Between full and non cooperation in the extraction of a natural resource

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Abstract

The model of resource exploitation developed in Lehvari and Mirman (1982) is considered and extended to account for the possibility of strategic interactions under incomplete information. Specifically, players are assumed to rely on simple non probabilistic beliefs on their opponents' behavior. Two cases are considered. In the first situation players use a simple learning procedure, the second one results in a dynamic game where states corresponding to the players' beliefs must be consistent with observed past plays. Closed form expressions of the optimal consumption plans and state dynamics are derived, and comparisons are made with the full information benchmark cases.

Keywords: Dynamic game; resource exploitation; non probabilistic beliefs.
JEL Classification: C73.

1 Introduction

The interplay between external and strategic effects is a main feature of many economic issues. An illustrative example is the model of great fish war, as developed by Levahri and Mirman in a seminal paper (1982). This study highlights the main issue created by this interplay. Specifically, non cooperative behaviors in problem of resource management leads to an overconsumption of the resource compared to a socially efficient (joint) management. Different studies have focused on this issue of resource management (REFERENCES). However, in most studies decision takers are assumed to have a

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perfect knowledge of the different characteristics of the setting, that is, the information is assumed to be complete.
We revisit the model of Levahri and Mirman when the information is incomplete, and decision takers do not have information on their opponents' preferences. A learning procedure is introduced where decision takers are assumed to form simple (non probabilistic) beliefs on their opponents' behaviors. Beliefs have to be reasonable, that is, they need to be consistent with observed policies. The procedure and feedback consistent solutions are defined; then convergence is studied, and the resulting policy is compared to the benchmark of complete information as developed by Levahri and Mirman both from an economic and environmental points of view. Closed form expressions of the solution are obtained in the linear growth model, and simulations are developed in the general growth model. Specifically, it is proved that there are situations where consistent solutions yield better outcomes in terms of environmental sustainability compared to the case of complete information Cournot Nash equilibrium.
There are different reasons for which we depart from the traditional bayesian framework. The most important one is that we are interested in deriving some insights regarding practical policy implications. As such we wanted to develop a simple learning procedure that might be used as a heuristic. The procedures developed in the present study might be quite useful because, in each case, there is an explicit link between the initial conditions and the resulting long run policies. As incomplete information is a usual characteristic of resource extraction, the knowledge of an explicit link between initial consumption and resulting long run dynamics might prove to be useful from a practical point of view. We elaborate further on this point at the end of section 4.

2 The model

The model developed in Lehvari and Mirman (1982) is briefly introduced. Let x_t be the stock of natural resource at time t. If $c_{i,t}$ denotes agent $i = 1, 2$ exxtraction at time t, then the evolution rule is given by

$$x_{t+1} = [x_t - c_{1,t} - c_{2,t}]^{\alpha}, \quad x(0) = x_0, \quad \alpha \in (0, 1]. \tag{1}$$

We will assume that c_i is the present consumption of country i and that the utility function of country i is logarithmic, i.e, $u_i(c_i) = log(c_i)$. Let $0 < \beta < 1$ be the discount factor for both agents. When dealing with the learning procedure we will consider the general biological rule introduced above. However, in the second situation we need to restrict to the linear case for analytical tractability. Let us first sum up some of the findings developed in Levahri and Mirman (1982), as these results will constitute our benchmark case.

3 Different types of behaviours

3.1 The cooperative case

In the case of a linear evolution rule the problem is:

$$\max_{\{c_{1,t};c_{2,t}\}} \sum_{t=0}^{\infty} \beta^t \left[log(c_{1,t}) + log(c_{2,t})\right]$$

subject to

$$x_{t+1} = x_t - c_{1,t} - c_{2,t}, \quad x_0 \text{ given.}$$

Considering $c_{1,t} = a_i x_t$, $i = 1, 2$, we obtain

$$x_{t+1} = (1 - a_1 - a_2)x_t, \quad x_t = (1 - a_1 - a_2)^t x_0.$$

The problem becomes:

$$\max_{\{a_1,a_2\}} \sum_{t=0}^{\infty} \beta^t \left[log(a_1) + log(a_2) + 2log(1 - a_1 - a_2)\right]$$

which implies:

$$a_1 = a_2 = (1 - \beta)/2, \quad x_t^c = \beta^t x_0, \quad c_{i,t}^c = \frac{1-\beta}{2}\beta^t x_0.$$

3.2 The learning procedure: definition and convergence results

We describe a situation of incomplete information and players are assumed to be adaptive, i.e. they simply adjust their behavior by trial and error. Player i has beliefs about the influence of its own consumption on the consumption of its opponent. We will assume a very simple relationship that will be described below.

At the beginning players have initial consumptions equal to $\bar{c^1}, \bar{c^2}$, which are publicly observed. We assume that there is only a first order effect and we assume that it is linear. Thus, the belief of player i on player j is of the following form:

$$c^j = \bar{c^j} + a^{ij}(c^i - \bar{c^i}). \tag{2}$$

a^{ij} is the idea of player i regarding the nature of the strategic interaction with its opponent j. Basically player i assumes that a variation of its own consumption has a first order linear effect on j's consumption. Even this form of beliefs is very simple, the players' behavior may be very diverse (ranging from purely non cooperative if $a = 0$ to purely cooperative if $a = 1$). Once the one shot game will be defined we will present the updating procedure which introduces a dynamic perspective and makes the connection from one period of consumption to the next one.
Taking beliefs defined by (2) into account player i determines its optimal consumption c_i as the solution of

$$\max_{c^i}(c^i, (\bar{c^j} + a^{ij}(c^i - \bar{c^i}))_{j \neq i}), \tag{3}$$

where

$$UTILITYFUNCTIONTOBEDEFINED.$$

Assuming that a suitable second order condition holds, the (necessary and sufficient) first order condition is

$$\frac{\partial \Pi_i}{\partial q_i} + \sum_{l \neq i} \frac{\partial \Pi_i}{\partial q_l} \frac{\partial q_l}{\partial q_i} = 0, \tag{4}$$

in the present case this is equal to

$$\begin{aligned} 0 = P(\overline{Q}_{-i} + \sum_{l \neq i} \alpha_{il}(q_i - \bar{q}_i) + q_i) + \\ + (\sum_{l \neq i} \alpha_{il} + 1) P'(\overline{Q}_{-i} + \sum_{l \neq i} \alpha_{il}(q_i - \bar{q}_i) + q_i) q_i - c_i'(q_i) - t_{i,M} e_i'(q_i). \end{aligned}$$

Given our assumptions the solution to this problem exists and is unique: it results in a function

$$c^i = r_i(\bar{c^i}, \bar{c^j}; (a^{il})_{l \neq i}). \tag{5}$$

The "one-shot game" has been described for each round k of consumption. Let us turn to the definition of the adjustment process. This process follows strictly the one developed in Jean-Marie and Tidball (2006).

At the end of period t, player i observes that the consumption of firm j is c_t^j. It concludes that its belief should have been $(a^{ij})'$ with

$$c_t^j = \bar{c^j} + (a^{ij})'(c_t^i - \bar{c^i}),$$

or

$$(a^{ij})' = \frac{c_t^j - \bar{c^j}}{c_t^i - \bar{c^i}}. \tag{6}$$

Each player needs to update its belief at the beginning of period $t+1$ to account for the difference between its beliefs at round t and the actual behavior of its opponent. It is assumed that each firm can estimate the consumption of its opponent at the end of period t. Therefore, it can estimate the beliefs that would have been actually optimal at round t (as obtained in equation (6)). It would adjust its beliefs according to this information. A weighted average of previous and optimal beliefs (using updating probability $0 < p_i < 1$) is used to account for possible uncertainties (like imperfect observation of outputs). Formally, the updating rule is:

$$a_{t+1}^{ij} = (1-p_i)a_t^{ij} + p_i \frac{c_t^j - \bar{c^j}}{c_t^i - \bar{c^i}}, \tag{7}$$

or

$$a_{t+1}^{ij} = (1-p_i)a_t^{ij} + p_i \frac{r_j(\bar{c^j}, \bar{c^i}; (a_t^{jl})_{l\neq j}) - \bar{c^j}}{r_i(\bar{c^i}, \bar{c^j}; (a_t^{il})_{l\neq i}) - \bar{c^i}}. \tag{8}$$

Remark 3.1. *(7) is defined only when r_i and r_{-i} have a value, and if $r_i(\bar{c^i}, \bar{c^j}; (a_t^{il})_{l\neq i}) \neq \bar{c^i}$.*

It is important to keep in mind that the procedure is iterative. In the iterative framework all players apply the same rule simultaneously. A simultaneous move game takes place every period.
We will focus on the steady-state of system (7). If we assume that anticipation (7) converges, that is: $a_t^{ij} \to a^{ij*}$ when $t \to \infty$, then, provided the functions $r_i(\bar{c^i}, \bar{c^j}; (a^{il})_{l\neq i})$ are continuous with respect to $(a^{il})_{l\neq i}$, we obtain:

$$a^{ij*} = (1-p_i)a^{ij*} + p_i \frac{r_j(\bar{c^j}, \bar{c^i}; (a^{jl*})_{l\neq j}) - \bar{c^j}}{r_i(\bar{c^i}, \bar{c^j}; (a^{il*})_{l\neq i}) - \bar{c^i}}.$$

Since $p_i \neq 0$, we obtain the fixed point equations:

$$a^{ij*} = \frac{r_j(\bar{c^j}, \bar{c^i}; (a^{jl*})_{l\neq j}) - \bar{c^j}}{r_i(\bar{c^i}, \bar{c^j}; (a^{il*})_{l\neq i}) - \bar{c^i}}. \tag{9}$$

If r_i is assumed to be continuous, $c^i = r_i(\bar{c^i}, \bar{c^j}; (a^{il^*})_{l \neq i})$ is the stationary consumption of player$i$.
Since the utility function of each player is twice continuously differentiable, and that the second order condition relative to (3) is fulfilled, the continuity of r_i follows from the implicit function theorem. Thus, equation (9) holds.
REGARDING THE ANALYSIS: try to write the utility function and what you've sent me in the pdf file. Here is what I found regarding the state dynamics:

Proposition 3.1. *Consider the consumption plan $\{c_t^c\}_t$ when players learn according to the process defined previously, that is, for any period t:*

$$c_t^c = \frac{x_t - (1 - a_t)\bar{c}}{(1 + a_t)(1 + \alpha\beta)}. \tag{10}$$

Now, suppose that the initial belief is such that $a < 1$; moreover, suppose that the initial conditions of the problem are such that

$$(1 + \alpha\beta)(1 + a_0) - 2 > 0. \tag{11}$$

Then, for any period t, the stock of the natural resource as implied by the above consumption plan x_t^c is such that $x_t^c > 0$.

Proof. We sketch the proof in the linear case for sake of simplicity, the same reasoning applies in the general case.
By contradiction, denote by $\bar{t} + 1$ the initial time at which the stock is exhausted, that is, we have $x_{\bar{t}+1}^c = 0$. Then, from the state dynamics and from the expression of the consumption plan as given in (10), this implies that we have

$$(1 + \beta)(1 + a_t)x_{\bar{t}} - 2x_{\bar{t}} + 2(1 - a_t)\bar{c} = 0,$$

or

$$[(1 + \beta)(1 + a_t) - 2]x_{\bar{t}} = -2(1 - a_t)\bar{c}. \tag{12}$$

Now, since $a_0 < 1$, it can be checked that the sequence $\{a_t\}_t$ is increasing. Moreover, combined with assumption (11) it implies that for any period t,

$$(1 + \beta)(1 + a_t) - 2 > 0.$$

Thus $(1 + \beta)(1 + a_{\bar{t}}) - 2 > 0$, which yields (since $x_{\bar{t}} > 0$ by definition of $\bar{t} + 1$):

$$[(1 + \beta)(1 + a_t) - 2]x_{\bar{t}} > 0.$$

But this inequality contradicts condition (12), as $\{a_t\}_t$ is an increasing sequence that converges to 1 (thus $(1 - a_t) > 0$) and $\bar{c} > 0$. By contradiction we conclude the proof. □

3.3 The case of state-based beliefs: Nash equilibria

For agents $i = 1, 2$, the problem now is:

$$\max_{\{c_{i,t}\}} \sum_{t=0}^{\infty} \beta^t log(c_{i,t})$$

subject to

$$x_{t+1} = x_t - c_{1,t} - c_{2,t}, \quad x_0 \text{ given.}$$

As below, considering $c_{1,t} = a_i x_t$, $i = 1, 2$ the problem becomes:

$$\max_{\{a_i\}} \sum_{t=0}^{\infty} \beta^t \big[log(a_i) + log(1 - a_1 - a_2)\big]$$

Which implies:

$$a_1 = a_2 = \frac{1-\beta}{2-\beta}, \quad x_t^N = \big(\frac{\beta}{2-\beta}\big)^t x_0, \quad c_{i,t}^N = \frac{1-\beta}{2-\beta}\big(\frac{\beta}{2-\beta}\big)^t x_0.$$

Remark 3.2. *Note that*

$$x_t^c = \beta^t x_0 > \big(\frac{\beta}{2-\beta}\big)^t x_0 = x_t^N, \quad \forall t. \tag{13}$$

3.4 The case of state and strategy based beliefs

3.4.1 Feedback consistent solutions

Following with the non-cooperative case, from Jean-Marie and Tidball (2005) [2] it is known that finding the feedback consistent solution to this problem is equivalent to finding the solution to:

$$\max_{\{c_t^i\}} \sum_{t=0}^{\infty} \beta^t log c_t^i,$$

subject to the following constraints:

$$x_{t+1} = x_t - a_j x_t - b_j y_t - c_t^i$$

and

$$y_{t+1} = c_t^i$$

where initial stock x_0 and initial catch c_0^i are common knowledge at the beginning of the problem. For instance, for $i = 1$, the dynamics of the equivalent problem can be rewritten as follows:

$$x_{t+1} - x_t = -a_2 x_t - b_2 y_t - c_t^1 \tag{14}$$

and

$$y_{t+1} - y_t = c_t^1 - y_t. \tag{15}$$

Now the principle of the maximum can be used, where the associated hamiltonian is (using (14) and (15)):

$$H = \beta^t log c_t^1 + \pi_t[-a_2 x_t - b_2 y_t - c_t^1] + \lambda_t[c_t^1 - y_t].$$

For any period t the first order conditions are:

$$\frac{\partial H}{\partial c_t^1} = \frac{\beta^t}{c_t^1} - \pi_t + \lambda_t = 0,$$

$$\pi_t - \pi_{t-1} = -\frac{\partial H}{\partial x_t} = a_2 \pi_t,$$

$$\lambda_t - \lambda_{t-1} = -\frac{\partial H}{\partial y_t} = b_2 \pi_t + \lambda_t. \tag{16}$$

From conditions (16) we obtain:

$$c_t^1 = \frac{\beta^t}{\pi_t - \lambda_t}; \quad \pi_t = \frac{\pi_0}{(1 - a_2)^t}; \quad \lambda_t = -b_2 \pi_{t+1}.$$

Using these expressions we can rewrite $\pi_t - \lambda_t$ as

$$\pi_t - \lambda_t = \frac{\pi_0}{(1 - a_2)^t} + \frac{b_2 \pi_0}{(1 - a_2)^{t+1}} = \frac{\pi_0}{(1 - a_2)^t} \frac{1 + b_2 - a_2}{1 - a_2}.$$

Thus, we obtain a final expression of the optimal consumption plan:

$$c_t^1 = [\beta(1 - a_2)]^t \frac{1 - a_2}{\pi_0(1 + b_2 - a_2)}. \tag{17}$$

Using this expression for $t = 0$, we can deduce an expression of the initial parameter π_0 as:

$$\pi_0 = \frac{1 - a_2}{c_0^1(1 + b_2 - a_2)}. \tag{18}$$

Now we can rewrite the expression of the state dynamics. Let us denote

$$A = \frac{b_2 + \beta(1 - a_2)}{\beta(1 + b_2 - a_2)\pi_0}; \tag{19}$$

now we can obtain that

$$b_2 c_{t-1}^1 + c_t^1 =$$

$$b_2(\beta(1 - a_2))^{t-1} \frac{1 - a_2}{\pi_0(1 + b_2 - a_2)} + (\beta(1 - a_2))^t \frac{1 - a_2}{\pi_0(1 + b_2 - a_2)} =$$

$$(\beta(1 - a_2))^t A.$$

Thus, the state dynamics can be rewritten as:

$$x_{t+1} = (1 - a_2)x_t - b_2 c_{t-1}^1 - c_t^1 = (1 - a_2)x_t - [\beta(1 - a_2)]^t A;$$

or

$$x_t = (1 - a_2)^t x_0 - A(1 - a_2)^{t-1} \sum_{i=0}^{t-1} \beta^i = (1 - a_2)^t x_0 - A(1 - a_2)^{t-1} \frac{1 - \beta^t}{1 - \beta}. \tag{20}$$

Now we can solve for the feedback consistent solutions. We consider symmetric solutions because we deal with a symmetric problem. Basically this amounts to assuming $c_0^1 = c_0^2 = c_0$; since the problem is symmetric the solution is symmetric too. This implies $c_t^1 = c_t^2 = c_t$ for any period t, hence $a_1 = a_2 = a$, and $b_1 = b_2 = b$.
Now we come back to the notion of feedback consistency. In the present case the state vector of the dynamics defined by equations (14) and (15). Specifically, at any period t the state of the process is given by (x_t, c_{t-1}). Thus, feedback consistency implies that the functional relationship that links (x_t, c_{t-1}) to the belief of each country regarding the opponent's optimal catch at period t corresponds to the actual optimal policy of this country. Formally, we must have:

$$a x_t + b c_{t-1} = c_t. \tag{21}$$

Thus, to obtain the feedback consistent solutions, one can rewrite condition (21) using the expressions of c_t and x_t obtained in (17) and (20), and then obtain the corresponding values of coefficients a and b.
Rewriting condition (21), we have:

$$c_t = [\beta(1 - a)]^t \frac{1 - a}{\pi_0(1 + b - a)} =$$

$$a[(1-a)^t x_0 - A(1-a)^{t-1}\frac{1-\beta^t}{1-\beta}] + b[\beta(1-a)]^{t-1}\frac{1-a}{\pi_0(1+b-a)}.$$

Using the expression of A given by (19) and rewriting, we obtain:

$$c_t = [\beta(1-a)]^t \frac{1-a}{\pi_0(1+b-a)} = (1-a)^t[ax_0 - \frac{a}{(1-a)(1-\beta)}\frac{b+\beta(1-a)}{\beta\pi_0(1+b-a)}]$$

$$+[\beta(1-a)]^t\{\frac{a[b+\beta(1-a)]}{\beta\pi_0(1-a)(1-\beta)(1+b-a)} + \frac{b(1-a)}{\beta\pi_0(1-a)(1+b-a)}\}.$$

Now feedback consistency is equivalent to looking for values of coefficients a and b such that:

$$ax_0 - \frac{a}{(1-a)(1-\beta)}\frac{b+\beta(1-a)}{\beta\pi_0(1+b-a)} = 0 \tag{22}$$

and

$$\frac{a[b+\beta(1-a)]}{\beta\pi_0(1-a)(1-\beta)(1+b-a)} + \frac{b(1-a)}{\beta\pi_0(1-a)(1+b-a)} = \frac{1-a}{\pi_0(1+b-a)}. \tag{23}$$

It is easily checked that conditions (22) and (23) yield the following solutions:

$$a = 0, b = \beta; \quad a = \frac{x_0\beta - x_0 + 2c_0}{x_0\beta}, b = \frac{(2c_0 - x_0)(c_0\beta - 2c_0 - x_0\beta + x_0)}{\beta c_0 x_0}.$$

NOTICE: I think there is a mistake in the above expression of b, the first term in the second expression must be $(x_0 - 2c_0)$ instead of $(2c_0 - x_0)$.

Remark 3.3. • *Note that the case $a < 0$ and $b < 0$ is not possible (because in thi case $c_{it} < 0$), then*

$$c_0 \geq \frac{1-\beta}{2}x_0.$$

NOTICE: AGAIN, DUE TO THE PREVIOUS NOTICE THERE IS ONE CASE THAT IS NOT CONSIDERED, $a < 0$ and $b > 0$ is a feasible case if my previous remark proves to be right. Indeed, $a < 0$ is equivalent to $c_0 < \frac{1-\beta}{2}x_0$, while $b > 0$ is equivalent to either $x_0 > 2c_0$ AND $c_0 < \frac{1-\beta}{2-\beta}x_0$, OR $x_0 < 2c_0$ AND $c_0 > \frac{1-\beta}{2-\beta}x_0$. The second case is *unfeasible, but the first case is feasible. If this is correct, then the case where* $c_0 < \frac{1-\beta}{2-\beta}x_0$ *is feasible and can be analysed. If, in this case, the optimal solution proves to correspond to*

$$a = \frac{x_0\beta - x_0 + 2c_0}{x_0\beta}, b = \frac{(2c_0 - x_0)(c_0\beta - 2c_0 - x_0\beta + x_0)}{\beta c_0 x_0},$$

then the corresponding consumption plan is

$$c_t = [\beta(1-a)]^t c_0 = [1 - 2\frac{c_0}{x_0}]^t c_0$$

and the state dynamics is

$$x_t = [\frac{x_0 - 2c_0}{\beta x_0}]^t \{1 - \beta x_0 c_0 (1 - \beta^t)\} x_0.$$

In that case, provided that $1 - \beta x_0 c_0 > 0$ *it can be checked that* $x_t \to_{t\to\infty} \infty$.

- *Note that* $a = 0$, $b = \beta$ *yields, for* $t \geq 1$*:*

$$c_t = \beta c_{t-1}, \quad x_t = x_0 - 2\frac{1-\beta^t}{1-\beta}c_0.$$

In order to have $x_t > 0$ *for all* t *we must ask for*

$$c_0 \leq \frac{1-\beta}{2}x_0.$$

Proposition 3.2.

- *If* $c_0 = \frac{1-\beta}{2}x_0$ *the feedback consistent solution is given by*

$$c_t^{fc} = \beta c_{t-1}^{fc}, \quad x_t^{fc} = \beta^t x_0.$$

- *Let* $\bar{t} = \dfrac{log[1 - (1-\beta)\frac{x_0}{2c_0}]}{log\beta}$. *If* $c_0 > \frac{1-\beta}{2}$ *the feedback consistent solution is given by*

$$\begin{cases} c_t^{fc} = \beta c_{t-1}^{fc}, & x_t^{fc} = x_0 - 2\frac{1-\beta^t}{1-\beta}c_0, \quad t \leq \bar{t} \\ c_t^{fc} = 0 & x_t^{fc} = 0 \qquad \forall t > \bar{t}. \end{cases}$$

Proof. By (17) and (18) we have that

$$c_t^1 = [\beta(1-a_2)]^t c_0.$$

Moreover, $c_0 \geq \frac{1-\beta}{2}x_0$ implies $a \geq 0$, as utility is increasing in c_t^1 the optimal solution is given by $a = 0$, $b = \beta$. The formula for x_t follows directly from the dynamics. □

4 Comparison

Proposition 4.1. *Let $c_0 = \theta x_0$, with $\theta \geq \frac{1-\beta}{2}$, then $\forall t$*

$$x_t^c = \beta^t x_0 \geq x_0 - 2\frac{1-\beta^t}{1-\beta}c_0 = x_t^{fc}$$

Proposition 4.2. *Let $c_0 = \theta x_0$ with $\theta \geq \frac{1-\beta}{2}$, then $\forall t$*

$$x_t^{fc} = \beta^t x_0 \geq x_0 - 2\frac{1-\beta^t}{1-\beta}c_0 \geq \big(\frac{\beta}{2-\beta}\big)^t x_0 = x_t^N \qquad \Longleftrightarrow \ \theta = \frac{1-\beta}{2}$$

Proposition 4.3.

- *Let $c_0 = \frac{1-\beta}{2}x_0$ then $x_t^{fc} = x_t^c$. If the initial comsunption corresponds to that of the cooperative case then the feedback consistent solution coincices with the cooperatif full information solution.*
- *Let $c_0 > \frac{1-\beta}{2}x_0$ then there exists $\tilde{t} < \bar{t}$ such that:*

$$\begin{cases} x_t^N < x_t^{fc} < x_t^c, & t \leq \tilde{t} \\ 0 < x_t^{fc} < x_t^N < x_t^c, & \tilde{t} < t \leq \bar{t} \\ 0 = x_t^{fc} < x_t^N < x_t^c & t > \bar{t}. \end{cases}$$

5 Concluding remarks

We study a problem of resource management under incomplete information when decision takers interact strategically and where each agent's consumption decision has an influence on the evolution of the size of the resource. A learning procedure is developed where agents form simple non probabilistic beliefs, which are updated according to available observations. The solution studied is such that beliefs must be consistent with observed behaviors. Closed form expressions of the optimal policies are obtained and compared to the benchmark studied by Levhari and Mirman. It is proved that there are cases where the consistent solution yields better outcomes regarding the resource management in the long run. This yields support to the procedure introduced in the present study. Due to its simplicity one can think of a potential practical application. From a practical point of view the present procedures and game theoretical situations might be quite useful because, in each case there is an explicit link between the initial conditions and the resulting long run policies. In other words, the dependence of convergence on the initial conditions explains which of the equilibria will prevail given the initial conditions. As incomplete information is a usual characteristic of resource extraction, the knowledge of an explicit link between initial consumption and resulting long run dynamics might prove to be useful from a practical point of view. Regarding potential applications, an important point is the speed of convergence of the procedure. This is an important issue since the growth rule might evolve as time goes by in practice. This deserves further research.

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