Ecosystem services in a general equilibrium setting: The case of insect pollination

Abstract

Insect pollination is widely used for agricultural production and contributes significantly to the global value of crops. In this study, the impact of insect pollinators on the social welfare is assessed within a general equilibrium. What would be the general consequences of a production loss due to pollinators decline? How are changes in profits distributed between producers of pollinated goods and other producers? These questions are studied within two alternative distribution of property rights over the firms: the case when agents possess and equal share of the firms (egalitarian ownership structure) and the case where each agent possesses one firm (polarized ownership structure). For each case, we consider the case when agent and firms are homogeneous, and the case when firms are heterogeneous. Under the egalitarian ownership structure, a pollinator decline will result in the decrease of the utility of both agents. When the distribution of the property right is polarized, the utility of the owner of the firm that produces the good which is not pollinator-dependent, will increase. Under specified condition, the social welfare might increase, especially if the production function of the firm of the non dependant sector is more efficient than the other.

1. Introduction

Since the *Millenium Ecosystem Assessment*, ecosystems service has become an important concept for linking the functioning of ecosystems to human welfare (MEA, 2003). Many difficulties remain nevertheless poorly solved since the multiple economic impacts of these services remain to be more precisely and globally understood (Dasgupta, 2000; Daily et al, 2000; MEA, 2005; Le Roux et al., 2008). Since the provocative paper of Costanza *et al.* (1997), the importance of ecosystem services had been highlighted, and, more recently, Richmond *et al.* (2007), shown that ecosystems contributed significantly to the world gross product. Fisher *et al.* (2009) identified more than 1000 studies that valued some ecosystem services since 1983, but very few of them allow to think further on the effective dependence (Daily et al., 1997) of economic activities upon these services.

To fix ideas, it is useful to focus on one quite important and rather well documented service: the case of pollination service. Insect pollination is widely used in agriculture since 84% of the crop species grown in Europe and 70% of those that are used directly to feed mankind need insect pollinators (Williams, 1994 ; Klein et *al.*, 2007). This pollination service contributes significantly to the total economic value of crop production and its share was respectively estimated at \$25 billion by Costanza et *al.* (1997), and at €250 billion by Pimentel et *al.* (1997), both converted in current US\$. A recent analysis (Gallai et *al.*, 2009) led to some €150 billion for the year 2005 (about US\$200 billion in current US\$).

A more appropriate economic valuation of insect pollination service is to assess the social welfare loss resulting from insect pollinator decline. A few studies estimate the welfare loss related to a pollinator decline, based upon partial equilibrium models focused on the reaction of consumers to the new production conditions (Southwick and Southwick, 1992; Gallai *et al.* 2009). This single-market simplification can be justified as an effort to get a quantitative measure of the direct welfare impact of such an ecological shock. But a partial equilibrium model ignores important effects regarding the indirect consequences of the shock on other markets that, in turn, will causes feedback effects on the economy.

Since the industrial revolution, many changes in the economy and the environment consisted in substituting ecosystem services by manmade productions. This evolution resulted in ambiguous effects since, on the one hand, this led to many aspects of the socioeconomic development and, on the other hand, to lesser attention to the situation of ecosystems that resulted in many harmful degradations. This article proposes to address this concern within a general equilibrium framework, that describes an economy where several markets make consistent, via an endogenous system of prices, multiple production and consumption plans. The vanishing of the pollination service due to the pollinators decline results in changes in the production technology which consequences are analyzed in terms of welfare variations.

There are few studies in the literature that address the issue of ecosystem services degradation into a general equilibrium framework. Some well known papers analyzes the effects of environmental policies, namely the effect of environmental taxes to highlight the question of double dividend (Bovenberg and Goulder, 1996), but very little have been devoted to the impact of ecosystem degradation (Tschirhart, 2000; Finnoff and Tschirhart, 2003; Eichner and Pethig, 2005; 2009), and, as far as we know, none is related to the welfare consequences of the vanishing of an ecosystem services.

What would be the consequences of a production loss due to an insect pollinator decline considering the adaptation of the overall economy and more particularly considering the possible spillovers on others markets? More specifically, how are the consequences on wages and the profits distributed between the producers of pollinated goods and other producers? These questions will be studied within two alternative scenarios for the distribution of property rights over the firms: the case when agents possess and equal share of the productive sector (the egalitarian ownership structure) and the case when each agent possesses one firm (the “polarized” ownership structure).

The article starts with a description of the general equilibrium dimension. It is done first for symmetric agents under, alternatively, the egalitarian and the polarized ownership structures. As it turns out, the ownership structure is crucial to appraise the effect of the ecological shock. The main result is that, under the egalitarian distribution of property rights, all the agents suffer from the shock, hence there is a reduction of welfare; by contrast, under the polarized structure, the agent who possesses the pollinated activity experiences an utility reduction, whereas the other agent can experience a higher utility. This result holds when: 1) either the elasticity of substitution between the two consumption goods is sufficiently high, 2) or when the non pollinated sector is relatively more productive than the pollinated sector. In either case, welfare can increase if the second agent is granted a relatively more important weight in the social welfare criterion. The last section discusses the results and suggests some perspectives.

2. The model

The economy has two firms *f* and *g,* using one input, to produce two goods $h = 1, 2$, enjoyed by two consumers c $= 1, 2$. The production of good 1 depends on insect pollination whereas the production of good 2 does not.

2.1. The production side

There are two technologies, called respectively *f* for firm *f* and *g* for firm *g*. The amount of input used by firm *f* (respectively by firm *g*) is z_f (resp. z_g). The total use of input is therefore $Z= z_f + z_g$.

Pollination is necessary for the production and reproduction of crops. A biologic ratio, called the dependence ratio or simply *D*, was created from a review by Klein et al. (2007, Appendix A). This ratio indicates the part of crop production dependent on insect pollination and is comprised between 0 and 1: it means that a total insect pollinator decline would reduce crop production by a factor *D*. Accordingly, the production function of good 1 that is dependent on insect pollinator is $f(z_f, D)$. Good 2 does not depend on insect pollinators and its production function is $g(z_g)$. We assume that *f(.,.)* and *g(.)* are concave functions, featuring decreasing returns to scale ($af(z_f,D)>f(az_f,D)$, for all $a>1$).

The production function has a Cobb-Douglas form for both firms. The production function of firm 1 is:

$$f(z_f, D) = (1-D)z_f^\beta \quad [1]$$

And the production function of firm 2 is:

$$g(z_g) = z_g^\beta \quad [2]$$

where β is a parameter chosen in the interval]0, 1[, which implies decreasing returns to scale.

The profit functions of firms, for given prices of output (p_1 and p_2) and input (a), are denoted Π^n. Those functions read as:

$$\Pi^1 = p_1 f(z_f, D) - az_f \quad [3]$$

$$\Pi^2 = p_2 g(z_g) - az_g \quad [4]$$

Firms use input in order to maximize profits. One can deduce the firms' demands of inputs as functions of the prevailing prices. Profits maximization result in demand functions for the first input, $z_f(p_1, D, a)$ and $z_g(p_2, a)$ for the second input. The total demand of input is simply $Z = z_f(p_1, D, a) + z_g(p_2, a)$. Also, plugging those decisions into the production functions, the supply for each consumption good, given the prevailing prices on the markets, will be $X_1(p_1, D, a)$ and $X_2(p_2, a)$.

2.2. The consumption side

Consumer 1 (respectively 2) is endowed with the first (resp. the second) production factor, $\overline{Z_1}$ (resp. $\overline{Z_2}$), which he supplies inelastically to firm *f* (resp. to firm *g*) and for the counterpart of which he receives wages. Hence the supply of inputs are constant, $z_f = \overline{Z_1}$ and $z_g = \overline{Z_2}$. Consumers are also endowed with a share of the firms. More precisely, consumer *c* works for one firm and we assume that he provides all the input. Then he receives the wage aZ_c. Furthermore the consumer owns a share (or the total) of firm *n*. Consequently he receives dividends that amounts to a share of the profits. Two ownership structures will be considered in turn. Under the egalitarian structure both consumers own 50% of both firms. Thus their revenues are:

$$\begin{aligned} R_1 &= 0.5\left(\Pi^1 + \Pi^2\right) + az_f \\ R_1 &= 0.5\left(p_1 f(z_{f1}, D) + p_2 g(z_g) + az_f - az_g\right) \end{aligned} \quad [5]$$

$$\begin{aligned} R_2 &= 0.5\left(\Pi^1 + \Pi^2\right) + az_g \\ R_2 &= 0.5\left(p_1 f(z_{f1}, D) + p_2 g(z_g) + az_g - az_f\right) \end{aligned} \quad [6]$$

The part of the revenue provided by firms is the same for both consumers. The difference in revenue is due to the possible difference in salaries. This distinction allows to isolate the impact of a pollinator decline on the workers revenue.

Under the polarized structure, Consumer *1* is the owner of firm 1 and Consumer *2* is the owner of firm 2. Formally:

$$R_1 = \Pi^1 + az_f = p_1 f(z_{f1}, D) \quad [7]$$

$$R_2 = \Pi^2 + az_g = p_2 g(z_g) \quad [8]$$

Here the impact of salaries on their revenue are eliminated because consumers are owner and worker of their own firm. Their only revenue comes from the gain of firms. This distinction allows to isolate the impact of a pollinator decline on the owner's revenue.

Whatever the ownership structure, consumer c faces the budget constraint $R_c \geq p_1 x_{c1} + p_2 x_{c2}$. For the time being, let us carry on with the egalitarian case.

The consumers' preferences are represented by a CES utility function:

$$U^c(x_{c1}, x_{c2}) = \frac{v x_{c1}^{\alpha}}{\alpha} + \frac{x_{c2}^{\alpha}}{\alpha} \quad [9]$$

with x_{c1} and $x_{c2} > 0$. The coefficient v is the relative weight of the utility derived from the consumption of the first good. This functional form allows for several degrees of substitutability between goods. When $\alpha = v = 1$, the case of perfect substitutability obtains. Those utility functions are concave and we let $\frac{\partial U^c}{\partial x_{c1}} = U_{c1}$ and $\frac{\partial U^c}{\partial x_{c2}} = U_{c2}$ stand for the marginal utilities of each good.

Consumers use their total revenue to buy goods in order to maximize their utility. Their maximization program ends up in individual demands for each good, denoted $x_{c1}(R_c, p_1, p_2)$ and $x_{c2}(R_c, p_1, p_2)$, configured by prices and income (Appendix B). And the total demand for good h, X_h , is the sum of the individual demands x_{ch} ($X_h = x_{1h} + x_{2h}$), where $x_{ch} \geq 0$.

2.3. The social welfare

The social welfare criterion (SWC) is a functional with consumers' utilities as arguments. An often used SWC is the generalized utilitarian criterion, which in our model is a convex combination of the two utilities:

$$W = \theta U^1(x_{11}, x_{12}) + (1-\theta) U^2(x_{21}, x_{22}) \quad [10]$$

where θ is a parameter chosen in the interval]0, 1[.

Then analyzing the impact of insect pollinator is a comparison between the state of economy after an insect pollinator decline *i.e.* when $D > 0$ and the state of the economy before insect pollinator decline *i.e.* when $D = 0$. And the impact on the social welfare is measured by:

$$\Delta W = W_{D>0} - W_{D=0} \quad [11]$$

3. The egalitarian ownership structure: the impact of pollinators decline on workers' revenues.

3.1. Results

At the initial state, *i.e.* before pollinators decline (D=0), and considering that preference (v) for insect pollinator dependent good (good 1) is equal to 1, the economy is perfectly symmetric since profit of firms *f* and *g* are equal and revenues of consumers 1 and 2 are also identical. Profit of firms at the equilibrium are described by the following expressions (see appendix 1 for details):

$$\Pi^1 = \frac{\bar{Z}}{\left(1+\frac{1}{Y}\right)}\left(\frac{1}{\beta}-1\right)$$
$$\Pi^2 = \frac{\bar{Z}}{(1+Y)}\left(\frac{1}{\beta}-1\right) \quad [12]$$

where Y = $(1-D)^{\frac{\alpha}{(1-\alpha\beta)}} v^{\frac{1}{1-\alpha\beta}}$. Thus when v = 1, Y is equal to 1 and the profit of firms is equal. The equilibrium revenues of consumers are represented by the following expressions:

$$R_1 = \frac{\bar{Z}}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)$$

$$R_2 = \frac{\bar{Z}}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)$$

When v = 1, the revenue of consumers is identical and equal to $R_i = \frac{\bar{Z}}{2\beta}$. Now, if consumers prefer good 1 to good 2, *i.e.* v>1, so Y > 0. Consequently, profit of firm 1 will increase and profit of firm 2 will decrease and revenue of consumer 1 will increase and the one of consumer 2 will decrease.

The impact of insect pollinators on the different functions of the economy depends on the ratio *D*. This ratio appears in all equilibrium function, such as revenues, prices of goods, exchanged quantities and in utilities (see Appendix 1). We analyzed all these functions deriving by D.

Thus considering a pollinator decline, *dD>0*, the production of good 1 will downsize and its price will increase. The consumption of good 2 will increase and consequently its price will increase. Thus firm 1's profit will decrease and firm 2's profit will increase. Inequalities will appear since the revenue of consumer 1 will decrease and the revenue of consumer 2 will increase (Appendix 1). As a consequence the consumer's capacity to consume goods would vary. Indeed consumer *1* could not buy as much as than before shock on production, while consumer *2* does not seems so impacted by pollinator decline. This leads to the following propositions:

Proposition 1: Let $\alpha \in]0, 1[$, $\beta \in]0, 1[$ and $\nu>0$. Then the larger the pollinator decline, the lower the consumption of good 1 by consumer 1 (see proof in Appendix 1).

Proposition 2: Let situations where 1) $\alpha \in]0, 1[$, $\beta \in]0, 1[$ and $\nu \in]0, 1[$, 2) $\alpha \in]0, \alpha^*[$, $\beta \in]0, 1[$ and $\nu \in]1; +\infty[$ and 3) $\alpha \in]0, 1[$, $\beta \in]0, \beta^*[$ and $\nu \in]1; +\infty[$. Then the larger the pollinator decline, the lower the consumption of good 1 by consumer 2. Let $\alpha \in]\alpha^*, 1[$, $\beta \in]\beta^*, 1[$ and $\nu \in]1; +\infty[$. Then the lower the pollinator decline, the larger the consumption of good 1 by consumer 2 (see proof in Appendix 1).

Proposition 3: Let $\alpha \in]0, 1[$, $\beta \in]0, 1[$ and $\nu]0, 1[$. Then the larger the pollinator decline, the lower the consumption of good 2 by consumer 1. Let $\alpha \in]0, 1[$, $\beta \in]0, 1[$ and $\nu \in]1, +\infty[$. Then the lower the pollinator decline, the larger the consumption of good 2 by consumer 1 (see proof in Appendix 1).

Proposition 4: Let $\alpha \in]0, 1[$, $\beta \in]0, 1[$ and $\nu>0$. Then the larger the pollinator decline, the larger the consumption of good 2 by consumer 2 (see proof in Appendix 1).

The impact of insect pollinators on the social welfare is measured by expression [11], as the variation of the sum of consumers' utilities after the pollinator decline. The consumers' utility depends on consumption of good 1 and good 2 (expression [9]). However, at the equilibrium, the production of both goods is influenced by *D* (Appendix 1), which means that both quantities exchanged would vary after a pollinator decline. But in which direction? The answer is given in the following two propositions:

Proposition 5: Let $\alpha \in]0, 1[$ and $\beta \in]0, 1[$. Then the larger the pollinators decline the lower the consumption of good 1 at the equilibrium (see proof in Appendix 1).

Proposition 6: Let $\alpha \in]0, 1[$ and $\beta \in]0, 1[$. Then the larger the pollinator decline the larger the consumption of good 2 at the equilibrium (see proof in Appendix 1).

These two propositions let assumed that the impact of a pollinator decline will be compensated by the existence of a second substitutable market. The impact of an insect pollinators decline on the consumers' utilities is determined by the difference between consumption losses of x_{c1} compare to consumption gain of x_{c2} and it can be measured by $\partial U^c / \partial D$. Thus we assume that:

H1: Utility of consumer 1, U^1, will increase after a pollinator decline when $v\left(\frac{x_{11}^*}{x_{12}^*}\right)^{\alpha-1} > -\frac{\partial x_{11}/\partial D}{\partial x_{12}/\partial D}$.

H2: Utility of consumer 1, U^2, will increase after a pollinator decline when $v\left(\frac{x_{21}^*}{x_{22}^*}\right)^{\alpha-1} > -\frac{\partial x_{21}/\partial D}{\partial x_{22}/\partial D}$.

Considering propositions 1 to 4, the H2 hypothesis is realizable, *i.e.* the utility of consumer 2 could increase after an insect pollinator decline. On the other hand, the H1 hypothesis will never be realized (see Appendix 1). The consequence of this result is summarized in the proposition 7:

<u>Proposition 7</u>: Under the egalitarian ownership structure and under the assumption H2, it exists θ such as a social welfare variation is positive (see proof in Appendix 1).

3.2. Interpretation

In this model we analyzed the impact of an insect pollinator decline in the society. The society is composed of producers and workers. In this specific model, it is assumed that profits of firms are distributed equally, which means that the only differences between agents of the economy comes from the salaries. We thus isolated the impact of pollinators decline on the agents considered as workers.

We found that insect pollinator decline in sector 1 will be compensated by substitutability of good 1 by good 2. In more detailed, we also found that consumer working on the sector depending on insect pollinators 1, *i.e.* consumer 1, will decrease his consumption of good 1 and compensate this loss in consuming good 2. But if the pollinator decline is too important, *i.e. D* tends to 1, the price's increase of goods would be too important compare to his capacity to consume and consequently he would not be able to buy good 2 at least as much as before the pollinator decline. Simultaneously, the increase of the consumer 2's revenue will enables him to compensate his loss in good 1 by buying more good 2. The pollinator loss will also oblige consumer 2 to decrease his consumption of good 1 except in a specific situation. This situation implies that consumers prefer good 1 to good 2 ($v>1$), the need to use inputs for firms is low, β tends to 1, and the need to use goods for consumers is low, α tends to 1. This leads to the conclusion that the insect pollinator decline will create inequalities in the society.

The impact of the ecological shock on the utilities of consumers will be negative except for consumer 2 in the specific case described in the preceding paragraph and explained in the proposition 3. In this situation and considering a low decline of pollinators ($D<D^*$), the utility of consumer 2 will increase. This possibility suggests that a gain in social welfare could appear after an insect pollinator decline. This gain would be possible when the social preferences encourage the non dependent on insect pollinator industry, *i.e.* when θ tends to 0.

4. The polarized ownership structure: the impact of pollinators decline on firm's owners.

4.1. Results

The revenue of consumers are described by the following expressions:

$$R_1 = \frac{\overline{Z}}{\beta\left(1+\frac{1}{Y}\right)}$$

$$R_2 = \frac{\overline{Z}}{\beta(1+Y)}$$

where Y = $(1-D)^{\frac{\alpha}{(1-\alpha\beta)}} v^{\frac{1}{1-\alpha\beta}}$. And are equal to the sales of firms as described by first part of expressions [12]. Thus consumers' revenues depend of the firms' profit. When Y increase, revenue of consumer 1 increase and revenue of consumer 2 decrease.

As in the preceding section, the at the initial state, *i.e.* before pollinators decline (D=0), and considering that preference (v) for insect pollinator dependent good (good 1) is equal to 1, the economy is perfectly symmetric.

As in the preceding model, the ecological shock will leads to an increase of prices of good 1 and 2, as well as exchanged quantities of good 2 and a decrease of exchanged quantities of good 1 (Appendix 2). It will imply that firm 1's profit will decrease and firm 2's profit will increase. However, in this model, consumers are firm owners: consumer 1 is the owner of firm 1 and consumer 2 is owner of firm 2. Thus the revenue of consumer 1 will decrease and revenue of consumer 2 will increase. Nevertheless we noted significant changes compared to the preceding case that are summary in the following propositions:

Proposition 8: Let $\alpha \in]0, 1[$, $\beta]0, 1[$ and $v>0$. In the case of a polarized ownership structure, the larger the pollinator decline, the lower the consumption of good 1 by consumer 2 (Proof: see Appendix 2).

Proposition 9: Let situations where 1) $\alpha \in]0, 1[$, $\beta]0, 1[$ and $v]0, 1[$ and 2) $\alpha \in]0, 1[$, $\beta]0, \beta^*[$ and $v]1; +\infty[$. Then the larger the pollinator decline, the lower the consumption of good 2 by consumer 1. Let $\alpha \in]0, 1[$, $\beta]\beta^*, 1[$ and $v]1; +\infty[$. Then the larger the pollinator decline the larger the consumption of good 2 by consumer 1. Let $\alpha \in]\alpha^*, 1[$, $\beta]\beta^*, 1[$ and $v]1; +\infty[$. Then the lower the pollinator decline, the larger the consumption of good 2 by consumer 1 (Proof: see Appendix 2).

The H1 hypothesis is not realizable, *i.e.* the utility of consumer 1 will decrease after pollinator decline (Appendix 2). On the other the utility of consumer 2 can increase in some cases: 1) Let v<1, the utility of consumer 2 will increase when β tends to 0, 2) let v>1, the utility of consumer 2 will increase when β tends to 0 and 3) let v>1, β tends to 1 and α tends to 1, the utility of consumer 2 will increase the pollinator decline D is low.

Finally the impact of the insect pollinator decline on the social welfare is negative except in the particular case where H2 is realized and combined with a θ tending to 0. Then the proposition 7 would be right.

4.2. Interpretation

In the model presented, we assumed that revenues of consumers are assimilated to profits of firms, where the profit of firm 1 is distributed to consumer 1 and the profit of firm 2 is distributed to consumer 2. Wages are eliminated of the study, such as consumers are assimilated to owners of firms. We thus isolated the impact of pollinators decline on the agents considered as owners of firms.

We found that the mechanism resulting of the pollinators decline is approximately the same than in the preceding section and thus the existence of a substitutable market limit the impact of the pollinators decline. Nevertheless, some adaptations of the agents are different and depend on the characteristics of the firm. Indeed, the consumption of good 2 by consumer 1 will increase when technological capacities of firm are high, i.e. when β tends to 1. The consumption of good 1 by consumer 2 will decrease in all situations while this could have been increase for specific situation in the preceding case. Indeed, in the polarized ownership structure the gain in revenue of consumer 2 due to pollinator decline would be lower than in the preceding case.

Consequently, the utility of consumer 1 will decrease after pollinator decline. The utility of consumer 2 that prefers the good 2 to good 1, $v>1$, will decrease in all situations. In the other hand, the utility of the consumer 2 that prefers the good 1 to good 2 will gain in utility for each situations, except when his need to consume goods is strong, α tends to 0, and the technological capacity of firms is strong, β tends to 1. This gain is due to the strong consumption of good 2 in order to compensate the loss in good 1. Finally the impact of an insect pollinators decline in the social welfare or the firm owner welfare will be negative except for some conditions that are less restrictive than in the egalitarian ownership structure. So the welfare of the firm owner, that depends on insect pollinations would be more vulnerable to pollinator decline than the firm owner that does not.

5. Discussion and perspective

The contribution of insect pollinators on the world agriculture has been evaluated at €153 billion (Gallai *et al.,* 2009). This value can be interpreted as a rough indicator of pollinator importance over the world. The consequence of such a dependence of insect pollination is the vulnerability of the social welfare confronted with a pollinator decline. Indeed, a decline of insect pollinator would impact prices of crop and in a second time the crop production exchanged in the market. This assessment of a pollinator loss impact on a single market has been evaluated at the scale of Australia (Gordon and Davis, 2003), United States (Southwick and Southwick, 1992) and the world level (Gallai *et al.,* 2009). By contrast, the present work qualifies those findings. Using general equilibrium with two markets, it is shown that while the pessimistic conclusion of an adverse consequence on welfare is somehow robust, it is not necessary. When several markets are taken into account in a general equilibrium, the ecological shock has redistributive effects. Often the shock makes every agent lose his purchasing power, hence the social satisfaction falls dawn. But sometimes, in both structure, there can be losers and winners. This is so because the second market, which does not depend on insect pollinator, cushions the economic consequences of a pollinator loss. Consumers compensate the loss of the pollinated good by consuming more of the other good and the welfare loss is softened. If the social "good" attaches more importance to those who do not possess the pollinated activity, and who see an increase in their revenue after the shock, there can even be a welfare improvement. This happens when the second sector is more productive or/and when the elasticity of substitution between goods is high enough.

Nevertheless, the new equilibrium found after the pollinator decline is not a Pareto optimal equilibrium since inequities in the profit of firms and in the consumers revenue. This means that we could improve the condition of one consumer without deteriorate the condition of another consumer.

Furthermore, the possible after gain in welfare is due to hypothesis of the models. First, we assumed substitutability between goods. This assumption is explained by the fact that market of good 1 represent all goods and services that depends on insect pollination and market of good 2 represents all other goods. Then possible weak substitution can exist between goods. However, another possible interpretation would be that market of good 1 would represent the agricultural sector and market of good 2 would represent the others markets. In this case, there is no possible substitution between goods. Then a pollinator decline would automatically negatively impact the social welfare. A way to model the economy within this assumption would be to attribute a Cobb-Douglas utility function to consumers.

In this general equilibrium model, we assume that the insect pollinator decline is exogenous. But the decline of pollinators is due to anthropogenic pressures. More particularly the use of agrochemicals in agriculture is responsible of a large part of their loss (Kuldna *et al.* 2009, National Research Council, 2007). But the use of these means in agriculture is important and it would be useful to study the optimal use of these inputs and the optimal use of the insect pollination input. Two modifications must be undertaken in order to introduce this question in our general equilibrium model. Firstly, the insect pollinators have to be taken as an endogenous variable. And secondly the relation between pollinators' abundance and the quantity of pesticide used in agriculture must be modeled.

Another way to improve the model would be to assume that only the wild pollinators could disappear which would mean that only the costly domestic pollinators would remain. Thus the shock would imply an increase of the production cost. Indeed, the insect pollinators are divided into two major categories: wild ones that are totally offered by Nature and domestic ones that are located by keepers for crop pollination or used for the honey production. However the wild pollinators decline is obvious (Biesmeijer *et al.* 2006), whereas the domestic pollinators decline is not, since it is possible to keep bee colonies. Thus Aizen and Harder (2009) demonstrated that the world stock of honeybees increased since 1961. A pessimistic scenario could be the total disappearance of the abundance and diversity of wild bees, which would lead to a total dependence of the crop pollination by domestic bees. Furthermore the abundance of these insects is not stationary since they suffer from the Varoa destructor and other diseases such as the Colony Collapse Disorder. Partial equilibrium models by Burgett *et al.* (2004) and Rucker *et al.* (2005) demonstrated that the impact of a variation of the bee population's density would imply changes in price of colonies, honey and crops. It would be interesting to use these results in a general equilibrium model introducing the beekeeper as a firm. Thus the gain of beekeepers due to decline of wild pollinators could reduce the welfare loss described in the model used here.

6. Conclusion

Generally, though not systematically, the social welfare decreases after an insect pollinator loss. This decrease goes through the modifications in the production capacity of firms and its extent depends on consumers' preferences on the pollinated good. Consequently, both firms and consumers are diversely affected by the ecological shock. This general message has been obtained and has been given a more precise content by using four slightly different general equilibrium models. Each has two consumers, two goods and two firms producing only one good each. The production of the first good depends on insect pollinators whereas the production of the second good does not. The first model considers identical consumers who have equal shares of the two firms (the egalitarian case). In the second model the ownership structure is polarized: each consumer possesses only one firm.

The main result is that, under the egalitarian distribution of property rights, all the agent suffer from the shock. Nevertheless the agents depending on the pollination industry suffer more than the other. In a specific case, the other agent could gain in welfare. Hence there is a reduction of welfare; by contrast, under the polarized structure, the agent who possesses the pollinated activity experiences an utility reduction, whereas the other agent can experience a higher utility. This result holds when: 1) either the elasticity of substitution between the two consumption goods is sufficiently high, 2) or when the non pollinated sector is relatively more productive than the pollinated sector. In either case, welfare can increase if the second agent is granted a relatively more important weight in the social welfare criterion. One policy implication from this general equilibrium appraisal is that the quest of efficiency is not the only justification for a public regulation in face of a pollinator shock. This reason may even collapse. A second justification, probably more robust, rests on distributive goals.

7. References

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Appendix 1: The model with egalitarian ownership structure

The model

- The supply side

The production function has a Cobb-Douglas form for both firms. Good 1 depends on insect pollination and is produced by firm 1 and good 2 does not depend on pollination and is produced by firm 2. Thus the production function of firm 1 is:

$$f(z_f, D) = (1-D)z_f^{\beta}$$

And the production function of firm 2 is:

$$g(z_g) = z_g^{\beta}$$

with β a parameter chosen in the interval]0, 1[, which implies decreasing returns to scale.

Profit function of firms 1 and 2, Π^1 and Π^2 are:

$$\Pi^1 = p_1 f(z_f, D) - a z_f = p_1(1-D)z_f^{\beta} - a z_f$$
$$\Pi^2 = p_2 f(z_g, D) - a z_g = p_2(1-D)z_g^{\beta} - a z_g$$

The profit of firm 1 is maximum when z_f verify:

$$\frac{\partial \Pi^1}{\partial z_f} = \beta p_1(1-D)z_f^{\beta-1} - a = 0$$
$$\Leftrightarrow z_f = \left(\frac{\beta p_1(1-D)}{a}\right)^{\frac{1}{1-\beta}}$$

The profit of firm 2 is maximum when z_g verify:

$$\frac{\partial \Pi^2}{\partial z_g} = \beta p_2 z_g^{\beta-1} - a = 0$$
$$\Leftrightarrow z_g = \left(\frac{\beta p_2}{a}\right)^{\frac{1}{1-\beta}}$$

Total demand of input, Z, is:

$$Z = z_f + z_g = \left(\frac{\beta}{a}\right)^{\frac{1}{1-\beta}}\left(\left(p_1(1-D)\right)^{\frac{1}{1-\beta}} + p_2^{\frac{1}{1-\beta}}\right)$$

We assume that the total demand of input is totally satisfied. The supply of input is offered by both consumers and is fixed $\overline{Z}$.

The total supply of good 1 is:

$$f(z_f, D) = (1-D)^{\frac{1}{1-\beta}} \left(\frac{\beta p_1}{a} \right)^{\frac{\beta}{1-\beta}}$$

The total supply of good 2 is:

$$g(z_g) = \left(\frac{\beta p_2}{a} \right)^{\frac{\beta}{1-\beta}}$$

- The demand side

Consumer maximizes his utility $U^c(x_{c1}, x_{c2}) = \frac{v x_{c1}^{\alpha}}{\alpha} + \frac{x_{c2}^{\alpha}}{\alpha}$ considering the budget constraint:

$$R_c \geq p_1 x_{c1} + p_2 x_{c2}$$

$$U_1^c = v x_{c1}^{\alpha-1} \qquad [3]$$

$$U_2^c = x_{c2}^{\alpha-1} \qquad [4]$$

At the equilibrium, consumer use all his revenue to consume x_{c1} and x_{c2} so that $R_c = p_1 x_{c1} + p_2 x_{c2}$ and consumption choices are done so that the marginal rate of substitution (MRS) x_{c1} and x_{c2} is equal to the slope of the budget curve which is p_1/p_2. We can define the optimal consumption of x_{c1} and x_{c2}:

$$MRS = \frac{\frac{\partial U}{\partial x_{c1}}}{\frac{\partial U}{\partial x_{c2}}} = \frac{v x_{c1}^{\alpha-1}}{x_{c2}^{\alpha-1}} = \frac{p_1}{p_2} \qquad [5]$$

$$\Leftrightarrow x_{c1} = x_{c2} \left(\frac{p_1}{v p_2} \right)^{\frac{1}{\alpha-1}}$$

$$R_c = p_1 x_{c2} \left(\frac{p_1}{v p_2} \right)^{\frac{1}{\alpha-1}} + p_2 x_{c2}$$

$$\Leftrightarrow x_{c2} = \frac{R_c}{p_2 + p_1 \left(\frac{p_1}{v p_2} \right)^{\frac{1}{\alpha-1}}} \qquad [6]$$

From expressions [3] and [4] it comes:

$$x_{c1} = \frac{R_c}{p_1 + p_2 \left(\frac{v p_2}{p_1} \right)^{\frac{1}{\alpha-1}}} \qquad [7]$$

- Revenues

$$R_1=\frac{1}{2}\left(\Pi^1+\Pi^2\right)+az_f=\frac{1}{2}\left(p_1(1-D)^{\frac{1}{1-\beta}}\left(\frac{\beta p_1}{a}\right)^{\frac{\beta}{1-\beta}}+p_2\left(\frac{\beta p_2}{a}\right)^{\frac{\beta}{1-\beta}}+a\left(\left(\frac{\beta p_1(1-D)}{a}\right)^{\frac{1}{1-\beta}}-\left(\frac{\beta p_2}{a}\right)^{\frac{1}{1-\beta}}\right)\right)$$

$$R_2=\frac{1}{2}\left(\Pi^1+\Pi^2\right)+az_g=\frac{1}{2}\left(p_1(1-D)^{\frac{1}{1-\beta}}\left(\frac{\beta p_1}{a}\right)^{\frac{\beta}{1-\beta}}+p_2\left(\frac{\beta p_2}{a}\right)^{\frac{\beta}{1-\beta}}+a\left(\left(\frac{\beta p_2}{a}\right)^{\frac{1}{1-\beta}}-\left(\frac{\beta p_1(1-D)}{a}\right)^{\frac{1}{1-\beta}}\right)\right)$$

$$R=R_1+R_2=\Pi^1+\Pi^2=p_1(1-D)^{\frac{1}{1-\beta}}\left(\frac{\beta p_1}{a}\right)^{\frac{\beta}{1-\beta}}+p_2\left(\frac{\beta p_2}{a}\right)^{\frac{\beta}{1-\beta}}$$

Equilibrium of the economy: total demand = total supply

- Prices a, p_1 and p_2

$$X_1=x_{11}+x_{21}=f(z_f,p_1,D)$$

$$\frac{p_1 f+p_2 g}{p_1+p_2\left(\frac{p_1}{vp_2}\right)^{\frac{1}{1-\alpha}}}=f$$

$$\Leftrightarrow g=f\left(\frac{p_1}{vp_2}\right)^{\frac{1}{1-\alpha}}$$

However $f(z_f,D)=(1-D)^{\frac{1}{1-\beta}}\left(\frac{\beta p_1}{a}\right)^{\frac{\beta}{1-\beta}}$ and $g(z_g)=\left(\frac{\beta p_2}{a}\right)^{\frac{\beta}{1-\beta}}$

$$p_2=\frac{(1-D)^{\frac{1-\alpha}{1-\alpha\beta}}}{v^{\frac{1-\beta}{1-\alpha\beta}}}p_1$$

By Walras' law the second equilibrium ($X_2=g$) is automatically satisfied. We assume that the price of input, a, is normalized to 1 (a=1). Using expression of the total input exchanged in the economy $Z=\overline{Z}$ we found p_1 and p_2:

$$\overline{Z} = \beta^{\frac{1}{1-\beta}} \left(\left(p_1(1-D)\right)^{\frac{1}{1-\beta}} + p_2^{\frac{1}{1-\beta}} \right)$$

$$\Leftrightarrow \beta^{\frac{1}{1-\beta}} \left(\left(p_1(1-D)\right)^{\frac{1}{1-\beta}} + p_1 \frac{(1-D)^{\frac{1-\alpha}{(1-\alpha\beta)(1-\beta)}}}{v^{\frac{1}{1-\alpha\beta}}} \right)$$

$$p_1 = \frac{\overline{Z}^{1-\beta}}{\beta(1-D)\left(1 + \frac{1}{\left((1-D)^{\alpha} v\right)^{\frac{1}{1-\alpha\beta}}}\right)^{1-\beta}}$$

$$\text{and } p_2 = \frac{\overline{Z}^{1-\beta}}{\beta\left((1-D)^{\alpha} v\right)^{\frac{1-\beta}{1-\alpha\beta}}\left(1 + \frac{1}{\left((1-D)^{\alpha} v\right)^{\frac{1}{1-\alpha\beta}}}\right)^{1-\beta}}$$

In order to simplify the writing we will set: $\left((1-D)^{\alpha} v\right)^{\frac{1}{1-\alpha\beta}} = Y$, where *Y(D)* is positive and decreasing (*dY/dD*<0).

- <u>Revenues</u>

Revenue of consumer 1:

$$R_1 = \frac{\overline{Z}}{2}\left(\frac{1}{\beta} + \frac{Y-1}{1+Y}\right)$$

Revenue of consumer 2:

$$R_2 = \frac{\overline{Z}}{2}\left(\frac{1}{\beta} + \frac{1-Y}{1+Y}\right)$$

Total revenues

$$R = \frac{\overline{Z}}{\beta}$$

- <u>Quantities exchanged of input, good 1 and good 2</u>

Quantities exchanged of input:

$$z_f = \frac{\overline{Z}}{1 + \frac{1}{Y}}$$

$$z_g = \frac{\overline{Z}}{1+Y}$$

Quantities exchanged of good 1:

$$x_{11}(D) = (1-D)\frac{\frac{\beta \overline{Z}^{\beta}}{2}\left(\frac{1}{\beta} + \frac{Y-1}{1+Y}\right)}{\left(1 + \frac{1}{Y}\right)^{\beta}}$$

$$x_{21}(D)=(1-D)\frac{\frac{\beta\bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)}{\left(1+\frac{1}{Y}\right)^{\beta}}$$

$$X_1(D)=(1-D)\frac{\bar{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{\beta}}$$

Quantities exchanged of good 2:

$$x_{12}(D)=\frac{\frac{\beta\bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)}{(1+Y)^{\beta}}$$

$$x_{22}(D)=\frac{\frac{\beta\bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)}{(1+Y)^{\beta}}$$

$$X_2(D)=\frac{\bar{Z}^{\beta}}{(1+Y)^{\beta}}$$

- Profit of firms

$$\Pi^1=\frac{\bar{Z}}{\left(1+\frac{1}{Y}\right)}\left(\frac{1}{\beta}-1\right)$$

$$\Pi^2=\frac{\bar{Z}}{(1+Y)}\left(\frac{1}{\beta}-1\right)$$

- Utilities

Utility of consumer 1:

$$U^1(D)=v\frac{x_{11}^{\alpha}(D)}{\alpha}+\frac{x_{12}^{\alpha}(D)}{\alpha}$$

$$=\frac{1}{\alpha}\left[v\left((1-D)\frac{\frac{\beta\bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)}{\left(1+\frac{1}{Y}\right)^{\beta}}\right)^{\alpha}+\left(\frac{\frac{\beta\bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)}{(1+Y)^{\beta}}\right)^{\alpha}\right]$$

$$=\frac{1}{\alpha}\left[v\left(\frac{\beta\bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)\right)^{\alpha}\left(\frac{(1-D)^{\alpha}}{\left(1+\frac{1}{Y}\right)^{\alpha\beta}}+\frac{1}{(1+Y)^{\alpha\beta}}\right)\right]$$

Utility of consumer 2:

$$U^2(D)=v\frac{x_{21}^{\alpha}(D)}{\alpha}+\frac{x_{22}^{\alpha}(D)}{\alpha}$$

$$=\frac{1}{\alpha}\left[v\left((1-D)\frac{\frac{\beta\overline{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)}{\left(1+\frac{1}{Y}\right)^{\beta}}\right)^{\alpha}+\left(\frac{\frac{\beta\overline{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)}{(1+Y)^{\beta}}\right)^{\alpha}\right]$$

$$=\frac{1}{\alpha}\left[v\left(\frac{\beta\overline{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)\right)^{\alpha}\left(\frac{(1-D)^{\alpha}}{\left(1+\frac{1}{Y}\right)^{\alpha\beta}}+\frac{1}{(1+Y)^{\alpha\beta}}\right)\right]$$

- Welfare

$$W(D)=\theta U^1(D)+(1-\theta)U^2(D)$$

$$=\frac{1}{\alpha}\left[\theta v\left(\frac{\beta\overline{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)\right)^{\alpha}\left(\frac{(1-D)^{\alpha}}{\left(1+\frac{1}{Y}\right)^{\alpha\beta}}+\frac{1}{(1+Y)^{\alpha\beta}}\right)+(1-\theta)v\left(\frac{\beta\overline{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)\right)^{\alpha}\left(\frac{(1-D)^{\alpha}}{\left(1+\frac{1}{Y}\right)^{\alpha\beta}}+\frac{1}{(1+Y)^{\alpha\beta}}\right)\right]$$

Impact of an insect pollinator decline

- Prices

$$\frac{\partial p_1}{\partial D}=\left(\frac{\overline{Z}^{1-\beta}}{\beta(1-D)\left(1+\frac{1}{Y}\right)^{1-\beta}}\right)'$$

$$\frac{\partial p_1}{\partial D}=\frac{\overline{Z}^{1-\beta}}{\beta(1-D)\left(1+\frac{1}{Y}\right)^{1-\beta}}\left(\frac{1}{1-D}+\frac{Y'}{Y(1+Y)}\right)$$

However *Y'*<0 so the sign of *dp1/dD* is not directly observable and need a study of tendencies.

When v < 1

α \ β	β tends to 0	β tends to 1
α tends to 0	*dp1/dD >0*	*dp1/dD >0*
α tends to 1	*dp1/dD >0*	*dp1/dD >0*

When v > 1

α \ β	β tends to 0	β tends to 1
α tends to 0	*dp1/dD >0*	*dp1/dD >0*
α tends to 1	*dp1/dD >0*	*dp1/dD >0*

We conclude that in the interval of the parameters, α, β and v, *dp1/dD >0*

$$\frac{\partial p_2}{\partial D} = \left(\frac{\overline{Z}^{1-\beta}}{\beta Y (1+Y)^{1-\beta}} \right)'$$

$$\frac{\partial p_2}{\partial D} = -\frac{\overline{Z}^{1-\beta}(1-\beta)Y'}{\beta Y^2 (1+Y)^{2-\beta}}$$

However *Y'*<0, α is comprised between 0 and 1 and β is comprised between 0 and 1. Considering these intervals dP_2/dD *is* positive.

- Total exchange quantities of good 1 and good 2

$$\frac{\partial X_1}{\partial D} = \frac{\overline{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{\beta}} \left(-1 + \frac{\beta(1-D)Y'}{Y(1+Y)} \right)$$

which is negative since *Y'*<0

$$\frac{\partial X_2}{\partial D} = \frac{-\overline{Z}^{\beta} Y'}{(1+Y)^{1+\beta}}$$

However *Y'*<0, α is comprised between 0 and 1 and β is comprised between 0 and 1. Considering these intervals dX_2/dD *is* positive.

- Exchange quantities of inputs z_f and z_g.

$$\frac{\partial z_f}{\partial D} = \frac{\overline{Z} Y'}{Y^2 \left(1+\frac{1}{Y}\right)^2}$$

$$\frac{\partial z_g}{\partial D} = -\frac{\overline{Z} Y'}{(1+Y)^2}$$

However *Y'*<0, which means that dz_f/dD is negative and dz_g/dD is positive.

- Revenues

Revenue of consumer 1:

$$\frac{\partial R_1}{\partial D} = \frac{\overline{Z} Y Y'}{(1+Y)^2} < 0 \text{ since } Y'<0$$

Revenue of consumer 2:

$$\frac{\partial R_2}{\partial D} = \frac{-\overline{Z} Y Y'}{(1+Y)^2} > 0 \text{ since } Y'<0$$

Total revenues

$R = \frac{\overline{Z}}{\beta}$. The total revenue will not move after a pollinator decline.

- Profit of firms

$$\frac{\partial \Pi^1}{\partial D} = \frac{\overline{Z} Y'}{Y^2 \left(1+\frac{1}{Y}\right)^2} \left(\frac{1}{\beta} - 1 \right) < 0 \text{ since } Y'<0$$

$$\frac{\partial \Pi^2}{\partial D} = -\frac{\bar{Z}Y'}{(1+Y)^2}\left(\frac{1}{\beta}-1\right) > 0 \text{ since } Y'<0$$

- Individual consumption of goods

Quantities exchanged of good 1:

$$\frac{\partial x_{11}}{\partial D} = \left[(1-D)\frac{\frac{\beta \bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)}{\left(1+\frac{1}{Y}\right)^{\beta}}\right]'$$

$$\frac{\partial x_{11}}{\partial D} = \frac{\beta \bar{Z}^{\beta}}{2\left(1+\frac{1}{Y}\right)^{\beta}}\left[\left(-\frac{1}{\beta}-\frac{Y-1}{1+Y}+\frac{2Y'(1-D)}{(1+Y)^2}\right)+\frac{\beta Y'}{Y^2\left(1+\frac{1}{Y}\right)}\left(\frac{1-D}{\beta}+\frac{(1-D)(Y-1)}{1+Y}\right)\right]$$

Considering that *Y'* is negative, dx_{11}/dD is negative.

$$\frac{\partial x_{21}}{\partial D} = \left[(1-D)\frac{\frac{\beta \bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)}{\left(1+\frac{1}{Y}\right)^{\beta}}\right]'$$

$$\frac{\partial x_{21}}{\partial D} = \frac{\beta \bar{Z}^{\beta}}{2\left(1+\frac{1}{Y}\right)^{\beta}}\left[\left(-\frac{1}{\beta}-\frac{1-Y}{1+Y}-\frac{2Y'(1-D)}{(1+Y)^2}\right)+\frac{\beta Y'}{Y^2\left(1+\frac{1}{Y}\right)}\left(\frac{1-D}{\beta}+\frac{(1-D)(1-Y)}{1+Y}\right)\right]$$

Considering that Y' is negative, we wondered if $-\frac{1}{\beta}-\frac{1-Y}{1+Y}-\frac{2Y'(1-D)}{(1+Y)^2}$ is negative or positive in the interval of the different parameters of the study and more particularly : α, β and *v*. If it is negative, dx_{21}/dD will be negative and if it is positive, dx_{21}/dD could be positive.

When v < 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dx_{21}/dD<0$	$dx_{21}/dD<0$
α tends to 1	$dx_{21}/dD<0$	$dx_{21}/dD<0$

When v > 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dx_{21}/dD<0$	$dx_{21}/dD<0$
α tends to 1	$dx_{21}/dD<0$	$D=]0;D^*[\Rightarrow dx_{21}/dD>0$ $D=D^* \Rightarrow dx_{21}/dD=0$ $D=]D^*;1[\Rightarrow dx_{21}/dD<0$

We find that when v > 1, $D =]0;D^*[$, α and β tends to 1 so $dx_{21}/dD>0$.

Quantities exchanged of good 2:

$$\frac{\partial x_{12}}{\partial D} = \left[\frac{\frac{\beta \bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)}{(1+Y)^{\beta}} \right]'$$

$$\frac{\partial x_{12}}{\partial D} = \frac{\beta \bar{Z}^{\beta}}{2(1+Y)^{\beta}}\left[\left(\frac{2Y'}{(1+Y)^2}\right) - \frac{\beta Y'}{(1+Y)}\left(\frac{1}{\beta}+\frac{(Y-1)}{1+Y}\right)\right]$$

The first part of this expression ($\frac{2Y'}{(1+Y)^2}$) is negative and the second part is positive ($\frac{-\beta Y'}{(1+Y)}\left(\frac{1}{\beta}+\frac{(Y-1)}{1+Y}\right)$). In order to find its sign after a pollinator decline we have to study it within the interval of the parameters.

When v < 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dx_{12}/dD<0$	$dx_{12}/dD<0$
α tends to 1	$dx_{12}/dD<0$	$dx_{12}/dD<0$

When v > 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$D=]0;D^*[=> dx_{12}/dD>0$ $D=D^*$(with D^* tends to 1)$=> dx_{12}/dD=0$ $D=]D^*;1[=> dx_{12}/dD<0$	$D=]0;D^*[=> dx_{12}/dD>0$ $D=D^*$(with D^* tends to 1)$=> dx_{12}/dD=0$ $D=]D^*;1[=> dx_{12}/dD<0$
α tends to 1	$D=]0;D^*[=> dx_{12}/dD>0$ $D=D^* => dx_{12}/dD=0$ $D=]D^*;1[=> dx_{12}/dD<0$	$D=]0;D^*[=> dx_{12}/dD>0$ $D=D^* => dx_{12}/dD=0$ $D=]D^*;1[=> dx_{12}/dD<0$

We find that when v > 1 and $D \in]0;D^*[$ so $dx_{12}/dD>0$. We observed that when α tends to 0, D^* tends to 1.

$$\frac{\partial x_{22}}{\partial D} = \left[(1-D)\frac{\frac{\beta \bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)}{(1+Y)^{\beta}} \right]'$$

$$\frac{\partial x_{22}}{\partial D} = \frac{\beta \bar{Z}^{\beta}}{2(1+Y)^{\beta}}\left[\left(-\frac{2Y'}{(1+Y)^2}\right) - \frac{\beta Y'}{(1+Y)}\left(\frac{1}{\beta}+\frac{(1-Y)}{1+Y}\right)\right]$$

The sign of dx_{22}/dD is positive since Y' is negative.

- Utilities

Utility of consumer 1:

$$\frac{\partial U^1}{\partial D}=\frac{vx_{11}'}{x_{11}^{1-\alpha}}+\frac{x_{12}'}{x_{12}^{1-\alpha}}$$

$$=\frac{\beta\bar{Z}^\beta}{2\left(1+\frac{1}{Y}\right)^\beta}\left[\frac{v\left[\left(-\frac{1}{\beta}-\frac{Y-1}{1+Y}+\frac{2Y'(1-D)}{(1+Y)^2}\right)+\frac{\beta Y'}{Y^2\left(1+\frac{1}{Y}\right)}\left(\frac{1-D}{\beta}+\frac{(1-D)(Y-1)}{1+Y}\right)\right]}{\left((1-D)\frac{\frac{\beta\bar{Z}^\beta}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)}{\left(1+\frac{1}{Y}\right)^\beta}\right)^{1-\alpha}}+\frac{\left[\left(\frac{2Y'}{(1+Y)^2}\right)-\frac{\beta Y'}{(1+Y)}\left(\frac{1}{\beta}+\frac{(Y-1)}{1+Y}\right)\right]}{\left(\frac{\frac{\beta\bar{Z}^\beta}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)}{(1+Y)^\beta}\right)^{1-\alpha}}\right]$$

When v < 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dU_1/dD<0$	$dU_1/dD<0$
α tends to 1	$dU_1/dD<0$	$dU_1/dD<0$

When v > 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dU_1/dD<0$	$dU_1/dD<0$
α tends to 1	$dU_1/dD<0$	$dU_1/dD<0$

Utility of consumer 1 will always be negative after a pollinator decline.

$$\frac{\partial U^2}{\partial D}=\frac{vx_{21}'}{x_{21}^{1-\alpha}}+\frac{x_{22}'}{x_{22}^{1-\alpha}}$$

$$=\frac{\beta\bar{Z}^\beta}{2\left(1+\frac{1}{Y}\right)^\beta}\left[\frac{v\left[\left(-\frac{1}{\beta}-\frac{1-Y}{1+Y}-\frac{2Y'(1-D)}{(1+Y)^2}\right)+\frac{\beta Y'}{Y^2\left(1+\frac{1}{Y}\right)}\left(\frac{1-D}{\beta}+\frac{(1-D)(1-Y)}{1+Y}\right)\right]}{\left((1-D)\frac{\frac{\beta\bar{Z}^\beta}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)}{\left(1+\frac{1}{Y}\right)^\beta}\right)^{1-\alpha}}+\frac{\left[\left(-\frac{2Y'}{(1+Y)^2}\right)-\frac{\beta Y'}{(1+Y)}\left(\frac{1}{\beta}+\frac{(1-Y)}{1+Y}\right)\right]}{\left(\frac{\frac{\beta\bar{Z}^\beta}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)}{(1+Y)^\beta}\right)^{1-\alpha}}\right]$$

When v < 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dU_2/dD<0$	$dU_2/dD<0$
α tends to 1	$dU_2/dD<0$	$dU_2/dD>0$

When v > 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dU_2/dD<0$	$dU_2/dD<0$
α tends to 1	$dU_2/dD<0$	$dU_2/dD>0$

Utility of consumer 2 can be positive when α and β tends to 1.

- Welfare

$$W(D)=\theta U^1(D)+(1-\theta)U^2(D)$$

$$=\frac{\beta\bar{Z}^{\beta}}{2\left(1+\frac{1}{Y}\right)^{\beta}}\left(\theta\left[\frac{v\left[\left(-\frac{1}{\beta}-\frac{Y-1}{1+Y}+\frac{2Y'(1-D)}{(1+Y)^2}\right)+\frac{\beta Y'}{Y^2\left(1+\frac{1}{Y}\right)}\left(\frac{1-D}{\beta}+\frac{(1-D)(Y-1)}{1+Y}\right)\right]}{\left((1-D)\frac{\frac{\beta\bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)}{\left(1+\frac{1}{Y}\right)^{\beta}}\right)^{1-\alpha}}+\frac{\left[\left(\frac{2Y'}{(1+Y)^2}\right)-\frac{\beta Y'}{(1+Y)}\left(\frac{1}{\beta}+\frac{(Y-1)}{1+Y}\right)\right]}{\left(\frac{\frac{\beta\bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{Y-1}{1+Y}\right)}{(1+Y)^{\beta}}\right)^{1-\alpha}}\right]\right.$$
$$\left.+(1-\theta)\left[\frac{v\left[\left(-\frac{1}{\beta}-\frac{1-Y}{1+Y}-\frac{2Y'(1-D)}{(1+Y)^2}\right)+\frac{\beta Y'}{Y^2\left(1+\frac{1}{Y}\right)}\left(\frac{1-D}{\beta}+\frac{(1-D)(1-Y)}{1+Y}\right)\right]}{\left((1-D)\frac{\frac{\beta\bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)}{\left(1+\frac{1}{Y}\right)^{\beta}}\right)^{1-\alpha}}+\frac{\left[\left(-\frac{2Y'}{(1+Y)^2}\right)-\frac{\beta Y'}{(1+Y)}\left(\frac{1}{\beta}+\frac{(1-Y)}{1+Y}\right)\right]}{\left(\frac{\frac{\beta\bar{Z}^{\beta}}{2}\left(\frac{1}{\beta}+\frac{1-Y}{1+Y}\right)}{(1+Y)^{\beta}}\right)^{1-\alpha}}\right]\right)$$

The two preceding expression of dU^1 and dU^2 suggest that dW would be negative whatever the amount of parameters α, β and v except when alpha and beta tends to 1. Considering this case, the sign of dW could be positive if θ is comprised between [0; θ*[where θ* is the value of θ for which dW=0

Appendix 2: The model with polarized ownership structure

The model

- <u>The supply side</u>

The production function has a Cobb-Douglas form for both firms. Good 1 depends on insect pollination and is produced by firm 1 and good 2 does not depend on pollination and is produced by firm 2. Thus the production function of firm 1 is:

$$f(z_f, D) = (1-D)z_f^{\beta}$$

And the production function of firm 2 is:

$$g(z_g) = z_g^{\beta}$$

with β a parameter chosen in the interval]0, 1[, which implies decreasing returns to scale.

Profit function of firms 1 and 2, Π^1 and Π^2 are:

$$\Pi^1 = p_1 f(z_f, D) - a z_f = p_1(1-D)z_f^{\beta} - a z_f$$
$$\Pi^2 = p_2 f(z_g, D) - a z_g = p_2(1-D)z_g^{\beta} - a z_g$$

The profit of firm 1 is maximum when z_f verify:

$$\frac{\partial \Pi^1}{\partial z_f} = \beta p_1(1-D)z_f^{\beta-1} - a = 0$$
$$\Leftrightarrow z_f = \left(\frac{\beta p_1(1-D)}{a}\right)^{\frac{1}{1-\beta}}$$

The profit of firm 2 is maximum when z_g verify:

$$\frac{\partial \Pi^2}{\partial z_g} = \beta p_2 z_g^{\beta-1} - a = 0$$
$$\Leftrightarrow z_g = \left(\frac{\beta p_2}{a}\right)^{\frac{1}{1-\beta}}$$

Total demand of input, Z, is:

$$Z = z_f + z_g = \left(\frac{\beta}{a}\right)^{\frac{1}{1-\beta}}\left((p_1(1-D))^{\frac{1}{1-\beta}} + p_2^{\frac{1}{1-\beta}}\right)$$

We assume that the total demand of input is totally satisfied. The supply of input is offered by both consumers and is fixed $\overline{Z}$.

The total supply of good 1 is:

$$f(z_f, D) = (1-D)^{\frac{1}{1-\beta}} \left(\frac{\beta p_1}{a} \right)^{\frac{\beta}{1-\beta}}$$

The total supply of good 2 is:

$$g(z_g) = \left(\frac{\beta p_2}{a} \right)^{\frac{\beta}{1-\beta}}$$

- <u>The demand side</u>

Consumer maximizes his utility $U^c(x_{c1}, x_{c2}) = \frac{v x_{c1}^{\alpha}}{\alpha} + \frac{x_{c2}^{\alpha}}{\alpha}$ considering the budget constraint:

$$R_c \geq p_1 x_{c1} + p_2 x_{c2}$$

$$U_1^c = v x_{c1}^{\alpha-1} \quad [8]$$

$$U_2^c = x_{c2}^{\alpha-1} \quad [9]$$

At the equilibrium, consumer use all his revenue to consume x_{c1} and x_{c2} so that $R_c = p_1 x_{c1} + p_2 x_{c2}$ and consumption choices are done so that the marginal rate of substitution (MRS) x_{c1} and x_{c2} is equal to the slope of the budget curve which is p_1/p_2. We can define the optimal consumption of x_{c1} and x_{c2}:

$$MRS = \frac{\frac{\partial U}{\partial x_{c1}}}{\frac{\partial U}{\partial x_{c2}}} = \frac{v x_{c1}^{\alpha-1}}{x_{c2}^{\alpha-1}} = \frac{p_1}{p_2}$$

$$\Leftrightarrow x_{c1} = x_{c2} \left(\frac{p_1}{v p_2} \right)^{\frac{1}{\alpha-1}} \quad [10]$$

$$R_c = p_1 x_{c2} \left(\frac{p_1}{v p_2} \right)^{\frac{1}{\alpha-1}} + p_2 x_{c2}$$

$$\Leftrightarrow x_{c2} = \frac{R_c}{p_2 + p_1 \left(\frac{p_1}{v p_2} \right)^{\frac{1}{\alpha-1}}} \quad [11]$$

From expressions [3] and [4] it comes:

$$x_{c1} = \frac{R_c}{p_1 + p_2 \left(\frac{v p_2}{p_1} \right)^{\frac{1}{\alpha-1}}} \quad [12]$$

- Revenues

$$R_1 = \Pi^1 + az_f = p_1(1-D)^{\frac{1}{1-\beta}}\left(\frac{\beta p_1}{a}\right)^{\frac{\beta}{1-\beta}}$$

$$R_2 = \Pi^2 + az_g = p_2\left(\frac{\beta p_2}{a}\right)^{\frac{\beta}{1-\beta}}$$

$$R = R_1 + R_2 = \Pi^1 + \Pi^2 = p_1(1-D)^{\frac{1}{1-\beta}}\left(\frac{\beta p_1}{a}\right)^{\frac{\beta}{1-\beta}} + p_2\left(\frac{\beta p_2}{a}\right)^{\frac{\beta}{1-\beta}}$$

Equilibrium of the economy: total demand = total supply

- Prices a, p_1 and p_2

$$X_1 = x_{11} + x_{21} = f(z_f, p_1, D)$$

$$\frac{p_1 f + p_2 g}{p_1 + p_2\left(\frac{p_1}{v p_2}\right)^{\frac{1}{1-\alpha}}} = f$$

$$\Leftrightarrow g = f\left(\frac{p_1}{v p_2}\right)^{\frac{1}{1-\alpha}}$$

However $f(z_f, D) = (1-D)^{\frac{1}{1-\beta}}\left(\frac{\beta p_1}{a}\right)^{\frac{\beta}{1-\beta}}$ and $g(z_g) = \left(\frac{\beta p_2}{a}\right)^{\frac{\beta}{1-\beta}}$

$$p_2 = \frac{(1-D)^{\frac{1-\alpha}{1-\alpha\beta}}}{v^{\frac{1-\beta}{1-\alpha\beta}}} p_1$$

By Walras' law the second equilibrium (X_2=g) is automatically satisfied. We assume that the price of input, a, is normalized to 1 (a=1). Using expression of the total input exchanged in the economy $Z=\overline{Z}$ we found p_1 and p_2:

$$\overline{Z} = \beta^{\frac{1}{1-\beta}}\left(\left(p_1(1-D)\right)^{\frac{1}{1-\beta}} + p_2^{\frac{1}{1-\beta}}\right)$$

$$\Leftrightarrow \beta^{\frac{1}{1-\beta}}\left(\left(p_1(1-D)\right)^{\frac{1}{1-\beta}} + p_1\frac{(1-D)^{\frac{1-\alpha}{(1-\alpha\beta)(1-\beta)}}}{v^{\frac{1}{1-\alpha\beta}}}\right)$$

$$p_1 = \frac{\overline{Z}^{1-\beta}}{\beta(1-D)\left(1+\frac{1}{\left((1-D)^{\alpha}v\right)^{\frac{1}{1-\alpha\beta}}}\right)^{1-\beta}}$$

$$\text{and } p_2 = \frac{\overline{Z}^{1-\beta}}{\beta\left((1-D)^{\alpha}v\right)^{\frac{1-\beta}{1-\alpha\beta}}\left(1+\frac{1}{\left((1-D)^{\alpha}v\right)^{\frac{1}{1-\alpha\beta}}}\right)^{1-\beta}}$$

In order to simplify the writing we will set: $\left((1-D)^{\alpha}v\right)^{\frac{1}{1-\alpha\beta}} = Y$, where *Y(D)* is positive and decreasing (*dY/dD*<0).

- <u>Revenues</u>

Revenue of consumer 1:

$$R_1 = \frac{\overline{Z}}{\beta\left(1+\frac{1}{Y}\right)}$$

Revenue of consumer 2:

$$R_2 = \frac{\overline{Z}}{\beta(1+Y)}$$

Total revenues

$$R = \frac{\overline{Z}}{\beta}$$

- <u>Quantities exchanged of input, good 1 and good 2</u>

Quantities exchanged of input:

$$z_f = \frac{\overline{Z}}{1+\frac{1}{Y}}$$

$$z_g = \frac{\overline{Z}}{1+Y}$$

Quantities exchanged of good 1:

$$x_{11}(D) = (1-D)\frac{\overline{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{1+\beta}}$$

$$x_{21}(D) = (1-D)\frac{\overline{Z}^{\beta}}{Y\left(1+\frac{1}{Y}\right)^{1+\beta}}$$

$$X_{1}(D) = (1-D)\frac{\overline{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{\beta}}$$

Quantities exchanged of good 2:

$$x_{12}(D) = \frac{\overline{Z}^{\beta}Y}{(1+Y)^{1+\beta}}$$

$$x_{22}(D) = \frac{\overline{Z}^{\beta}}{(1+Y)^{\beta}}$$

$$X_{2}(D) = \frac{\overline{Z}^{\beta}}{(1+Y)^{\beta}}$$

- Profit of firms

$$\Pi^{1} = \frac{\overline{Z}}{\left(1+\frac{1}{Y}\right)}\left(\frac{1}{\beta}-1\right)$$

$$\Pi^{2} = \frac{\overline{Z}}{(1+Y)}\left(\frac{1}{\beta}-1\right)$$

- Utilities

Utility of consumer 1:

$$U^{1}(D) = v\frac{x_{11}^{\alpha}(D)}{\alpha} + \frac{x_{12}^{\alpha}(D)}{\alpha}$$
$$= \frac{YZ^{\alpha\beta}}{\alpha(1+Y)^{\alpha(1+\beta)}}\left[v\left((1-D)Y^{\beta}\right)^{\alpha}+1\right]$$

Utility of consumer 2:

$$U^{2}(D) = v\frac{x_{21}^{\alpha}(D)}{\alpha} + \frac{x_{22}^{\alpha}(D)}{\alpha}$$
$$= \frac{Z^{\alpha\beta}}{\alpha(1+Y)^{\alpha(1+\beta)}}\left[v\left((1-D)Y^{\beta}\right)^{\alpha}+1\right]$$

- Welfare

$$W(D) = \theta U^{1}(D) + (1-\theta)U^{2}(D)$$
$$= \theta\frac{YZ^{\alpha\beta}}{\alpha(1+Y)^{\alpha(1+\beta)}}\left[v\left((1-D)Y^{\beta}\right)^{\alpha}+1\right] + (1-\theta)\frac{Z^{\alpha\beta}}{\alpha(1+Y)^{\alpha(1+\beta)}}\left[v\left((1-D)Y^{\beta}\right)^{\alpha}+1\right]$$
$$= \frac{Z^{\alpha\beta}}{\alpha(1+Y)^{\alpha(1+\beta)}}\left(v\left((1-D)Y^{\beta}\right)^{\alpha}+1\right)\left(Y\theta+(1-\theta)\right)$$

Impact of an insect pollinator decline

- Prices

$$\frac{\partial p_1}{\partial D}=\left(\frac{\overline{Z}^{1-\beta}}{\beta(1-D)\left(1+\frac{1}{Y}\right)^{1-\beta}}\right)'$$

$$\frac{\partial p_1}{\partial D}=\frac{\overline{Z}^{1-\beta}}{\beta(1-D)\left(1+\frac{1}{Y}\right)^{1-\beta}}\left(\frac{1}{1-D}+\frac{Y'}{Y(1+Y)}\right)$$

However $Y'<0$ so the sign of *dp1/dD* is not directly observable and need a study of tendencies.

When v < 1

α \ β	β tends to 0	β tends to 1
α tends to 0	*dp1/dD >0*	*dp1/dD >0*
α tends to 1	*dp1/dD >0*	*dp1/dD >0*

When v > 1

α \ β	β tends to 0	β tends to 1
α tends to 0	*dp1/dD >0*	*dp1/dD >0*
α tends to 1	*dp1/dD >0*	*dp1/dD >0*

We conclude that in the interval of the parameters, α, β and v, *dp1/dD >0*

$$\frac{\partial p_2}{\partial D}=\left(\frac{\overline{Z}^{1-\beta}}{\beta Y(1+Y)^{1-\beta}}\right)'$$

$$\frac{\partial p_2}{\partial D}=-\frac{\overline{Z}^{1-\beta}(1-\beta)Y'}{\beta Y^2(1+Y)^{2-\beta}}$$

However $Y'<0$, α is comprised between 0 and 1 and β is comprised between 0 and 1. Considering these intervals dP_2/dD *is* positive.

- Total exchange quantities of good 1 and good 2

$$\frac{\partial X_1}{\partial D}=\frac{\overline{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{\beta}}\left(-1+\frac{\beta(1-D)Y'}{Y(1+Y)}\right)$$

which is negative since $Y'<0$

$$\frac{\partial X_2}{\partial D}=\frac{-\overline{Z}^{\beta}Y'}{(1+Y)^{1+\beta}}$$

However $Y'<0$, α is comprised between 0 and 1 and β is comprised between 0 and 1. Considering these intervals dX_2/dD *is* positive.

- Exchange quantities of inputs z_f and z_g.

$$\frac{\partial z_f}{\partial D} = \frac{\bar{Z}Y'}{Y^2\left(1+\frac{1}{Y}\right)^2}$$

$$\frac{\partial z_g}{\partial D} = -\frac{\bar{Z}Y'}{(1+Y)^2}$$

However $Y'<0$, which means that dz_f/dD is negative and dz_g/dD is positive.

- Revenues

Revenue of consumer 1:

$$\frac{\partial R_1}{\partial D} = \frac{\bar{Z}Y'}{\beta(1+Y)\left(1+\frac{1}{Y}\right)} < 0 \text{ since } Y'<0$$

Revenue of consumer 2:

$$\frac{\partial R_2}{\partial D} = \frac{-\bar{Z}Y'}{\beta(1+Y)^2} > 0 \text{ since } Y'<0$$

Total revenues

$R = \frac{\bar{Z}}{\beta}$. The total revenue will not move after a pollinator decline.

- Profit of firms

$$\frac{\partial \Pi^1}{\partial D} = \frac{\bar{Z}Y'}{Y^2\left(1+\frac{1}{Y}\right)^2}\left(\frac{1}{\beta}-1\right) < 0 \text{ since } Y'<0$$

$$\frac{\partial \Pi^2}{\partial D} = -\frac{\bar{Z}Y'}{(1+Y)^2}\left(\frac{1}{\beta}-1\right) > 0 \text{ since } Y'<0$$

- Individual consumption of goods

Quantities exchanged of good 1:

$$\frac{\partial x_{11}}{\partial D} = \left[(1-D)\frac{\bar{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{\beta}}\right]'$$

$$\frac{\partial x_{11}}{\partial D} = \frac{\bar{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{\beta}}\left[-1+\frac{(1-D)(1+\beta)Y'}{Y(1+Y)}\right]$$

Considering that Y' is negative, dx_{11}/dD is negative.

$$\frac{\partial x_{21}}{\partial D}=\left[(1-D)\frac{\bar{Z}^{\beta}}{Y\left(1+\frac{1}{Y}\right)^{1+\beta}}\right]'$$

$$\frac{\partial x_{21}}{\partial D}=\frac{\bar{Z}^{\beta}}{Y^2\left(1+\frac{1}{Y}\right)^{1+\beta}}\left(-Y-(1-D)Y'+\frac{(1+\beta)Y'}{1+Y}\right)$$

Considering that Y' is negative, we wondered if $-Y-(1-D)Y'+\frac{(1+\beta)Y'}{1+Y}$ is negative or positive in the interval of the different parameters of the study and more particularly : α, β and v. If it is negative, dx_{21}/dD will be negative and if it is positive, dx_{21}/dD could be positive.

When v < 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dx_{21}/dD<0$	$dx_{21}/dD<0$
α tends to 1	$dx_{21}/dD<0$	$dx_{21}/dD<0$

When v > 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dx_{21}/dD<0$	$dx_{21}/dD<0$
α tends to 1	$dx_{21}/dD<0$	$dx_{21}/dD<0$

Considering the interval of α, β and v, dx_{21}/dD will always be negative.

Quantities exchanged of good 2:

$$\frac{\partial x_{12}}{\partial D}=\left[\frac{\bar{Z}^{\beta}Y}{(1+Y)^{1+\beta}}\right]'$$

$$\frac{\partial x_{12}}{\partial D}=\frac{\bar{Z}^{\beta}Y'}{(1+Y)^{\beta}}\left(1-\frac{Y(1+\beta)}{1+Y}\right)$$

The sign of dx_{12}/dD depends on the expression $1-\frac{Y(1+\beta)}{1+Y}$. In order to find its sign after a pollinator decline we have to study it within the interval of the parameters.

When v < 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dx_{12}/dD<0$	$dx_{12}/dD<0$
α tends to 1	$dx_{12}/dD<0$	$dx_{12}/dD<0$

When v > 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dx_{12}/dD<0$	$dx_{12}/dD>0$
α tends to 1	$dx_{12}/dD<0$	$D=]0;D^*[=>dx_{12}/dD>0$ $D=D^*=>dx_{12}/dD=0$ $D=]D^*;1[=>dx_{12}/dD<0$

We find that when v > 1 and $D=]0;D^*[$ so $dx_{12}/dD>0$. We observed that when α tends to 0, D^* tends to 1.

$$\frac{\partial x_{22}}{\partial D}=\left[\frac{\overline{Z}^{\beta}}{(1+Y)^{\beta}}\right]'$$

$$\frac{\partial x_{22}}{\partial D}=-\frac{\overline{Z}^{\beta}(1+\beta)Y'}{(1+Y)^{2+\beta}}$$

The sign of dx_{22}/dD is positive since Y' is negative.

- Utilities

Utility of consumer 1:

$$\frac{\partial U^1}{\partial D}=\frac{vx_{11}'}{x_{11}^{1-\alpha}}+\frac{x_{12}'}{x_{12}^{1-\alpha}}$$

$$=\frac{\overline{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{\beta}}\left[\frac{v\left(\left[-1+\frac{(1-D)(1+\beta)Y'}{Y(1+Y)}\right]\right)}{\left((1-D)\frac{\overline{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{\beta}}\right)^{1-\alpha}}+\frac{Y^{\beta}Y'\left(1-\frac{Y(1+\beta)}{1+Y}\right)}{\left(\frac{\overline{Z}^{\beta}Y}{(1+Y)^{1+\beta}}\right)^{1-\alpha}}\right]$$

When v < 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dU_1/dD<0$	$dU_1/dD<0$
α tends to 1	$dU_1/dD<0$	$dU_1/dD<0$

When v > 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dU_1/dD<0$	$dU_1/dD<0$
α tends to 1	$dU_1/dD<0$	$dU_1/dD<0$

Utility of consumer 1 will always be negative after a pollinator decline.

$$\frac{\partial U^2}{\partial D}=\frac{vx_{21}{}'}{x_{21}^{1-\alpha}}+\frac{x_{22}{}'}{x_{22}^{1-\alpha}}$$

$$=\frac{\overline{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{1+\beta}}\left(\frac{\frac{v}{Y^2}\left(-Y-(1-D)Y'+\frac{(1+\beta)Y'}{1+Y}\right)}{\left((1-D)\frac{\overline{Z}^{\beta}}{Y\left(1+\frac{1}{Y}\right)^{1+\beta}}\right)^{1-\alpha}}-\frac{(1+\beta)Y^{2+\beta}Y'}{\left(\frac{\overline{Z}^{\beta}}{(1+Y)^{\beta}}\right)^{1-\alpha}}\right)$$

When v < 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dU_2/dD>0$	$dU_2/dD<0$
α tends to 1	$dU_2/dD>0$	$dU_2/dD<0$

When v > 1

α \ β	β tends to 0	β tends to 1
α tends to 0	$dU_2/dD>0$	$dU_2/dD<0$
α tends to 1	$dU_2/dD>0$	$D=]0;D^*[=>dx_{12}/dD>0$ $D=D^*=>dx_{12}/dD=0$ $D=]D^*;1[=>dx_{12}/dD<0$

Utility of consumer 2 can be positive when α and β tends to 1.

- Welfare

$$W(D)=\theta U^1(D)+(1-\theta)U^2(D)$$

$$=\frac{\overline{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{1+\beta}}\left[\theta\left[\frac{v\left(\left[-1+\frac{(1-D)(1+\beta)Y'}{Y(1+Y)}\right]\right)}{\left((1-D)\frac{\overline{Z}^{\beta}}{\left(1+\frac{1}{Y}\right)^{\beta}}\right)^{1-\alpha}}+\frac{Y^{\beta}Y'\left(1-\frac{Y(1+\beta)}{1+Y}\right)}{\left(\frac{\overline{Z}^{\beta}Y}{(1+Y)^{1+\beta}}\right)^{1-\alpha}}\right]+(1-\theta)\left(\frac{\frac{v}{Y^2}\left(-Y-(1-D)Y'+\frac{(1+\beta)Y'}{1+Y}\right)}{\left((1-D)\frac{\overline{Z}^{\beta}}{Y\left(1+\frac{1}{Y}\right)^{1+\beta}}\right)^{1-\alpha}}-\frac{(1+\beta)Y^{2+\beta}Y'}{\left(\frac{\overline{Z}^{\beta}}{(1+Y)^{\beta}}\right)^{1-\alpha}}\right)\right]$$

The two preceding expression of dU^1 and dU^2 suggest that dW would be negative whatever the amount of parameters α, β and v except when alpha and beta tends to 1. Considering this case, the sign of dW could be positive if θ is comprised between [0; θ*[where θ* is the value of θ for which dW=0