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An introduction to valued constraint satisfaction

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An introduction to Valued Constraint Satisfaction

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with contributed slides by P. Jeavons (Univ. Oxford), M. Cooper (Univ. Toulouse)
Javier Larrosa (UPC, Spain), S. de Givry, D. Allouche & A. Favier (INRA, France),
R. Dechter (UCI, USA), R. Marinescu (4C, Ireland)

Valued Constraint Satisfaction

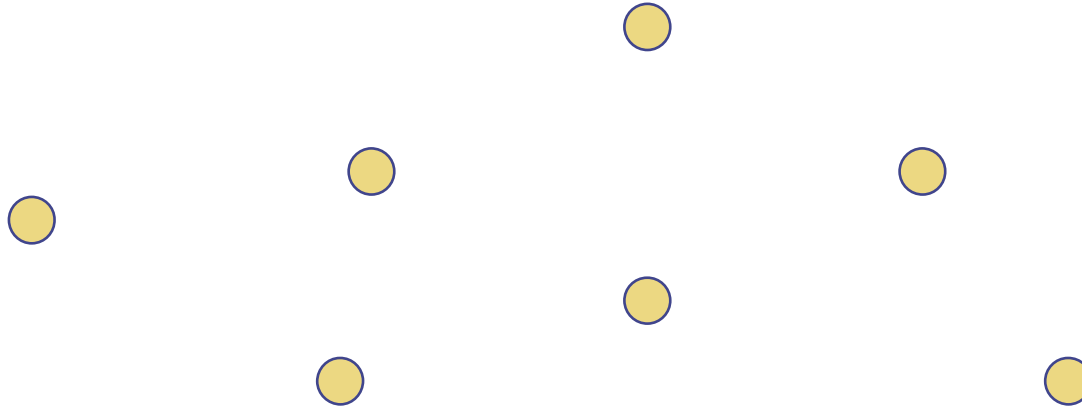
- ◆ What is it and why do we need it?
- ◆ Can it be done efficiently?
- ◆ Search
- ◆ Problem transformations
- ◆ Open problems

Chapter 1. What is it?

Motivation,
Definitions,
Some general theorems

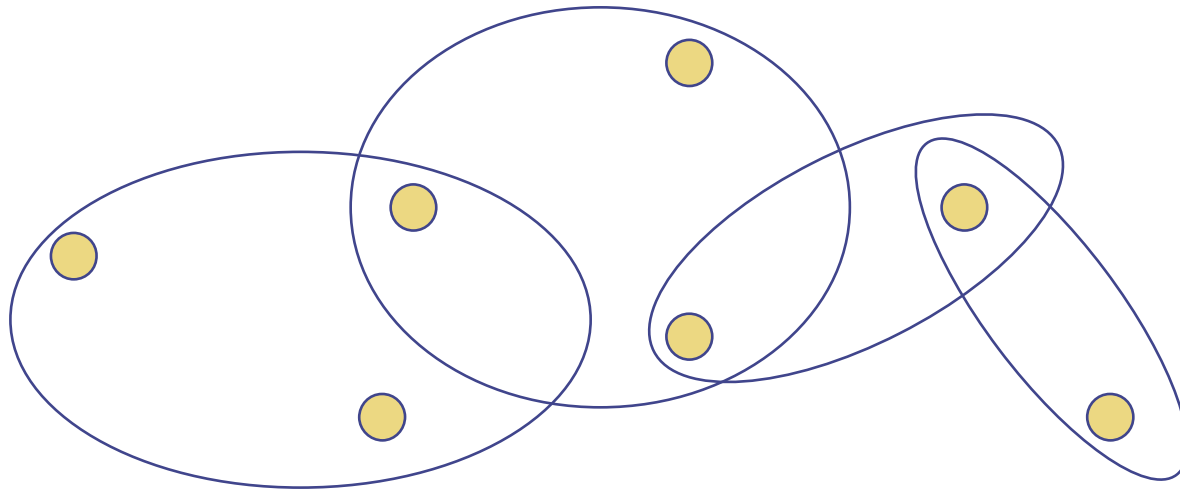
Constraint Satisfaction Problem

A unifying abstraction



- Variables ● = Talks to be scheduled at conference
Transmitters to be assigned frequencies
Amino acids to be located in space
Circuit components to be placed on a chip

A unifying abstraction



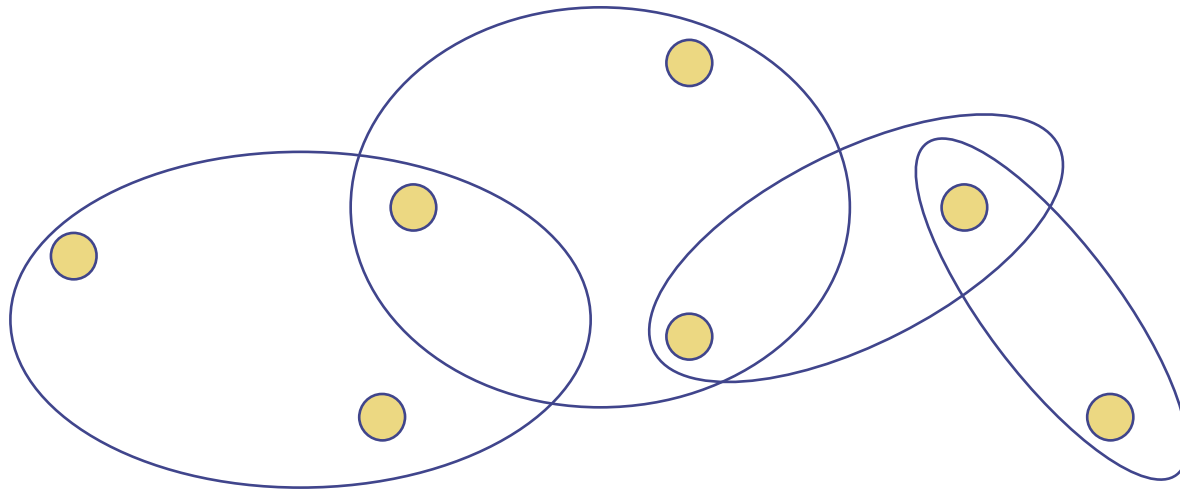
Constraints \circ = All invited talks on different days

No interference between near transmitters

$$x + y + z > 0$$

Foundations dug before walls built

A unifying abstraction



A **solution** is an assignment of values to variables that satisfies all the constraints

Constraint programming (OR, Ilog Solver...)

But what if...

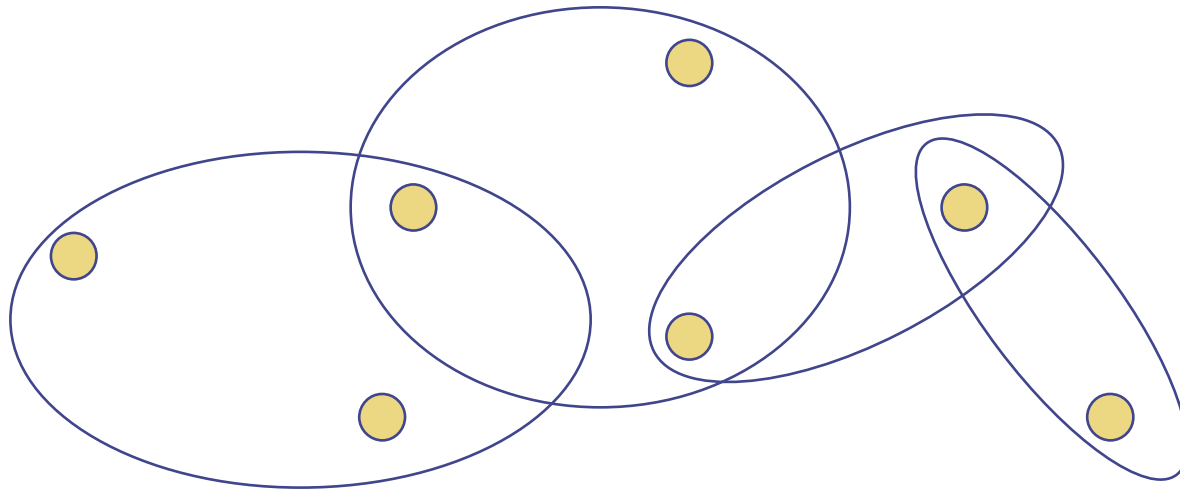
- ◆ There are lots of solutions, but some are better than others?
- ◆ There are no solutions, but some assignments satisfy more constraints than others?
- ◆ We don't know the exact constraints, only probabilities, or fuzzy membership functions?
- ◆ We're willing to violate some constraints if we can get a better overall solution that way?

Fragmentation/Heterogeneity

- ◆ Fuzzy CSP (easier to solve, Rosenfeld 76)
- ◆ Max, Weighted, Partial CSP (Shapiro 81, Freuder 91)
- ◆ Weighted Max-SAT
- ◆ Constraint Optimization Problems
- ◆ Lexicographic CSP
- ◆ Hierarchical Constraint Logic Programming (Borning et al)

- ◆ Pseudo-Boolean Optimisation
- ◆ Bayesian Networks
- ◆ Random Markov Fields
- ◆ Factor Graphs
- ◆ Integer Programming
- ◆ 2D grammars...

A unifying abstraction



“Constraints”  associate costs with each assignment

A solution is an assignment of values to variables that minimises the combined costs

Definition of a VCSP instance

(IJCAI 1995)

- ◆ a set of n variables X_i with domains d_i
- ◆ a set of e cost functions, each having a
 - scope (list of variables)
- ◆ cost functions map assignments to costs

It only remains to specify what the possible costs are,
and how to combine them

Definition of a valuation structure

- ◆ a set S of costs
- ◆ a total order $<$
- ◆ minimum and maximum elements:
we denote these by 0 and ∞
- ◆ an aggregation operator \oplus which is commutative, associative, monotonic, and such that $\forall \alpha, \alpha \oplus 0 = \alpha$

Examples of valuation structures

- ◆ If $S = \{0, \infty\}$, then VCSP \equiv CSP
- ◆ If $S = \{0, 1, 2, \dots, \infty\}$, and \oplus is addition, then VCSP generalizes MAX-CSP
- ◆ If $S = [0,1]$, and \oplus is max, then VCSP \equiv Fuzzy CSP
- ◆ If $S = \{0, 1, \dots, k\}$, and \oplus is bounded addition $+_k$ where $\alpha +_k \beta = \min\{k, \alpha + \beta\}$, then VCSP \equiv Weighted CSP

Families of valuation structures

A valuation structure is **idempotent** if

$$\forall \alpha, \alpha \oplus \alpha = \alpha$$

All idempotent valuation structures
are equivalent to Fuzzy CSP

(as in CSP redundancy of information is fine)

Families of valuation structures

A valuation structure is **strictly monotonic** if

$$\forall \alpha < \beta, \forall \gamma < \infty, \alpha \oplus \gamma < \beta \oplus \gamma$$

A valuation structure is **fair** if

aggregation has a partial inverse, that is,

$$\forall \alpha \geq \beta, \exists \gamma \text{ such that } \beta \oplus \gamma = \alpha$$

All strictly monotonic valuation structures
can be embedded in a fair valuation structure

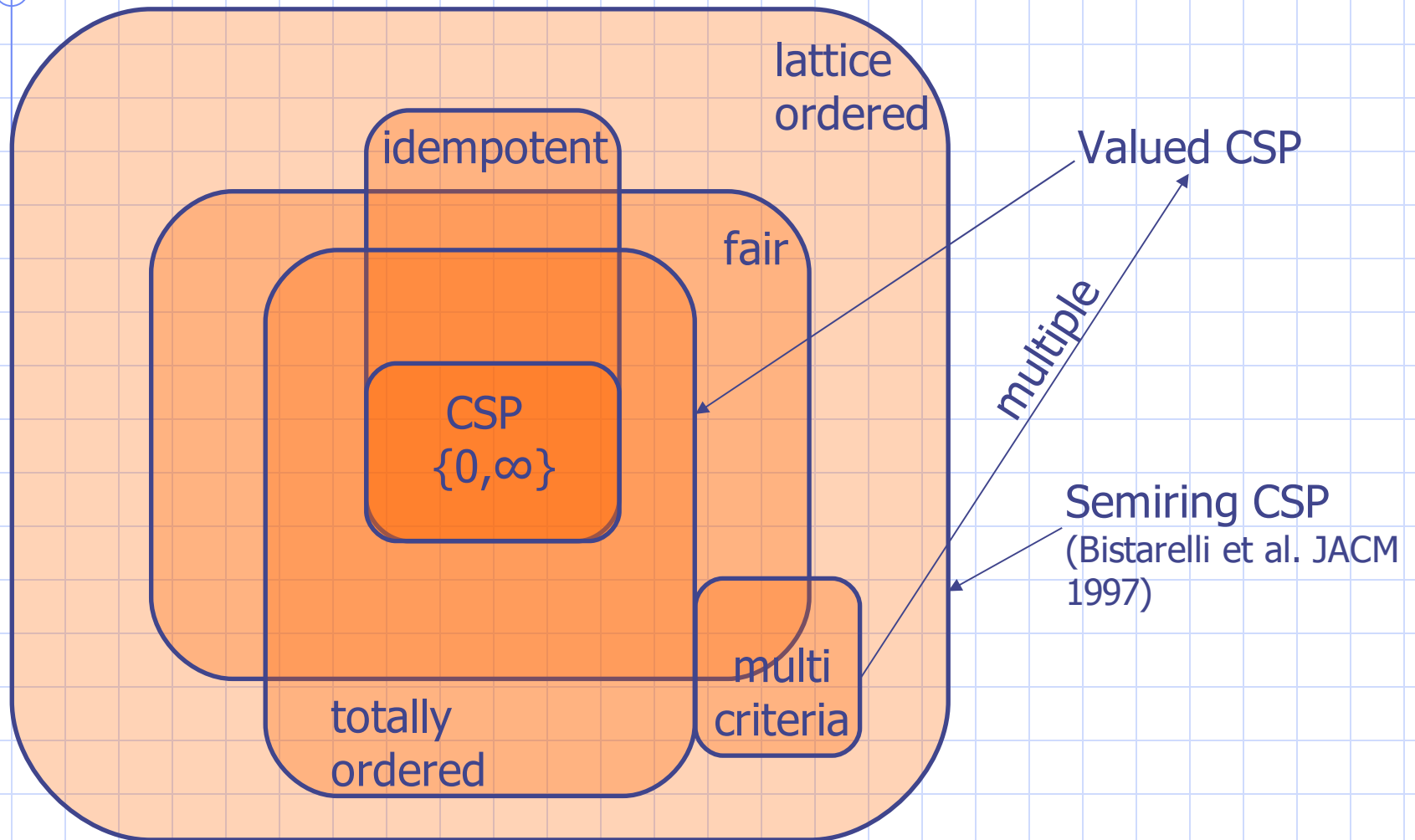
Families of valuation structures

A valuation structure is **discrete** if between any pair of finite costs there are finitely many other costs

All discrete and fair valuation structures
can be decomposed into
a contiguous sequence of valuation structures
with aggregation operator \oplus_k

(interacting as fuzzy CSP)

General frameworks and cost structures



Bibliography

- ◆ For general background on VCSP and other formalisms for soft constraints, see the chapter on “Soft Constraints” by Meseguer, Rossi and Schiex, in the *Handbook of Constraint Programming*, Elsevier, 2006.
- ◆ For classification results on valuation structures see “Arc Consistency for Soft Constraints”, *Cooper & Schiex, AIJ*, 2004.

Chapter 2. Efficiency

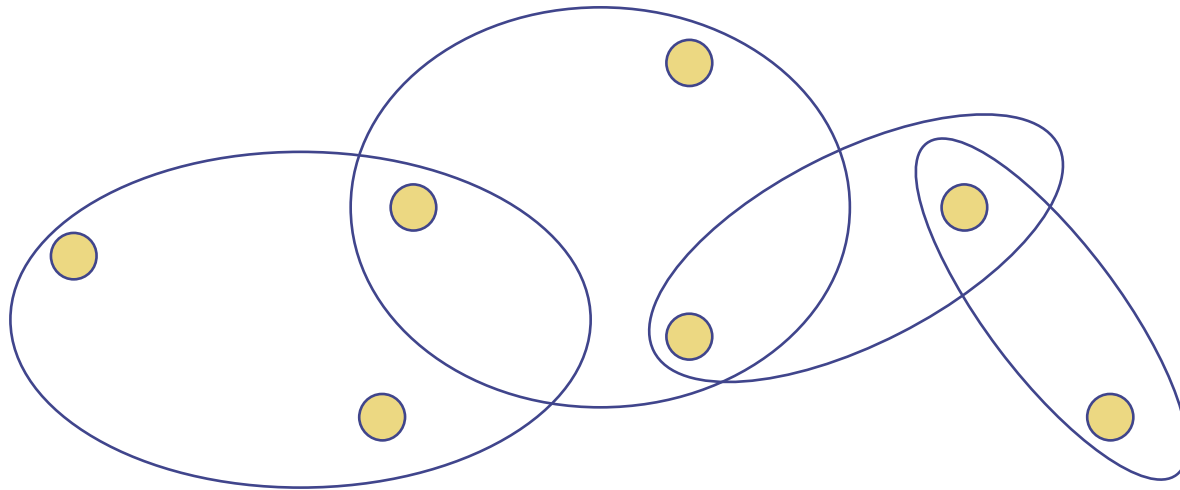
Structural restrictions,
Valued constraint languages

General question

Having a unified formulation allows us to ask *general* questions about efficiency:

When is the VCSP
tractable?

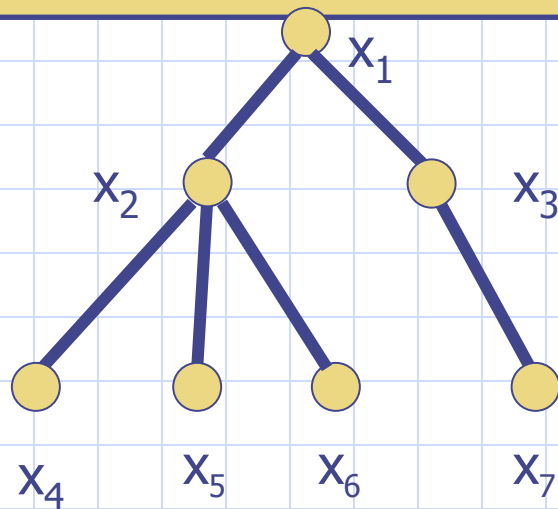
Problem features



- ◆ This picture illustrates the constraint *scopes*
- ◆ The set of scopes is sometimes called the *constraint hypergraph*, or the *scheme*
- ◆ Restricting the scheme can lead to tractability, as in the standard CSP

Structural tractability

Tree-structured binary VCSPs are tractable



Time complexity $O(e d^2)$
Space complexity $O(n d)$

n: number of variables
d: maximum domain size
e: number of cost functions

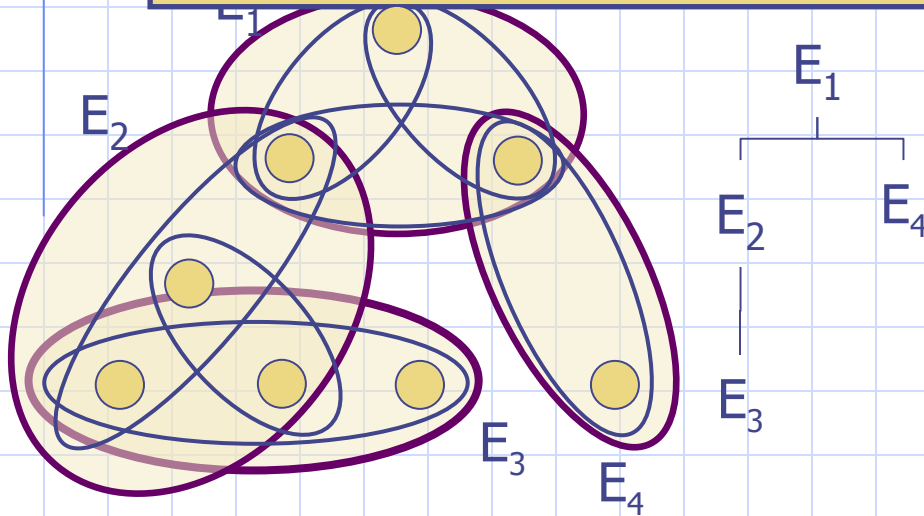
Proceed from the leaf nodes to a chosen root node

Project out leaf nodes by minimising over possible assignments

Tree decomposition

Bounded treewidth VCSPs are tractable

Time complexity $O(e d^{w+1})$
Space complexity $O(n d^s)$



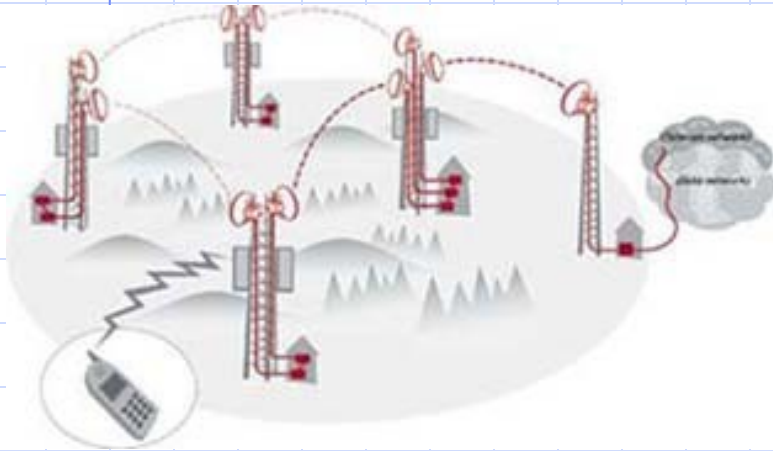
w : bounded treewidth
 $= \max |E_i| - 1$

s : $\max \{|E_i \cap E_j| : i \neq j\}$

Finding a tree decomposition with minimum w^* is NP-hard!

Radio Link Frequency Assignment Problem

(Capon et al., *Constraints* 1999) (Koster et al., *4OR* 2003)



- Given a telecommunication network
- ...find the **best** frequency for each communication link, avoiding interferences

- **Best** can be:
 - Minimize the maximum frequency, no interference (max operator)
 - **Minimize the global interference (sum operator)**
- Generalizes graph coloring problems: $|f_i - f_j| \geq a$

CELAR problem size: $n=100-458$; $d=44$; $e=1,000-5,000$

Tree decomposition example

Benchmark problem
assigning frequencies
to transmitters
to minimise total interference

CELAR scen06r

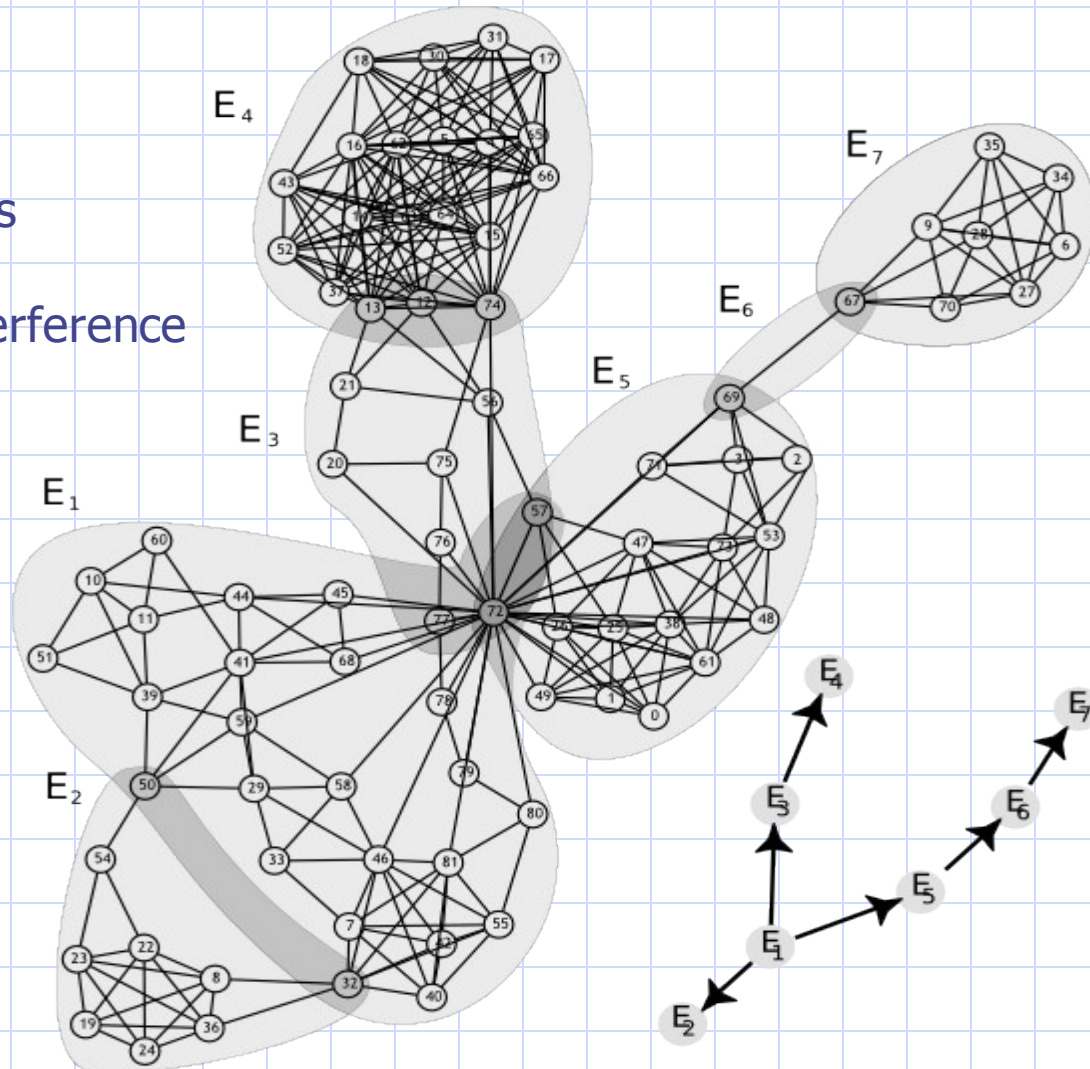
$n = 82$

$d = 44$

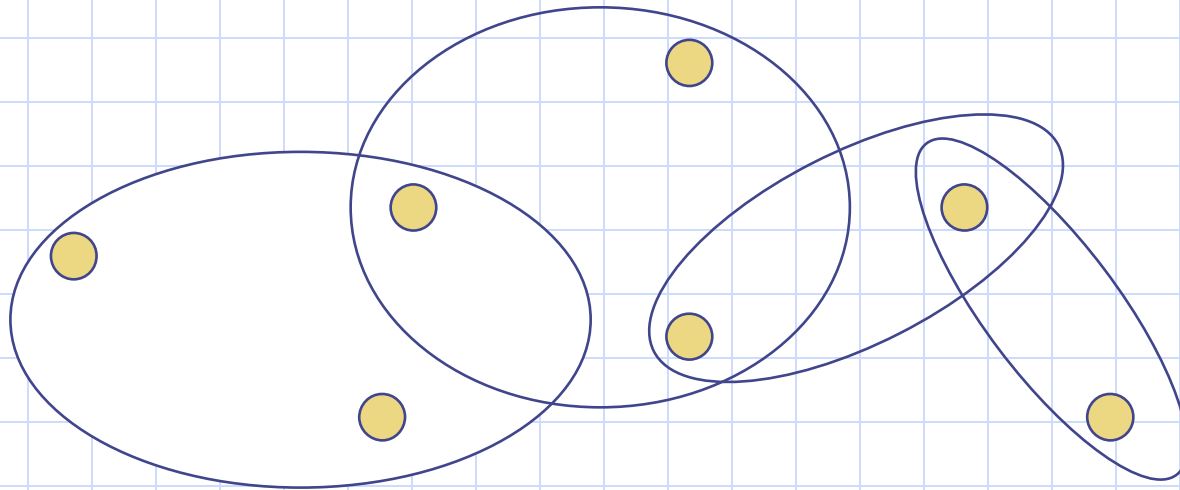
$e = 327$

$w = 26$

$s = 3$

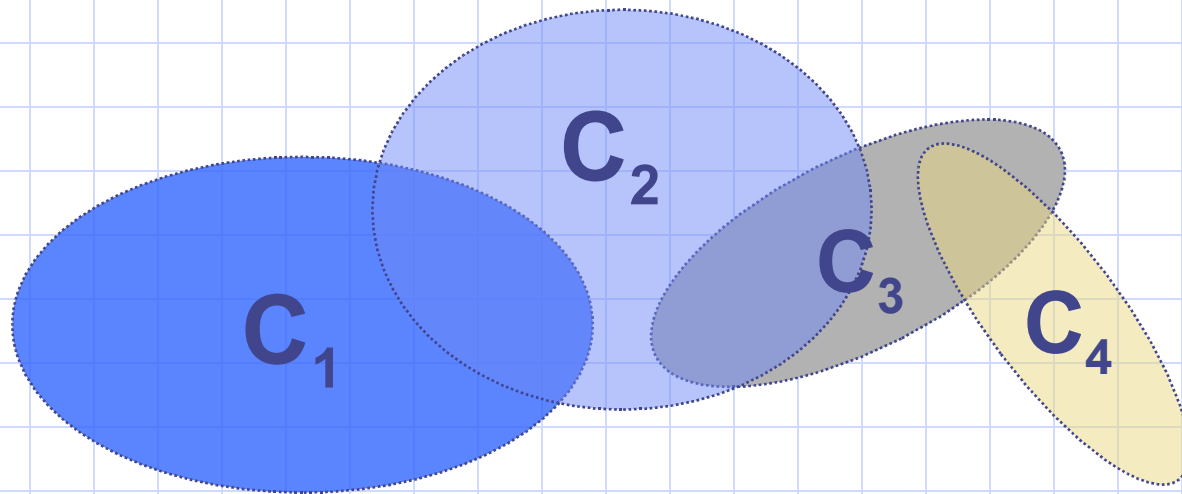


Problem features



- ◆ We have seen that structural features of a problem can lead to tractability
- ◆ This is very similar to the standard CSP
- ◆ What about other kinds of restrictions to the VCSP?

More problem features



- ◆ The picture now emphasises the cost functions
- ◆ Restricting the cost functions we allow can also lead to tractability

Valued constraint languages

- ◆ A set of cost functions is called a **valued constraint language**
- ◆ $\text{VCSP}(\Gamma)$ represents the set of VCSP instances whose cost functions belong to the valued constraint language Γ
- ◆ For some choices of Γ , $\text{VCSP}(\Gamma)$ is tractable
- ◆ We will consider some examples where the valuation structure contains non-negative real values and infinity, and aggregation is standard addition

Submodular functions

A class of functions that has been widely studied in OR is the submodular functions...

A cost function c is **submodular** if $\forall \mathbf{s}, \mathbf{t}$

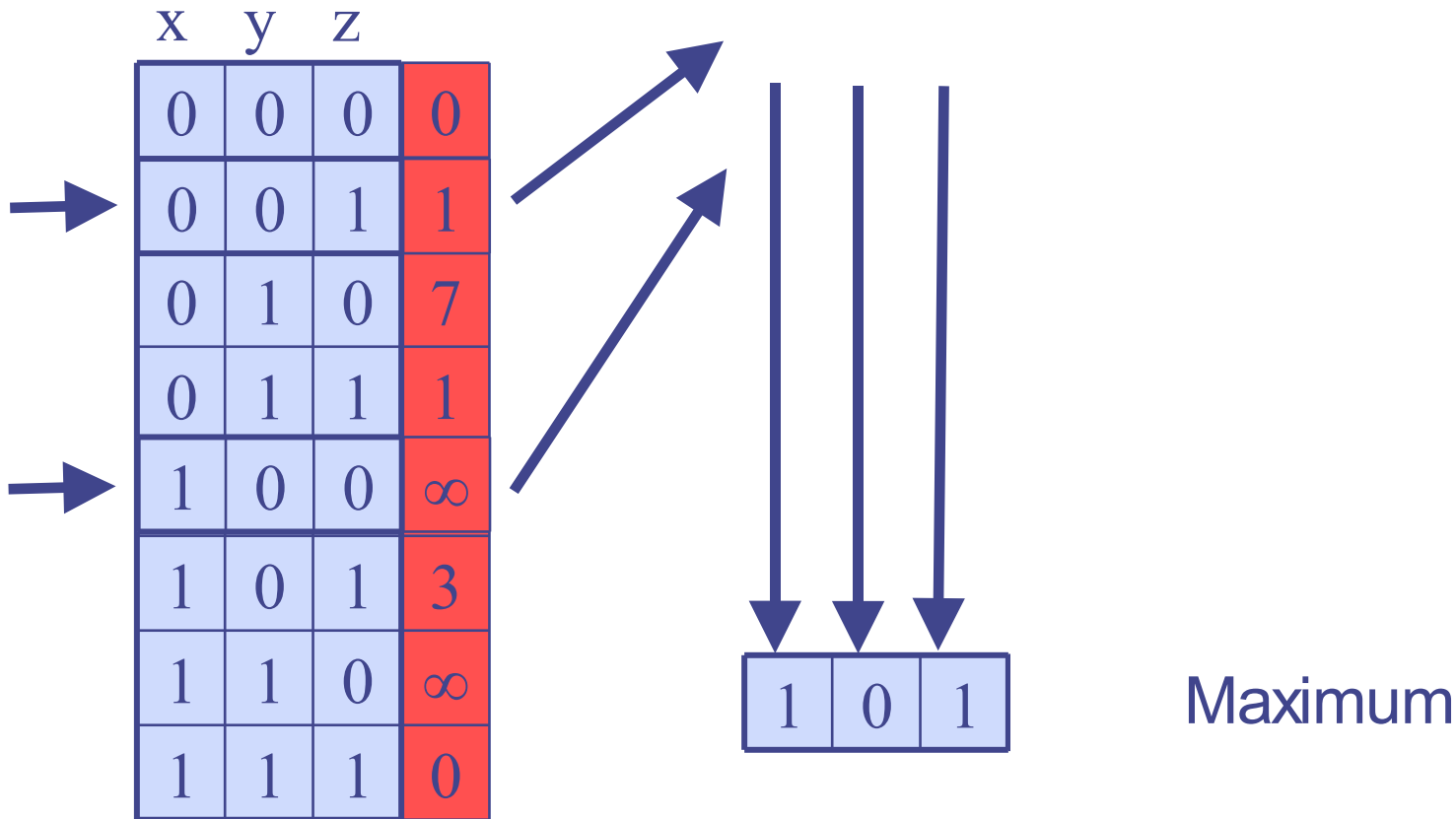
$$c(\min(\mathbf{s}, \mathbf{t})) + c(\max(\mathbf{s}, \mathbf{t})) \leq c(\mathbf{s}) + c(\mathbf{t})$$

where \min and \max are applied component-wise, i.e.

$$\min(\langle s_1, \dots, s_k \rangle, \langle t_1, \dots, t_k \rangle) = \langle \min(s_1, t_1), \dots, \min(s_k, t_k) \rangle$$

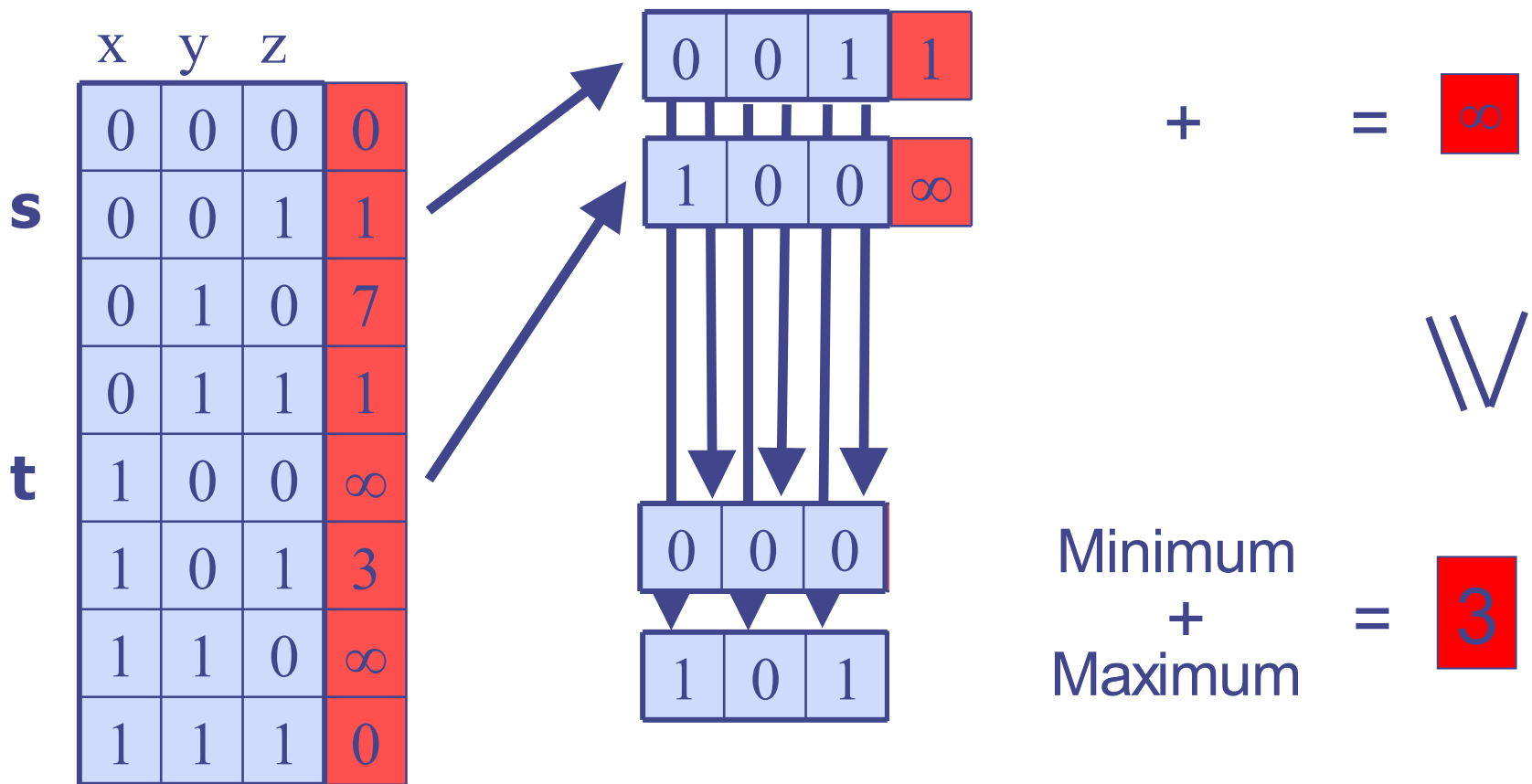
VCSP($\Gamma_{\text{submodular}}$) is tractable

Examples of submodular functions



Examples of submodular functions

$$\forall s, t \quad \text{Cost}(\text{Min}(s, t)) + \text{Cost}(\text{Max}(s, t)) \leq \text{Cost}(s) + \text{Cost}(t)$$



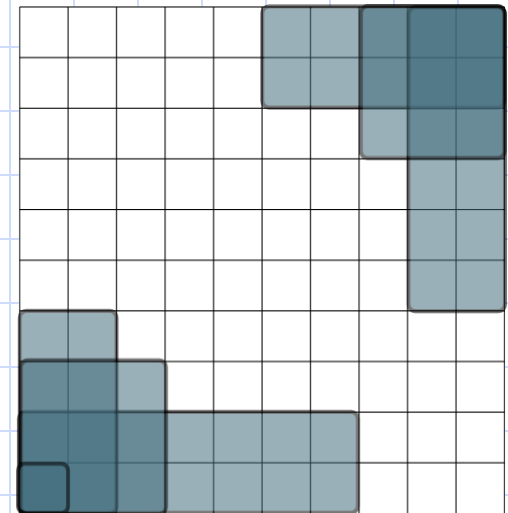
Examples of submodular functions

- ◆ all unary functions
- ◆ all linear functions (of any arity)
- ◆ the binary function ϕ_{cut}
where $\phi_{\text{cut}}(a,b)=1$ if $(a,b)=(0,1)$ (0 otherwise)
- ◆ the rank function of a matroid
- ◆ the Euclidean distance function between two points $(x_1, x_2), (x_3, x_4)$ in the plane
- ◆ $\phi(x,y)=(x-y)^r$ if $x \geq y$ (∞ otherwise) for $r \geq 1$
(compare “Simple Temporal CSPs with strictly monotone preferences”
Khatib et al, IJCAI 2001)

Binary submodular functions

Binary submodular functions over any finite domain can be expressed as a sum of "Generalized Interval" functions

(they correspond to Monge matrices)



Binary VCSP($\Gamma_{\text{submodular}}$) is $O(n^3d^3)$

See Cohen et al "A maximal tractable class of soft constraints", JAIR 2004

Beyond submodularity

$$\forall s,t \text{ Cost}(\text{Min}(s,t)) + \text{Cost}(\text{Max}(s,t)) \leq \text{Cost}(s) + \text{Cost}(t)$$

x	y	z	
0	0	0	0
0	0	1	1
0	1	0	7
0	1	1	1
1	0	0	∞
1	0	1	3
1	1	0	∞
1	1	1	0

The cost function has the *multimorphism* (Min,Max)

By choosing *other* functions, we can obtain other tractable valued constraint languages...

Known tractable cases

If the cost functions all have one of these eight multimorphisms, then the problem is tractable:

- 1) (Min,Max)
- 2) (Max,Max)
- 3) (Min,Min)
- 4) (Majority,Majority,Majority)
- 5) (Minority,Minority,Minority)
- 6) (Majority,Majority,Minority)
- 7) (Constant 0)
- 8) (Constant 1)

See Cohen et al "The complexity of soft constraint satisfaction", AIJ 2006

A dichotomy theorem

If the cost functions all have one of these eight multimorphisms, then the problem is tractable:

- 1) (Min,Max)
- 2) (Max,Max)
- 3) (Min,Min)
- 4) (Majority,Majority,Majority)
- 5) (Minority,Minority,Minority)
- 6) (Majority,Majority,Minority)
- 7) (Constant 0)
- 8) (Constant 1)

For Boolean cost functions...

In all other cases the cost functions have **no** significant common multimorphisms and the VCSP problem is **NP-hard**.

See Cohen et al "The complexity of soft constraint satisfaction", AIJ 2006

Benefits of a general approach

- ◆ The dichotomy theorem immediately implies earlier results for SAT, MAX-SAT, Weighted Min-Ones and Weighted Max-Ones

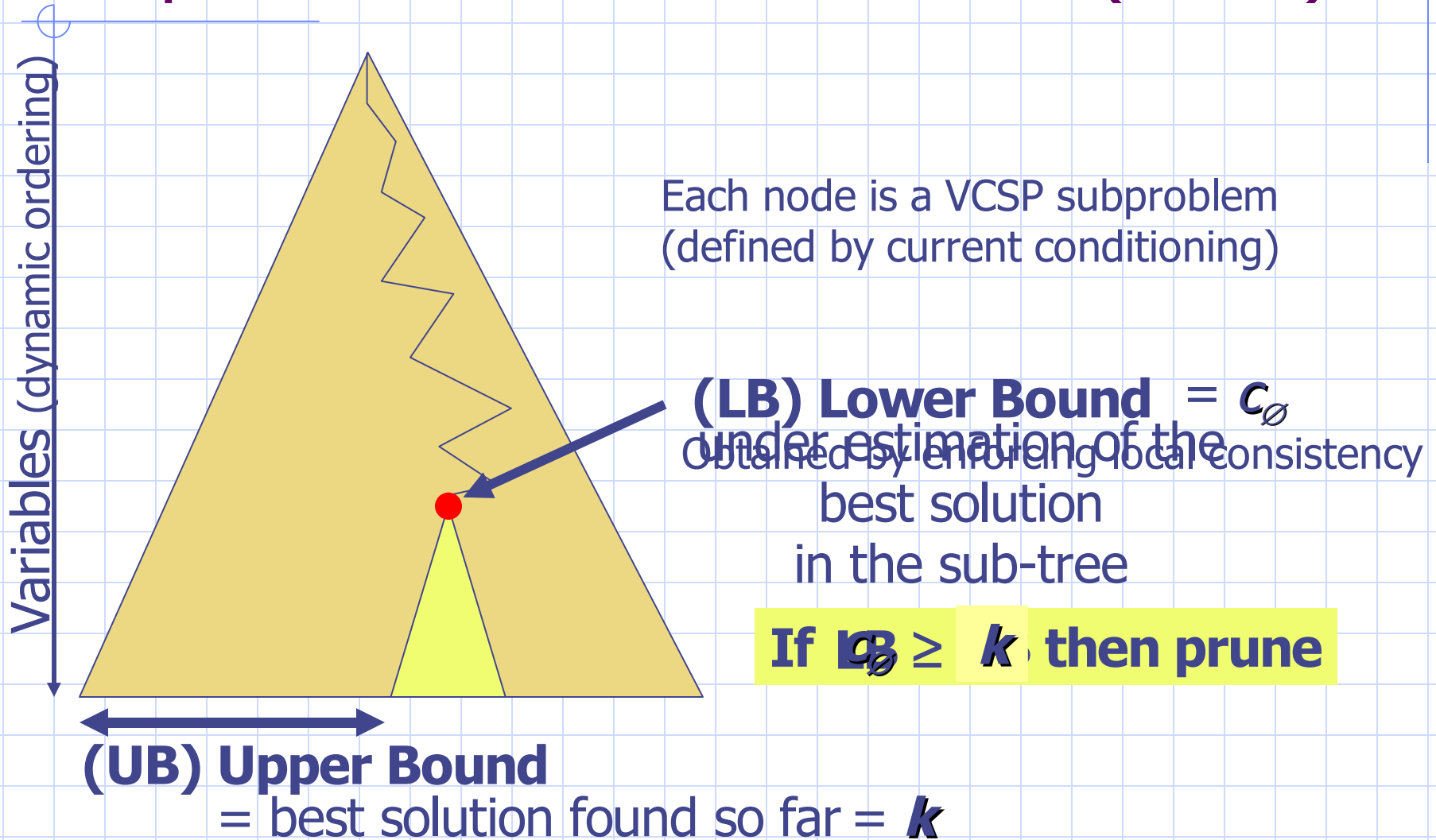
Bibliography

- ◆ For general background on tractable structures, see the chapter on “Tractable Structures” by Dechter, in the *Handbook of Constraint Programming*, Elsevier, 2006.
- ◆ For tractable valued constraint languages see “The complexity of soft constraint satisfaction”, Cohen, Cooper, Jeavons & Krokhin, AIJ 2006.

Chapter 3. Search using problem transformations

Branch and Bound,
Equivalence-preserving operations,
Local consistency (node, arc, directional,
virtual, optimal),
Global cost functions.

Depth-First Branch and Bound (DFBB)



Local consistency

A property that says that the network is “explicit” enough at a local level

Filtering algorithm: transforms a network in an equivalent network that satisfies the property (closure)

CSP: pol. time, confluent, incremental

Equivalence-preserving transformations (EPT)

- ◆ An **EPT** transforms VCSP instance P1 into another VCSP instance P2 with the same objective function.
- ◆ Examples of EPTs:
 - Propagation of inconsistencies (∞ costs)
 - UnaryProject
 - Project/Extend

INCREMENTALITY!

UnaryProject(i, α)

Precondition: $0 \leq \alpha \leq \min\{c_i(a) : a \in d_i\}$

$c_0 := c_0 + \alpha ;$

for all $a \in d_i$ **do**

$c_i(a) := c_i(a) - \alpha ;$

Increases the lower bound c_0 if all unary costs $c_i(a)$ are non-zero.

Project(M, i, a, α)

Precondition: $i \in M, a \in d_i, -c_i(a) \leq \alpha \leq \min\{c_M(x) : x[i]=a\}$

$c_i(a) := c_i(a) + \alpha ;$

for all $x \in \text{labelings}(M)$ s.t. $x[i]=a$ **do**

$c_M(x) := c_M(x) - \alpha ;$

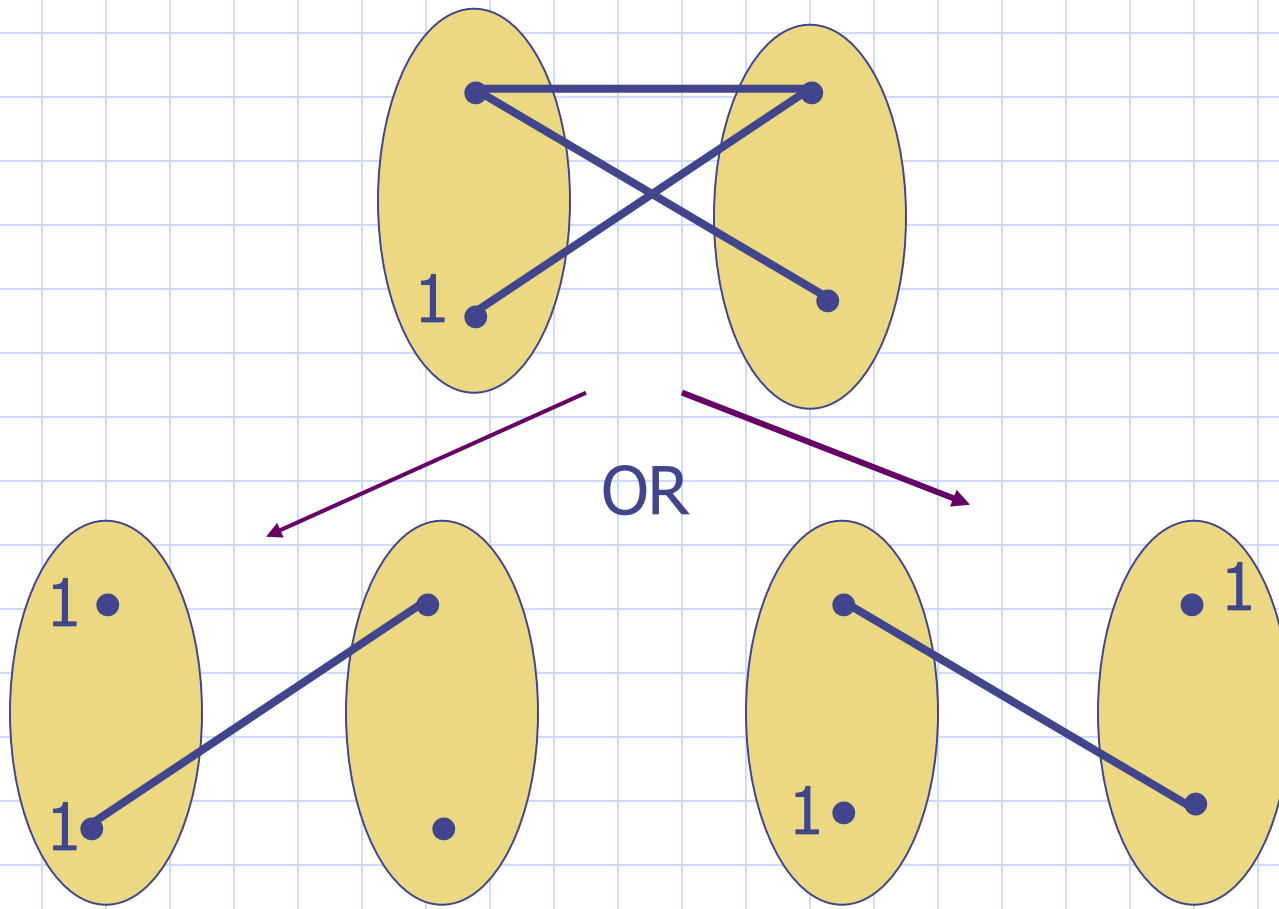
If $\alpha > 0$, this **projects** costs from c_M to c_i

If $\alpha < 0$, this **extends** costs from c_i to c_M

Node and soft arc consistency

- ◆ Node consistent (NC) if $\forall i$
no UnaryProject(i, α) is possible for $\alpha > 0$ and
no propagation of ∞ costs possible between c_i
and c_0 (forbidden values removed if $c_i + c_0 \geq k$)
- ◆ Soft arc consistent (SAC) if $\forall M, i, a$
no Project(M, i, a, α) is possible for $\alpha > 0$

The SAC closure is not unique



Finding the best order of integer EPT application is NP-hard (Cooper, Schiex 2004)

Different soft AC notions:

- ◆ **Directional:** send costs from X_j to X_i if $i < j$ (in the hope that this will increase c_0)
- ◆ **Existential:** $\forall i$, send costs to X_i simultaneously from its neighbor variables if this increases c_0
- ◆ **Virtual:** no *sequence* of Projects/Extends increases c_0
- ◆ **Optimal:** no *simultaneous set* of Projects/Extends increases c_0

Directional Arc Consistency

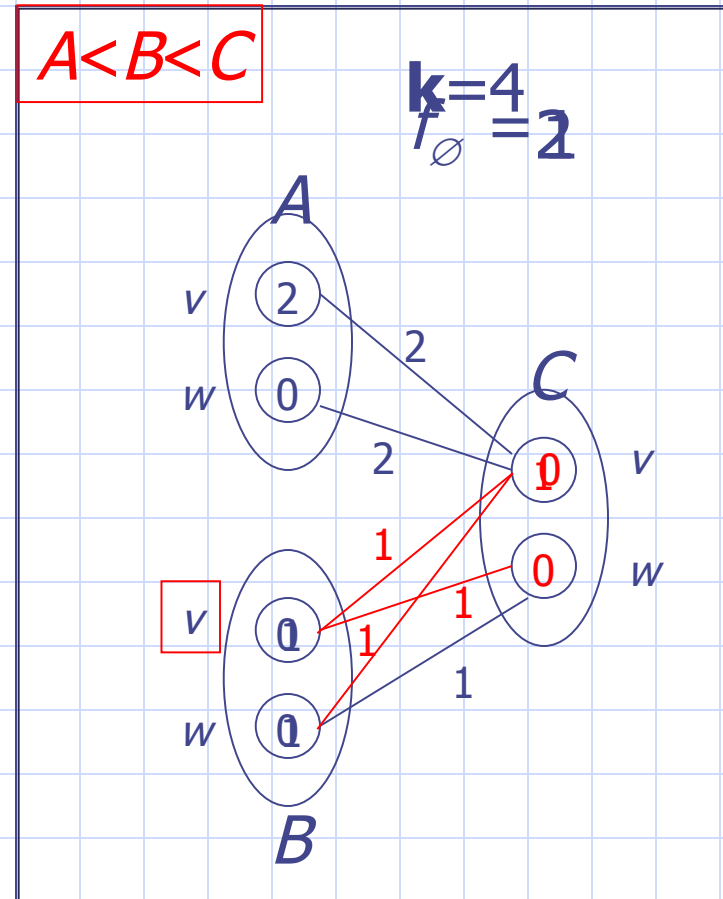
- ◆ for all $i < j$, $\forall a \in d_i \exists b \in d_j$ such that $c_{ij}(a,b) = c_j(b) = 0$.
- ◆ Solves tree-structured VCSPs
- ◆ **FDAC** (Full Directional AC) =
Directional AC + Soft AC
- ◆ FDAC can be established in $O(\text{end}^3)$ time (or in $O(ed^2)$ time if $+_k$ is $+$)

Directional AC (DAC*)

- NC^*
- For all f_{AB} ($A < B$)
 - ◆ $\forall a \exists b$

$$f_{AB}(a,b) + f_B(b) = 0$$
- b is a *full-support*
- complexity:

$$O(ed^2)$$

Shift($f_{BC}, (C,v), -1$)Shift($f_{BC}, (B,v), 1$)Shift($f_{BC}, (B,w), 1$)Shift($f_B, \emptyset, 1$)Shift($f_A, \emptyset, -2$)Shift($f_A, \emptyset, 2$)

Existential Arc Consistency

- ◆ node consistent and $\forall i, \exists a \in d_i$ such that $c_i(a) = 0$ and for all cost functions c_{ij} , $\exists b \in d_j$ such that $c_{ij}(a, b) = c_j(b) = 0$
- ◆ **EDAC** = Existential AC + FDAC
- ◆ EDAC can be established in $O(ed^2 \max\{nd, k\})$ time

Virtual Arc Consistency (VAC)

(Cooper et al, 2008)

- ◆ If P is a VCSP instance then $\text{Bool}(P)$ is the CSP instance whose allowed tuples are the zero-cost tuples in $P-c_0$
- ◆ If $\text{Bool}(P)$ has a solution, then P has a solution of cost c_0 (but usually $\text{Bool}(P)$ has no solution)
- ◆ Definition: P is VAC if $\text{Bool}(P)$ is AC.

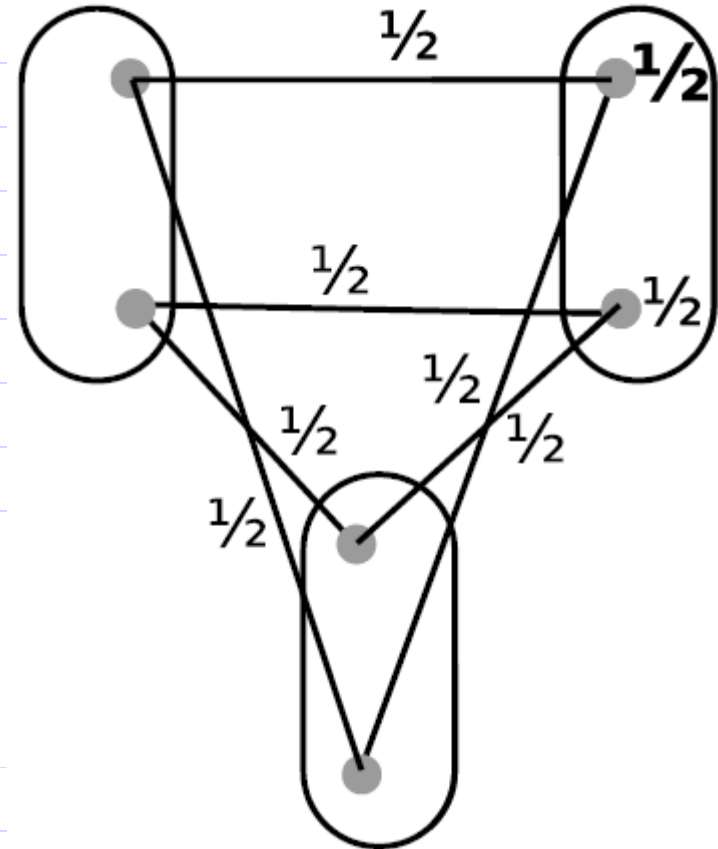
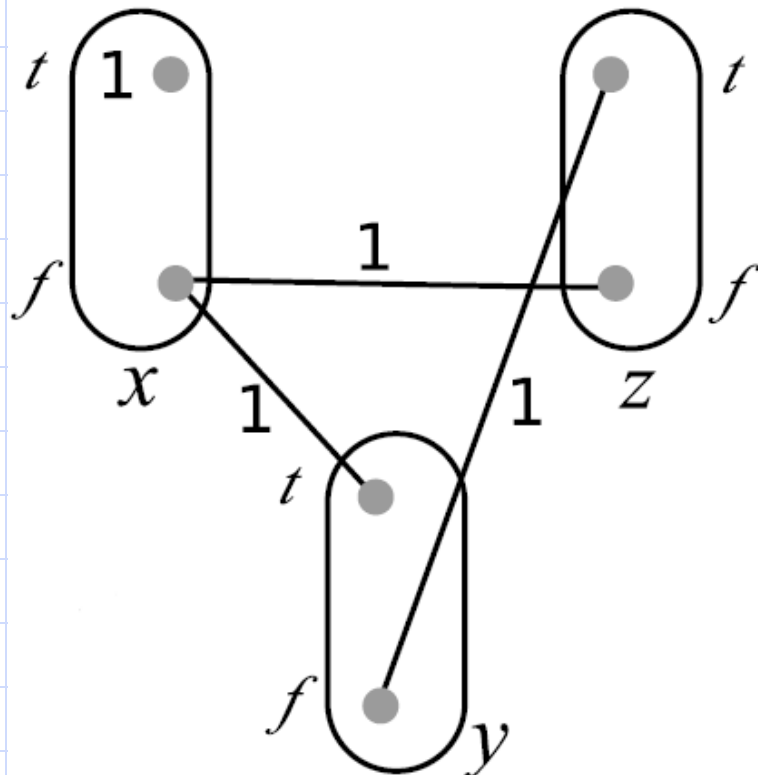
Approximating VAC

(similar to Augmenting DAG, Schlesinger et al, 30 years before)

- ◆ If a sequence of AC operations in $\text{Bool}(P)$ leads to a domain wipe-out, then a similar sequence of SAC operations in P increases c_0
- ◆ But, in this sequence, costs may need to be sent in more than one direction from the same $c_M \Rightarrow$ **Introduction of *fractional weights***
- ◆ VAC_ε algorithm may converge to a local minimum (and more, an instance P' which is *not VAC*)
- ◆ VAC_ε can be established in $O(ed^2 k/\varepsilon)$ time

Enforcing VAC

AC, DAC, FDAC, EDAC



Optimal Soft Arc Consistency

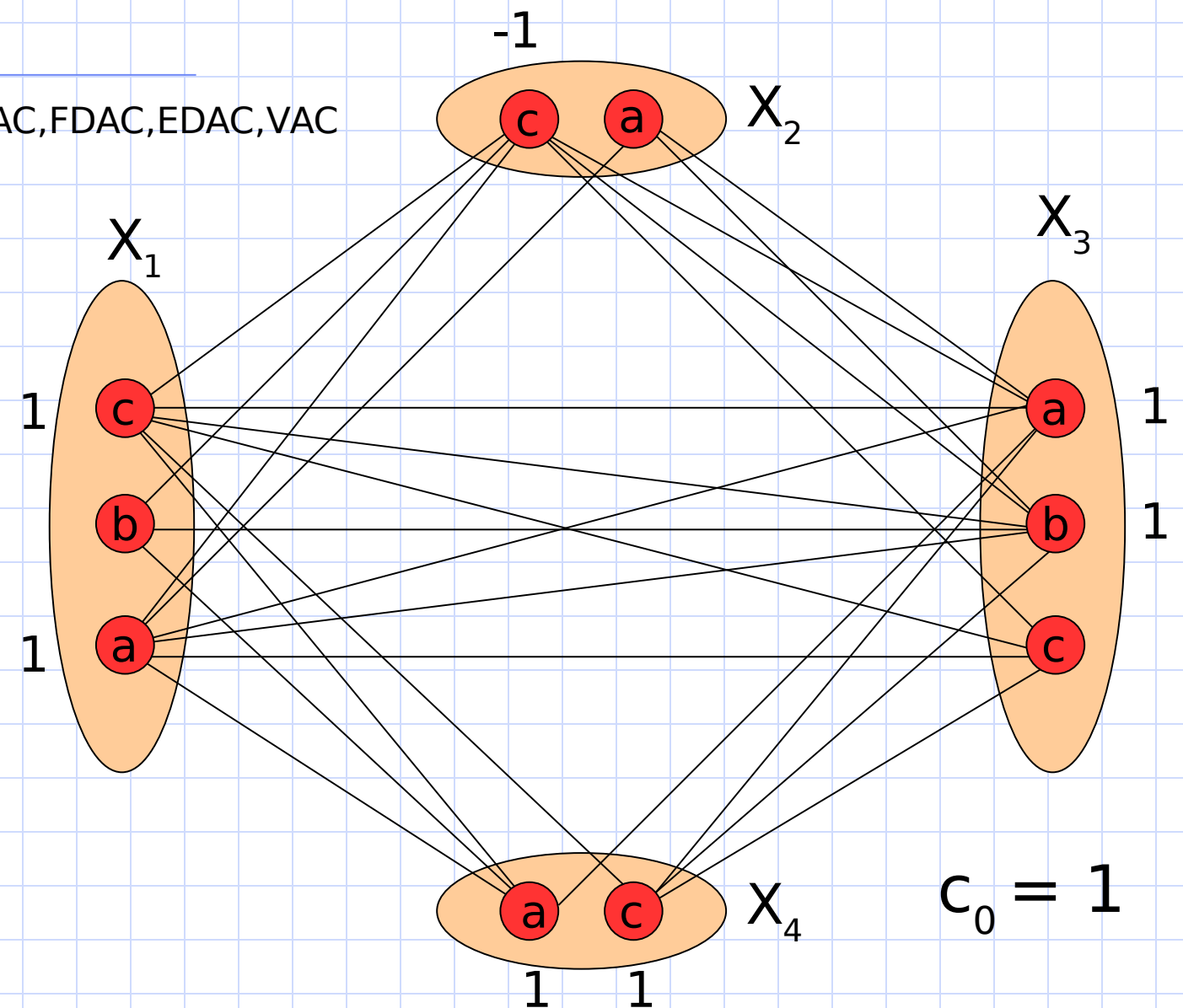
(Cooper et al. 2007), similar to (Schlesinger, 30 years before)

- ◆ We can overcome this problem of convergence by solving a LP to find the *set* of simultaneous UnaryProject and Project operations which maximises c_0 .
- ◆ The resulting VCSP instance is **OSAC** (Optimal Soft Arc Consistent).
- ◆ OSAC is strictly stronger than VAC.
- ◆ Unfortunately, the LP has $O(\text{edr}+n)$ variables and $O(\text{edr}+nd)$ constraints(pre-processing).

$$p_{2c}^{23} = p_{3a}^{34} = p_{3b}^{31} = p_{1a}^{12} = p_{1c}^{14} = -1$$

$$p_{3a}^{23} = p_{3b}^{23} = p_{4c}^{34} = p_{1a}^{31} = p_{1c}^{31} = p_{2c}^{12} = p_{4a}^{14} = u_4 = 1$$

AC, DAC, FDAC, EDAC, VAC



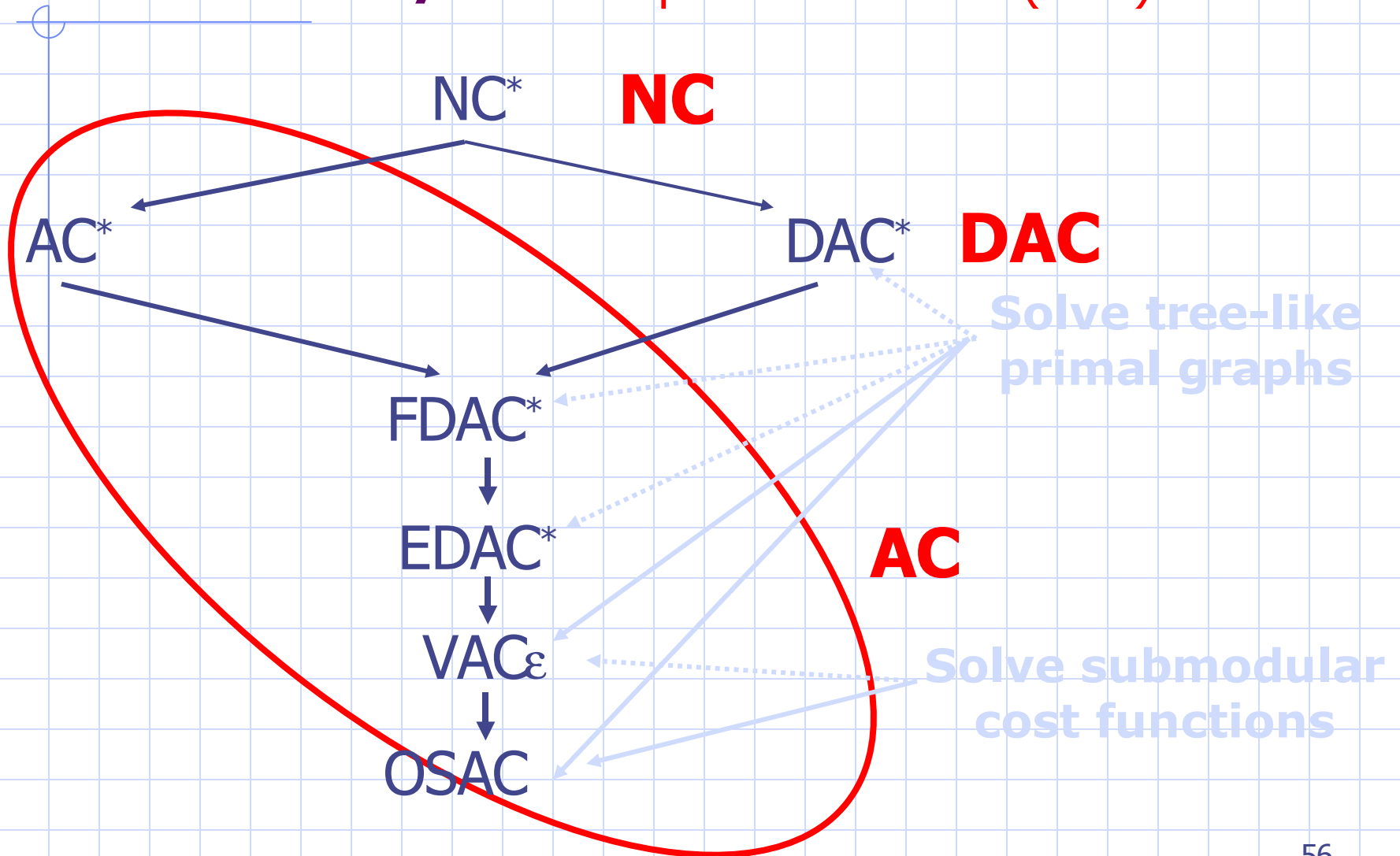
Virtual Arc Consistency solves (locally-defined) submodular VCSP

If $P \in \text{VCSP}(\Gamma_{\text{sub}})$ and P is VAC,
then $\text{Bool}(P)$ is arc consistent, max-closed.
Hence, $\text{Bool}(P)$ has a solution.
This solution has cost c_0 in P and is thus
necessarily optimal.

Thus OSAC solves SFM since Project and
UnaryProject preserve submodularity.
Also permuted submodular (some technicalities)

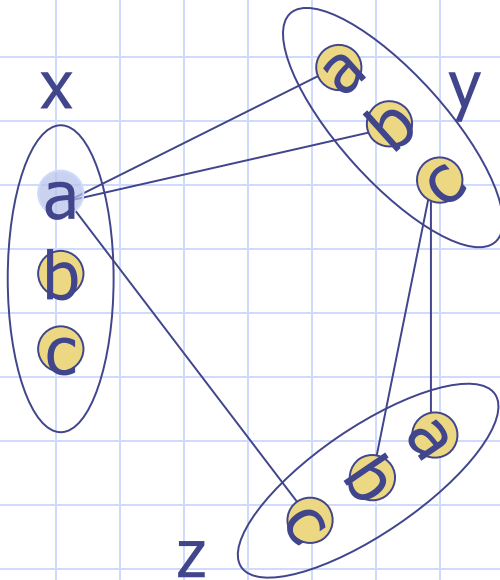
Hierarchy

Special case: CSP ($k=1$)



Beyond Arc Consistency

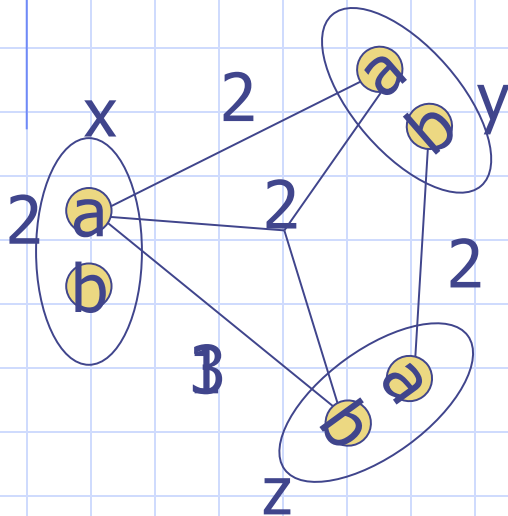
◆ Path inverse consistency (Debryune & Bessière)



(x, a) can be pruned because there are two other variables y, z such that (x, a) cannot be extended to any of their values.

Beyond Arc Consistency

◆ Soft Path inverse consistency

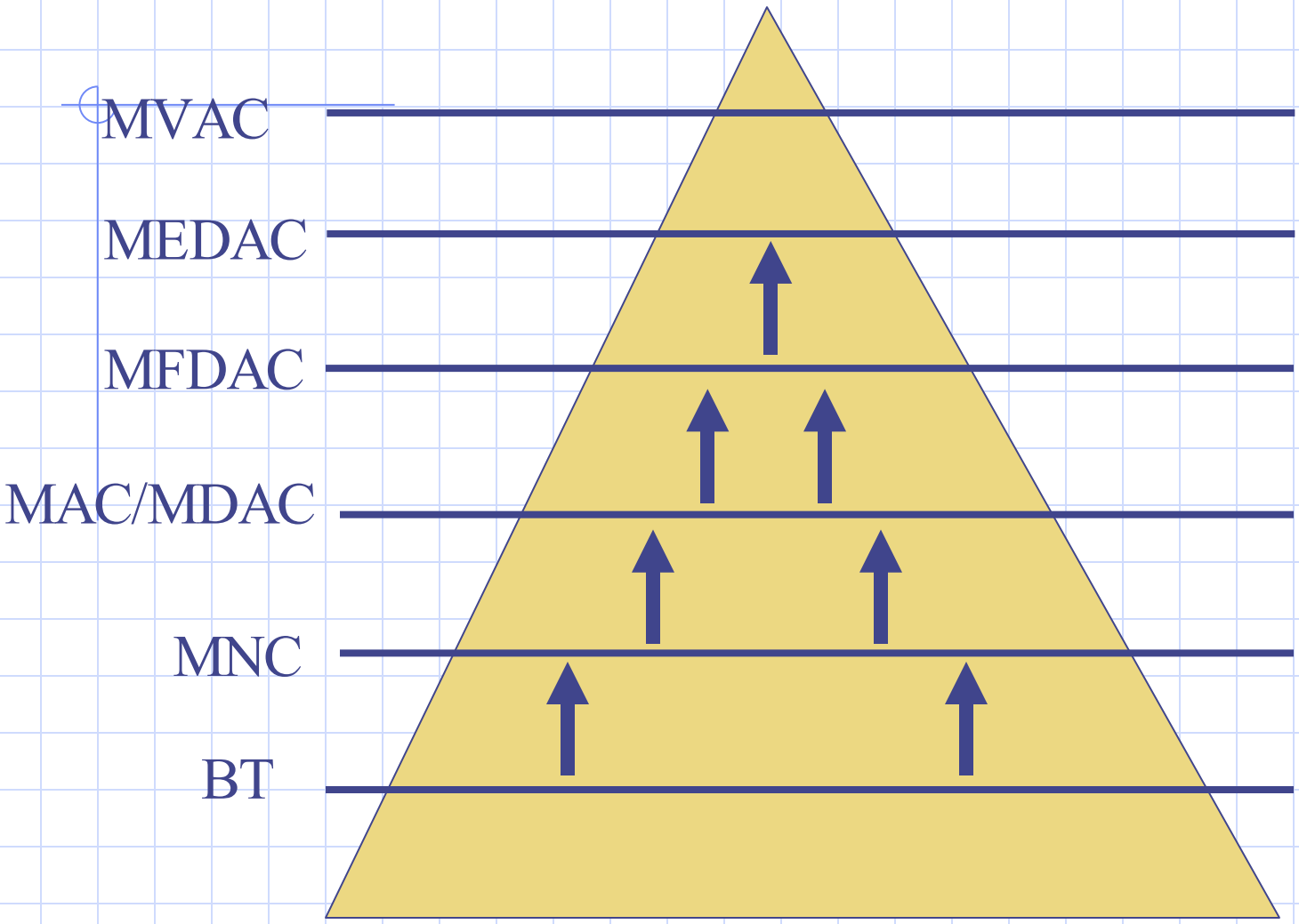


$$f_y \oplus f_z \oplus f_{xy} \oplus f_{xz} \oplus f_{yz}$$

x	y	z	
a	a	a	0
a	a	b	3
a	b	a	0
a	b	b	1
b	a	a	0
b	a	b	0
b	b	a	2
b	b	b	0

$$(f_y \oplus f_z \oplus f_{xy} \oplus f_{xz} \oplus f_{yz})[x]$$

a	
a	2
b	0



Some practical observations

- ◆ For very hard-to-solve instances, maintaining VAC provides a significant speed-up (closed RLFAP graph11/13).
- ◆ For many problems, maintaining a simpler form of soft arc consistency (e.g. EDAC) is faster.
- ◆ Unary costs $c_i(a)$ and EAC value inform value and variable ordering heuristics

RLFAP: CELAR 6 results since 1993

n. of vars: $n=100$, domain size: $d=44$, n. of cost functions: $e=1222$

Time of optimality proof	Method(s) used	Publication
26 days (SUN UltraSparc 167 MHz)	Ad-hoc problem decomposition & Russian Doll Search (<i>22 vars only</i>)	(de Givry, Verfaillie, Schiex, CP 1997)
3 days (SUN Sparc 2)	Ad-hoc problem decomposition & PFC-MRDAC (<i>22 vars only</i>)	(Larrosa, Meseguer, Schiex, AIJ 1998)
8 hours (DEC Alpha 500MP)	Preprocessing rules & BbB Elimination	(Koster PhD thesis, 1999)
3 hours (PC 2.4 GHz)	B&B with EDAC & tree decomposition (BTD)	(de Givry, Schiex, Verfaillie, AAAI 2006)
1' 26" (PC 2.5 GHz) 25000x 16 x	BTD-RDS & variable ordering heuristics & dichotomic branching	(Sanchez, Allouche, de Givry, Schiex, IJCAI 2009)

CELAR 7 (n=200) solved in 4.5 days (Sanchez et al, IJCAI 2009)

CELAR 8 (n=458) solved in < 2 days (127 days)

2010 Approximate Inference Evaluation (results given at UAI'10)

Networks – by domain (1 hour)

Network	PR	MAR	MPE
CSP	8	8	55
Grids	20	20	40
Image Alignment			10
Medical Diagnosis	26	26	
Object Detection	96	96	92
Pedigree	4	4	
Protein Folding			21
Protein-Protein Interaction			8
Segmentation	50	50	50

Summary of the results

Seconds	PR	MAR	MPE
20	Arthur Choi (UCLA)	Arthur Choi (UCLA)	Joris Mooij (Max Planck)
1200	Vibhav Gogate (UW+UCI)	Vibhav Gogate (UW+UCI)	Thomas Schiex (INRA)
3600	Vibhav Gogate (UW+UCI)	Vibhav Gogate (UW+UCI)	Joris Mooij (Max Planck)

toulbar2 was also first at UAI'08 Evaluation, MaxCSP'06,'08 Competition

Winning Teams

- (MAR) **IJGP** by Vibahv Gogate (UW), Andrew Gefland, Natasha Flerova and Rina Dechter (UCI):
Anytime iterative GBP based algorithm
- (PR) **Vgogate** by Vibahv Gogate, Pedro Domingos (UW), Andrew Gefland and Rina Dechter (UCI):
Formula based importance sampling
- (PR+MAR) **EDBP** by Arthur Choi, Adnan Darwiche, with support from Glen Lenker and Knot Pipatsrisawat (UCLA):
Anytime BP based anytime thickening of structure
- (MAP) **libDAI** by Joris Mooij (Max Planck):
junction tree, LBP/MP, double-loop GBP, Gibbs, decimation

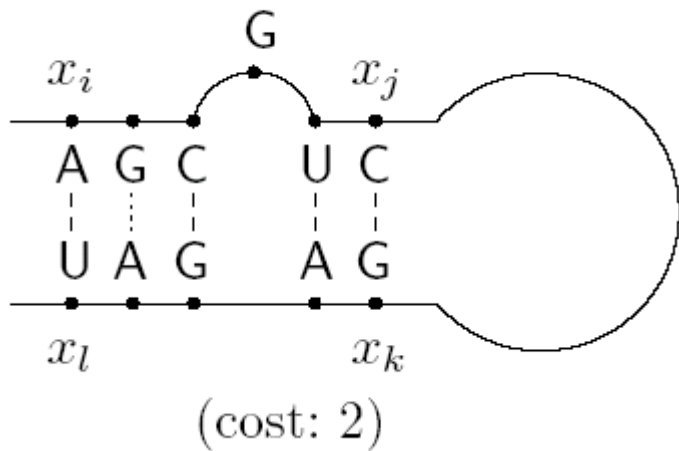
- (MAP) **toulbar2** by Thomas Schiex et al (INRA)
Anytime branch and bound weighted CSP solver

73% instances
solved exactly

AC for global cost functions

- ◆ **Global constraints:** specific family of constraints on an unbounded number of variables with efficient local consistency filtering.
 - Example: AllDifferent (max matching)
- ◆ Same for global cost functions
 - Example: # of variables with the same value (van Hoeve et al, J. Heur. 2006) (Lee & Leung, IJCAI'09)

RNA gene finding (Zytnicki et al, 2008)



- Given a sequence and an RNA gene descriptor
- ...find all the occurrences of the descriptor with at most k mismatches

- NP-complete for $k=0$ (Vialeto, 2004)
- Sort solutions by their number of mismatches

RNA problem sizes: $n=20$; $d > 100$ million! ; $e(4)=10$

Bound arc consistency

- ◆ Goal: space complexity independent of the domains
- ◆ BAC^\emptyset (Zytnicki et al, *JAIR*, 2010)
 - Avoid EPTs, except those shifting cost to
 - Prune *extremity* domain values only
 - Complexity
 - ◆ Time $\mathbf{O}(n^2 r^2 d^{r+1})$ and space $\mathbf{O}(n+er)$
with maximum constraint arity r
 - BAC^\emptyset is confluent
 - Can be specialized for semi convex cost functions (d to d^2 speedup on binary CF)

RNA gene finding

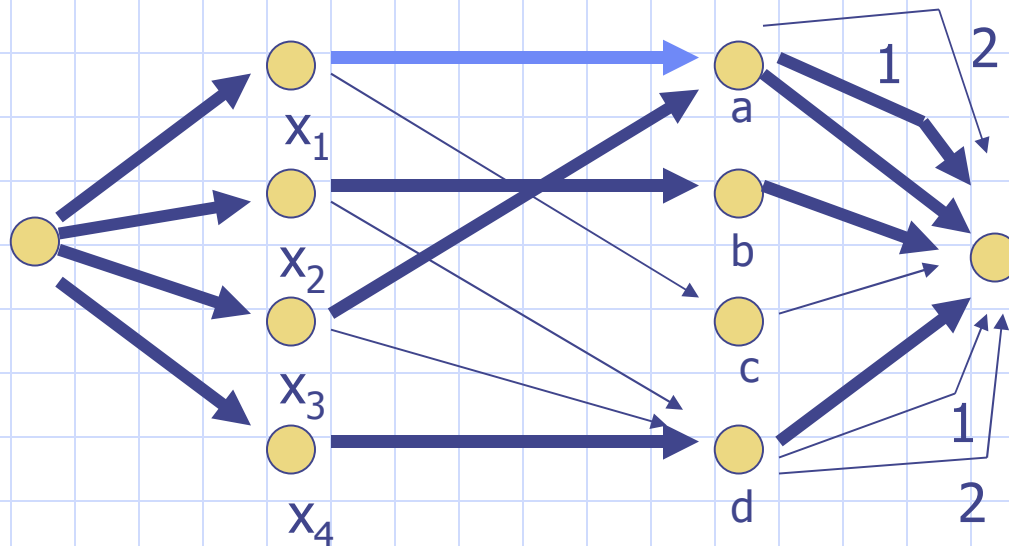
Domain size	10k	50k	100k	500k	1M	4.9M
Nb. of solutions	32	33	33	33	41	274
AC*						
Time	1hour 25min.	44hours	-	-	-	-
Nb. of backtracks	93	101	-	-	-	-
BAC						
Time (sec.)	0.016	0.036	0.064	0.25	0.50	2.58
Nb. of backtracks	93	101	102	137	223	1159

Fig. 6. Searching all the solutions of a tRNA motif in *Escherichia coli* genome.

Darn! solver (Zytnicki et al, Constraints 2008)

Network representing “min number of variables with same value”

(Beldiceanu & Petit, CPAIOR'04)



All edge capacities are equal to 1

All edge costs are 0 if not indicated

Flow shown is a min-cost max-flow with $x_1 = a$.

We can project 1 from c_M to $c_1(a)$ by reducing the cost of the light blue edge from 0 to -1 .

Latin Square N x N with costs

Example of solution for N = 5:

2	1	3	5	4
4	2	1	3	5
1	5	4	2	3
5	3	2	4	1
3	4	5	1	2

All variables take a different value in each row and each column

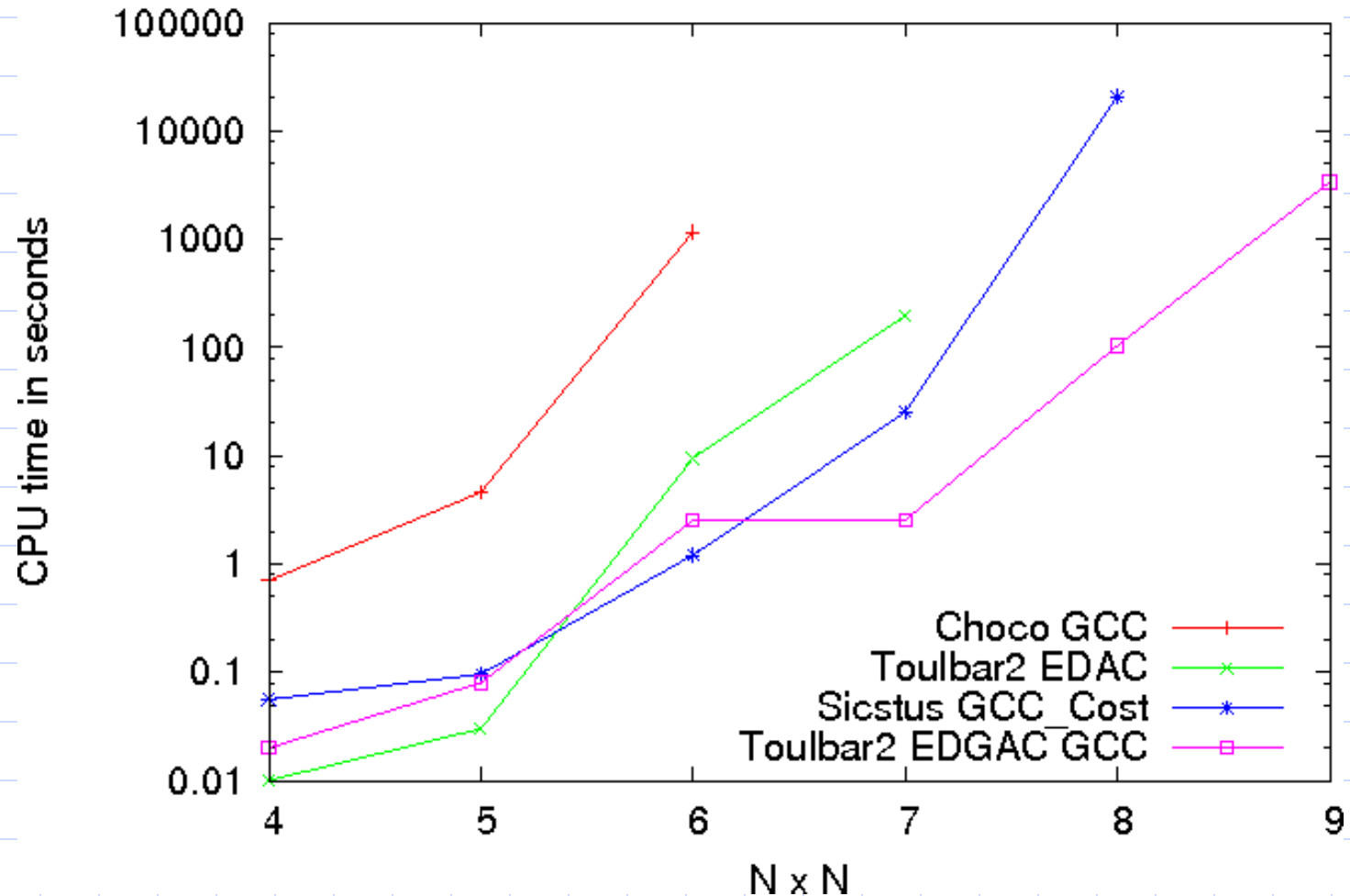
A unary cost function for each cell $f_{i,j}(x_{i,j}) : D \rightarrow [0, \text{MaxCost}[$

Objective: 49

$$\text{Objective} = \sum_i \sum_j f_{i,j}(x_{i,j})$$

Latin Square with costs

Latin Square with unary costs



Bibliography

- ◆ For an overview of soft local consistencies, see "*Soft arc consistency revisited*", Cooper, de Givry, Sanchez, Schiex, Zytnicki & Werner, AIJ 2010.
- ◆ For soft global constraints (FDGAC), see "*Towards Efficient Consistency Enforcement for Global Constraints in Weighted Constraint Satisfaction*", Lee & Leung, IJCAI 2009.

Chapter 4. Open problems

Concerning problem definition,
search, transformations, tractable
classes

Possible extensions to VCSP

- ◆ Partial order instead of total order
- ◆ 2 arbitrary binary operators (e.g. calculating the sum of products instead of the min of the sum subsumes #CSP)
- ◆ Objective function not constructible using a binary aggregation operator (e.g. the median of the set of costs)

Tractability

- ◆ Can we characterize/unify all tractable classes of VCSP over non-Boolean domains?
- ◆ Are there interesting tractable classes apart from submodular functions?
- ◆ Are there more efficient algorithms for submodular function minimisation?

New problem transformations

- ◆ Global cost functions
- ◆ Decomposition into several problems whose sum is equal to the original VCSP
- ◆ Transformations which preserve at least one solution (if it exists) but do not necessarily all costs (substituability).
- ◆ Applying rules involving ≥ 2 constraints

Conclusion

- ◆ VCSP combines CSP and optimisation in a unified way
- ◆ Technology is usable and useful, and still maturing
- ◆ Something different: Structure estimation in Gene Networks