table 2: Summary of results for nonlinear cost $c'(\cdot) < 0$

Objective	D	α	p
Enclosed Stock	$+$	$+$ if $D \geq Q$ $-$ if D small	$+$ if D small $-$ if Q small
Aggregate Stock	$+$	$+$ if $D \geq Q$ or D small	$+$ if D small
Enclosed Profit	$+$	$+$ if $D \geq Q$ $-$ if D small	$+$
Aggregate Profit	$+$ if $D \geq Q$	$+$ if $D \geq Q$ or D small	?

While these results are more nuanced than those derived when $c'(\cdot) = 0$, they are largely consistent. Three exceptions are worth pointing out. First, if the objective is to maximize enclosed patch stock, then the effect of higher biological growth on the optimal patch to enclose can flip depending on the value of self-retention. When self retention is large (in particular, when it is larger than Q) it is always optimal to enclose the patch with high biological growth (intuitively, because the high value of D allows the enclosed patch to capture most of the benefits of its larger α). But if self retention is sufficiently small, it is optimal to enclose a patch with lower self-retention: When self retention is small, the stock in a patch derives primarily from large values of residual stock in *other* patches. Thus, the enclosure owner would like his benefactor to have a high value of α (because he would claim the spill-over). The other exceptions involve the parameter p. When $c'(\cdot) = 0$, resource stock in all patches is unaffected by price, so no preference is given for enclosure. But when $c'(\cdot) < 0$ the optimal enclosure again depends on the extent of the spatial externality. To maximize enclosed patch stock, when self retention is small, one would like to enclose the patch with high price. This is because a higher price causes the enclosed patch owner to decrease his residual stock (see Lemma 3), but since D is small, this has little effect on his own stock. Instead, if the high price patch is open access, it will cause the open access patch to reduce its residual stock and when D is small, this has a greater (negative) impact on the enclosed patch stock. Similar reasoning explains why enclosing a low price patch is optimal when out-of-patch migration (Q) is small. A similar argument explains the effects of price on aggregate resource stock. The general case of price and its effect on aggregate profit cannot be signed, except for in special cases.

3 Illustrative Example

To illustrate the results of this analysis, we now present two versions of an example, which are easily replicated in a simple spreadsheet. We loosely base these examples on spatial analysis of fisheries near the Channel Islands, California (e.g. see White and Casselle 2008 [37]), which was explored by Costello and Kaffine 2009 [8]; we focus on the 13 patches of roughly 210 km^2 each surrounding the Northern Channel Islands (see Figure 1). The equation of motion in patch i is given by:

$$x_{it+1} = \sum_{j=1}^{13} D_{ji} \underbrace{\left[e_{jt} + r_j e_{jt} \left(1 - \frac{e_{jt}}{K_j} \right) \right]}_{g(e_j, \alpha_j)}. \tag{6}$$

Under this functional specification, both the intrinsic growth rate r_j and the carrying capacity K_j conform to the requirements of the general parameter α_j from Section 2.[10]

3.1 Heterogeneous patches

The first example allows the patches to be heterogeneous and calculates the benefits, and optimal siting, of enclosure. Dispersal (D_{ij}), growth (r_j), and carrying capacity (K_j) parameters are drawn loosely from real data in the region. Dispersal from patch j to i is given by the *dispersal kernel*, and is loosely based on White et al. 2013 [38]. We assume that both growth and carrying capacity are positively related to the fraction of the patch that is covered by kelp ($L_j \leq 1$), according to these relationships:

$$r_j = 0.4 + L_j^{1/2} \tag{7}$$
$$K_j = 100 + 1000 L_j \tag{8}$$

[10] Under this logistic growth model, $g(e) = e + re(1 - e/K)$. Both r and K adhere to requirements for α: $\frac{\partial g}{\partial r} = e(1 - e/K) > 0$, $\frac{\partial g}{\partial K} = re^2/K^2 > 0$, $\frac{\partial^2 g}{\partial r \partial e} = 1 - \frac{2e}{K} > 0$ and $\frac{\partial^2 g}{\partial K \partial e} = \frac{2re}{K^2} > 0$.

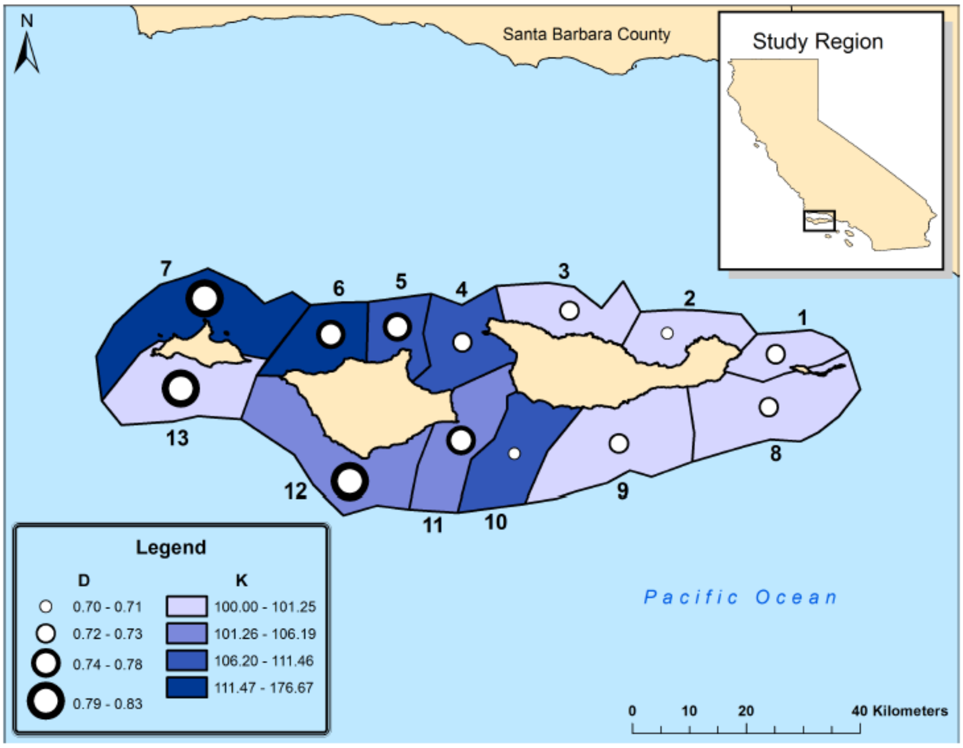
In this example, this produces intrinsic growth rates $r_j \in [0.40, 0.68]$ and carrying capacities $K_j \in [100, 177]$.[11] We assume marginal harvest cost is given by the function $c(s) = \theta/s$, and we use $\theta = 15$ (see White and Costello [39]). Price is set to unity, and we use $\delta = .9$.

Under this model, the open access equilibrium residual stock is given by: $\hat{e} = 15$ in all patches (see Proposition 1), which is confirmed by this numerical application. Thus, under this parameterization, the open access residual stock is 8%-15% of carrying capacity (depending on the patch). Enclosing patches one-by-one, while the other 12 patches remain open access, generates optimal residual stock level in the enclosed patch of between 18.7 (patch 1) and 47.1 (patch 7) with an average of 26.3 (see Lemma 1). It is not immediately obvious which patch to enclose to achieve different objectives. Using the guidance from Propositions 4-6, patch 12 has the highest self-retention and patch 6 has the largest values of r and K. However, while patch 7 does not have the largest value of any single parameter, it does have relatively large values of all parameters. Stock in all non-enclosed patches increases as a consequences of enclosure (see Proposition 2). In our numerical example, the increase in system-wide stock arising from partial enclosure depends on which patch is enclosed. It ranges from 4.1 (when enclosing patch 1, representing just a 2% increase in stock) to 38 (when enclosing patch 7, representing a 16% increase). Consistent with Proposition 3, equilibrium profit in all non-enclosed patches increases as a consequence of enclosure. As was the case with stock, the increase in system-wide profit arising from partial enclosure ranges from .2% (when enclosing patch 1) to 5% (when enclosing patch 6).

In this example, it turns out that for three of the four objectives (enclosed patch stock, aggregate stock, and enclosed patch profit), patch 7 is the optimal enclosure. The optimal enclosure to maximize aggregate profit is patch 6. Figure 1 shows the study area and displays the values of D in circles and K (which is correlated with r in this example) colored shading. From this figure, it is clear why patches 6 and 7 are good candidates for enclosure.

[11] We provide in the Appendix all input parameters necessary to replicate all of these results.

Figure 1: Study region, TURF delineation, and optimal enclosure site (shaded).



3.2 Homogeneous patches

Our second example illustrates the comparative statics of optimal enclosure derived in Section 2.5. It builds on the first example, but adopts the starting point from Section 2.4 that all patches are symmetric. For r_i and K_i we assume that each patch has the average value of those parameters from the previous example (so $r = .49$, $K = 114.2$). For D_{ij} we assume that all off-diagonal terms are $Q = .06$ and that the diagonal terms are $D = .20$ (though we also explore values of $D \in [0, .30]$). We continue to assume $\delta = 0.9$, $p = 1$, and $\theta = 15$. Together, these assumptions yield a completely symmetric set of patches. From this starting point, it is equally desirable to enclose any one of the 13 patches, though doing so only leads to a small increase in stock (by 1%) and equilibrium profit (by 3%).

Following the theoretical treatment, we numerically calculate the comparative statics associated with a 10% increase in each parameter in a single patch. We do so by incrementing a single parameter in patch 1 only, holding all other parameters at their initial values, and calculating the subsequent effects on the entire system. Our main focus is on how this change in a parameter will affect the optimal patch to enclose under the various objectives spelled out above.

3.2.1 Comparative statics: D_{11}

When the self-retention parameter in patch 1 is increased, it implies that patch 1 is a stronger residual claimant of conservative harvesting behavior than are the other patches. Thus, consistent with Proposition 4 and with the extended results in Table 2, we find that patch 1 is the optimal enclosure. This result holds across all four objectives. When patch 1 is enclosed, a 10% increase in D_{11} (from 0.2 to 0.22) leads to $< 1\%$ increases in stock and profit.

3.2.2 Comparative statics: r_1 and K_1

The parameters r_i and K_i are both special cases of the more general biological growth parameter α_i considered in the analytical model. Thus, the comparative statics in Section 2.5 apply to both r and K. To maximize enclosed patch stock, we find that a larger value of r or a larger value of K promotes a patch for enclosure, provided $D > .06$, and it relegates a patch for enclosure if $D < .06$. We obtain a very similar result for the objective of maximizing enclosed patch profit, though the cutoff values of D are slightly larger. To maximize aggregate stock we found a similar result for r (though the cutoff is $D = .01$), and we found that higher value of K always promotes a patch for enclosure. Finally, when the goal is to maximize aggregate profit, we found that higher values of r or K always promote a patch for enclosure. This result holds for all values of D. Again, in the homogeneous patch case considered here, we find small elasticities: a 10% increase in r or K in a patch leads to less than 1% increases in aggregate stock or profit.

3.2.3 Comparative statics: p_1

Finally we consider numerically the effects of an increase in price. When p_1 is increased, the optimal patch to enclose depends on the objective being pursued and on the extent of the spatial externality, via D and Q. If the objective is to maximize profit in the enclosed patch or aggregate stock, we find an unambiguous result that higher p promotes a patch for enclosure. If the goal is to maximize enclosed patch stock, we find a mixed result: when $D < .06$, we find that higher p promotes a patch for enclosure. But when $D > .06$, higher price relegates a patch. This is consistent with the theoretical finding reported in Table 2 that small D promotes a patch for enclosure and small Q relegates a patch for enclosure. Finally, if the objective is to maximize aggregate profit, we find the unambiguous result that higher p always relegates a patch for enclosure, regardless of the value of D.

Each of these results is consistent with the theoretical results reported in Table 2. While the example is meant to be illustrative only, and not to provide specific policy guidance for the Channel Islands, it does demonstrate the ease and utility with which the model developed here can be applied in a real world context.

4 Conclusion

Partial enclosure of the commons is perhaps the most common institutional arrangement for governing renewable natural resources, yet it has received almost no attention in the literature. We define it as a circumstance in which *part* of a spatially-connected resource is controlled by a sole owner, but the remainder is competed for by an open access fringe. Because the resource is mobile, each group imposes an externality on others. We develop a spatial bioeconomic model to address questions such as: Under what conditions will *partial enclosure* lead to aggregate (or individual) welfare gains? What will be the consequences of *partial enclosure* on the open access fringe? What are the ecological effects of *partial enclosure*? And, for different objectives, in which patches should *partial enclosure* be undertaken? The framework allows us to make sharp analytical predictions, which are then illustrated with a numerical example of a spatially-connected fishery surrounding the Northern Channel Islands.

Perhaps the most salient welfare implication of partial enclosure is that it always leads to a strict Pareto Improvement over open access. This conclusion holds whether the agents' objectives are based solely on profit or are motivated by conservation, and it holds even when one assigns partial enclosure haphazardly. We also explored the environmental (via D_{ij}), biological (via α_i), and economic (via p_i) characteristics that make a patch a particularly good candidate for enclosure. We found that, ceteris paribus, if a patch has higher self-retention, it is a good candidate for enclosure regardless of one's objective. Here, there is strong consistency between individual and aggregate welfare and between stock and profit as objectives. But we found that patches with higher biological parameters (such as growth rate or patch size) may not be ideal candidates, depending on one's objective. In that case, the optimal enclosure location can be reversed depending on whether one is interested in enclosed patch outcomes or aggregate outcomes. Finally, economic returns and resource growth have potentially opposite comparative effects.

Overall, these findings suggest that partial enclosure of the commons is a potentially valuable (though second best) institutional arrangement with positive economic and environmental consequences. Our comparative results emphasize that optimal siting of enclosures are often consistent between individual and societal objectives and between conservation and profit motives. But the analysis also illuminates interesting tensions where the optimal siting of partial enclosure can impact negatively on some agents. In those cases, policy interventions such as monetary transfers may be designed to remove these tensions.

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5 Appendix

Proof of Lemma 1

The result follows immediately from the first order conditions.

Proof of Lemma 2

By the first order condition in any period t, open access optimal residual stock levels are independent of all stock levels (the state vector) in that same period. The same result holds for the optimal residual stock level of the enclosed patch. Let patch j be enclosed. When the optimal residual stock level is positive, the optimality condition is necessary and sufficient. The term on the left hand side of this condition reflects the marginal contribution of residual stock to current period payoff, and is independent of x_t by inspection. The derivative of the payoff function in period $t+1$ depends on the period $t+1$ state, but is independent of the period t state. Since we know that an interior solution satisfies $\bar{e}_{jt} < \bar{x}_{jt}$ and using Expression 2, $\bar{x}_{jt+1}$ is a function of $\bar{e}_{jt}$ but not of $\bar{x}_{jt}$. Therefore, the term on the right hand side in the first order condition is independent of $\bar{x}_{jt}$, and the period t problem of the sole owner has state independent control.

Second, we must prove that optimal residual stock levels are time independent. Again, since economic returns are independent of time, open access optimal residual stock levels are time independent. Now, regarding patch j, we just proved that $\bar{e}_{jt}$ is independent of $\bar{x}_{jt}$. This implies that a change in stock in the next period affects the payoff function in $t+1$ only through the term relating to $\bar{x}_{jt+1}$. Since biological growth, dispersal, and economic returns are time-independent, the optimal choice, $\bar{e}_{jt}$, is also time-independent.

Proof of Proposition 2

Let $\hat{e}_i$ denote residual stock in patch i when all patches are open access and let e_i denote residual stock in patch i when patch j is enclosed, but all other patches i are open access. First, note that $\hat{e}_i = e_i$ for $i \neq j$ (because $p_i = c_i(\hat{e}_i) = c_i(e_i)$). For patch j, the residual stock under open access is $p_j - c_j(\hat{e}_j) = 0$. But under enclosure, residual stock in j is given by the FOC: $p_j - c_j(\bar{e}_j) = \delta D_{jj}\left[p_j - c_j(\bar{x}_j)\right] g_{\bar{e}}(\bar{e}_j, \alpha_j)$. The right hand side of this expression is > 0, thus $p_j - c_j(\bar{e}_j) > 0$, so $\bar{e}_j > \hat{e}_j$. The difference in patch i stock in the enclosed case minus the open access case is simply $D_{ji}g_j(\bar{e}_j) - D_{ji}g_j(\hat{e}_j)$. Because $g_e > 0$, stock is higher in all patches i such that $D_{ji} > 0$ and is unchanged in all patches i such that $D_{ji} = 0$.

Proof of Proposition 3

Without loss of generality, let patch j be enclosed. If patch j chooses the open access residual stock level, then all patches are indifferent to the enclosure. But if patch j chooses a different residual stock level, then patch j must do so to increase her profit. The proof to Proposition 2 shows that patch j chooses a residual stock larger than the open access level, so patch j must be better off under the enclosure. Again the proof to Proposition 2 shows that if $D_{ji} > 0$, then patch i receives a higher stock, and is thus better off under the enclosure. Instead, if $D_{ji} = 0$, then patch i stock is unchanged under the enclosure, and thus patch i is indifferent to the enclosure.

Proof of Corollary 1

Follows immediately from the evolution rule of the stock levels.

Proof of Lemma 3

Applying the implicit function theorem to the first order condition gives the relevant total derivatives:

$$\begin{aligned}
\frac{d\bar{e}_j}{dD_j} &= -\frac{\delta g_{\bar{e}}(e_j,\alpha_j)\left[p_j - c_j(\bar{x}_j) - D_{jj}c_j'(\bar{x}_j)\cdot g(\bar{e}_j,\alpha_j)\right]}{SOC} \\
\frac{d\bar{e}_j}{dp_j} &= \frac{1-\delta D_{jj} g_{\bar{e}}(\bar{e}_j,\alpha_j)}{SOC} \\
\frac{d\bar{e}_j}{d\alpha_j} &= -\frac{\delta D_{jj}\left[-g_{\bar{e}}(\bar{e}_j,\alpha_j)\cdot g_\alpha(\bar{e}_j,\alpha_j)c_j'(\bar{x}_j)D_{jj} + (p_j - c_j(\bar{x}_j))\, g_{\bar{e}\alpha}(\bar{e}_j,\alpha_j)\right]}{SOC}
\end{aligned}$$

with $SOC = c_j'(\bar{e}_j) + \delta D_{jj}\left[-D_{jj}c_j'(\bar{x}_j)\left(g_{\bar{e}_j}(\bar{e}_j,\alpha_j)\right)^2 + (p_j - c_j(\bar{x}_j))\, g_{\bar{e}_j\bar{e}_j}(\bar{e}_j,\alpha_j)\right] < 0$, which is the second order condition. The numerators of the fraction in the expressions of $\frac{d\bar{e}_j}{dD_{jj}}$ and $\frac{d\bar{e}_j}{d\alpha_j}$ are unambiguously non-negative. Therefore, $\frac{d\bar{e}_j}{dD_{jj}} \geq 0$ and $\frac{d\bar{e}_j}{d\alpha_j} \geq 0$. Finally, by the first order condition for $\bar{e}_j$, the numerator of $\frac{d\bar{e}_j}{dp_j}$ is non-negative, thus $\frac{d\bar{e}_j}{dp_j} \leq 0$.

Proof of the claim in Section 2.4

First, let us assume that patch j is enclosed while all other patches are under open access. Now, for $i \neq j$, we have :

$$\bar{x}_j = D_{jj}g\left(\bar{e}_j,\alpha_j\right) + \sum_{k\neq j}^{N} D_{kj}g\left(e_k,\alpha_k\right) \;\; ; \;\; x_i = D_{ji}g\left(\bar{e}_j,\alpha_j\right) + \sum_{k\neq j}^{N} D_{ki}g\left(e_k,\alpha_k\right)$$

If enclosing patch j yields the highest value of the aggregate stock level, the following inequality is satisfied :

$$\bar{x}_j + \sum_{l\neq j}^{N} x_l \geq \max\left\{\bar{x}_1 + \sum_{k=2}^{N} x_k; \ldots; \bar{x}_N + \sum_{l\neq N}^{N} x_l\right\}$$

Let us assume that this inequality holds. We are going to compare the expression of aggregate stock levels when patch j is enclosed and when another patch (say i) is enclosed (assuming in both cases that all other patches remain under open access). We obtain the equivalent inequalities :

$$
\begin{aligned}
& D_{jj} g\left(\bar{e}_j, \alpha_j\right)+\sum_{k \neq j}^N D_{kj} g\left(e_k, \alpha_k\right)+\sum_{l \neq j}^N\left(D_{jl} g\left(\bar{e}_j, \alpha_j\right)+\sum_{k \neq j}^N D_{kl} g\left(e_k, \alpha_k\right)\right) \\
> \; & D_{ii} g\left(\bar{e}_i, \alpha_i\right)+\sum_{k \neq i}^N D_{ki} g\left(e_k, \alpha_k\right)+\sum_{l \neq i}^N\left(D_{il} g\left(\bar{e}_i, \alpha_i\right)+\sum_{k \neq i}^N D_{kl} g\left(e_k, \alpha_k\right)\right) \\
\Leftrightarrow \; & g\left(\bar{e}_j, \alpha_j\right)+\sum_{l \neq j}^N g\left(e_l, \alpha_l\right)>g\left(\bar{e}_i, \alpha_i\right)+\sum_{l \neq i}^N g\left(e_l, \alpha_l\right) \\
\Leftrightarrow \; & g\left(\bar{e}_j, \alpha_j\right)+g\left(e_i, \alpha_i\right)>g\left(\bar{e}_i, \alpha_i\right)+g\left(e_j, \alpha_j\right) \\
\Leftrightarrow \; & g\left(\bar{e}_j, \alpha_j\right)-g\left(e_j, \alpha_j\right)>g\left(\bar{e}_i, \alpha_i\right)-g\left(e_i, \alpha_i\right)
\end{aligned}
$$

Setup of proofs to Propositions 4-6

Without loss of generality, we will assume that a single parameter is elevated in patch 1 and we explore the consequences of enclosing patch 1 or patch 2. We indicate the enclosed patch by placing a bar over its relevant variables ($\bar{x}$, $\hat{e}$, $\bar{\alpha}$, $\hat{D}$ and $\hat{p}$). We indicate an open access patch (which may have an elevated parameter) without a bar, e.g. x. Finally, we must also account for the other $N-2$ patches $3, 4, ..., N$ which neither have elevated parameters nor are enclosed. We denote the representative patch with a tilde, e.g. $\tilde{x}$. Prior to any change in parameters, the three equations of motion are given by:

$$\bar{x} = Dg(\bar{e}, \bar{\alpha}) + (N-1)Qg(e, \alpha) \tag{9}$$

$$x = [D + (N-2)Q]\, g(e, \alpha) + Qg(\bar{e}, \bar{\alpha}) \tag{10}$$

$$\tilde{x} = [D + (N-2)Q]\, g(e, \alpha) + Qg(\bar{e}, \bar{\alpha}) \tag{11}$$

By Lemma 3, the optimal residual stock is time-independent, which implies that the profit expressions are given by :

$$\bar{\Pi} = \bar{p}(x_0 - \bar{e}) - \int_{\bar{e}}^{x_0} c(s)ds + \frac{\delta}{1-\delta}\left[\bar{p}(\bar{x} - \bar{e}) - \int_{\bar{e}}^{\bar{x}} c(s)ds\right] \tag{12}$$

$$\Pi = p(x_0 - e) - \int_{e}^{x_0} c(s)ds + \frac{\delta}{1-\delta}\left[p(x - e) - \int_{e}^{x} c(s)ds\right] \tag{13}$$

$$\tilde{\Pi} = \tilde{p}(x_0 - \tilde{e}) - \int_{\tilde{e}}^{x_0} c(s)ds + \frac{\delta}{1-\delta}\left[\tilde{p}(\tilde{x} - \tilde{e}) - \int_{\tilde{e}}^{\tilde{x}} c(s)ds\right] \tag{14}$$

where x_0 is the initial stock level. We examine comparative statics for three parameters (D, α, and p). For any parameter $\theta \in \{D, \alpha, p\}$, total differentiation gives the expressions in Table 3.

To determine whether it is advantageous to enclose patch 1 (the patch with the elevated

Table 3: Comparative statics

Enclosed	Profit	Stock	# patches
1	1: $\frac{d\bar{\Pi}}{d\bar{\theta}} = \frac{\partial\bar{\Pi}}{\partial\bar{\theta}}$	1: $\frac{d\bar{x}}{d\bar{\theta}} = \frac{\partial\bar{x}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\bar{\theta}} + \frac{\partial\bar{x}}{\partial\bar{\theta}}$	1
	2: $\frac{d\Pi}{d\bar{\theta}} = \frac{\partial\Pi}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\bar{\theta}} + \frac{\partial\Pi}{\partial\bar{\theta}}$	2: $\frac{dx}{d\bar{\theta}} = \frac{\partial x}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\bar{\theta}} + \frac{\partial x}{\partial\bar{\theta}}$	$N-1$
2	1: $\frac{d\Pi}{d\theta} = \frac{\partial\Pi}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\Pi}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\theta} + \frac{\partial\Pi}{\partial\theta}$	1: $\frac{dx}{d\theta} = \frac{\partial x}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial x}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\theta} + \frac{\partial x}{\partial\theta}$	1
	2: $\frac{d\bar{\Pi}}{d\theta} = \frac{\partial\bar{\Pi}}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\bar{\Pi}}{\partial\theta}$	2: $\frac{d\bar{x}}{d\theta} = \frac{\partial\bar{x}}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\bar{x}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\theta} + \frac{\partial\bar{x}}{\partial\theta}$	1
	k: $\frac{d\tilde{\Pi}}{d\theta} = \frac{\partial\tilde{\Pi}}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\tilde{\Pi}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\theta} + \frac{\partial\tilde{\Pi}}{\partial\theta}$	k: $\frac{d\tilde{x}}{d\theta} = \frac{\partial\tilde{x}}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\tilde{x}}{\partial\bar{e}}\frac{\partial\bar{e}}{\partial\theta} + \frac{\partial\tilde{x}}{\partial\theta}$	$N-2$

parameter) or patch 2 (a patch without the elevated parameter), we compute the differences:

$$\text{Enclosed Stock: } \Delta_1(\theta) \equiv \frac{d\bar{x}}{d\bar{\theta}} - \frac{d\bar{x}}{d\theta} \tag{15}$$

$$\text{Aggregate Stock: } \Delta_2(\theta) \equiv \frac{d\bar{x}}{d\bar{\theta}} + (N-1)\frac{dx}{d\bar{\theta}} - \left(\frac{d\bar{x}}{d\theta} + \frac{dx}{d\theta} + (N-2)\frac{d\tilde{x}}{d\theta}\right) \tag{16}$$

$$\text{Enclosed Profit: } \Delta_3(\theta) \equiv \frac{d\bar{\Pi}}{d\bar{\theta}} - \frac{d\bar{\Pi}}{d\theta} \tag{17}$$

$$\text{Aggregate Profit: } \Delta_4(\theta) \equiv \frac{d\bar{\Pi}}{d\bar{\theta}} + (N-1)\frac{d\Pi}{d\bar{\theta}} - \left(\frac{d\bar{\Pi}}{d\theta} + \frac{d\Pi}{d\theta} + (N-2)\frac{d\tilde{\Pi}}{d\theta}\right) \tag{18}$$

And we can use the total derivative calculations in the table 3 to re-write Equations 15-18 as partial derivatives. For example, $\Delta_3(\theta) = \frac{\partial\bar{\Pi}}{\partial\bar{\theta}} - \frac{\partial\bar{\Pi}}{\partial e}\frac{\partial e}{\partial\theta} + \frac{\partial\bar{\Pi}}{\partial\theta}$.

Depending on the parameter being examined, many of these partial derivative terms are zero. For example, price in an open access patch has no direct influence on an enclosed patch stock, so $\frac{\partial\bar{x}}{\partial p} = 0$. All of the terms in the following table equal zero:

Table 4: Conditions

	D	α	p
Enclosed Stock	$\frac{\partial e}{\partial D}, \frac{\partial\bar{e}}{\partial D}, \frac{\partial\bar{x}}{\partial D}$	$\frac{\partial e}{\partial\alpha}$	$\frac{\partial\bar{x}}{\partial\bar{p}}, \frac{\partial\bar{x}}{\partial p}, \frac{\partial\bar{e}}{\partial p}, \left(\frac{\partial e}{\partial p}\right)$
Aggregate Stock	$\frac{\partial e}{\partial D}, \frac{\partial\bar{e}}{\partial D}, \frac{\partial\bar{x}}{\partial D}, \frac{\partial x}{\partial\bar{D}}, \frac{\partial\tilde{x}}{\partial D}$	$\frac{\partial e}{\partial\alpha}, \left(\frac{\partial\bar{e}}{\partial\alpha}\right), \left(\frac{\partial\bar{x}}{\partial\alpha}\right)$	$\frac{\partial\bar{x}}{\partial\bar{p}}, \frac{\partial\bar{x}}{\partial p}, \frac{\partial\bar{e}}{\partial p}, \left(\frac{\partial e}{\partial p}\right), \frac{\partial x}{\partial\bar{p}}, \frac{\partial x}{\partial p}, \frac{\partial\tilde{x}}{\partial p}$
Enclosed Profit	$\frac{\partial e}{\partial D}, \frac{\partial\bar{\Pi}}{\partial D}$	$\frac{\partial e}{\partial\alpha}$	$\frac{\partial\bar{\Pi}}{\partial p}$
Aggregate Profit	$\frac{\partial e}{\partial D}, \frac{\partial\bar{\Pi}}{\partial D}, \frac{\partial\bar{e}}{\partial D}, \frac{\partial\Pi}{\partial\bar{D}}$	$\frac{\partial e}{\partial\alpha}, \left(\frac{\partial\bar{e}}{\partial\alpha}\right), \left(\frac{\partial\bar{x}}{\partial\alpha}\right)$	$\frac{\partial\bar{\Pi}}{\partial p}, \frac{\partial\Pi}{\partial\bar{p}}, \frac{\partial\tilde{\Pi}}{\partial p}, \frac{\partial\bar{e}}{\partial p}$

The parenthetical terms (e.g. $\frac{\partial e}{\partial p}$) equal zero only if $= 0$. We will make extensive use of Table 4 in the proofs that follow. For each of the four objectives (enclosed stock, aggregate stock, enclosed profit, aggregate profit) and for each of the three parameters (D, α, and p) we analyze the effects of enclosing patch 1 minus patch 2. This difference is given in Equations 15-18.

Proof of Proposition 4 (self-retention, D)

1. **Enclosed patch stock**
   Adopting the conditions in Table 4, the difference 15 is $\Delta_1(D) = \frac{d\bar{x}}{d\bar{D}} > 0$.
2. **Aggregate stock**
   Adopting the conditions in Table 4, the difference 16 is:

   $$\Delta_2(D) \quad = \quad \frac{\partial \bar{e}}{\partial \bar{D}}\left(\frac{\partial \bar{x}}{\partial \bar{e}} + (N-1)\frac{\partial x}{\partial \bar{e}}\right) + \frac{\partial \bar{x}}{\partial \bar{D}} - \frac{\partial x}{\partial D} = \frac{\partial \bar{e}}{\partial \bar{D}}\left(\frac{\partial \bar{x}}{\partial \bar{e}} + (N-1)\frac{\partial x}{\partial \bar{e}}\right) + g(\bar{e},\bar{\alpha}) - g(e,\alpha)$$

   Each of these terms is positive since the growth rate function is increasing and $\bar{e} > e$ and $\bar{\alpha} = \alpha$ (because in this case the variable of interest is D), so $\Delta_2(D) > 0$.
3. **Enclosed patch profit**
   Adopting the conditions in Table 4, the difference 17 is $\Delta_3(D) = \frac{d\bar{\Pi}}{d\bar{D}} > 0$.
4. **Aggregate profit**
   Adopting the conditions in Table 4, the difference 18 is:

   $$\Delta_4(D) = \frac{\partial \bar{\Pi}}{\partial \bar{D}} - \frac{\partial \Pi}{\partial D} + (N-1)\frac{\partial \Pi}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \bar{D}}.$$

   First, note that the last term is always positive. Now, regarding the first term, we have :

   $$\frac{\partial \bar{\Pi}}{\partial \bar{D}} - \frac{\partial \Pi}{\partial D} = [p - c(\bar{x})]g(\bar{e},\bar{\alpha}) - [p - c(x)]g(e,\alpha)$$

   Note that if $c' = 0$, this term is $(p-c)[g(\bar{e},\bar{\alpha}) - g(e,\alpha)] > 0$ since g is increasing and $\bar{e} > e$. This implies that $\Delta_4(D) > 0$. If $c' < 0$, because $g_e > 0$, a sufficient condition for this term being positive is $\bar{x} > x$. The difference between Equations 9 and 10 is:

   $$\bar{x} - x = (D - Q)\left[g(\bar{e},\bar{\alpha}) - g(e,\alpha)\right]$$

   If $D \geq Q$ then the term is positive and $\Delta_4(D) > 0$.

Proof of Proposition 5 (biological growth, α)

We will use the following notations for the derivatives in the remainder of this appendix: $g_e = g_e(e,\alpha)$, $g_{\bar{e}} = g_e(\bar{e},\bar{\alpha})$, $g_\alpha = g_\alpha(e,\alpha)$, $g_{\bar{\alpha}} = g_\alpha(\bar{e},\bar{\alpha})$, and $g_{\bar{e},\bar{\alpha}} = g_{e\alpha}(\bar{e}\bar{\alpha})$.

1. **Enclosed patch stock**
   Adopting the conditions in Table 4 the difference 15 is:

   $$\Delta_1(\alpha) = \frac{\partial \bar{x}}{\partial \bar{e}}\underbrace{\left(\frac{\partial \bar{e}}{\partial \bar{\alpha}} - \frac{\partial \bar{e}}{\partial \alpha}\right)}_{\equiv A} + \left(\frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha}\right). \tag{19}$$

First note that if $c' = 0$, $\frac{\partial \bar{x}}{\partial \alpha} = \frac{\partial \bar{e}}{\partial \alpha} = 0$ and $\Delta_1(\alpha) > 0$. If $c' < 0$, then using Equation 9, we have $\frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha} = Dg_{\bar{\alpha}} - Qg_{\alpha}$. This term is positive provided $D \geq Q$, and is negative for sufficiently small D. The other parenthetical term involves analyzing

$$\begin{aligned}
\frac{\partial \bar{e}}{\partial \bar{\alpha}} &= \frac{\delta D \left(g_{\hat{e}} c'(\bar{x}) \frac{\partial \bar{x}}{\partial \bar{\alpha}} - [p - c(\bar{x})] g_{\bar{e}\bar{\alpha}} \right)}{SOC} > 0 \qquad (20) \\
\frac{\partial \bar{e}}{\partial \alpha} &= \frac{\delta D \left(g_{\hat{e}} c'(\bar{x}) \frac{\partial \bar{x}}{\partial \alpha} \right)}{SOC} > 0 \qquad (21)
\end{aligned}$$

The denominator of both terms is the second order condition, which is negative. Subtracting the expressions gives:

$$\frac{\partial \bar{e}}{\partial \bar{\alpha}} - \frac{\partial \bar{e}}{\partial \alpha} = \frac{\delta D \left(g_{\bar{e}} c'(\bar{x}) (\frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha}) - [p - c(\bar{x})] g_{\bar{e}\bar{\alpha}} \right)}{SOC} \qquad (22)$$

Because $g_{\bar{e}\bar{\alpha}} > 0$, a sufficient condition for this term being positive is $D \geq Q$. Instead, suppose $D = 0$. In that case, term $A = 0$ and $\frac{\partial \bar{x}}{\partial \bar{\alpha}} = 0$. The entire expression is negative. By a continuity argument, this implies that $\Delta_1(\alpha) < 0$ for sufficiently small values of D. To summarize, if $= 0$, we have $\Delta_1(\alpha) > 0$; if < 0 then if $D \geq Q$, we have $\Delta_1(\alpha) > 0$, and if D sufficiently small, we have $\Delta_1(\alpha) < 0$.

2. **Aggregate stock**
   Adopting the conditions in Table 4, the difference 16 is:

$$\begin{aligned}
\Delta_2(\alpha) &= \frac{\partial \bar{x}}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \bar{\alpha}} + \frac{\partial \bar{x}}{\partial \bar{\alpha}} + (N-1) \left(\frac{\partial x}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \bar{\alpha}} + \frac{\partial x}{\partial \bar{\alpha}} \right) \\
&- \left[\frac{\partial x}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \bar{\alpha}} + \frac{\partial x}{\partial \alpha} + \frac{\partial \bar{x}}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \alpha} + \frac{\partial \bar{x}}{\partial \alpha} + (N-2) \left(\frac{\partial \tilde{x}}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \alpha} + \frac{\partial \tilde{x}}{\partial \alpha} \right) \right]
\end{aligned}$$

We use the fact that $\frac{\partial x}{\partial \bar{e}} = \frac{\partial \tilde{x}}{\partial \bar{e}}$ to rewrite $\Delta_2(\alpha)$:

$$\Delta_2(\alpha) = \underbrace{\left(\frac{\partial \bar{x}}{\partial \bar{e}} + (N-1) \frac{\partial x}{\partial \bar{e}} \right)}_{>0} \underbrace{\left(\frac{\partial \bar{e}}{\partial \bar{\alpha}} - \frac{\partial \bar{e}}{\partial \alpha} \right)}_{A} + (N-2) \left(\frac{\partial x}{\partial \bar{\alpha}} - \frac{\partial \tilde{x}}{\partial \alpha} \right) + \frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha} + \frac{\partial x}{\partial \bar{\alpha}} - \frac{\partial x}{\partial \alpha}$$

First note that the last term composed by the derivatives of stocks (w.r.t α) are unambiguously positive since it involves analyzing Equations 9 and 10 as follows:

$$\frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha} + \frac{\partial x}{\partial \bar{\alpha}} - \frac{\partial x}{\partial \alpha} = (D + Q)(g_{\bar{\alpha}} - g_{\alpha}) > 0.$$

Then the other parenthetical term involves analyzing Equations 9 and 11 such that: $\frac{\partial x}{\partial \bar{\alpha}} - \frac{\partial \tilde{x}}{\partial \alpha} = Q(g_{\bar{\alpha}} - g_{\alpha})$. We now provide sufficient conditions that ensure that term A is positive, which would enable us to conclude that $\Delta_2(\alpha)$ is positive. First, note that if $c' = 0$, $\frac{\partial \bar{x}}{\partial \alpha} = \frac{\partial \bar{e}}{\partial \alpha} = 0$, then the difference $\Delta_2(\alpha)$ is unambiguously positive. If $c' < 0$, then the difference $\Delta_2(\alpha)$ is positive provided $D \geq Q$ or $D = 0$ which, by a continuity argument, enables to conclude that $\Delta_2(\alpha) > 0$ for sufficiently small value of D.

3. **Enclosed patch profit**
   Adopting the conditions in Table 4, the difference 17 is:

$$\Delta_3(\alpha) = \frac{d\bar{\Pi}}{d\bar{\alpha}} - \frac{\partial \bar{\Pi}}{\partial \alpha} = \frac{\delta}{1-\delta}(p - c(\bar{x})) \left(\frac{\partial \bar{x}}{\partial \bar{\alpha}} - \frac{\partial \bar{x}}{\partial \alpha} \right) = \frac{\delta}{1-\delta}(p - c(\bar{x}))(Dg_{\bar{\alpha}} - Qg_{\alpha})$$

First note that if $c' = 0$ then $\frac{\partial \bar{x}}{\partial \alpha} = 0$ and $\Delta_3(\alpha)$ is unambiguously positive. If $c' < 0$, then this term is positive provided $D \geq Q$, and is negative for sufficiently small D.

4. **Aggregate profit**
Adopting the conditions in Table 4, the difference 18 is:

$$\begin{aligned}\Delta_4(\alpha) &= \frac{\partial \bar{\Pi}}{\partial \bar{\alpha}} + (N-1)\left(\frac{\partial \Pi}{\partial \bar{e}} + \frac{\partial \bar{e}}{\partial \alpha} + \frac{\partial \Pi}{\partial \bar{\alpha}}\right) \\ &- \left[\frac{\partial \Pi}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \alpha} + \frac{\partial \Pi}{\partial \alpha} + \frac{\partial \bar{\Pi}}{\partial \alpha} + (N-2)\left(\frac{\partial \tilde{\Pi}}{\partial e}\frac{\partial e}{\partial \alpha} + \frac{\partial \tilde{\Pi}}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \alpha} + \frac{\partial \tilde{\Pi}}{\partial \alpha}\right)\right]\end{aligned}$$

We use the fact that $\frac{\partial \Pi}{\partial \bar{e}} = \frac{\partial \tilde{\Pi}}{\partial \bar{e}}$ to rewrite $\Delta_4(\alpha)$:

$$\Delta_4(\alpha) = (N-1)\frac{\partial \Pi}{\partial \bar{e}}\underbrace{\left(\frac{\partial \bar{e}}{\partial \bar{\alpha}} - \frac{\partial e}{\partial \alpha}\right)}_{A} + (N-2)\underbrace{\left(\frac{\partial \Pi}{\partial \bar{\alpha}} - \frac{\partial \tilde{\Pi}}{\partial \alpha}\right)}_{=(p-c(x))Q(g_{\bar{\alpha}}-g_{\alpha})>0} + \frac{\partial \bar{\Pi}}{\partial \bar{\alpha}} - \frac{\partial \bar{\Pi}}{\partial \alpha} + \frac{\partial \Pi}{\partial \bar{\alpha}} - \frac{\partial \Pi}{\partial \alpha}$$

The last term composed by the derivatives of profits (with respect to α) is unambiguously positive since it involves analyzing Equations 12 and 13 as follows:

$$\frac{\partial \bar{\Pi}}{\partial \bar{\alpha}} - \frac{\partial \bar{\Pi}}{\partial \alpha} + \frac{\partial \Pi}{\partial \bar{\alpha}} - \frac{\partial \Pi}{\partial \alpha} = (g_{\bar{\alpha}} - g_{\alpha})\left[D\left(p - c(\bar{x})\right) + Q\left(p - c(x)\right)\right] > 0.$$

As previously, a sufficient condition to sign this difference depends on the term A. First, note that if $c'(\cdot) = 0$, the difference $\Delta_4(\alpha)$ is unambiguously positive. If $c'(\cdot) < 0$, if $D \geq Q$ then $\Delta_4(\alpha) > 0$. Instead, suppose $D = 0$. In that case, the term A is equal to zero and $\Delta_4(\alpha) > 0$ which, by a continuity argument, enables to conclude that $\Delta_4(\alpha) > 0$ for sufficiently small value of D.

Proof of Proposition 6 (price, p)

1. **Enclosed patch stock**
Adopting the conditions in Table 4 the difference 15 is:

$$\Delta_1(p) = \frac{\partial \bar{x}}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \bar{p}} - \frac{\partial \bar{x}}{\partial e}\frac{\partial e}{\partial p} \tag{23}$$

First note that if $c' = 0$, $\frac{\partial e}{\partial p} = \frac{\partial \bar{e}}{\partial \bar{p}} = 0$ so $\Delta_1(p) = 0$. If $c'(\cdot) < 0$, then the difference can be written:

$$\Delta_1(p) = Dg_{\bar{e}}\frac{\partial \bar{e}}{\partial \bar{p}} - Qg_e\frac{\partial e}{\partial p} \tag{24}$$

This term is positive for sufficiently small D and negative for sufficiently small Q.

2. **Aggregate stock**
Adopting the conditions in Table 4, the difference 16 is:

$$\begin{aligned}\Delta_2(p) &= \frac{\partial \bar{x}}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \bar{p}} + \frac{\partial \bar{x}}{\partial \bar{p}} + (N-1)\left(\frac{\partial x}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial \bar{p}} + \frac{\partial x}{\partial \bar{p}}\right) \\ &- \left[\frac{\partial x}{\partial e}\frac{\partial e}{\partial p} + \frac{\partial x}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial p} + \frac{\partial x}{\partial p} + \frac{\partial \bar{x}}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial p} + \frac{\partial \bar{x}}{\partial e}\frac{\partial e}{\partial p} + \frac{\partial \bar{x}}{\partial p} + (N-2)\left(\frac{\partial \tilde{x}}{\partial \bar{e}}\frac{\partial \bar{e}}{\partial p} + \frac{\partial \tilde{x}}{\partial e}\frac{\partial e}{\partial p} + \frac{\partial \tilde{x}}{\partial p}\right)\right]\end{aligned}$$

We use the facts that $\frac{\partial \bar{x}}{\partial e} = \frac{\partial \tilde{x}}{\partial e}$ and $\frac{\partial \tilde{x}}{\partial \bar{e}} = \frac{\partial x}{\partial \bar{e}}$ to rewrite $\Delta_2(p)$:

$$\Delta_2(p) = [D + (N-1)Q] \left[g_{\bar{e}} \left(\frac{\partial \bar{e}}{\partial \bar{p}} - \frac{\partial \bar{e}}{\partial p} \right) - g_e \frac{\partial e}{\partial p} \right].$$

If $c'(\cdot) = 0$ then the first order condition for the enclosed patch is $1 = \delta D g_{\bar{e}}$, which is independent of the market price in patch 1 in any case. This implies that $\frac{\partial \bar{e}}{\partial \bar{p}} = \frac{\partial \bar{e}}{\partial p} = 0$. Thus, if $c'(\cdot) = 0$, $\Delta_2(p) = 0$.

Instead if $c'(\cdot) < 0$ we have

$$\frac{\partial \bar{e}}{\partial \bar{p}} = \frac{1 - \delta D g_{\bar{e}}}{SOC} < 0 \; ; \; \frac{\partial \bar{e}}{\partial p} = \frac{\delta D Q g_{\bar{e}} g_e c'(\bar{x}) \frac{\partial e}{\partial p}}{SOC} < 0$$

Because $g_{\bar{e}} < g_e$ (as $\bar{\alpha} = \alpha$ and $g_{ee} < 0$), we have: $g_{\bar{e}} \left(\frac{\partial \bar{e}}{\partial \bar{p}} - \frac{\partial \bar{e}}{\partial p} \right) - g_e \frac{\partial e}{\partial p} > g_{\bar{e}} \left(\frac{\partial \bar{e}}{\partial \bar{p}} - \frac{\partial e}{\partial p} \right)$. When D gets close to zero, $\frac{\partial \bar{e}}{\partial \bar{p}} - \frac{\partial e}{\partial p}$ gets close to zero, which by a continuity argument, implies that $g_{\bar{e}} \left(\frac{\partial \bar{e}}{\partial \bar{p}} - \frac{\partial \bar{e}}{\partial p} \right) - g_e \frac{\partial e}{\partial p} \geq 0$ (and $\Delta_2(p) > 0$) for sufficiently small values of D.

3. **Enclosed patch profit**
Adopting the conditions in Table 4 the difference 17 is:

$$\Delta_3(p) = \frac{\partial \bar{\Pi}}{\partial \bar{p}} - \frac{\partial \bar{\Pi}}{\partial e} \frac{\partial e}{\partial p} \tag{25}$$

The first term is positive. If $c'(\cdot) = 0$, $\frac{\partial e}{\partial p} = 0$, so $\Delta_3(p) > 0$. If $c'(\cdot) < 0$, then the second term, equal to $\frac{(p - c(\bar{x})) Q g_e}{c'(e)}$, is negative. Thus the difference $\Delta_3(p) > 0$.

4. **Aggregate profit**
Adopting the conditions in Table 4, the difference 18 is:

$$\begin{aligned} \Delta_4(p) &= \frac{\partial \bar{\Pi}}{\partial \bar{p}} + (N-1) \frac{\partial \Pi}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \bar{p}} \\ &- \left[\frac{\partial \Pi}{\partial e} \frac{\partial e}{\partial p} + \frac{\partial \Pi}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial p} + \frac{\partial \Pi}{\partial p} + \frac{\partial \bar{\Pi}}{\partial e} \frac{\partial e}{\partial p} + \frac{\partial \bar{\Pi}}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial p} + (N-2) \left(\frac{\partial \tilde{\Pi}}{\partial e} \frac{\partial e}{\partial p} + \frac{\partial \tilde{\Pi}}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial p} \right) \right] \end{aligned}$$

Which can be rewritten:

$$\begin{aligned} \Delta_4(p) &= x_0 - \bar{e} + \frac{\delta}{1-\delta}(\bar{x} - \bar{e}) + (N-1) \frac{\delta}{1-\delta} (p - c(x)) Q g_{\bar{e}} \frac{\partial \bar{e}}{\partial \bar{p}} \\ &- \left[x_0 - e + \frac{\delta}{1-\delta}(x - e) + \frac{\delta}{1-\delta} (p - c(x)) D g_e \frac{\partial e}{\partial p} \right] \\ &- \left[\frac{\delta}{1-\delta} (p - c(x)) Q g_{\bar{e}} \frac{\partial \bar{e}}{\partial p} + \frac{\delta}{1-\delta} (p - c(\bar{x})) Q g_e \frac{\partial e}{\partial p} \right] \\ &- (N-2) \left(\frac{\delta}{1-\delta} (p - c(\tilde{x})) Q g_e \frac{\partial e}{\partial p} + \frac{\delta}{1-\delta} (p - c(\tilde{x})) Q g_{\bar{e}} \frac{\partial \bar{e}}{\partial p} \right) \end{aligned}$$

If $c'(\cdot) = 0$, then we use the facts that $\frac{\partial \bar{e}}{\partial \bar{p}} = \frac{\partial \bar{e}}{\partial p} = \frac{\partial e}{\partial p} = 0$, which imply that

$$\begin{aligned}\Delta_4(p) &= e-\bar{e}+\frac{\delta\,(\bar{x}-\bar{e}-x+e)}{1-\delta}=\frac{(e-\bar{e})}{1-\delta}+\frac{\delta\,(\bar{x}-x)}{1-\delta}\\ &= \frac{(e-\bar{e})}{1-\delta}+\frac{\delta\,(D-Q)\,[g(\bar{e},\bar{\alpha})-g(e,\alpha)]}{1-\delta}.\end{aligned}$$

The first conclusion is that $\Delta_4(p) < 0$ as long as $Q \geq D$ or δ is small. Indeed, when $Q \geq D$, the second term in the expression of $\Delta_4(p)$ is negative, and the first term is obviously negative as $\bar{e} > e$. Now, when δ is small, the sign of $\Delta_4(p)$ is given by that of the following expression:

$e-\bar{e}+\delta(D-Q)\,[g(\bar{e},\bar{\alpha})-g(e,\alpha)] = (\bar{e}-e)\left(-1+\frac{\delta(D-Q)[g(\bar{e},\bar{\alpha})-g(e,\alpha)]}{\bar{e}-e}\right)$. When $\delta \to 0$, we have $\bar{e} \to e$: in this case, this implies that $\frac{g(\bar{e},\bar{\alpha})-g(e,\alpha)}{\bar{e}-e} \to g_e > 0$ (and finite). All together, when δ converges to zero, we have: $-1+\frac{\delta(D-Q)[g(\bar{e},\bar{\alpha})-g(e,\alpha)]}{\bar{e}-e} \to -1 < 0$. Thus, a continuity argument enables us to conclude that $\Delta_4(p) \leq 0$ for sufficiently small values of δ.

Moreover, we know that $e = 0$, which implies that $g(e,\alpha) = g(0,\alpha) = 0$ and $\bar{x} = Dg(\bar{e},\bar{\alpha})$. Plugging these expressions into the above equality and simplifying, we obtain:

$$\Delta_4(p) = -\frac{\bar{e}}{1-\delta}+\frac{\delta\,(D-Q)\,g(\bar{e},\bar{\alpha})}{1-\delta}=\frac{1}{1-\delta}\left[-\bar{e}+\delta\left(1-\frac{Q}{D}\right)\bar{x}\right].$$

If Q gets close to zero and δ gets close to one, then the sign of $\Delta_4(p)$ is that of $-\bar{e}+\bar{x}$, which is positive. By a continuity argument, we conclude that $\Delta_4(p) \geq 0$ for sufficiently small values of Q and large values of δ.

Input parameters for numerical example from Section 3.1

$D =$

$$\begin{pmatrix}
.728 & .013 & .039 & .008 & .002 & .005 & .016 & .018 & .011 & .005 & .002 & .003 & .002\\
.008 & .697 & 0.056 & .007 & .002 & .005 & .025 & .026 & .013 & .004 & .002 & .004 & .002\\
.010 & .025 & .722 & .007 & .002 & .005 & .028 & .026 & .013 & .005 & .002 & .004 & .002\\
.004 & .015 & .039 & .717 & .002 & .006 & .020 & .018 & .014 & .007 & .001 & .004 & .002\\
.002 & .010 & .022 & .003 & .765 & .004 & .013 & .015 & .006 & .005 & .001 & .003 & .001\\
.002 & .009 & .013 & .002 & .001 & .777 & .015 & .015 & .006 & .003 & .002 & .004 & .001\\
.003 & .004 & .008 & .003 & .001 & .003 & .803 & .010 & .005 & .004 & .002 & .004 & .001\\
.009 & .016 & .046 & .008 & .002 & .005 & .015 & .717 & .015 & .007 & .003 & .003 & .003\\
.005 & .015 & .044 & .005 & .002 & .004 & .013 & .019 & .731 & .006 & .003 & .002 & .002\\
.008 & .014 & .032 & .011 & .002 & .005 & .014 & .020 & .021 & .711 & .007 & .002 & .002\\
.005 & .007 & .013 & .004 & .001 & .003 & .010 & .012 & .013 & .009 & .772 & .002 & .001\\
.003 & .002 & .001 & .001 & .000 & .000 & .003 & .004 & .007 & .002 & .001 & .825 & .001\\
.004 & .003 & .003 & .001 & .000 & .001 & .003 & .005 & .004 & .001 & .001 & .001 & .824
\end{pmatrix}$$

$r = [.4297; .4254; .4329; .4980; .5034; .6769; .6460; .4000; .4353; .5070; .4758; .4787; .4000]$

$K = [100.88; 100.65; 101.08; 109.60; 110.68; 176.67; 160.53; 100.00; 101.25; 111.46; 105.75; 106.19; 100.00]$

$\delta = 0.90$, $p = 1.0$, $\theta = 15$.

Input parameters for numerical example from Section 3.2

$Q = .06$, $D = .20$ though we also examine values of $D \in [0, .3]$, $r = .485$, $K = 114.21$, $\delta = 0.90$, $p = 1.0$, $\theta = 15$.

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