An introduction to valued constraint satisfaction

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An introduction to Valued Constraint Satisfaction

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Valued Constraint Satisfaction

- What is it and why do we need it?
- Can it be done efficiently?
- Search
- Problem transformations
- Open problems

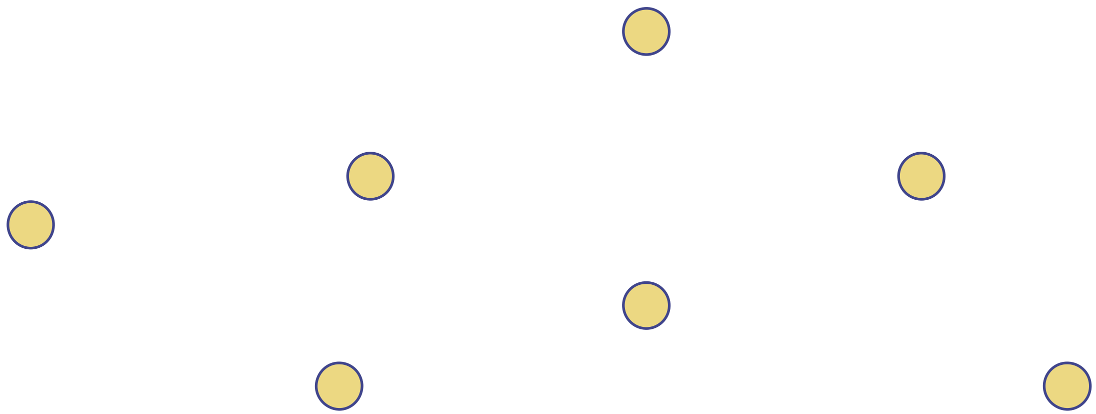
Chapter 1. What is it?

Motivation,

Definitions,

Some general theorems

Constraint Satisfaction Problem
A unifying abstraction



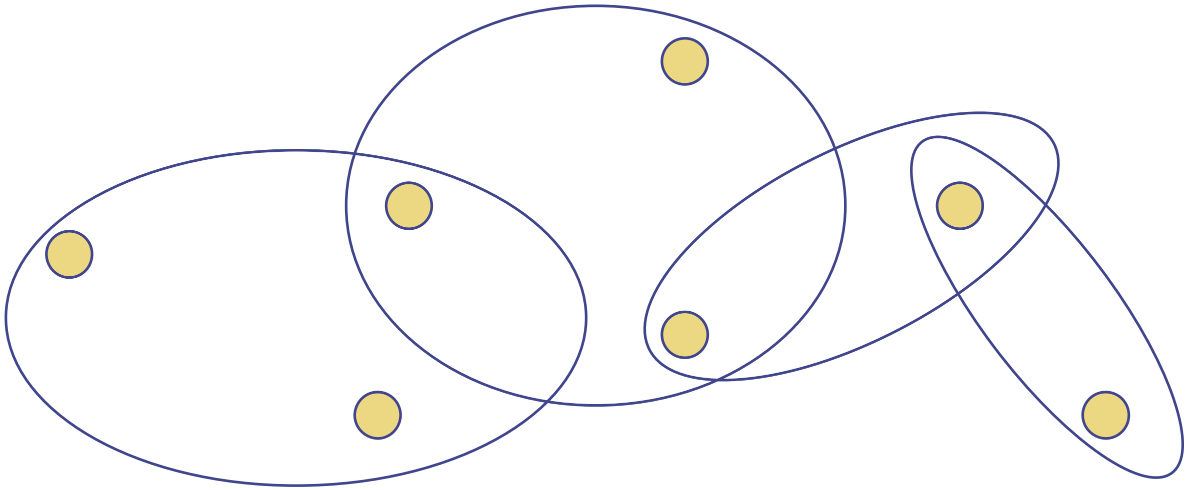
Variables = Talks to be scheduled at conference

Transmitters to be assigned frequencies

Amino acids to be located in space

Circuit components to be placed on a chip

A unifying abstraction



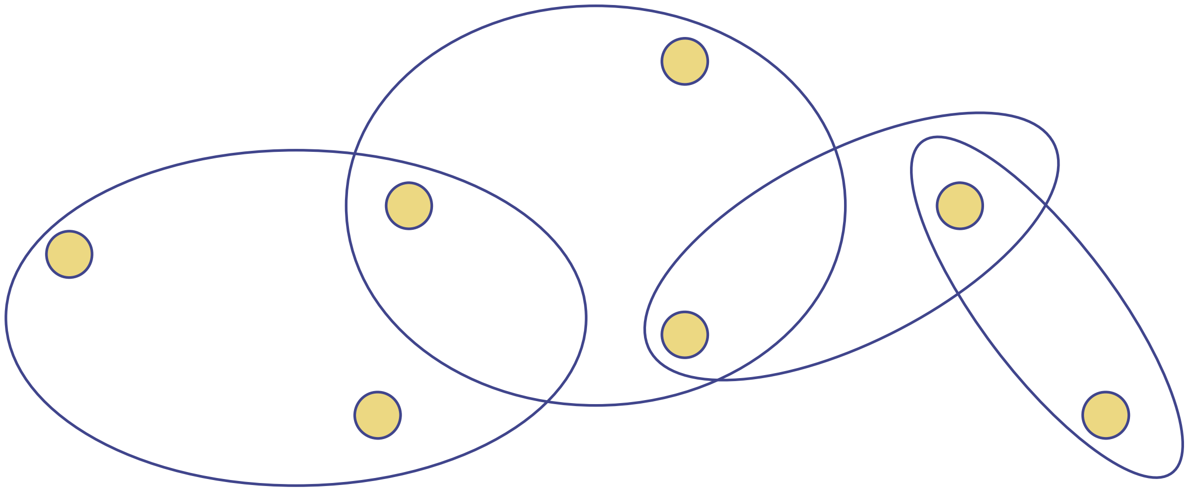
Constraints = All invited talks on different days

No interference between near transmitters

$x + y + z > 0$

Foundations dug before walls built

A unifying abstraction



A solution is an assignment of values to variables that satisfies all the constraints

Constraint programming (OR, Ilog Solver...)

But what if…

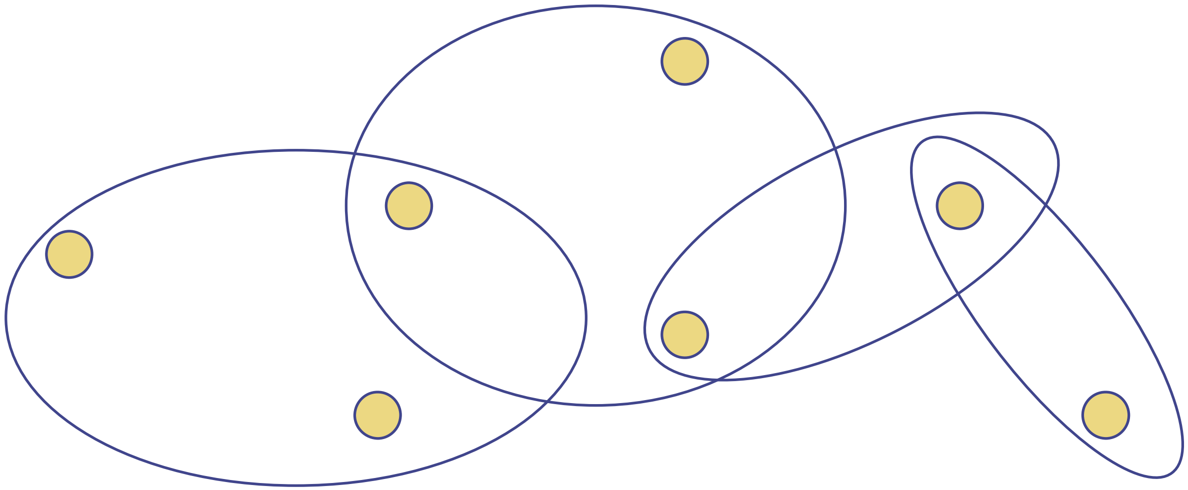
- There are lots of solutions, but some are better than others?
- There are no solutions, but some assignments satisfy more constraints than others?
- We don’t know the exact constraints, only probabilities, or fuzzy membership functions?
- We’re willing to violate some constraints if we can get a better overall solution that way?

Fragmentation/Heterogeneity

- Fuzzy CSP (easier to solve, Rosenfeld 76)
- Max, Weighted, Partial CSP (Shapiro 81, Freuder 91)
- Weighted Max-SAT
- Constraint Optimization Problems
- Lexicographic CSP
- Hierarchical Constraint Logic Programming (Borning et al)

- Pseudo-Boolean Optimisation
- Bayesian Networks
- Random Markov Fields
- Factor Graphs
- Integer Programming
- 2D grammars...

A unifying abstraction



"Constraints" associate costs with each assignment

A solution is an assignment of values to variables that minimises the combined costs

Definition of a VCSP instance
(IJCAI 1995)

- a set of n variables X_i with domains d_i
- a set of e cost functions, each having a
  - scope (list of variables)
- cost functions map assignments to costs

It only remains to specify what the possible costs are, and how to combine them

Definition of a valuation structure

- a set S of costs
- a total order <
- minimum and maximum elements:

  we denote these by 0 and ∞
- an aggregation operator $\oplus$ which is commutative, associative, monotonic, and such that $\forall\alpha,\ \alpha\oplus 0=\alpha$

Examples of valuation structures

- If $S = \{0, \infty\}$, then VCSP $\equiv$ CSP

- If $S = \{0, 1, 2, \ldots, \infty\}$, and $\oplus$ is addition,
  then VCSP generalizes MAX-CSP

- If $S = [0,1]$, and $\oplus$ is max, then VCSP $\equiv$ Fuzzy CSP

  - If $S = \{0, 1, \ldots., k\}$, and $\oplus$ is bounded addition $+_k$
    where $\alpha +_k \beta = \min\{k, \alpha+\beta\}$,
    then VCSP $\equiv$ Weighted CSP

Families of valuation structures

A valuation structure is idempotent if
$\forall \alpha,\ \alpha \oplus \alpha = \alpha$

All idempotent valuation structures are equivalent to Fuzzy CSP

(as in CSP redundancy of information is fine)

Families of valuation structures

A valuation structure is strictly monotonic if
$\forall\alpha<\beta, \forall\gamma<\infty, \ \alpha\oplus\gamma < \beta\oplus\gamma$

A valuation structure is fair if
aggregation has a partial inverse, that is,
$\forall\alpha\geq\beta, \exists\gamma$ such that $\beta\oplus\gamma=\alpha$

All strictly monotonic valuation structures can be embedded in a fair valuation structure

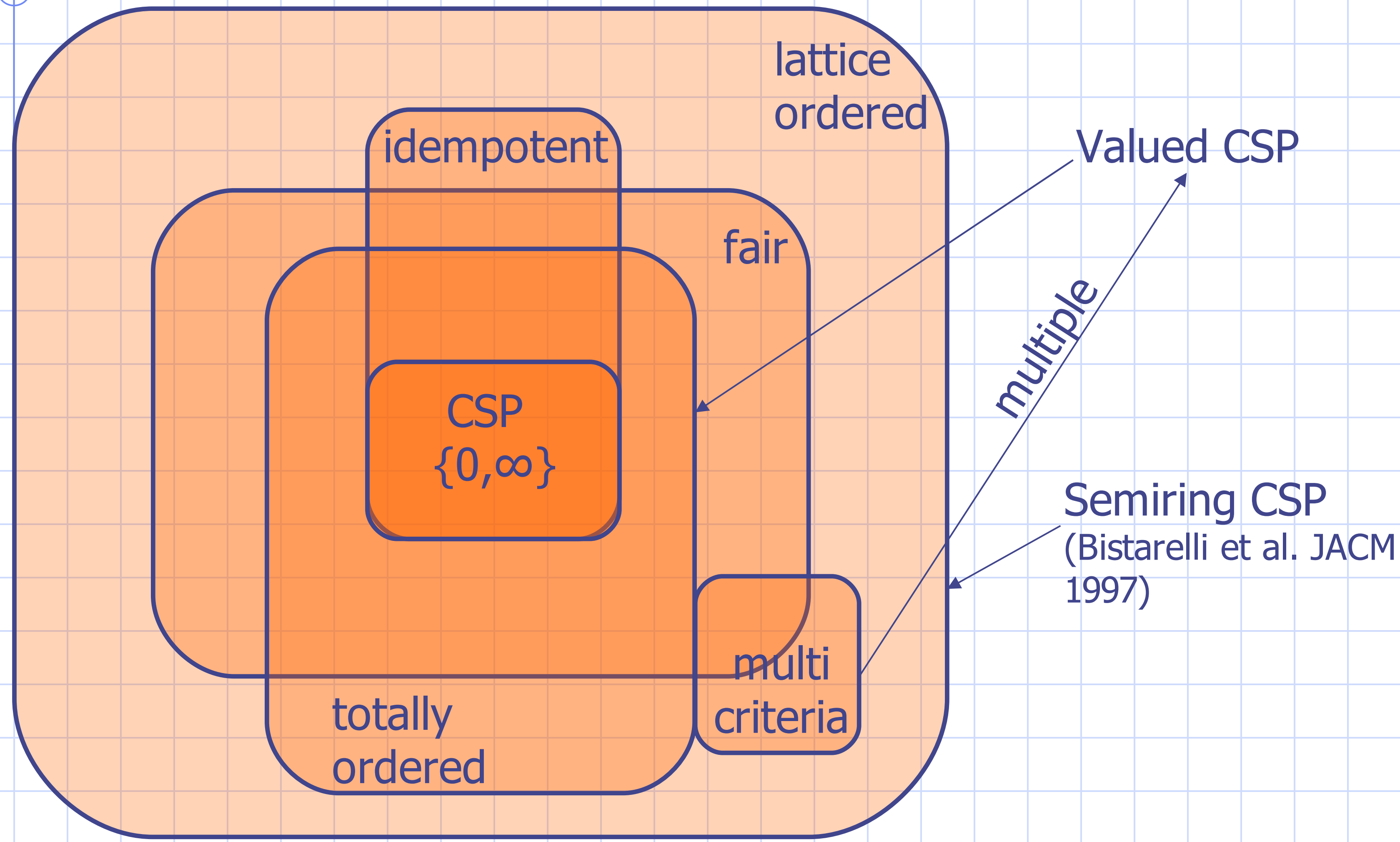
Families of valuation structures

A valuation structure is discrete if between any pair of finite costs there are finitely many other costs

All discrete and fair valuation structures
can be decomposed into
a contiguous sequence of valuation structures
with aggregation operator $+_k$

(interacting as fuzzy CSP)

General frameworks and cost structures



Bibliography

- For general background on VCSP and other formalisms for soft constraints, see the chapter on "Soft Constraints" by Meseguer, Rossi and Schiex, in the *Handbook of Constraint Programming*, Elsevier, 2006.

- For classification results on valuation structures see "Arc Consistency for Soft Constraints", *Cooper & Schiex,* AIJ, 2004.

Chapter 2. Efficiency

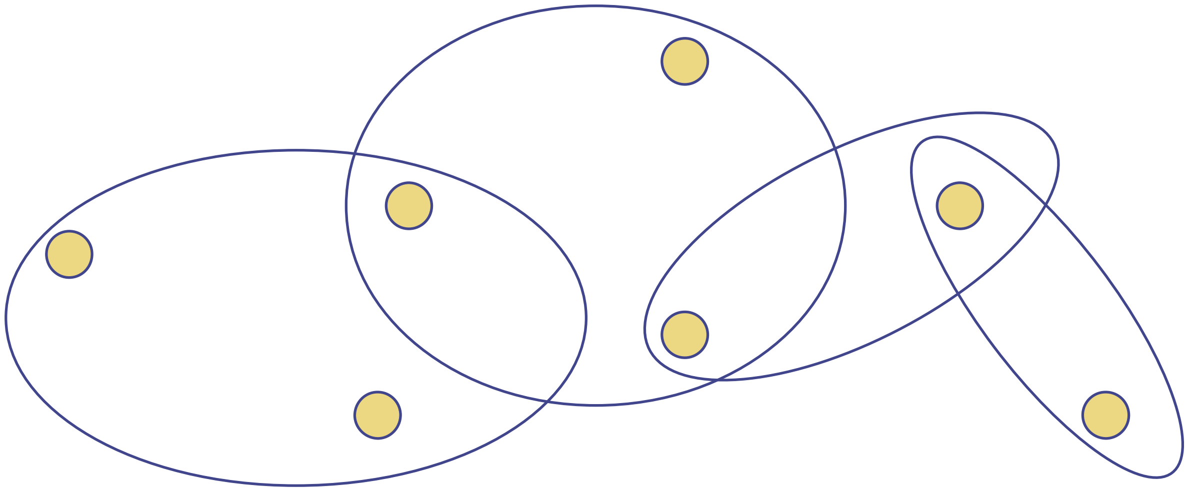
Structural restrictions,
Valued constraint languages

General question

Having a unified formulation allows us to ask *general* questions about efficiency:

When is the VCSP tractable?

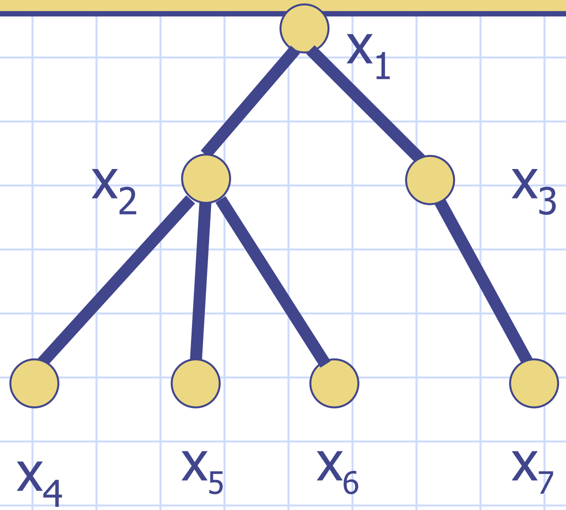
Problem features



- This picture illustrates the constraint *scopes*
- The set of scopes is sometimes called the *constraint hypergraph*, or the *scheme*
- Restricting the scheme can lead to tractability, as in the standard CSP

Structural tractability

Tree-structured binary VCSPs are tractable



Time complexity $O(e\ d^2)$
Space complexity $O(n\ d)$

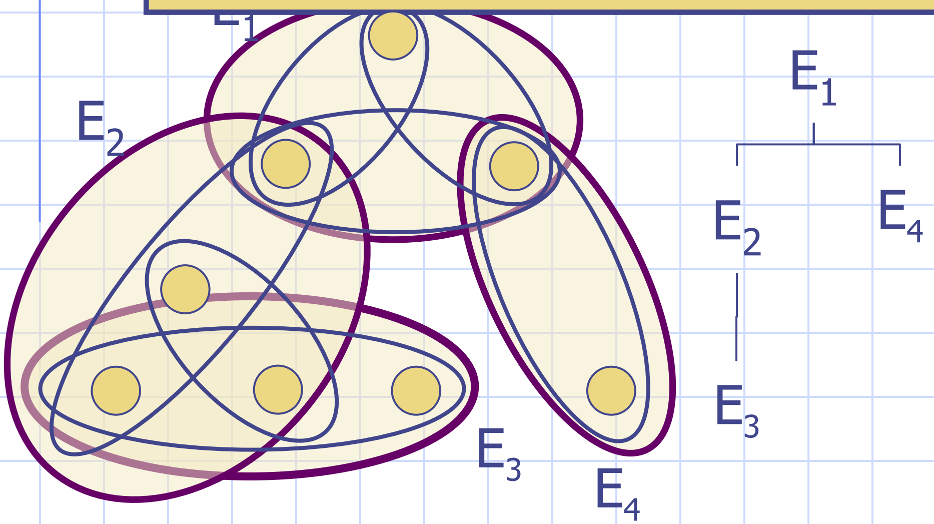
n: number of variables
d: maximum domain size
e: number of cost functions

Proceed from the leaf nodes to a chosen root node

Project out leaf nodes by minimising over possible assignments

Tree decomposition

Bounded treewidth VCSPs are tractable



Time complexity $O(e\, d^{w+1})$
Space complexity $O(n\, d^s)$

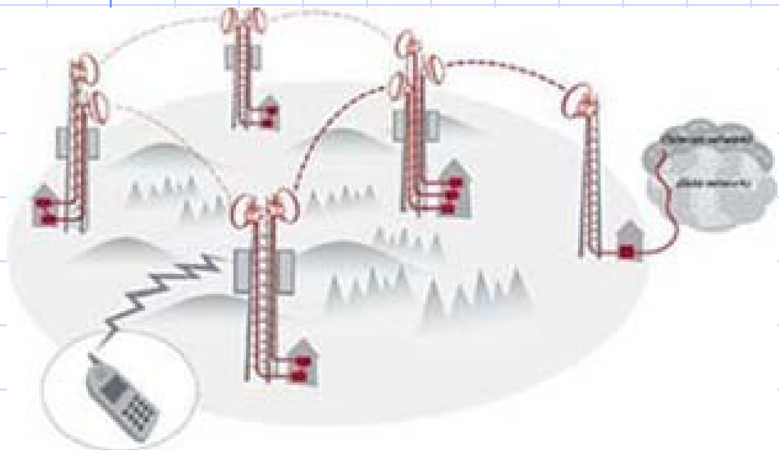
w: bounded treewidth
$= \max |E_i| - 1$

s: $\max \{|E_i \cap E_j| : i \neq j\}$

Finding a tree decomposition with minimum w* is NP-hard!

Radio Link Frequency Assignment Problem

(Cabon et al., *Constraints* 1999) (Koster *et al.*, *4OR* 2003)



- Given a telecommunication network
- …find the best frequency for each communication link, avoiding interferences

- Best can be:
  - Minimize the maximum frequency, no interference (max operator)
  - **Minimize the global interference (sum operator)**
- Generalizes graph coloring problems: $|f_i - f_j| \geq a$

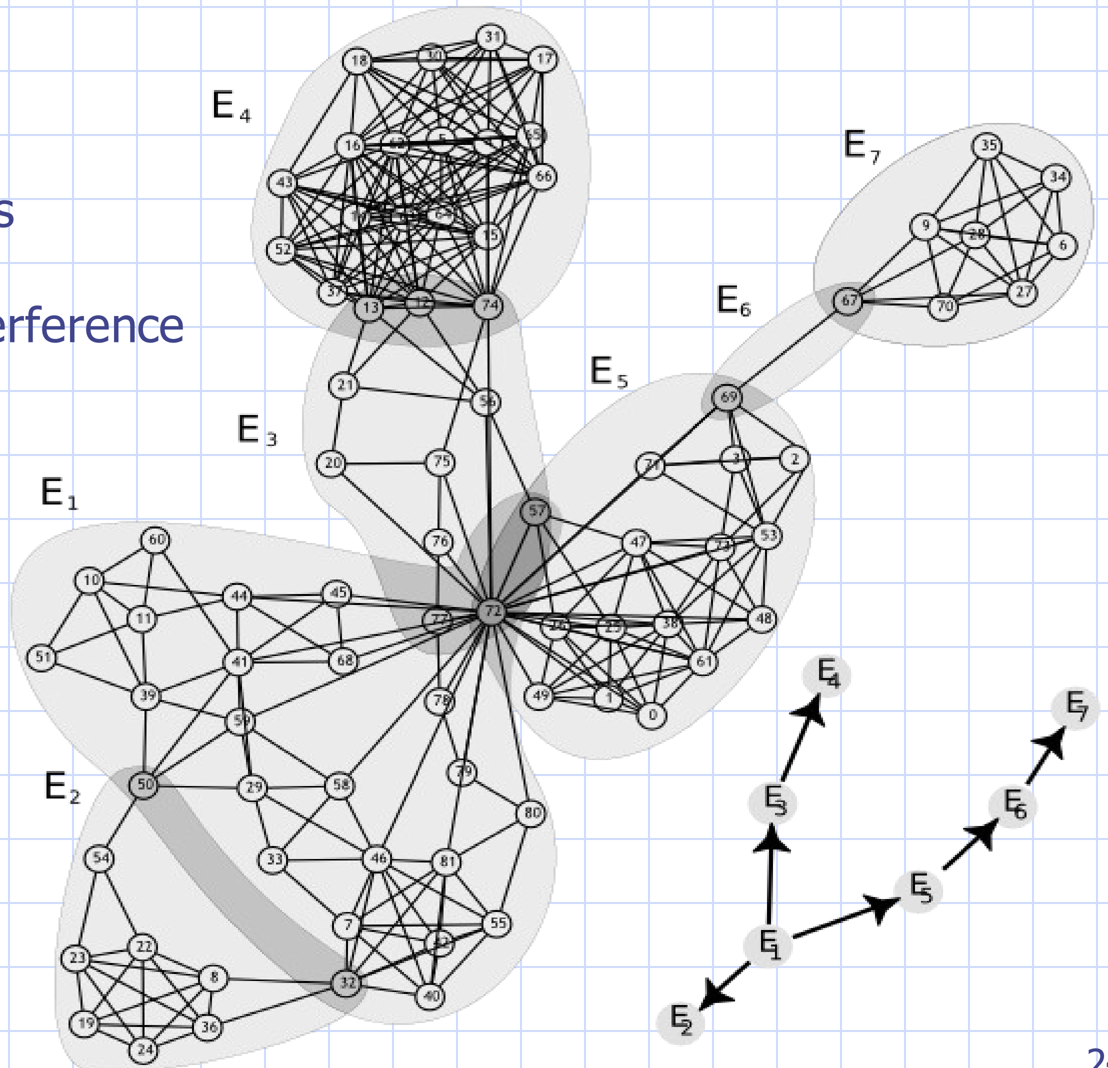
***CELAR** problem size: n=100—458 ; d=44 ; e=1,000—5,000*

Tree decomposition example

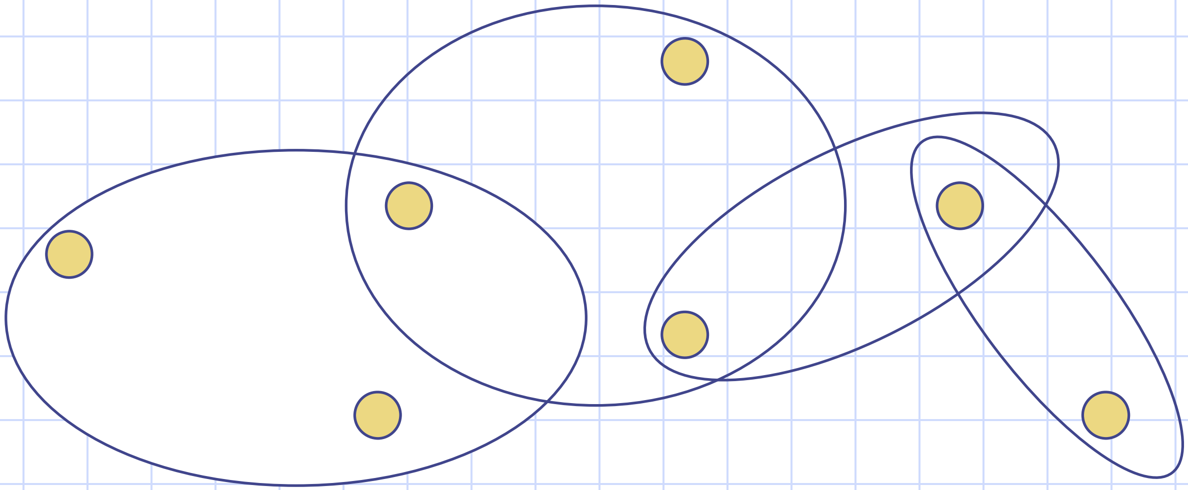
Benchmark problem
assigning frequencies
to transmitters
to minimise total interference

CELAR scen06r
n = 82
d = 44
e = 327

w = 26
s = 3

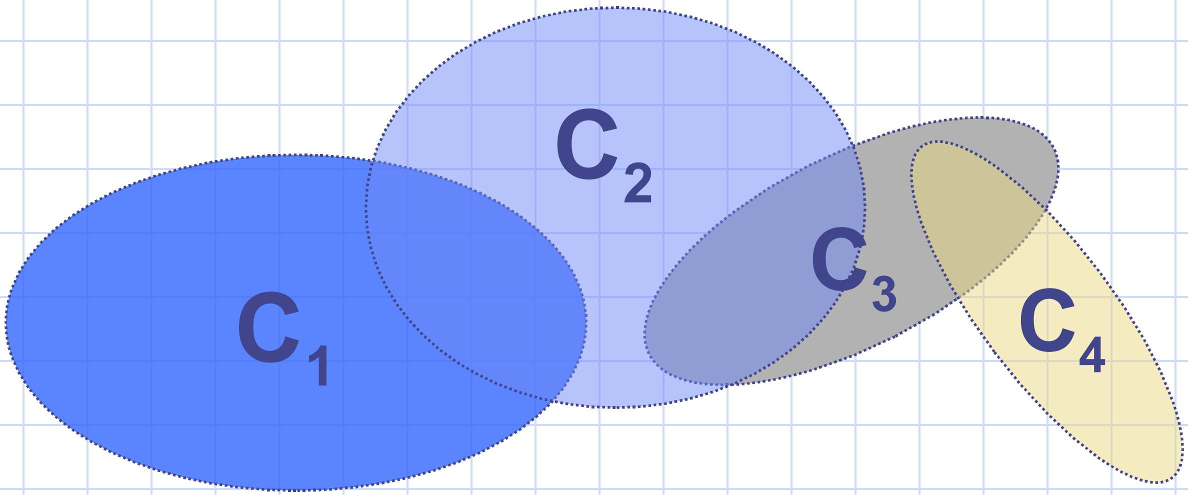


Problem features



- We have seen that structural features of a problem can lead to tractability
- This is very similar to the standard CSP
- What about other kinds of restrictions to the VCSP?

More problem features



- The picture now emphasises the cost functions
- Restricting the cost functions we allow can also lead to tractability

Valued constraint languages

- A set of cost functions is called a valued constraint language
- VCSP(Γ) represents the set of VCSP instances whose cost functions belong to the valued constraint language Γ
- For some choices of Γ, VCSP(Γ) is tractable
- We will consider some examples where the valuation structure contains non-negative real values and infinity, and aggregation is standard addition

Submodular functions

A class of functions that has been widely studied in OR is the submodular functions…

A cost function c is submodular if $\forall$**s**,**t**

$$c(\min(\mathbf{s},\mathbf{t})) + c(\max(\mathbf{s},\mathbf{t})) \leq c(\mathbf{s}) + c(\mathbf{t})$$

where min and max are applied component-wise, i.e.

$$\min(\langle s_1,\ldots,s_k\rangle,\langle t_1,\ldots,t_k\rangle) = \langle \min(s_1,t_1),\ldots,\min(s_k,t_k)\rangle$$

$\mathrm{VCSP}(\Gamma_{\mathrm{submodular}})$ is tractable

Examples of submodular functions

x	y	z	
0	0	0	0
0	0	1	1
0	1	0	7
0	1	1	1
1	0	0	∞
1	0	1	3
1	1	0	∞
1	1	1	0

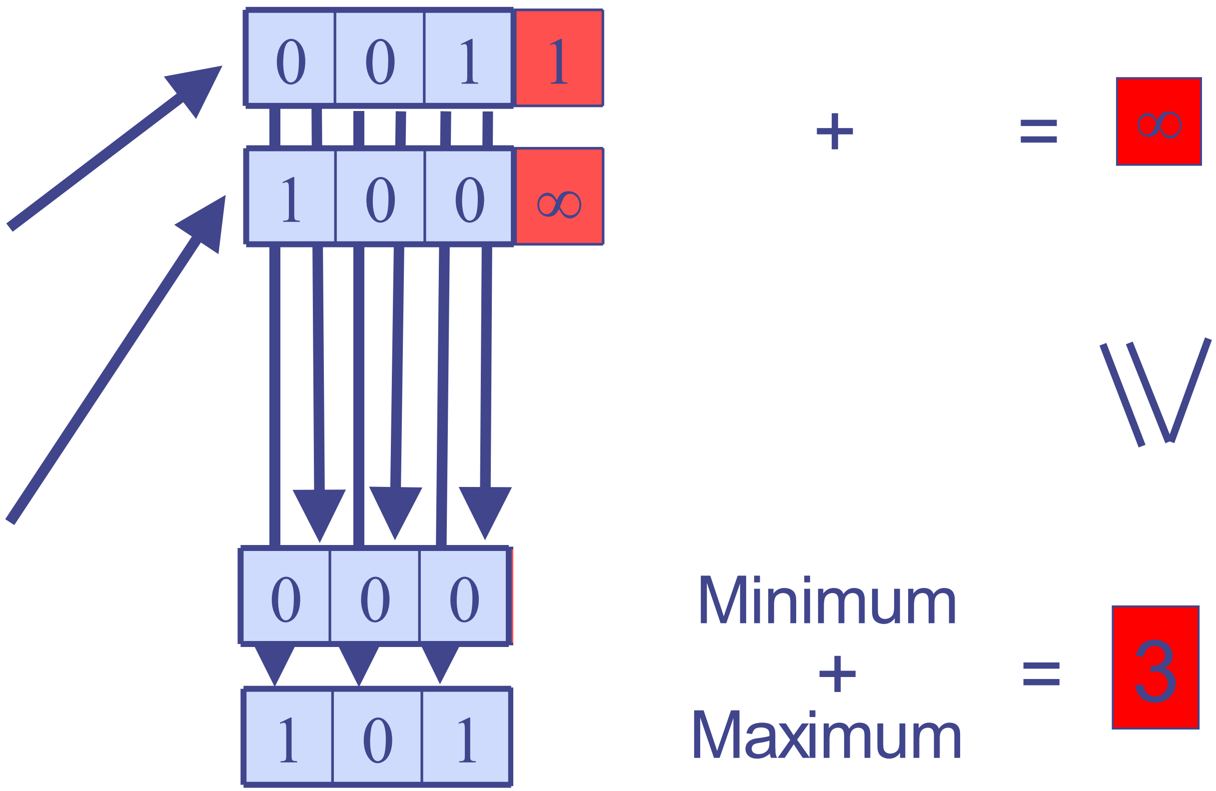
1	0	1

Maximum

Examples of submodular functions

$\forall s,t \;\; \mathrm{Cost}(\mathrm{Min}(s,t)) + \mathrm{Cost}(\mathrm{Max}(s,t)) \leq \mathrm{Cost}(s) + \mathrm{Cost}(t)$

	x	y	z	
	0	0	0	0
s	0	0	1	1
	0	1	0	7
	0	1	1	1
t	1	0	0	∞
	1	0	1	3
	1	1	0	∞
	1	1	1	0



Examples of submodular functions

- all unary functions
- all linear functions (of any arity)
- the binary function ϕ_{cut}

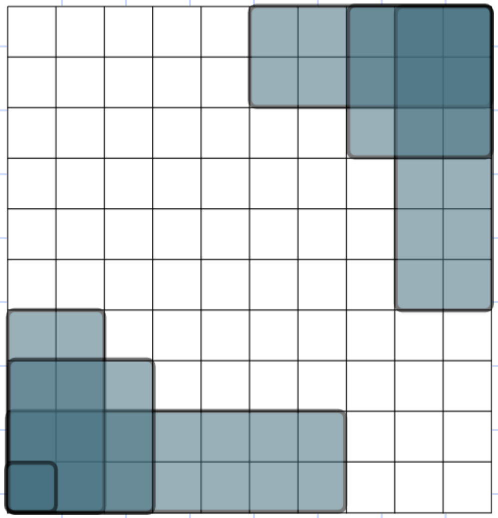
  where $\phi_{cut}(a,b)=1$ if $(a,b)=(0,1)$ (0 otherwise)
- the rank function of a matroid
- the Euclidean distance function between two points (x_1, x_2), (x_3, x_4) in the plane
- $\phi(x,y)=(x-y)^r$ if $x \geq y$ (∞ otherwise) for $r \geq 1$

  (compare "*Simple Temporal CSPs with strictly monotone preferences*"

  Khatib et al, IJCAI 2001)

Binary submodular functions

Binary submodular functions
over any finite domain
can be expressed as a sum of
"Generalized Interval" functions



(they correspond to Monge matrices)

Binary VCSP($\Gamma_{\text{submodular}}$) is $O(n^3d^3)$

See Cohen et al "A maximal tractable class of soft constraints", JAIR 2004

Beyond submodularity

$$\forall s,t \;\; \mathrm{Cost}(\mathrm{Min}(s,t)) + \mathrm{Cost}(\mathrm{Max}(s,t)) \leq \mathrm{Cost}(s) + \mathrm{Cost}(t)$$

x	y	z	
0	0	0	0
0	0	1	1
0	1	0	7
0	1	1	1
1	0	0	∞
1	0	1	3
1	1	0	∞
1	1	1	0

The cost function has the ***multimorphism*** (Min,Max)

By choosing *other* functions, we can obtain other tractable valued constraint languages…

Known tractable cases

If the cost functions all have one of these eight multimorphisms, then the problem is tractable:

1) (Min,Max)
2) (Max,Max)
3) (Min,Min)
4) (Majority,Majority,Majority)
5) (Minority,Minority,Minority)
6) (Majority,Majority,Minority)
7) (Constant 0)
8) (Constant 1)

See Cohen et al "The complexity of soft constraint satisfaction", AIJ 2006

A dichotomy theorem

If the cost functions all have one of these eight multimorphisms, then the problem is tractable:

1) (Min,Max)
2) (Max,Max)
3) (Min,Min)
4) (Majority,Majority,Majority)
5) (Minority,Minority,Minority)
6) (Majority,Majority,Minority)
7) (Constant 0)
8) (Constant 1)

For Boolean cost functions…

In all other cases the cost functions have **no** significant common multimorphisms and the VCSP problem is **NP-hard**.

See Cohen et al "The complexity of soft constraint satisfaction", AIJ 2006

Benefits of a general approach

- The dichotomy theorem immediately implies earlier results for SAT, MAX-SAT, Weighted Min-Ones and Weighted Max-Ones

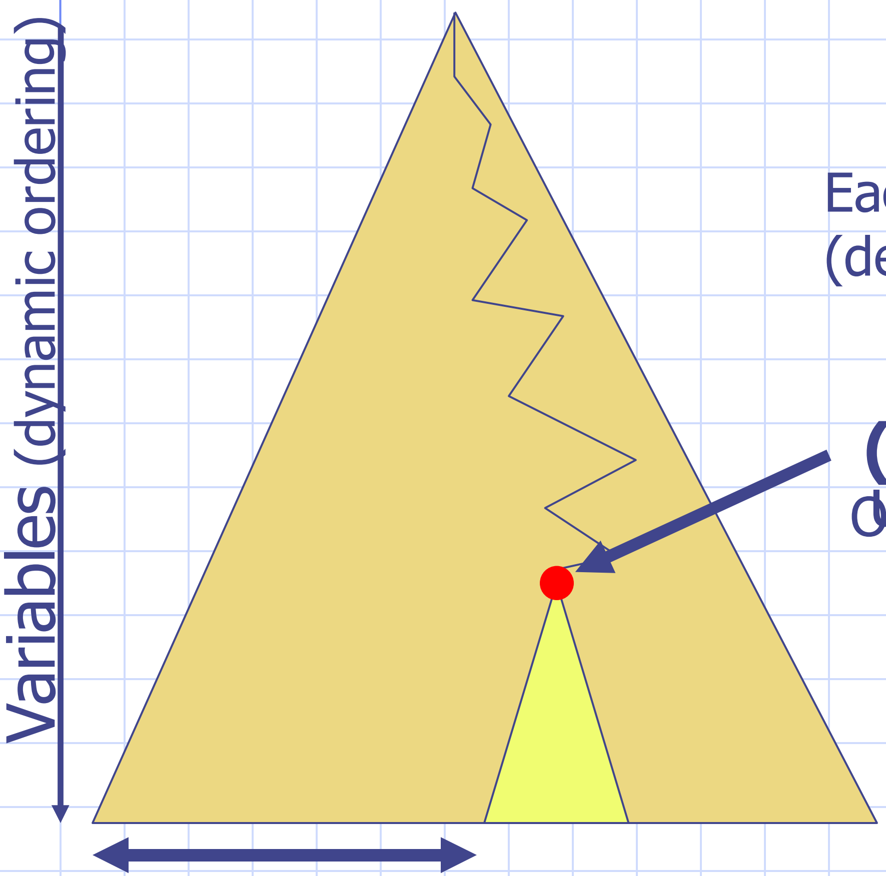
Bibliography

- For general background on tractable structures, see the chapter on "Tractable Structures" by Dechter, in the *Handbook of Constraint Programming*, Elsevier, 2006.
- For tractable valued constraint languages see "The complexity of soft constraint satisfaction", Cohen, Cooper, Jeavons & Krokhin, AIJ 2006.

Chapter 3. Search using problem transformations

Branch and Bound,
Equivalence-preserving operations,
Local consistency (node, arc, directional, virtual, optimal),
Global cost functions.

Depth-First Branch and Bound (DFBB)



Each node is a VCSP subproblem
(defined by current conditioning)

(LB) Lower Bound = $c_\varnothing$

under estimation of the best solution in the sub-tree

Obtained by enforcing local consistency

If LB $\geq$ *k* then prune

If $c_\varnothing \geq k$ then prune

(UB) Upper Bound
= best solution found so far = ***k***

Local consistency

A property that says that the network is "explicit" enough at a local level

Filtering algorithm: transforms a network in an equivalent network that satisfies the property (closure)

CSP: pol. time, confluent, incremental

Equivalence-preserving transformations (EPT)

- An EPT transforms VCSP instance P1 into another VCSP instance P2 with the same objective function.
- Examples of EPTs:

- Propagation of inconsistencies (∞ costs)

- UnaryProject

- Project/Extend

INCREMENTALITY!

UnaryProject(i,α)

Precondition: $0 \le \alpha \le \min\{c_i(a) : a \in d_i\}$

$c_0 := c_0 + \alpha$;

for all $a \in d_i$ **do**

$\quad c_i(a) := c_i(a) - \alpha$;

Increases the lower bound c_0 if all unary costs $c_i(a)$ are non-zero.

Project(M,i,a,α)

Precondition: $i \in M$, $a \in d_i$, $-c_i(a) \le \alpha \le \min\{c_M(x): x[i]=a\}$

$c_i(a) := c_i(a) + \alpha$;
for all $x \in$ labelings(M) s.t. x[i]=a **do**
    $c_M(x) := c_M(x) - \alpha$;

If $\alpha>0$, this projects costs from c_M to c_i

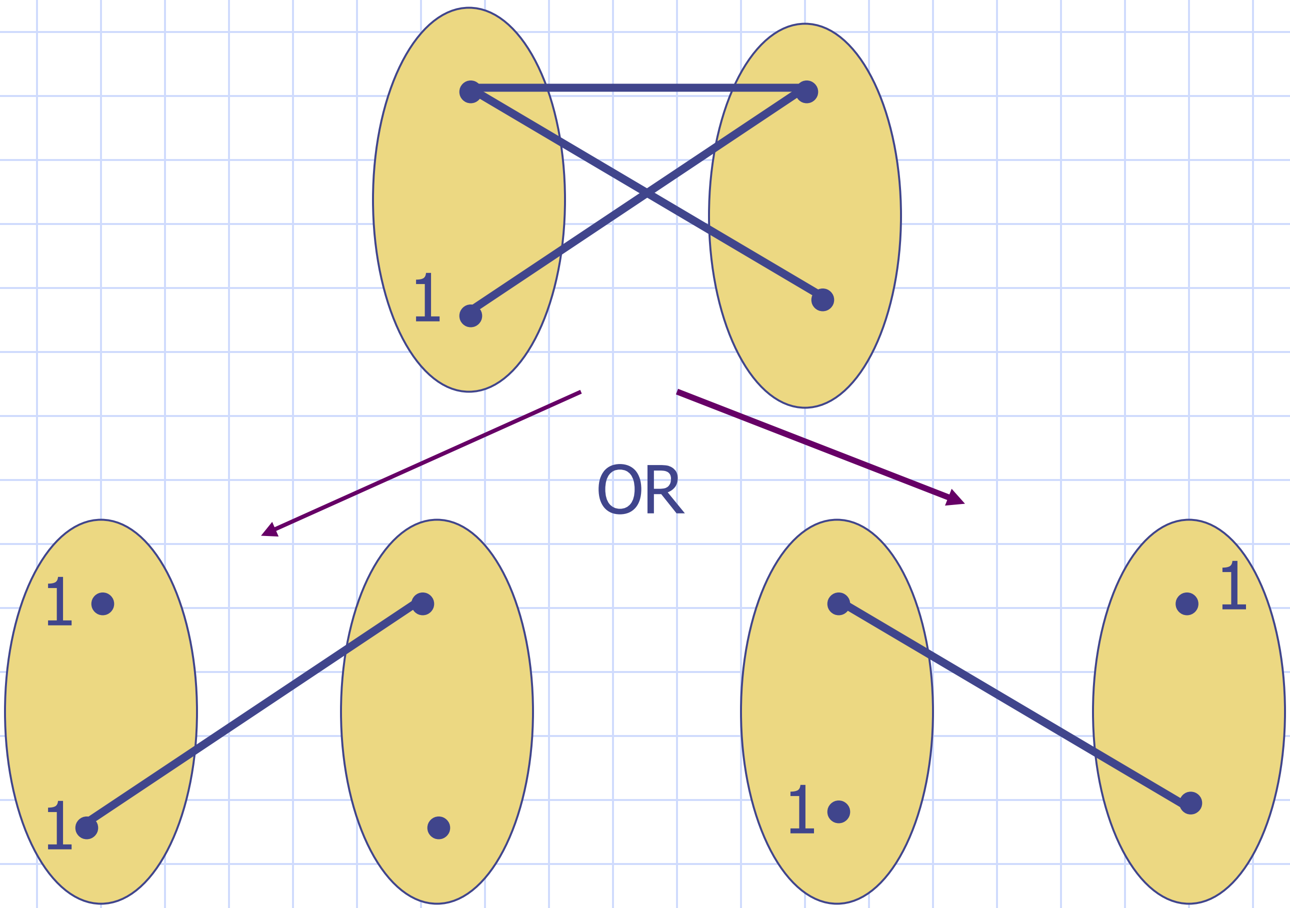
If $\alpha<0$, this extends costs from c_i to c_M

Node and soft arc consistency

- Node consistent (NC) if $\forall i$
  no UnaryProject(i,α) is possible for $\alpha>0$ and no propagation of ∞ costs possible between c_i and c_0 (forbidden values removed if $c_i+c_0 \geq k$)

- Soft arc consistent (SAC) if $\forall M,i,a$
  no Project(M,i,a,α) is possible for $\alpha>0$

The SAC closure is not unique



Finding the best order of integer EPT application is NP-hard (Cooper, Schiex 2004)

Different soft AC notions:

- Directional: send costs from X_j to X_i if $i<j$ (in the hope that this will increase c_0)
- Existential: $\forall i$, send costs to X_i simultaneously from its neighbor variables if this increases c_0
- Virtual: no *sequence* of Projects/Extends increases c_0
- Optimal: no *simultaneous set* of Projects/Extends increases c_0

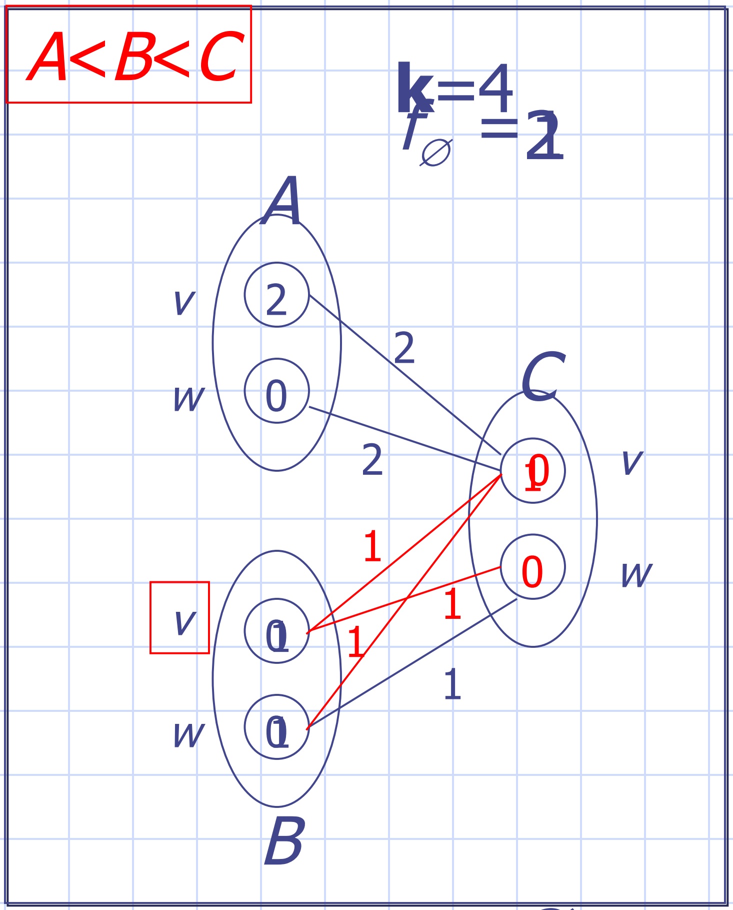
Directional Arc Consistency

- for all $i<j$, $\forall a \in d_i \; \exists b \in d_j$ such that $c_{ij}(a,b) = c_j(b) = 0$.
- Solves tree-structured VCSPs
- FDAC (Full Directional AC) = Directional AC + Soft AC
- FDAC can be established in $O(end^3)$ time (or in $O(ed^2)$ time if $+_k$ is $+$)

(Cooper, Fuzzy Sets and Systems 2003)

Directional AC (DAC*)

- NC*
- For all f_{AB} (*A*<*B*)
  - $\forall a \, \exists \, b$
    $f_{AB}(a,b) + f_B(b) = 0$
- *b* is a *full-support*
- complexity:
  $\mathbf{O}(ed^2)$



Shift(f_{BC},(C,v),-1) Shift(f_{BC},(B,v),1) Shift(f_{BC},(B,w),1) Shift(f_B, $\varnothing$,1)

Shift(f_A,$\varnothing$, -2) Shift(f_A,$\varnothing$, 2)

Existential Arc Consistency

- node consistent and $\forall i, \exists a \in d_i$ such that $c_i(a) = 0$ and for all cost functions c_{ij} , $\exists b \in d_j$ such that $c_{ij}(a, b) = c_j(b) = 0$
- EDAC = Existential AC + FDAC
- EDAC can be established in

  $O(ed^2 \max\{nd,k\})$ time

Virtual Arc Consistency (VAC)
(Cooper et al, 2008)

- If P is a VCSP instance then Bool(P) is the CSP instance whose allowed tuples are the zero-cost tuples in P-c_0
- If Bool(P) has a solution, then P has a solution of cost c_0 (but usually Bool(P) has no solution)
- Definition: P is VAC if Bool(P) is AC.

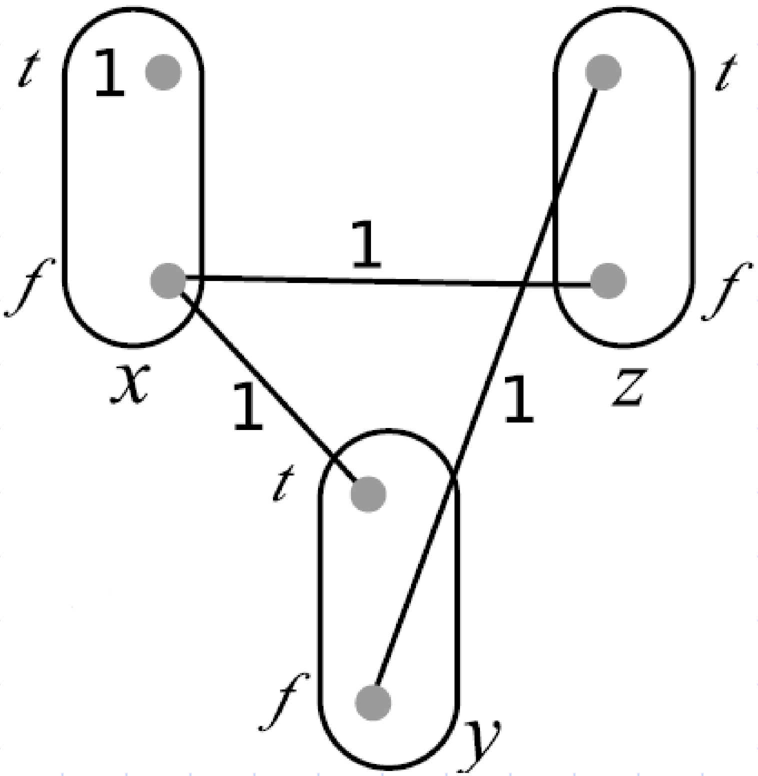
Approximating VAC

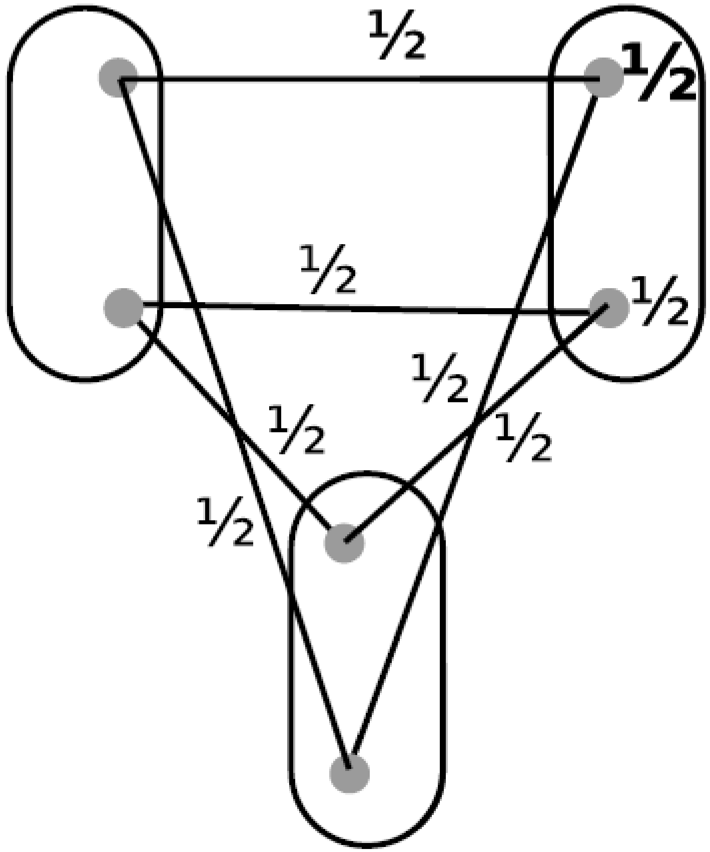
(similar to Augmenting DAG, Schlesinger et al, 30 years before)

- If a sequence of AC operations in Bool(P) leads to a domain wipe-out, then a similar sequence of SAC operations in P increases c_0
- But, in this sequence, costs may need to be sent in more than one direction from the same c_M $\Rightarrow$ Introduction of *fractional* weights
- VACε algorithm may converge to a local minimum (and more, an instance P′ which is *not* VAC)
- VACε can be established in $O(ed^2\, k/\varepsilon)$ time

Enforcing VAC

AC,DAC,FDAC,EDAC





Optimal Soft Arc Consistency

(Cooper et al. 2007), similar to (Schlesinger, 30 years before)

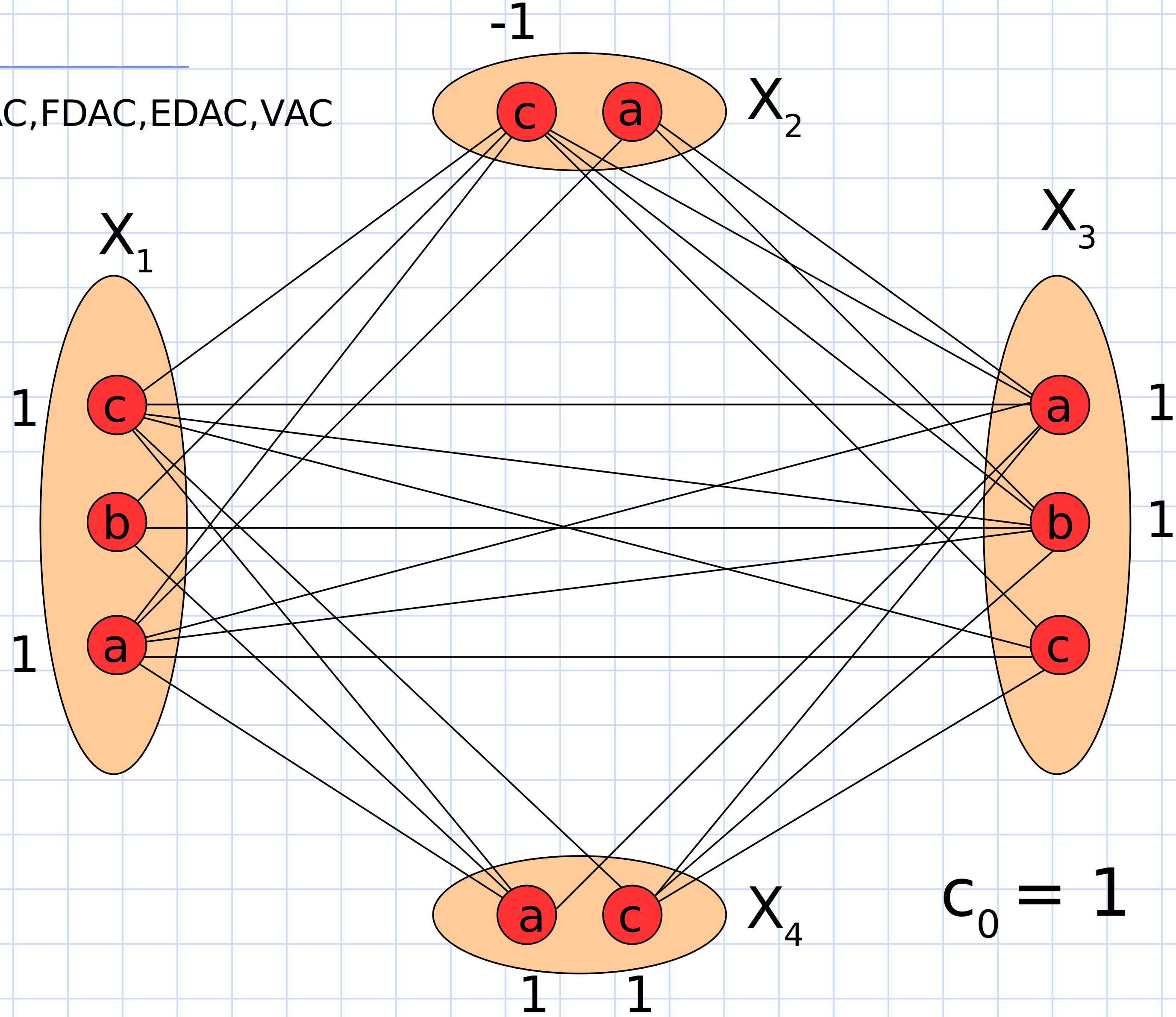
- We can overcome this problem of convergence by solving a LP to find the *set* of simultaneous UnaryProject and Project operations which maximises c_0.

- The resulting VCSP instance is OSAC (Optimal Soft Arc Consistent).
- OSAC is strictly stronger than VAC.
- Unfortunately, the LP has O(edr+n) variables and O(ed^r+nd) constraints(pre-processing).

$$p^{23}_{2c} = p^{34}_{3a} = p^{31}_{3b} = p^{12}_{1a} = p^{14}_{1c} = -1$$

$$p^{23}_{3a} = p^{23}_{3b} = p^{34}_{4c} = p^{31}_{1a} = p^{31}_{1c} = p^{12}_{2c} = p^{14}_{4a} = u_4 = 1$$

AC,DAC,FDAC,EDAC,VAC



$c_0 = 1$

Virtual Arc Consistency solves (locally-defined) submodular VCSP

If $P \in \text{VCSP}(\Gamma_{\text{sub}})$ and P is VAC,

then Bool(P) is arc consistent, max-closed.

Hence, Bool(P) has a solution.

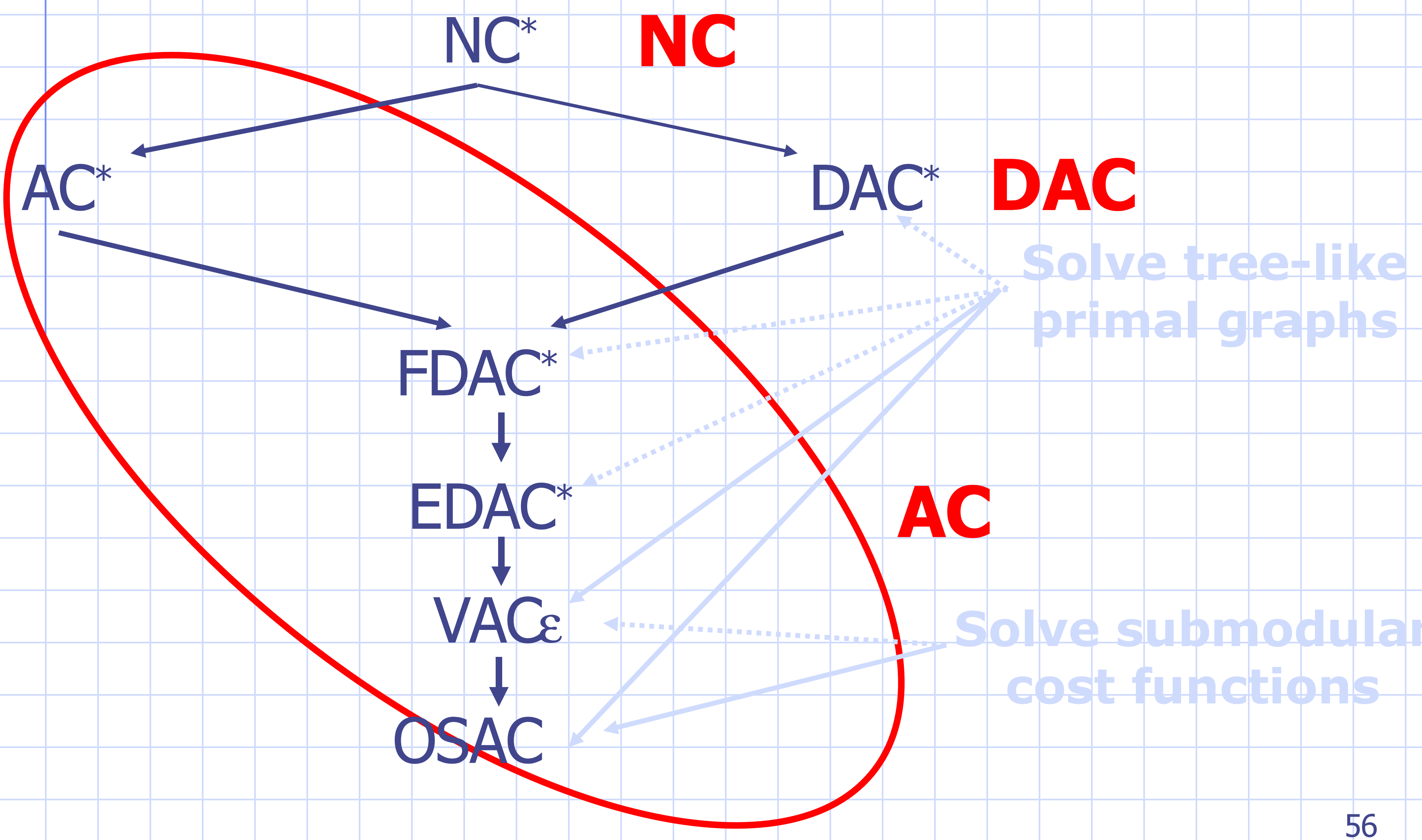
This solution has cost c_0 in P and is thus necessarily optimal.

Thus OSAC solves SFM since Project and UnaryProject preserve submodularity.

Also permutated submodular (some technicalities)

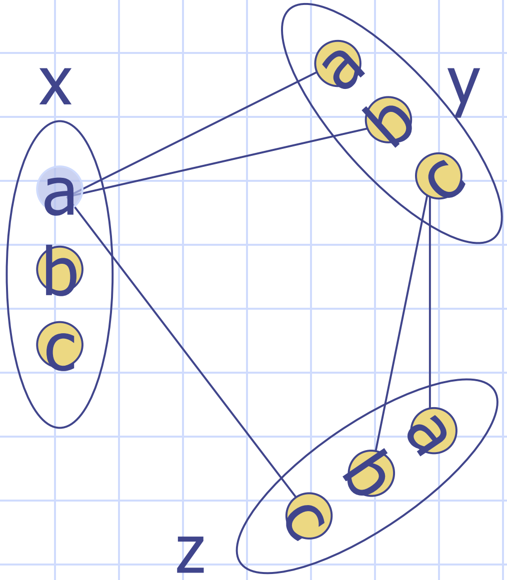
Hierarchy

Special case: CSP (**k**=1)



Beyond Arc Consistency

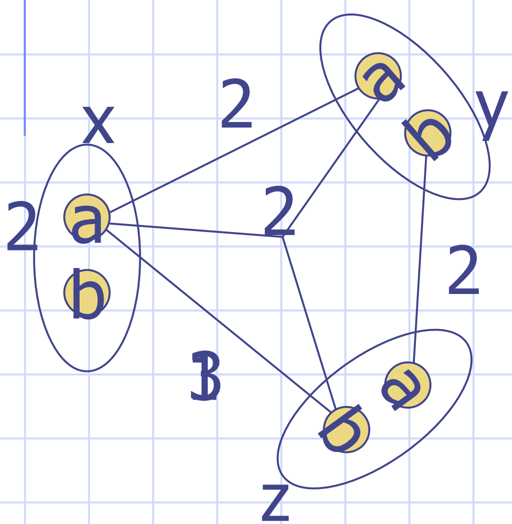
- Path inverse consistency (Debryune & Bessière)



(x,a) can be pruned because there are two other variables y,z such that (x,a) cannot be extended to any of their values.

Beyond Arc Consistency

- Soft Path inverse consistency

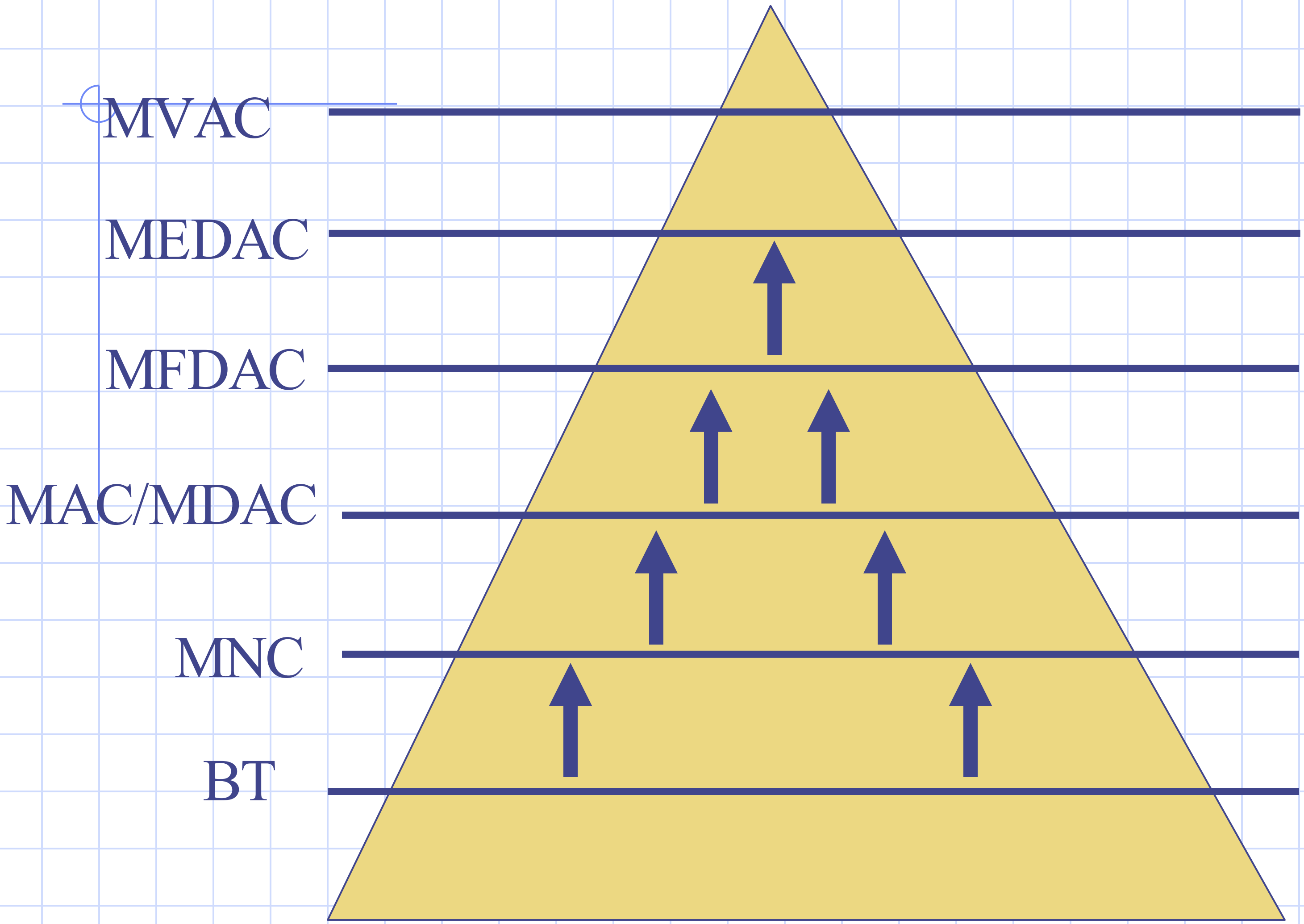


$f_y \oplus f_z \oplus f_{xy} \oplus f_{xz} \oplus f_{yz}$

x	y	z	
a	a	a	0
a	a	b	3
a	b	a	0
a	b	b	1
b	a	a	0
b	a	b	0
b	b	a	2
b	b	b	0

$(f_y \oplus f_z \oplus f_{xy} \oplus f_{xz} \oplus f_{yz})[x]$

x	
a	2
b	0


MVAC
MEDAC
MFDAC
MAC/MDAC
MNC
BT

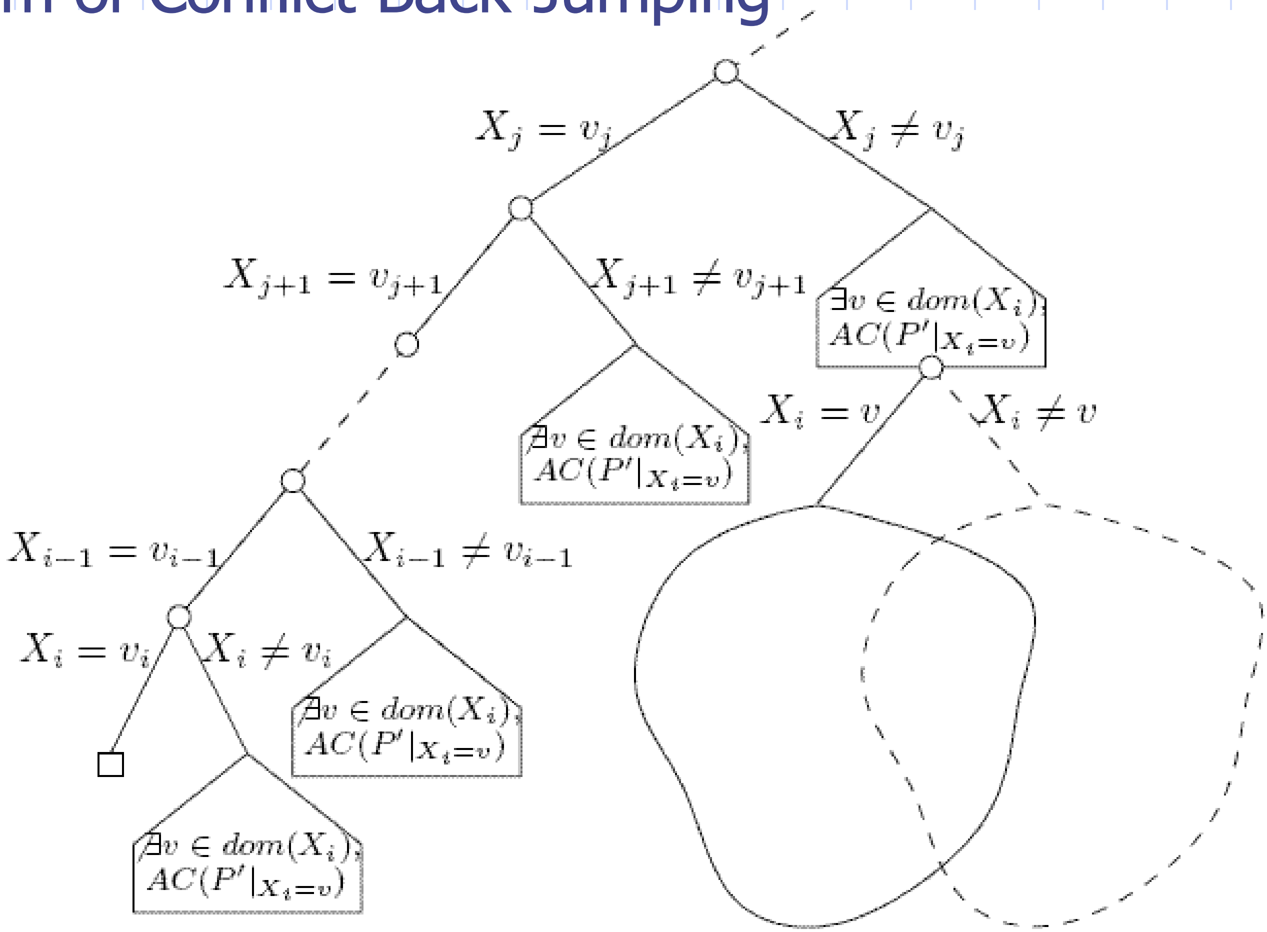
Some practical observations

- For very hard-to-solve instances, maintaining VAC provides a significant speed-up (closed RLFAP graph11/13).
- For many problems, maintaining a simpler form of soft arc consistency (e.g. EDAC) is faster.
- Unary costs $c_i(a)$ and EAC value inform value and variable ordering heuristics

Variable heuristic

Last-conflict (Lecoutre et al, ECAI 2006)

Basic form of Conflict Back-Jumping



Used in combination with domain size / weighted degree (Lecoutre et al, AIJ 2009), breaking ties with max unary cost

RLFAP: CELAR 6 results since 1993

n. of vars: n=100, domain size: d=44, n. of cost functions: e=1222

Time of optimality proof		Method(s) used	Publication
26 days **(SUN UltraSparc 167 MHz)**		Ad-hoc problem decomposition & Russian Doll Search *(22 vars only)*	(de Givry, Verfaillie, Schiex, CP 1997)
3 days (SUN Sparc 2)		Ad-hoc problem decomposition & PFC-MRDAC *(22 vars only)*	(Larrosa, Meseguer, Schiex, AIJ 1998)
8 hours (DEC Alpha 500MP)		Preprocessing rules & BbB Elimination	(Koster PhD thesis, 1999)
3 hours (PC 2.4 GHz)		B&B with EDAC & tree decomposition (BTD)	(de Givry, Schiex, Verfaillie, AAAI 2006)
1’ 26’’ **(PC 2.5 GHz)**	**25000x** **16 x**	BTD-RDS & variable ordering heuristics & dichotomic branching	(Sanchez, Allouche, de Givry,Schiex, IJCAI 2009)

CELAR 7 (n=200) solved in 4.5 days (Sanchez et al, IJCAI 2009)

CELAR 8 (n=458) solved in < 2 days (127 days)

2010 Approximate Inference Evaluation (results given at UAI'10)

Networks – by domain (1 hour)

Network	PR	MAR	MPE
CSP	8	8	55
Grids	20	20	40
Image Alignment			10
Medical Diagnosis	26	26	
Object Detection	96	96	92
Pedigree	4	4	
Protein Folding			21
Protein-Protein Interaction			8
Segmentation	50	50	50

Summary of the results

Seconds	PR	MAR	MPE
20	Arthur Choi (UCLA)	Arthur Choi (UCLA)	Joris Mooij (Max Planck)
1200	Vibhav Gogate (UW+UCI)	Vibhav Gogate (UW+UCI)	Thomas Schiex (INRA)
3600	Vibhav Gogate (UW+UCI)	Vibhav Gogate (UW+UCI)	Joris Mooij (Max Planck)

toulbar2 was also first at UAI'08 Evaluation, MaxCSP'06,'08 Competition

Winning Teams

- (MAR) **IJGP** by Vibahv Gogate (UW), Andrew Gefland, Natasha Flerova and Rina Dechter (UCI):
  Anytime iterative GBP based algorithm
- (PR) **Vgogate** by Vibahv Gogate, Pedro Domingos (UW),, Andrew Gefland and Rina Dechter (UCI):
  Formula based importance sampling
- (PR+MAR) **EDBP** by Arthur Choi, Adnan Darwiche, with support from Glen Lenker and Knot Pipatsrisawat (UCLA):
  Anytime BP based anytime thickening of structure
- (MAP) **libDAI** by Joris Mooij (Max Planck):
  junction tree, LBP/MP, double-loop GBP, Gibbs, decimation
- (MAP) **toulbar2** by Thomas Schiex et al (INRA)
  Anytime branch and bound weighted CSP solver

73% instances solved exactly

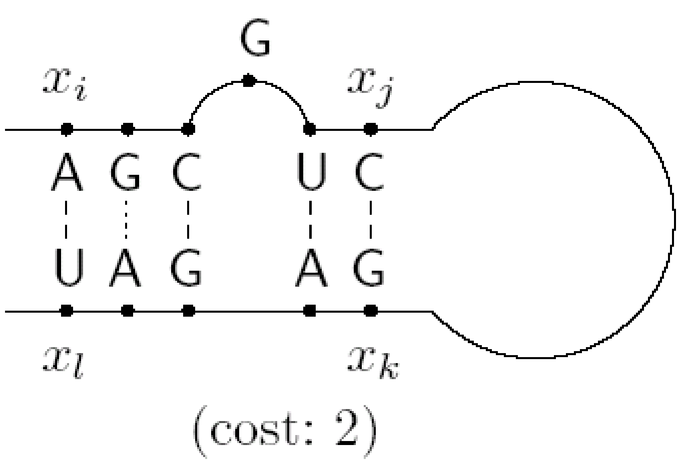
AC for global cost functions

- **Global constraints**: specific family of constraints on an unbounded number of variables with efficient local consistency filtering.
  - Example: AllDifferent (max matching)

- Same for global cost functions
  - Example: # of variables with the same value

    (van Hoeve et al, J. Heur. 2006) (Lee & Leung, IJCAI'09)

RNA gene finding (Zytnicki et al, 2008)



- Given a sequence and an RNA gene descriptor
- …find all the occurrences of the descriptor with at most *k* mismatches
  - NP-complete for *k*=0 (Vialette, 2004)
  - Sort solutions by their number of mismatches

***RNA** problem sizes: n=20 ; d>100 million! ; e(4)=10*

Bound arc consistency

- Goal: space complexity independent of the domains
- BAC∅ (Zytnicki et al, *JAIR, 2010*)
  - Avoid EPTs, except those shifting cost to
  - Prune *extremity* domain values only
  - Complexity
    - Time **O**($n^2 r^2 d^{r+1}$) and space **O**($n+er$)

    with maximum constraint arity r
  - BAC∅ is confluent
  - Can be specialized for semi convex cost functions (d to d^2 speedup on binary CF)

RNA gene finding

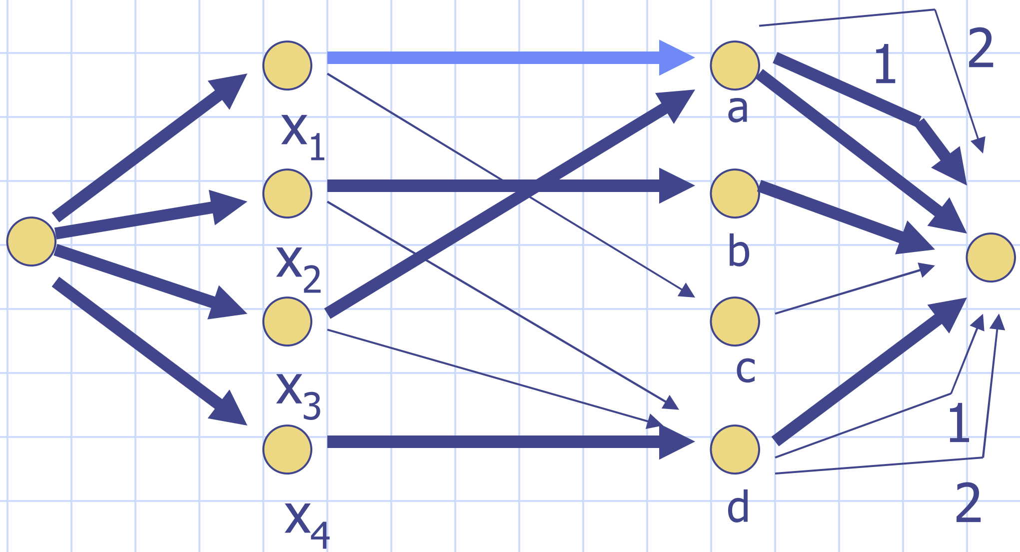
Domain size	10k	50k	100k	500k	1M	4.9M
Nb. of solutions	32	33	33	33	41	274
AC*						
Time	1hour 25min.	44hours	-	-	-	-
Nb. of backtracks	93	101	-	-	-	-
BAC						
Time (sec.)	0.016	0.036	0.064	0.25	0.50	2.58
Nb. of backtracks	93	101	102	137	223	1159

Fig. 6. Searching all the solutions of a tRNA motif in *Escherichia coli* genome.

Darn! solver (Zytnicki et al, Constraints 2008)

Network representing "min number of variables with same value"

(Beldiceanu & Petit, CPAIOR'04)



All edge capacities are equal to 1

All edge costs are 0 if not indicated

Flow shown is a min-cost max-flow with x_1=a.

We can project 1 from c_M to $c_1(a)$ by reducing the cost of the light blue edge from 0 to –1.

Latin Square N x N with costs

Example of solution for N = 5:

2 1 3 5 4
4 2 1 3 5
1 5 4 2 3
5 3 2 4 1
3 4 5 1 2

All variables take a different value in each row and each column

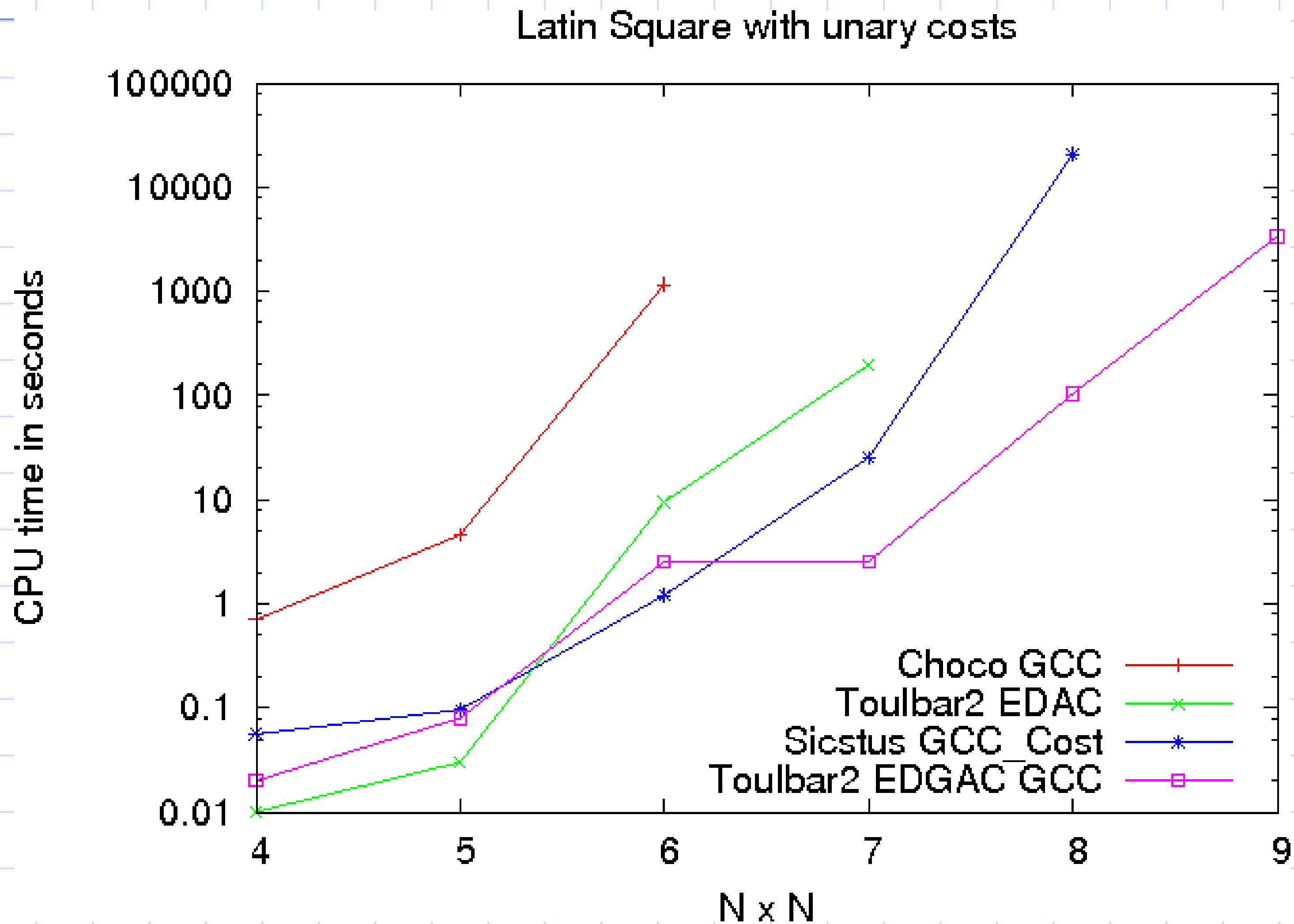
A unary cost function for each cell $f^{i,j}(x_{i,j}) : D \rightarrow [0,MaxCost[$

Objective: 49

$$\text{Objective} = \sum_i \sum_j f^{i,j}(x_{i,j})$$

GCC_Cost (Régin, *Constraints* 2002)
EDGAC (Lee & Leung, *AAAI* 2010)

Latin Square with costs



choco v2.1.1, toulbar2 v0.9.3, sicstus v4.1.2 on linux PC 2.66 Ghz 64GB

Bibliography

- For an overview of soft local consistencies, see "*Soft arc consistency revisited*", Cooper, de Givry, Sanchez, Schiex, Zytnicki & Werner, AIJ 2010.

- For soft global constraints (FDGAC), see "*Towards Efficient Consistency Enforcement for Global Constraints in Weighted Constraint Satisfaction*", Lee & Leung, IJCAI 2009.

Chapter 4. Open problems

Concerning problem definition, search, transformations, tractable classes

Possible extensions to VCSP

- Partial order instead of total order
- 2 arbitrary binary operators (e.g. calculating the sum of products instead of the min of the sum subsumes #CSP)
- Objective function not constructible using a binary aggregation operator (e.g. the median of the set of costs)

Tractability

- Can we characterize/unify all tractable classes of VCSP over non-Boolean domains?
- Are there interesting tractable classes apart from submodular functions?
- Are there more efficient algorithms for submodular function minimisation?

New problem transformations

- Global cost functions
- Decomposition into several problems whose sum is equal to the original VCSP
- Transformations which preserve at least one solution (if it exists) but do not necessarily all costs (substituability).
- Applying rules involving ≥2 constraints

Conclusion

- VCSP combines CSP and optimisation in a unified way
- Technology is usable and useful, and still maturing
- Something different: Structure estimation in Gene Networks