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On the (In-)Efficiency of Unanimity in Multilateral Bargaining with Endogenous Recognition*

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Abstract

In this paper, we study the (symmetric) equilibria of a model of multilateral bargaining where players are heterogeneous regarding their time preferences, and make costly efforts at the beginning of the process in order to influence their probabilities of being the proposer for all stages of the negotiation process. We analyse whether the optimality of the unanimity rule (as the voting rule minimizing the social cost resulting from the agents' willingness to buy influence) characterised in Yildirim (2007) extends to the present situation. In the case of weakly heterogeneous agents, we show that k-majority rules may actually become strictly optimal. Then we provide numerical examples that suggest that there are situations where each type of voting rule (unanimity and strict k-majority) may be socially optimal.

Keywords: Sequential bargaining; Persistent recognition; Transitory recognition; Heterogeneity; Rent seeking contests.
JEL Classification Codes: C70; D72.

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1 Introduction

Negotiations are common in many important economic problems, such as legislative bargaining (Snyder et al. (2005)), international environmental agreements, litigation processes, issues of corporate governance. Agents taking part in such processes have incentives to gain power in order to influence the outcome of the process. There are plenty of evidence suggesting that agents exert (costly) efforts to promote their most preferred alternative. For instance, agents can provide services and contributions to the functioning of their organization in order to increase their chances to be elected as members of the executive committee. This in turn will enable them to influence the system of decision-making.

The agents' incentives to buy influence have been studied in certain contexts. Grossman and Helpman (2002) analyse settings where special interest groups might influence the outcome of legislative bargaining by compensating other parties or the agenda setter. Their main focus is on the effect of such process of vote buying on the characteristics of the policies that are implemented. In Evans (1997), Anbarci et al. (2002), or Board and Zwiebel (2005), agents exert unproductive efforts to influence their rights to propose. All these contributions do not compare different voting rules with respect to the social cost resulting from influence activities, which is the main goal of the present paper. As such, the closest references are Yildirim (2007, 2010), where the author analyses a sequential bargaining situation in which agents compete in order to influence their chances to become the proposer. Competition can take place at a pre-bargaining stage (persistent recognition) or at each stage of the negotiations (transitory recognition). In Yildirim (2007) the author characterizes unanimity as the unique voting rule minimizing the social cost resulting from influence activities when agents are identical and recognition is transitory. Then, in Yildirim (2010), he compares both recognition systems for a given rule (unanimity).

The present contribution complements the above two papers by comparing the optimality of the different voting rules when agents may become heterogeneous and recognition is persistent. A striking conclusion is obtained: when heterogeneity is weak, there are cases where strict k-majority rules may become optimal. It is further highlighted that it is not possible to conclude that this is a generic property of the model. Specifically, we analyse a situation with weak heterogeneity (a case where the difference between the values of the agents' discount factors is small (see Ryvkin (2007)) where k-majority rules are strictly optimal. Then we provide numerical examples that suggest that unanimity may become optimal too.

Unlike Yildirim (2007), the present paper focuses on the case of persistent recognition, where agents exert efforts to influence their chances to become proposers at the beginning of the process, i.e before the first round of negotiation. This is mainly because this type of recognition seems to be appropriate when considering many important real world processes, such as legislative bargaining or executive committees in organizations.

Since this type of recognition has not been considered in Yildirim (2007), the first step of the analysis consisted in studying the existence and characterization of the symmetric equilibria of the homogeneous case. This is provided in a companion paper (Quérou and Soubeyran (2010)), where it is highlighted that the case of persistent recognition differs substantially. Indeed, an equivalence property is obtained. While unanimity is the unique optimal voting rule when recognition is transitory, voting rules yield the same social cost (provided that an equilibrium exists) when agents are identical and exert efforts only once at the beginning of the process. This interesting result leads us to wonder if one may obtain the strict optimality of k-majority rules in heterogeneous situations.

In the second stage of the analysis (which is provided in the present paper), we introduce heterogeneity in the model by considering that agents may have different time preferences. This is modeled by assuming that the value of any agent i's discount factor δ_i is either high (δ_H) or low (δ_L), where $\delta_H > \delta_L$ are positive. We mainly focus on the case of weak heterogeneity where $\delta_H = \delta_L + \varepsilon$, with ε a positive parameter which is assumed to be sufficiently small. In this case, we highlight, by focusing on a simple case where one agent is more patient than the others, that there are situations where k-majority voting rules may become strictly optimal. Finally, we provide (numerical) examples in more general cases (regarding the agents' degree of heterogeneity) that suggest that unanimity and k-majority voting rules may become optimal too.

The intuition is as follows. Unlike the case where agents are identical, the heterogeneous case exhibits the following property. If an equilibrium level of efforts of a specific type of agent increases when moving from the unanimity to a strict k-majority voting rule, then the other type finds it optimal to decrease it. The net effect on the overall social cost is ambiguous, and each type of voting rule may become optimal.

Even though the overall conclusion is important, there are two points that we would like to stress. First, we use a specific form of recognition function in the present paper, which is yet the most widely used form in the literature on rent seeking (see Tullock (1980)). Second, the analysis provided here is not exhaustive. Specifically, we do not provide a full characterization of the cases where k-majority voting rules become optimal. We rather focus on the case where all agents become more and more impatient.

The reasons are as follows. The main goal of the analysis is to highlight the fact that the case of persistent recognition is quite specific, since the conclusions obtained here differ sensibly from those obtained in the transitory case. In order to fulfil this goal, one has to contrast results in the homogeneous and heterogeneous cases. Compared to Yildirim (2007), the introduction of heterogeneity makes it much more difficult to derive analytically tractable expressions that enable to compare the different voting rules. The logit form of the recognition function used in the present paper enables us to provide informative results, while keeping technical difficulties at a reasonable level. Developing the comparison of the voting rules in the heterogeneous case would be very difficult from a technical point of view, while (according to us) not adding much to the main message of the contribution.

The remainder of the paper is organized as follows. The model is introduced in Section 2. The characterization of the (symmetric) SSPE is provided in Sections 3 and 4. The comparison of unanimity and k-majority rules is provided in Section 5, where it is highlighted that strict k-majority rules may become optimal. Numerical examples are provided in Section 6. Section 7 concludes. Most proofs are relegated in an appendix.

2 Description of the model

We consider the problem introduced by Yildirim (2007, 2010) where agents may have different time preferences. Specifically, we assume that $n \geq 2$ agents (who belong to $N = \{1, ..., n\}$)

bargain over the allocation of a surplus of fixed size (normalized to one). They have possibly different time preferences (we denote by $0 < \delta_i < 1$ agent i's discount factor). Agents negotiate according to a bargaining protocol *a la* Rubinstein (1982), except that their recognition probabilities are endogenous. Each agent exerts effort at the beginning of the process, and relative efforts determine each agent's recognition probability for all periods.

We assume that, provided agent i exerts effort x_i at the beginning of the process, his recognition probability is given by $p_i \equiv p(x_i, x_{-i})$, where x_{-i} is the vector of efforts of the $n-1$ other players ($\boldsymbol{x}$ will denote the vector of efforts of the n players). We will have to impose more structure on the recognition probabilities in parts of the analysis, especially to characterize the social cost. We will use a Tullock contest success function by assuming the following form:

Assumption: *Let the recognition probability be such that, for* $\boldsymbol{x} \geq 0$,

$$p(x_i, x_{-i}) = \begin{cases} \frac{x_i}{\sum_{l=1}^{n} x_l} \text{ if } \boldsymbol{x} \neq \mathbf{0}, \\ \frac{1}{n} \text{ if } \boldsymbol{x} = \mathbf{0}, \end{cases} \tag{1}$$

This function has been introduced by Tullock and it has been widely used in the literature on contests. This is the simplest form of contest success functions with axiomatic foundations (Skaperdas, 1996).

Efforts are costly and, in order to keep the analysis as simple as possible, we will assume that the cost of effort is linear, that all agents have the same marginal cost of effort, and that this cost is denoted by the positive parameter c.

In order to be consistent with Yildirim (2007, 2010), we will focus on stationary subgame perfect equilibria (SSPE). With this equilibrium notion, it will be easily checked that (since $\delta_i < 1$) agent i will always have incentives to make an offer that is immediately accepted.

Before proceeding with the analysis, let us stress once again the main point of the present paper. We want to analyse the optimal voting rule under endogenous recognition. Yildirim (2007) proved that, in the homogeneous case, the unanimity rule minimizes the social cost resulting from the agents' incentives to exert unproductive efforts to become the proposer during the negotiations. He focused on the case where the contest takes place at each stage of the process (*transitory recognition*). In the present paper, we ask whether unanimity minimizes the total cost of recognition efforts when agents may be heterogeneous. It will be proved that the conclusion provided by Yildirim (on the strict optimality of the unanimity rule) has to be contrasted when recognition is persistent. Specifically, a situation will be analysed where k-majority rules become strictly optimal.

Let us now describe the different steps adopted in the present paper. In the next sections, we will use backward reasoning to characterize the SSPE and more specifically the agents' equilibrium payoffs, expected shares of the surplus, and levels of effort. We will focus on symmetric equilibria, i.e. equilibria where identical players make the same effort.

In Section 3, the optimal strategies of the negotiation stage will be characterized. In order to rule out cases where agents might be indifferent between certain strategies, we will have to rely on a tie-breaking rule that will be described in the same section.

In Section 4, we will analyse the initial stage of recognition, and we will characterize the equilibrium candidate. Then, we will complete the analysis by providing a condition that will ensure that this candidate is immune to unilateral deviations. At this stage of the analysis, we will focus on the case of weakly heterogeneous agents (as discussed by Ryvkin (2007)), and we will assume that agents become more and more impatient (their discount factors become arbitrarily small).

In Section 5, we will compare the resulting social costs and we will show that there are conditions under which strict k-majority rules become strictly optimal. We will then provide numerical examples in Section 6 in order to stress that unanimity may become optimal too.

Let us now proceed with the analysis of the negotiation stage.

3 The negotiation phase: second stage

We will use backward induction in order to solve for the SSPE of the present two stage game. We will first analyse the final stage of the game where agents negotiate in order to allocate the surplus. Some notations and definitions will be needed: they will be introduced in the first subsection. Then we will proceed with the analysis.

One important point has to be stressed. In the present section, we will study the general situation without imposing weak heterogeneity. This assumption will be needed (and introduced) later.

3.1 Notations and definitions

In order to analyse the negotiation stage we will have to characterize the agents' optimal strategies and the equilibrium shares. Each agent's strategy can be characterized by the probabilities to include other agents in his winning coalition. Let $\overline{\boldsymbol{\psi}}_i = \left(\overline{\psi}_{ij}\right)_{j \in N \setminus \{i\}}$ denote the vector of inclusion probabilities, where $\overline{\psi}_{ij}$ denotes specifically the probability that player i includes player j in his winning coalition. Under a given k-majority rule, we must have $\overline{\psi}_{ij} \in [0,1]$ for all $i, j \in N$, $i \neq j$ and $\sum\limits_{j \in N \setminus \{i\}} \overline{\psi}_{ij} = k - 1$ for all $i \in N$. It is convenient to define $\overline{\boldsymbol{\psi}}_{-i} = \left(\overline{\boldsymbol{\psi}}_1, \overline{\boldsymbol{\psi}}_2, ..., \overline{\boldsymbol{\psi}}_{i-1}, \overline{\boldsymbol{\psi}}_{i+1}, ..., , \overline{\boldsymbol{\psi}}_n\right)$.

Now let us define the shares induced by the agents' strategies. If the resulting vector of such shares is denoted by $\overline{\boldsymbol{s}} = (\overline{s}_1, ..., \overline{s}_n)$, it is easily checked that each individual share s_i is characterized by:

$$\overline{s}_i = p_i \left(1 - \overline{w}_i\right) + \overline{\mu}_i \delta_i \overline{s}_i, \text{ for } i = 1, ..., n. \tag{2}$$

where,

$$\overline{w}_i = \sum_{j \neq i} \overline{\psi}_{ij} \delta_j \overline{s}_j \text{ and } \overline{\mu}_i = \sum_{j \neq i} p_j \overline{\psi}_{ji}. \tag{3}$$

In other words, agent i's share is induced by the agents' strategies as follows. When agent i becomes the proposer (which happens with probability p_i) he will include a certain set of agents in his winning coalition. In order to do so, he will offer them their continuation value. Now, with the remaining probability agent i is not the proposer. In this case, provided that he belongs to the proposer's winning coalition (in case his own equilibrium continuation value $\delta_i s_i$ is sufficiently low) he will be offered his own continuation value.

In the next subsection we will characterize the agents' equilibrium strategies.

3.2 Second stage equilibrium

In the present subsection we characterize the agents' optimal strategies during the negotiation process (taking into account that their recognition probabilities are fixed). At this stage of the analysis we will introduce a tie breaking rule that will explain what type of behavior is assumed from identical agents.

We now proceed with the analysis. Fix $\boldsymbol{p} = (p_1, ..., p_n)$ and $\overline{\boldsymbol{s}} = (\overline{s}_1, ..., \overline{s}_n)$. The second stage equilibrium is characterized by $\boldsymbol{\psi} = (\boldsymbol{\psi}_1, ..., \boldsymbol{\psi}_n)$ such that:

$$\boldsymbol{\psi}_i = Arg \max_{\overline{\boldsymbol{\psi}}_i} \left\{ p_i[1 - \sum_{j \neq i} \overline{\psi}_{ij} \delta_j \overline{s}_j] + \sum_{j \neq i} p_j \psi_{ji} \delta_i \overline{s}_i \right\}; \tag{4}$$

The best reply of player i in the second stage of the game is given by:

$$\forall i, \forall j \neq i, \psi_{ij} = \begin{cases} 1 \text{ if } \delta_j \overline{s}_j < \delta_k \overline{s}_k \\ \leq 1 \text{ if } \delta_j \overline{s}_j = \delta_k \overline{s}_k \\ 0 \text{ if } \delta_k \overline{s}_k < \delta_j \overline{s}_j \end{cases} \tag{5}$$

This reasoning leads to the following preliminary result:

Lemma 1 : *In the equilibrium of the second stage, the vector of probabilities of inclusion,* $\psi = (\boldsymbol{\psi}_i)_{i \in N}$, *and the vector of shares* $s = (s_i)_{i \in N}$ *are functions of* $(\boldsymbol{x}, \boldsymbol{\delta})$, *with* $\psi_i = \psi_i(\boldsymbol{x}, \boldsymbol{\delta})$ *and* $s_i = s_i(\boldsymbol{x}, \boldsymbol{\delta})$ *for all* $i \in N$. *The vector of shares* s *is the solution of*

$$s_i = p_i (1 - w_i) + \mu_i \delta_i s_i, \text{ for } i = 1, ..., n, \tag{6}$$

where,

$$w_i = \sum_{j \neq i} \psi_{ij} \delta_j s_j \text{ and } \mu_i = \sum_{j \neq i} p_j \psi_{ji}, \tag{7}$$

and,

$$\forall i, \forall j \neq i, \psi_{ij} = \begin{cases} 1 \text{ if } \delta_j s_j < \delta_k s_k \\ \leq 1 \text{ if } \delta_j s_j = \delta_k s_k \\ 0 \text{ if } \delta_k s_k < \delta_j s_j \end{cases} \tag{8}$$

The second stage equilibrium strategies are characterized implicitly.

Before deriving the first stage equilibrium, we will show an interesting property of the second stage equilibrium. In order to prove the result we need to introduce a tie-breaking rule (TBR) that will specify what happens in situations where the votes of two players have the same cost. This is required in order to specify the agents' strategies when they could be indifferent between two potential candidates for their winning coalition. The tie-breaking rule can be described as follows:

Assumption (TBR): If $\delta_l s_l = \delta_k s_k = \delta_i s_i$ with $i \neq l$, then $\psi_{ji} = \psi_{jl} = \psi_{jk}$ if $j \neq i, l, k$, $\psi_{ki} = \psi_{kl}$ and $\psi_{ik} = \psi_{lk}$ if $k \neq i, l$.

This rule is quite natural and intuitive: it implies that identical players are treated the same way, and behave the same way. Equipped with this rule, we can now prove the following result:

Proposition 1: *Assume that* $\delta_i \leq \delta_l$ *and assumption (TBR) holds. In the second stage equilibrium,* $p_i < p_l \Rightarrow \delta_i s_i < \delta_l s_l$ *and* $p_i \leq p_l \Rightarrow \delta_i s_i \leq \delta_l s_l$.

Proof of Proposition 1: In the appendix. □

The above Proposition will now be used to prove the following important property:

Corollary 1: *In the second stage equilibrium, if* $\delta_i = \delta_l$, then $x_i = x_l \Longleftrightarrow p_i = p_l \Rightarrow s_i = s_l$.

Proof of Corollary 1: Assume $\delta_i \leq \delta_l$. According to Proposition 1, if $x_i = x_l$ then $\delta_i s_i = \delta_l s_l$. *Hence, if* $\delta_i = \delta_l$, we have $s_i = s_l$.□

All the above results provide us with a characterisation of the agents' optimal strategies at the last stage of the game. Now we go backward and analyse the initial stage of the game, where agents exert efforts in order to influence their recognition probability.

4 Recognition: first stage

4.1 General characterisation

Reasoning backward, the first stage equilibrium of the game is the equilibrium of a one-shot situation where the payoff of player i is given by the difference between his equilibrium share and the cost incurred, that is $v_i(\boldsymbol{x}, \boldsymbol{\delta}) = s_i(\boldsymbol{x}, \boldsymbol{\delta}) - cx_i$, with $x_i \geq 0$. The rest of the paper will focus on the symmetric equilibrium of this one shot game, where such symmetric equilibrium is characterized by a vector of effort levels $\boldsymbol{x}^* = (x^*, ..., x^*)$ satisfying the following conditions

$$v_i(\boldsymbol{x}^*, \boldsymbol{\delta}) \geq v_i\left(x', \boldsymbol{x}^*_{-i}, \boldsymbol{\delta}\right), \text{ for all } i, \tag{9}$$

with,

$$s_i = p_i(1 - w_i) + \mu_i \delta_i s_i, \textit{ for } i = 1, ..., n. \tag{10}$$

Using Lemma 1, we obtain:

$$w_i = \begin{cases} w_k + \delta_k s_k - \delta_i s_i \text{ if } \delta_i s_i \leq \delta_k s_k \\ w_k \text{ if } \delta_k s_k \leq \delta_i s_i \end{cases}, \tag{11}$$

and,

$$\mu_i \begin{cases} = 1 - p_i \text{ if } \delta_i s_i < \delta_k s_k \\ \leq 1 - p_i \text{ if } \delta_i s_i = \delta_k s_k \\ 0 \text{ if } \delta_k s_k < \delta_i s_i \end{cases}, \tag{12}$$

The above expressions result from an intuitive idea. On one side, agents choose the cheapest winning coalition when they become proposers. On the other side, only those agents with the cheapest votes will be included in the winning coalition when they are not recognized as proposers. This characterization will be used to derive the equilibrium candidates (specifically, the levels of effort chosen by the different types).

Before proceeding with this part of the analysis, it may be interesting to briefly come back to the homogeneous case. Quérou and Soubeyran (2010) analysed the existence and characterisation of symmetric equilibria in this case. They proved that, as long as a symmetric equilibrium exists, voting rules yield the same social cost. This equivalence property is substantially different from the case of transitory recognition (Yildirim 2007) where unanimity is strictly optimal.

We will now consider the case where players are heterogeneous. In the present setting, we will highlight that the conclusion reached by Yildirim may be reversed when recognition is persistent.

4.2 Recognition with weak heterogeneity from above

We analyse the case where one agent is more patient than the others. We first consider the unanimity voting rule and then k-majority rules with $k \leq n-1$. As mentioned in the introduction, we will assume that agents are weakly heterogeneous (Ryvkin (2007)), that is:

$$\delta_H = \delta_L + \varepsilon, \tag{13}$$

where ε is a positive constant assumed to be sufficiently small (we will elaborate on this point when required in the analysis). We assume that $n-1$ agents have a low discount factor (δ_L) and one agent has a high discount factor δ_H (where $0 < \delta_L < \delta_H < 1$). We assume, without loss of generality, that the n^{th} agent is the agent with the highest discount factor. We denote his share, his probability of recognition and his effort by s_H, p_H and x_H, respectively.

4.2.1 The unanimity rule

When an agreement requires unanimous consent, it is relatively easy to characterize the optimal continuation values. We first analyse the negotiation game. The expected share of the surplus secured by agent i satisfies the following equality:

$$s_i = p_i[1 - \sum_{j \neq i} \delta_j s_j] + \sum_{j \neq i} p_j \delta_i s_i; \tag{14}$$

In other words, agent i offers to each agent his expected (discounted) share of the surplus when recognized as the proposer (which happens with probability p_i). When another agent j is recognized, agent i receives his expected (discounted) share of the surplus (equal to $\delta_i s_i$). The above expression can be rewritten as follows:

$$s_i = \frac{p_i}{1-\delta_i}[1 - \sum_{j=1}^{n} \delta_j s_j]. \tag{15}$$

Multiplying (14) by δ_i, then summing over i, we obtain:

$$\sum_{j=1}^{n} \delta_j s_j = \frac{\sum_{j=1}^{n} \frac{\delta_j p_j}{1-\delta_j}}{\sum_{j=1}^{n} \frac{\delta_j p_j}{1-\delta_j} + 1}. \tag{16}$$

Coming back to expression (15) and using $\sum_{j=1}^{n} p_j = 1$, we have:

$$s_i = \frac{\frac{p_i}{1-\delta_i}}{\sum_{j=1}^{n} \frac{p_j}{1-\delta_j}}. \tag{17}$$

Using $p_i(x_i, x_{-i}) = \frac{x_i}{\sum_{j=1}^{n} x_j}$ for $\boldsymbol{x} \neq \mathbf{0}$, we finally obtain:

$$s_i = \frac{\frac{x_i}{1-\delta_i}}{\sum_{j=1}^{n} \frac{x_j}{1-\delta_j}}. \tag{18}$$

Now we come back to the first stage of the process in order to derive a closed form expression of the optimal levels of effort. At this stage, the problem of any agent i is to exert the optimal level of effort in order to maximize his expected payoff (taking into account that other agents exert efforts too). In other words, agent i's optimal level of effort solves:

$$\max_{x_i \geq 0} \pi_i(x_i, x^u_{-i}) = s_i(x_i, x^u_{-i}) - cx_i, \tag{19}$$

where $(x^u_1, ..., x^u_n)$ denotes a vector of optimal levels of effort. It is easily checked that $\pi_i(., x^*_{-i})$ is twice continuously differentiable and (strictly) concave. Thus, we know that a vector of equilibrium levels of effort exists (and is unique), and that it is characterized by the following first order conditions:

$$\frac{1}{1-\delta_i} \frac{\sum_{j \neq i} \frac{x^u_j}{1-\delta_j}}{\left[\sum_{j=1}^n \frac{x^u_j}{1-\delta_j}\right]^2} - c \leq 0, \tag{20}$$

for any agent i, where the inequality is binding if and only if the equilibrium level x^u_i is positive. Since we concentrate on symmetric equilibria where the low type agents have the same share, we must have $x^u_i = x^u_L$. The following result provides a characterization of the equilibrium levels of effort under weak heterogeneity:

Proposition 2: *At the symmetric equilibrium, both types of agents exert positive levels of effort. When the degree of heterogeneity is weak, they are given by the following expressions:*

$$x^u_L \simeq \frac{n-1}{n^2 c} \left(1 - \frac{(n-2)}{n(1-\delta_L)} \varepsilon\right), \tag{21}$$

and,

$$x^u_H \simeq \frac{n-1}{n^2 c} \left(1 + \frac{n-2}{n(1-\delta_L)} (n-1) \varepsilon\right) \tag{22}$$

Proof of Proposition 2: In the appendix. □

The above proposition states that both types of agents will exert positive levels of effort (since $\varepsilon << 1$). An increase in the degree of heterogeneity ε decreases the effort of the low types agents whereas it increases the effort of the high type agent. The general formula (without approximation) of social costs is

$$\begin{aligned} SC^u &= \sum_{i=1}^n cx^u_i = c[x^u_H + (n-1) x^u_L] \\ &= (n-1)(1-\delta_H) \frac{(n-1)(1-\delta_L) + (1-\delta_H + (n-1)(\delta_H - \delta_L))}{((n-1)(1-\delta_L) + (1-\delta_H))^2} \end{aligned} \tag{23}$$

Let us now analyse the effect of heterogeneity on the above expression of the social cost. A marginal increase in the degree of heterogeneity induces a decrease of size $\frac{(n-2)(n-1)^2}{n^3(1-\delta_L)c}$ in the cumulated effort of the low type agents $(n-1) x^u_L$ and an increase of size $\frac{(n-2)(n-1)^2}{n^3(1-\delta_L)c}$ in the effort of the high type agent. Hence, heterogeneity has no first order effect on the social cost. However, the following result highlights that an effect of second order exists:

Proposition 3: *Under unanimity, for a weak degree of heterogeneity ε, the social cost is (approximately):*

$$SC^u \simeq \frac{n-1}{n} - \frac{(n-1)^3}{n^3 (1-\delta_L)^2}\varepsilon^2, \tag{24}$$

which increases as the degree of heterogeneity ε increases *.*
Proof: Differentiating the expression of social costs with respect to ε (twice) yields the above expression. □

This proposition shows that an increase of heterogeneity will decrease social cost (with an effect of second order). Hence, the decrease in the effort of the $n-1$ low type agents outweighs the increase in the effort of the high type agent whatever the number of low type agents.[1] Specifically, we obtain the following result:

Corollary 2: *A second order approximation of SC^u around $(\delta_L, \delta_H) \to (0, 0)$ leads to:*

$$SC^u \simeq \frac{n-1}{n} - \left(\frac{n-1}{n}\right)^3 \left[(\delta_H)^2 + (\delta_L)^2\right] \tag{25}$$

In order to summarize our findings, we characterized the optimal levels of effort, derived the expression of the resulting social cost, and analysed the influence of the degree of heterogeneity when an agreement requires unanimous consent. In the next section we will proceed with the analysis of the cases where strict majorities are required.

4.2.2 Strict k-majority rules

We now proceed with the characterization of the symmetric SSPE under strict k-majority rules and weak heterogeneity from above. We still consider that the high type player is the n^{th} player. The first result implies that the optimal continuation value of the high type player must be higher than that of the low type agents. Specifically, we have:
Lemma 2: *When players are heterogeneous (from above), in a symmetric equilibrium:*

$$\delta_L s_i = \delta_L s_L < \delta_H s_H \text{ for all } i \leq n-1 \tag{26}$$

Proof of Lemma 2: In the appendix. □

The above lemma will enable us to proceed with the characterization of the equilibrium candidate. The next proposition describes the unique candidate for a symmetric equilibrium.
Proposition 4 : *When players are heterogeneous (from above), under a strict k-majority rule, the levels of effort are (in a symmetric equilibrium):*

$$x_L^k = \frac{(1-\delta_L \frac{k-1}{n-1})(n-1)}{c[n-\delta_L(k-1)]^2}; x_H^k = \frac{(1-\delta_L(k-1))(n-1)}{c[n-\delta_L(k-1)]^2}, \tag{27}$$

[1] A second order approximation of the efforts leads to: $x_L^u \simeq \frac{(n-1)}{cn^2} - \frac{(n-1)(n-2)}{cn^3(1-\delta)}\varepsilon - \frac{1}{c}\frac{(n-1)(2n-3)}{n^4(1-\delta)^2}\varepsilon^2$ and $x_H^u \simeq \frac{n-1}{cn^2} + \left(\frac{1}{c}\frac{(n-1)^2(n-2)}{n^3(1-\delta)}\right)\varepsilon - \frac{(n-1)^2}{n^4(1-\delta)^2}\left(n^2-3n+3\right)\varepsilon^2$.

the agents' equilibrium payoffs are:

$$v_H^k = \left(\frac{1 - \delta_L (k-1)}{n - \delta_L (k-1)} \right)^2, \tag{28}$$

and,

$$v_L^k = \frac{1}{(n - \delta_L (k-1))^2}, \tag{29}$$

Proof of Proposition 4: In the appendix. □

The above result identifies the unique equilibrium candidate. However, it does not prove that this candidate is actually an equilibrium. Specifically, up to this point, we assumed that the symmetric equilibrium exists. It remains to rule out potential unilateral deviations.

In order to do so, we will rewrite the discount factors as follows: $\delta = \delta_L = \frac{\delta_H}{\lambda}$ with $\lambda > 1$, since these expressions will be easier to use in the proofs of the next results.

The following condition ensures the existence of the equilibrium:

Lemma 3 : *If* $\delta \to 0$, $x_H^k, x_L^k > 0$ *and the corresponding shares are such that* $\delta_H s_H > \delta_L s_L$.

Thus, when agents become sufficiently impatient one can ensure that the unique candidate is actually immune to unilateral deviations. We can now proceed with the last step of the analysis and compare the social costs resulting from unanimity and k-majority voting rules.

5 Comparison of social costs

5.1 Optimality of strict k-majority rules

As in the previous section we denote $\delta_H = \lambda\delta$ and $\delta_L = \delta$. They will enable us to provide a simple comparison of the social costs. We obtain the following expressions:

$$SC^u = (n-1)(1-\lambda\delta) \frac{(n-1)(1-\delta) + (1 - \lambda\delta + (n-1)(\lambda-1)\delta)}{((n-1)(1-\delta) + (1-\lambda\delta))^2} \tag{30}$$

and

$$SC^k = (n-1) \frac{n - 2\delta(k-1)}{(n - \delta(k-1))^2} \tag{31}$$

Relying on the assumption of weak heterogeneity (and that all agents become arbitrarily impatient), we can compare these costs by using their second order approximations around $\delta = 0$:

$$SC^u \simeq \frac{n-1}{n} - (\lambda-1)^2 \left(\frac{n-1}{n} \right)^3 \delta^2 \tag{32}$$

$$SC^k \simeq \frac{n-1}{n} - (k-1)^2 \frac{n-1}{(n)^3} \delta^2 \tag{33}$$

and

$$SC^u - SC^k \simeq \frac{n-1}{n^3} \delta^2 \left[(k-1)^2 - (\lambda-1)^2 (n-1)^2 \right] \tag{34}$$

It is immediately checked that $SC^u > SC^k$ if and only if $1 + \frac{k-1}{n-1} > \lambda$. In other words, there exists a threshold value regarding δ such that k-majority rules become strictly optimal for any value of δ lying below this threshold. This implies that, in weakly heterogeneous situations where agents become arbitrarily impatient, unanimity is eventually dominated by strict k-majority rules.

5.2 Intuition and implications of the result

The main intuition is as follows. In the present case, the optimal strategies of both types of agent (regarding their level of effort) exhibit the following property. When the optimal level of effort of a specific type increases as a result of the introduction of a new voting rule, the second type of agents will find it beneficial to decrease his level of effort. Specifically, under a k-majority rule and heterogeneity from above, the benefit of being the proposer for a low type agent is (when $s_i = s_L$ for each low type agent i):

$$\Delta_L = 1 - (k-1)\left(\frac{n-1}{n-2} + \frac{1}{n-1}\right)\delta_L s_L \tag{35}$$

while the benefit for a high type agent is:

$$\Delta_H = 1 - (k-1)\,\delta_L s_L. \tag{36}$$

It can be checked that the efforts of all players cannot simultaneously increase when moving from unanimity to k-majority.

In order to illustrate this property of the game, let us consider the following example: $n = 3$, the other parameters being $\delta_L = 1/10$, $c = 1$, $\delta_H = 1/9$, and we consider simple majority voting. We plot the reaction functions under k-majority and unanimity of player 1 (low type) and player 3 (high type) (denoted by, respectively, $R_1^k\left(x_2^k, x_H\right)$, $R_H^k\left(x_1, x_2^k\right)$, $R_1^u\left(x_2^u, x_H\right)$ and $R_H^u\left(x_1, x_2^u\right)$), while the effort level of player 2 (low type) is fixed to its equilibrium value. We obtain the following graph:

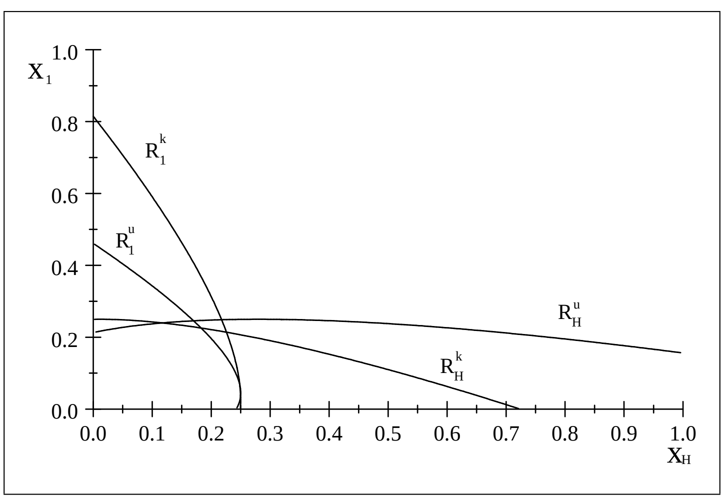

Fig. 1: Reaction functions and voting rule

This graph illustrates our main point: an increase in the optimal level of effort of one type of agent implies necessarily that the equilibrium level of effort decreases for the other type. The above result highlights that there are cases where strict k-majority voting rules may become optimal.

This conclusion may have diverse interpretations. For example, it highlights the fact that the use of executive committees in economic organizations may be efficient (provided that agents are heterogeneous) as it minimizes the agents' unproductive efforts to increase their influence. This is so because it excludes some members from the collective decision making process. Nonetheless, we do not claim that this is the unique conclusion.

Indeed, the previous analysis is restricted to the case of weak heterogeneity because the analysis of the general case becomes quickly untractable. Nonetheless, numerical examples will be provided in the next section in order to highlight that each type of voting rules may become optimal in more general cases.

6 The case of higher degrees of heterogeneity

Before concluding the paper, we would like to provide several examples in order to highlight the fact that the situation remains highly non monotonic in general heterogeneous cases. Before providing these numerical examples, we must stress the following important point. In the examples we focus on the equilibrium candidate described in the previous sections. However, we did not check whether this candidate is actually an equilibrium. The purpose of the examples is to highlight that there are situations where this candidate yields a strictly higher social cost compared to the unanimity rule. As such, they suggest that there could be situations where unanimity might become optimal as well.

Let us now provide the examples, which can be summarised as follows:

n_L	n_H	δ_H	δ_L	SC^u	SC^{n-1}	SC^{n-2}	SC^{n-3}	SC^{n-4}	SC^{n-5}	Optimal rule
2	1	1/8	1/10	.6664	.6659	.6666	–	–	–	2-majority
2	1	1/2	1/3	.645	no sym. eq.	.667	–	–	–	Unanimity
3	1	1/2	1/3	.720	no sym. eq.	no sym. eq.	.750	–	–	Unanimity
2	2	1/2	1/3	.704	.742	.731	.750	–	–	Unanimity
4	1	1/5	1/7	.774	.793	no sym. eq.	.799	.800	–	Unanimity
4	1	1/3	1/7	.772	.793	.797	.799	.800	–	Unanimity
2	3	1/5	1/7	.796	.800	.794	.796	.800	–	3-majority
3	2	1/5	1/7	.796	.798	no sym. eq.	.798	.800	–	Unanimity
2	4	1/5	1/7	.829	.822	.831	.824	no sym. eq.	.833	5-majority

Fig.2: Optimal rule: numerical examples

The above description should make it clear that the main insights gained from the analysis are robust to the introduction of a general degree of heterogeneity. One important conclusion is that one should not expect the unanimity rule to always minimize the agents' incentives to exert unproductive efforts in order to influence the outcome of the process. While this conclusion is valid when recognition is transitory and agents are identical, this is not valid under persistent recognition.

7 Conclusion

The issue of buying influence in collective decision making is extremely important as it is prevalent in many real world economic situations (lobbying in legislative bargaining, international negotiations, composition of executive committees in economic organizations). There are many questions related to this issue. The present contribution analyses a multilateral bargaining situation where recognition is persistent and endogenous and compares voting rules with respect to the social cost resulting from them. It is proved that this comparison differs depending on the type of recognition that is considered. Moreover, it is highlighted that each type of voting rule may become optimal when agents exert efforts at a pre-bargaining stage as soon as heterogeneity is brought into the picture. This stresses the fact that one should be very cautious when thinking about the choice of the appropriate voting rule in collective decision making situations where influence activities might be used. In such situations, the specifics of both the recognition process (recognition versus transitory) and of the agents' characteristics may matter.

Appendix

Proof of Proposition 1

The equivalence $x_i \leq x_l \iff p_i \leq p_l$ is directly deduced from our assumptions on the form of the recognition probabilities.Assume $\delta_i \leq \delta_l$. To show the first implication of the proposition, $p_i < p_l \Rightarrow \delta_i s_i < \delta_l s_l$, fix $\boldsymbol{p}$ and $\boldsymbol{s}$ with $p_i < p_l$ and $\delta_l s_l \leq \delta_i s_i$. We distinguish three cases.
Case 1: $\delta_l s_l \leq \delta_i s_i < \delta_k s_k$. The shares of players i and j are given by:

$$\begin{aligned} s_i &= p_i\left(1 - w_k - \delta_k s_k + \delta_i s_i\right) + \left(1 - p_i\right)\delta_i s_i \\ &= p_i\left(1 - w_k - \delta_k s_k\right) + \delta_i s_i \end{aligned} \tag{37}$$

and,

$$\begin{aligned} s_l &= p_l\left(1 - w_k - \delta_k s_k + \delta_l s_l\right) + \left(1 - p_l\right)\delta_l s_l \\ &= p_l\left(1 - w_k - \delta_k s_k\right) + \delta_l s_l \end{aligned} \tag{38}$$

Since $\sum_i s_i = 1$, we have $1 - w_k - \delta_k s_k > 0$. Since $p_i < p_l$, we have

$$\left(1 - \delta_i\right) s_i < \left(1 - \delta_l\right) s_l \tag{39}$$

Combining condition (39) with $\delta_l s_l \leq \delta_i s_i$, we have

$$\frac{1 - \delta_i}{1 - \delta_l} < \frac{s_l}{s_i} \leq \frac{\delta_i}{\delta_l}, \tag{40}$$

which implies

$$\delta_l < \delta_i, \tag{41}$$

a contradiction.
Case 2: $\delta_k s_k < \delta_l s_l \leq \delta_i s_i$. The shares of players i and j are given by:

$$s_i = p_i\left(1 - w_k\right) \tag{42}$$

and,

$$s_l = p_l\left(1 - w_k\right). \tag{43}$$

Since $p_i < p_l$, we have $s_i < s_l$ and then $\delta_l < \delta_i$, a contradiction.
Case 3: $\delta_l s_l \leq \delta_k s_k \leq \delta_i s_i$. The share of players i is such that:

$$s_i = p_i\left(1 - w_k\right) + \mu_i \delta_i s_i \leq p_i\left(1 - w_k\right) + \left(1 - p_i\right)\delta_i s_i, \tag{44}$$

The share of player l is such that:

$$\begin{aligned} p_l\left(1 - w_k - \delta_k s_k + \delta_l s_l\right) &\leq s_l = p_l\left(1 - w_k - \delta_k s_k + \delta_l s_l\right) + \mu_l \delta_l s_l && (45) \\ &\leq p_l\left(1 - w_k - \delta_k s_k + \delta_l s_l\right) + \left(1 - p_l\right)\delta_l s_l, && (46) \end{aligned}$$

with equality when $\delta_l s_l < \delta_k s_k$.

When $\delta_l s_l < \delta_k s_k < \delta_i s_i$, we have

$$s_i = p_i \left(1 - w_k\right), \tag{47}$$

and,

$$s_l = p_l \left(1 - w_k\right) + \delta_l s_l - p_l \delta_k s_k. \tag{48}$$

Combining the two equations, we obtain:

$$s_i = \frac{p_i}{p_l} \left(1 - \delta_l\right) s_l + p_i \delta_k s_k < \frac{p_i}{p_l} \left(1 - \delta_l\right) s_l + p_i \delta_i s_i, \tag{49}$$

or,

$$\left(1 - p_i \delta_i\right) s_i < \frac{p_i}{p_l} \left(1 - \delta_l\right) s_l, \tag{50}$$

Since $p_i < p_l \leq 1$, we must have

$$\left(1 - \delta_l\right) s_l > \left(1 - \delta_i\right) s_i, \tag{51}$$

hence $\delta_i < \delta_l$, a contradiction.

When $\delta_l s_l = \delta_k s_k < \delta_i s_i$, we have

$$s_i = p_i \left(1 - w_k\right), \tag{52}$$

and,

$$p_l \left(1 - w_k\right) \leq s_l \leq p_l \left(1 - w_k\right) + \left(1 - p_l\right) \delta_l s_l. \tag{53}$$

Pluging the first equation into the first inequality, we have:

$$\frac{p_l}{p_i} \leq \frac{s_l}{s_i}, \tag{54}$$

because $p_i > 0$ (indeed, if $p_i = 0$, then $s_i = 0$ and $\delta_l s_l < 0$, a contradiction). Since $p_i < p_l$, we have $s_i < s_l$. Since $\delta_l s_l < \delta_i s_i$, we must have $\delta_l < \delta_i$, a contradiction.

When $\delta_l s_l < \delta_k s_k = \delta_i s_i$, we have

$$s_i \leq p_i \left(1 - w_k\right) + \left(1 - p_i\right) \delta_i s_i. \tag{55}$$

and,

$$\left(1 - \delta_l\right) s_l = p_l \left(1 - w_k\right) - p_l \delta_k s_k. \tag{56}$$

This equation can be rewritten as:

$$\left(1 - w_k\right) = \frac{\left(1 - \delta_l\right)}{p_l} s_l + \delta_i s_i. \tag{57}$$

Hence,

$$s_i \leq \frac{p_i}{p_l} \left(1 - \delta_l\right) s_l + \delta_i s_i. \tag{58}$$

Since $p_i < p_l$, we have $\left(1 - \delta_i\right) s_i < \left(1 - \delta_l\right) s_l$. Since $\delta_l s_l < \delta_i s_i$, we must have $\delta_l < \delta_i$, a contradiction.

Finally, when $\delta_l s_l = \delta_k s_k = \delta_i s_i$, the shares are given by

$$s_i = p_i (1 - w_k) + \sum_{j \neq i} p_j \psi_{ji} \delta_i s_i, \tag{59}$$

and,

$$s_l = p_l (1 - w_k) + \sum_{j \neq l} p_j \psi_{jl} \delta_l s_l. \tag{60}$$

If $i, j \neq k$, we have:

$$s_i = p_i (1 - w_k) + \sum_{j \neq i,l,k} p_j \psi_{ji} \delta_i s_i + (p_l \psi_{li} + p_k \psi_{ki}) \delta_i s_i, \tag{61}$$

and,

$$s_l = p_l (1 - w_k) + \sum_{j \neq i,l,k} p_j \psi_{jl} \delta_l s_l + (p_i \psi_{il} \delta_l s_l + p_k \psi_{kl}) \delta_l s_l. \tag{62}$$

Using assumption TBR, we have:

$$s_i = p_i (1 - w_k) + \sum_{j \neq i,l,k} p_j \psi_{ji} \delta_i s_i + (p_l \psi_{il} + p_k \psi_{kl}) \delta_k s_k, \tag{63}$$

and,

$$s_l = p_l (1 - w_k) + \sum_{j \neq i,l,k} p_j \psi_{jl} \delta_l s_l + (p_i \psi_{il} + p_k \psi_{kl}) \delta_k s_k. \tag{64}$$

If $i = k$ or $j = k$: assume $i = k$ without loss of generality, then, using assumption TBR:

$$s_k = p_k (1 - w_k) + \sum_{j \neq l,k} p_j \psi_{ji} \delta_i s_i + p_l \psi_{kl} \delta_k s_k, \tag{65}$$

and,

$$s_l = p_l (1 - w_k) + \sum_{j \neq l,k} p_j \psi_{jl} \delta_l s_l + p_k \psi_{kl} \delta_k s_k. \tag{66}$$

Hence, for both cases (i.e. for $i \neq j$), the difference of the shares is:

$$s_i - s_l = (p_i - p_l) (1 - w_k - \psi_{kl} \delta_k s_k). \tag{67}$$

Since $\sum_i s_i = 1$, we have $1 - w_k - \psi_{kl} \delta_k s_k > 0$. Since $p_i < p_l$ we have $\frac{s_i}{s_l} < 1$. Hence $\frac{\delta_l}{\delta_i} < 1$, a contradiction.

The proof of the second result, $x_i \leq x_l \, (\iff p_i \leq p_l) \Rightarrow \delta_i \bar{s}_i \leq \delta_l \bar{s}_l$, is very similar to the proof of the first result of the proposition and is not reported here. The main difference is that the proof of the second result does not rely on assumption (TBR).□

Proof of Proposition 2

Suppose that $x_L^u = 0 < x_H^u$, then the FOC for the high type agent becomes $-c \leq 0$, and he has an incentive to choose $x_H^u = 0$, which is a contradiction. Now suppose $x_H^u = 0 < x_L^u$, then the FOCs become:

$$\frac{1}{1-\delta_H}\frac{1}{(n-1)\,x_L^u} \leq c \text{ and } \frac{n-2}{c\,(n-1)^2} = x_L^u. \tag{68}$$

Combining these two expressions and simplifying, we obtain $1-\delta_H \geq \frac{n-1}{n-2}$, which is impossible since $\delta_H < 1$. Hence, all agents exert positive levels of effort at the equilibrium. This implies that all first order conditions are satisfied as equalities. Rewriting conditions (20) we obtain:

$$\frac{1}{1-\delta_L}\frac{(n-2)\,\frac{x_L^u}{1-\delta_L} + \frac{x_H^u}{1-\delta_H}}{\left((n-1)\,\frac{x_L^u}{1-\delta_L} + \frac{x_H^u}{1-\delta_H}\right)^2} = c, \tag{69}$$

and,

$$\frac{1}{1-\delta_H}\frac{(n-1)\,\frac{x_L^u}{1-\delta_L}}{\left((n-1)\,\frac{x_L^u}{1-\delta_L} + \frac{x_H^u}{1-\delta_H}\right)^2} = c. \tag{70}$$

Condition (69) is the first order condition corresponding to a low type agent's problem. A first implication is that we have necessarily:

$$\frac{1}{1-\delta_L}\left((n-2)\,\frac{x_L^u}{1-\delta_L} + \frac{x_H^u}{1-\delta_H}\right) = \frac{n-1}{1-\delta_H}\frac{x_L^u}{1-\delta_L}, \tag{71}$$

which provides us with the expression of $\frac{x_H^u}{1-\delta_H}$ as a function of x_L^u:

$$\frac{x_H^u}{1-\delta_H} = \frac{x_L^u}{1-\delta_L}\left(\frac{n-1}{1-\delta_H} - \frac{n-2}{1-\delta_L}\right)(1-\delta_L)\,. \tag{72}$$

Plugging this expression into (69), we deduce the expression of the effort of a low type agent:

$$x_L^u = \frac{\left(\frac{1-\delta_L}{1-\delta_H}\right)(n-1)}{c\left(1+\left(\frac{1-\delta_L}{1-\delta_H}\right)(n-1)\right)^2}. \tag{73}$$

Now, using (72), we have:

$$x_H^u = \frac{(n-1)\left(\left(\frac{1-\delta_L}{1-\delta_H}\right)(n-1) - (n-2)\right)}{c\left(1+\left(\frac{1-\delta_L}{1-\delta_H}\right)(n-1)\right)^2}. \tag{74}$$

Using linear approximations of x_L^u and x_H^u around $\varepsilon = 0$, we have:

$$x_L^u \simeq \frac{n-1}{cn^2} - \frac{1}{c}\frac{(n-1)(n-2)}{n^3(1-\delta_L)}\varepsilon = \frac{n-1}{cn^2}\left(1 - \frac{n-2}{n(1-\delta_L)}\right)\varepsilon, \tag{75}$$

and,

$$x_H^u \simeq \frac{n-1}{cn^2} + \frac{1}{c}\frac{(n-1)^2}{n^3(1-\delta_L)}(n-2)\,\varepsilon = \frac{n-1}{cn^2}\left(1 + \frac{n-1}{n(1-\delta_L)}(n-2)\,\varepsilon\right). \tag{76}$$

This concludes the proof. □

Proof of Lemma 2

In a symmetric equilibrium, we must have

$$x_i = x_L \text{ for all } i \leq n-1. \tag{77}$$

Using Corollary 1, we have

$$s_i = s_L \text{ for all } i \leq n-1. \tag{78}$$

Assume that $\delta_L s_L > \delta_H s_H$. According to Proposition 1, we must have $x_L > x_H$. The shares are given by:

$$s_i = p_i \left[1 - \delta_H s_H - \frac{k-2}{n-2}\delta_L \sum_{j \leq n-1, j \neq i} s_j\right] + p_H \frac{k-1}{n-1}\delta_L s_i + (1 - p_i - p_H)\frac{k-2}{n-2}\delta_L s_i, \tag{79}$$

for $i \leq n-1$, and,

$$s_H = p_H \left[1 - \frac{k-1}{n-1}\delta_L \sum_{j \leq n-1} s_j\right] + (1 - p_H)\,\delta_H s_H. \tag{80}$$

Thus,

$$s_H = \frac{p_H}{1 - (1 - p_H)\,\delta_H}\left[1 - \frac{k-1}{n-1}\delta_L \sum_{j \leq n-1} s_j\right], \tag{81}$$

and,

$$\left(1 - p_H \frac{k-1}{n-1}\delta_L - (1-p_H)\frac{k-2}{n-2}\delta_L\right) s_i = p_i \left[\begin{array}{c} 1 - \frac{\delta_H p_H}{1-(1-p_H)\delta_H} \\ + \left(\frac{\delta_H p_H}{1-(1-p_H)\delta_H}\frac{k-1}{n-1} - \frac{k-2}{n-2}\right)\delta_L \sum_{j \leq n-1} s_j \end{array}\right], \tag{82}$$

for $i \leq n-1$.

Summing the last condition over $i \leq n-1$, we have:

$$\sum_{j \leq n-1} s_j = \frac{(1-p_H)(1-\delta_H)}{1 - \delta_H + \left(\delta_H - \frac{k-1}{n-1}\delta_L\right) p_H}. \tag{83}$$

Hence,

$$s_H = \frac{\left(1 - \frac{k-1}{n-1}\delta_L\right) p_H}{1 - \delta_H + \left(\delta_H - \frac{k-1}{n-1}\delta_L\right) p_H}, \tag{84}$$

and,

$$s_i = \frac{(1-\delta_H)\, p_i}{1 - \delta_H + \left(\delta_H - \frac{k-1}{n-1}\delta_L\right) p_H}, \tag{85}$$

for $i \leq n-1$.

Using the Tullock function, we have:

$$v_H = \frac{\left(1 - \frac{k-1}{n-1}\delta_L\right) x_H}{(1-\delta_H)\, X + \left(\delta_H - \frac{k-1}{n-1}\delta_L\right) x_H} - c x_H, \tag{86}$$

and,

$$v_i = \frac{(1-\delta_H)\, x_i}{(1-\delta_H)\, X + \left(\delta_H - \frac{k-1}{n-1}\delta_L\right) x_H} - c x_i, \tag{87}$$

for $i \leq n-1$.

Assume $x_H > 0$, the FOCs are given by:

$$\left(1 - \frac{k-1}{n-1}\delta_L\right) \frac{(1-\delta_H)\,(X - x_H)}{\left[(1-\delta_H)\, X + \left(\delta_H - \frac{k-1}{n-1}\delta_L\right) x_H\right]^2} = c, \tag{88}$$

and,

$$(1-\delta_H)\, \frac{(1-\delta_H)\,(X - x_i) + \left(\delta_H - \frac{k-1}{n-1}\delta_L\right) x_H}{\left[(1-\delta_H)\, X + \left(\delta_H - \frac{k-1}{n-1}\delta_L\right) x_H\right]^2} = c. \tag{89}$$

In a symmetric equilibrium, we have:

$$\left(1 - \frac{k-1}{n-1}\delta_L\right)(1-\delta_H)(n-1)\, x_L = c\left[(1-\delta_H)(n-1)\, x_L + \left(1 - \frac{k-1}{n-1}\delta_L\right) x_H\right]^2, \tag{90}$$

and,

$$\begin{aligned} & (1-\delta_H)\left[(1-\delta_H)(n-2)\, x_L + \left(1 - \frac{k-1}{n-1}\delta_L\right) x_H\right] \\ = \ & c\left[(1-\delta_H)(n-1)\, x_L + \left(1 - \frac{k-1}{n-1}\delta_L\right) x_H\right]^2. \end{aligned} \tag{91}$$

Combining the two conditions, we have:

$$\frac{x_L}{x_H} = \frac{1 - \frac{k-1}{n-1}\delta_L}{\left(1 - \frac{k-1}{n-1}\delta_L\right)(n-1) - (1-\delta_H)(n-2)}. \tag{92}$$

The Right hand side is striclty larger than 1 if and only if

$$\frac{k-1}{n-1}\delta_L > \delta_H, \tag{93}$$

a contradiction.

Now assume $x_H = 0$, the FOCs are:

$$\left(1 - \frac{k-1}{n-1}\delta_L\right) \frac{(1-\delta_H)(n-1)\, x_L}{\left[(1-\delta_H)(n-1)\, x_L\right]^2} < c, \tag{94}$$

and,

$$(1-\delta_H)\, \frac{(1-\delta_H)(n-2)\, x_L}{\left[(1-\delta_H)(n-1)\, x_L\right]^2} = c. \tag{95}$$

Hence,

$$x_L = \frac{1}{c}\frac{n-2}{(n-1)^2}, \tag{96}$$

and condition (94) becomes:

$$\delta_L > \frac{\delta_H (n-2) + 1}{k-1}. \tag{97}$$

Since $k \leq n-1$, we have $\frac{\delta_H(n-2)+1}{k-1} > \delta_H$ and (97) implies $\delta_L > \delta_H$, a contradiction.

Finally, we must have $\delta_H s_H > \delta_L s_L$.□

Proof of Proposition 4

We first deal with the most general case where the voting rule is not equivalent to a dictatorship of the proposer and the number of players is at least 3. Then, we will prove that the general characterization of the equilibrium remains valid for the case where $k = 1$. Consequently, the statement of the proposition holds for $n = 2$ (because $k = 1$ is the only strict k-majority for $n = 2$).

Let us proceed with the first step of the proof. We know from Lemma 2 that $\delta_L s_L < \delta_H s_H$, that is, the low type agent is the cheapest one. This means that any agent will include $k-1$ low type agents in his winning coalition. Moreover, the only chance that the high type agent gets a positive share of the surplus is that he becomes the proposer. The shares can thus be rewritten as follows (we still assume that agents $i = 1, ..., n-1$ constitute the sub-population of low types):

$$s_i = p_i[1 - \sum_{j \neq i, j \leq n-1} \frac{k-1}{n-2}\delta_j s_j] + \sum_{j \neq i, j \leq n-1} p_j \frac{k-1}{n-2}\delta_i s_i + p_H \frac{k-1}{n-1}\delta_i s_i, \tag{98}$$

for $i = 1, ..., n-1$ and

$$s_H = p_H[1 - \sum_{j \leq n-1} \frac{k-1}{n-1}\delta_j s_j], \tag{99}$$

for the high type agent.

Let us elaborate on both expressions. Any low type agent i includes $k-1$ low type agents in his winning coalition when he becomes the proposer. The tie-breaking rule is such that each remaining low type agent $j \neq i$ is included in the winning coalition with the same probability $\frac{k-1}{n-2}$. Now, when another low type agent $j \neq i$ becomes the proposer, he includes i in his winning coalition with probability $\frac{k-1}{n-2}$. Finally, when the high type agent becomes the proposer, i is included in the winning coalition with probability $\frac{k-1}{n-1}$. This explains expression (98). Now, when the high type agent is the proposer, he includes $k-1$ low type agents in his winning coalition. Again, the probability that any low type agent be included in the winning coalition is the same, but now it is $\frac{k-1}{n-1}$. When a low type agent is the proposer, the high type agent is never in the winning coalition (since $k \leq n-1$).

Using $\delta_j = \delta_L$ for all $j \leq n-1$ and $\sum_{j \neq i, j \leq n-1} p_j = 1 - p_i - p_H$, (98) can be rewritten as follows:

$$s_i = \frac{p_i}{1 - \delta_L\left((1-p_H)\frac{k-1}{n-2} + p_H\frac{k-1}{n-1}\right)}\left(1 - \delta_L\frac{k-1}{n-2}\sum_{j \leq n-1} s_j\right), \tag{100}$$

Summing (98) over $i = 1, ..., n-1$ and simplifying, we obtain:

$$\sum_{i=1}^{n-1} s_i = \frac{\sum_{i=1}^{n-1} p_i}{1 - \delta_L \frac{k-1}{n-1} p_H} = \frac{1 - p_H}{1 - \delta_L \frac{k-1}{n-1} p_H}. \tag{101}$$

In order to keep the expressions simple, let us denote $q = \frac{k-1}{n-1}$, which is the "quota" of the k majority rule. Coming back to expression (99) we obtain:

$$s_H = \frac{(1 - \delta_L q)\, p_H}{1 - \delta_L q p_H} \tag{102}$$

and, plugging (101) into (98), we have

$$s_i\left(1-\delta_L\frac{k-1}{n-2}\left(1-p_H\right)-\delta_L q p_H\right)=p_i\left(1-\delta_L\frac{k-1}{n-2}\frac{1-p_H}{1-\delta_L q p_H}\right) \tag{103}$$

for any low type agent i. This implies that, for any $i\leq n-1$ we have:

$$s_i=p_i\frac{1-\delta_L\frac{k-1}{n-1}\frac{1-p_H}{1-\delta_L q p_H}}{1-\delta_L\frac{k-1}{n-2}(1-p_H)-\delta_L q p_H}. \tag{104}$$

Rewriting and simplifying, we obtain:

$$s_i=\frac{p_i}{1-\delta_L q p_H}, \tag{105}$$

for any agent $i=1,...,n-1$.

Reasoning backward, we can now analyse the first stage where players compete in a contest for recognition. Using the Tullock function, the shares can be written as functions of the levels of effort $(x_1,...x_{n-1},x_H)$:

$$s_i=\frac{x_i}{X-\delta_L q x_H}, \tag{106}$$

for any agent $i=1,...,n-1$, with $X=\sum_{j=1}^{n}x_j$, and

$$s_H=\frac{\left(1-\delta_L q\right)x_H}{X-\delta_L q x_H}, \tag{107}$$

for the high type agent. At this stage, the problem of any agent i is to exert the optimal level of effort in order to maximize his expected payoff (taking into account that other agents exert efforts too), or in other words:

$$\max_{x_i\geq 0}\pi_i(x_i,x_{-i}^k)=s_i(x_i,x_{-i}^k)-cx_i, \tag{108}$$

where $(x_1^k,...x_{n-1}^k,x_H^k)$ denotes a vector of optimal levels of effort. As in the case of unanimous consent, we easily check that each agent's expected payoff is strictly concave in x_i, that a unique equilibrium candidate (characterized by a vector of effort levels) exists, and that the corresponding effort levels are characterized by the following first order conditions:

$$\frac{X^k-\delta_L q x_H^k-x_i^k}{\left(X-\delta_L q x_H^k\right)^2}\leq c, \tag{109}$$

for any agent $i=1,...,n-1$, where $X^k=\sum_{j=1}^{n-1}x_j^k+x_H^k$ and the inequality is binding if and only if the equilibrium level x_i^k is positive. And,

$$\frac{\left(1-\delta_L q\right)\left(X^k-x_H^k\right)}{\left(X^k-\delta_L q x_H^k\right)^2}\leq c, \tag{110}$$

for the high type agent. Since we focus on symmetric equilibria where the low type agents have the same share, we must have $x_i^k = x_L^k$ for any low type agent. Hence, in equilibrium, we have,

$$\frac{(n-2)\,x_L^k + (1-\delta_L q)\,x_H^k}{\left((n-1)\,x_L + (1-\delta_L q)\,x_H^k\right)^2} \leq c, \tag{111}$$

and,

$$\frac{(1-\delta_L q)\,(n-1)\,x_L^k}{\left((n-1)\,x_L^k + (1-\delta_L q)\,x_H^k\right)^2} \leq c, \tag{112}$$

Assume first that $x_L^k = 0 < x_H^k$: then condition (112) becomes $0 \leq c$ and the high type player has no incentive to make a positive effort, $x_H^k = 0$, which is a contradiction. Now assume that $x_H^k = 0 < x_L^k$, then $s_H = 0 < s_L = \frac{1}{n-1}$. This contradicts our initial assumption that $\delta_L s_L < \delta_H s_H$. Hence, in equilibrium, we have $x_L^k, x_H^k > 0$, i.e. the equilibrium is interior and the FOCs are binding:

$$\frac{(n-2)\,x_L^k + (1-\delta_L q)\,x_H^k}{\left((n-1)\,x_L^k + (1-\delta_L q)\,x_H^k\right)^2} = c, \tag{113}$$

and,

$$\frac{(1-\delta_L q)\,(n-1)\,x_L^k}{\left((n-1)\,x_L^k + (1-\delta_L q)\,x_H^k\right)^2} = c. \tag{114}$$

A necessary condition is then:

$$(1-\delta_L q)\,x_H^k = (1-\delta_L q\,(n-1))\,x_L^k. \tag{115}$$

Substituting this expression into (114), we have:

$$x_L^k = \frac{(1-\delta_L q)\,(n-1)}{c\left((n-1)\,(1-\delta_L q)+1\right)^2}. \tag{116}$$

Using the necessary condition, we obtain:

$$x_H^k = \frac{(n-1)\,(1-\delta_L q\,(n-1))}{c\left((n-1)\,(1-\delta_L q)+1\right)^2}. \tag{117}$$

This concludes the proof for $2 \leq k \leq n-1$. Let us now check that the characterization remains valid when $k = 1$ for $n \geq 2$. In this specific case, the characterization of the optimal strategies at the second stage is straightforward. Specifically, the share of any agent i is given by the following expression:

$$s_i = p_i. \tag{118}$$

Then, using the expression of the recognition function, checking that agent i's payoff is strictly concave and deriving the first order conditions, we conclude immediately that the unique equilibrium candidate for a symmetric SSPE is such that $x_i = \frac{1}{c}\frac{n-1}{n^2}$ for any agent $i = 1, ..., n$. This expression corresponds to the case $k = 1$ when using the general characterization introduced in the statement of the proposition. Moreover, it is easily checked that this candidate is actually an equilibrium. Indeed, this is true since the characterization of all agents' optimal shares during the second stage does not change even in case of a unilateral deviation (because $k=1$). Thus, the fact that the vector of effort levels satisfies the first order conditions enables us to conclude the proof. □

Proof of Lemma 3

The proof results from the combination of the three following lemmas:

Lemma 4 : *If* $\delta \rightarrow 0$, the levels of effort x_H^k, x_L^k are positive *and the corresponding shares are such that* $\delta_H s_H > \delta_L s_L$.

Proof: Since $x_L^k > x_H^k$, the condition is equivalent to $x_H^k > 0$, that is,

$$\frac{1}{k-1} > \delta. \tag{119}$$

When $\delta \rightarrow 0$, this condition is valid.

Condition $\delta_H s_H > \delta_L s_L$ is equivalent to

$$\delta_H \left(1 - \delta_L \frac{k-1}{n-1}\right) x_H^k > \delta_L x_L^k \tag{120}$$

or,

$$\left(1 - \delta(k-1)\right)\lambda - 1 > 0. \tag{121}$$

When $\delta \rightarrow 0$, this condition is valid. This concludes the proof. □

Thus we know that the candidate described in Proposition 4 is consistent with situations where all agents become very impatient. To ensure that the candidate is indeed an equilibrium, we just need to check that it is robust to all possible unilateral deviations. This is checked for each type of agent in Lemmas 5 and 6. Specifically, we have:

Lemma 5: *Under a strict* k*-majority rule, when* $\delta \rightarrow 0$*, the high type agent has no incentive to deviate from* $\boldsymbol{x}^k = \left(x_L^k, ..., x_L^k, x_H^k\right)$.

Proof: We already know from the proof of Proposition 4 that there is no profitable deviation such that $\delta_L s_i = \delta_L s_L < \delta_H s_H$ for all $i \leq n-1$.

Let us first consider the case where the H-type agent (agent n) deviates by exerting an effort such that $\delta_H s_H = \delta_L s_L$. Using assumption (TBR), the shares are given by the following expressions:

$$\begin{aligned} s_H &= p_H[1-(k-1)\delta_L s_L] + (1-p_H)\frac{k-1}{n}\delta_H s_H; \\ s_L &= p_L[1-(n-1)\frac{k-1}{n}\delta_L s_L - \frac{k-1}{n}\delta_H s_H] + (1-p_L)\frac{k-1}{n}\delta_L s_L \end{aligned} \tag{122}$$

Rearranging these expressions, we obtain:

$$\begin{aligned} s_H &= p_H[1-(k-1)\delta_L s_L - \frac{k-1}{n}\delta_H s_H] + \frac{k-1}{n}\delta_H s_H; \\ s_L &= p_L[1-(k-1)\delta_L s_L - \frac{k-1}{n}\delta_H s_H] + \frac{k-1}{n}\delta_L s_L \end{aligned} \tag{123}$$

Then

$$s_H = \frac{n - \delta_L(k-1)}{n - \delta_H(k-1)} s_L. \tag{124}$$

Since $\delta_H s_H = \delta_L s_L$, we must have

$$\delta_L s_L = \delta_H \frac{n - \delta_L(k-1)}{n - \delta_H(k-1)} s_L. \tag{125}$$

Now assume $s_L = 0$, then $s_H = 0$ and $s_L = p_L$ must be strictly positive because $x_L^k > 0$. Hence, we must have

$$\delta_L = \delta_H \frac{n - \delta_L(k-1)}{n - \delta_H(k-1)}, \tag{126}$$

which is equivalent to $\delta_L = \delta_H$, a contradiction.

Now consider that the high type agent deviates by exerting an effort x_n such that $\delta_H s_H < \delta_L s_L$. Equations (86) and (87) yield the following expressionfs of the payoff functions:

$$v_H = \frac{\left(1 - \frac{k-1}{n-1}\delta_L\right) x_n}{\left(1 - \delta_H\right)\left(n-1\right) x_L^k + \left(1 - \frac{k-1}{n-1}\delta_L\right) x_n} - c x_n, \tag{127}$$

and,

$$v_i = \frac{\left(1 - \delta_H\right) x_L^k}{\left(1 - \delta_H\right)\left(n-1\right) x_L^k + \left(1 - \frac{k-1}{n-1}\delta_L\right) x_n} - c x_i, \tag{128}$$

for $i \leq n-1$. Condition $\delta_H s_H < \delta_L s_L$ is equivalent to:

$$\delta_H \left(1 - \frac{k-1}{n-1}\delta_L\right) x_n < \delta_L \left(1 - \delta_H\right) x_L^k, \tag{129}$$

or,

$$x_n < \frac{\delta_L \left(1 - \delta_H\right)}{\delta_H} \frac{\left(n-1\right)}{c\left(n - \delta_L\left(k-1\right)\right)^2} \equiv \overline{x}. \tag{130}$$

The optimal (interior) $\widehat{x}_n$ is such that

$$\widehat{x}_n = \frac{1}{c} \frac{\left(n-1\right)\sqrt{\left(1-\delta_H\right)}}{n - \delta_L\left(k-1\right)} \left[1 - \frac{\sqrt{\left(1-\delta_H\right)}\left(n-1\right)}{n - \delta_L\left(k-1\right)}\right]. \tag{131}$$

It is easily checked that $\widehat{x}_n > 0$ is equivalent to

$$\delta_H > 1 - \left(\frac{n - \delta_L\left(k-1\right)}{n-1}\right)^2 = \left(\frac{-1 + \delta_L\left(k-1\right)}{n-1}\right)\left(1 + \frac{n - \delta_L\left(k-1\right)}{n-1}\right). \tag{132}$$

Condition 1 ensures that the RHS is negative and then $\widehat{x}_n > 0$.

Condition (130) can be rewritten as:

$$1 - \frac{\left(n-1\right)\sqrt{\left(1-\delta\lambda\right)}}{n - \delta\left(k-1\right)} < \frac{1}{\lambda} \frac{\sqrt{\left(1-\delta\lambda\right)}}{n - \delta\left(k-1\right)}. \tag{133}$$

For $\delta = 0$, this condition becomes $\frac{1}{n} < \frac{1}{\lambda}\frac{1}{n}$, which is impossible. Since both sides are continuous functions of δ, when δ is sufficiently close to 0, the condition does not hold.

This enables us to conclude that the payoff the high type player can get is bounded above by the value of v_H at $\overline{x}$ (which we denote by $\overline{v}$). Formally,

$$v_H < \overline{v} = \frac{\delta_L}{(n-1)\,\delta_H + \delta_L} - \frac{\delta_L\,(1-\delta_H)}{\delta_H}\frac{(n-1)}{(n-\delta_L\,(k-1))^2}, \tag{134}$$

or,

$$\overline{v} - v_H^k = \frac{\delta_L}{(n-1)\,\delta_H + \delta_L} - \frac{\frac{\delta_L(1-\delta_H)}{\delta_H}(n-1) + (1-\delta_L(k-1))^2}{(n-\delta_L\,(k-1))^2} \tag{135}$$

This difference is negative only if

$$\begin{aligned}\delta_L\,(n-\delta_L\,(k-1))^2 \quad \leq \quad & \frac{\delta_L\,(1-\delta_H)}{\delta_H}\,((n-1)\,\delta_H+\delta_L)\,(n-1) \\ & + ((n-1)\,\delta_H+\delta_L)\,(1-\delta_L(k-1))^2\,,\end{aligned} \tag{136}$$

or,

$$\begin{aligned} f\,(\delta,\lambda) \quad &= \quad \lambda\,(n-\delta\,(k-1))^2 - (1-\lambda\delta)\,((n-1)\,\lambda+1)\,(n-1) \\ -\lambda\,((n-1)\,\lambda+1)\,(1-\delta(k-1))^2 \quad &\leq \quad 0 \end{aligned} \tag{137}$$

where $\delta = \delta_L$ and $\lambda\delta = \delta_H$. Since f is a continuous function of δ and $f\,(0,\lambda) = -\,(\lambda-1)^2\,(n-1)$, we conclude that $f\,(\delta,\lambda) \leq 0$ when δ is sufficiently close to 0. This rules out this type of unilateral deviations, which concludes the proof. □

Now, we proceed with the possible deviations of a low type player. Without loss of generality, we assume that player 1 deviates and chooses an effort denoted $x_1 \neq x_L^k$. The efforts of the other players are fixed: $x_i = x_L^k$ for $2 \leq i \leq n-1$ and $x_n = x_H^k$ for the high type player. Thus, whatever x_1, we have $p_i = p_L^k$ for $2 \leq i \leq n-1$ and $p_n = p_H^k$ for the high type player. Using Corollary 1, we have also $s_i = s_L$ for $2 \leq i \leq n-1$. We obtain the following result:

Lemma 6: *Under a strict k-majority rule, when $\delta \to 0$, any low type agent has no incentive to deviate from $\boldsymbol{x}^k = \left(x_L^k, ..., x_L^k, x_H^k\right)$.*

Proof: First remark that, when the players become very impatient, the cost of the votes become negligeable and the share of each player comes close to his recognition probability:

$$\lim_{\delta\to 0} s_i = \lim_{\delta\to 0} p_i. \tag{138}$$

In the proof we will also use that

$$\lim_{\delta\to 0} x_L^k = \lim_{\delta\to 0} x_H^k = \frac{n-1}{cn^2}. \tag{139}$$

Notice that a deviation by player 1 will modify the shares of all other players. However, the resulting shares of the other low type agents will still be indentical (see Corollary 1). We distinguish 2 different cases:

Case 1: $\delta_L s_L < \delta_L s_1$. According to Proposition 1, since $\delta_L s_L < \delta_L s_1$, we must have $x_L^k < x_1$. Then, let $x_1 = \gamma x_L^k$ with $\gamma > 1$. Using (138) and (139), we have:

$$\lim_{\delta \to 0} v_1 = \frac{\gamma}{\gamma + (n-1)} - c\gamma \frac{n-1}{cn^2}, \tag{140}$$

and,

$$\lim_{\delta \to 0} v_L^k = \frac{1}{n^2}. \tag{141}$$

For $\delta \to 0$, player 1 has an incentive to play x_1 rather than x_L^k if and only if:

$$\lim_{\delta \to 0} v_1 - \lim_{\delta \to 0} v_L^k \geq 0, \tag{142}$$

or,

$$0 \geq (n-1)^2 [\gamma - 1]^2, \tag{143}$$

a contradiction.

Case 2: $\delta_L s_1 < \delta_L s_L$. According to Proposition 1, since $\delta_L s_1 < \delta_L s_L$, we must have $x_1 = \beta x_L^k$ with $\beta < 1$. Then it is easily checked that the following condition holds:

$$\lim_{\delta \to 0} v_1 = \frac{\beta}{\beta + n - 1} - \beta \frac{n-1}{n^2}. \tag{144}$$

For $\delta \to 0$, player 1 has an incentive to play x_1 rather than x_L^k if and only if:

$$\lim_{\delta \to 0} v_1 - \lim_{\delta \to 0} v_L^k \geq 0, \tag{145}$$

or,

$$0 \geq (\beta - 1)^2 (n-1), \tag{146}$$

a contradiction. This concludes the proof. □

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