Voting rules in bargaining with costly persistent recognition: a field experiment on motivational crowding-outta analysis

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Voting Rules in Bargaining with Costly Persistent Recognition

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Abstract

In this paper, we consider a model of multilateral bargaining where homogeneous agents may exert effort before negotiations in order to influence their chances to become the proposer. Effort levels have a permanent effect on the recognition process (persistent recognition). We prove two main results. First, all voting rules are equivalent (that is, they yield the same social cost) when recognition becomes persistent. Secondly, an equilibrium may fail to exist, because players may have more incentives to reduce their effort level (in order to be included in winning coalitions) than to increase it (in order to increase their proposal power). Both results differ greatly from the case where recognition is transitory: Yildirim (2007) shows that unanimity is the unique strictly optimal rule, and that an equilibrium always exists (under mild assumptions) in such a setting. Moreover, our second conclusion is quite different from the one obtained in most of the existing literature on bargaining (which assumes an exogenous recognition process), where it is generally considered that it is always in an agent's best interest to have a proposal power as high as possible.

JEL Classification Codes: C70; D72.

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1 Introduction

Negotiations are common in many important economic problems, such as legislative bargaining (Baron and Ferejohn (1989), Snyder et al. (2005)), international environmental agreements, litigation processes, issues of corporate governance. Agents taking part in such processes have incentives to gain power in order to influence the outcome of the process. There are plenty of real life situations where agents exert (costly) efforts to promote their most preferred alternative. For instance, agents can provide services and contributions to the functioning of their organization in order to increase their chances to be elected as members of the executive committee. This in turn will enable them to influence the system of decision-making.

The agents' incentives to buy influence have been studied in certain contexts. Grossman and Helpman (2002) analyse settings where special interest groups might influence the outcome of legislative bargaining by compensating other parties or the agenda setter. Their main focus is on the effect of such process of vote buying on the characteristics of the policies that are implemented. In Evans (1997), Anbarci et al. (2002), or Board and Zwiebel (2005), agents exert unproductive efforts to influence their rights to propose. All these contributions do not compare different voting rules with respect to the social cost resulting from influence activities, which is the main goal of the present paper. As such, the closest references are Yildirim (2007, 2010), where the author analyses a sequential bargaining situation in which agents compete in order to influence their chances to become the proposer. Competition can take place at a pre-bargaining stage (persistent recognition) or at each stage of the negotiations (transitory recognition). In Yildirim (2007) the author characterizes unanimity as the voting rule minimizing the social cost resulting from influence activities when agents are identical and recognition is transitory. Then, in Yildirim (2010), he compares both recognition systems for a given rule (unanimity).

Unlike Yildirim (2007), the present paper focuses on the case of persistent recognition, where agents exert efforts to influence their chances to become proposers at the beginning of the process, i.e before the first round of negotiation. This is mainly because this type of recognition seems to be the most appropriate when considering many important real world processes, such as legislative bargaining or executive committees in organizations.

The present contribution complements Yildirim (2007) by comparing the optimality of the different voting rules when recognition is persistent. The first contribution is an equivalence result regarding the social cost resulting from voting rules. While unanimity is the unique optimal voting rule when recognition is transitory, all voting rules are equivalent (in that they yield the same social cost) when agents are identical and exert efforts only once at the beginning of the process. Our second contribution relates to the analysis of incentives to deviate from the (unique) candidate for a symmetric equilibrium. We show that, when the (symmetric) equilibrium fails to exist, this is *not* because players have incentives to increase their chances to become the proposer (through an increase of effort), but rather because they prefer to lower their effort level in order to be included in the winning coalition. In other words, a qualitative property of the present model is that the ability to propose is less important than the ability to be included in winning coalitions for the players. This is quite unexpected, as one of the main conclusions of the traditional literature on bargaining highlights the dominance of proposal rights to define political power (see Eraslan (2002), or Kalandrakis (2006)).

Let us describe the analysis provided more specifically. We focus on the (symmetric) stationary subgame perfect equilibria (SSPE) of the game (as in Yildirim (2007, 2010)) and we characterize the conditions under which such equilibria exist for general voting rules. These conditions are shown to depend on the type of voting rules that is considered. We proceed by backward induction and we first characterize the (only) potential candidate. We characterize the resulting social costs and we prove that they coincide for all voting rules. Then we prove that this equilibrium exists under the unanimity rule, and under the dictatorship rule. We show that, for any other k-majority voting rule, the symmetric equilibrium fails to exist, because the players have an incentive to reduce their effort in order to be included in the winning coalition by the others.

The trade-off the agents face with endogenous recognition is the following. On one side, as they increase their effort, the chances that they become the proposer increase, which might result in a higher payoff. On the other side, a higher probability to become the proposer makes an agent's vote more expensive, which decreases his chance to be included in a winning coalition (in case a strict k-majority rule is used) if this agent is not the proposer, and this might result in a lower payoff. The general conclusion is that the second effect dominates the first one in the present situation.

This paper provides an interesting contribution since it highlights two main conclusions. First, the strict optimality of the unanimity rule under endogenous recognition is not a general property, even when agents are identical. Second, under strict k-majority rules, the incentives to be included in winning coalitions by the other players dominate individual incentives and create a "race to the bottom", which eventually destabilises the unique (symmetric) equilibrium candidate.

Before moving on to the formal description of the model, there are two important points that have to be stressed. First, we use a specific form of recognition function in the present paper, which is yet the most widely used form in the literature on rent seeking (see Tullock (1980)). Second, the analysis provided here is not exhaustive, that is, we do not analyse asymmetric equilibria.

The reasons are as follows. The main goal of the analysis is to highlight the fact that the case of persistent recognition is quite specific, since voting rules are shown to be equivalent in terms of the resulting social cost, and that there are cases where a race to the bottom emerges at the recognition stage. Compared to Yildirim (2007), the persistent nature of the recognition process makes it much more difficult to derive analytically tractable expressions that enable to compare the different voting rules. The logit form of the recognition function used in the present paper enables us to provide informative results, while keeping technical difficulties at a reasonable level.

The remaining of the paper is organized as follows. The model is introduced in Section 2 and an illustrative example highlights the main difference between transitory recognition and persistent recognition. The unique equilibrium candidate is characterised in Section 3. The equivalence result prevailing in the homogeneous case is provided in the same section. The (non) existence problem is analysed in Section 4. Section 5 concludes. Proofs that do not appear in the body of the paper are relegated to an appendix.

2 The model

2.1 Description of the model

We consider the problem introduced by Yildirim (2007, 2010) where agents are identical (same time preferences, same marginal cost of effort). Specifically, we assume that $n \geq 2$ agents belonging to a set $N = \{1, ..., n\}$ bargain over the allocation of a surplus of fixed size (normalized to one). Agents negotiate according to a bargaining protocol *a la* Rubinstein (1982), except that their recognition probabilities are endogenous. Each agent exerts effort at the beginning of the process, and relative efforts determine each agent' bargaining power (their recognition probability) for all periods. We assume that, provided agent i exerts effort x_i at the beginning of the process, his recognition probability is given by $p_i \equiv p(x_i, x_{-i})$, where x_{-i} is the vector of efforts of the $n-1$ other players. Let $\boldsymbol{x}$ denote the vector of efforts of the n players.

We will have to impose more structure on the recognition probabilities, especially to characterize the social cost. We will use a Tullock contest success function (TCSF):

Assumption (TCSF): *Let the recognition probability be such that, for* $\boldsymbol{x} \geq 0$,

$$p(x_i, x_{-i}) = \begin{cases} \frac{x_i}{\sum_{l=1}^{n} x_l} \text{ if } \boldsymbol{x} \neq \boldsymbol{0}, \\ \frac{1}{n} \text{ if } \boldsymbol{x} = \boldsymbol{0}, \end{cases} \tag{1}$$

This function has been introduced by Tullock and it has been widely used in the literature on contests. This is the simplest form of contest success functions with axiomatic foundations (Skaperdas, 1996). Efforts are costly and, in order to keep the analysis as simple as possible, we assume that the cost of effort is linear (and that all agents have the same marginal cost of effort), and this cost is denoted by the positive parameter $c < 1$.

To be consistent with Yildirim (2007, 2010), we will focus on stationary subgame perfect equilibria (SSPE) in the remainder of the paper. With this equilibrium notion, it is easily checked that (since $\delta_i < 1$) agent i has incentives to make an offer that is immediately expected.

In the next sections, we will use backward reasoning to characterize the SSPE and more specifically the agents' equilibrium payoffs, expected shares of the surplus, and levels of effort. We will focus on symmetric equilibria, i.e. equilibria where two identical players are treated the same (same share of the pie, same effort).

In Section 3 the optimal strategies of the negotiation stage are characterized. In order to rule out cases where agents might be indifferent between certain strategies, we will have to rely on a tie breaking rule that will be described in this section.

In Section 4, we will analyse the initial stage of recognition, and we will characterize the equilibrium candidate. Then we will complete the analysis by ruling out the potentially beneficial unilateral deviations. At this stage of the analysis, we will provide the conditions on the value of the discount parameter δ that ensure that the symmetric equilibrium exists for general k-majority rules. We will then show that the social cost resulting from the equilibrium is the same for all voting rules.

2.2 Comparison of transitory and persistent recognition: the illustrative two player case

The present section illustrates how the ranking of voting rules (in terms of social costs) is affected when recognition is assumed persistent instead of transitory in the case of two players. The case of transitory recognition differs from the model presented above because instead of choosing an effort x_i one for all before the negotiation phase as in the case of persistent recognition (as considered in Yildirim 2010 and in the present paper), players exert efforts at each step of the negotiation process (which is the case analysed in Yildirim 2007).

With two players, there are only two possible voting rules, unanimity and dictatorship. In the case of unanimity, a proposal needs the approval of the other player to be implemented and in the case of dictatorship, the proposer can share the pie without the agreement of the other player.

Both the transitory recognition model and the persistent recognition model coincide in the case of dictatorship. Indeed, when the proposer does not need the agreement of the other players, he keeps the whole pie. Both coincide with the standard rent-seeking model. Hence, in the case of two homogeneous players, the payoff of player $i = 1, 2$ is given by:

$$v_i = p_i(x_1, x_2) - cx_i.$$

Using (TCSF), it is easy to check that the unique equilibrium is such that $x_1^* = x_2^* = \frac{1}{4c}$. The social cost is then $SC = c(x_1^* + x_2^*) = \frac{1}{2}$.

Transitory recognition with two players: Under unanimity, the expected equilibrium payoff of player 1 satisfies (see Yildirim 2007):

$$v_1 = \max_{x_1 \geq 0} \{p_1 [1 - \delta v_2] + (1 - p_1) \delta v_1 - cx_1\}$$

With probability p_1, player 1 becomes the proposer and under unanimity he needs to compensate player 2 (in paying him δv_2) and player 2 accepts. With probability $1 - p_1$, player 2 is the proposer. Player 1 aggrees on the sharing and receives δv_1. Similarly, the expected equilibrium payoff of player 2 satisfies

$$v_2 = \max_{x_2 \geq 0} \{p_2 [1 - \delta v_1] + (1 - p_2) \delta v_2 - cx_2\}.$$

The equilibrium effort of player $i = 1, 2$ satisfies:

$$\frac{\partial p_i}{\partial x_i} \times [1 - \delta v_1 - \delta v_2] - c \leq 0 \; (= 0 \text{ if } x_i > 0).$$

One can easely show that both players exert a positive effort, $x_i^* = \frac{1}{4c}(1 - \delta(1 - c))$ and the social cost is $SC = c(x_1^* + x_2^*) = \frac{1}{2}(1 - \delta(1 - c))$. Since $\delta > 0$ and $c < 1$, the social cost is strictly lower under unanimity than under dictatorship.

Persistent recognition with two players: the game has two stages, in the first stage players choose their effort and in the second stage players bargain. We use backward induction to solve the game. In the second stage, given the efforts x_1 and x_2, the shares of the players, s_1 and s_2 satisfy:

$$s_1 = p_1 [1 - \delta s_2] + (1 - p_1) \delta s_1,$$

and,

$$s_2 = p_2 [1 - \delta s_1] + (1 - p_2) \delta s_2.$$

Solving this set of two equations, we obtain $s_i = p_i$ for $i = 1, 2$.

Now consider the first stage where the players choose their efforts. The expected payoff of player $i = 1, 2$ is given by:

$$v_i = s_i - cx_i = p_i - cx_i$$

Hence, as in the case of dictatorship, the game coincides with the standard rent-seeking model.

This illustrative example highlights an important difference between the two models. Whereas unanimity is the voting rule that (strictly) minimize social costs in the case of transitory recognition, the voting rule does not affect social costs in the case of persistent recognition.

In the rest of the paper, we concentrate on the persistent recognition model where players choose an effort in the first stage and then the negotiation process takes place. In order to solve for the SSPE of the present two stage game, we use backward induction. We will first analyse the final stage of the game where agents negotiate in order to allocate the surplus.

3 Symmetric equilibrium

3.1 The negotiation stage

Let us first introduce some notations and definitions. Let $\overline{\boldsymbol{\psi}}_i = \left(\overline{\psi}_{ij}\right)_{j \in N \setminus \{i\}}$ where $\overline{\psi}_{ij}$ is the probability that player i includes player j in his winning coalition. Under a given k-majority rule, we must have $\overline{\psi}_{ij} \in [0,1]$ for all $i, j \in N$, $i \neq j$ and $\sum\limits_{j \in N \setminus \{i\}} \overline{\psi}_{ij} = k - 1$ for all $i \in N$. It is convenient to define $\overline{\boldsymbol{\psi}}_{-i} = \left(\overline{\boldsymbol{\psi}}_1, \overline{\boldsymbol{\psi}}_2, ..., \overline{\boldsymbol{\psi}}_{i-1}, \overline{\boldsymbol{\psi}}_{i+1}, ..., , \overline{\boldsymbol{\psi}}_n\right)$. The shares $\overline{\boldsymbol{s}} = (\overline{s}_1, ..., \overline{s}_n)$ are characterized by:

$$\overline{s}_i = p_i \left(1 - \overline{w}_i\right) + \delta \overline{\mu}_i \overline{s}_i, \text{ for } i = 1, ..., n. \tag{2}$$

where,

$$\overline{w}_i = \sum_{j \neq i} \overline{\psi}_{ij} \delta \overline{s}_j \text{ and } \overline{\mu}_i = \sum_{j \neq i} p_j \overline{\psi}_{ji}.$$

We now characterize the agents' optimal strategies during the negotiation process (taking into account that their recognition probabilities are fixed). At this stage of the analysis we will introduce a tie breaking rule that will explain what type of behavior is assumed from identical agents. It will be helpful to avoid cases where agents might be indifferent between two different strategies.

We now proceed with the analysis. Fix $\boldsymbol{p} = (p_1, ..., p_n)$ and $\boldsymbol{s} = (s_1, ..., s_n)$. The second stage equilibrium is characterized by $\boldsymbol{\psi} = (\boldsymbol{\psi}_1, ..., \boldsymbol{\psi}_n)$ such that:

$$\boldsymbol{\psi}_i = Arg \max_{\overline{\boldsymbol{\psi}}_i} \left\{ p_i [1 - \delta \sum_{j \neq i} \overline{\psi}_{ij} \overline{s}_j] + \delta \sum_{j \neq i} p_j \psi_{ji} \overline{s}_i \right\};$$

The best reply of player i in the second stage of the game is given by:

$$\forall i, \forall j \neq i, \psi_{ij} = \begin{cases} 1 \text{ if } \overline{s}_j < \overline{s}_k \\ \leq 1 \text{ if } \overline{s}_j = \overline{s}_k \\ 0 \text{ if } \overline{s}_k < \overline{s}_j \end{cases} \tag{3}$$

This reasoning leads to the following preliminary result:

Lemma 1 : *In the equilibrium of the second stage, the vector of probabilities of inclusion, $\psi = (\boldsymbol{\psi}_i)_{i\in N}$, and the vector of shares $s = (s_i)_{i\in N}$ are functions of $(\boldsymbol{x}, \delta)$, with $\psi_i = \psi_i(\boldsymbol{x}, \delta)$ and $s_i = s_i(\boldsymbol{x}, \delta)$ for all $i \in N$. The vector of shares s is the solution of*

$$s_i = p_i (1 - w_i) + \delta \mu_i s_i, \textit{ for } i = 1, ..., n, \tag{4}$$

where,

$$w_i = \delta \sum_{j \neq i} \psi_{ij} s_j \textit{ and } \mu_i = \sum_{j \neq i} p_j \psi_{ji},$$

and,

$$\forall i, \forall j \neq i, \psi_{ij} = \begin{cases} 1 \textit{ if } s_j < s_k \\ \leq 1 \textit{ if } s_j = s_k \\ 0 \textit{ if } s_k < s_j \end{cases} \tag{5}$$

The second stage equilibrium strategies are characterized implicitly.

Before deriving the first stage equilibrium, we will show an interesting property of the second stage equilibrium. To show the result, we need to fix a tie breaking rule (TBR) regarding situations where the votes of two players have the same cost:

Assumption (TBR): If $s_l = s_k = s_i$ with $i \neq l$, then $\psi_{ji} = \psi_{jl} = \psi_{jk}$ if $j \neq i, l, k$, $\psi_{ki} = \psi_{kl}$ and $\psi_{ik} = \psi_{lk}$ if $k \neq i, l$.

We use the following preliminary result from Quérou and Soubeyran (2010):

Lemma 2 : *Assume that assumption (TBR) holds. In the second stage equilibrium, if $\delta_i = \delta_l$, then $x_i = x_l \Longleftrightarrow p_i = p_l \Rightarrow s_i = s_l$.*

This result enables us to characterize the agents' optimal strategies at the last stage of the game. In order to solve for the SSPE we go backward and analyse the initial stage of the game where agents exert efforts in order to influence their recognition probabilities. In the rest of the paper, we assume that assumptions (TCSF) and (TBR) hold.

3.2 The recognition stage

The present sub section characterize the potential candidate for a (symmetric) SSPE of the two stage game by characterizing the first order conditions of the investment game. The analysis of the existence of the equilibrium is left to the next section.

Reasoning backward, the first stage equilibrium of the game is the equilibrium of a one-shot game where the payoff of player i is given by $v_i(\boldsymbol{x},\delta) = s_i(\boldsymbol{x},\delta) - cx_i$, with $x_i \geq 0$. The rest of the paper will focus on the symmetric equilibrium of this one shot game.

A symmetric equilibrium of the one shot game is characterized by $\boldsymbol{x}^* = (x^*, ..., x^*)$ such that

$$v_i(\boldsymbol{x}^*,\delta) \geq v_i\left(x', \boldsymbol{x}^*_{-i}, \delta\right), \text{ for all } i,$$

with,

$$s_i = p_i(1-w_i) + \delta\mu_i s_i, \textit{ for } i = 1, ..., n. \tag{6}$$

Using Lemma 1 and by relabelling the players in increasing share, we have:

$$w_i = \begin{cases} w_k + \delta(s_k - s_i) \text{ if } s_i \leq s_k \\ w_k \text{ if } s_k \leq s_i \end{cases},$$

and,

$$\mu_i \begin{cases} = 1 - p_i \text{ if } s_i < s_k \\ \leq 1 - p_i \text{ if } s_i = s_k \\ 0 \text{ if } s_k < s_i \end{cases},$$

3.3 Voting rules and social cost

In the present section, we characterize the unique candidate for a symmetric equilibrium and compare the social cost for the different possible voting rules.

Proposition 1: *Whatever the voting rule ($1 \leq k \leq n$), the only candidate for a symmetric equilibrium is the vector of efforts $\boldsymbol{x}^* = (x^*, ..., x^*)$ such that:*

$$x^* = \frac{1}{c}\frac{n-1}{n^2}.$$

We have shown that $\boldsymbol{x}^*$ is the unique candidate for a symmetric equilibrium. In other words, there is at most one symmetric equilibrium and it is $\boldsymbol{x}^*$. As mentioned previously, the above result has an interesting implication. Endogenous recognition has different effects depending on whether it is transitory or persistent. Specifically, under transitory recognition the unanimity rule yields the lowest social cost resulting from influence activities. Under persistent recognition, all voting rules yield the same cost.

4 Existence of the symmetric equilibrium

In the case of unanimity and dictatorship, it is easy to check that the symmetric equilibrium always exists. Indeed, in both cases, the payoff of player i is given by

$v_i(\boldsymbol{x}, \boldsymbol{\delta}) = \frac{x_i}{\sum_j x_j} - cx_i$ (when at least one of the players' effort is strictly positive) and it is concave in x_i. This proves the first claim of the following proposition:
Proposition 2: *Under the unanimity rule and the dictatorship rule, the symmetric equilibrium always exists. Under the strict k-majority rule ($2 \leq k \leq n-1$), the symmetric equilibrium fails to exist because each player has incentives to lower his effort.*

This result also highlights an important difference between transitory and persistent recognition. Whereas an equilibrium generally exists when recognition is transitory, it fails to exist for majority rules in the case of persistent recognition. Moreover, the reason why the symmetric equilibrium does not exist is an interesting qualitative property of negotiations where recognition is persistent.

The proof of proposition 2 shows that players have incentives to deviate from the symmetric equilibrium candidate by reducing their effort. The main intuition is that the agents face a trade-off with endogenous recognition. On one side, as they increase their effort, the chances that they become the proposer increase, which might result in a higher payoff. On the other side, a higher probability to become the proposer makes an agent's vote more expensive, which decreases his chance to be included in a winning coalition (in case a strict k-majority rule is used) if this agent is not the proposer, and this might result in a lower payoff. At the symmetric equilibrium candidate, the second effect dominates. This effect is strong, since a very small decrease in the player's effort induces a small decrease in his probability of being the proposer; but then his vote becomes the cheapest one, and he is included with certainty in all the winning coalitions during the negotiation stage.

The following result completes this analysis:
Lemma 3: *Under the strict k-majority rule ($2 \leq k \leq n-1$), no player has an incentive to deviate from the symmetric equilibrium by increasing his effort.*

This Lemma and Proposition 2 show that the equilibrium fails to exist only because of the race to the bottom described above. Players have no incentive to increase their effort from the symmetric equilibrium (candidate). This is a quite unexpected and interesting result in the bargaining literature because players usually benefit from being the proposer (see Eraslan (2002)).

5 Concluding remarks

The issue of buying influence in collective decision making is extremely important as it is prevalent in many real world economic situations (lobbying in legislative bargaining, international negotiations, composition of executive committees in economic organizations). There are many questions related to this issue. The present contribution analyses a multilateral bargaining situation where recognition is persistent and endogenous, and compares voting rules with respect to the social cost resulting from them. It is proved that this comparison differs notably depending on the type of recognition that is considered. While unanimity is the only strictly optimal rule when recognition is transitory, all voting rules become equivalent as soon as recognition becomes persistent (provided a symmetric equilibrium exists). We also show that (unlike with transitory recognition) the symmetric equilibrium fails to exist because of a race to the bottom. Players have incentives to reduce their proposal power in order to be included in the winning coalition. This stresses the fact that one should be cautious when thinking about the choice of the appropriate voting rule in collective decision making situations, especially when influence activities might be used.

Different lines of research may extend and complete the present analysis. First, we have not analysed the asymmetric equilibrium (asymmetric equilibria may exist even when symmetric equilibrium fails to exist, since the payoffs are not continuous). However, as in the present paper, the comparison of social costs (for asymmetric equilibria) requires to fully characterize the social cost for each voting rule (because it is not a continuous function of the number k associated to the voting rule). Unfortunately, this seems untractable.

A second line of research is to analyse the problem with heterogeneous players. We contribute to this line in a companion paper (Quérou and Soubeyran 2010). Finally, a third interesting point would be to bring some uncertainty to the model. For instance, assuming that players do not know perfectly the cost of the other players' vote would be a realistic assumption, which would smooth the players' payoffs and could solve the existence problem. These issues are left for future research.

Appendix

Proof of Proposition 1: First consider the case of unanimity, $k = n$. The shares of the players are characterized by (see Yildirim 2010), for all i:

$$s_i = \frac{p_i}{\sum_j p_j} = p_i.$$

Hence, assuming $\boldsymbol{x} \neq \mathbf{0}$, the equilibrium effort of player i is given by:

$$x_i^* = \arg\max_{x_i \geq 0} \left(\frac{x_i}{x_i + \sum_j x_j^u} - cx_i \right)$$

In an interior equilibrium

$$\sum_j x_j^* - x_i^* = c\left(x_i^* + \sum_j x_j^*\right)^2, \text{ for all } i.$$

Hence any interior equilibrium is necessarily symmetric with $x_i^* = x^* = nx^* - cn^2 (x^*)^2$ and then $x^* = \frac{n-1}{cn^2}$. Remark that the objective of player i is concave in x_i for $\sum\limits_j x_j > 0$ then $\boldsymbol{x}^*$ is an equilibrium. To complete the proof, remark that $\boldsymbol{x} = (0, ..., 0)$ cannot be an equilibrium since any player has an incentive to deviate from this situation and make an infinitesimal effort.

Now assume that $k \leq n-1$. In a symmetric equilibrium, players' efforts are the same, $x_i = x_j$ for all $i, j \in N$. According to Lemma 2, the players have same share, $s_i = s_j$ for all $i, j \in N$. The share of player $i \in N$ is then given by:

$$s_i = p_i[1 - \frac{k-1}{n-1}\sum_{j\neq i}\delta s_j] + \frac{k-1}{n-1}\sum_{j\neq i} p_j \delta s_i; \tag{7}$$

Using the fact that $\sum_{j\neq i} p_j = 1 - p_i$ and rearranging terms, we have:

$$s_i = \frac{p_i}{1 - \frac{k-1}{n-1}\delta}[1 - \frac{k-1}{n-1}\delta \sum_j s_j]. \tag{8}$$

Summing over the set of agents, we obtain:

$$\sum_j s_j = 1 \tag{9}$$

Thus, the share of player i is given by:

$$s_i = p_i. \tag{10}$$

In the first stage of the game, players compete in a contest. Player i maximizes

$$\underset{x_i \geq 0}{Max} \ \left(s_i\left(x_i, x_{-i}\right) - cx_i\right). \tag{11}$$

The optimal strategy is then

$$x_i^* = x^* = \frac{1}{c}\frac{n-1}{n^2}, \text{ for all } i \in N.$$

Thus, $\boldsymbol{x}^u$ is the unique candidate for a symmetric equilibrium.□

Proof of Proposition 3: Assume that $k \leq n-1$. We know that $\boldsymbol{x}^*$ is the unique candidate for a symmetric equilibrium. At this point, the payoff of each player is given by

$$v_1^*(\boldsymbol{x}^*) = \frac{1}{n^2}.$$

Now we study players' unilateral incentives to deviate from this candidate. Let us consider that agent 1 deviates by choosing $x_1 < x^*$. This implies that $s_1 < s$, where s denotes the equilibrium share of all agents $2, ..., n$ and s_1 denotes the equilibrium share of agent 1. This implies that the new equilibrium strategies at the second period are:

$$\psi_{i1} = \psi_{j1} = 1; \;\; \psi_{ij} = \psi_{ji} = \frac{k-2}{n-2}; \;\; \psi_{1i} = \frac{k-1}{n-1},$$

where $i \neq j$ and $i = 2, ..., n$. This yields the following characterisation of the equilibrium shares:

$$s_1 = p_1[1 - \delta(k-1)s] + (n-1)p\delta s_1,$$

and

$$s = p[1 - \delta s_1 - \delta(k-2)s] + (n-2)p\frac{k-2}{n-2}\delta s + p_1\frac{k-1}{n-1}\delta s.$$

Solving the above set of equations, we obtain:

$$s_1\left(x_1, \boldsymbol{x}_{-1}^*\right) = [n-1-\delta(k-1)]\frac{x_1}{(n-1)(1-\delta)[(n-1)x^* + x_1] + \delta x_1(n-k)}.$$

Then, coming back to the first agent's expected payoffs:

$$v_1\left(x_1, \boldsymbol{x}_{-1}^*\right) = [n-1-\delta(k-1)]\frac{x_1}{(n-1)(1-\delta)[(n-1)x^* + x_1] + \delta x_1(n-k)} - cx_1.$$

Let $x_1 = \gamma x^*$ with $0 \leq \gamma < 1$. The gain from deviating can then be written as follows:

$$\begin{aligned}
\Delta(\gamma) &\equiv v_1\left(\gamma x^*, \boldsymbol{x}_{-1}^*\right) - v_1^*(\boldsymbol{x}^*) \\
&= [n-1-\delta(k-1)]\frac{\gamma}{(n-1)(1-\delta)[n-1+\gamma] + \delta\gamma(n-k)} - \gamma\frac{n-1}{n^2} - \frac{1}{n^2}.
\end{aligned}$$

Notice that the above function is defined even at point $\gamma = 1$. At this point its value is:

$$\begin{aligned}
\Delta(1) &= \frac{n-1-\delta(k-1)}{(n-1)(1-\delta)n + \delta(n-k)} - \frac{n-1}{n^2} - \frac{1}{n^2} \\
&= \frac{1}{n}\delta(n-k)\frac{n-1}{(n-1)(1-\delta)n + \delta(n-k)} > 0
\end{aligned}$$

Since Δ is a continuous function of γ, there exists $\overline{\gamma} < 1$, such that $\Delta(\gamma) > 0$ for all $\gamma \in (\overline{\gamma}, 1)$. This concludes the proof.$\square$

Proof of Lemma 3: Assume that one agent (say 1) deviates from $\boldsymbol{x}^*$ by exerting effort $x_1 > x^*$. According to Proposition 1, this implies $\delta s_1 > \delta s$, where s_1 denotes the share of player 1 and s denotes the share of all agents $2, ..., n$ and s_1 denotes the share of agent 1 (those shares resulting from the vector of effort $(x_1, x^*, x^*, ..., x^*)$). This and assumption TBR imply that the new equilibrium strategies at the second period are:

$$\psi_{i1} = 0; \ \psi_{ij} = \psi_{ji} = \frac{k-1}{n-2}; \ \psi_{1i} = \frac{k-1}{n-1},$$

where $i = 2, ..., n$ and $j \neq i$. This yields the following characterisation of the equilibrium shares:

$$s_1 = p_1[1 - \delta(k-1)s],$$

and

$$s = p[1 - \delta(k-1)s] + (n-2)p\frac{k-1}{n-2}\delta s + p_1\frac{k-1}{n-1}\delta s,$$

where p_1 is the probability that player 1 is the proposer and p is the probability of each agent $i \geq 2$. Solving the above set of equations, we obtain:

$$s_1 = p_1 \frac{n-1-\delta(k-1)}{n-1-\delta(k-1)p_1},$$

Replacing $p_1 = \frac{x_1}{(n-1)x^* + x_1}$, we have:

$$s_1\left(x_1, \boldsymbol{x}^*_{-1}\right) = [n-1-\delta(k-1)]\frac{x_1}{(n-1)[(n-1)x^* + x_1] - \delta x_1(k-1)}.$$

Then, coming back to the first agent's expected payoffs:

$$v_1\left(x_1, \boldsymbol{x}^*_{-1}\right) = [n-1-\delta(k-1)]\frac{x_1}{(n-1)[(n-1)x^* + x_1] - \delta x_1(k-1)} - cx_1.$$

The above function is easily checked to be strictly concave. Moreover, we obtain the following expression of marginal expected payoffs:

$$\frac{\partial v_1}{\partial x_1}\left(x_1, \boldsymbol{x}^*_{-1}\right) = [n-1-\delta(k-1)]\frac{(n-1)^2 x^*}{[(n-1)((n-1)x^* + x_1) - \delta x_1(k-1)]^2} - c.$$

The first order condition for an interior equilibrium yields the following equality:

$$[n-1-\delta(k-1)](n-1)^2 x^* = c[(n-1)((n-1)x^* + x_1) - \delta x_1(k-1)]^2.$$

The solution to this equation is:

$$x_1 = \frac{1}{c}\frac{n-1}{n}\frac{\sqrt{(n-1)[n-1-\delta(k-1)]} - \frac{(n-1)^2}{n}}{n-1-\delta(k-1)}.$$

Thus x_1 is the optimal deviation provided $x_1 > x^u$ which is equivalent to:

$$-\delta\left(k-1\right)\left(\left(n^2-3n+2\right)n+\left(k-1\right)\delta\right) > 0.$$

Remark that $n^2 - 3n + 2$ is non negative for $n \geq 2$. Since $\delta > 0$, the inequality never holds, which enables us to rule out the possibility of a profitable deviation for values higher than x^*.$\square$

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