Consistent conjectures in a dynamic model of non-renewable resource management

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Abstract

We consider a dynamic model of non-renewable resource extraction under the assumption that players do not know their opponents' utility functions. Agents form conjectures on the behavior of others. Two forms of beliefs are introduced, namely, beliefs based on the state of the resource, and beliefs based on the state of the resource and on the strategy of other agents (their consumption). We focus on consistent equilibria, where states corresponding to the players' beliefs must be consistent with observed past plays. Closed form expressions of the optimal consumption plans and of the state dynamics are derived. Comparisons are made with the full information benchmark case. It is proved that, depending on the initial consumption levels, the present process might lead to different patterns of management of the resource in the long run. With strategy and state based beliefs, the agents behave more (respectively, less) agressively than in the non-cooperative benchmark when initial consumption is high (respectively, intermediate). When initial consumption is low, the optimal consumption path lies below that of the cooperative benchmark.

Keywords: Dynamic game; resource extraction; non probabilistic beliefs, conjectural variations.
JEL Classification: C72; C73; Q30.

1 Introduction

The interplay between external and strategic effects is a main feature of many economic issues. An important example of this fact is the model of resource extraction studied by Levhari and Mirman (1980).[1] This study highlights

[1] An extension of this study can be found in Datta and Mirman (1999).

the main issue created by this interplay. Specifically, non-cooperative behavior in the problem of resource management leads to overconsumption of the resource compared to a socially efficient (joint) management. Different studies have confirmed this issue in problems of resource management (see for instance Fisher and Mirman (1992, 1996), or Houba et al (2000)). Several conclusions follow. Efficient consumption plans do exist, but this typically requires the use of discontinuous strategies (Dutta and Sundaram (1993)), or the threat of punishment (Benhabib and Radner (1992)). In this literature decision takers are assumed to have perfect knowledge of the different characteristics of the setting. In other words, the information is assumed to be complete. Moreover, they have the ability to anticipate perfectly the influence of their present consumption policy on all future consumption decisions (they are perfectly rational). In practice, agents might lack information regarding the other people's preferences, and their rationality might be bounded.

We revisit the problem of non-renewable resource management where information is incomplete, and consequently depart from the traditional complete information approach adopted in the above studies. Specifically, in our procedure, decision makers know the evolution rule of the resource but do not have information about their opponents' preferences and operate based on simple beliefs about their opponents' behaviour. An important feature is that players rely on non-probabilistic beliefs, which we also refer to as conjectures (see for instance Figuières et al (2004)). Basically, players assume that a variation of their own consumption has a first order linear effect on the consumption of others.

The notion of conjectures is used in the empirical literature as it seems to provide a general framework to model imperfect competition (see Slade (1995) as an example). The theoretical literature on conjectural variation equilibrium has thus focused on several questions in order to rationalize this notion. A part of the literature tries to provide support for this notion in static games by introducing different forms of consistency. However, Lindh (1992) shows that endogeneizing conjectures by using notions of consistency does not rationalize conjectural equilibria in a static setting. The only way to rationalize this type of equilibrium is to develop a dynamic notion.

Thus, another part of the literature focuses on the notion of conjectural variations in dynamic settings. There are two different types of contributions. Some papers provide support for conjectural variations by considering evolutionary games with myopic agents and showing that consistent conjectures are evolutionary stable. This is shown by Muller and Normann (2005) in linear quadratic oligopoly models (and further generalized by Possajennikov (2009)). Dixon and Somma (2003) provide an evolutionary process converging to consistent conjectures in an oligopolistic model. Other papers consider a repeated game setting where dynamic notions of conjectural variations are introduced and analysed. Recent contributions are Friedman and Mezzetti (2002), and Jean-Marie and Tidball (2005), where the main focus is on repeated oligopoly games. Finally, learning procedures based on conjectures are developed in Jean-Marie and Tidball (2006) and Quérou and Tidball (2009). In these papers agents assume that a variation of their own strategy has a first order linear effect on the strategy of others, and beliefs are updated based on observations over time.

The present paper belongs to the second part of this literature, and complements the existing contributions by analysing a situation where there are state dynamics. More specifically, both Friedman and Mezzetti (2002) and Jean-Marie and Tidball (2005) analyse purely repeated games. By contrast we analyze a discounted discrete-time, infinite-horizon extraction game with conjectures, where the state of the resource is subject to a dynamic evolution rule and players use feedback strategies.

We consider two different types of beliefs. In one case agents assume that the strategy of others is a function of the state of the resource at the previous period (state based beliefs). In the other case they assume that their opponents' behavior is a function of the state of the resource and of the strategy of the other agents (state and strategy based beliefs).

Beliefs are required to be consistent with observed policies (we use a concept of consistency that has been developed in Jean-Marie and Tidball (2005)).[2] More specifically, consistency requires that the paths corresponding to the players' beliefs must be consistent with observed past plays. Jean-Marie and Tidball (2005) show (in theorem 5.1) in a finite horizon, repeated game setting that, with state based beliefs[3] the consistent equilibrium coincides with the feedback Nash equilibrium (under complete information). This result provides support (based on rational grounds) for this notion of consistency. We will prove (in proposition 4.2) that the same result is valid in the present setting (keeping in mind that we consider a fully dynamic game here). We will further develop the contribution by analysing the case of state and strategy based beliefs.

Let us describe the main parts of the analysis. The procedure and consistent solutions are defined. Then convergence is studied, and the resulting policy is compared to the benchmark of complete information both from an economic and environmental point of view. Closed form expressions of the solutions

[2]Other studies have defined notions of consistency in dynamic games, as Friedman (1977), Fershtman and Kamien (1985), or Laitner (1980). These notions raise issues regarding the computability of solutions. We may refer to Friedman and Mezzetti (2002) who propose another possible model too.

[3]The state refers to the usual definition when a repeated game is considered.

are derived. It is proved that there are situations where consistent solutions yield better outcomes in terms of environmental sustainability compared to the case of complete information Cournot Nash equilibrium. This is shown to rely on the agents' initial levels of consumption.
The paper is organized as follows. The model and the main results of the benchmark case of full information are introduced in sections 2 and 3 (respectively). The situations with state based and state and strategy based beliefs are analyzed in section 4. Section 5 concludes. Technical details of one proof are provided in an appendix in Section 6.

2 The benchmark case of complete information

The model of resource extraction analysed by Lehvari and Mirman (1980) is briefly introduced in the first sub section. The players' utility functions and the specifics of the model are described and the notations explained.

2.1 Dynamics

We consider a 2-agent problem of natural resource extraction, where the resource is subject to a dynamic evolution rule. The time horizon of the problem is infinite. Let x_t be the stock of natural resource at time t. If $c_{i,t}$ denotes agent $i = 1, 2$ extraction at time t, then the evolution rule is given by

$$x_{t+1} = [x_t - c_{1,t} - c_{2,t}]^{\alpha}, \quad x(0) = x_0, \quad \alpha \in (0, 1]. \tag{1}$$

In expression (1) parameter α denotes the measure of the rate of regeneration of the natural resource. In the remaining of the paper we will restrict the analysis to the case of a non renewable resource (that is, a situation where $\alpha = 1$). This is made for analytical tractability[4].

Let us discuss the other assumptions of the present model. First, the biological rule is strictly concave and extraction is assumed to be linear. This is assumed in models of resource extraction to ensure that the situation remains analytically tractable (see Arnason et al. (2004) for an empirical application). The two assumptions imply that a (unique) interior solution to the problem of optimal extraction will exist. In the present contribution

[4]The case with $0 < \alpha < 1$ will be analyzed numerically in a future work.

this leads to the existence of an (unique) interior optimal consumption path.[5] This enables us to make a precise comparison with the complete information model.[6]

Second, players share the same extraction technology. Again this ensures that the optimal consumption policies can be characterized analytically. The present contribution constitutes the first application of the present notion of consistent beliefs to such dynamic models. Thus analytical results are required to highlight its main characteristics (link between initial conditions and optimal consumption policies, comparison with the benchmark case). Applying it to situations with heterogeneous agents is the next step of the project, but this will rely on a numerical approach.

In the next sub section the agents' benefit functions are defined, and we elaborate on the implications resulting from this specification.

2.2 The agents' payoff functions

Each agent derives utility from present and future consumptions. A given agent's consumption policy influences obviously the consumption of the other by its influence on the evolution of the resource. As such we consider an infinite horizon dynamic game. As in the contribution of Levhari and Mirman (1980) we will assume that the relationship (for agent i) between consumption at period t and the level of instantaneous utility derived is given by the following specification:

$$u_i(c_{i,t}) = log(c_{i,t}),$$

where $c_{i,t}$ denotes the consumption of agent i at period t. This is consistent with our willingness to highlight the main characteristics of our learning procedure by comparing our findings with those of the benchmark case in terms of the long run management strategies that result from each contribution.

Agents are assumed to choose their optimal consumption policies in order to maximize the discounted sum of their instantaneous utility derived from consumption, that is, to maximize

$$\sum_{t=0}^{\infty} \beta^t u_i(c_{i,t}),$$

[5] When agents have perfect foresight, the existence of a Markov perfect equilibrium is ensured.

[6] Let us keep in mind that Levhari and Mirman (1980) do not analyse the case of a non renewable resource. We use the present specification as it is simple and will enable us to derive closed form expressions of the optimal policies.

taking into account the evolution of the resource (and the influence of the other agents' consumption strategy on this evolution as well). In the above specification of payoffs we assume that $0 < \beta < 1$ denotes the measure of the agents' time preferences. In the case of Levhari and Mirman (1980) the information is assumed to be complete, while in the present contribution agents do not know their opponent's utility function and form beliefs about their behavior. We will later assume two simple forms of such beliefs and we will solve for the above specification of the dynamic game.

In the next sections we will characterize the optimal policies corresponding to the complete information benchmark case,[7] then we will define the notions of state based and strategy and state based beliefs (as introduced in Jean-Marie and Tidball (2005)) and we will apply these notions to the present model. This will enable us to derive comparisons between our findings and the benchmark case.

3 The full information benchmark case

Let us be more specific about the way we proceed with the analysis. As a first step we consider the complete information benchmark case. We characterize the optimal consumption policies (and related state dynamics) under joint and non-cooperative managements (in sections 3.1 and 3.2 respectively).

Then in the next section (section 4) players are assumed to lack information on their opponent's utility function and to form conjectures about their behavior. We first analyze the dynamic game where agents' conjectures are a function of the level of the stock only (state based beliefs). We then consider a situation where players' beliefs at period t are a function of their own previous consumption decision and of the current state of the resource (state and strategy based beliefs). We require conjectures to be consistent with past observed decisions.[8] In the context of state and strategy based beliefs, consistency requires that the functional relationship that links (x_t, c_{t-1}) to an agent's belief regarding the opponent's optimal catch at period t corresponds to the actual optimal policy of this country.

In both cases we derive the closed form expressions of the optimal consumption policies. As a final step we compare the present findings with

[7]Because this characterization is straightforward, we will omit the full proofs.

[8]If optimal decisions (according to beliefs) were inconsistent with past observed decisions then the players would have no incentive to keep on using this scheme.

the benchmark case and we highlight the role of initial conditions (here the initial levels of consumption) in this comparison.

Let us now proceed with the analysis of the case of joint management.

3.1 The cooperative case

In this section the information is assumed to be complete. In the case of a linear evolution rule the problem for a given agent is the following:

$$\max_{\{c_{1,t};c_{2,t}\}} \sum_{t=0}^{\infty} \beta^t \left[log(c_{1,t}) + log(c_{2,t})\right]$$

subject to

$$x_{t+1} = x_t - c_{1,t} - c_{2,t}, \quad x_0 \text{ given.}$$

We restrict to feedback strategies, that is, we assume that $c_{i,t} = a_i x_t$, for any player $i = 1, 2$. Plugging this expression into the above equality, we obtain

$$x_{t+1} = (1 - a_1 - a_2)x_t, \quad x_t = (1 - a_1 - a_2)^t x_0.$$

The problem then becomes:

$$\max_{\{a_1,a_2\}} \sum_{t=0}^{\infty} \beta^t \left[log(a_1) + log(a_2) + 2log(1 - a_1 - a_2).\right]$$

Soving for a_1 and a_2 it is easily checked that the optimal consumption policies are characterized by

$$a_1 = a_2 = (1 - \beta)/2, \quad x_t^c = \beta^t x_0, \quad c_{i,t}^c = \frac{1-\beta}{2}\beta^t x_0,$$

where $c_{i,t}^c$ denotes the optimal consumption policy of agent i (at period t) under joint management of the resource. In the next sub-section we derive the closed form expression of the optimal consumption policies when agents behave non-cooperatively.

3.2 The non-cooperative case: Nash equilibria

We want to calculate the non-cooperative optimal consumption policies. So, for players $i = 1, 2$, the problem is specified as follows:

$$\max_{\{c_{i,t}\}} \sum_{t=0}^{\infty} \beta^t log(c_{i,t})$$

subject to

$$x_{t+1} = x_t - c_{1,t} - c_{2,t}, \quad x_0 \text{ given.}$$

In this setting we again consider feedback strategies, thus $c_{i,t} = a_i x_t$, $i = 1, 2$. Plugging this expression into the above equality, the problem becomes:

$$\max_{\{a_i\}} \sum_{t=0}^{\infty} \beta^t \big[log(a_i) + log(1 - a_1 - a_2).\big]$$

Solving for a_1 and a_2, we obtain the following non-cooperative consumption policies:

$$a_1 = a_2 = \frac{1-\beta}{2-\beta}, \quad x_t^N = \big(\frac{\beta}{2-\beta}\big)^t x_0, \quad c_{i,t}^N = \frac{1-\beta}{2-\beta}\big(\frac{\beta}{2-\beta}\big)^t x_0,$$

where $c_{i,t}^N$ denotes the optimal consumption policy of agent i (at period t) under non-cooperative management of the resource. Notice that

$$x_t^c = \beta^t x_0 > \big(\frac{\beta}{2-\beta}\big)^t x_0 = x_t^N, \quad \forall t. \tag{2}$$

In other words, strategic considerations lead to a more intensive extraction of the resource than under joint management. This is the main conclusion provided by Levhari and Mirman (1980) in the case of a renewable resource.

We would like to stress the following point. We could have used the Bellman equation associated to the above non-cooperative problem in order to solve for the optimal consumption policy at each period of the game. Instead we use the linear structure of the game. In such a case the optimal solution is linear, and this enables us to simply identify the (optimal) coefficients. The same method will be used in Section 4.1 to derive the consistent solution with state based beliefs.

In the next section we will consider the case where information is incomplete.

4 The situation of incomplete information with conjectures

In this section we consider that agents do not have information on the other agent's preferences. However, they all know the evolution rule of the resource. We first analyse the case where players use state-based beliefs, that is, where beliefs are a function of the remaining stock of the resource at any given period (in section 4.1). We confirm the result of Jean-Marie and Tidball (2005, theorem 5.1) who show (in a **repeated game** setting) that the consistent equilibrium with state based conjectures coincides with the complete information feedback Nash equilibrium. Then we analyse the case of strategy and state based beliefs (in section 4.2) and we compare the optimal consumption policies (corresponding to the consistent solution) with those of the full information benchmark case.

4.1 The case of state based beliefs

Let us consider in a first step the situation where any player i conjectures that player j's consumption policy at period t is a function of x_t only. Specifically, we consider that agent 1's beliefs regarding the behavior of agent 2 have the following form:

$$c_{2,t}^{1} = a_2 x_t.$$

In other words, agent 1 conjectures that the optimal consumption of agent 2 at any given period t is a function of the stock of the resource at the beginning of the same period. It is implicitly assumed that each agent can observe the stock of the resource at the end of each period.

Now we look for consistent solutions, that is, solutions such that the conjectured policy corresponds to the actual optimal policy. This means that conjecture a_2 must be such that the conjectured value of consumption of agent 2 (as implied by the above expression) corresponds to the solution to the optimal problem of agent 2 (for any given period).

It can be checked that a straightforward application of this definition of consistency leads to the following conclusion.[9]

Proposition 4.1. *Let us denote by $c_{i,t}^{fc}$ the (feedback) consistent optimal consumption policy of agent i ($i = 1, 2$) at period t. When agents use state based beliefs, the optimal consumption policy is characterized, for any period t and any agent i, by:*

$$c_{i,t}^{fc} = c_{i,t}^{N},$$

where $c_{i,t}^{N}$ denotes the non-cooperative optimal consumption policy in the benchmark (full information) case.

[9] As said at the end of the previous section, we use the specific structure of the game as it enables us to simply identify the optimal coefficients characterizing the solution.

The proof is omitted as it is immediately checked that one has to solve the same system of first order conditions in both situations. Because of the assumptions there is a unique optimal policy that satisfies these conditions, which yields the conclusion.

The implication of this result is that the non-cooperative management of the resource is equivalent under both complete and incomplete information. Even though this conclusion might be perceived as disappointing at first sight, there is a notable difference between the two cases. Indeed, requiring consistency enables to get rid of the assumption of perfect knowledge about the agents' utility functions. Thus, the case of state based beliefs provides some support for the notion of consistent conjectures. In this simple case conjectures are rationalized in that the notion of consistency leads to the well know concept of feedback Nash equilibrium.

We will now introduce a second type of beliefs, which are defined as a function of the state of the resource and of the agents' consumption strategy. We will then compare the optimal consumption policies with those of the benchmark case.

4.2 The case of state and strategy based beliefs

We first consider a type of beliefs where a conjecture is a function of both the state of the resource and of the consumption pattern. We will see that this definition allows for a rich pattern of potential behaviors. Moreover it will be stressed that the optimal consumption policy (and thus the management of the resource along the equilibrium path) depends on the initial consumption levels. This influence will be characterised as well.

We now proceed with the definition of strategy and state based beliefs. Let us consider the case of player 1. This player conjectures that player 2's consumption decision at period t is given by:

$$c^1_{2,t} = a_2 x_t + b_2 c_{1,t-1},$$

where a_2 and b_2 model the player's beliefs. In other words, agent 1 assumes that the consumption policy of agent 2 at period t is a function of the state of the resource at period t and of agent 1's consumption strategy at period $t-1$.[10]

[10] One should keep in mind that x_t is the state of the resource at the beginning of period t. As such, it is observable by the agent before he chooses his optimal consumption policy

This form deserves further comment. Firstly, it is implicitly assumed that agents observe the level of the resource and the consumption of others at every period. Secondly, it is worth explaining why we choose this form. This is the simplest form of beliefs that can be thought of when beliefs are defined as a function of the characteristics of the situation at each period, namely, the state of the resource and the strategy of the other agents. Since agents are assumed to learn the behavior of others, it seems natural to assume that they define their beliefs as a function of the only characteristic of the other agents that they can observe. We consider the simplest form of relationship to define conjectures by assuming that the consumption of one agent has a first order linear effect on the behavior of the others. A natural implication is that agents are (to some extent) boundedly rational. The other way to think about the definition of the conjectures is to consider the present learning procedure as a potential heuristic that could be used to guide individual decision making. It could be useful if it proves to be reasonably efficient. We will come back to this question during the analysis of the process.[11]

Let us come to the notion of feedback consistency. If there is a persisting difference between the consumption of player 2 as conjectured by player 1 and the actual optimal policy of player 2, then player 1 would have incentives to stop relying on this scheme. This is why we require that solutions be consistent. Consistent solutions are such that the conjectured policy corresponds to the actual optimal policy. This notion of consistency enables us to introduce an equivalent formulation of the problem.[12]

Specifically, from Jean-Marie and Tidball (2005) it is known that finding the feedback consistent solution to the present problem is equivalent to finding the solution to:

$$\max_{\{c_t^i\}} \sum_{t=0}^{\infty} \beta^t log c_{i,t},$$

subject to the following constraints:

$$x_{t+1} = x_t - a_j x_t - b_j y_t - c_{i,t}$$

at this period, together with consumptions at period $t-1$. Thus, his belief at period t is based on the data that are observable at the beginning of this period.

[11] Let us notice that the notion of strategy and state based beliefs shares a feature in common with the notion introduced by Friedman and Mezzetti (2002). Specifically, they use the other agents' strategies at period $t-1$ in order to define the agents' conjectures at period t. We will see that the present paper is consistent with their analysis, as we will check that there is a link between the initial conditions (the consumption levels) and the optimal consumption path.

[12] We elaborate on the characterization of consistent solutions by using the equivalent formulation of the problem.

and

$$y_{t+1} = c_{i,t}$$

where initial stock x_0 and initial catch $c_{i,0}$ are common knowledge at the beginning of the problem. For instance, for $i = 1$, the dynamics of the equivalent problem can be rewritten as follows:

$$x_{t+1} - x_t = -a_2 x_t - b_2 y_t - c_{1,t} \tag{3}$$

and

$$y_{t+1} - y_t = c_{1,t} - y_t. \tag{4}$$

In the present case the state vector of the dynamics is defined by equations (3) and (4). Specifically, at any period t the state of the process is given by $(x_t, c_{1,t-1}, c_{2,t-1})$. Requiring feedback consistency amounts to requiring that the functional relationship that links $(x_t, c_{1,t-1}, c_{2,t-1})$ to the belief of each agent regarding the opponent's optimal catch at period t corresponds to the actual optimal policy of this country.

From this definition of consistency the remaining steps in calculations are straightforward. When we derive the optimality conditions associated to the above problem, we will obtain a set of parameters (a_2, b_2) (respectively, a set of pairs (a_1, b_1) for agent 2) that are potential solutions. Then we will look for the specific values that correspond to the feedback consistent solutions.

The above specification of the problem will enable us to derive the closed form expression of the optimal consumption path. Then we will compare the optimal policies with those of the benchmark case provided in Section 3. Specifically, we will prove that the symmetric consistent optimal consumption policy (and resulting stock level) is characterized by

$$c_t^{fc} = [\beta(1-a)]^t c_0, \quad x_t^{fc} = \frac{c_t^{fc} - b c_{t-1}^{fc}}{a},$$

for any period t (c_t^{fc} denotes the symmetric (feedback) consistent optimal consumption policy at period t with strategy and state based beliefs).

We can now state the main result of this section. We provide an explicit link between the level of initial consumption and the closed form expressions of optimal consumption policies and corresponding state dynamics. This enables us to compare the different policies to the benchmark cases of sections 3.1 and 3.2.[13] The following remark will prove to be useful in this analysis.

[13] A general conclusion will be that the resource will be exhausted eventually (since optimal consumption is positive and the resource is non renewable). The important point is to compare the findings under incomplete and full information.

Remark 4.1. *First notice that we deal with values of* c_0 *such that* $x_0 > 2c_0$*, otherwise the stock will be exhausted before the beginning of the process. Second, one could notice that, since the discount factor* β *lies between 0 and 1, we have the following inequalities:*

$$\frac{1-\beta}{2}x_0 < \frac{1-\beta}{2-\beta}x_0 < \frac{1}{2}x_0. \tag{5}$$

These inequalities will be useful in the next result.

The main conclusions are described as follows.

Proposition 4.2. *The following results hold:*

- *If* $c_0 = \frac{1-\beta}{2}x_0$*, then* $a = 0$ *and* $b = \beta$*. The feedback consistent solution is given by :*

  $$c_t^{fc} = \beta c_{t-1}^{fc}, \quad x_t^{fc} = \beta^t x_0.$$

  In this case $x_t^{fc} = x_t^c$*. In others words, if the level of initial consumption corresponds to that of the full information cooperative case, then the feedback consistent solution coincices with the benchmark solution under joint management.*

- *If* $c_0 \neq \frac{1-\beta}{2}x_0$ *then :*

  $$c_t = [1 - 2\frac{c_0}{x_0}]^t c_0, \quad x_t = [1 - 2\frac{c_0}{x_0}]^t x_0.$$

  Moreover, we can state the following comparisons:

  – *if* $c_0 < \frac{1-\beta}{2}x_0$ *then* $a < 0$ *and* $b > 0$ *and* $x_t^{fc} > x_t^c > x_t^N$*;*

  – *if* $\frac{1-\beta}{2}x_0 < c_0 < \frac{1-\beta}{2-\beta}x_0$ *then* $a > 0$*,* $b > 0$ *and* $x_t^N < x_t^{fc} < x_t^c$*;*

  – *if* $\frac{1-\beta}{2-\beta}x_0 < c_0 < \frac{1}{2}x_0$ *then* $a > 0$ *and* $b < 0$ *and* $x_t^{fc} < x_t^N < x_t^c$*.*

Proof. See Appendix. □

Let us elaborate on the implications of the above result. First, the optimal consumption policies (and thus, the stock of the resource along the equilibrium path) depend on the initial consumption level. This is an important feature of the present model. The observation of this initial condition can help one to anticipate which of the patterns of consumption will prevail. From another perspective, this implies that one might influence the consumption path that will be chosen by focusing on the initial level of consumption.

Second, the proposition leads to several predictions. The first part of the result states that the feedback consistent solution coincides with the cooperative solution under complete information provided that the initial level of consumption is the same in both cases. This is intuitively appealing. If agents cooperate initially (even though they might not know it due to incomplete information) then asking for consistency ensures that cooperation will be sustained in the long run.

If the initial consumption is too high, then the present procedure leads to a more aggressive pattern than even in the non-cooperative case under full information. In such a case the effects of strategic behaviors are reinforced by incomplete information. However, provided that agents' consumption level lies initially below a threshold value the previous conclusion is reversed. Specifically, for moderate values, the stock of the resource lies in between non-cooperative and joint management patterns under full information.

Finally, when initial consumption is sufficiently low, we obtain the surprising conclusion that the present procedure leads to an under-exploitation of the resource compared to the full information cooperative benchmark (if one focuses on economic efficiency only, that is, on getting the most utility out of consumption). The intuition is as follows. Unlike the situation of complete information, the choice of the initial consumption level influences the shape of the optimal consumption path. The consistency requirement enables one to select a pattern of individual behavior that comes close to pure cooperation. Then, the choice of a sufficiently low initial consumption level enables the agents to select a path that is parallel to the full information cooperative benchmark, but that lies below it (due to the choice of a lower initial consumption level). This result has an interesting implication. There are situations where the management of the resource could be subject to other criteria than economic efficiency. For instance, it could be the case that the focus be on sustainable management. In such a case, the use of the procedure would be clearly interesting as it would lead to a better management of the resource (with regard to sustainability) compared to the cooperative benchmark. This suggests that in the third case incomplete information makes the emergence of alternative (and more cautious) patterns of behavior plausible, and our procedure takes advantage of this possibility.

There is a final insight to be noticed. For such a management of the resource to be feasible, initial consumption must be kept below the level that corresponds to a joint management under complete information. It could be the case that some (public) intervention be required to achieve this task. An appropriate extraction tax combined with the use of the procedure might thus contribute to a better management outcome.

5 Concluding remarks

We study a problem of resource management under incomplete information when decision takers interact strategically and where each agent's consumption decision has an influence on the evolution of the size of the resource. A learning procedure is developed where agents form conjectures which are updated according to available observations. We consider two types of conjecture, one based only on the state of the resource, the other based on the state and the consumption strategy of the other agents. The solution studied is such that beliefs must be consistent with observed behaviors.

Closed form expressions of the optimal policies are obtained and compared to the benchmark case (as provided in Section 3). It is proved that the optimal policies correspond to the full information, non-cooperative solution when conjectures are based on the state of the resource only. In the second case (with state and strategy based beliefs), the consistent solution is shown to yield better outcomes regarding the resource management in the long run compared to joint management if initial consumption is sufficiently low, or to lead to a more aggressive pattern than the non-cooperative benchmark if initial consumption is too high. For intermediate values of initial consumption, the optimal path lies in between the non-cooperative and cooperative benachmark cases.

From a practical point of view the present procedure provides an explicit link between the initial conditions and the resulting long run policies. In other words, the dependence of convergence on the initial conditions explains which of the equilibria will prevail given the initial conditions. As incomplete information is a usual characteristic of resource extraction, the knowledge of an explicit link between initial consumption and resulting long run dynamics is a potentially useful property. If one thinks about this learning procedure as a heuristic to be used to guide individual decision making, then this link is useful as it provides potential policy makers with a way to induce agents to adopt different patterns of management of the resource.

Even though the present contribution highlights some interesting properties of the learning procedure, we abstract from several issues. A first extension would be to analyse cases where agents could be heterogeneous. For instance, they could use different extraction technologies. The investigation of situations where the resource is renewable is an important case as well. These points are left for future research.

6 Appendix

In this appendix we now provide the proof of proposition 4.2. We proceed in three steps. First, we derive the optimality conditions associated to the problem. This enables us to characterize the optimal consumption path. Secondly, we characterize the solutions that satisfy our requirement of consistency. Finally, the conclusions of proposition 4.2 follow immediately from this characterization and the results obtained in Section 3.

Let us first derive the optimality conditions associated to the problem. With this new specification (associated to the new state dynamics (3, 4)), the principle of the maximum can be used.[14] The associated hamiltonian is (using (3) and (4)):

$$H = \beta^t log c_{1,t} + \pi_t[-a_2 x_t - b_2 y_t - c_{1,t}] + \lambda_t[c_{1,t} - y_t],$$

where λ_t and π_t denote the lagrangian parameters associated to the two constraints of the present optimization problem. For any period t the first order conditions are:

$$\frac{\partial H}{\partial c_{1,t}} = \frac{\beta^t}{c_{1,t}} - \pi_t + \lambda_t = 0,$$

$$\pi_t - \pi_{t-1} = -\frac{\partial H}{\partial x_t} = a_2 \pi_t,$$

$$\lambda_t - \lambda_{t-1} = -\frac{\partial H}{\partial y_t} = b_2 \pi_t + \lambda_t. \tag{6}$$

From conditions (6) we obtain:

$$c_{1,t} = \frac{\beta^t}{\pi_t - \lambda_t}; \;\; \pi_t = \frac{\pi_0}{(1-a_2)^t}; \;\; \lambda_t = -b_2 \pi_{t+1}.$$

[14] It should be noted that the use of conjectures enables us to move from a dynamic game to a problem of optimal control. This is why we can use the principle of the maximum here.

Using these expressions we can rewrite $\pi_t - \lambda_t$ as

$$\pi_t - \lambda_t = \frac{\pi_0}{(1-a_2)^t} + \frac{b_2\pi_0}{(1-a_2)^{t+1}} = \frac{\pi_0}{(1-a_2)^t}\frac{1+b_2-a_2}{1-a_2}.$$

Thus, we obtain a final expression of the optimal consumption plan:

$$c_{1,t} = [\beta(1-a_2)]^t \frac{1-a_2}{\pi_0(1+b_2-a_2)}. \tag{7}$$

Using this expression for $t = 0$, we can deduce an expression of the initial parameter π_0 as:

$$\pi_0 = \frac{1-a_2}{c_{1,0}(1+b_2-a_2)}. \tag{8}$$

Now we can rewrite the expression of the state dynamics. Let us denote

$$A = \frac{b_2+\beta(1-a_2)}{\beta(1+b_2-a_2)\pi_0}; \tag{9}$$

now we can obtain that

$$b_2c_{1,t-1} + c_{1,t} =$$

$$b_2(\beta(1-a_2))^{t-1}\frac{1-a_2}{\pi_0(1+b_2-a_2)} + (\beta(1-a_2))^t\frac{1-a_2}{\pi_0(1+b_2-a_2)} =$$

$$(\beta(1-a_2))^t A.$$

Thus, the state dynamics can be rewritten as:

$$x_{t+1} = (1-a_2)x_t - b_2c_{1,t-1} - c_{1,t} = (1-a_2)x_t - [\beta(1-a_2)]^t A;$$

or

$$x_t = (1-a_2)^t x_0 - A(1-a_2)^{t-1}\sum_{i=0}^{t-1}\beta^i = (1-a_2)^t x_0 - A(1-a_2)^{t-1}\frac{1-\beta^t}{1-\beta}. \tag{10}$$

Up to now there are several potential solutions to the problem (that is, several possible pairs of parameters (a_2, b_2)).

Now we solve for the feedback consistent solutions. We consider symmetric solutions because we deal with a symmetric problem. Basically this amounts

to assuming $c_{1,0} = c_{2,0} = c_0$, that is, the initial level of consumption is the same for both agents. Since the problem is symmetric, we focus on a symmetric solution too. This implies $c_{1,t} = c_{2,t} = c_t$ for any period t, hence $a_1 = a_2 = a$, and $b_1 = b_2 = b$.
Now we come back to feedback consistency. As said previously, the state of the process is given (for any period t) by (x_t, c_{t-1}). Feedback consistency requires that an agent's consumption strategy as conjectured by the other agent corresponds to his actual optimal policy. Formally, we must have:

$$ax_t + bc_{t-1} = c_t. \tag{11}$$

Thus, to obtain the feedback consistent solutions, one can rewrite condition (11) by using the expressions of c_t and x_t obtained in (7) and (10), and then obtain the corresponding values of coefficients a and b.
Rewriting condition (11), we have:

$$c_t = [\beta(1-a)]^t \frac{1-a}{\pi_0(1+b-a)} =$$

$$a[(1-a)^t x_0 - A(1-a)^{t-1}\frac{1-\beta^t}{1-\beta}] + b[\beta(1-a)]^{t-1}\frac{1-a}{\pi_0(1+b-a)}.$$

Using the expression of A given by (9) and rewriting, we obtain:

$$c_t = [\beta(1-a)]^t \frac{1-a}{\pi_0(1+b-a)} = (1-a)^t[ax_0 - \frac{a}{(1-a)(1-\beta)}\frac{b+\beta(1-a)}{\beta\pi_0(1+b-a)}]$$

$$+[\beta(1-a)]^t\{\frac{a[b+\beta(1-a)]}{\beta\pi_0(1-a)(1-\beta)(1+b-a)} + \frac{b(1-a)}{\beta\pi_0(1-a)(1+b-a)}\}.$$

Now feedback consistency is equivalent to looking for values of coefficients a and b such that:

$$ax_0 - \frac{a}{(1-a)(1-\beta)}\frac{b+\beta(1-a)}{\beta\pi_0(1+b-a)} = 0 \tag{12}$$

and

$$\frac{a[b+\beta(1-a)]}{\beta\pi_0(1-a)(1-\beta)(1+b-a)} + \frac{b(1-a)}{\beta\pi_0(1-a)(1+b-a)} = \frac{1-a}{\pi_0(1+b-a)}. \tag{13}$$

It is easily checked that conditions (12) and (13) yield the following solutions:

$$a = \frac{x_0\beta - x_0 + 2c_0}{x_0\beta}, b = \frac{(x_0 - 2x_0)(c_0\beta - 2c_0 - x_0\beta + x_0)}{\beta c_0 x_0} = \frac{x_0 - 2x_0}{x_0} - \frac{x_0 - 2x_0}{c_0}a. \tag{14}$$

We now prove proposition 4.2. The proof is straigthforward because by construction

$$c_t^{fc} = [\beta(1-a)]^t c_0, \quad x_t^{fc} = \frac{c_t^{fc} - bc_{t-1}^{fc}}{a}.$$

The optimality of corresponding policies results from the signs of a and b, which in turn follow from (14) and (5). The comparisons follow immediately from the forms of optimal policies derived in sections 3.1 and 3.2. This concludes the proof.

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