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LAMETA

Laboratoire Montpelliérain d'Economie Théorique et Appliquée

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Nicola GALLAI

Jean-Michel SALLES

Charles FIGUIERES

Bernard E. VAISSIERE

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Faculté de Sciences Economiques - Espace Richter

Avenue de la Mer - Site de Richter C.S. 79606

3 4 9 6 0 MONTPELLIER CEDEX 2

Tél: 33(0)467158495 Fax: 33(0)467158467

E-mail: lameta@lameta.univ-montp1.fr

Université Montpellier 1 · agro Montpellier Ecole Nationale Supérieure Agronomique · CNRS Centre National de la Recherche Scientifique · INRA

Economic assessment of an insect pollinator decline:
A general equilibrium analysis

Nicola Gallai[1,4*]*, Jean-Michel Salles[2], Charles Figuières[3] & Bernard E. Vaissière[4]

[1] Université Montpellier 1, UMR LAMETA, 2 place Viala, 34060 Montpellier Cedex 1, France

[2] CNRS, UMR LAMETA, 2 place Viala, 34060 Montpellier Cedex 1, France

[3] INRA, UMR LAMETA, 2 place Viala, 34060 Montpellier Cedex 1, France

[4] INRA, Laboratoire Pollinisation & Ecologie des Abeilles, UMR406 Abeilles & Environnement, 84914 Avignon Cedex 9, France

Abstract

Insect pollination service is widely used in agriculture. This pollination service contributes significantly to the total economic value of crop production. A better economic valuation is to assess the welfare loss resulting from insect pollinator decline, this welfare loss being the sum of producer and consumer surplus variation. In this study, we assess the impact of insect pollinators on the social welfare within a general equilibrium analysis. What would be the consequences of a production loss due to an insect pollinator decline considering the adaptation of the overall economy and more particularly considering the possible spillovers on others markets? How are changes in profits distributed between producers of pollinated goods and other producers? These two questions will be studied within two alternative scenarios for the distribution of property rights over the firms: the case where agents possess and equal share of the productive sector (the egalitarian ownership structure) and the case where each agent possess one firm (the polarized ownership structure). For each scenario, we considered two states of the economy. In the first state, agent and firms are homogeneous. In the second state, firms are heterogeneous. The social welfare is a function of the profile of consumers' utilities. We will analyze and measure the variation of the social welfare after the insect pollinator decline. Under the egalitarian ownership structure, we found that an insect pollinator decline will cause a social welfare loss. However this loss is reduced by the possibility of agents to consume the good, whose production does not depends on insect pollination. This result no longer holds when the distribution of the property right is heterogeneous between agents. In this case the owner of the firm that does not produce the pollination-dependent good, would experience a gain in utility. As a result, the social welfare could increase after a pollinator decline. This social welfare gain would rise if the production function of the firm of the non agricultural sector would be more efficient than the agricultural one.

[*] Corresponding author: Nicola Gallai, UMR LAMETA, INRA, 2 place Viala, 34060 Montpellier Cedex 1, France, Tel. (33) [0]4.99.61.29.67, Fax (33) [0]4.67.54.58.05, e-mail: gallai@supagro.inra.fr

1. Introduction

Insect pollination service is widely used in agriculture since 84% of the crop species grown in Europe and 70% of those that are used directly to feed mankind need insect pollination (Williams, 1994 ; Klein et *al.*, 2007). This pollination service contributes significantly to the total economic value of crop production and its share was estimated between €14 to €200 billion (Costanza et *al.*, 1997; Pimentel et *al.*, 1997) and more recently at US$190 billion for 2005 (Gallai et *al.*, 2009). Yet a more appropriate economic valuation of insect pollination service would be to assess the welfare loss resulting from insect pollinator decline, this welfare loss being the sum of producer and consumer surplus variation.

In existing analyses, the welfare variation, after a pollinator decline, is studied in a single market where farmers could adapt their production to the new conditions (Southwick and Southwick, 1992; Gallai *et al.* 2009). This single-market simplification is somehow justified as an effort to assess, as simply as possible, the impact of this ecological shock. But a single market model ignores important subtleties regarding the indirect consequences of the shock on other markets that, in turn, will causes a wealth of feedback effects on the overall economy.

This article makes a first step to address this concern. We propose a general equilibrium framework that describes an economy where several markets make consistent, via an endogenous system of prices, multiple production and consumption plans. What would be the consequences of a production loss due to an insect pollinator decline considering the adaptation of the overall economy and more particularly considering the possible spillovers on others markets? How are changes in profits distributed between producers of pollinated goods and other producers? These two questions will be studied within two alternative scenarios for the distribution of property rights over the firms: the case where agents possess and equal share of the productive sector (the egalitarian ownership structure) and the case where each agent possess one firm (the polarized ownership structure). Within a general equilibrium the social welfare is a function of the profile of consumers' utilities. We will analyze and measure the variation of the social welfare after the insect pollinator decline.

The article starts with a general description, using unspecified functions for utilities and technologies, with two goods, two consumers and two producers, and it offers a coarse account of how the economy react after a pollinator decline. In a second step, we will retain more specific functions. This is clearly a simplification; still, the general equilibrium dimension is there and we gain the possibility to grasp more information about the sensitivity and the intensity of the reactions following the shock. We do so first for symmetric agents under, alternatively, the egalitarian and the polarized ownership structures. Then we introduce heterogeneity between producers, due to a technological parameter, and we revisit again, alternatively, the two ownership structures. As it turns out, the ownership structure is crucial to appraise the effect of the ecological shock. The main result is that, under the egalitarian distribution of property rights, all the agents suffer from the shock, hence there is a reduction of welfare; by contrast, under the polarized structure, the agent who possesses the pollinated activity experiences an utility reduction, whereas the other agent can experience a higher utility. This result holds when: 1) either the elasticity of substitution between the two consumption goods is sufficiently high, 2) or when the non pollinated sector is relatively more productive than the pollinated sector. In either case, welfare can increase if the second agent is granted a relatively more important weight in the social welfare criterion. The last section discusses the results and suggests some perspectives.

2. The impact of an insect pollinator decline: a general model

2.1. The model

The economy has two firms f and g, using two inputs $k = 1, 2$, to produce two goods $h = 1, 2$, enjoyed by two consumers c $= 1, 2$. The production of good 1 depends on insect pollination whereas the production of good 2 does not.

2.1.1. The production side

There are two technologies, called respectively f for firm f and g for firm g. Both technologies use the two inputs. The amount of input k used by firm 1 (respectively by firm 2) is z_{fk} (resp. z_{gk}). The total use of input k is therefore $Z_k = z_{fk} + z_{gk}$.

Pollination is necessary for the production and reproduction of crops. A biologic ratio, called a dependence ratio or simply D, was created from a review by Klein et al. (2007, Appendix A). This ratio indicates the part of crop production dependent on insect pollination and is comprised between 0 and 1: it means that a total insect pollinator decline would reduce crop production by a factor D. Accordingly, the production function of good 1 that is dependent on insect pollinator is $f\left(z_{f1}, z_{f2}, D\right)$. Good 2 does not depend on insect pollinators and its production function is $g(z_{g1}, z_{g2})$. We assume that *f(.,.,.)* and *g(.,.)* are concave functions, featuring decreasing returns to scale ($af(z_{f1},z_{f2},D)>f(az_{f1},az_{f2},D)$, for all $a>1$).

The profit functions of firms, for given prices of output (p_G) and input (a_H), are denoted Π^n. Those functions read as:

$$\Pi^1 = p_1 f(z_{f1}, z_{f2}, D) - a_1 z_{f1} - a_2 z_{f2} \quad [1]$$

$$\Pi^2 = p_2 g(z_{g1}, z_{g2}) - a_1 z_{g1} - a_2 z_{g2} \quad [2]$$

Firms pick the best combination of inputs in order to maximize profits. As explained in Appendix A, one can deduce the firms' demands of inputs as functions of the prevailing prices. Profits maximization result in demand functions $z_{f1}(p_1, D, a_1, a_2)$ and $z_{g1}(p_2, a_1, a_2)$ for the first input, $z_{f2}(p_1, D, a_1, a_2)$ and $z_{g2}(p_2, a_1, a_2)$ for the second input. The total demand of inputs are simply $Z_1 = z_{f1}(p_1, D, a_1, a_2) + z_{g1}(p_2, a_1, a_2)$ and $Z_2 = z_{f2}(p_1, D, a_1, a_2) + z_{g2}(p_2, a_1, a_2)$. Also, plugging those decisions into the production functions, the supply for each consumption good, given the prevailing prices on the markets, will be $X_1(p_1, D, a_1, a_2)$ and $X_2(p_2, a_1, a_2)$.

2.1.2. The consumption side

Consumer 1 (respectively 2) is endowed with the first (resp. the second) production factor, $\overline{Z_1}$ (resp. $\overline{Z_2}$), which he supplies inelastically and for the counterpart of which he receives wages. Hence the supply of inputs are constant, $Z_1 = \overline{Z_1}$ and $Z_2 = \overline{Z_2}$. Consumers are also endowed with a share of the firms. More precisely, consumer *c* works for both firms and we assume that he provides all the input *k*. Then he receives the wage $a_k Z_k$ *i.e.* $a_k(z_{fk} + z_{gk})$. Furthermore the consumer owns a share (or the total) of firm *n*. Consequently he receives dividends that amounts to a share of the profits. Two ownership structures will be considered in turn. Under the egalitarian structure both consumers own 50% of both firms. Thus their revenues are:

$$\begin{aligned} R_1 &= 0.5(\Pi^1 + \Pi^2) + a_1 z_{f1} + a_1 z_{g1} \\ R_1 &= 0.5\left(p_1 f(z_{f1}, z_{f2}, D) + p_2 g(z_{g1}, z_{g2}) + a_1 z_{f1} + a_1 z_{g1} - a_2 z_{f2} - a_2 z_{g2}\right) \end{aligned} \quad [3]$$

$$\begin{aligned} R_2 &= 0.5(\Pi^1 + \Pi^2) + a_2 z_{f2} + a_2 z_{g2} \\ R_2 &= 0.5\left(p_1 f(z_{f1}, z_{f2}, D) + p_2 g(z_{g1}, z_{g2}) + a_2 z_{f2} + a_2 z_{g2} - a_1 z_{f1} - a_1 z_{g1}\right) \end{aligned} \quad [4]$$

Under the polarized structure, Consumer *1* is the owner of firm 1 and Consumer *2* is the owner of firm 2. Formally:

$$R_1 = \Pi^1 + a_1 z_{f1} + a_1 z_{g1} = p_1 f(z_{f1}, z_{f2}, D) - a_2 z_{f2} + a_1 z_{g1} \quad [5]$$

$$R_2 = \Pi^2 + a_2 z_{f2} + a_2 z_{g2} = p_2 g(z_{g1}, z_{g2}) - a_1 z_{g1} + a_2 z_{f2} \quad [6]$$

Whatever the ownership structure, consumer c faces the budget constraint $R_c \geq p_1 x_{c1} + p_2 x_{c2}$. For the time being, let us carry on with the egalitarian case.

Consumers' preferences over baskets of consumption goods are represented by utility functions that are denoted $U^c(x_{c1}, x_{c2})$. Those utility functions are concave and we let $\frac{\partial U^c}{\partial x_{c1}} = U_{c1}$ and $\frac{\partial U^c}{\partial x_{c2}} = U_{c2}$ stand for the marginal utilities of each good.

Consumers use their total revenue to buy goods in order to maximize their utility. Their maximization program ends up in individual demands for each good, denoted $x_{c1}(R_c, p_1, p_2)$ and $x_{c2}(R_c, p_1, p_2)$, configured by prices and income (Appendix B). And the total demand for good h, X_h , is the sum of the individual demands x_{ch} ($X_h = x_{1h} + x_{2h}$), where $x_{ch} \geq 0$.

2.1.3. The social welfare

The social welfare criterion (SWC) is a functional with consumers' utilities as arguments. An often used SWC is the generalized utilitarian criterion, which in our model is a convex combination of the two utilities:

$$W = \theta U^1(x_{11}, x_{12}) + (1-\theta) U^2(x_{21}, x_{22}) \quad [7]$$

where θ is a parameter chosen in the interval]0, 1[.

Then analyzing the impact of insect pollinator is a comparison between the state of economy after an insect pollinator decline *i.e.* when $D > 0$ and the state of the economy before insect pollinator decline *i.e.* when $D = 0$. And the impact on the social welfare is measured by:

$$\Delta W = W_{D>0} - W_{D=0} \quad [8]$$

2.2. The total consumers revenue

The social revenue is the sum of individual revenues, which are given by expressions [3] and [4]. The total revenue is written as follows: $R = R_1 + R_2 = p_1 f(z_{f1}, z_{f2}, D) + p_2 g(z_{g1}, z_{g2})$. Generally, the social revenue will be impacted by decline of pollinators since it depends on *D*.

2.3. Determining equilibrium prices p_1, p_2, a_1 and a_2 and exchanged quantities

At the equilibrium, the demand of good *h* is equal to the production of good *h* and the demand for input *k* is equal to the supply of input *k.* The price of input 2 is normalized to unity. So we find that

$$X_1(p_1, a_1, D) = x_{11}(R_1(p_1, D, a_1), p_1, p_2) + x_{21}(R_2(p_2, D, a_1), p_1, p_2)$$

$$X_2(p_2, a_2, D) = x_{12}(R_1(p_1, D, a_1), p_1, p_2) + x_{22}(R_2(p_2, D, a_1), p_1, p_2)$$

$$\overline{Z}_1 = z_{f1}(p_1, D, a_1) + z_{g1}(p_2, D, a_1)$$

$$\overline{Z}_2 = z_{f2}(p_1, D, a_1) + z_{g2}(p_2, D, a_1)$$

Solving the above system:

$$p_1(D, \overline{Z}_1, \overline{Z}_2)$$

$$p_2(D, \overline{Z}_1, \overline{Z}_2)$$

$$a_1(D, \overline{Z}_1, \overline{Z}_2)$$

The exchanged quantities in the economy depend on p_1, p_2 and a_1. In turn, those prices p_1, p_2 and a_1 depend on *D,* $\overline{Z}_1$ and $\overline{Z}_2$. Overall, the equilibrium quantities of interest in the economy depend solely on *D,* $\overline{Z}_1$ and $\overline{Z}_2$.

2.4. What is the impact of the pollinator decline on the social welfare?

Utilities are function of quantities realized through markets. As shown above, equilibrium quantities depend on *D,* $\overline{Z}_1$ and $\overline{Z}_2$. The variation of the social welfare when *D* jumps upward from zero will be used to assess the impact of a pollinator decline on the economy.

Unfortunately, this measure can hardly be analyzed qualitatively with the general utility and production functions used so far. The problem is that, with unspecified functional forms, very little can be deduced about the sensitivity of x_{c1} and x_{c2}, at the general equilibrium, with respect to a variation of D. But one can refine the analysis by using explicit functions. Furthermore, by using specific functions one could integrate in the study the role played by preferences and technological parameters, regarding the adaptation of the economy confronted with an ecological shock.

3. A perfectly symmetric model with explicit functions of productions and utilities

3.1. The model

The production function has a Cobb-Douglas form for both firms. Recall that good 1 depends on insect pollination and is produced by firm 1 and good 2 does not depend on pollination and is produced by firm 2. Thus the production function of firm 1 is:

$$f(z_{f1}, z_{f2}, D) = (1-D) z_{f1}^{\beta} z_{f2}^{\beta} \quad [9]$$

And the production function of firm 2 is:

$$g(z_{g1}, z_{g2}) = z_{g1}^{\beta} z_{g2}^{\beta} \quad [10]$$

where β is a parameter chosen in the interval]0, 1/2[, which implies decreasing returns to scale.

With such a formulation, Appendix C shows that firm 1's demands for inputs are:

$$z_{f1} = \frac{\left(p_1 \beta (1-D)\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{1-\beta}{1-2\beta}} a_2^{\frac{\beta}{1-2\beta}}} \quad [11]$$

$$z_{f2} = \frac{\left(p_1 \beta (1-D)\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{\beta}{1-2\beta}} a_2^{\frac{1-\beta}{1-2\beta}}} \quad [12]$$

As one may expect, the higher the sale price p_1, the larger the demand of inputs. And the more expensive the inputs, the lower their demand. Finally, the larger the shock D > 0, the lower the demands for inputs.

The firm 2's demands are:

$$z_{g1} = \frac{(p_2\beta)^{\frac{1}{1-2\beta}}}{a_1^{\frac{1-\beta}{1-2\beta}} a_2^{\frac{\beta}{1-2\beta}}} \quad [13]$$

$$z_{g2} = \frac{(p_2\beta)^{\frac{1}{1-2\beta}}}{a_1^{\frac{\beta}{1-2\beta}} a_2^{\frac{1-\beta}{1-2\beta}}} \quad [14]$$

The consumers' preferences are represented by a CES utility function:

$$U^c(x_{c1}, x_{c2}) = \frac{v x_{c1}^{\alpha}}{\alpha} + \frac{x_{c2}^{\alpha}}{\alpha} \quad [15]$$

where x_{c1} and $x_{c2} > 0$. The coefficient v is the relative weight of the utility derived from the consumption of the first good. This functional form allows for several degrees of substitutability between goods. When $\alpha = v = 1$, the case of perfect substitutability obtains.

Under the CES specification, demands for consumption goods are (Appendix B):

$$x_{c1} = \frac{R_c}{p_1 + p_2\left(\frac{v p_2}{p_1}\right)^{\frac{1}{\alpha-1}}} \quad [16]$$

$$x_{c2} = \frac{R_c}{p_2 + p_1\left(\frac{p_1}{v p_2}\right)^{\frac{1}{\alpha-1}}} \quad [17]$$

3.2. The consumers revenue

When consumers own 50% of both firms, revenues are expressed according to [3] for consumer 1 and [4] for consumer 2:

$$R_1 = 0.5\left(p_1(1-D)^{\frac{1}{1-2\beta}}\left(\frac{(\beta p_1)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}} + p_2\left(\frac{(\beta p_2)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}} + a_1 z_{f1} + a_1 z_{g1} - a_2 z_{f2} - a_2 z_{g2}\right) \quad [18]$$

$$R_2 = 0.5\left(p_1(1-D)^{\frac{1}{1-2\beta}}\left(\frac{(\beta p_1)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}} + p_2\left(\frac{(\beta p_2)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}} + a_2 z_{f2} + a_2 z_{g2} - a_1 z_{f1} - a_1 z_{g1}\right) \quad [19]$$

The total social revenue is the sum of R_1 and R_2:

$$R = p_1(1-D)^{\frac{1}{1-2\beta}}\left(\frac{(\beta p_1)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}} + p_2\left(\frac{(\beta p_2)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}}$$
$$\Leftrightarrow R = \left(\frac{(\beta)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}}\left((p_1(1-D))^{\frac{1}{1-2\beta}} + p_2^{\frac{1}{1-2\beta}}\right) \qquad [20]$$

3.3. The equilibrium of the economy

3.3.1. Prices: p_1, p_2, a_1 and a_2

At an equilibrium the supply of goods equalizes the total demand for goods, hence $X_1 = x_{11} + x_{12} = f(z_{f1},z_{f2},D)$ and $X_2 = x_{21} + x_{22} = g(z_{g1},z_{g2})$. These expressions determine the relation between p_1 and p_2:

$$X_1 = f(z_{f1}, z_{f2}, D)$$
$$\Leftrightarrow \frac{R_1 + R_2}{p_1 + p_2\left(\frac{\nu p_2}{p_1}\right)^{\frac{1}{\alpha-1}}} = (1-D)^{\frac{1}{1-2\beta}}\frac{(\beta p_1)^{\frac{2\beta}{1-2\beta}}}{(a_1 a_2)^{\frac{\beta}{1-2\beta}}} \qquad [21]$$

Using expressions [21], one finds the relation between p_1 and p_2

$$p_2 = p_1\left(\frac{(1-D)^{1-\alpha}}{\nu^{1-2\beta}}\right)^{\frac{1}{1-2\alpha\beta}} \qquad [22]$$

By Walras' law, the second equilibrium condition is automatically satisfied, $X_2 = g(z_{g1},z_{g2})$.

We assume that the price of input a_2 is normalized to 1 ($a_2 = 1$). So using expressions [18] and [19] of the Appendix C and equation [22] one can determine the equilibrium prices a_1^*, p_1^* and p_2^*:

$$\beta^{\frac{1}{1-2\beta}}\left(((1-D)p_1)^{\frac{1}{1-2\beta}} + p_2^{\frac{1}{1-2\beta}}\right) = a_1^{\frac{1-\beta}{1-2\beta}}\overline{Z}_1 = a_1^{\frac{\beta}{1-2\beta}}\overline{Z}_2 \quad \Leftrightarrow \quad a_1^* = \frac{\overline{Z}_2}{\overline{Z}_1} \qquad [23]$$

$$p_1^* = \frac{\overline{Z}_1^{-\beta}\,\overline{Z}_2^{1-\beta}}{\beta\left((1-D)^{\frac{1}{1-2\beta}} + \frac{(1-D)^{\frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)}}}{\nu^{\frac{1}{1-2\alpha\beta}}}\right)^{1-2\beta}} \qquad [24]$$

$$p_2^* = \frac{(1-D)^{\frac{1-\alpha}{1-2\alpha\beta}} v^{\frac{1-2\beta}{2\alpha\beta-1}} \overline{Z}_1^{-\beta} \overline{Z}_2^{1-\beta}}{\beta\left((1-D)^{\frac{1}{1-2\beta}} + \frac{(1-D)^{\frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)}}}{v^{\frac{1}{1-2\alpha\beta}}}\right)^{1-2\beta}} \quad [25]$$

3.3.2. Exchanged quantities

The equilibrium prices determine all variables, R, R_1, R_2, $f(.)$, $g(.)$, X_1, X_2, x_{11}, x_{21}, x_{12}, and x_{22}, as functions of parameters v, β, α, D, $\overline{Z}_1$ and $\overline{Z}_2$.

Following expression [18], [19], [20], [23], [24] and [25]:

$$R^* = \frac{\overline{Z}_2}{\beta} \quad [26]$$

$$R_1^* = \frac{\overline{Z}_2}{2\beta} \quad [27]$$

$$R_2^* = \frac{\overline{Z}_2}{2\beta} \quad [28]$$

At the equilibrium, exchanged quantities of good 1 are X^*_1, which can be decomposed into x^*_{11}, the amount consumed by agent 1 and x^*_{21} is the quantity enjoyed by agent 2.

$$X_1^* = f(.,.,D)^* = (\overline{Z}_1\overline{Z}_2)^{\beta} \frac{(1-D)}{\left(1 + (1-D)^{\frac{1-\alpha}{(1-2\alpha\beta)}} v^{\frac{1}{2\alpha\beta-1}}\right)^{2\beta}} \quad [29]$$

$$X_2^* = g(.,.)^* = \frac{(\overline{Z}_1\overline{Z}_2)^{\beta}}{v^{\frac{2\beta}{1-2\alpha\beta}}} \frac{1}{\left((1-D)^{\frac{\alpha}{1-2\alpha\beta}} + \frac{1}{v^{\frac{1}{1-2\alpha\beta}}}\right)^{2\beta}} \quad [30]$$

$$x_{c1}^* = \frac{1}{2} X_1^* \quad [31]$$

$$x_{c2}^* = \frac{1}{2} X_2^* \quad [32]$$

3.4. What is the impact of the pollinator decline on the social welfare?

The impact of insect pollinators on the different variables of the economy depends on the ratio D. This ratio appears in all equilibrium variables, except for revenues and prices of inputs, which means that they would not vary after a pollinator decline. But the ratio appears in all other functions at the equilibrium such as prices of goods, exchanged quantities and in utilities.

The impact of insect pollinators on the social welfare is measured by expression [8], as the variation of the sum of consumers' utilities after the pollinator decline. The consumers' utility depends on consumption of good 1 and good 2 (expression [15]). However, at the equilibrium, the production of both goods is influenced by D (expressions [29] and [30]), which means that both quantities exchanged would vary after a pollinator decline. But in which direction? The answer is given in the following two propositions:

Proposition 1: Let $\alpha \in]0, 1[$ and $\beta]0, ½[$. Then the larger the pollinators decline the lower the consumption of good 1 at the equilibrium (Proof: see Appendix D).

Proposition 2: Let $\alpha \in]0, 1[$ and $\beta]0, ½[$. Then the larger the pollinator decline the larger the consumption of good 2 at the equilibrium (Proof: see Appendix D).

Consequently the impact of an insect pollinators decline on the consumers' utilities is determined by the difference between consumption losses of x_{c1} compare to consumption gain of x_{c2} and it can be measured by $\partial U^c / \partial D$. Thus we assume that:

H1: Utility of consumer 1, U^1, will increase after a pollinator decline when $v\left(\frac{x_{11}^*}{x_{12}^*}\right)^{\alpha-1} > -\frac{\partial x_{11}/\partial D}{\partial x_{12}/\partial D}$.

H2: Utility of consumer 1, U^2, will increase after a pollinator decline when $v\left(\frac{x_{21}^*}{x_{22}^*}\right)^{\alpha-1} > -\frac{\partial x_{21}/\partial D}{\partial x_{22}/\partial D}$.

Those analytical properties do not give general results about the impact of insect pollinator decline on the welfare since x_{c1} will decrease after pollinator decline and x_{c2} will increase. At this point, we cannot conclude because the loss in utility due to the decrease of the good dependent on insect pollinator could be offset by a gain in utility due to increase consumption of the good that does not depend on pollinators. This ambiguity stems from a possible compensation by others markets. To obtain clear-cut answers, we shall now use a numerical example.

3.5. Numeric example

This section gives specific values to the following parameters: *v, β, α, D,* $\overline{Z}_1$ and $\overline{Z}_2$ (Table 1). It also analyzes the evolution of the main variables of interest: $U(.)$, $U^1(.)$,$U^2(.)$, $X_1(.)$, $X_2(.)$, $x_{11}(.)$, $x_{21}(.)$, $x_{12}(.)$, $x_{22}(.)$, $p_1(.)$, $p_2(.)$, $\Pi_1(.)$ and $\Pi_2(.)$. Finally, we analyze the influence of parameters *v, β* and *α* parameters on these variables.

Table 1 – Value of parameters

α	β	D	$\overline{Z}_1$	$\overline{Z}_2$	v
1/2	1/3	1/2	1	1	1

Results are summarized in the table 2 and exposed in more detailed in appendix 1. This numerical exercise confirms that exchanged quantities of good 2 increase and exchanged quantities of good 1 decrease for both consumers after the pollinator decline. However from the value of table 1, it turns out that both consumers' utilities decrease after pollinator decline. Consequently the overall social welfare decreases. We also note that firm 1's profit decreases while firm 2's profit increases. And all prices in the economy increase after the pollinator loss.

Proposition 3: Under the symmetric ownership structure, H1 and H2 are never realized which imply that social welfare variation will always be negative and this regardless of value of the parameter *α* under the interval [0, 1] and the parameter *β* under the interval [0, 1/2].

Table 2 indicates in which state of parameters, the intensity of welfare loss would be more or less important. For instance, the first line is to be read as follows: "the variation of welfare is affected by changes in parameters *v*, *α* and *β.* When *v* increases*,* the welfare loss is less important, starting to zero loss and ending to a negative value. When *α* or *β* increases, the welfare loss jumps to higher (but still negative) levels." The welfare loss is the same for both consumers. The individual and the social welfare loss are minimal when *v* tends to 0, that is when consumers are concerned mainly by the non pollinated good. On the other hand the social welfare loss increases when *v* increase. Considering *α,* the welfare loss is minimal when this parameter tends to 0 and maximal when *α* tends to 0. Regarding *β,* the welfare loss is stronger when this parameter tends to 0 and is minimal when it tends to 0.5.

The impact of insect pollinators on firms' profit is symmetric. The loss of profit for firms and the gain of profit for firm 2 would be stronger when $v = 1$. But when v tends to 0 or tends to infinity, the variation of profits tends to 0. The profit loss of firm 1 and the gain of firm 2 would be stronger when α tends to 1 and β tends to 0. The increase of prices is positively related to increase of v.

Table 2 – Representation of intensity and sensitivity of pollinator decline impact (ΔD) on the variation of the main variables of the economy: $\Delta W(.)$, $\Delta U^1(.)$, $\Delta U^2(.)$,$\Delta \Pi_1(.)$, $\Delta \Pi_2(.)$, $\Delta X_1(.)$, $\Delta X_2(.)$, $\Delta x_{11}(.)$, $\Delta x_{21}(.)$, $\Delta x_{12}(.)$, $\Delta x_{22}(.)$, $\Delta p_1(.)$ and $\Delta p_2(.)$ face to evolution of parameters v, α and β. The impact of pollinator decline is indicated with a "+" when it is > 0, a "-" when it is < 0 and 0 when it is null. The sensitivity of variables with respect to parameters is indicating with a "□" when pollinator impact increases, a "□" when it decreases and a "→" when it does not vary.

Functions	v	α	β
$\Delta W(.)$	0 ↘ -	- ↗ -	- ↗ -
$\Delta U^i(.)$	0 ↘ -	- ↗ -	- ↗ -
$\Delta U^j(.)$	0 ↘ -	- ↗ 0	- ↗ -
$\Delta x_{i1}(.)$	- ↘ -	- ↘ -	- ↗ -
$\Delta x_{j1}(.)$	- ↘ -	- ↘ -	- ↗ -
$\Delta X_1(.)$	- ↘ -	- ↘ -	- ↗ -
$\Delta x_{i2}(.)$	0 ↗ + ↘ 0	0 ↗ +	0 ↗ +
$\Delta x_{j2}(.)$	0 ↗ + ↘ 0	0 ↗ +	0 ↗ +
$\Delta X_2(.)$	0 ↗ + ↘ 0	0 ↗ +	0 ↗ +
$\Delta \Pi_1(.)$	0 ↘ - ↗ 0	0 ↘ -	- ↗ 0
$\Delta \Pi_2(.)$	0 ↗ + ↘ 0	0 ↗ +	+ ↘ 0
$\Delta p_1(.)$	0 ↗ +	+ ↘ 0	+ ↘ 0
$\Delta p_2(.)$	0 ↗ + ↘ +	0 ↗ +	+ ↘ 0

3.6. Discussion and conclusion

We used a symmetric model with two consumers and two firms to study the impact of an insect pollinator decline on the social welfare. The pollinator decline will downsize the production of good 1 and will increase its price. The consumption of good 2 will increase and consequently its price will increase. Thus firm 1's profit will decrease and firm 2's profit will increase. Furthermore the consumers' revenues will not change after the decline. So their capacity to consume goods would be reduced, which explains why the consumers welfare decreases.

Using a numerical specification, we found that welfare loss would be reduced when consumers have weaker preferences on good 1. We also found that technological capacities of firms allow to reduce the impact of pollinator decline because it reduced the increase of prices and consequently the welfare loss decrease.

4. A specified symmetric model with a polarized ownership structure

The model of this section uses the same basis as the previous one, except for the distribution of revenues, which now corresponds to the polarized case. For that purpose, we will consider that consumer 1 is the owner of firm 1 and consumer *2* is the owner of firm 2, as by expressions [5] and [6]. Recall that the social revenue R is the sum of individual revenue R_c and is equal to: $R = R_1 + R_2 = p_1 f\left(z_{f1}, z_{f2}, D\right) + p_2 g\left(z_{g1}, z_{g2}\right)$.

Thus consumer *1*'s revenue is:

$$R_1 = \Pi^1 + a_1\left(z_{f1} + z_{g1}\right) - a_2 z_{f2} = p_1 f(.) + a_1 z_{g1} - a_2 z_{f2} \qquad [33]$$

and consumer *2*'s revenue is:

$$R_2 = \Pi^2 + a_2\left(z_{f2} + z_{g2}\right) - a_1 z_{g1} = p_2 g(.) + a_2 z_{f2} - a_1 z_{g1} \qquad [34]$$

The total revenues are

$$R = R_1 + R_2 = p_1 f(.) + p_2 g(.)$$
$$\Leftrightarrow \beta^{\frac{2\beta}{1-2\beta}} \left(\frac{(1-D)^{\frac{1}{1-2\beta}} p_1^{\frac{1}{1-2\beta}} + p_2^{\frac{1}{1-2\beta}}}{(a_1 a_2)^{\frac{\beta}{1-2\beta}}} \right)$$

Note that the social revenue is the same as in the symmetric model. Normalize again the price of the second input, so that $a_2 = 1$. As a consequence, neither the equilibrium prices p_1, p_2, a_1 (Expression [23], [24] and [25]), nor the total quantity exchanged X_1 and X_2 (Expressions [29] and [30]), will differ from that derived in the previous section. What remains to be done is to determine the individual exchanged quantities and the revenues.

4.1. The equilibrium of the economy

4.1.1. Revenues at the equilibrium

Consumer *1*'s revenue is described in expression [33] as a function of the production of good 1, the price of good 1, the price of both inputs, the quantities of inputs 1 and 2 consumed by firm 1. Thus using expressions [9], [10] and [16] of the Appendix C and [23], [24] we can give the revenue of consumer *1* when the economy is at the equilibrium:

$$R_1^* = \left(\frac{P_1(1-D)}{\left(\frac{Z_2}{Z_1}\right)^{\beta}} \right)^{\frac{1}{1-2\beta}} \left(\frac{1}{\beta} - 1 + \frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}} v^{\frac{1}{1-2\alpha\beta}}} \right) \qquad [35]$$

Consumer *2*'s revenue is described by expression [34] as a function of the price of good 2, the prices of both inputs, the quantity of input 1 consumed by firm 2 and the quantities of both inputs consumed by firm 2. Using expressions [14], [15] and [17] of the appendix C and [23], [25] we can give consumer *2*'s revenue when the economy is at the equilibrium:

$$R_2^* = \left(\frac{P_1(1-D)}{\left(\frac{Z_2}{Z_1}\right)^{\beta}} \right)^{\frac{1}{1-2\beta}} \left(\frac{1}{\beta(1-D)^{\frac{\alpha}{1-2\alpha\beta}} v^{\frac{1}{1-2\alpha\beta}}} + 1 - \frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}} v^{\frac{1}{1-2\alpha\beta}}} \right) \qquad [36]$$

The social revenue at the equilibrium R_c^* is equal to the sum of R_1^* and R_2^* and is equal to $R^* = \frac{\overline{Z}_2}{\beta}$ (see expression [26]). The social revenue would not be impacted by an insect pollinator decline. But, unlike the previous case, the coefficient D appears in the individual revenues. Consumer's revenues would be diversely impacted by a pollinator decline.

4.1.2. Exchanged quantities at the equilibrium

A consequence of the inequality of revenue is the difference on the distribution of consumption of goods 1 and 2 between both consumers. We can assert this by considering expressions of the exchanged quantities of good 1 and good 2 for consumers *1* and *2,* from expressions [24], [25], [35] and [36].

$$x_{11}^{*}=\left(\frac{P_1(1-D)}{\left(\frac{Z_2}{Z_1}\right)^{\beta}}\right)^{\frac{1}{1-2\beta}}\frac{\left(\frac{1}{\beta}-1+\frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}}v^{\frac{1}{1-2\alpha\beta}}}\right)}{\left(1+\frac{1}{v^{\frac{1}{1-2\alpha\beta}}(1-D)^{\frac{\alpha}{1-2\alpha\beta}}}\right)} \quad [37]$$

$$x_{12}^{*}=\left(\frac{P_1(1-D)}{\left(\frac{Z_2}{Z_1}\right)^{\beta}}\right)^{\frac{1}{1-2\beta}}\frac{\left(\frac{1}{\beta}-1+\frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}}v^{\frac{1}{1-2\alpha\beta}}}\right)}{\left(\frac{(1-D)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}}+v^{\frac{2\beta}{1-2\alpha\beta}}(1-D)^{\frac{1}{1-2\alpha\beta}}\right)} \quad [38]$$

$$x_{21}^{*}=\left(\frac{P_1(1-D)}{\left(\frac{Z_2}{Z_1}\right)^{\beta}}\right)^{\frac{1}{1-2\beta}}\frac{\left(\frac{1}{\beta(1-D)^{\frac{\alpha}{1-2\alpha\beta}}v^{\frac{1}{1-2\alpha\beta}}}+1-\frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}}v^{\frac{1}{1-2\alpha\beta}}}\right)}{\left(1+\frac{1}{v^{\frac{1}{1-2\alpha\beta}}(1-D)^{\frac{\alpha}{1-2\alpha\beta}}}\right)} \quad [39]$$

$$x_{22}^{*}=\left(\frac{P_1(1-D)}{\left(\frac{Z_2}{Z_1}\right)^{\beta}}\right)^{\frac{1}{1-2\beta}}\frac{\left(\frac{1}{\beta(1-D)^{\frac{\alpha}{1-2\alpha\beta}}v^{\frac{1}{1-2\alpha\beta}}}+1-\frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}}v^{\frac{1}{1-2\alpha\beta}}}\right)}{\left(\frac{(1-D)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}}+v^{\frac{2\beta}{1-2\alpha\beta}}(1-D)^{\frac{1}{1-2\alpha\beta}}\right)} \quad [40]$$

We also observed that these four expressions are dependent on the ratio *D*, which means that they will all vary after an insect pollinator decline.

4.2. What is the impact of pollinator decline on the social welfare?

In this model we assumed an asymmetric distribution of consumers' revenues. Regarding expressions 37 to 40 we note that individual exchanged quantities are different compared to the symmetric case (see expressions 29 to 32). Thus the impact of an insect pollinator decline will be modified considering the individual consumption of goods. This new results are summarized in the following propositions:

Proposition 4: Let $\alpha \in]0, 1[$ and $\beta\,]0, ½[$. Then the larger the pollinators decline the lower the consumption of good 1 at the equilibrium for consumer 1 and 2 (Proof: see Appendix E).

Proposition 5: Let $\alpha \in]0, 1[$ and $\beta\,]0, ½[$. Then the larger the pollinator decline the larger the consumer 2's consumption of good 2 at the equilibrium (Proof: see Appendix E).

Proposition 6: Let $\alpha \in]0, 1[$ and $\beta\,]0, \beta^*[$. Then the larger the pollinator decline the lower the consumer 1's consumption of good 2 at the equilibrium. And let $\alpha \in]0, 1[$ and $\beta\,]\beta^*, 1/2\, [$. Then the larger the pollinator decline the larger the consumer 1's consumption of good 2 at the equilibrium. (Proof: see Appendix E).

4.1. Numeric example

Again, we analyze the evolution of the main variables: $U^1(.)$, $U^2(.)$,$x_{11}(.)$, $x_{21}(.)$, $x_{12}(.)$, $x_{22}(.)$, $R_1(.)$ and $R_2(.)$. As in the preceding part, we focus the analysis on the influence of parameters v, β, α on these variables. Values attributed to parameters for which no change is considered are listed in Table 1.

Results are summarized in the table 3. Consumer *1*'s utility decreases after the pollinator decline. The larger v the larger this decrease. Also, the utility loss decreases when α and β increase. Consumer *1's* loss of utility is always higher than that of consumer *2*. The welfare of consumer *2* would decrease regardless the values of v and β. But the evolution of welfare would be positive with low values of α and positive with high values of α. In the numerical example, the welfare of consumer *2* decreases when alpha is lower than 0.94 and increases when it is higher.

The consumed quantity of good 1 would decrease after pollinator decline for both consumers. The consumed quantity of good 2 would always increase for consumer *2*. But it can decrease for consumer *1* when v is weak (less than 0.05 with our numerical example) and/or when β is lower than 0.25. Revenues of both consumers are symmetric: the revenue of consumer *1* will decrease in any case while revenue of consumer *2* will always increase, which imply that total social revenue will not vary after pollinator decline.

Considering these results, the H2 hypothesis is realizable *i.e.* that the utility of consumer 2 could increase after an insect pollinator decline. The consequence of this result is summarized in the next proposition 7:

Proposition 7: Under the polarized ownership structure and under the assumption H2, it exists θ such as social welfare variation is positive.

Table 3 – Representation of intensity and sensitivity of pollinator decline impact (ΔD) on the variation of the main variables of the economy: $\Delta U^1(.)$, $\Delta U^2(.)$, $\Delta x_{11}(.)$, $\Delta x_{21}(.)$, $\Delta x_{12}(.)$, $\Delta x_{22}(.)$, $\Delta R_1(.)$ and $\Delta R_2(.)$ face to evolution of parameters v, α and β. The impact of pollinator decline is indicated with + when it is > 0, - when it is < 0 and 0 when it is null. The sensitivity of variables face to parameters is indicating with □when pollinator impact increases, □ when it decreases and → when it does not vary.

Functions	v	α	β
$\Delta U^1(.)$	0 ↘ -	- ↗ 0	- ↗ -
$\Delta U^2(.)$	0 ↘ -	- ↗ +	- ↘ -
$\Delta x_{11}(.)$	0 ↘ -	- ↘ -	- ↗ -
$\Delta x_{21}(.)$	0 ↘ -	- ↘ -	- ↗ -
$\Delta x_{12}(.)$	0 ↘ - ↗ +	0 ↗ +	- ↗ +
$\Delta x_{22}(.)$	0 ↗ + ↘ 0	0 ↗ +	+ ↗ +
$\Delta R_1(.)$	0 ↘ - ↗ -	0 ↘ -	- ↗ 0
$\Delta R_2(.)$	0 ↗ + ↘ +	0 ↗ +	+ ↘ +

4.2. Discussion and conclusion

In this model we assumed an asymmetry between the revenues of consumers. The impact of the pollinator decline cannot be done using analytical result. But using numerical example it appears that, after a pollinator decline, welfare loss of consumer *1* would be higher than welfare loss of consumer *2*.

As we already studied it in the preceding case, the pollinator decline will cause a decrease of production of good 1 and an increase of production of good 2 it implies that prices of goods will both increase. We also found that profit of firm 1 will decrease and profit of firm 2 will increase. On the contrary of the preceding analysis, consumers do not have the same origin of revenue. The revenue of consumer *1* depends on profit of firm 1 and the revenue of consumer *2* depends on profit of firm 2. As a consequence the revenue of consumer *1* decreases and revenue of consumer *2* increases. So their capacity to consume goods would vary. Indeed consumer *1* could not buy as much as than before shock on production and as much as than in the preceding case. On the other hand, the consumer *2* could buy much more than in the preceding case. Consequently, the welfare of consumer *1* would decrease much more than in the preceding case while the welfare of consumer *2* would increase compared to the preceding case.

We also found that the welfare of consumer *2* could increase after pollinator decline in the case of α close to 1. However α represents the needs to consume a certain quantity of good in order to satisfy the utility. The more α is closer to 1, the less we need to consume the good for the same utility. So a consumer *2* with a high α do not need to consume a lot of goods in order to maintain his utility at the same level than before the pollinator decline. A consequence of the increase of revenue is that consumer *2* could increase his utility compared to the one before the shock.

The utility of consumers is also related to the technological capacity of firms, β. But contrarily of the preceding case the influence of β on the consumer utility is different since it could decrease the loss in utility of the consumer *1* and it could increase the loss of consumer *2*. We explained this difference because when β tends to 0.5, the adaptation of firms to pollinator loss would be better and the profit loss of firm 1 would decrease. Consequently revenue loss of consumer *1* decreases, which implies that he could buy more goods. As the quantity of goods is limited on the economy, the consequence of the improvement of the purchasing power of consumer *1* is the decrease of the utility of consumer *2*.

Finally since the asymmetry of revenue was solely due to change in profit of firms, while wages stay the same than in the perfectly symmetric case, a consequence of a pollinator decline could be interpreted as the consequence on the firm owner's welfare. So the welfare of the firm owner, that depends on insect pollinations would be more vulnerable to pollinator decline than the firm owner that does not.

5. A specified asymmetric model with egalitarian ownership structure

5.1. The model

We analyzed the insect pollinator decline impact considering that producers and consumers were homogeneous *i.e.* that both producers share the same capacities to produce and that both consumers have the same preferences on goods. However we demonstrated that impact of pollinator decline would be reduced when production capacities of firm is high (β tends to ½). We also found that profit of firms would change in a symmetric way after pollinator decline *i.e.* the increase of profit of firm 2 is equal to the decrease of profit of firm 1. Considering different production capacities would imply an asymmetry in the adaptation of firms to the loss of production of good 1. Consequently the welfare loss due to pollinator decline could be worsened or on the contrary it could be improved. We will analyze the different cases using explicit functions.

The model exposed in this part is based on the preceding models: the economy has two goods $h = 1, 2$, two inputs $k = 1, 2$, produced by two firms $n = 1, 2$ and two consumers $c = 1, 2$. The goods 1 is dependent on insect pollination and good 2 is not. This model is perfectly symmetric because consumers are owners of an equal part of firms.

We introduce a parameter $T_n > 0$, representing a technological scale parameter. We assume that this technological aspect is a second difference between firms, in addition to the fact that that only the production of good 1 by the first firm depends on insect pollination.

The technological parameter is incorporated as follows on the production functions (see expressions [9] and [10]):

$$f(z_{f1}, z_{f2}, D) = (1 - D) T_1 z_{f1}^{\beta} z_{f2}^{\beta} \quad [41]$$

And production function of firm 2 is:

$$g(z_{g1}, z_{g2}) = T_2 z_{g1}^{\beta} z_{g2}^{\beta} \quad [42]$$

With such a formulation, Appendix F shows that firm 1's demands for inputs are:

$$z_{f1} = \frac{\left(p_1 T_1 \beta (1 - D)\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{1-\beta}{1-2\beta}} a_2^{\frac{\beta}{1-2\beta}}} \quad [43]$$

$$z_{f2} = \frac{\left(p_1 T_1 \beta (1-D)\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{\beta}{1-2\beta}} a_2^{\frac{1-\beta}{1-2\beta}}} \qquad [44]$$

and firm 2's demands are:

$$z_{g1} = \frac{\left(p_2 T_2 \beta\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{1-\beta}{1-2\beta}} a_2^{\frac{\beta}{1-2\beta}}} \qquad [45]$$

$$z_{g2} = \frac{\left(p_2 T_2 \beta\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{\beta}{1-2\beta}} a_2^{\frac{1-\beta}{1-2\beta}}} \qquad [46]$$

The consumers' welfare does not change compared of preceding models and the explicit function is described by expression [15] and consequently their individual demand of goods 1 and 2 is described by expressions [16] and [17]. The function of revenue is the one explained in expression [3] and [4].

5.2. The consumer revenues

We considered that consumers were owners of 50% of both firms, so we used expressions [3] and [4] to express respectively the revenue of consumer *1* and *2*.

$$R_1 = 0.5\left(\left(p_1 T_1 (1-D)\right)^{\frac{1}{1-2\beta}} \left(\frac{(\beta)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}} + \left(T_2 p_2\right)^{\frac{1}{1-2\beta}} \left(\frac{(\beta)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}} + a_1 z_{f1} + a_1 z_{g1} - a_2 z_{f2} - a_2 z_{g2}\right) \qquad [47]$$

$$R_2 = 0.5\left(\left(p_1 T_1 (1-D)\right)^{\frac{1}{1-2\beta}} \left(\frac{(\beta)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}} + \left(T_2 p_2\right)^{\frac{1}{1-2\beta}} \left(\frac{(\beta)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}} + a_2 z_{f2} + a_2 z_{g2} - a_1 z_{f1} - a_1 z_{g1}\right) \qquad [48]$$

The total social revenue is the sum of R_1 and R_2:

$$R = \left(p_1 T_1 (1-D)\right)^{\frac{1}{1-2\beta}} \left(\frac{(\beta)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}} + \left(T_2 p_2\right)^{\frac{1}{1-2\beta}} \left(\frac{(\beta)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}}$$

$$\Leftrightarrow R = \left(\frac{(\beta)^2}{a_1 a_2}\right)^{\frac{\beta}{1-2\beta}} \left(\left(T_1 p_1 (1-D)\right)^{\frac{1}{1-2\beta}} + \left(T_2 p_2\right)^{\frac{1}{1-2\beta}}\right) \qquad [49]$$

5.3. Determining equilibrium of the economy

5.3.1. Prices: p_1, p_2, a_1 and a_2

We assumed that the price of input 2 is normalized to 1. So using expressions [26] and [27] of the Appendix F and the relation between p_1 and p_2 obtained due to equality between supply and demand for good we found that:

$$a_1^* = \frac{\overline{Z}_2}{\overline{Z}_1} \quad [50]$$

$$p_1^* = \frac{\overline{Z}_1^{-\beta}\,\overline{Z}_2^{1-\beta}}{\left(\beta\left(\left(T_1(1-D)\right)^{\frac{1}{1-2\beta}} + T_2^{\frac{1}{(1-2\beta)^2}}\,\frac{\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)}}}{v^{\frac{1}{1-2\alpha\beta}}}\right)\right)^{1-2\beta}} \quad [51]$$

$$p_2^* = \frac{(1-D)^{\frac{1-\alpha}{1-2\alpha\beta}}\,v^{\frac{1-2\beta}{2\alpha\beta-1}}\,\overline{Z}_1^{-\beta}\,\overline{Z}_2^{1-\beta}}{\left(\beta\left(\left(T_1(1-D)\right)^{\frac{1}{1-2\beta}} + T_2^{\frac{1}{(1-2\beta)^2}}\,\frac{\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)}}}{v^{\frac{1}{1-2\alpha\beta}}}\right)\right)^{1-2\beta}} \quad [52]$$

5.3.2. Exchanged quantities

Now we can determine all variables, R, R_1, R_2, $f(.)$, $g(.)$, X_1, X_2, x_{11}, x_{21}, x_{12}, and x_{22} function of parameters v, β, α, D, T_1, T_2, $\overline{Z}_1$ and $\overline{Z}_2$.

Following expressions [47], [48], [49], [50], [51] and [52]:

$$R^* = \frac{\overline{Z}_2}{\beta} \quad [53]$$

$$R_1^* = \frac{\overline{Z}_2}{2\beta} \quad [54]$$

$$R_2^* = \frac{\overline{Z}_2}{2\beta} \quad [55]$$

The consumed quantities in the economy are:

$$X_1^* = f(.,.,D)^* = T_1^{\frac{2\beta}{1-2\beta}} \left(\overline{Z}_1 \overline{Z}_2\right)^{\beta} \frac{(1-D)}{\left(T_1 + T_1^{\frac{2\beta(1-\alpha)}{1-2\alpha\beta}} T_2^{\frac{1-2\beta}{1-2\alpha\beta}} (1-D)^{\frac{1-\alpha}{(1-2\alpha\beta)}} v^{\frac{1}{2\alpha\beta-1}}\right)^{2\beta}} \quad [56]$$

$$X_2^* = g(.,.)^* = \frac{\left(\overline{Z}_1 \overline{Z}_2\right)^{\beta}}{v^{\frac{2\beta}{1-2\alpha\beta}}} \frac{T_2^{\frac{2\beta}{1-2\beta}}}{\left(T_1 (1-D)^{\frac{\alpha}{1-2\alpha\beta}} + \frac{T_1^{\frac{2\beta(1-\alpha)}{1-2\alpha\beta}} T_2^{\frac{1-2\beta}{1-2\alpha\beta}}}{v^{\frac{1}{1-2\alpha\beta}}}\right)^{2\beta}} \quad [57]$$

$$x_{c1}^* = \frac{1}{2} X_1^* \qquad x_{c2}^* = \frac{1}{2} X_2^* \quad [58 \;\&\; 59]$$

As in the preceding section, the revenue of consumers will not vary after pollinator loss. We also observe that the parameters T_1 and T_2 appear on the functions of quantities exchanged. This result confirms that technological parameters could change equilibrium after pollinator decline compared to equilibrium with homogeneous firms.

5.4. What is the impact of pollinator decline on the social welfare?

We are now in position to study the impact of insect pollinators decline on the social welfare and individual utilities and the evolution of the welfare variation as function of T_1 and T_2. We assume that T_2 is equal to 1 and we study the evolution of T_1 between 0 and 10 i.e. when T_1 can be lower or higher than T_2. Values are attributed to the following parameters: v, β, α, D, $\overline{Z}_1$ and $\overline{Z}_2$. We also analyze the evolution of the main variables: $U(.)$, $U^1(.)$, $U^2(.)$, $X_1(.)$, $X_2(.)$, $x_{11}(.)$, $x_{21}(.)$, $x_{12}(.)$, $x_{22}(.)$, $p_1(.)$, $p_2(.)$, $\Pi_1(.)$ and $\Pi_2(.)$. Values attributed to parameters that are not subject to variations are listed in Table 4.

Table 4 – Value of parameters

α	β	D	$\overline{Z}_1$	$\overline{Z}_2$	v	T_2
1/2	1/3	1/2	1	1	1	1

The social and individual welfare losses increase when T_1 is higher than T_2. The same property holds as for consumed quantities of good 1 (Table 5). Consumed quantities of good 2 increase after pollinator decline. The intensity of this increase is maximum when T_1 = 2.

The profit of firm 1 decreases after a pollinator decline. This loss is maximum when $T_1 = T_2$ and is null when T_1 tends to 0 or tends to infinity. The profit of firm 2 increases after a pollinator decline. This gain is maximum when $T_1 = T_2$ and is null when T_1 tends to 0 or tends to infinity. Both prices increase after a pollinator loss. The increase of price of good 1 is higher when T_1 is lower than T_2. The increase of price of good 2 is higher when $T_1 > T_2$.

Table 5 – Representation of intensity and sensitivity of pollinator decline impact (ΔD) on the variation of the main variables of the economy: $\Delta W(.)$, $\Delta U^1(.)$, $\Delta U^2(.)$, $\Delta \Pi_1(.)$, $\Delta \Pi_2(.)$, $\Delta X_1(.)$, $\Delta X_2(.)$, $\Delta x_{11}(.)$, $\Delta x_{21}(.)$, $\Delta x_{12}(.)$, $\Delta x_{22}(.)$, $\Delta p_1(.)$ and $\Delta p_2(.)$ face to evolution of parameter T_1 compared to T_2. The impact of pollinator decline is indicated with + when it is >0, - when it is <0 and 0 when it is null. The sensitivity of variables face to parameters is indicating with □when pollinator impact increases, □ when it decreases and → when it does not vary.

Functions	T_1
$\Delta W(.)$	0 ↘ -
$\Delta U^1(.)$	0 ↘ -
$\Delta U^2(.)$	0 ↘ -
$\Delta x_{11}(.)$	0 ↘ -
$\Delta x_{21}(.)$	0 ↘ -
$\Delta X_1(.)$	0 ↘ -
$\Delta x_{12}(.)$	0 ↗ + ↘ 0
$\Delta x_{22}(.)$	0 ↗ + ↘ 0
$\Delta X_2(.)$	0 ↗ + ↘ 0
$\Delta \Pi_1(.)$	0 ↘ - ↗ 0
$\Delta \Pi_2(.)$	0 ↗ + ↘ 0
$\Delta p_1(.)$	+ ↘ 0
$\Delta p_2(.)$	0 ↗ +

5.5. Discussion and conclusion

The last model generalizes the analyses by incorporating a second technological difference between firms that owes nothing to pollinators. Consequently we observed that the welfare loss observed so far could decrease depending on ratio between technology of firm 1 and technology of firm 2. In the case where the technology of firm 1 is more productive than that of firm 2, the welfare loss would be higher than when firms were homogeneous or when firm 2 is more productive.

Let us explain this paradoxical result by considering than firm 1 has a better technology than firm 2. Then a pollinator decline implies a decrease of production of good 1 and an increase of production of good 2 and a rise of all market prices. Regarding the curve of good 1's price (Table 5), the increase of technology would imply a decrease of price because of the decrease of the cost related to the use of inputs. Thus the decrease of profit of firm 1 is dampened. This improvement of profit would not benefit to consumers because it will be associated to a decrease of wages. At the same time, the price of good 2 would increase and would be higher than price of good 1. Consumers would not benefit of lower price of good 1 because of limited quantity of good 1 and would have to consume good 2, which is more expensive. Consequently their capacity to consume and, thus their utilities, would be reduced compared to the case with homogeneous firms and the case where firm 2 is more productive.

6. A specified asymmetric model with a polarized ownership structure

6.1. The model

We found that when property rights are heterogeneously distributed in the economy, consumer *2* could gain depending on the value of α. Under this condition, social welfare loss is reduced, or possibly increased. We also found that the heterogeneity of firms due to productivity differences could reduce the welfare loss. We now combine those two possibilities to analyze their interplay.

We use the same model as in Section 4, but we assumed that revenues are the ones described by expressions [5] and [6]. It implies that aggregated to the total economy will be the same than in the preceding model: input used (expressions [43], [44], [45] and [46]), total quantities exchanged (expressions [56] and [57]), prices ($a_2 = 1$, expressions [50], [51] and [52]) and total revenue (expression [53]).

In brief, we studied in which conditions the heterogeneity of firms' technologies, considering an economy with heterogeneous source of revenue of consumers, would reduce or improve the negative consequences of a pollinator decline. Firstly we calculate the individual consumer revenues and the individual exchanged quantities. Then we study the impact of a pollinator decline under different values for T_1 and T_2.

6.1. The equilibrium of the economy

6.1.1. Revenues at the equilibrium

Consumer 1's revenue is described by expression [5], as a function of the production of good 1, the price of good 1, the prices of inputs 1 and 2, the quantity of inputs 1 and 2 consumed by firm 1. Thus using expressions [20] and [21] of the Appendix F, [56], [50] and [51] we can give consumer 1's revenue when the economy is at the equilibrium:

$$R_1^* = \frac{Z_2\left(\frac{((1-D)T_1)^{\frac{1}{1-2\beta}}}{\beta} - ((1-D)T_1)^{\frac{1}{1-2\beta}} + \frac{\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}}\right)}{((1-D)T_1)^{\frac{1}{1-2\beta}} + \frac{T_2^{\frac{1}{(1-2\beta)^2}}\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)}}}{v^{\frac{1}{1-2\alpha\beta}}}} \qquad [59]$$

Consumer 2's revenue is described in the expression [6] as a function of the price of good 2, the price of both inputs, the quantities of both inputs consumed by firm 2. Using expressions [22], [23] of the Appendix F, [57], [50] and [52] we can give consumer 2's revenue when the economy is at the equilibrium:

$$R_2^* = \frac{Z_2\left(\frac{T_2^{\frac{1}{(1-2\beta)^2}}\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)}}}{\beta v^{\frac{1}{1-2\alpha\beta}}} - \frac{\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}} + ((1-D)T_1)^{\frac{1}{1-2\beta}}\right)}{((1-D)T_1)^{\frac{1}{1-2\beta}} + \frac{T_2^{\frac{1}{(1-2\beta)^2}}\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)}}}{v^{\frac{1}{1-2\alpha\beta}}}} \qquad [60]$$

The social revenue at the equilibrium R_c* is equal to the sum of R_1* and R_2* and is equal to $R^* = \frac{\overline{Z}_2}{\beta}$ (expression [26]). The social revenue would not be impacted by an insect pollinator decline. Regarding the individual revenues, the coefficients D, T_1 and T_2 appear, on the contrary of the preceding case, which means that consumer's revenues would be impacted by a pollinator decline and this impact would be different following the technology of firms.

6.1.2. Exchanged quantities

The exchanged quantities of good 1 and good 2 for consumers *1* and *2* are:

$$x_{11}^* = \frac{\left(\beta^{\frac{1}{1-2\beta}} p_1^{*\frac{2\beta}{1-2\beta}}\right)}{\left(\frac{Z_2}{Z_1}\right)^{\frac{\beta}{1-2\beta}}} \left(\frac{\frac{\left((1-D)T_1\right)^{\frac{1}{1-2\beta}}}{\beta} - \left((1-D)T_1\right)^{\frac{1}{1-2\beta}} + \frac{\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}}}{1 + \frac{1}{v^{\frac{1}{1-2\alpha\beta}}\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{\alpha}{1-2\alpha\beta}}}} \right) \quad [61]$$

$$x_{12}^* = \frac{\left(\beta^{\frac{1}{1-2\beta}} p_1^{*\frac{2\beta}{1-2\beta}}\right)}{\left(\frac{Z_2}{Z_1}\right)^{\frac{\beta}{1-2\beta}}} \left(\frac{\frac{\left((1-D)T_1\right)^{\frac{1}{1-2\beta}}}{\beta} - \left((1-D)T_1\right)^{\frac{1}{1-2\beta}} + \frac{\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}}}{\frac{\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}} + v^{\frac{2\beta}{1-2\alpha\beta}}\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1}{1-2\alpha\beta}}} \right) \quad [62]$$

$$x_{21}^* = \frac{\left(\beta^{\frac{1}{1-2\beta}} p_1^{*\frac{2\beta}{1-2\beta}}\right)}{\left(\frac{Z_2}{Z_1}\right)^{\frac{\beta}{1-2\beta}}} \left(\frac{\frac{T_2^{\frac{1}{(1-2\beta)^2}}\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)}}}{\beta v^{\frac{1}{1-2\alpha\beta}}} - \frac{\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}} + \left((1-D)T_1\right)^{\frac{1}{1-2\beta}}}{1 + \frac{1}{v^{\frac{1}{1-2\alpha\beta}}\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{\alpha}{1-2\alpha\beta}}}} \right) \quad [63]$$

$$x_{22}^{*}=\frac{\left(\beta^{\frac{1}{1-2\beta}}p_1^{*\frac{2\beta}{1-2\beta}}\right)}{\left(\frac{Z_2}{Z_1}\right)^{\frac{\beta}{1-2\beta}}}\left(\frac{\dfrac{T_2^{\frac{1}{(1-2\beta)^2}}\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)}}}{\beta v^{\frac{1}{1-2\alpha\beta}}}-\dfrac{\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}}+\left((1-D)T_1\right)^{\frac{1}{1-2\beta}}}{\dfrac{\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}}+v^{\frac{2\beta}{1-2\alpha\beta}}\left(\frac{T_1}{T_2}(1-D)\right)^{\frac{1}{1-2\alpha\beta}}}\right) \quad [64]$$

6.2. What is the impact of pollinator decline on the social welfare?

We study in this section the impact of an insect pollinator decline on the social welfare and individual utilities and the evolution of the welfare variation in function of T_1 and T_2. As in the preceding section, we assume that T_2 is equal to 1 and we study the evolution of T_1 between 0 and 10 *i.e.* when T_1 was lower or higher than T_2. Furthermore, in the section 4 *i.e.* when T_1 was equal to T_2, we found that when α was equal or higher than 0.94 the utility of consumer *2* would be equal to 0 or increase after pollinator decline. For this numerical example we assumed that $\alpha = 0.94$. Thus we could analyze in which conditions of T_1 and T_2 the welfare gain of consumer *2* could increase or decrease. We also attribute values to following parameters: *v, β, D,* $\overline{Z}_1$ and $\overline{Z}_2$ (Table 4). Again we analyze the evolution of the main variables: $U^1(.), U^2(.), x_{11}(.), x_{21}(.), x_{12}(.), x_{22}(.), R_1(.)$ and $R_2(.)$.

The utility of consumer *1* decreases in relation to the increase of T_1, whatever the value of T_2 (Table 6). The utility of consumer *2* increases when $T_1 < T_2$ and decreases when $T_1 > T_2$. The quantity consumed of good 1 by consumer *1* decreases in relation to T_1 and his consumption of good 2 decreases when $T_1 < T_2$ and increases $T_1 > T_2$. The consumption of good 1 by consumer *2* decreases when $T_1 < T_2$ and increases when $T_1 > T_2$. His consumption of good 2 increases in relation to an increase in T_1. Finally consumer 1's revenue decreases whereas consumer 2's revenue increases. Consumer 1's loss of revenue and consumer 2's gain is maximum when $T_1 = T_2$.

Table 6 – Representation of intensity and sensitivity of pollinator decline impact (ΔD) on the variation of the main variables of the economy: $\Delta W(.)$, $\Delta U^1(.)$, $\Delta U^2(.)$,$\Delta \Pi_1(.)$, $\Delta \Pi_2(.)$, $\Delta X_1(.)$, $\Delta X_2(.)$, $\Delta x_{11}(.)$, $\Delta x_{21}(.)$, $\Delta x_{12}(.)$, $\Delta x_{22}(.)$, $\Delta p_1(.)$ and $\Delta p_2(.)$ face to evolution of parameter T_1 compared to T_2. The impact of pollinator decline is indicated with + when it is >0, - when it is <0 and 0 when it is null. The sensitivity of variables face to parameters is indicating with □when pollinator impact increases, □ when it decreases and → when it does not vary.

Functions	T_1
$\Delta U^1(.)$	0 ↘ -
$\Delta U^2(.)$	0 ↗ + ↘ -
$\Delta x_{11}(.)$	0 ↘ -
$\Delta x_{21}(.)$	0 ↘ - ↗ + ↘ 0
$\Delta x_{12}(.)$	0 ↗ + ↘ 0
$\Delta x_{22}(.)$	0 ↗ + ↘ 0
$\Delta R_1(.)$	0 ↘ - ↗ 0
$\Delta R_2(.)$	0 ↗ + ↘ 0

6.3. Discussion and conclusion

The impact of an insect pollinator decline on the social welfare is not necessarily negative, since consumer 2' s utility can increase when the second firm technology is more productive.

The adaptation of the economy after an insect pollinator decline under heterogeneous revenues is characterized by a decrease of exchanged quantities of good 1 and increase of all market prices. Also, consumer *1* loses some revenue, which implies that he cannot maintain his previous level of satisfaction. The adaptation of consumer *2* depends on the relative productivities of firms. Let us start with the case where firm 2 is more productive, *i.e.* for the same use of input the production of firm 2 would be higher than the production of firm 1. The market price of good 2 is lower than the price of good 1. Consequently consumer *2* buys more of good 2 and even if his consumption of good 1 decrease his utility after a pollinator loss is larger than before. On the other hand, when firm 1 is more productive, the price of good 2 is higher than the price of good 1. Consequently consumers both reallocate their consumptions with a bias towards good 1. However the profit of firm 2 decreases compared to the case where $T_1 < T_2$. Consumer *2*'s revenue decreases as well. His purchasing power is cut down and he enjoys a lower level of utility.

7. Discussion and perspective

The contribution of insect pollinators on the world agriculture has been evaluated at €153 billion (Gallai *et al.,* 2009). This value can be interpreted as a rough indicator of pollinator importance over the world. The consequence of such a dependence of insect pollination is the vulnerability of the social welfare confronted with a pollinator decline. Indeed, a decline of insect pollinator would impact prices of crop and in a second time the crop production exchanged in the market. This assessment of a pollinator loss impact on a single market has been evaluated at the scale of Australia (Gordon and Davis, 2003), United States (Southwick and Southwick, 1992) and the world level (Gallai *et al.,* 2009). By contrast, the present work qualifies those findings. Using general equilibrium with two markets, it is shown that while the pessimistic conclusion of an adverse consequence on welfare is somehow robust, it is not necessary. When several markets are taken into account in a general equilibrium, the ecological shock has redistributive effects. Often the shock makes every agent lose his purchasing power, hence the social satisfaction falls dawn. But sometimes, actually when the ownership structure is polarized, there can be losers and winners. This is so because the second market, which does not depend on insect pollinator, cushions the economic consequences of a pollinator loss. Consumers compensate the loss of the pollinated good by consuming more of the other good and the welfare loss is softened. If the social "good" attaches more importance to those who do not possess the pollinated activity, and who see an increase in their revenue after the shock, there can even be a welfare improvement. This happens when the second sector is more productive or/and when the elasticity of substitution between goods is high enough.

In this general equilibrium model, we assume that the insect pollinator decline is exogenous. But the decline of pollinators is due to anthropogenic pressures. More particularly the use of agrochemicals in agriculture is responsible of a large part of their loss (Kuldna *et al.* 2009, National Research Council, 2007). But the use of these means in agriculture is important and it would be useful to study the optimal use of these inputs and the optimal use of the insect pollination input. Two modifications must be undertaken in order to introduce this question in our general equilibrium model. Firstly, the insect pollinators have to be taken as an endogenous variable. And secondly the relation between pollinators' abundance and the quantity of pesticide used in agriculture must be modelized.

Another way to improve the model would be to assume that only the wild pollinators could disappear which would mean that only the costly domestic pollinators would remain. Thus the shock would imply an increase of the production cost. Indeed, the insect pollinators are divided into two major categories: wild ones that are totally offered by Nature and domestic ones that are located by keepers for crop pollination or used for the honey production. However the wild pollinators decline is obvious (Biesmeijer *et al.* 2006), whereas the domestic pollinators decline is not, since it is possible to keep bee colonies. Thus Aizen and Harder (2009) have demonstrated that the world stock of honeybees increased since 1961. A pessimistic scenario could be the total disappearance of the abundance and diversity of wild bees, which would lead to a total dependence of the crop pollination by domestic bees. Furthermore the abundance of these insects is not stationary since they suffer from the Varoa destructor and other diseases such as the Colony Collapse Disorder. Partial equilibrium models by Burgett *et al.* (2004) and Rucker *et al.* (2005) demonstrated that the impact of a variation of the bee population's density would imply changes in price of colonies, honey and crops. It would be interesting to use these results in a general equilibrium model introducing the beekeeper as a firm. Thus the gain of beekeepers due to decline of wild pollinators could reduce the welfare loss described in the model used here.

8. Conclusion

Generally, though not systematically, the social welfare decreases after an insect pollinator loss. This decrease goes through the modifications in the production capacity of firms and its extent depends on consumers' preferences on the pollinated good. Consequently, both firms and consumers are diversely affected by the ecological shock. This general message has been obtained and has been given a more precise content by using four slightly different general equilibrium models. Each has two consumers, two goods and two firms producing only one good each. The production of the first good depends on insect pollinators whereas the production of the second good does not. The first model considers identical consumers who have equal shares of the two firms (the egalitarian case). In the second model the ownership structure is polarized: each consumer possesses only one firm. The third and fourth model introduces a second dimension of heterogeneity between producers, related to a productivity parameter.

The main result is that, under the egalitarian distribution of property rights, all the agents suffer from the shock, hence there is a reduction of welfare; by contrast, under the polarized structure, the agent who possesses the pollinated activity experiences an utility reduction, whereas the other agent can experience a higher utility. This result holds when: 1) either the elasticity of substitution between the two consumption goods is sufficiently high, 2) or when the non pollinated sector is relatively more productive than the pollinated sector. In either case, welfare can increase if the second agent is granted a relatively more important weight in the social welfare criterion. One policy implication from this general equilibrium appraisal is that the quest of efficiency is not the only justification for a public regulation in face of a pollinator shock. This reason may even collapse. A second justification, probably more robust, rests on redistributive goals.

9. References

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10. Appendix

10.1. Appendix A

The profit of firm 1 is maximum when

$$\frac{\partial \Pi_1}{\partial z_{f1}} = p_1 f'\left(z_{f1}, z_{f2}, D\right) - a_1 = 0$$

$$\frac{\partial \Pi_1}{\partial z_{f2}} = p_1 f'\left(z_{f1}, z_{f2}, D\right) - a_2 = 0$$

The optimal use of inputs by firm 1 are $z^*_{f1}(p_1, D, a_1, a_2)$ and $z^*_{f2}(p_1, D, a_1, a_2)$.

The profit of firm 2 is maximum when

$$\frac{\partial \Pi_2}{\partial z_{g1}} = p_2 g'\left(z_{g1}, z_{g2}\right) - a_1 = 0$$

$$\frac{\partial \Pi_2}{\partial z_{g2}} = p_2 g'\left(z_{g1}, z_{g2}\right) - a_2 = 0$$

The optimal use of input by firm 1 are $z^*_{g1}(p_2, a_1, a_2)$ and $z^*_{g2}(p_2, a_1, a_2)$

As a result the total production plan of good 1 and 2 at the prevailing prices can be detailed. Indeed we can write $f(p_1, D, a_1, a_2)$ and $g(p_2, a_1, a_2)$. The total demand of input $Z_1 = z_{f1}(p_1, D, a_1, a_2) + z_{g1}(p_2, a_1, a_2)$ and $Z_2 = z_{g1}(p_1, D, a_1, a_2) + z_{g2}(p_2, a_1, a_2)$. On the other side of the markets of inputs k are constant, $Z_1 = \overline{Z}_1$ and $Z_2 = \overline{Z}_2$.

10.2. Appendix B

Consumer maximizes his utility $U^c(x_{c1}, x_{c2}) = \frac{v x_{c1}^{\alpha}}{\alpha} + \frac{x_{c2}^{\alpha}}{\alpha}$ considering the budget constraint:

$$R_c \geq p_1 x_{c1} + p_2 x_{c2}$$

$$U_1^c = v x_{c1}^{\alpha - 1} \quad [1]$$

$$U_2^c = x_{c2}^{\alpha - 1} \quad [2]$$

At the equilibrium, consumer use all his revenue to consume x_{c1} and x_{c2} so that $R_c = p_1 x_{c1} + p_2 x_{c2}$ and consumption choices are done so that the marginal rate of substitution (MRS) x_{c1} and x_{c2} is equal to the slope of the budget curve which is p_1/p_2. We can define the optimal consumption of x_{c1} and x_{c2}:

$$MRS = \frac{\frac{\partial U}{\partial x_{c1}}}{\frac{\partial U}{\partial x_{c2}}} = \frac{v x_{c1}^{\alpha-1}}{x_{c2}^{\alpha-1}} = \frac{p_1}{p_2}$$
$$\Leftrightarrow x_{c1} = x_{c2}\left(\frac{p_1}{v p_2}\right)^{\frac{1}{\alpha-1}} \quad [3]$$

$$R_c = p_1 x_{c2}\left(\frac{p_1}{v p_2}\right)^{\frac{1}{\alpha-1}} + p_2 x_{c2}$$
$$\Leftrightarrow x_{c2} = \frac{R_c}{p_2 + p_1\left(\frac{p_1}{v p_2}\right)^{\frac{1}{\alpha-1}}} \quad [4]$$

From expressions [3] and [4] it comes:

$$x_{c1} = \frac{R_c}{p_1 + p_2\left(\frac{v p_2}{p_1}\right)^{\frac{1}{\alpha-1}}} \quad [5]$$

10.3. Appendix C

For the specific functional forms used in this chapter, the profit of firm 1 is:

$$\Pi^1 = p_1(1-D) z_{f1}^{\beta} z_{f2}^{\beta} - a_1 z_{f1} - a_2 z_{f2}$$

This profit is maximum when z_{f1} and z_{f2} verify:

$$\frac{\partial \Pi^1}{\partial z_{f1}} = p_1 \beta (1-D) z_{f1}^{\beta-1} z_{f1}^{\beta} - a_1 = 0 \quad [6]$$

$$\frac{\partial \Pi^1}{\partial z_{f2}} = p_1 \beta (1-D) z_{f1}^{\beta} z_{f2}^{\beta-1} - a_2 = 0 \quad [7]$$

$$\Leftrightarrow p_1\beta(1-D) = az_{f1}^{1-\beta}z_{f2}^{-\beta} = bz_{f1}^{-\beta}z_{f2}^{1-\beta}$$
$$\Leftrightarrow a_1 z_{f1} = a_2 z_{f2} \quad [8]$$

Using expressions [6], [7] and [8] we determine z_{f1} and z_{f2}, the optimal demand of input 1 and 2 for the firm 1 production.

$$z_{f1} = \frac{(p_1\beta(1-D))^{\frac{1}{1-2\beta}}}{a_1^{\frac{1-\beta}{1-2\beta}} a_2^{\frac{\beta}{1-2\beta}}} \quad [9]$$

$$z_{f2} = \frac{(p_1\beta(1-D))^{\frac{1}{1-2\beta}}}{a_1^{\frac{\beta}{1-2\beta}} a_2^{\frac{1-\beta}{1-2\beta}}} \quad [10]$$

The profit of firm 2 is

$$\Pi^2 = p_2 z_{g1}^{\beta} z_{g2}^{\beta} - a_1 z_{g1} - a_2 z_{g2}$$

This profit is maximum when z_{g1} and z_{g2} verify:

$$\frac{\partial\Pi^2}{\partial z_{g1}} = p_2\beta z_{g1}^{\beta-1} z_{g2}^{\beta} - a_1 = 0 \quad [11]$$

$$\frac{\partial\Pi^2}{\partial z_{g2}} = p_2\beta z_{g1}^{\beta} z_{g2}^{\beta-1} - a_2 = 0 \quad [12]$$

$$\Leftrightarrow p_2\beta = az_{g1}^{1-\beta}z_{g2}^{-\beta} = bz_{g1}^{-\beta}z_{g2}^{1-\beta}$$
$$\Leftrightarrow a_1 z_{g1} = a_2 z_{g2} \quad [13]$$

Using expressions [11], [12] and [13] we determine z_{g1} and z_{g2}, the optimal demand of input 1 and 2 for the production of firm 2.

$$z_{g1} = \frac{(p_2\beta)^{\frac{1}{1-2\beta}}}{a_1^{\frac{1-\beta}{1-2\beta}} a_2^{\frac{\beta}{1-2\beta}}} \quad [14]$$

$$z_{g2} = \frac{(p_2\beta)^{\frac{1}{1-2\beta}}}{a_1^{\frac{\beta}{1-2\beta}} a_2^{\frac{1-\beta}{1-2\beta}}} \quad [15]$$

We can now calculate the total production of goods 1 using expressions [9] and [10] and good 2 using expressions [14] and [15]:

$$f(z_{f1}, z_{f2}) = (1-D)^{\frac{1}{1-2\beta}} \left(\frac{(\beta p_1)^2}{a_1 a_2} \right)^{\frac{\beta}{1-2\beta}} \quad [16]$$

$$g(z_{g1} z_{g2}) = \left(\frac{(\beta p_2)^2}{a_1 a_2} \right)^{\frac{\beta}{1-2\beta}} \quad [17]$$

The total demand of input Z_1 is calculated using expressions [9] and [14]:

$$Z_1 = z_{f1} + z_{g1} = \beta^{\frac{1}{1-2\beta}} \left(\frac{((1-D)p_1)^{\frac{1}{1-2\beta}} + p_2^{\frac{1}{1-2\beta}}}{a_1^{\frac{1-\beta}{1-2\beta}} a_2^{\frac{\beta}{1-2\beta}}} \right) = \overline{Z}_1 \quad [18]$$

The total demand of input Z_2 is calculated using expressions [10] and [15]:

$$Z_2 = z_{f2} + z_{g2} = \beta^{\frac{1}{1-2\beta}} \left(\frac{((1-D)p_1)^{\frac{1}{1-2\beta}} + p_2^{\frac{1}{1-2\beta}}}{a_1^{\frac{\beta}{1-2\beta}} a_2^{\frac{1-\beta}{1-2\beta}}} \right) = \overline{Z}_2 \quad [19]$$

10.4. Appendix D

In order to study the variations of quantities exchanged for both goods and consumers, we differentiate x_{ch} with respect to D.

The derivative of x_{c1}(D) is:

$$\frac{\partial x_{C1}(D)}{\partial D} = \frac{(\overline{Z}_1 \overline{Z}_2)^{\beta}}{\left(1 + \frac{(1-D)^{\frac{1-\alpha}{(1-2\alpha\beta)}}}{v^{\frac{1}{1-2\alpha\beta}}}\right)^{3\beta}} \left(-1 - \frac{(1-D)^{\frac{1-\alpha}{(1-2\alpha\beta)}}}{v^{\frac{1}{1-2\alpha\beta}}} \left(1 - \frac{2\beta(1-\alpha)}{(1-2\alpha\beta)} \right) \right)$$

The sign of $x_{c1}'(D)$ depends on the sign of:

$$-1 - \frac{(1-D)^{\frac{1-\alpha}{(1-2\alpha\beta)}}}{v^{\frac{1}{1-2\alpha\beta}}} \left(1 - \frac{2\beta(1-\alpha)}{(1-2\alpha\beta)} \right).$$

Considering the interval of $\alpha \in \,]0,1[$ and $\beta \in \,]0,1/2[$, the derivative of x_{c1}(D) is negative, which means that a pollinator decline will decrease the consumption of good 1.

The differential of $x_{c2}(D)$ is:

$$x_{C2}(D) = \frac{\left(\overline{Z}_1\overline{Z}_2\right)^{\beta}}{2v^{\frac{2\beta}{1-2\alpha\beta}}} \frac{\dfrac{\alpha}{1-2\alpha\beta}}{\left((1-D)^{\frac{\alpha}{1-2\alpha\beta}} + \dfrac{1}{v^{\frac{1}{1-2\alpha\beta}}}\right)^3}$$

Considering the interval of $\alpha \in \,]0,1[$ and $\beta \in \,]0,1/2[$, the derivative of x_{c2}(D) is positive, which means that a pollinator decline will always increase the consumption of good 2.

10.5. Appendix E

In order to study the variations of quantities exchanged for both goods and consumers we differentiate $x_{ch}(D)$ with respect to D.

The derivative of $x_{11}(D)$ is:

$$\frac{\partial x_{11}}{\partial D} = \frac{\left(P_1(1-D)\right)^{\frac{1}{1-2\beta}}}{\left(1+\dfrac{1}{v^{\frac{1}{1-2\alpha\beta}}(1-D)^{\frac{\alpha}{1-2\alpha\beta}}}\right)^2 \left(\dfrac{Z_2}{Z_1}\right)^{\frac{\beta}{1-2\beta}}} \left(\left[\left[\left(\frac{1}{(1-2\beta)} \frac{P_1'(D)}{P_1(D)} \left(\frac{1}{\beta} - 1 + \frac{1}{(1-D)^{\frac{\alpha}{(1-2\alpha\beta)}} v^{\frac{1}{1-2\alpha\beta}}} \right) + \left(\frac{1}{1-D}\left(1-\frac{1}{(1-2\beta)\beta}\right) - \frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)} \frac{1}{(1-D)^{\frac{\alpha}{(1-2\alpha\beta)}+1} v^{\frac{1}{1-2\alpha\beta}}} \right) \right) \right] \times \left(1+\frac{1}{v^{\frac{1}{1-2\alpha\beta}}(1-D)^{\frac{\alpha}{1-2\alpha\beta}}}\right) \right] + \left[\left(\frac{1}{\beta} - 1 + \frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}} v^{\frac{1}{1-2\alpha\beta}}} \right) \left(\frac{\alpha}{(1-2\alpha\beta) v^{\frac{1}{1-2\alpha\beta}} (1-D)^{\frac{\alpha}{1-2\alpha\beta}-1}} \right) \right] \right)$$

The trend of the derivative x_{11} following extreme values of parameters α and β as explained in the table 1 show that it is always negative which imply that a pollinator decline will decrease the consumption of good 1 for consumer 1.

Table 1 – Sensitivity and intensity of exchanged quantities of good 1 by consumer 1, x_{11}, following values of α and β.

	β tends to 0	β tends to 1/2
α tends to 0	$dx_{11}<0$	$dx_{11}<0$
α tends to 1	$dx_{11}<0$	$dx_{11}<0$

The derivative of $x_{12}(D)$ is:

$$x_{12}^{*} = \frac{\left(P_1(1-D)\right)^{\frac{1}{1-2\beta}}}{\left(\frac{(1-D)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}} + v^{\frac{2\beta}{1-2\alpha\beta}}(1-D)^{\frac{1}{1-2\alpha\beta}}\right)^{2}\left(\frac{Z_2}{Z_1}\right)^{\frac{\beta}{1-2\beta}}} \left(\left[\left(\frac{1}{(1-2\beta)}\frac{P_1^{'}(D)}{P_1(D)}\left(\frac{1}{\beta}-1+\frac{1}{(1-D)^{\frac{\alpha}{(1-2\alpha\beta)}}v^{\frac{1}{1-2\alpha\beta}}}\right) + \left(\frac{1}{1-D}\left(1-\frac{1}{(1-2\beta)\beta}\right) - \frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)}\frac{1}{(1-D)^{\frac{\alpha}{(1-2\alpha\beta)}+1}v^{\frac{1}{1-2\alpha\beta}}}\right)\right) \times \left(\frac{(1-D)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}} + v^{\frac{2\beta}{1-2\alpha\beta}}(1-D)^{\frac{1}{1-2\alpha\beta}}\right) \right] + \left[\left(\frac{1}{\beta}-1+\frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}}v^{\frac{1}{1-2\alpha\beta}}}\right)\left(\frac{(1-\alpha)(1-D)^{\frac{1-\alpha}{1-2\alpha\beta}-1}}{(1-2\alpha\beta)v^{\frac{1-2\beta}{1-2\alpha\beta}}} + \frac{v^{\frac{2\beta}{1-2\alpha\beta}}}{1-2\alpha\beta}(1-D)^{\frac{1}{1-2\alpha\beta}-1}\right) \right] \right)$$

The trend of the derivative x_{12} following extreme values of parameters α and β as explained in the table 2 show that it is negative for a low value of β and positive for a high value of β. However the function $dx_{12}(D)$ is continuous considering the interval D=[0,1], which imply that it exist a value β^* for which $dx_{12}(D) = 0$. Consequently a pollinator decline will decrease the consumption of good 2 for consumer 1 when β is lower than β^* and increase when β is higher than β^*.

Table 2 – Sensitivity and intensity of exchanged quantities of good 2 by consumer 1, x_{12}, following values of α and β.

	β tends to 0	$\beta = \beta^*$	β tends to 1/2
α tends to 0	$dx_{12}<0$	$dx_{12}=0$	$dx_{12}>0$
α tends to 1	$dx_{12}<0$	$dx_{12}=0$	$dx_{12}>0$

The derivative of $x_{21}(D)$ is:

$$x_{21}^* = \frac{\left(P_1(1-D)\right)^{\frac{1}{1-2\beta}}}{\left(1+\frac{1}{v^{\frac{1}{1-2\alpha\beta}}(1-D)^{\frac{\alpha}{1-2\alpha\beta}}}\right)^2\left(\frac{Z_2}{Z_1}\right)^{\frac{\beta}{1-2\beta}}} \left(\left[\left(\frac{1}{(1-2\beta)} \frac{P_1^{'}(D)}{P_1(D)} \left(\frac{1}{\beta(1-D)^{\frac{\alpha}{1-2\alpha\beta}} v^{\frac{1}{1-2\alpha\beta}}} + 1 - \frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}} v^{\frac{1}{1-2\alpha\beta}}} \right) + \left(\frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)(1-D)^{\frac{\alpha}{1-2\alpha\beta}-1} v^{\frac{1}{1-2\alpha\beta}}} \left(1-\frac{1}{\beta}\right) - \frac{1}{(1-2\beta)(1-D)} \right) \right) \times \left(1+\frac{1}{v^{\frac{1}{1-2\alpha\beta}}(1-D)^{\frac{\alpha}{1-2\alpha\beta}}}\right) \right] + \left[\left(\frac{1}{\beta} - 1 + \frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}} v^{\frac{1}{1-2\alpha\beta}}} \right) \left(\frac{\alpha}{(1-2\alpha\beta) v^{\frac{1}{1-2\alpha\beta}} (1-D)^{\frac{\alpha}{1-2\alpha\beta}-1}} \right) \right] \right)$$

The trend of the derivative x_{21} following extreme values of parameters α and β as explained in the table 3 show that it is always negative which imply that a pollinator decline will decrease the consumption of good 1 for consumer 2.

Table 3 – Sensitivity and intensity of exchanged quantities of good 1 by consumer 2, x_{21}, following values of α and β.

	β tends to 0	β tends to 1/2
α tends to 0	$dx_{21}<0$	$dx_{21}<0$
α tends to 1	$dx_{21}<0$	$dx_{21}<0$

The derivative of $x_{21}(D)$ is:

$$
x_{22}^{*}=\frac{\left(P_1(1-D)\right)^{\frac{1}{1-2\beta}}}{\left(1+\frac{1}{v^{\frac{1}{1-2\alpha\beta}}(1-D)^{\frac{\alpha}{1-2\alpha\beta}}}\right)^2\left(\frac{Z_2}{Z_1}\right)^{\frac{\beta}{1-2\beta}}}
\left(
\begin{array}{l}
\left[\left(\frac{1}{(1-2\beta)}\frac{P_1^{'}(D)}{P_1(D)}\left(\frac{1}{\beta(1-D)^{\frac{\alpha}{1-2\alpha\beta}}v^{\frac{1}{1-2\alpha\beta}}}+1-\frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}}v^{\frac{1}{1-2\alpha\beta}}}\right)+\left(\frac{1-\alpha}{(1-2\beta)(1-2\alpha\beta)(1-D)^{\frac{\alpha}{1-2\alpha\beta}-1}v^{\frac{1}{1-2\alpha\beta}}}\left(1-\frac{1}{\beta}\right)-\frac{1}{(1-2\beta)(1-D)}\right)\right)\right. \\
\left.\times\left(\frac{(1-D)^{\frac{1-\alpha}{1-2\alpha\beta}}}{v^{\frac{1-2\beta}{1-2\alpha\beta}}}+v^{\frac{2\beta}{1-2\alpha\beta}}(1-D)^{\frac{1}{1-2\alpha\beta}}\right)\right] \\
+\left[\left(\frac{1}{\beta}-1+\frac{1}{(1-D)^{\frac{\alpha}{1-2\alpha\beta}}v^{\frac{1}{1-2\alpha\beta}}}\right)\left(\frac{(1-\alpha)(1-D)^{\frac{1-\alpha}{1-2\alpha\beta}-1}}{(1-2\alpha\beta)v^{\frac{1-2\beta}{1-2\alpha\beta}}}+\frac{v^{\frac{2\beta}{1-2\alpha\beta}}}{1-2\alpha\beta}(1-D)^{\frac{1}{1-2\alpha\beta}-1}\right)\right]
\end{array}
\right)
$$

The trend of the derivative x_{22} following extreme values of parameters α and β as explained in the table 4 show that it is always positive which imply that a pollinator decline will increase the consumption of good 2 for consumer 2.

Table 4 – Sensitivity and intensity of exchanged quantities of good 2 by consumer 2, x_{22}, following values of α and β.

	β tends to 0	β tends to 1/2
α tends to 0	$dx_{22}>0$	$dx_{22}>0$
α tends to 1	$dx_{22}>0$	$dx_{22}>0$

10.6. Appendix F

For the specific functional forms used in this section 5 and 6, the profit of the firm 1 is:

$$\Pi^1 = p_1(1-D)T_1 z_{f1}^{\beta} z_{f2}^{\beta} - a_1 z_{f1} - a_2 z_{f2}$$

Without further details, which are already explained in appendix C, we give the optimal use of inputs by firm 1:

$$z_{f1} = \frac{\left(p_1 T_1 \beta (1-D)\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{1-\beta}{1-2\beta}} a_2^{\frac{\beta}{1-2\beta}}} \qquad [20]$$

$$z_{f2} = \frac{\left(p_1 T_1 \beta (1-D)\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{\beta}{1-2\beta}} a_2^{\frac{1-\beta}{1-2\beta}}} \qquad [21]$$

The profit of the firm 2 is:

$$\Pi^2 = p_2 z_{g1}^{\beta} z_{g2}^{\beta} - a_1 z_{g1} - a_2 z_{g2}$$

The optimal demand of inputs for the production of the good 2.

$$z_{g1} = \frac{\left(p_2 T_2 \beta\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{1-\beta}{1-2\beta}} a_2^{\frac{\beta}{1-2\beta}}} \qquad [22]$$

$$z_{g2} = \frac{\left(p_2 T_2 \beta\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{\beta}{1-2\beta}} a_2^{\frac{1-\beta}{1-2\beta}}} \qquad [23]$$

We can now calculate the total production of good 1 using expressions [22] and [23] and good 2 using expressions [24] and [25]:

$$f(z_{f1}, z_{f2}) = \left(T_1(1-D)\right)^{\frac{1}{1-2\beta}} \left(\frac{(\beta p_1)^2}{a_1 a_2} \right)^{\frac{\beta}{1-2\beta}} \qquad [24]$$

$$g(z_{g1}^{\beta} z_{g2}^{\beta}) = T_2^{\frac{1}{1-2\beta}} \left(\frac{(\beta p_2)^2}{a_1 a_2} \right)^{\frac{\beta}{1-2\beta}} \qquad [25]$$

The total demand of input Z_1 is calculated using expressions [22] and [24]:

$$Z_1 = z_{f1} + z_{g1} = \beta^{\frac{1}{1-2\beta}} \left(\frac{\left((1-D)T_1 p_1\right)^{\frac{1}{1-2\beta}} + \left(T_2 p_2\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{1-\beta}{1-2\beta}} a_2^{\frac{\beta}{1-2\beta}}} \right) = \overline{Z}_1 \qquad [26]$$

The total demand of input Z_2 is calculated using expressions [23] and [25]:

$$Z_2 = z_{f2} + z_{g2} = \beta^{\frac{1}{1-2\beta}} \left(\frac{\left((1-D)T_1 p_1\right)^{\frac{1}{1-2\beta}} + \left(T_2 p_2\right)^{\frac{1}{1-2\beta}}}{a_1^{\frac{\beta}{1-2\beta}} a_2^{\frac{1-\beta}{1-2\beta}}} \right) = \overline{Z}_2 \qquad [27]$$

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[1] La liste intégrale des Documents de Travail du LAMETA parus depuis 1997 est disponible sur le site internet : http://www.lameta.univ-montp1.fr

Contact :

Stéphane MUSSARD : mussard@lameta.univ-montp1.fr