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«Public Infrastrucuture, Strategic Interactions and Endogenous Growth»

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Public Infrastructure, Strategic Interactions and Endogenous Growth[1]

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ABSTRACT.- This paper develops a two-country general equilibrium model with endogenous growth where governements behave strategically in the provision of productive infrastructure. The public capitals enter both national and foreign production as an external input and, they are financed by a flat tax on income. In the private sector, firms and households take the public policy as given when making their decisions. It is shown that both a Markov Perfect Equillibrium (MPE) and a Centralized Solution (CS) exist, even when the parameters allow for endogenous growth, therefore explosive paths for the state variables. And the dynamic analysis reveals three important features. Firstly, under constant returns, the two countries' growth rates differ during the transition but are identical on the balanced growth path. Secondly, due to the infrastructure externality, assuming away constant returns to scale a country with decreasing returns can experience sustained growth provided that the other grows at a positive constant rate. Thirdly, Nash growth rates are compared with the centralized rates. We show that cooperation in infrastructure provision does not necessarily lead to higher growth for each country. We also show that, in some configurations of households' preferences and initial conditions, cooperation would call for a recession in the initial stages of development, whereas strategic investments would not. Lastly, depending also on the configuration of preferences, we show that cooperation can increase or decrease the gap between countries' growth rates.

Key words: infrastructure, transboundary externalities, strategic behaviour, endogenous growth

JEL codes: D9, E6, H5, C73.

1 Introduction

Do governments invest too little in public infrastructure? Do they thereby give up important opportunities to generate growth? More precisely, what are the consequences, as far as growth is concerned, of lacking cooperation in public investments made by uncoordinated countries? In our mind infrastructure refers more specifically to *green* infrastructure, as measured by the flow of public expenditures to finance purification stations for air or waters, though a more comprehensive list typically includes sewer systems, roads, public transports, airports, harbors, hospitals, public schools, public sectors R&D, military buildings and so on...

The interest in these questions dates back at least to Arrow and Kurz's path-breaking book (1970), but it was sparked again 20 years later by Aschauer's empirical papers (1989a, 1989b), who suggested a very powerful role for public infrastructure in the productivity of private capital and lamented an under-investment problem in the United States. As surveyed by Gramlich (1994), because of mixed evidence regarding the level of impact, a more balanced view has developed, where public capital does affect growth, though probably less strongly than initially suggested.[1]

On its theoretical side, this literature attempts to clarify the economic role of public infrastructure. To do so, it often introduces it as an externality in the production function. Different versions exist, depending on whether public infrastructure enters as a flow or as a capital into the production function, whether there is congestion, whether there are constant returns to the augmentable factors, and so on. The insights one can expect from this approach are about the nature of dynamic responses of macroeconomics variables, such as consumption, output, unemployment, interest rates, etc. after a change in the public investment decisions. The insights are also about the policy implications of the suboptimality of decentralized private decisions (because of externalities) and about the issue of optimal size of the public sector. Regarding the latter, the taxation to finance public infrastructure typically has two opposite effects: first, a higher tax rate means, *ceteris paribus*, larger public capital, so higher rate of private profit and growth; but second, it reduces the incentives of private activities and therefore growth. Clearly, there is an optimal tax rate. But those policy implications are far too simple for they neglect possible failures in the public sector itself, due to external effects that may spread far beyond the area of competence of local public decision makers. For those situations, a well-grounded approach would first identify a benchmark investment path, with a normative appeal that takes into account overall economic effects, against which any uncoordinated investment plans could be compared. This is the challenge of this paper.

Research by Barro (1990), Glomm and Ravikumar (1994) and Shibata (2001) has some bearing on the above concern. Shibata (2001) analyzes a partial equilibrium model where two

[1] With annual data on the United States from 1949 to 1985, Aschauer finds an elasticity of aggregate product with respect to public capital as high as 0.39, actually higher than the elasticity with respect to the private capital!

decision makers strategically choose their public investments. The role played by the information structure is emphasized. If policy makers can commit to investment paths, that is if they use open-loop strategies, competition ends up in only one equilibrium with growth. If policy makers use markov strategies, there are multiple equilibria, some with growth, others without. But no comparison is made between those non cooperative equilibria and Pareto optimal paths to assess welfare losses. Anyway, such a comparison would be subject to usual criticisms of welfare analysis in partial equilibrium models; besides the direct effect on production, public investment also alters the trade-offs between private investment and consumption, at home and abroad, which has an effect on equilibrium prices that in turn affects trade-offs and so on... All those indirect general equilibrium effects should also be accounted for when estimating the consequences of lack of cooperation in public sectors. Barro (1990) and Glomm and Ravikumar (1994), in continuous and discrete time formulations respectively, do handle general equilibrium frameworks, but with only one country, therefore no cooperation issue arises in their analysis.

In this paper we begin to fill these important gaps in the theoretical literature. More precisely, we examine the consequence of the lack of cooperation among governments in the first framework that combines:

i) dynamic strategic interactions,

ii) general equilibrium effects,

iii) endogenous growth[2].

It is relatively easy to construct *ad hoc* dynamics with surprising properties. But it is more useful, and demanding, to nest such dynamics into a meaningful model with micro-foundations, so that particular growth regimes could be associated with well-identified economic logics, and their normative properties be assessed. Fortunately, this turns out to be possible and delivers a range of results, in particular:

1. under specific conditions, there is too little (respectively too much) balanced growth at a Markov Perfect Equilibrium, compared to the centralized solution, when consumers prefer the domestic good (respectively the foreign good);

2. when households value more the foreign good than their domestic good, cooperation may call for an economic recession in the early stage of development, whereas strategic investmens would not; this possibility occurs under a range of initial imbalances between private capital stocks;

[2] Shibata (2001) captures points *i)* and *iii)*, Barro (1990) or Glomm and Ravikumar (1994) captures points *ii)* and *iii)*; some papers like Datta and Mirman (2000) deal with *i)* and *ii)*. But no paper, before the present one, encompasses *i)*, *ii)* and *iii)*.

3. in the case of bilateral technological externalities, the assumption of constant returns to scale forces countries to tend to the same balanced growth rate, a property that rules out a widely used argument to explain the observations of different growth rates;

4. relaxing the assumption of constant returns to scale, countries experiences different balanced growth rates; and cooperation increases (respectively decreases) the gap between countries' growth rates when households value more (respectively less) their domestic good than the foreign good.

The discussion develops as follows. Section 2 constructs a dynamic general equilibrium model with two strategic governments. In Section 3, two possible rationales for financing public capitals are considered: a non cooperative one and a centralized one. Section 4 then compares the resulting tax rates and Section 5 compares the respective growth rates they generate. Section 6 summarizes the results. When too technical or too long, proofs are relegated to an appendix .

2 Public infrastructure in a two-country model

A general equilibrium model with two strategic countries or regions will serve as a conceptual vehicle for the analysis (in the rest of the paper we use the terms country and region interchangeably). Within each country, a representative firm and a representative consumer form the private sector, whereas a local government captures the logic of the public sector. The study proceeds in three steps. First, for arbitrary policies in the public sector, we model individual decisions in the private sectors and we characterize the resulting equilibrium. Then we rationalize the public investments of the two countries in infrastructure, *i.e.* the decisions in the public sector, taking into account their effects on the equilibrium. Two scenarios are envisionned here: *i)* the case where local governments make cooperative decisions regarding the sequences of public investments, *ii)* the case where they behave non cooperatively. Finally, we compare the outcomes of the two scenarios, with a particular attention to the growth rates they generate.

2.1 Agents

2.1.1 Firms

The representative firm in country i produces a homogenous good (Y_i), which can be consumed locally (c_{ii}) or abroad (c_{ji})[3], or invested (I_i). The production technology uses two private inputs, capital (K_i) and labour (L_i); local public infrastructure (G_i) enhance the productivity of the private factors, and for this reason it can be considered a production factor. In addition, infrastructure generates cross-border spillovers, which means that the production possibilities of a

[3] From now on, whenever i and j appears in the same expression, it is implicitely assumed that $i \neq j$.

country are affected by the infrastructure G_j of the other country. Formally, those assumptions are captured by the following production functions[4]:

$$Y_{it} = A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} L_{it}^{1-\alpha_i} \ , \quad i,j = 1,2, \tag{1}$$

with α_i, θ_i and $\rho_i \subset [0,1]$.

The transboundary externality G_{jt} is akin to an additional and costless input for country i. All the production factors are immobile.

Firms are competitive: they take as given the factor prices, the levels of infrastructure and they choose labor and private capital to maximize profits,

$$\max_{L_{it},K_{it}} A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} L_{it}^{1-\alpha_i} - w_{it} L_{it} - r_{it} K_{it} \ , \tag{2}$$

with w_{it} the wage rate and r_{it} the interest rate. Under the assumption of complete depreciation of capital after one period, profit maximization ends up in the usual equality between prices and marginal productivities :

$$w_{it} = (1-\alpha_i) A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} L_{it}^{-\alpha_i} \ , \tag{3}$$

$$r_{it} = \alpha_i A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i - 1} L_{it}^{1-\alpha_i} \ . \tag{4}$$

2.1.2 Households

In country i, consumption and investment decisions come from a representative infinitely-lived household. His utility in each period is defined over the consumption of the two commodities produced in the economy, according to:

$$U_i(c_{iit}, c_{ijt}) = \nu_i \ln c_{iit} + \ln c_{ijt} \ , \tag{5}$$

where c_{iit} (resp. c_{ijt}) corresponds to the consumption of the domestic (resp. foreign) commodity, and $\nu_i > 0$ is the relative weight given to the local commodity. In the following, it will be crucial to distinguish the situations where the representative household values more the domestic good ($\nu_i > 1$), from the situations where it values more the foreign good ($\nu_i < 1$). Presumably, the first possibility occurs when the two goods are close substitutes; there is then a sort of national preference. Whereas the second possibility could be relevant for instance when the foreign good fullfils basic needs while the domestic good satisfies more evolved needs.

[4] This Cobb-Douglas formulation for production functions is widely used. Yet it implies foreign infrastructure is a necessary input, which may or may not be a sensible property, depending on the particular kind of infrastructure one has in mind. One may impose however that public capitals never reach zero values. This would be an innocuous constraint since, as derived in Section 3, the production of infrastructure is always positive. Or similarly, it is as if the technology were of the following form, with the possibility of positive production at zero foreign infrastructure:

$$Y_{it} = A_i G_{it}^{\theta_i} (\varepsilon_i + G_{jt})^{\rho_i} K_{it}^{\alpha_i} L_{it}^{1-\alpha_i} \ , \quad \varepsilon_i > 0, \ i,j = 1,2, \ i \neq j.$$

Since the two commodities are different, there is trade on two interregional markets. Trade activities create a second source of externalities between countries. Let us denote p_t as the relative price of the foreign commodity and τ_{it} the income tax rate. The representative agent supplies inelastically one unit of labor, and earns the returns on investment. His total income (net of taxes) is used for the purchase of the two commodities and for the investment in capital, over the life-cycle:

$$K_{it+1} = (1 - \tau_{it})(w_{it}L_{it} + r_{it}K_{it}) - c_{iit} - p_t c_{ijt} \; . \tag{6}$$

Is is worth noting that the budget constraint depends on the regional government taxation policy, which the agent takes as given.

The agent allocates his resources between consumptions and investment to maximize the sum of his discounted per period utilities; if $\beta \in (0,1)$ is the discount factor, his problem is to solve:

$$\max_{\{c_{iit}, c_{ijt}, K_{it+1}\}} \sum_{t=0}^{+\infty} \beta^t \left(\nu_i \ln c_{iit} + \ln c_{ijt}\right) \tag{7}$$

given K_{i0}, $\{w_{it}, r_{it}, \tau_{it}, p_t\}_{t=0}^{\infty}$, subject to $c_{iit}, c_{ijt}, K_{it+1} \geq 0$, $\forall t$, and the budget constraint (6). To summarize, the consumer has to cope with two distinct trade-offs. First, there is the classical question of how to allocate optimally his consumption possibilities over the life cycle, *i.e.* the optimal choice between current consumption and investment. Then there is the question of how to split optimally his consumption expenses between the home commodity and the foreign one.

For reasons to be clarified later, we shall impose, $\forall i = 1, 2$:

$$\alpha_i + \theta_i + \rho_i \leq \frac{1}{\beta} \; , \tag{8}$$

which means that the inverse of the discount factor, $\beta^{-1} > 1$, places an upper bound on returns to scale. However, this does not rule out increasing returns.

2.1.3 The public sector

Each local government is responsible for the financing and production of the local public infrastructure. To do so, it levies a share $\tau_{it} \in [0,1]$ of the representative agent's income. The focus of the paper is on infrastructure as flows of public expenses, therefore:

$$G_{it+1} = \tau_{it}(w_{it}L_{it} + r_{it}K_{it}) \; . \tag{9}$$

Once profits are maximized, the resulting quantity of the public capital can be expressed as a share of the national product

$$G_{it+1} = \tau_{it} A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} L_{it}^{1-\alpha_i}. \tag{10}$$

The following section studies the competitive equilibrium. The constraints and trade-offs in the private sector are detailed.

2.2 The equilibrium

Given an arbitrary vector of public policies $\pi = \{\tau_{it}, G_{it}, \tau_{jt}, G_{jt}\}_{t=0}^{\infty}$, a world competitive equilibrium makes consistent all the decisions undertaken in the private sectors.

Definition 1 *Given the public policies π, a world competitive equilibrium π-CE, is a sequence of aggregated variables*

$$\{c_{iit}, c_{jit}, c_{ijt}, c_{jjt}, K_{it}, L_{it}, K_{jt}, L_{jt}\}_{t=0}^{\infty} ,$$

and a sequence of prices

$$\{w_{it}, r_{it}, w_{jt}, r_{jt}, p_t\}_{t=0}^{\infty}$$

such that:

(i) agents, in each country, are at their optimum,

(ii) the factor markets clear: $L_{it} = N_i = 1$, $K_{it+1} = I_{it}$ $\forall i = 1, 2$,

(iii) the markets of goods are balanced, i.e. the relative price p_t is such that $c_{ijt} = Y_{jt} - c_{jjt}$.

2.2.1 Two artificial problems

Inspired by Glomm and Ravikumar (1994), it is possible to formulate two *artificial problems*, one for each country, with their solutions giving the demand functions for the consumption goods and the investment decisions. In country i, the artificial problem is as follows:

$$\max_{\{c_{iit}, c_{ijt}\}} \sum_{t=0}^{+\infty} \beta^t (\nu_i \ln c_{iit} + \ln c_{ijt}) , \qquad s.t. \begin{cases} c_{iit} + p_t c_{ijt} + K_{it+1} = (1 - \tau_{it}) A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} , \\ K_{i0}, G_{i0}, G_{j0}, p_t \text{ given.} \end{cases} \tag{11}$$

Appendix A shows the unique solution to those planning programs consists of linear functions of the output net of taxes:

$$c_{iit} = \frac{\nu_i}{1 + \nu_i}(1 - \alpha_i\beta)(1 - \tau_{it}) A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} , \tag{12}$$

$$K_{it+1} = \beta\alpha_i(1 - \tau_{it}) A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} , \tag{13}$$

for $i, j = 1, 2$. And foreign consumptions are given by:

$$c_{ijt} = \frac{1}{(1 + \nu_i)p_t}(1 - \alpha_i\beta)(1 - \tau_{it}) A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} , \tag{14}$$

$$c_{jit} = \frac{p_t}{1 + \nu_j}(1 - \alpha_j\beta)(1 - \tau_{jt}) A_j G_{jt}^{\theta_j} G_{it}^{\rho_j} K_{jt}^{\alpha_j} . \tag{15}$$

Proposition 1 *Assume the sequences* $\{G_{it}\}_{t=0}^{\infty}$ *and* $\{G_{jt}\}_{t=0}^{\infty}$ *are bounded above respectively by* $\{\eta^t G_{i0}\}_{t=0}^{\infty}$ *and* $\{\eta^t G_{j0}\}_{t=0}^{\infty}$ *for some* $\eta \geq 1$. *Then, the sequences of individual decisions* $\{c_{iit}, c_{ijt}, c_{jit}, c_{jjt}\}_{t=0}^{\infty}$ *given by (12), (14) and (15), and aggregated variables* $\{K_{it}, K_{jt}\}_{t=0}^{\infty}$ *given by (13), are the unique solutions to the artificial problems.*

Proof. Follows the same logic as Glomm et Ravikumar (1994). ■

The foreign consumptions, (14) and (15), depend on the relative price p_t. To characterize complety the decisions, it remains to determine the equilibrium prices on the markets for those goods.

2.2.2 The equilibrium relative price

At the equilibrium, supply and demand for good j are identical, *i.e.*

$$c_{ijt} = Y_{jt} - K_{jt+1} - c_{jjt} \ .$$

Given the demands (14) and (12), evaluated for j, the equilibrium price is therefore:

$$p_t = \frac{(1+\nu_j)(1-\alpha_i\beta)(1-\tau_{it})A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i}}{(1+\nu_i)(1-\alpha_j\beta)(1-\tau_{jt})A_j G_{jt}^{\theta_j} G_{it}^{\rho_j} K_{jt}^{\alpha_j}} \ . \tag{16}$$

Inserting expression (16) into (14) and (15) gives the individual choices for foreign consumptions,

$$c_{ijt} = \frac{1}{1+\nu_j}(1-\alpha_j\beta)(1-\tau_{jt})A_j G_{jt}^{\theta_j} G_{it}^{\rho_j} K_{jt}^{\alpha_j} \ , \tag{17}$$

for $i, j = 1, 2$. Those consumptions appear, at each date, as fractions of the foreign productions.

The following section studies the non cooperative behaviors of regional governments, with the purpose of comparing the Markov Perfect Equilibrium with the centralized solution.

3 Two rationales for taxation and provision of infrastructure

By substituting the equilibrium decisions (12) and (17) into preferences, the per-period indirect utility function for consumer i is given by:

$$V_i(K_{it}, K_{jt}, G_{it}, G_{jt}) = \left\{ \begin{array}{l} \nu_i \ln(1-\tau_{it}) + (\nu_i\theta_i + \rho_j)\ln G_{it} + \nu_i\alpha_i \ln K_{it} \\ + \ln(1-\tau_{jt}) + (\nu_i\rho_i + \theta_j)\ln G_{jt} + \alpha_j \ln K_{jt} + \gamma_i \end{array} \right\} ,$$

where γ_i is a constant. The sum of the discounted functions $V_i(.,.,.,.), i = 1, 2$, are the objectives in the public authorities' optimization problems.

3.1 Markov Perfect Equilibrium

In a markov perfect equilibrium (MPE), each government chooses the sequence of tax rates $\{\tau_{it}\}_{t=0}^{+\infty}$ that maximizes the discounted sum of per-period indirect utilities, given the markov decision rule of the other country and the private and public capitals dynamics. In other words:

$$\max_{\{\tau_{it}\}} \sum_{t=0}^{+\infty} \beta^t V_i(K_{it}, K_{jt}, G_{it}, G_{jt}) \ ,$$

$$s.t \begin{cases} G_{it+1} = \tau_{it} A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} \ , \\ K_{it+1} = \alpha_i \beta (1 - \tau_{it}) \ A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} \ , \\ i, j = 1, 2. \end{cases}$$

Using dynamic programing tools, the MPE tax rates obtained are :

$$\tau_i^N = \beta\theta_i + \beta\rho_j \left(\frac{1 - \beta\alpha_i - \beta\theta_i + \beta\rho_i\nu_i}{(1 - \beta\alpha_j - \beta\theta_j)\nu_i + \beta\rho_j} \right) \ , \tag{18}$$

for $i, j = 1, 2$ (see Appendix B). Also, as can be seen from the details given in Appendix B, the MPE is an equilibrium in dominant strategies.

The first component $\beta\theta_i$ precisely corresponds to the solution with no interactions at all between countries as studied by Glomm and Ravikumar (1994): the higher the impact of infrastructure in production (measured by θ_i), the higher the tax rate and the provision of the domestic public good. Also, the lower the degree of impatience (lower discount factor) the lower the tax rates and the investments.

More interestingly, with interacting countries there is a second term that reflects the interaction between them: the larger the impact of domestic infrastructure on foreign production (represented by ρ_j), the higher the Nash tax rate τ_i^N. This property is due to the fact that country i's contribution tends to increase country j's production and thus the amount of resources that it will be willing to allocate to its own public good provison. In turn, the rise in the stock G_j will benefit the production in country i through the infrastructure externality channel. Moreover, there exists an additional positive effect that results from the consumption side: the production of foreign good is also consumed at home. Thus, more foreign production means more utility. The government takes into account this feeback effect[5] and provide a quantity of public good higher than the one chosen in the case of pure autarky.

Moreover, one observes that τ_i^N decreases with the relative weight ν_i of the domestic good in preferences once the following holds:

$$(1 - \beta\alpha_j - \beta\theta_j)((1 - \beta\alpha_i - \beta\theta_i) - \beta^2\rho_i\rho_j > 0 \ . \tag{19}$$

This inequality is satisfied under the assumption (8) on technology. Actually, a fall in ν_i means that the consumer attaches less importance to the domestic good. Public authorities have then

[5] The benefits are perceived two periods after the investment.

the incentive to reinforce the fiscal policy at the expense of the national product. This decision implies a reduction of the resources devoted to both investment and global consumption but, it also goes with an increase in the stock G_i meant to stimulate foreign production. Therefore, preferences abroad remaining unchanged, this policy leads to a rise in the amount of the good available on the market which, combined with a fall in the relative price p_t, allows the domestic consumer to effectively change his consumption basket by purchasing a higher quantity of his most desired good.

3.2 The centralized solution

The centralized solution (CS) singles out the sequences of tax rates $\{\tau_{it}\}_{t=0}^{+\infty}$ and $\{\tau_{jt}\}_{t=0}^{+\infty}$ that maximize the sum of the two representative agents' overall utilities. It appears as a natural benchmark to assess the impact of strategic interaction and can be interpreted as a form of cooperation[6] in the production of infrastructure. The problem to solve is given by:

$$\max_{\{\tau_{it},\tau_{jt}\}} \sum_{t=0}^{\infty} \beta^t \left[V_i(K_{it}, K_{jt}, G_{it}, G_{jt}) + V_j(K_{jt}, K_{it}, G_{jt}, G_{it}) \right] \ ,$$

$$s.t \begin{cases} G_{it+1} = \tau_{it} A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} \ , \\ K_{it+1} = \alpha_i \beta (1 - \tau_{it}) \ A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} \ , \\ i, j = 1, 2. \end{cases}$$

As before, using dynamic programing the expressions of the CS tax rates follow:

$$\tau_i^C = \beta\theta_i + \beta\rho_j \left(\frac{(1+\nu_j)(1-\beta\alpha_i-\beta\theta_i) + (1+\nu_i)\beta\rho_i}{(1+\nu_i)(1-\beta\alpha_j-\beta\theta_j) + (1+\nu_j)\beta\rho_j} \right) \ , \tag{20}$$

for $i, j = 1, 2$.

The following section compares the MPE and the CS tax rates, not only in the general framework with *diversified consumers* developed until now, but also when the agents value only their domestic good, a case we refer to as *domestic-prone consumers*, when there are only production externalities and countries live in autarky as far as consumption is concerned. The corresponding outcomes (with the superscript "A" for autarky) are obtained by letting the relative weights ν_i and ν_j tend to infinity in expressions (18) and (20):

$$\tau_{it}^{AN} = \beta\theta_i + \frac{\beta^2 \rho_i \rho_j}{1 - \beta\alpha_j - \beta\theta_j} \ ,$$

$$\tau_{it}^{AC} = \beta\theta_i + \beta\rho_j \frac{1 - \beta\alpha_i - \beta\theta_i + \beta\rho_i}{1 - \beta\alpha_j - \beta\theta_j + \beta\rho_j} \ .$$

[6] However it does not give a Pareto optimal outcome. This is due to the instrument under consideration: a flat tax on income modifies private agents' decisions. A lump-sum tax would avoid those distortions...

4 Strategic taxations and departure from efficiency

To understand how strategic incentives fail to realize the centralized optimum and the consequences on growth rates, it is important to add more precisions about tax levels, under both the non cooperative scenario and the centralized one. The goal is to rank MPE and CS tax rates. The results are summarized in the following proposition.

Proposition 2 *Under Assumption (8):*

(i) with domestic-prone consumers, MPE tax rates are lower than CS tax rates:

$$\tau_i^{AN} < \tau_i^{AC} \ , \quad \forall i = 1, 2 \ .$$

(ii) with diversified consumers, the ranking depends on preferences:

$$\tau_i^N \geq (\leq) \tau_i^C \Leftrightarrow \nu_i \nu_j \leq (\geq) 1, \ \forall i, j = 1, 2 \ .$$

Proof. part (i): proving $\tau_i^{AN} < \tau_i^{AC}$ boils down to verifying the following inequality:

$$(1 - \beta\alpha_j - \beta\theta_j)(1 - \beta\alpha_i - \beta\theta_i) - \beta^2 \rho_i \rho_j > 0 \ ,$$

which is guaranteed under the assumption of weakly increasing returns to scale (8).

part (ii), $\tau_i^N \geq \tau_i^C \Longleftrightarrow$

$$\left[(1 - \beta\alpha_j - \beta\theta_j)(1 - \beta\alpha_i - \beta\theta_i) - \beta^2 \rho_i \rho_j\right](1 - \nu_i \nu_j) \geq 0 \ ,$$

Since the first term of the above product is positive under Assumption (8), the ranking is given by the sign of $1 - \nu_i \ \nu_j$. ■

With domestic-prone consumers there is no trade, and spillovers disseminate only through the channel of production technologies. This is a positive externalities framework and, as expected, non-cooperative countries ignore their positive impact on the other country and invest too little in infrastructure.

With diversified consumers, there exists a second channel of interaction, namely the consumption of the good produced abroad. As a result, the ranking between Nash and centralized tax rates is crucially bound to preferences. For instance, if each country prefers its own good ($\nu_i, \nu_j \geq 1$), then we have $\tau_i^N \leq \tau_i^C$, $\forall i = 1, 2$. In a sense, the concern for the foreign good is too small to modify the previous logic of positive input externalities. But, when each country pays more attention to the good produced abroad ($\nu_i, \nu_j \leq 1$), the ranking of tax rates is reversed. There is overcontribution to public infrastructure compared to the socially optimal level, that is $\tau_i^N \geq \tau_i^C$, $\forall i = 1, 2$. The intuition is as follows. Country i neglects its home production to invest heavily in infrastructure, for this is a way to induce a large production of the good it values the most produced in country j; and this exerts a downward pressure on price (see expression 16.)

Country j does the same reasoning and both countries settle for too much consumption of their home commodity along with inefficiently high tax rates.

Finally, in the mixed cases where one country prefers the domestic good whereas the other country prefers the foreign good, for instance $\nu_1 > 1$, $\nu_2 < 1$, the two previous logics are at work and the sign of $1 - \nu_i\nu_j$ indicates which one prevails.[7]

The next part of the analysis deals with dynamics. We more precisely focus on several scenarios regarding the conditions of sustained growth in the two countries.

5 Growth

In the world of interdependant economies depicted here, two questions about growth come to mind. First, one may wonder how technology interdependancy itself affects the prospects of growth? Second, given the strategic incentives of each local government to free-ride on foreign investments, what role for coordination arises regarding growth? Popular wisdom would probably reply: "from cooperation one expect increased growth rates, or at least avoidance of recessions". The answers are more subtle, and sometimes surprising...

With a view to answering those questions, the previous sections have provided two important pieces of information: *i)* under both the non cooperative and the centralized scenario, tax rates are constant, *ii)* those tax rates can be ranked.

Under constant tax rates implemented in each country, the dynamics in the private and the public sectors are:

$$K_{it+1} = \alpha_i\beta(1-\tau_i)A_iG_{it}^{\theta_i}G_{jt}^{\rho_i}K_{it}^{\alpha_i} \ , \tag{21}$$

and

$$G_{it+1} = \tau_iA_iG_{it}^{\theta_i}G_{jt}^{\rho_i}K_{it}^{\alpha_i} \ . \tag{22}$$

Using these equations,

$$\frac{G_{it+1}}{K_{it+1}} = \frac{\tau_i}{\alpha_i\beta(1-\tau_i)} = \mu_i \ , \quad \forall i = 1,2, \tag{23}$$

which means that the infrastructure-capital ratio is constant over time. Thus, private and public capitals stocks grow at the same rate. The study of the economic dynamics then boils down, for instance, to the analysis of capital accumulation. Substituting the expression of G_{it} given by (23) in (21) yields:

$$K_{it+1} = \Gamma_iK_{it}^{\alpha_i+\theta_i}K_{jt}^{\rho_i} \ , \quad i,j = 1,2, \tag{24}$$

[7] We note that result *i)* and *ii)* are akin to Datta and Mirman (2000)'s conclusions. These authors show, in a dynamic game of investment, that if regions have identical preferences (which would mean here $\nu_i\nu_j = 1$), then the Nash equilibrium coincides with the centralized solution.

with,

$$\Gamma_i = A_i \frac{(\alpha_i\beta)^{1-\theta_i}}{(\alpha_j\beta)^{\rho_i}}(1-\tau_i)^{1-\theta_i}\tau_i^{\theta_i}\left(\frac{\tau_j}{1-\tau_j}\right)^{\rho_i} , \tag{25}$$

for $i, j = 1, 2$.

Expressions (24) and (25) summarize the dynamic links between the two countries. Clearly, country i's conditions of growth will depend not only on the technology parameters (and particularly the returns to scale) but also on public policies undertaken in each country (through the coefficient Γ_i).

In the rest of the paper, we scrutinize economies with constant returns to scale before considering more diversified economies, where one country has diminishing returns while the other country has increasing or constant returns. In each case, we start with the analysis of the dynamic properties shared by MPE and CS paths, as long as the associated tax rates fall into specified intervals. Then we study the differences between the two paths to shed light on the consequences of uncoordinated public investments.

5.1 Economies with constant returns and catching up

The literature on endogenous growth, with a single independant country, has focused heavily on the assumption of constant returns to scale for a reason that appears clearly from expression (24). Setting $\rho_i = 0$ to rule out cross-country technical links, $\alpha_i + \theta_i = 1$ is necessary for the dynamics to follow a balanced growth path (BGP in the sequel). With $\alpha_i + \theta_i < 1$, capital stocks converge to steady state values and there is no growth except in the transition. With $\alpha_i + \theta_i > 1$, capital stocks grow at an ever increasing rate.

At least for the purpose of comparison with this literature, in this section we also assume constant returns with respect to the augmentable factors:

$$\alpha_i + \theta_i + \rho_i = 1, \quad \rho_i > 0, \quad \forall i = 1, 2 . \tag{26}$$

5.1.1 Long term growth *versus* transitory growth

The imbalance of the initial conditions in the capital stocks is crucial to explain the transition. Define the variable $u_t = K_{it} \ / \ K_{jt}$ as a *measure of imbalance*. From equality (26), $\alpha_i + \theta_i - \rho_j = \alpha_j + \theta_j - \rho_i = \phi < 1$, and using (24) the evolution of imbalance can be written as:

$$\frac{K_{it+1}}{K_{jt+1}} = \frac{\Gamma_i}{\Gamma_j}\left(\frac{K_{it}}{K_{jt}}\right)^{\phi} , \tag{27}$$

or:

$$u_{t+1} = \frac{\Gamma_i}{\Gamma_j} u_t^{\phi} . \tag{28}$$

The solution $\{\tilde{u}_t\}_{t=0}^{\infty}$ to this equation converges toward a unique limit $\tilde{u} = \frac{\Gamma_i}{\Gamma_j}^{\frac{1}{1-\phi}}$, this convergence being monotonic and increasing (resp. decreasing) when $u_0 < \tilde{u}$ (resp. when $u_0 > \tilde{u}$).

Let g_{kt} be country k's growth rate at date t:

$$g_{kt} = \frac{K_{kt+1}}{K_{kt}} - 1, \qquad k = i, j.$$

Inserting $K_{jt} = K_{it} \ / \ u_t$ into (24), and using the fact that $\alpha_i + \theta_i + \rho_i = 1$, one can get the expression of growth rates in country i during the transition and along the BGP:

$$g_{it} = \Gamma_i \left(\frac{1}{u_t}\right)^{\rho_i} - 1 \ , \tag{29}$$

$$g_{jt} = \Gamma_j u_t^{\rho_j} - 1 \ , \tag{30}$$

$$\lim_{t \to +\infty} g_{it} = g_i = \lim_{t \to +\infty} g_{jt} = g_j = \Gamma_i^{\frac{\rho_j}{\rho_i + \rho_j}} \Gamma_j^{\frac{\rho_i}{\rho_i + \rho_j}} - 1 \ . \tag{31}$$

Proposition 3 *Under the following conditions on parameters*

$$\theta_i \geq \frac{\alpha_i}{1 + \beta\alpha_i}, \ A_i \geq \frac{1}{\beta\alpha_i(1 - \beta\theta_i)} \ ,$$

for any tax rate $\beta\theta_i \leq \tau_i \leq \theta_i$ *, one has* $\Gamma_i \geq 1$*, therefore* $g_i \geq 0, \forall i = 1, 2$

Proof. see Appendix C. ■

The two above conditions on parameters are sufficient to ensure that, for taxes in the specified intervals, each country grows, in the long run, at a positive constant rate since the parameters Γ_i, the constant part of the growth rates, are greater than one. From the expression of MPE tax rates and CS tax rates, (18) and (20), notice that necessarily $\beta\theta_i \leq \tau_i^C, \tau_i^N$. But it need not be true that $\tau_i^C, \tau_i^N \leq \theta_i$.

Once these conditions are set, we are able to deal with the differences in growth rates.

Proposition 4 *The two countries' growth rates differ during the transition but are indentical in the long run.*

Proof. see expressions (29), (30), (31). ■

So, in contrast with Glomm and Ravikumar (1994), there exists transitional dynamics. Due to the existing heterogeneity, both in terms of public policies and initial dotations in capital, the two countries experiment different growh paths during the transition. In fact, it is possible to distinguish several cases, depending of the initial imbalance:

Proposition 5 *Assume positive long run growth rates. Then:*

1. *when the initial imbalance falls short of the long run imbalance,* $u_0 < \tilde{u}$*, the sequence of growth rates in country* i *is decreasing while the sequence of growth rates in country* j *is increasing. Besides, when* $u_0 < \Gamma_j^{-\frac{1}{\rho_j}}$*, growth rates are always non negative in country* i*,*

$g_{it} \geq 0$, *but country* j *experiences an initial recession, i.e. there exists a date* $\bar{t}$ *such that* $g_{jt} < 0$, $\forall t < \bar{t}$ *and* $g_{jt} \geq 0$, $\forall t \geq \bar{t}$. *When* $\Gamma_j^{-\frac{1}{\rho_j}} \leq u_0$, *growth rates are always non negative in both countries,* $g_{it} \geq 0, g_{jt} \geq 0$.

2. *when the initial imbalance exceeds the long run imbalance,* $\tilde{u} < u_0$, *the sequence of growth rates in country* i *is increasing while the sequence of growth rates in country* j *is decreasing. Besides, when* $u_0 < \Gamma_i^{-\frac{1}{\rho_j}}$, *growth rates are always non negative in country* i, $g_{it} \geq 0$, *but country* j *experiences a intial recession, i.e. there exists a date* $\bar{t}$ *such that* $g_{jt} < 0$, $\forall t < \bar{t}$ *and* $g_{jt} \geq 0$, $\forall t \geq \bar{t}$. *When* $\Gamma_i^{-\frac{1}{\rho_j}} \leq u_0$, *growth rates are always non negative in both countries,* $g_{it} \geq 0, g_{jt} \geq 0$.

Proof. see Appendix D. ■

To summarize, according to the initial gap in capital dotations and the sequences of tax rates, one of the two countries grows at an increasing rate while the other country's growth rate is decreasing until the BGP is reached. And one country can experience an initial recession, depending on the initial imbalance.

We conclude this section with the most important comment on Proposition 4: in the long run, both countries follow the same BGP since their initial differences progressively vanish. This important property contradicts previous arguments found in the literature to explain empirical observations of different growth rates for different countries. From conceptual frameworks using single independant countries, this stylized fact is explained by different technological or preference paremeters (see for instance Glomm and Ravikumar, 1994, on page 1182, or Mankiw, 1995). Interdependancy of economies with constant returns to scale rules out such an explanation. Production possibilities in such a case cannot be considered at the regional level. Rather they are linked in such a way to form a unique production set at the interregional level, despite local differences. But, relaxing the assumption of constant returns in one country, we shall discover in Section 5.2 other explanations for different growth rates.

5.1.2 Impact of strategic behavior on growth

The properties of economic dynamics drawn so far apply both to strategic investments and to centralized investments. It remains to investigates what distinguishes the two scenarii. For instance, could strategic investments in infrastructure improve (or on the contrary jeopardize) the two countries' prospects of growth? A technical property is first required.

Proposition 6 *Assume constant returns to scale in both countries. Growth rates in country* i *are all increasing in* τ_i *iff* $\tau_i \leq \theta_i$. *Under the same condition, growth rates in country* j *are all increasing in* τ_i.

Proof. See appendix E. ■

Requiring a positive impact of taxation on all growth rates is of course very demanding. For tax rates that would exceed the required thresholds, a small variation of taxes could have a negative impact on growth at some date and a positive impact at another date. For instance assume an increase in the tax rate τ_i, from period 0 onwards. It does not necessarily benefit to capital accumulation in country i. Actually, this rise has two opposite effects. Other things equal, it implies a rise in the stock of infrastructure available at the next period (G_{i1}) which tends to increase production (Y_{i1}). This increase in production stimulates investment (I_{i1}) in physical capital at period 1 and capital accumulation in next period (K_{i2}). And it also means a rise in the tax base that positively affects the financing of infrastructure (G_{i2}). On the other hand, the increase in τ_i comes at the expense of current consumption and investment. This reduction of capital at period 1 (K_{i1}), and therefore of production (Y_{i1}) leads two periods ahead to a fall in both capital stock (K_{i2}) and the public good (G_{i2})... In this context, imposing a tax rate lower than θ_i is a mean to ensure the positive effect dominates.

Interestingly enough, the same property, following the same condition, appears in Barro (1990). But it is restricted to the long run growht rates. This generalization in the two-country framework (and for growth rates at any date) was not obvious in the first place, but upon reflection it comes as no surprise. Due to the transboundary externalities, the effects of a rise in τ_i do not stop at country i's frontier. More domestic infrastructure (G_{i1}) means more foreign production (Y_{j1}, to an extent measured by ρ_j). Consequently, it favours both capital accumulation (K_{j2}) and infrastructure provision (G_{j2}). In other words, at a two periods horizon, the increase in τ_i indirectly benefits to country i through the positive externality that links the production to the stock of public good.

The analysis of the consequences of a rise in the tax rate τ_j on g_{it} is very similar. An increase in τ_j has first a positive direct effect on production and capital accumulation in this country once $\tau_j \leq \theta_j$. It tends to reduce the ratio K_{i2}/K_{j2} and so to improve the potential of growth in country i (see equation (29)). Moreover, it stimulates the provision of infrastructure at home (G_{j1}) and positively affects production, capital accumulation and growth in country i. Therefore, the single condition $\tau_j \leq \theta_j$ garantees that an increase in τ_j amounts to a rise in g_{it}.

It is now possible to compare the growth rates obtained in the four possible configurations, domestic-prone consumers *versus* diversified consumers, Nash *versus* cooperation.

Proposition 7 *If $\nu_i > 1$ for $i = 1, 2$, with $\nu_i < \nu_j$, and if furthermore $\tau_i^{AC} \leq \theta_i$ and $\tau_j^C \leq \theta_j$, then*

i) *with domestic-prone consumers, there is not enough growth at MPE tax rates: $g_{it}^{AN} < g_{it}^{AC}$ for $i = 1, 2$,*

ii) *with diversified consumers, a similar ranking holds, $g_{it}^N < g_{it}^C$ for $i = 1, 2$.*

Proof. If $\nu_i > 1$ for $i = 1, 2$ and, for instance, $\nu_i < \nu_j$, then it is possible to rank the tax rates associated with all possible configurations: $\tau_i^{AN} < \tau_i^N < \tau_i^C < \tau_i^{AC}$ and $\tau_j^{AN} < \tau_j^N < \tau_j^{AC} < \tau_j^C$. Therefore it is sufficient to impose $\tau_i^{AC} \leq \theta_i$ and $\tau_j^C \leq \theta_j$ for Proposition 6 to apply. ■

Remark 1 *The statement of Proposition 5 rests on the condition $\tau_i^{AC} \leq \theta_i$ and $\tau_j^C \leq \theta_j$, which is an assumption on endogenous variables. Those endogenous variables are of course functions of the model parameters, and one may prefer a statement that makes explicit the conditions on those parameters underwhich Proposition 7 holds. This can be done as follows. First define the functions*

$$\Theta_i(x, y) = \beta\rho_j \frac{(1 - \beta\alpha_i - \beta\theta_i)(1 + y) + \beta\rho_i(1 + x)}{(1 - \beta)\left[(1 - \beta\alpha_j - \beta\theta_j)(1 + x) + \beta\rho_j(1 + y)\right]}, \quad i, j = 1, 2, \quad i \neq j.$$

and then replace the assumption $\tau_i^{AC} \leq \theta_i$ and $\tau_j^C \leq \theta_j$ by $\theta_i \geq \Theta_i(0, 0)$ and $\theta_j \geq \Theta_j(\nu_j, \nu_i)$. From a methodological point of view, this might be prefered, but it is more difficult to interpret.

Remark 2 *The ranking of strategic and centralized growth rates applies not only in the long run but also in the transition.*

The most spectacular consequence of the under-investment problem exhibited in Proposition 7 is when centralized decisions allow for growth whereas Nash decisions does not.

Corollary 1 ***(Sustainability and cooperation)*** *As in Proposition 7 assume $\nu_i > 1$ for $i = 1, 2$, with $\nu_i < \nu_j$, and $\tau_i^{AC} \leq \theta_i$ and $\tau_j^C \leq \theta_j$. Assume also the scale parameters A_i are such that:*

$$A_i = \left\{ \frac{(\alpha_i\beta)^{1-\theta_i}}{(\alpha_j\beta)^{\rho_i}} (1 - \tau_i^N)^{1-\theta_i} \left(\tau_i^N\right) \left(\frac{\tau_j^N}{1 - \tau_j^N}\right)^{\rho_i} \right\}^{-1}, \quad i = 1, 2,$$

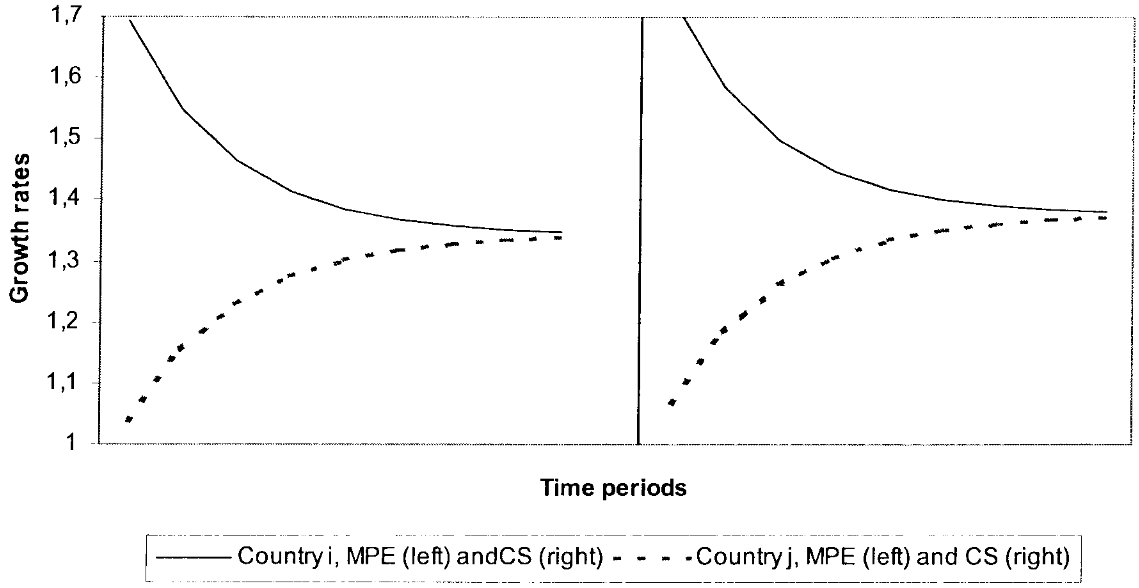
then there is no long run growth at MPE tax rates, whereas countries experience positive long run growth rates under the centralized scenario.

Proof. Under the mentioned conditions on parameters A_i the functions Γ_1, Γ_2 evaluated at MPE tax rates are equal to unity, therefore the long run growth rate are zero. On the other hand, according to Proposition 6 growth rates are larger, therefore positive, under the centralized scenario when $\nu_i > 1$ for $i = 1, 2$, with $\nu_i < \nu_j$, and $\tau_i^{AC} \leq \theta_i$ and $\tau_j^C \leq \theta_j$ (or equivalently $\theta_i \geq \Theta_i(0, 0)$ and $\theta_j \geq \Theta_j(\nu_j, \nu_i)$). ■

There are many ways to define sustainability. If it is understood as the simple idea of "enduring growth", then it is clear that in some circumstances sustainability does not rest only on production possibilities: it also requires cooperation.

It should be stressed that the conditions of Proposition 7 (and Corollary 1) are sufficient but not necessary for the property of too little growth at MPE public investments. Figure 1 illustrates this, with numerical values such that both MPE tax rates and CS tax rates fall outside the set of values for which the proposition applies.

Figure 1: too little growth at non cooperative public investments



Parameter values for Figure 1

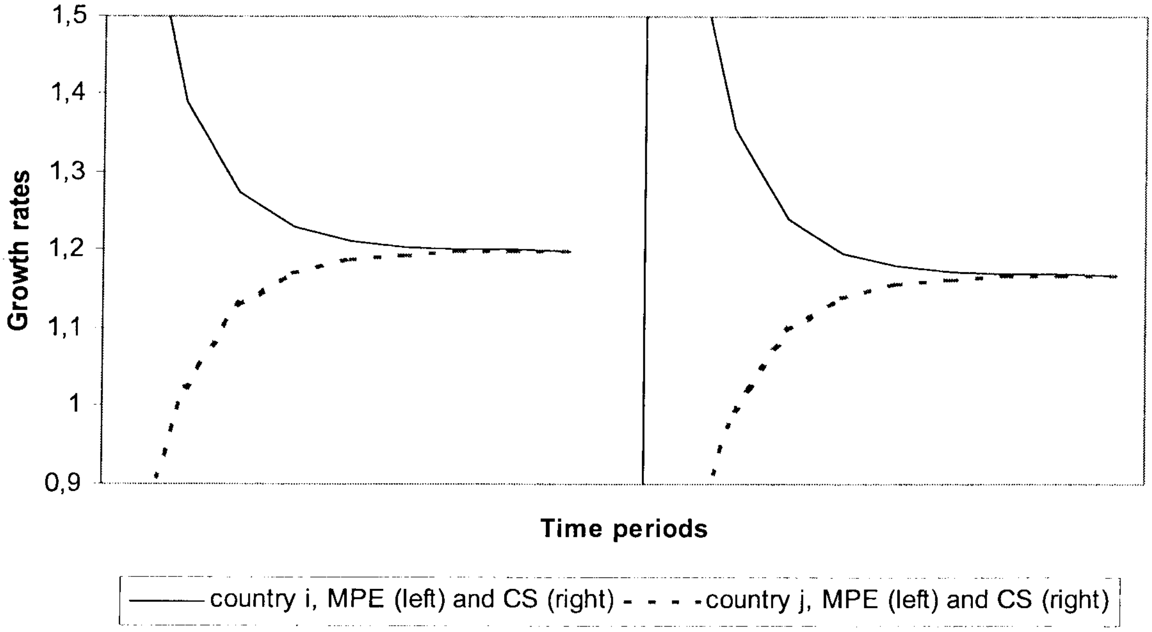
$\alpha_i = 0.3$	$\theta_i = 0.5$	$\rho_i = 0.2$	$\beta_i = \beta = 0.8$	$A_i = 7$	$k_{i0} = 1$	$\nu_i = 1.5$
$\alpha_j = 0.3$	$\theta_j = 0.5$	$\rho_j = 0.2$	$\beta_j = \beta = 0.8$	$A_j = 7$	$k_{j0} = 2$	$\nu_j = 1.5$

Tax rates for Figure 1

$$\tau_i^N \simeq 0.53, \quad \tau_j^N \simeq 0.53, \quad \tau_i^C \simeq 0.56, \quad \tau_i^C \simeq 0.56.$$

But for too large values of tax rates, outside the admissible range, the negative effect of taxation on growth rates dominates and there is too much growth at MPE tax rates, even with consumers who prefer their domestic good, as illustrated in the second example below.

Figure 2: too much growth at non cooperative public investments



Parameter values for Figure 2

$\alpha_i = 0.4$	$\theta_i = 0.3$	$\rho_i = 0.3$	$\beta_i = \beta = 0.8$	$A_i = 7$	$k_{i0} = 1$	$\nu_i = 1.5$
$\alpha_j = 0.4$	$\theta_j = 0.3$	$\rho_j = 0.3$	$\beta_j = \beta = 0.8$	$A_j = 7$	$k_{j0} = 2$	$\nu_j = 1.5$

Tax rates for Figure 2

$$\tau_i^N \simeq 0.45, \quad \tau_j^N \simeq 0.45, \quad \tau_i^C \simeq 0.48, \quad \tau_i^C \simeq 0.48.$$

Another rationale for too much growth at MPE is when households value less their domestic good.

Proposition 8 *With diversified consumers who prefer the foreign good ($\nu_i < 1$), if $\tau_i^N \leq \theta_i$ for $i = 1, 2$ there is too much growth at MPE tax rates: $g_{it}^C < g_{it}^N$ for $i = 1, 2$.*

Proof. If $\nu_i, \nu_j < 1$ and, for instance, $\nu_i < \nu_j$, again it is possible to rank the tax rates associated with all possible configurations: $\tau_i^{AN} < \tau_i^C < \tau_i^{AC} < \tau_i^N$ and $\tau_j^{AN} < \tau_j^{AC} < \tau_j^C < \tau_j^N$ (since $\tau_i^N > \tau_i^{AN}$, $i = 1, 2$). For each country, τ_i^N is the highest rate. Therefore, it is sufficient to impose $\tau_i^N \leq \theta_i$, $i = 1, 2$ to satisfy the conditions of Proposition 6 and to conclude. ■

Remark 3 *Here again it is possible to state this result by making explicit the required assumptions on parameters, since $\tau_i^N \leq \theta_i \Leftrightarrow \Theta_i(\nu_i - 1, 0) \leq \theta_i$.*

There exists the widespread belief, as far as growth is concerned, that more is necessarily better. The last proposition destroys this belief. The intuition is simple: investment is required for growth, which is good for future consumption and welfare, but it comes at the expense of current generations, that is the generations that are valued the most in the discounted criterion used to assess efficiency. Clearly it is possible to invest too much. Proposition 8 pins down this possibility. At non cooperative tax rate, because domestic households prefer the foreign good, local decision maker i neglects domestic consumption and favor investment as an indirect way to increase the foreign production and to consume more of it. Decision maker j behaves similarly and both countries settle for too much production of their domestic good resulting from too high investments therefore too much growth.

Cooperation has also a role to play in the transition. To see this, two particular cases are worth noting, for their ability to be easily interpreted and for the conclusions they deliver. Let the capital stocks at date zero and tax rates be such that the initial imbalance falls short of the long run imbalance, both under MPE and CS scenarios:

Condition 1 $u_0 < \tilde{u}^N = \left(\Gamma_i^N / \Gamma_j^N\right)^{1/(1-\phi)}$ *and* $u_0 < \tilde{u}^C = \left(\Gamma_i^C / \Gamma_j^C\right)^{1/(1-\phi)}$.

Let also the households prefer their domestic good, so that long run growth factors at MPE are too small (Proposition 7) and assume finally the initial imbalance lies in a specific interval:

Condition 2 $\left(1/\Gamma_j^C\right)^{1/\rho_j} < u_0 < \left(1/\Gamma_j^N\right)^{1/\rho_j}$.

Under Conditions 1 and 2, Proposition 5 indicates that country i has positive growth rates at any date whereas country j experiences a initial recession at MPE tax rates; Proposition 5 also states that both countries have positive growth rates at any date under CS tax rates. This proves the following:

Proposition 9 *Assume constant returns to scale. Let the households prefer their domestic good. Let the initial imbalance of private capital stock satisfy Conditions 1 and 2. Then cooperation prevents country j from undergoing an economic recession during the first stages of development.*

There is not enough investment at MPE tax rates. Increased efficiency calls for higher growth rates and no recession in the economy.

The second interesting example is obtained when households prefer the foreign good; MPE tax rates are too large, which can be compatible with an initial imbalance such that:

Condition 3 $\left(1/\Gamma_j^N\right)^{1/\rho_j} < u_0 < \left(1/\Gamma_j^C\right)^{1/\rho_j}$.

Substituting Condition 3 for 2, while maintaining Condition 1, we learn from Proposition 5 that both countries have positive growth rates at any date at MPE tax rates; as for CS tax rates, Proposition 5 states that country i has positive growth rates at any dates, but country j experiences negative growth rates before recovering. Thus:

Proposition 10 *Assume constant returns to scale. Let the households prefer the foreign good. Let the initial imbalance of private capital stocks satisfy Conditions 1 and 3. Then cooperation calls for an initial recession in country j whereas strategic investement does not.*

In the case described in Proposition 10, the centralized level of imbalance is so far away from the initial level of imbalance that an initial recession in country j is called upon to reduce the gap between capital stocks; at the same time the discrepancy between the MPE imbalance at the initial level is not so large to call also for an initial recession.

5.2 Economies with different balanced growth rates

The economy just analyzed has two distinguishing features: bilateral externalities and constant returns to scale. It is important to unravel the role played by those specificities in the striking result of different countries having the same BGP.

5.2.1 Relaxing the assumption of constant returns to scale

One may investigate first the dynamic properties of the system (24) when the assumption of constant returns to scale is relaxed. Working with growth factors, the dynamics are:

$$\begin{cases} 1 + g_{1t} = (1 + g_{1t-1})^{\alpha_1+\theta_1} (1 + g_{2t-1})^{\rho_1} \ , & g_{10} = \Gamma_1 K_{10}^{\alpha_1+\theta_1-1} K_{20}^{\rho_1} - 1 \ , \\ 1 + g_{2t} = (1 + g_{2t-1})^{\alpha_2+\theta_2} (1 + g_{1t-1})^{\rho_2} \ , & g_{20} = \Gamma_2 K_{20}^{\alpha_2+\theta_2-1} K_{10}^{\rho_2} - 1. \end{cases}$$

From well-established properties of planar systems (see for instance Azariadis, 1993, Chapter 4), some conclusions immediately follow. Under decreasing returns, $\alpha_i + \theta_i + \rho_i < 1$, the steady state with no growth, $g_{it} = 0$, is globally stable. With constant returns, as previously shown both economies converges to the same BGP. It remains to analyze the case of increasing returns, $\alpha_i + \theta_i + \rho_i > 1$.The steady state without growth becomes saddle path stable. More interesting are of course the possibilities for other steady states growth rates. A necessary condition for the existence of those other steady states is

$$(1 - \alpha_1 - \theta_1)(1 - \alpha_2 - \theta_2) = \rho_1 \rho_2 \ . \tag{32}$$

We discard the cases where one or both countries exhibits constant returns with respect to its national factors, and the cases where $\rho_1 = 0$ and/or $\rho_2 = 0$. The details about those last cases are postponed to the next subsection, where the important situations of unidirectional externalities are discussed.

When (32) holds, $\alpha_1+\theta_1 \neq 1$, $\alpha_2+\theta_2 \neq 1$ and $\rho_1, \rho_2 \neq 0$, there is a one-dimensional manifold of steady states defined by

$$1+g_j = (1+g_i)^{\frac{1-\alpha_i-\theta_i}{\rho_i}} .$$

In our two-country framework, equality (32) is a key condition for positive balanced growth rates. As in two-sector models of endogenous growth (see Mulligan and Sala-I-Martin, 1993), it does not imply constant returns to scale. For instance it is consistent with diminishing returns in country 1 provided it is offset by appropriate increasing returns in country 2 : $\alpha_1+\theta_1+\rho_1 < 1$, $\alpha_2+\theta_2+\rho_2 > 1$ and (32) hold together. But if there are constant returns in one country, there must be constant returns in the other.

Condition (32) does not imply either that long run growth rates be identical, except when there are constant returns to scale in both countries, thus $(1-\alpha_i-\theta_i)/\rho_i = \rho_j/(1-\alpha_j-\theta_j) = 1$, or when the parameters are such that $\alpha_1+\theta_1 = \alpha_2+\theta_2$ and $\rho_1=\rho_2$.

The stability of those steady states for growth rates can be infered from the topologically equivalent linear system that obtains by logarithmic transformation, $u_t = \log(1+g_{1t})$, $v_t = \log(1+g_{2t})$:

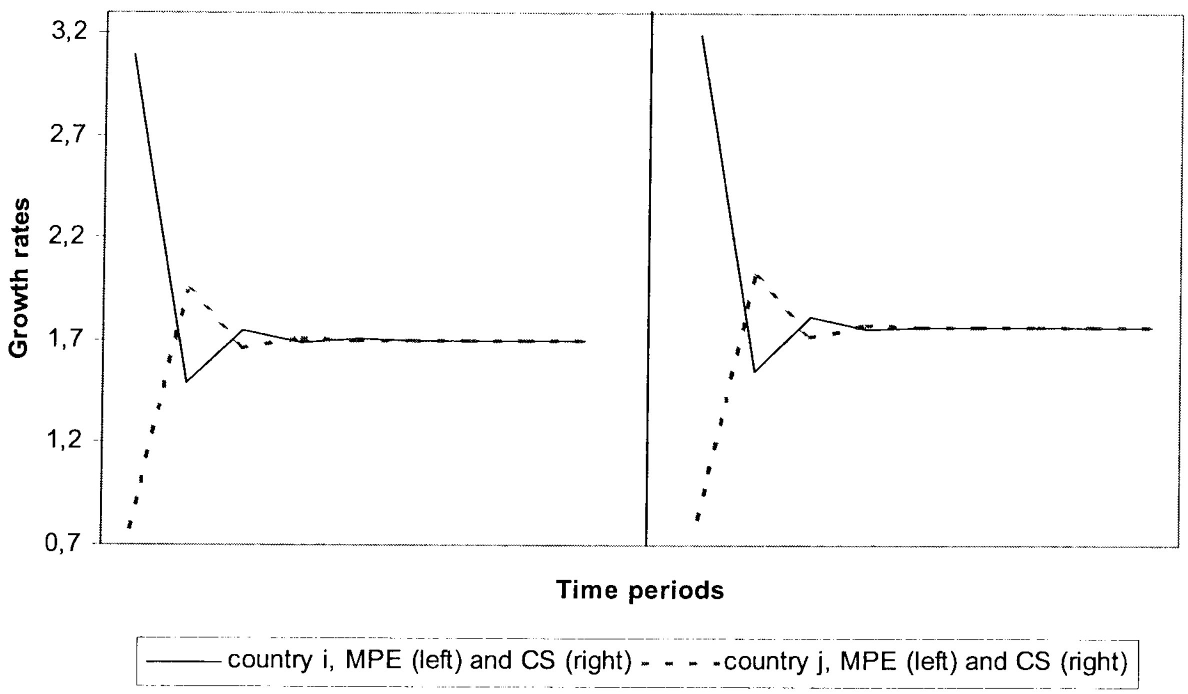
$$\begin{cases} u_t = (\alpha_1+\theta_1)\,u_{t-1} + \rho_1 v_{t-1} \ , & u_0 = \log\left(\Gamma_1 K_{10}^{\alpha_1+\theta_1-1} K_{20}^{\rho_1}\right) \ , \\ v_t = (\alpha_2+\theta_2)\,v_{t-1} + \rho_2 u_{t-1} \ , & v_0 = \log\left(\Gamma_2 K_{20}^{\alpha_2+\theta_2-1} K_{10}^{\rho_2}\right) . \end{cases} \tag{33}$$

It has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = \alpha_1+\theta_1+\alpha_2+\theta_2-1$. Stability depends crucially on λ_2, which can cross over several bifurcation values. When $0 < \alpha_1+\theta_1+\alpha_2+\theta_2 < 1$, the dynamics exhibits dampened oscillations around a BGP (this possibility is illustrated on figure 3); when $\alpha_1+\theta_1+\alpha_2+\theta_2 = 1$, the second eigenvalue is zero, there is no transitional dynamics, variables jump directly to a BGP; when $1 < \alpha_1+\theta_1+\alpha_2+\theta_2 < 2$, there is a transitional dynamics toward a BGP; when $\alpha_1+\theta_1+\alpha_2+\theta_2 = 2$ the second eigenvalue is also equal to 1, the steady states are unstable[8]; finally, when $\alpha_1+\theta_1+\alpha_2+\theta_2 > 2$ the BGP are also unstable. When stability obtains, the initial conditions pick up a unique path that converges to a unique BGP on the unidimensional manifold: therefore BGP depends both on initial capital stocks and on tax policies.

[8]In that case, the solutions are:

$$\begin{aligned} u_t &= u_0 + [\rho_1 v_0 - (1-\alpha_1-\theta_1)\ u_0]\ t \ , \\ v_t &= u_0 + [\rho_1 v_0 - (1-\alpha_1-\theta_1)\ u_0]\ t \ . \end{aligned}$$

Figure 3: convergence to BGP with dampened oscillations



Parameter values for Figure 3

$\alpha_i = 0.2$	$\theta_i = 0.2$	$\rho_i = 0.6$	$\beta_i = \beta = 0.8$	$A_i = 7$	$k_{i0} = 1$	$\nu_i = 1.5$
$\alpha_j = 0.2$	$\theta_j = 0.2$	$\rho_j = 0.6$	$\beta_j = \beta = 0.8$	$A_j = 7$	$k_{j0} = 2$	$\nu_j = 1.5$

Tax rates for Figure 3

$$\tau_i^N \simeq 0.6, \quad \tau_j^N \simeq 0.6, \quad \tau_i^C \simeq 0.64, \quad \tau_i^C \simeq 0.64.$$

Proposition 11 *Assume parameters allows for BGP, i.e. (32) holds. Also, let there be increasing returns in one country and decreasing returns in the other country. Then:*

1. *the highest long run growth rate is associated to the country with increasing returns;*
2. *cooperation increases (respectively decreases) the gap between balanced growth rates when consumers prefer their domestic commodity (respectively the foreign commodity).*

   Proof. *Appendix F.* ■

So, a country with decreasing returns can experience a positive BGP! Actually, the positive externality in production plays an essential role insofar as it allows, say, country j to benefit from the economic development in country i. In this context, the engine of growth for country j is the growth in country i that stimulates, through the infrastructure externality channel, both its production and its capital accumulation.

5.2.2 The case of unidirectional externalities

When there are no externalities at all and constant returns to scale, countries have independent dynamics and different technologies or preferences may end up in different BGP. With bilateral externalities, the heterogeneity of BGP disappear. But what for the intermediate case of an unilateral externality? This is illustrative of the bulk of externality problems endowed with geographical attributes. An international river is a good example: any public investment made in the upstream country to improve the water quality benefits the downstream country, while the converse is not true. There are about 200 such international rivers in the world, distributed across the African, Asian, American and European continents. Egypt is the most spectacular example with 97 % of its water resources originating outside its borders.

The focus is now on the special case of one-way technological externality. Assume in addition the technology in one of the two countries exhibits constant returns to scale. More precisely:

$$\begin{array}{l} \alpha_i + \theta_i = 1,\ \rho_i = 0\ , \\ \alpha_j + \theta_j + \rho_j \lesseqgtr 1\ . \end{array} \tag{34}$$

Country i is assumed to have a technology with constant returns to domestic inputs, but is not subject to the transboundary externality ($\rho_i = 0$). The other country benefits from the positive effects of the foreign investment in infrastructure ($\rho_j > 0$).

In this context, from (24), the equations describing capital accumulation become, respectively for i and j[9]:

$$K_{it} = \Gamma_i^t K_{i0} \tag{35}$$

$$K_{jt+1} = \Gamma_j \left(\Gamma_i^t K_{i0}\right)^{\rho_j} K_{jt}^{\alpha_j+\theta_j} \tag{36}$$

The main consequence of the absence of an externality, for country i, is that it directly follows a BGP where the economy grows at a constant rate $g_i = \Gamma_i - 1$ (positive under the assumptions of Proposition 3). What about the dynamics in country j?

Proposition 12 *Assume Country j has decreasing returns with respect to the domestic factors ($\alpha_j + \theta_j < 1$). Country j experiences a process of sustained growth, and its balanced growth rate is:*

i) lower than country i's balanced growth rate when $\alpha_j + \theta_j + \rho_j < 1$;

ii) equal to country i's balanced growth rate when $\alpha_j + \theta_j + \rho_j = 1$;

iii) larger than country i's balanced growth rate when $\alpha_j + \theta_j + \rho_j > 1$

[9] It is worth noting that the central argument used in the proof of the existence of a world equilibrium (see proposition 1) lies in the fact that the two objectives are finite. This result is straighforward once we consider the dynamics (35) in region i and the utility function. It is also true for the dynamics (36) since proposition 1 applies by replacing the condition on the sequences $\{G_{it}\}_{t=0}^{\infty}$ and $\{G_{jt}\}_{t=0}^{\infty}$ by a single condition on the sequence $\{K_{it}\}_{t=0}^{\infty}$. Therefore, the dynamics given by (35)-(36) clearly corresponds to the world equilibrium defined in section 3.1 when the restrictions (34) are set.

Proof. The solution to the difference equation (36) can be written as follows:

$$K_{jt} = \left(\Gamma_j \left(K_{i0}\right)^{\rho_j}\right)^{\frac{1-\eta^t}{1-\eta}} \Gamma_i^{\rho j \sum_{k=1}^{t} \eta^{k-1}(t-k)} K_{j0}^{\eta^t}$$

with $\eta = \alpha_j + \theta_j < 1$ (to find this expression, simply express K_{j1} as a function of K_{j0}, then K_{j2} as a function of $K_{j1}(K_{j0})$, thus as a function $K_{j2}(K_{j0})$, and so on and so forth until date t.)

Using this expression, the growth factor at date $t+1$ is

$$\begin{aligned} \frac{K_{jt+1}}{K_{jt}} &= \frac{\left(\Gamma_j \left(K_{i0}\right)^{\rho_j}\right)^{\frac{1-\eta^{t+1}}{1-\eta}} \Gamma_i^{\rho j \sum_{k=1}^{t+1} \eta^{k-1}(t+1-k)} K_{j0}^{\eta^{t+1}}}{\left(\Gamma_j \left(K_{i0}\right)^{\rho_j}\right)^{\frac{1-\eta^t}{1-\eta}} \Gamma_i^{\rho j \sum_{k=1}^{t} \eta^{k-1}(t-k)} K_{j0}^{\eta^t}}, \\ &= \left(\Gamma_j \left(K_{i0}\right)^{\rho_j}\right)^{\eta^t} \Gamma_i^{\rho j \frac{1-\eta^t}{1-\eta}} K_{j0}^{\eta^t(\eta-1)}. \end{aligned}$$

Therefore, because $\eta < 1$

$$\lim_{t \to +\infty} \frac{K_{jt+1}}{K_{jt}} = \Gamma_i^{\frac{\rho j}{1-\eta}}.$$

When $\rho_j/(1-\eta) < 1$ (this is equivalent to the assumption $\alpha_j + \theta_j + \rho_j < 1$) and $\Gamma_i > 1$, necessarily $1 < \Gamma_i^{\frac{\rho j}{1-\eta}} < \Gamma_i$. When $\rho_j/(1-\eta) = 1$ (or equivalently $\alpha_j + \theta_j + \rho_j = 1$), then $\Gamma_j = \Gamma_i^{\frac{\rho j}{1-\eta}} = \Gamma_i$. Finally, when $\rho_j/(1-\eta) > 1$ (or $\alpha_j + \theta_j + \rho_j > 1$), then $\Gamma_j = \Gamma_i^{\frac{\rho j}{1-\eta}} > \Gamma_i$. ■

So the property that country j's capital stock can indefinitely grow despite decreasing returns, already found in the case of bilateral externalities, hold also with unidirectional externalities.

Figure 4 provides an illustration. Notice that there is too little growth at MPE tax rates. This is not surprising since the consumers of this example prefer the domestic good (remember Proposition 5). Also, it seems in this example that the gap between growth rates is larger under cooperation. The following statement clarify this last property of cooperation:

Proposition 13 *Let the parameters be as in (34) but without constant returns in country j. Then cooperation increases (respectively decreases) the gap between balanced growth rates when consumers prefer their domestic commodity (respectively the foreign commodity).*

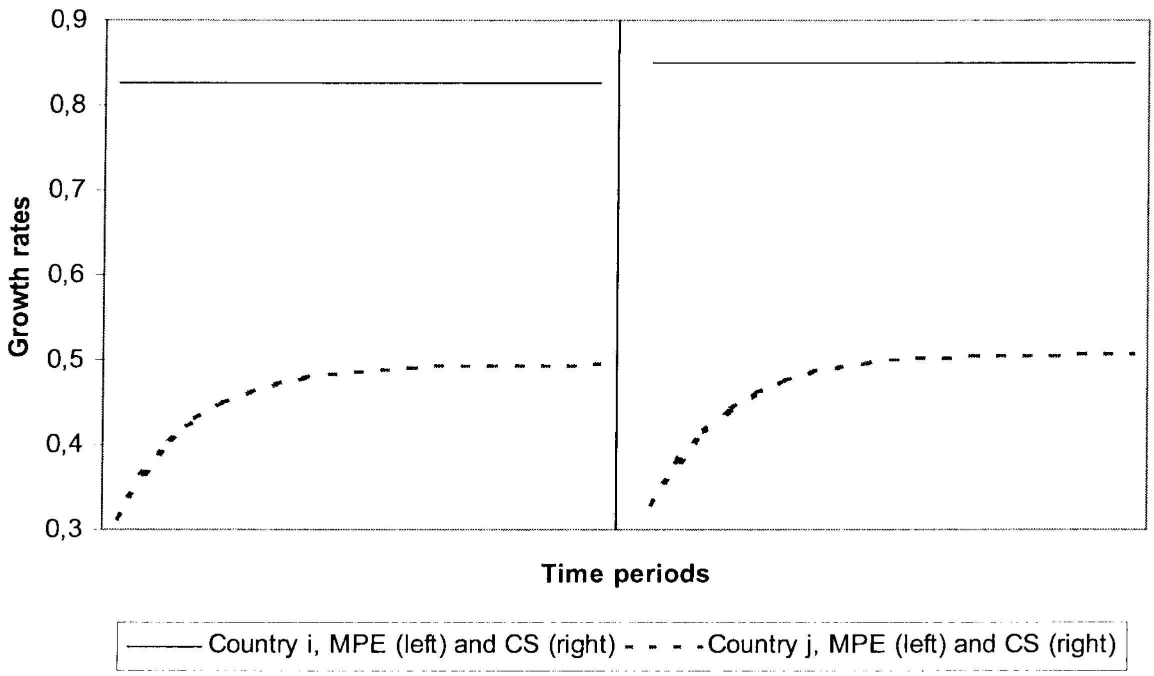
Proof. As soon as $\rho_j/(1-\eta) \neq 1$ (or equivalently $\alpha_j + \theta_j + \rho_j \neq 1$), the gap

$$|\Gamma_j - \Gamma_i| = \left|\Gamma_i^{\frac{\rho j}{1-\eta}} - \Gamma_i\right|$$

is an increasing function of $\Gamma_i \in [1, +\infty[$; and we know from Proposition 5 that Γ_i computed at CS tax rates is larger (respectively lower) than MPE tax rates when consumers prefer the domestic commodity (respectively the foreign commodity). ■

In the asymmetric situation under consideration, MPE tax rates and CS tax rates for country j are the same (to check that, see the expressions for tax rates when $\rho_i = 0$). Therefore country j growth rates are also the same under either scenario. When consumers prefer the domestic good, cooperation requires to increase country i growth rate, hence a higher gap. On the contrary, when consumers value more the foreign good, efficiency calls for a lower growth rate for country i, therefore a smaller gap between countries' growth rates.

Figure 4: different balanced growth rates



Parameter values for Figure 4

$\alpha_i = 0.5$	$\theta_i = 0.5$	$\rho_i = 0$	$\beta_i = \beta = 0.6$	$A_i = 7$	$k_{i0} = 1$	$\nu_i = 1.5$
$\alpha_j = 0.3$	$\theta_j = 0.4$	$\rho_j = 0.2$	$\beta_j = \beta = 0.6$	$A_j = 7$	$k_{j0} = 3$	$\nu_j = 1.5$

Tax rates for Figure 4

$$\tau_i^N \simeq 0.35, \quad \tau_j^N \simeq 0.24, \quad \tau_i^C \simeq 0.37, \quad \tau_i^C \simeq 0.24.$$

A word of warning: the previous example might give the impression that, with interdependent economies, growth is guaranted when at least one country has constant or increasing returns. A counter-example is provided here. Assume:

$$\begin{aligned} &\alpha_i + \theta_i < 1, \ \rho_i = 0 \ , \\ &\alpha_j + \theta_j + \rho_j = 1 \ . \end{aligned} \tag{37}$$

The first country evolves independently under a regime of decreasing returns: it has no growth in the long run. As a consequence, because infrastructure are necessary for the production in country j, there is no growth there as well in the long run.

6 Conclusion

This paper deals with the consequences of strategic public investments on growth. To do so it constructs a two-country model with public infrastructure as inputs in the production technologies. Each country has three types of agents: firms, households and a local government. Local governments levy a share of the domestic households' income to finance the provision of infrastructure that improves the efficiency of private inputs in production. In addition, public investment in one country is assumed to produce positive spillovers on the foreign production. Public authorities behave strategically when they choose the amount to invest in infrastructure while private agents take the public policy as given when making their trade-offs.

In this setting, the main results can be summarized as follows.

First, when technologies exhibit constant returns to scale in reproducible inputs, we show that the two countries' growth rates differ during the transitional dynamics. This gap in growth performance results from the existing heterogeneity among countries. In fact, countries are endowed with different initial capital stocks, have different technologies and preferences and therefore implement different public policies. Due to the interaction between countries, these differences play no role in the long run and countries tend to the same balanced growth rate. However, there is no convergence in levels of consumption and output since there remains a discrepancy in production levels that is explained by distinct political orientations. Next, we prove that the quest for efficiency does not necessarily means higher growth rates. More precisely, when households in each country prefer the commodity produced abroad, local governments have the incentive to strenghten their fiscal policy to promote the production of their citizens' most preferred good, namely the foreign good. This strategy goes hand-in-hand with an overcontribution to infrastructure and implies that Nash growth rates are higher than the centralized ones. It is also established that cooperation can prevent an economic recession in one country in the early stages of development when households prefer their domestic good, but on the contrary cooperation may call for an initial recession, that would not occur under strategic investments, when households prefer the foreign good.

Second, assuming away constant returns to scale, growth in both countries is still possible, even when one country has diminishing returns to scale provided that it can benefit from a growing externality from the the other country. Countries cease to converge towards the same growth rates. The country with the most advantagous technology grows faster. Finally, it is established that cooperation increases (respectively decreases) the gap between growth rates when households prefer the domestic (respectively the foreign) commodity.

References

[1] Arrow K. and M. Kurtz (1970), "Public investment, the rate of return and, optimal fiscal policy". John Hopkins University Press.

[2] Aschauer D. (1989a), "Is public expenditure productive ?", *Journal of Monetary Economics*, vol. 23, pp. 177-200.

[3] Aschauer D. (1989b), "Does public capital crowd out private capital ?", *Journal of Monetary Economics*, vol. 24, pp. 171-188.

[4] Azariadis C. (1993), "Intertemporal macroeconomics", Blackwell Publishers, Oxford UK & Cambridge USA.

[5] Barro R. (1990), "Government spending in a simple model of endogenous growth", *Journal of Political Economy*, vol. 98, pp. 103-125.

[6] Datta M. and L. Mirman (2000), "Dynamic externalities and policy coordination". *Review of International Economics*, pp. 44-59.

[7] Glomm G. and B. Ravikumar (1994), "Public investment in infrastructure in a simple growth model". *Journal of Economic Dynamics and Control*, pp. 1173-1187.

[8] Gramlich E. (1994), "Infrastructure investment: a review essay", *Journal of Economic Literature*, vol. 32, pp. 1176-1196.

[9] Mankiw G. (1995), "The growth of nations", *Brookings Papers on Economic Activities*, vol. 1, pp. 275-326.

[10] Mulligan C. B. and X. Sala-I-Martin (1993), "Transitional Dynamics in two-sector models of endogenous growth". *Quaterly Journal of Economics*, pp. 739-773.

Appendix

A Derivation of the π-CE

The Hamiltonian associated with the artificial planning problem (11) reads as:

$$\begin{aligned} H^i\left(c_{iit}, c_{ijt}, K_{it}, \lambda_{t+1}\right) &= \beta^t\left(\nu_i \ln c_{iit} + \ln c_{ijt}\right) \\ &\quad +\lambda_{t+1}\left[(1-\tau_{it})A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} - c_{iit} - p_t c_{ijt}\right] , \end{aligned}$$

where λ_{t+1} is the shadow price of the resource constraint.

The first order conditions are:

$$\frac{\partial H^i}{\partial c_{iit}} = 0 \Leftrightarrow \frac{\beta^t \nu_i}{c_{iit}} = \lambda_{t+1} , \tag{38}$$

$$\frac{\partial H^i}{\partial c_{ijt}} = 0 \Leftrightarrow \frac{\beta^t}{c_{ijt}} = p_t \lambda_{t+1} , \tag{39}$$

and

$$\lambda_t = \frac{\partial H^i}{\partial K_{it}} = \lambda_{t+1}\alpha_i(1-\tau_{it})A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i - 1} . \tag{40}$$

A relationship between the optimal consumptions of the two goods is obtained from (38) and (39):

$$c_{ijt} = \frac{c_{iit}}{\nu_i p_t} . \tag{41}$$

As in Glomm and Ravikimar (1994) let us postulate, and afterwards confirm, that optimal decisions are linear functions of the after tax income. In particular

$$c_{iit} = m_i R_{i,t} , \tag{42}$$

where $R_{i,t} = (1-\tau_{it})A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i}$. Therefore, using (41) and the dynamic equation of the capital stock:

$$c_{ijt} = \frac{m_i}{\nu_i p_t} R_{i,t} , \tag{43}$$

$$K_{i,t+1} = \left(1 - m_i - m_i \nu_i^{-1}\right) R_{i,t} , \tag{44}$$

From (38) and (42), one can write:

$$\lambda_{t+1} = \frac{\beta^t \nu_i}{m_i R_{i,t}} .$$

Inserting this expression in (40), one also has:

$$\begin{aligned} \lambda_t &= \frac{\beta^t \nu_i}{m_i R_{i,t}} \alpha_i (1-\tau_{it}) A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i - 1} , \\ &= \frac{\beta^t \nu_i}{m_i R_{i,t}} * \frac{\alpha_i R_{i,t}}{K_{i,t}} , \\ &= \frac{\beta^t \nu_i \alpha_i}{m_i K_{i,t}} . \end{aligned}$$

Or, changing the time period

$$\lambda_{t+1} = \beta^{t+1}\frac{\nu_i\alpha_i}{m_iK_{i,t+1}} .$$

Therefore,

$$\begin{aligned}
K_{i,t+1} &= R_{i,t} - c_{ii,t} - p_t c_{ij,t} , \\
&= R_{i,t} - \frac{\beta^t\nu_i}{\lambda_{t+1}} - \frac{\beta^t}{\lambda_{t+1}} , \\
&= R_{i,t} - \frac{\beta^t\nu_i}{\beta^{t+1}\frac{\nu_i\alpha_i}{m_i}}K_{i,t+1} - \frac{\beta^t}{\beta^{t+1}\frac{\nu_i\alpha_i}{m_i}}K_{i,t+1} , \\
&= R_{i,t} - \frac{m_i}{\alpha_i\beta}K_{i,t+1} - \frac{m_i}{\alpha_i\nu_i\beta}K_{i,t+1}.
\end{aligned}$$

So, rearranging this last expression

$$\begin{aligned}
K_{i,t+1} &= \left[1 + \frac{m_i}{\alpha_i\beta} + \frac{m_i}{\alpha_i\nu_i\beta}\right]^{-1} R_{i,t} , \\
&= \frac{\alpha_i\nu_i\beta}{\alpha_i\nu_i\beta + m_i\nu_i + m_i} R_{i,t} .
\end{aligned}$$

By identification of this last expression with (44), a simple equation for m_i is obtained:

$$\frac{\alpha_i\nu_i\beta}{\alpha_i\nu_i\beta + m_i\nu_i + m_i} = 1 - m_i - m_i\nu_i^{-1} ,$$

whose solution is

$$m_i = \frac{\nu_i}{1+\nu_i}(1-\alpha_i\beta) .$$

To summarize:

$$\begin{aligned}
c_{iit} &= \frac{\nu_i}{1+\nu_i}(1-\alpha_i\beta)R_{i,t} , \\
c_{ijt} &= \frac{\nu_i}{\nu_i p_t(1+\nu_i)}(1-\alpha_i\beta)R_{i,t} , \\
K_{i,t+1} &= \alpha_i\beta R_{i,t} ,
\end{aligned}$$

as given by (12), (14) and (13) in the text.

The interested reader may want to check that this planning problem combines a static optimization problem (how to allocate optimally at each period the resources net of investment between the consomption of the two goods) with an intertemporal problem (the trade-off between consumption and investment).

B Markov perfect equilibrium tax rates

Let $v_i(K_{it}, G_{it}, K_{jt}, G_{jt})$ be country i's value function for the subgame starting at date t with stock variables $K_{it}, G_{it}, K_{jt}, G_{jt}$ inherited from past decisions. In the Cobb-Douglas game framework at hand, it makes sense to guess value functions of the following form:

$$v_i(K_{it}, G_{it}, K_{jt}, G_{jt}) = D_i \ln K_{it} + F_i \ln G_{it} + H_i \ln K_{jt} + J_i \ln G_{jt} , \quad i,j = 1,2, \quad i \neq j,$$

where D_i, F_i, H_i and J_i are some constants to be determined. At a subgame perfect equilibrium country i's tax rate at date t solves the Bellman equation:

$$v_i(k_{it}, G_{it}, k_{jt}, G_{jt}) = \max_{\tau_{it}} \{V_i(K_{it}, K_{jt}, G_{it}, G_{jt}) + \beta v_i(K_{it+1}, G_{it+1}, K_{jt+1}, G_{jt+1})\},$$

$$\text{where} \begin{cases} G_{it+1} = \tau_{it} A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} , \\ K_{it+1} = \alpha_i \beta (1 - \tau_{it})\, A_i G_{it}^{\theta_i} G_{jt}^{\rho_i} K_{it}^{\alpha_i} , \\ i, j = 1, 2, \\ \tau_{jt} \text{ given.} \end{cases}$$

The first order condition for the maximization of the r.h.s. of the two Bellman equations are:

$$\frac{-\nu_i}{1 - \tau_{it}} - \beta D_i \frac{\alpha_i \beta Y_{it}}{\alpha_i \beta (1 - \tau_{it}) Y_{it}} + \beta F_i \frac{Y_{it}}{\tau_{it} Y_{it}} = 0, \quad i = 1, 2.$$

Their solutions read as:

$$\tau_{it} = \frac{\beta F_i}{\nu_i + \beta Di + \beta F_i}, \quad i = 1, 2, \quad \forall t. \tag{45}$$

Inserting those expressions into the Bellman equations, and because those equations hold for any values of the stock variables, identification of similar terms ends up in the following system of equations:

$$D_i = \alpha_i(\nu_i + \beta D_i + \beta F_i) , \tag{46}$$

$$F_i = \theta_i(\nu_i + \beta D_i + \beta F_i) + \rho_j (1 + \beta H_i + \beta J_i) , \tag{47}$$

$$H_i = \alpha_j (1 + \beta H_i + \beta J_i) , \tag{48}$$

$$J_i = \rho_i (\nu_i + \beta D_i + \beta F_i) + \theta_j (1 + \beta H_i + \beta J_i) . \tag{49}$$

Note, from (46):

$$\nu_i + \beta D_i + \beta F_i = \frac{D_i}{\alpha_i} = \frac{\nu_i + \beta F_i}{1 - \alpha_i \beta} , \tag{50}$$

and from (48):

$$1 + \beta H_i + \beta J_i = \frac{H_i}{\alpha_j} = \frac{1 + \beta J_i}{1 - \alpha_j \beta} . \tag{51}$$

Substituting the l.h.s. of those expressions into the system above, a simpler two dimensional system for F_i and J_i obtains:

$$F_i = \frac{\nu_i + \beta F_i}{1 - \alpha_i \beta} \theta_i + \frac{1 + \beta J_i}{1 - \alpha_j \beta} \rho_j ,$$

$$J_i = \frac{\nu_i + \beta F_i}{1 - \alpha_i \beta} \rho_i + \frac{1 + \beta J_i}{1 - \alpha_j \beta} \theta_j .$$

Solving this system of equations, one finds:

$$F_i = \frac{\nu_i \left[\theta_i (1 - \alpha_j \beta - \theta_j \beta) + \beta \rho_i \rho_j\right] + \rho_j (1 - \alpha_i \beta)}{(1 - \alpha_i \beta - \theta_i \beta)(1 - \alpha_j \beta - \theta_j \beta) - \beta^2 \rho_i \rho_j} . \tag{52}$$

Using (45) and (50) to get rid off D_i in the expression of τ_{it}, one has:

$$\tau_i = \frac{\beta F_i (1 - \alpha_i \beta)}{\nu_i + \beta F_i} \ .$$

Plugging (52) into the above expression and simplifying:

$$\tau_i = \frac{\beta \theta_i \nu_i [1 - \beta (\theta_j + \alpha_j)] + \beta^2 \nu_i \rho_i \rho_j + \beta \rho_j (1 - \beta \alpha_i)}{\nu_i [1 - \beta (\theta_j + \alpha_j)] + \beta \rho_j} \ ,$$

which is equivalent to:

$$\tau_i = \frac{\beta \theta_i \{\nu_i [1 - \beta (\theta_j + \alpha_j)] + \beta \rho_j\} - \beta^2 \theta_i \rho_j + \beta^2 \nu_i \rho_i \rho_j + \beta \rho_j (1 - \beta \alpha_i)}{\nu_i [1 - \beta (\theta_j + \alpha_j)] + \beta \rho_j} \ ,$$

therefore:

$$\tau_i = \beta \theta_i + \beta \rho_j \frac{1 - \beta \alpha_i - \beta \theta_i + \beta \nu_i \rho_i}{\nu_i [1 - \beta (\theta_j + \alpha_j)] + \beta \rho_j} \ ,$$

as reported in the text.

C Conditions for balanced growth

A sufficient condition for balanced growth is when $\Gamma_i \geq 1$, $\forall i = 1, 2$. From (25), remember that

$$\Gamma_i = \Gamma_i(\tau_i, \tau_j) = A_i \alpha_i \beta (1 - \tau_i) \left(\frac{\tau_i}{\alpha_i \beta (1 - \tau_i)} \right)^{\theta_i} \left(\frac{\tau_j}{\alpha_j \beta (1 - \tau_j)} \right)^{\rho_j} \ .$$

The logic of the proof is to exhibit conditions underwhich $\Gamma_i = \Gamma_i(\tau_i, \tau_j)$ is increasing in both arguments and bounded below by 1.

The partial derivative $\frac{\partial \Gamma_i}{\partial \tau_j}$ is always positive, whereas $\frac{\partial \Gamma_i}{\partial \tau_i} \geq 0$ for all $\tau_i \leq \theta_i$.

Under assumption $\theta_i \geq \frac{\alpha_i}{1 + \beta \alpha_i}$, one can write

$$A_i \alpha_i \beta (1 - \beta \theta_i) \left(\frac{\theta_i}{\alpha_i (1 - \beta \theta_i)} \right)^{\theta_i} \geq A_i \alpha_i \beta (1 - \beta \theta_i) \ .$$

In addition, when $A_i \geq \frac{1}{\beta \alpha_i (1 - \beta \theta_i)}$ then

$$A_i \alpha_i \beta (1 - \beta \theta_i) \geq 1 \ ,$$

therefore

$$A_i \alpha_i \beta (1 - \beta \theta_i) \left(\frac{\theta_i}{\alpha_i (1 - \beta \theta_i)} \right)^{\theta_i} \geq 1 \ .$$

Under the assumption $\theta_j \geq \frac{\alpha_j}{1 + \beta \alpha_j}$, we also have

$$\Gamma_i(\beta \theta_i, \beta \theta_j) \geq A_i \alpha_i \beta (1 - \beta \theta_i) \left(\frac{\theta_i}{\alpha_i (1 - \beta \theta_i)} \right)^{\theta_i} \geq 1 \ .$$

Finally, restricting attention to tax rates (τ_i, τ_j) that belongs to $[\beta \theta_i, \theta_i] \times [\beta \theta_j, \theta_j]$, since $\frac{\partial \Gamma_i}{\partial \tau_i}$, $\frac{\partial \Gamma_i}{\partial \tau_j} \geq 0$, one has $\Gamma_i = \Gamma_i(\tau_i, \tau_j) \geq \Gamma_i(\beta \theta_i, \beta \theta_j) \geq 1$, $\forall k$.

The same logic applies to ascertain that $\Gamma_i \geq 1$.

D Proof of Proposition 5 (transitional growth)

Remember that the expressions of growth rates are given by:

$$g_{it} = \Gamma_i u_t^{-\rho_i} - 1 \tag{53}$$

$$g_{jt} = \Gamma_j u_t^{\rho_j} - 1 \tag{54}$$

The sign and the evolution of both growth rates mainly follow from the properties of the sequence $\{u_t\}$: if $u_0 \leq \tilde{u}$ then u_t is monotonically increasing until it reaches its steady state level $\tilde{u} = (\Gamma_i/\Gamma_j)^{\frac{1}{1-\phi}}$. Otherwise ($u_0 > \tilde{u}$), u_t is monotonically decreasing towards $\tilde{u}$.

Assume first that $u_0 \leq \tilde{u}$. Then $u_0 \leq u_1 \leq u_2 \leq ... \leq \tilde{u}$. According to (53) and (54), it implies that g_{it} is decreasing while g_{jt} is increasing during the transition.

- By assumption $g_i \geq 0$, which is equivalent to $\Gamma_i \geq \tilde{u}^{\rho_i}$. Thus $\Gamma_i \geq \tilde{u}^{\rho_i} \geq u_t^{\rho_i}$ $\forall t$ since $u_t \leq \tilde{u}$ $\forall t$, which means $g_{it} \geq 0$, $\forall t$.

- when in addition $u_0^{-\rho_j} \leq \Gamma_j$, or equivalently $g_{j0} \geq 0$, one has $\Gamma_j \geq u_0^{-\rho_j} \geq u_t^{-\rho_j}$ $\forall t$ since $u_0 \leq u_t$ $\forall t$. This means $g_{jt} \geq 0$ $\forall t$.

- on the contrary when $\Gamma_j < u_0^{-\rho_j}$, or equivalently $g_{j0} < 0$, since $u_0^{-\rho_j} \geq u_1^{-\rho_j} \geq u_2^{-\rho_j} \geq ...$ and by assumption $g_j \geq 0$, necessarily $\exists \bar{t}$ such that $\Gamma_j < u_t^{-\rho_j}$ for all $t < \bar{t}$ (so $g_{jt} < 0,\ t < \bar{t}$), and $u_t^{-\rho_j} \leq \Gamma_j$ for all $t \geq \bar{t}$ (so $g_{jt} \geq 0,\ t \geq \bar{t}$).

The case where $\tilde{u} < u_0$ can be analyzed along similar lines and is left to the reader.

E Proof of Proposition 6

Solving the difference equation in $u_t = k_{it}/k_{jt}$ given by (27) yields:

$$u_t = \left(\frac{\Gamma_i}{\Gamma_j}\right)^{\frac{1-\phi^t}{1-\phi}} u_0^{\phi^t} .$$

Subsituting this expression in equations (53) gives:

$$g_{it} = \Gamma_i \left(\left(\frac{\Gamma_i}{\Gamma_j}\right)^{\frac{1-\phi^t}{1-\phi}} u_0^{\phi^t}\right)^{-\rho_i} - 1 = \Gamma_i \left(\left(\frac{\Gamma_i}{\Gamma_j}\right)^{\frac{1-\phi^t}{1-\phi}} \left(\frac{K_{i0}}{K_{j0}}\right)^{\phi^t}\right)^{-\rho_i} - 1 ,$$

where parameters Γ_i and Γ_j are:

$$\Gamma_i = A_i'(1-\tau_i)^{1-\theta_i}\tau_i^{\theta_i}\left(\frac{\tau_j}{1-\tau_j}\right)^{\rho_i} ,$$

$$\Gamma_j = A_j'(1-\tau_j)^{1-\theta_j}\tau_j^{\theta_j}\left(\frac{\tau_i}{1-\tau_i}\right)^{\rho_i} ,$$

with A_i' and A_j' some constants.

Let Ψ corresponds to the ratio Γ_i/Γ_j:

$$\Psi = \frac{A_i'(1-\tau_i)}{A_j'(1-\tau_j)} \left(\frac{\tau_i}{1-\tau_i}\right)^{\theta_i-\rho_j} \left(\frac{\tau_i}{1-\tau_i}\right)^{\rho_i-\theta_j} .$$

The derivative of g_{it} with respect to τ_i writes as:

$$\frac{\partial g_{it}}{\partial \tau_i} = \frac{\partial \Gamma_i}{\partial \tau_i} u_t^{-\rho_i} - \Gamma_i \rho_i \frac{1-\phi^t}{1-\phi} u_0^{\phi^t} \frac{\partial \Psi}{\partial \tau_i} \Psi^{\frac{1-\phi^t}{1-\phi}-1} u_t^{-\rho_i-1} , \tag{55}$$

with

$$\frac{\partial \Gamma_i}{\partial \tau_i} = \frac{\Gamma_i}{\tau_i(1-\tau_i)}(\theta_i - \tau_i) ,$$

$$\frac{\partial \Psi}{\partial \tau_i} = \frac{\Gamma_i}{\tau_i(1-\tau_i)\Gamma_j}(\theta_i - \rho_j - \tau_i) .$$

Substituting these derivatives in (55) and rearranging the expression yields:

$$\frac{\partial g_{it}}{\partial \tau_i} = \frac{\Gamma_i u_t^{-\rho_i}}{(\rho_i+\rho_j)\tau_i(1-\tau_i)} \left((\rho_i+\rho_j)(\theta_i-\tau_i) - \rho_i(1-\phi^t)(\theta_i-\rho_j-\tau_i)\right) .$$

Direct calculations show that $\frac{\partial g_{it}}{\partial \tau_i} \geq 0 \ \forall t$ is equivalent to:

$$\tau_i \leq \theta_i + \frac{\rho_i \rho_j (1-\phi^t)}{\rho_i \phi^t + \rho_j} , \quad \forall t .$$

Evaluated at t = 0, this condition becomes $\tau_i \leq \theta_i$, which is therefore necessary to ensure $\frac{\partial g_{it}}{\partial \tau_i} \geq 0 \ \forall t$.

Concerning the derivative with respect to τ_j one has:

$$\frac{\partial g_{it}}{\partial \tau_j} = \frac{\partial \Gamma_i}{\partial \tau_j} u_t^{-\rho_i} - \Gamma_i \rho_i \frac{1-\phi^t}{1-\phi} u_0^{\phi^t} \frac{\partial \Psi}{\partial \tau_j} \Psi^{\frac{1-\phi^t}{1-\phi}-1} u_t^{-\rho_i-1} , \tag{56}$$

with

$$\frac{\partial \Gamma_i}{\partial \tau_j} = \frac{\rho_i \Gamma_i}{\tau_j(1-\tau_j)} ,$$

$$\frac{\partial \Psi}{\partial \tau_j} = \frac{\Gamma_i}{\tau_j(1-\tau_j)\Gamma_j}(\tau_j - \theta_j + \rho_i) .$$

Finally, the derivative is given by:

$$\frac{\partial g_{it}}{\partial \tau_j} = \frac{\rho_i \Gamma_i u_t^{-\rho_i}}{(\rho_i+\rho_j)\tau_j(1-\tau_j)} \left((\rho_i+\rho_j) - (1-\phi^t)(\tau_j-\theta_j+\rho_i)\right) ,$$

and the following equivalence holds: $\frac{\partial g_{it}}{\partial \tau_j} \geq 0 \leftrightarrow$

$$\tau_j \leq \theta_j + \frac{\rho_i \phi^t + \rho_j}{1-\phi^t}$$

which is always verified when $\tau_j \leq \theta_j$, because the second term in the LHS is positive.

F Proof of Proposition 11

The proof comes from the analysis of the system (33). At a steady state (u, v) of the log transforms of growth rates (so at a BGP), necessarily:

$$u = \frac{\rho_1}{1 - \alpha_1 - \theta_1} v = \frac{1 - \alpha_2 - \theta_2}{\rho_2} v = \kappa v \ ,$$

with $\rho_i \neq 0, \alpha_i + \theta_i \neq 0, \ i = 1, 2$. It is easy to check that $\kappa > 1$ (respectively $\kappa < 1$) when there are increasing (decreasing) returns to scale in country 1 while there are decreasing (increasing) returns to scale in country 2. Then any steady state is such that $u > v$ (respectively $u < v$), which proves point 1 of the proposition.

The demonstration of point 2 is made recursively. Note first that u_0 and v_0 are increasing functions of Γ_1 and Γ_2 respectively, which in turn are increasing functions of τ_1 and τ_2 provided that $\tau_i \leq \theta_i$, $i = 1, 2$ (see the details given in Appendix C). From (33) observe also that u_1 and v_1 are increasing functions of u_0 and v_0. Therefore u_1 and v_1 are increasing functions of τ_1 and τ_2. Assume next that this property holds for u_t and v_t ; to complete the proof it remains to show that the property necessarily hold for u_{t+1} and v_{t+1}. But this is obvious, by inspection again of the dynamic system (33) that shows u_{t+1} and v_{t+1} are increasing functions of u_t and v_t . So,

$$\frac{\partial u_t}{\partial \tau_i} \geq 0 \ , \ \frac{\partial u_t}{\partial \tau_i} \geq 0 \ , \ \forall t.$$

This property holds also at steady states, and because $u = \kappa v$, with $\kappa \neq 1$, an increase in tax rates produces an increase of the gap between steady states. Finally, when there is under-taxation at MPE (when consumers prefer their domestic good), cooperation increases the gap between growth rates. When there is over-taxation at MPE (when consumers prefer the foreign good), cooperation decreases the gap.

Documents de Recherche parus en 2007[1]

[1] La liste intégrale des Documents de Travail du LAMETA parus depuis 1997 est disponible sur le site internet : http://www.lameta.univ-montp1.fr

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