Hedonic Prices for Timber Auctions with Endogenous Participation

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Abstract

How much is the timber from public forests worth? How can the Public Forest Service define a fair market price for standing timber lots? To estimate the value of a timber lot we adopt the transaction-evidence appraisal approach using data from timber auctions in Lorraine (Eastern France) accounting for the facts that: (*i*) the seller's reserve prices are secret, (*ii*) there remain many unsold lots, and (*iii*) the number of bidders varies from one auction to another. We estimate parameters of a sample selection model in which the hedonic equation includes an endogenous ordinal explanatory variable (the intensity of participation).

Key words: Timber auctions, hedonic prices, sample selection, endogenous participation, secret reserve price.

Code JEL: C11, C15, C34, C35, C63, D44, L73.

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1 Introduction

How much is the timber from public forests worth? How can the Public Forest Service define a fair market price for standing timber lots? Answering these questions is challenging. First, it is difficult to refer to production costs. Indeed, a forest takes time to grow and expand. Timber supply is more a harvesting decision based on silvicultural motives and related to the management of a renewable natural resource, than just a question of wood production. Secondly, the seller (the Public Forest Service) wants to maximize sales receipts, but also has other objectives, such as securing the timber supply to the wood local industry at a price that allows them to remain competitive on international markets and/or against other industries (steel, aluminum, etc.) Thus, the objectives of the seller might be multiple and contradictory. Third, standing timber is different from perishable goods. The optimal time for harvesting might have passed if the lot remains unsold for many years, but the trees continue to grow and the forest still offers other values (recreation, carbon sequestration, biodiversity, water filtration, …) that are difficult to take into account when defining the value of a timber lot. To sum up, it is difficult for the seller to evaluate her own reservation value for a lot in standing timber sales.

Yet, even if the Public Forest Service uses an auction system to fix the selling price, the marketing director needs to set a relevant reserve price for each timber lot that he wants to sell. Given that assessing the value of a standing timber lot is challenging, the seller needs to refer to demand factors such as: lot quality, species composition, lot location, harvesting conditions, etc. In this article, we use the so called "transaction evidence appraisal" (TEA) reduced form method, *i.e.* we estimate timber value from market prices obtained during past timber auctions in France.

Most French timber sales are sequential first-price sealed-bid auctions of heterogeneous lots. Heterogeneity in the product is probably the most important feature of standing timber sales. Lots differ from each other with respect to volume, composition, location, harvesting conditions, etc. (inter lots heterogeneity). But a lot is also composed of trees of different species and qualities (intra lot heterogeneity). These inter- and intra-lot heterogeneities raise various questions about the valuation of the lots that are put on sale and about their optimal composition. Heterogeneity of timber lots makes the hedonic price function approach useful in order to infer appraisal value since many characteristics may influence the stumpage price. The hedonic price method is based on the implicit price of each characteristic and determines how the market values a lot as a set of characteristics.

There are two problems that arise when we analyze timber auction data sets. Both arise from the endogenous participation of the bidders in the auctions. First, there are many lots for which there is no bid and there are good reasons to think that this outcome is not random: bidders may not bid on timber lots that are of bad quality or have difficult harvesting conditions, etc. It is important to note that in French timber auctions, the seller does not announce any reserve price. The seller might withdraw the lot if she thinks the highest bid is too low, but the reserve price is kept secret before the auction. Thus, the lack of bids can not be explained by a reserve price that is set too high, since no minimum amount is required to bid for a lot. Of course, lots with no submission remain unsold. However, we have to take lots without bids into account in the estimation procedure in order to prevent a possible sample selection bias. Secondly, when there are bids submitted for the lot, the degree of competition varies from one auction to another. According to the independent private values auction model, the number of bidders has a positive impact on the bidding strategies in first-price auctions. Indeed, bidders bid more aggressively when the number of bidders increases. Moreover, there are many auctions with only one bidder. This special case needs to be analyzed with caution. Remember that the number of bids cannot be explained by the value of the reserve price here, so it is sensible to think that the number of bidders is driven by the characteristics of the lot. In other words, the number of bidders has to be included in the hedonic price equation as an endogenous explanatory variable.

From an econometric point of view, the main problem is related to the correlation between unobservable variables that determine the participation process and the auction result. We solve this challenge by specifying a 3-equation model: equation (1) defines the probability that there is no bid, equation (2) determines among submitted lots the degree of competition, and equation (3) is the hedonic price equation that explains the auction result. We estimate parameters of this system of simultaneous equations using a Bayesian Monte Carlo Markov

Chain (MCMC) simulation algorithm, which simplifies inferences in latent discrete variables models.[1]

Our empirical work contributes to the literature on timber value appraisal by explicitly modeling the fact that the seller's reserve price is not announced. This is the main difference with the existing stumpage appraisal literature (discussed in the next section) that uses the Tobit two-stage procedure. Indeed, we can not explain bidders' participation by the level of the reserve price. In this article, bidder participation directly depends on the characteristics of the timber lot. Secondly, we contribute to the empirical auction literature since we propose a methodology to assess the value of heterogeneous goods from sequential auctions with secret reserve price and endogenous participation.

In the next section, we specify our objective and our empirical approach; in particular, we explain why the structural econometrics of an auction model does not fit our purpose. Section 3 describes the institutional framework of French public timber auctions and the data set. The methodology is detailed in section 4 and section 5 presents the results. Section 6 concludes our research.

2 Timber appraisal

Our methodology uses a reduced form procedure that does not rely on the structural approach to the econometrics of auctions. First, we explain why we do not estimate a theoretical model of auction. Then we present previous works on timber transaction evidence appraisal and we clarify the relationship between seller's value, reserve price, bidder's value and highest bid.

[1] See Poirier and Tobias (2007) for instance for a general introduction on this topic. The idea is to replace methods based on maximum likelihood that often do not converge in complicated settings.

2.1 Structural econometrics of auction

There is a recent but important and growing literature on structural econometrics of auctions. See for example Laffont and Vuong (1996), Perrigne and Vuong Q. (1999), Athey and Haile (2002), Paarsch and Hong (2006) for surveys. The aim of this highly technical and sophisticated literature is to estimate the structural parameters of a well defined theoretical auction model. In an auction model, the structural parameters are the parameters of the distribution function $F(.)$ of the random bidders' values. In a private values auction model[2], the bidder's value v_j is the private valuation each potential bidder j receives for the object. Using theoretical equilibrium bidding strategies, the objective of the econometrician is to infer unobserved bidders' private values v_j from their observed bids b_j so as to estimate the parameters of the bidders' private values distribution $F(.)$.

The main drawback of this approach is that a tractable theoretical model needs first to be solved at least partially. Auction theory has developed a great deal during the last three decades, nevertheless our understanding of real auction sales is far from complete. Many strong and restrictive assumptions need to be made in order to have tractable theoretical auction models. Our objective here is not to list all of them, but to point out some particularly awkward assumptions for timber auctions. The sequential and dynamic aspect is not taken into account in the auction model of structural econometric studies. The different auctions, indexed by i, $i = 1, \ldots, I$, are treated as independent from each other and the heterogeneity between auctions is mostly considered as noise. Yet, two important components vary from one auction to another: the number of bids and the characteristics of the auctioned good. The number of bidders N_i in an auction i is usually assumed to be known and exogenous in most structural econometrics studies of auctions. Second, as we discussed before, lot heterogeneity is one of the most important features in timber auctions. Our timber appraisal approach focuses on this lot heterogeneity and does not rely on the estimation of the distribution function of the random components of the bidders' values. To better understand this point, we decompose the value of timber lot i as:

[2] Most timber auction studies rely on this paradigm, but some uses a common value auction model, such as Chatterjee and Harrison (1988) and Athey and Levin (2001). Of course, real timber auctions contain both private and common value component. Nevertheless, hardwood timber sales (we use in this study) fit the private value better since it is a less standard product than softwood and may have much diverse uses.

$v_i = \mathrm{x}_i\, \beta + \varepsilon_i \qquad i = 1, \ldots, I$

where β is a vector of unknown parameters which are the implicit prices of the characteristics of lot i collected in the vector x_i; and ε_i is an unobservable variable.

In structural econometric papers, bidders' private values are thus defined as:

$v_{i,j} = \mathrm{x}_i\, \beta + \varepsilon_{i,j} \qquad i = 1, \ldots, I \quad \text{and} \quad j = 1, \ldots, N_i.$

The main objective in structural econometrics of auctions is to infer the parameters of the distribution function of the random bidders' values. Thus the object characteristics x_i essentially represent nuisance parameters and not much attention is dedicated to β. But the heterogeneity in auctions (number of actual bidders N_i and observed characteristics x_i) is central to our study. It is the heterogeneity of the auctioned object that makes it difficult for the seller to determine her own private valuation $v_{i,0}$ of the timber lot i she wants to sell. Actually, it is our main objective to better estimate the weight, β, of each characteristic in the timber lot valuation v_i. In other words, we are more interested in the implicit price of each lot characteristic so as to infer a better price function for timber lot, than in the parameters of the distribution function from which all potential buyers' valuations are drawn.

In auction theory, the shape of the distribution of the bidders' valuations is important in order to determine the optimal selling mechanism. However, as we said previously, the private value of the seller v_0 is also crucial to determine the optimal reserve price (even when the reserve price is not announced before the auction). We already argued that the seller's value v_0, *i.e.* her reservation value, is not well defined and, at least, is not precisely known at the time of the auction. We present empirical evidence of this claim in the data section. Consequently, without any clear information about v_0, it is not surprising that the seller has difficulties in setting the optimal secret reserve price under which a lot should not be sold.

Moreover, most structural econometric studies consider symmetric bidders (*i.e.* bidders' valuations are drawn from the same distribution function) or at most only two groups of bidders. But we believe that the valuation of each characteristic differs from one potential buyer to another. Indeed, in timber auctions there is an important heterogeneity among the lots, but there is also an important heterogeneity (asymmetry) among the buyers. Some of them are interested in high quality timber, some only in given species, or have strict constraints on timber diameters, etc. Thus, there are many different types of bidders (small saw mills, big paper companies, merchants, etc.) and each potential buyer might valuate each characteristic differently. As a result, a more elaborated model could be:

$v_{i,j} = x_i \beta_j + \varepsilon_{i,j} \qquad i = 1, \ldots, I \quad \text{and} \quad j = 1, \ldots, N_i.$

Naturally this more complex model raises important identification problems within a structural econometric approach. Nevertheless, when buyers have different valuations for each characteristic, lot heterogeneity might explain why there is a varying number of bidders across auctions even when the reserve price is not announced. This contrasts with the literature on structural econometrics of auctions where lot heterogeneity is generally considered as noise and does not have any impact on the number of bidders. Yet, it is more likely that more valuable lots attract more bidders than less valuable sales, or that certain types of lots fit the demand of particular buyers better than other lots, or that certain lot characteristics are unacceptable for some buyers but not for others… Hence, participation in timber auctions and lots characteristics are strongly related.[3]

Although timber auctions have many special features that distinguish them from most theoretical auction models – those special features are described in section 3 for French timber sales – many articles in the structural econometrics of auctions literature rely on timber auction data: Mead (1967), Hansen (1985, 1986), Paarsch (1991, 1997), Elyakime et al. (1994, 1997), Baldwin, Marshall and Richard (1997), Athey and Levin (2001), Haile (2001), Li and Perrigne (2003), Athey, Levin and Siera (2004), and Campo, Guerre, Perrigne and Vuong (2006), among others. In most structural timber auction studies, as in our data, there is a variable called the seller's estimate or the appraisal value. Usually that value is found to be a summary of all the other variables (such as quality, species composition, harvesting condition, etc). To avoid any correlation problem, most papers use only that variable to take into account the object heterogeneity between auctions. In contrast, we want to improve on the appraisal value of the seller. Therefore, we will focus on all other variables that might influence her reservation value.

[3] Buyers' characteristics could be included in the hedonic price function to take into account the buyer's impact, unfortunately, in our data set, bidders are anonymous.

To sum up, a structural econometric model of auction data does not fit our objective to improve timber appraisal from the seller's point of view. We believe that a more straightforward use of the data could be more helpful to build a price indicator for the seller.

2.2 The transaction evidence appraisal approach (TEA)

The TEA method relies on the results of past timber sales, usually auctions, for predicting stumpage prices.[4] Unsold timber lots were not considered in early regression-based models (e.g. Jackson and McQuillan, 1979, McQuillan and Johnson-True, 1988). Prescott and Puttock (1990) and Puttock, Prescott and Meilke (1990) propose a standard hedonic price function to forecast stumpage prices in Southern Ontario timber sales; there was no unsold lots in their data. Buongiorno and Young (1984) modeled winning bids using OLS conditional on timber auctions that received at least two bids. However, as Huang and Buongiorno (1986) argued, the fact that some timber lots remained unsold is important market information. Thus, the following transaction evidence appraisal models include this market information to prevent biased predictions of market values. Since the reserve price is known and announced before the auctions in U.S. timber sales, it is assumed that the reserve price explains why some lots are not sold. Therefore, to take into account unsold lots, censored regressions (Tobit model) have been conducted (Huang and Buongiorno, 1986). Niquidet and van Kooten (2004) do not have sufficient information on no-bid auctions (or non-submitted lots), so they seek to predict a fair market value of standing timber in British Columbia using a two-stage truncated regression procedure.

Beyond the treatment of unsold lots, the number of bidders also appears as an important variable in the estimation of the winning bid in stumpage appraisal literature. Indeed, the degree of competition in auctions has an impact on the bidding strategies. Participants do not necessarily know the actual number of bidders, but they bid according to the expected or potential competition (Brannman 1996). Many studies on timber auctions such as Johnson (1979), Hansen (1986), and Sendack (1991) empirically support the auction theory prediction

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[4] Before the TEA method was introduced in the 1980's, the residual value approach was used. The residual value mainly represents the price of all the products likely to result from a particular timber lot, subtracting all the processing costs. (Nautiyal, 1980)

that there is a positive relationship between the number of bidders and the value of the highest bid. Sendack (1991) explicitly examines the impact of the number of bidders on the winning bid by including a transformation of the number of bids submitted as an explanatory variable. Assigning a dummy variable to each number of bidders ($n = 1$, $n = 2$, …, $n = 11$), Brannman, Klein and Weiss (1987) obtained estimated coefficients that support first-price auction theory: bid shading is decreasing with the number of bidders. None of these studies endogenize participation. However, to use stumpage appraisal models as predictive tools it is necessary to endogenize the actual number of bidders. Examining the impact of the (announced) reserve prices in sealed-bid Federal timber auctions, Carter and Newman (1998) endogenize the number of bidders in a simultaneous-two-equations Tobit framework, but the expected number of bidders is determined strictly by the reserve price.[5] Of course, this model does not fit French timber auctions since the reserve price is secret.

2.3 Seller's value, reserve price, bidders' values and highest bid

It is not straightforward for the seller to know below which price she should not sell a timber lot, even when she sees the highest bid. The seller's (reservation) value v_0 corresponds to the price under which the seller would get no profit from the transaction. The reserve price is chosen by the seller, contrary to v_0 which is exogenous. The seller commits not to sell the good below the reserve price. Of course, if the seller has perfect information on her private value v_0, the reserve price can not be lower than v_0. Actually, the seller's value might be seen as a "pseudo common value" imperfectly known but correlated with the bidders' private values. We can see it as the best expected price the seller could obtain in a future sale. That value depends on many features. For example, it depends not only on future global market conditions and macro variables, but also on how the market is valuing each characteristic of the lot. It is with respect to the latter feature that we want to improve timber appraisal. Our objective is to use the results of past timber auctions to build a hedonic price equation.

From a buyer's point of view, the estimated value of a lot is different than from the seller's point of view. Buyers have information on harvesting costs, on what they will produce with

[5] They treat the number of bidders as a continuous variable.

the wood and at what price they will be able to sell their products. It is therefore easier for them than for sellers to estimate their reservation value. Therefore, as in auction models, we believe that each bidder knows his private reservation value for a particular lot. That value depends on the characteristics of the lot, but may also depend on his inventory (*i.e.* on which lots the buyer already bought, and on whether he still needs wood).

We propose to estimate a hedonic price function based on the highest bids. The highest bid of an auction is not necessary a winning bid (and thus a market price) since the seller might withdraw the lot if she believes that the highest bid is too low. However, we choose to estimate the highest bid and not the sale price because the sale price is not independent from the seller's decision (because of the secrete reserve price) and thus is less informative about market demand.[6]

3 Data on French timber auctions

Competitive bidding is widely used in timber sales in France. In particular, the French National Public Forest Service (ONF[7]) uses first-price sealed-bid auctions to sell timber from public forest. Timber auctions of ONF, which represent 40% of the timber sold each year in France, generally concern standing timber. The auction mechanism seems to be the best way to determine an "objective" or a "fair" market value for such a heterogeneous product. Before presenting the data set on fall timber auctions in Lorraine conducted by ONF in 2003, we describe the institutional framework of French timber auctions.

[6] There is a difference between the highest bid and the private value of the highest bidder. According to auction theory, buyers in first-price sealed-bid auctions do not bid their valuation. However, we do not attempt to infer the bidders' private value here. Our aim is more to build a price equation for a fair market price based on lots characteristics.

[7] ONF stands for Office National des Forêts.

3.1 Institutional framework

Timber auctions are sequential auctions of heterogeneous goods since many different lots (usually more than one hundred) are put on sale one after the other; the result of the auction of a lot is given before the next lot is put on sale. The first lot is usually randomly drawn, next the auctioneer follows the catalogue order. The sale catalogue details all the lots. During a sale, bidders are not interested in every lot. Each bidder has a specific demand about species, volume, and quality. Thus, the number of bidders for a particular lot is fairly small and it is quite usual that there is only one bid or even no bid at all. Besides, bidders are asymmetric; they have different goals (sawyer, merchant, etc.), different business sizes, different needs and different locations.

At harvesting time, ONF does not choose the characteristics of the products. It has to sell what came out of the forest, which is heterogeneous by nature. Thus, lots are heterogeneous (different from one another), but they are also made up of heterogeneous wood. In particular in standing timber sales, a lot may contain many species of different diameter and of different quality. Auctioning such a product raises the problem of the optimal lot composition. The successive auctions correspond to different lots, but lots might be interrelated. Some lots may be close substitutes while others may present synergies. For example, it may be only profitable for some buyers to harvest two or more lots that are close to each other.

Taking into account the heterogeneity of the lots raises practical issues. Potential buyers visit the lots that they intend to buy, although there is a catalogue that details the characteristics of the lots. Moreover, bidders have to prospect 5 to 10 times as many lots as they intend to buy since they are not guaranteed to obtain the lots they want. This leads to non-negligible prospecting costs for the bidders. These search costs, which are linked to the heterogeneity of the product, are wasteful from a social perspective. Reducing the cost of preparing a bid in timber auctions may increase the number of bidders.

Contrary to North American timber auctions, the reserve price of the seller is not announced in French public timber auctions. It is kept secret. This singular practice has been studied in the literature, but is difficult to justify theoretically. Elyakime, Laffont, Loisel and Vuong (1994) show in an independent private value auction model that the seller is always better off announcing her reserve price. Nevertheless, the practice of a secret reserve price is sometimes justified either by the fact that announcing a reserve price reduces the participation of the bidders or by a common value component (Vincent, 1995). Risk aversion is also mentioned to justify a secret reserve price (Li and Tan, 2000). A lack of competition for some lots and ONF's willingness to maintain a reasonable timber price may also explain this practice. Finally, a secret reserve price may be used to prevent collusion between bidders at the reserve price. When the reserve price is not announced, the optimal secret reserve price is equal to the seller's reservation value v_0.

As a matter of fact, we believe that the seller prefers not to announce and commit to any reserve price mainly because she does not know her reservation value at the auction time as claimed in the previous section. Indeed, a secret reserve price is reported for each lot in the database, but this price is not the seller's reservation value since many auctioned lots (about 40% in our data set) are sold under this reserve price (which should theoretically be equal to the seller's private valuation v_0). This means that the French public Forest Service decides to sell or not a lot at the last moment and does not commit to any reserve price before the auction. So, the seller uses the bids to adjust her valuation v_0 of the lot. With this privilege, the seller keeps a certain flexibility to manage the sale, but that practice may be costly for the seller from an auction theoretical point of view. Without firm and credible commitment, ONF may lose a part of the benefit of an auction. If the bidders anticipate that the seller can modify the rules of the game, then they will modify their bidding strategy, which may lower the efficiency of the bidding mechanism. Nevertheless, the fact that the seller updates her reserve prices shows her difficulty to assess her value v_0 of a lot. Hence, announcing a reserve price might have negative consequences if the model used to set reserve prices is mis-specified. Indeed, a reserve price set too high can result in no bids, while a reserve price set too low may result in too much bid shading especially since the number of bidders is usually low in timber auctions.

3.2 Data

The data set we use is part of the data collected by Costa and Préget (2004). It relies on the auction results of the ten fall 2003 timber sales of Lorraine, a Region of the eastern part of France. A total of 2262 lots were put on sale. Since there are many differences between hardwood and softwood valuations, we select only pure hardwood lots, i.e. lots that are composed of more than 99% of hardwood. Between September 9th and October 28th 2003, 1205 hardwood lots have been put on sale. Lots may be very heterogeneous and made up of many species. The Herfindahl index is used to measure intra lot heterogeneity.[8] Out of the 1205 hardwood lots put on sale, only 52% of the lots are put on sale for the first time; thus 48% of the lots correspond to previously unsold lots.

At the end of the auctions, lots may be classified according to the auction results. A lot sold during the auction is said to be "auctioned", whereas the others are called "unsold lots". The percentage of unsold lots is 42% and shows a relatively difficult wood market environment in the Lorraine area during that period. It is useful to distinguish between lots that got one or more bids but have nevertheless been withdrawn by the seller and lots that got no bid at all, referred to as the "no bid" category. Table 1 presents sale results.

Table 1. Timber auction results

Auctioned lots		695 (58%)
Unsold lots	Withdrawn lots	318 (26%)
	No bid	192 (16%)
Total		1205 (100%)

The database of Costa and Préget (2004) includes more than one hundred variables that represent a large part of the information available in the catalogues. It also includes private information from ONF (harvesting conditions, quality of the lot, secret reserve price), data about the auction results (the number of bids, the auctioned prices) and computed data such as the Herfindahl index. This database is particularly rich. Moreover, it is exhaustive since it contains all the standing timber lots from public forests put on sale in the region during the fall of 2003. Nevertheless, the data set does not contain any information about the bidders. The following two tables give summary statistics of variables used in our econometric study.

[8] The Herfindahl index is the sum of the square volume proportion of each species. Here the number of species is limited to 7, then the Herfindahl index varies from 0.14 to 1. The more homogeneous the lot, the closer is the index to one.

Table 2. Descriptive statistics for binary variables

Variable	%
No restrictions	37.18
Cutting	
arranged cutting	52.70
other cutting	4.40
selection cutting	1.08
accidental products	2.74
regeneration cutting	39.09
Previously unsold	48.22
Harvesting conditions	
easy logging & extraction	27.22
normal logging	58.76
difficult logging	2.74
difficult logging & extraction	7.97
very difficult logging & extraction	3.15
Mitraille (scrap-iron, grape-shot from the first world war)	
no mitraille	77.56
light mitraille	13.72
average mitraille	05.99
heavy mitraille	2.74
Stand, crop	
high forest	29.71
conversion of a stand	62.41
coppice forest	0.58
coppice with standards	7.30
state-owned forest	25.89
community-owned forest	74.11
Landing area	
unarranged	80.41
arranged	15.93
none	3.65
Quality	
very good	4.07
good	34.85
normal	45.64
mediocre	12.61
bad	2.66

Table 3. Descriptive statistics for continuous variables

Variable	Mean	Std. Dev.	Min	Max
Surface (in hectare)	12.41	10.38	0.20	104.04
Number of trees	238.27	205.63	21	2259
Number of poles	267.07	663.76	0	11366
Herfindahl index	0.6007	0.1949	0.3337	1.0000
Stem volume of the mean-tree	1.0623	0.7314	0.0596	4.7190
Oak volume without crown	94.51	115.98	0	859.98
Beech volume without crown	136.83	164.09	0	1365.80
Other hardwood volume without crown	67.66	97.25	0	838.60
Crown hardwood volume	166.62	153.64	0	1196.47
Coppice volume	0.33	5.39	0	153.83
Relative order of the auction	0.50	0.29	0	1

All continuous variables are defined in logs except variables in percentage such as the Herfindahl index and the variable used to give the relative order of the auction in the sale and the stem volume of the mean-tree. Thirty six percents of the auctioned lots are sold at a price lower than the seller reserve price. These figures show that the seller does not commit to a credible reserve price and takes her decision to accept or not the highest bid at the last moment. Thus, the “a priori” reserve price of our data set has no clear significance.

Table 4 reports the number of lots according to the number of bidders. In the data there are up to 13 bids for a lot, but the most frequent case is when there is only one bid.

Table 4. Number of lots according to the number of bidders

Number of bids	0	1	2	3 and more	Total
Number of lots	192 (16%)	227 (19%)	183 (15%)	603 (50%)	1205 (100%)

In our empirical application, we propose to distinguish timber lots which received no bid and lots for which we observe at least one bid. Among the submitted lots, we distinguish 3 categories depending on the level of competition (i.e. the number of bidders):

i) there is no competition: 1 bid,

ii) there is limited competition: 2 bids,

iii) there is strong competition for the lot: 3 bids or more.

4 Methodology

Participation in timber auctions raises two econometric problems. First, many lots receive no bid and thus remain unsold at the end of the sale. Secondly, the number of bidders in an auction has an impact on the result of the auction: it makes a big difference if there is only one bidder (no competition) or if there are two or more bidders that compete for the same lot.[9] Nevertheless, participation depends on the characteristics of the lots and thus is endogenous from an econometric point of view. We propose a reduced form econometric methodology that simultaneously deals with non-submitted lots (sample selection) and an endogenous number of bidders in the hedonic price function. We explicitly model participation by constructing J categories; but as announced before, we will consider 3 categories in our application: 1 bid, 2 bids, and 3 bids or more. We explain the intensity of participation by the characteristics of the lots in an ordinal probit framework.

We propose a Bayesian Monte Carlo Markov Chain (MCMC) sampling algorithm. We know that classical maximum likelihood procedures might be unreliable, even when we analyze the issues of sample selection and endogenous explanatory variable separately. We are not aware of any study that deals with both issues at the same time as it would require three correlation coefficients to estimate. The existing maximum likelihood estimation procedures (such as simulated maximum likelihood) do not perform well with multiple correlation coefficients and sample selection (see Waelbroeck, 2005). This justifies our Bayesian algorithm that is more reliable to produce robust correlation coefficients. The idea is to simulate the (latent) variables that determine the participation outcomes, which greatly simplifies the analysis of the joint posterior distribution of the parameters.[10] We propose a slightly different MCMC algorithm for the sample selection part of the model than Van Hasselt (2005). We write the latent model as a SUR model with an unequal number of observations; and thus inference on the coefficients of the observed equation only relies on observations that are not censored.

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[9] Even when there is only one bidder, the submitted bid can not be too low because it has to reach the secret reserve price of the seller in order to become a winning bid.

[10] Indeed, latent variables can be simulated and, conditional on these variables, the model is a simple Seemingly Unrelated Regression (SUR) model that is easy to deal with. We use a Metropolis step to draw from the conditional posterior distribution of the elements of the covariance matrix of the unobservable variables.

Despite the importance of the issue of sample selection with endogenous variable, we are not aware of a study that deals simultaneously with these two issues. On the one hand, the problem of sample selection has been widely analyzed in the econometrics literature starting with the seminal work of Nobel price winner James Heckman, who proposed a method (Heckit) to correct sample selection bias. Van Hasselt (2005) has proposed a Bayesian Monte Carlo Markov Chain (MCMC) algorithm to make inference on the correlation coefficient of the sample selection model. The author conducts a Monte Carlo study that shows that Gibbs sampling algorithm performs well regardless of whether the parameters of the model are fully identified or not.[11] On the other hand, Chakravarty and Li (2003) propose a Bayesian algorithm to test the effect of an endogenous binary variable on the profits of a trader (we are not aware of another similar study). They propose a simple Gibbs sampling algorithm that alternates between conditional posterior probability distribution of the parameters. They find no evidence of significant correlation between traders' private information and their profits.

We contribute to the econometric literature on two points. First, we deal with three correlation coefficients because we have three unobservable variables in our model, while Chakravarty and Li (2003) and Van Hasselt (2005) only have to deal with one correlation coefficient. Secondly, both articles reparameterize the elements of the covariance matrix that simplify the sampling procedure and speed up the rate of convergence of the simulated Markov chain. Their algorithms might not be optimal with likelihood functions of irregular shapes. We have included a Metropolis step from the conditional posterior distribution of the covariance matrix that sometimes accepts draws that decrease the likelihood function.[12]

We analyze endogenous participation in French public timber auctions using a system of three equations. Equation (1) determines the selection process. In other words, it is the probability that there is at least one bid. In case the bidders do not participate in the auction (no bid), the expected payoff of participating, $w_{1,i}$, is zero or negative. Thus, we define $y_{1,i} = 1$ if at least one bidder participates in the auction and $y_{1,i} = 0$ otherwise where i indexes the i^{th} lot.

[11] The Gibbs algorithm is an MCMC algorithm that iteratively draws from the conditional posterior distributions of the parameters and always accepts such draws.
[12] See Chen *et al.* (2000).

$$y_{1,i} = \begin{cases} 1 & \text{if } w_{1,i} > 0 \\ 0 & \text{if } w_{1,i} \leq 0 \end{cases} \tag{1}$$

where $w_{1,i} = x_{1,i}'\, \beta_1 + \varepsilon_{1,i}$, β_1 is of dimension k_1 and $x_{1,i}$ is a set of control variables.

Equation (2) determines the outcome of the endogenous ordinal variable in the selected sample.[13] We define $y_{2,i}$ as an ordinal variable that can take on J values (in the application J = 3).

$$y_{2,i} = \begin{cases} 1 & \text{if } w_{2,i} \leq \alpha_1 \\ \ldots & \\ j & \text{if } \alpha_{j-1} < w_{2,i} \leq \alpha_j \\ \ldots & \\ J & \text{if } w_{2,i} > \alpha_{J-1} \end{cases} \qquad \text{if } y_{1,i} = 1 \tag{2}$$

where $w_{2,i} = x_{2,i}'\, \beta_2 + \varepsilon_{2,i}$, β_2 is of dimension k_2 and $x_{2,i}$ is a set of control variables. We define $\alpha = (\alpha_1, ..., \alpha_{J-1})'$ as the vector of cutoff parameters to be estimated.

Finally, equation (3) is the hedonic price equation that explains the highest bid $w_{3,i}$ as a function of lot characteristics and the endogenous ordinal participation variable $y_{2,i}$ included as a set of J–1 binary variables.[14] Equation (3) is only observed for lots that have received at least one bid ($y_{1,i} = 1$).

$$w_{3,i} = z_{3,i}'\, \gamma_3 + z_{2,i}'\, \delta_2 + \varepsilon_{3,i} = x_{3,i}'\, \beta_3 + \varepsilon_{3,i} \quad \text{observed for } y_{1,i} = 1 \tag{3}$$

where $z_{2,i} = (z_{2,2,i}, \ldots, z_{2,J,i})'$ with $z_{2,j,i} = 1$ if $y_{2,i} = j$ (and $z_{2,j,i} = 0$ otherwise, $j = 2, ..., J$), δ_2 is a vector of parameters of dimension J–1, $x_{3,i} = (z_{3,i}', z_{2,i}')'$ and $\beta_3 = (\gamma_3', \delta_2)'$.

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[13] Generally, we only observe the endogenous ordinal variable (2) in the selected sample. For instance, in the application, the ordinal variable is the extent of auction participation, which is only observed for lots that received at least one bid. The observed equation (3) explains the highest bid.

[14] We decompose the ordinal variable in a set of binary variables so that our results do not depend on the way we have coded the ordinal variable. This is not an issue in equation (2) since the methodology automatically determine the cut-off points regardless of the values of the ordinal variable.

We assume that $\varepsilon_i = (\varepsilon_{1,i}', \varepsilon_{2,i}', \varepsilon_{3,i}')'$ is normally distributed with mean $(0, 0, 0)'$ and covariance Σ for $i = 1, \ldots, n$:

$$\Sigma = \begin{bmatrix} 1 & \rho_{12} & \rho_{13}\sigma_3 \\ \rho_{12} & 1 & \rho_{23}\sigma_3 \\ \rho_{13}\sigma_3 & \rho_{23}\sigma_3 & \sigma_3^2 \end{bmatrix}$$

Parameters ρ_{12}, ρ_{13} and ρ_{23} represent the correlations between the unobservable variables. Hence, ρ_{13} is the correlation coefficient of the Heckman sample selection procedure, while ρ_{23} is related to the lack of competition for the lot in the hedonic price equation. Parameter σ_3^2 is the variance of $\varepsilon_{3,i}$. Since probit equations (1) and ordinal probit equation (2) are not identified, we had to impose two restrictions. We chose to normalize the variances of the selection equation and of the endogenous binary variable to 1. These are standard restrictions in probit models.[15]

We always observe $(x_{1,i}, y_{1,i})$, but we only observe $y_{2,i}$ and $w_{3,i}$ when $y_{1,i} = 1$.[16] Moreover, the variables $w_{1,i}$ and $w_{2,i}$ are latent. The vector of explanatory variables can be stacked in order to write the (partially) latent model as a Seemingly Unrelated Regressions (SUR) model with an unequal number of observations. Let n_1 be the number of observations for which $y_{1,i} = 0$ and n_2 the number of observations such that $y_{1,i} = 1$, with $n = n_1+n_2$. We now assume for notational convenience that the data have been sorted according to the values of y_1. We also note the vector of binary dependent variables as $y = (y_1', y_2')'$. Let $\beta = (\beta_1', \beta_2', \beta_3')'$, $w_1 = (w_{1,1}, \ldots, w_{1,n})'$, $w_2 = (w_{2,1}, \ldots, w_{2,n_2})'$, $w_3 = (w_{3,1}, \ldots, w_{3,n_2})'$ and define $w = (w_1', w_2', w_3')'$. We define ε_1, ε_2, ε_3 and ε in a similar fashion.

For notational convenience, we decompose the vectors of unobservable variables according to the selection process: $\varepsilon = (\varepsilon_{11}', \varepsilon_{12}', \varepsilon_2', \varepsilon_3')'$, where the second index equals 1 if $y_{1,i} = 0$ and equals 2 if $y_{1,i} = 1$. Thus the covariance of the unobservable variables is simply

[15] See Wooldridge (2002) or any other textbook on the econometrics of qualitative dependant variable.
[16] Depending on the data set, we always observe $x_{2,i}$ and $x_{3,i}$, or we might only observe them when $y_{1,i} = 1$. However, in the former case we do not use censored data to make inference in equations (2) and (3).

$$\Omega = E\varepsilon\varepsilon' = \begin{bmatrix} I_{n_1} & 0 \\ 0 & \Sigma\otimes I_{n_2} \end{bmatrix}$$

where I_j denotes the identity matrix of dimension $j\times j$. Thus Ω^{-1} is readily obtained. We also decompose and stack the vector of the partially latent dependent variables as $w = (w_{11}', w_{12}', w_2', w_3')'$ and define similarly

$$X = \begin{bmatrix} x_{11} & 0 & 0 \\ x_{12} & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{bmatrix} \qquad (n_1+3n_2)\times(k_1+k_2+k_3)$$

The (partially) latent model can be written in matrix format:

$$w = X\beta + \varepsilon \tag{4}$$

Hence conditional on w and Ω, the estimates of β are simply obtained by a Generalized Least Squares (GLS) regression of (4).[17] Moreover, the matrices $X'\Omega^{-1}X$ and $X'\Omega^{-1}w$ required for the GLS estimates of the parameters of the model are easily computed.

The 4 steps of the Metropolis-Gibbs algorithm are described in appendix 1, and the computation of the partial effects can be found in appendix 2.

5 Results

We first estimate the probit equation (1) and the ordinal probit equation (2) separately and run a Heckit procedure using sample selection equation (1) and hedonic bid equation (3) as

[17] Since each stage contains different number of observations and generally different sets of explanatory variables, we can not estimate the SUR model with ordinary least squares regression applied to each latent equation separately.

benchmarks. Secondly, we compare these estimation results with the Bayesian estimation of parameters of equations (1), (2) and (3) using the MCMC algorithm. All the variables available have been used to build the following model but only significant variables have been kept in each equation. The signs of the estimated coefficients are coherent and intuitive, except for the variable ‘no restriction’ for which the coefficient is surprisingly negative in equation (3).

Table 5 gives the probability a lot will receive at least one bid.

Table 5 - Equation (1) Probit regression results of y_1

y_1		Coef.	Std. Dev.
selection cutting & other cutting	**	-0.4673	0.2169
accidental products	***	-1.2117	0.2925
previously unsold	***	-2.7786	0.4056
difficult & very difficult logging & extraction	*	-0.2852	0.1509
Herfindahl index	**	0.6278	0.3172
mitraille	**	-0.3110	0.1388
number of trees	***	0.3481	0.0769
arranged landing area	***	0.5224	0.1639
normal quality	***	-0.4989	0.1428
mediocre & bad quality	***	-0.5153	0.1837
beech volume without crown	*	0.0731	0.0375
first sale	***	-1.3975	0.1854
_cons	***	1.5638	0.6218

Log-lik = -307.44

Table 6 gives the intensity of competition (*i.e* the number of bidders) for a lot: (*i*) probability that there is no competition, i.e. only 1 bid, (*ii*) probability that there are 2 bids, and (*iii*) probability that there are 3 or more bids.

Table 6 - Equation (2) ordinal Probit regression results of y_2

y_2		Coef.	Std. Dev.
selection cutting & other cutting	**	-0.4892	0.1956
previously unsold	***	-0.7330	0.0823
normal logging	***	-0.3639	0.1037

difficult & very difficult logging & extraction	***	-0.5633	0.1375
Herfindahl index	***	1.9817	0.3301
light mitraille	***	-0.4393	0.1237
average mitraille	***	-0.4522	0.1745
heavy mitraille	***	-0.7958	0.2415
relative order of the auction	***	0.4558	0.1428
conversion of a stand	**	0.2068	0.0990
arranged landing area	***	0.4127	0.1140
normal quality	***	-0.2909	0.0921
mediocre & bad quality	***	-0.6665	0.1336
surface	***	-0.2642	0.0818
other hardwood volume without crown	***	0.1604	0.0367
oak volume without crown	***	0.2574	0.0364
beech volume without crown	***	0.2088	0.0328
first sale	***	-0.5276	0.1785
α_1	***	1.4253	0.4089
α_2	***	2.0603	0.4108

Log-lik = -830.39

Table 7 gives the estimation results of the observed equation obtained by the Heckman methodology using the method of maximum likelihood. The hedonic equation is the estimated value of the log of the highest bid (equation (3)). The selection equation, which gives the factors that influence whether a lot will receive at least one bid or not, and estimated coefficients were already reported in Table 5. We also ran an OLS regression of equation (3) but results were similar and are not reported. This is expected since the coefficient associated with the inverse Mills ratio is not significantly different from zero in Table 7. However, this result is not robust and depends on the variables used to build the model. Actually, if we use only variables that are available in the sale catalogue, we may observe a selection bias. Such a model is presented in Appendix 3 where we only use variables from the sale catalogue. However, we found that estimations of the correlation coefficients ρ_{13} using the Heckit procedure can lead to misleading inference. In model specification of Appendix 3, the Heckit procedure leads to significant sample selection bias (.68), while the Bayesian procedure does not detect any problem of sample selection bias (the correlation coefficient is .30 but is not significantly different from 0), which confirms results from this section.

Table 7 - Equation (3) Heckman regression results of w_3

w_3 = log highest bid		Coef.	Std. Dev.
no restrictions	***	-0.0882	0.0303
accidental products	***	-0.4529	0.1100
regeneration cutting	***	0.1264	0.0307
previously unsold	***	-0.1098	0.0322
Density	***	0.0053	0.0011
difficult & very difficult logging & extraction	***	-0.0929	0.0368
Herfindahl index	***	0.9389	0.1263
Mitraille	**	-0.0786	0.0324
number of trees	***	0.3735	0.0369
relative order of the auction	***	0.1662	0.0431
conversion of a stand	***	0.1425	0.0339
coppice forest & coppice with standards	***	0.1980	0.0533
no landing area	**	-0.1570	0.0674
normal quality	***	-0.1182	0.0277
mediocre & bad quality	***	-0.2308	0.0424
surface	***	0.2336	0.0426
other hardwood volume without crown	***	0.0593	0.0149
oak volume without crown	***	0.1899	0.0151
crown hardwood volume	***	0.0643	0.0097
beech volume without crown	***	0.0977	0.0129
stem volume of the mean-tree	***	0.4505	0.0265
first sale	*	0.1102	0.0567
last sale	***	0.1608	0.0347
y_2 one bid	***	-0.2143	0.0382
y_2 three or more bids	***	0.3595	0.0335
_cons	***	3.4724	0.1426
ρ_{13}		-0.0467	0.1366
σ_3	***	0.3758	0.0084
λ		-0.0176	0.0514

Table 8 gives the Bayesian estimation of the 3-equation model. For each equation, we used exactly the same variables as before.

Convergence of the MCMC algorithm was reach quickly. We removed the first 100000 iterations and kept the next 1000000 iterations for inference. In Appendix 4, Figures 1 to 3 display the marginal posterior distribution of the correlation coefficients. They all have a single mode.

Controlling for endogenous participation and for the characteristics of the lots, we find that, compared to the highest bid for lots with two bids, on average: (*i*) lots with only one bid receive a highest bid that is 22.31% below and (*ii*) lots with three or more bids receive a highest bid that is 37.09% higher. Compared to these results, the heckit procedure slightly underestimates the effect of the endogenous variable, nevertheless coefficients are quite similar in both methodology.

Table 8 - Bayesian estimation of the 3-equation model

Variable		Coef.	Std. Dev.
Equation (1)			
selection cutting & other cutting	**	-0.4762	0.2188
accidental products	***	-1.2381	0.2957
previously unsold	***	-2.9745	0.4589
difficult & very difficult logging & extraction	*	-0.2824	0.1513
Herfindahl index	**	0.6432	0.3182
mitraille	**	-0.3139	0.1393
number of trees	***	0.3526	0.0773
arranged landing area	***	0.5380	0.1654
normal quality	***	-0.5020	0.1429
mediocre & bad quality	***	-0.5174	0.1842
beech volume without crown	*	0.0726	0.0375
first sale	***	-1.4172	0.1860
_cons	***	1.7395	0.6587
Equation (2)			
selection cutting & other cutting	***	-0.4899	0.1959
previously unsold	***	-0.7265	0.0881
normal logging	***	-0.3643	0.1033
difficult & very difficult logging & extraction	***	-0.5620	0.1375
Herfindahl index	***	1.9769	0.3322
light mitraille	***	-0.4324	0.1263
average mitraille	***	-0.4563	0.1753
heavy mitraille	***	-0.7949	0.2410
relative order of the auction	***	0.4587	0.1430
conversion of a stand	**	0.2062	0.0989

arranged landing area	***	0.4106	0.1148
normal quality	***	-0.2891	0.0924
mediocre & bad quality	***	-0.6658	0.1340
surface	***	-0.2640	0.0818
other hardwood volume without crown	***	0.1603	0.0367
oak volume without crown	***	0.2575	0.0363
beech volume without crown	***	0.2094	0.0330
first sale	***	-0.5220	0.1811
α_1	***	1.4265	0.4095
α_2	***	2.0618	0.0437
Equation (3)			
no restrictions	***	-0.0884	0.0308
accidental products	***	-0.4538	0.1116
regeneration cutting	***	0.1258	0.0311
previously unsold	***	-0.1049	0.0366
density	***	0.0053	0.0011
difficult & very difficult logging & extraction	**	-0.0910	0.0384
Herfindahl index	***	0.9270	0.1412
mitraille	**	-0.0763	0.0348
number of trees	***	0.3735	0.0374
relative order of the auction	***	0.1635	0.0461
conversion of a stand	***	0.1413	0.0350
coppice forest & coppice with standards	***	0.1978	0.0541
no landing area	**	-0.1563	0.0682
normal quality	***	-0.1159	0.0298
mediocre & bad quality	***	-0.2261	0.0485
surface	***	0.2348	0.0438
other hardwood volume without crown	***	0.0581	0.0162
oak volume without crown	***	0.1885	0.0171
crown hardwood volume	***	0.0646	0.0099
beech volume without crown	***	0.0964	0.0145
stem volume of the mean-tree	***	0.4507	0.0269
first sale	*	0.1139	0.0589
last sale	***	0.1617	0.0355
y_2 one bid	***	-0.2231	0.0581
y_2 three or more bids	***	0.3709	0.0657
_cons	***	3.4837	0.1526
ρ_{12}		-0.0147	0.0581
ρ_{13}		-0.0482	0.1296
ρ_{23}		-0.0254	0.1242

σ_3	***	0.3837	0.0093

Two results deserve special mention. First, the degree of intra-lot heterogeneity is a significant variable in all 3 equations: the Herfindahl index has a significant positive effect. Thus, during an auction with at least one bidder, competition increases for lots that are more homogenous in species, *i.e.* with an Herfindahl index closer to one. In addition, a higher Herfindahl index increases the highest bid. Thus, concentrated lots with a Herfindahl index close to 1 (in other words lots that are not heterogeneous) have a sale premium. Boltz, Carter and Jacobson (2002) highlight the importance of intra-lot heterogeneity on auction prices of mixed species lots from timber auctions in North Carolina. Their Tobit estimation results (the reserve price being announced) show that increased heterogeneity leads to lower sale prices. In some way, they interpret such decrease in the revenue as an opportunity cost for ecosystem management where biodiversity is a desired constraint. Here, the opportunity cost of maintaining mixed forest can be estimated from the partial effect associated to the Herfindahl index: increasing the index by 1% increases the expected highest bid by 0.9164%. This figure can be found in Table 9 below which gives the partial effect for every variable used in this model.

Second, the coefficient associated with the 'relative position of a lot' in the sale is significantly positive in equation (2) and (3). This indicates that lots put on the market at the end of a sale have a higher probability to receive more bids and to obtain a better highest bid than lots auctioned in the beginning of the sale, after we control for quality differences. This last result implies that the decline in prices often observed in sequential auctions is not present in our sample of timber auctions. On the contrary, prices tend to increase for hardwood lots during a sale. This could be due to cautious behavior of the bidders in the beginning of the auctions and more aggressive bids at the end of the auctions. This interpretation is confirmed by two additional results. First, the probability that a lot receives bids is significantly lower in the first sale of the campaign: the variable 'first sale' has a significant negative impact in equation (1) and (2). Bidders wait and see at the beginning of the timber sale campaign. Second, the variable 'last sale' has a significant positive impact in the hedonic bid equation (3). This result reinforces the 'relative position of a lot' variable on a larger scale. Indeed, the highest bid increases during a sale (which is composed of many timber lots put on sale the same day), moreover the highest bids tend to be higher in the tenth sale (the one that took place the last day of the timber sale campaign).

Table 9 – Partial effects

		Partial effects	Std. Dev.
no restrictions	***	-0.0887	0.0313
selection cutting & other cutting		-0.0011	0.0230
accidental products	***	-0.4139	0.1163
regeneration cutting	***	0.1253	0.0312
previously unsold	**	-0.1725	0.0745
density	***	0.0053	0.0011
normal logging	*	-0.0102	0.0059
difficult & very difficult logging & extraction	***	-0.1156	0.0411
Herfindahl index	***	0.9164	0.1580
mitraille	*	-0.0708	0.0384
light mitraille	*	-0.0115	0.0068
average mitraille		-0.0121	0.0080
heavy mitraille	*	-0.0174	0.0103
number of trees	***	0.3352	0.0469
relative order of the auction	***	0.1791	0.0459
conversion of a stand	***	0.1472	0.0342
coppice forest & coppice with standards	***	0.1989	0.0553
arranged landing area		0.0140	0.0249
no landing area	**	-0.1565	0.0667
normal quality	***	-0.1125	0.0368
mediocre & bad quality	***	-0.2595	0.0531
surface	***	0.2265	0.0448
other hardwood volume without crown	***	0.0631	0.0159
oak volume without crown	***	0.1963	0.0166
crown hardwood volume	***	0.0648	0.0100
beech volume without crown	***	0.0949	0.0171
stem volume of the mean-tree	***	0.4516	0.0276
first sale		0.1168	0.0787
last sale	***	0.1618	0.0348

6 Conclusion

Using detailed data set on timber auctions in Lorraine, we have highlighted the importance of endogenous participation on auction results, focusing on lots that do not receive any bids and on the degree of competition when lots receive at least one bid. We have proposed a methodology to deal with both issues at the same time. The econometric method can easily be extended to deal with truncated or censored dependant variables in the hedonic price equation, when the reserve price is announced.

Our results can help public forest services to determine a relevant reserve price for each lot according to its characteristics. In order to avoid auctions with 1 bid or less, the methodology could also be used to propose more attractive lots and to better understand demand factors. Our hedonic price function for stumpage value gives interesting information about the implicit price of each lot characteristic for the optimal lot composition. We have discussed the impact of the relative order of the lot in the sale and the impact of the intra lot heterogeneity, but our results show that many variables have a significant impact on the participation process and on the auctioned price including the type of cutting, the type of stand, the harvesting conditions, the volume and the composition of the lot. These results can help the forest public services to manage forest more efficiently so as to offer more attractive lots.

This methodology can also be useful for bidders to define a bid that increases their probability of winning at a lower cost. Models can be elaborated according to which variables are available to the agent just before the auction.

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Appendix 1: The Metropolis-Gibbs sampling algorithm

The Metropolis-Gibbs sampling algorithm proceeds in 4 steps. The first step is a standard data augmentation step. We use a uniform prior for β, α, ρ_{12}, ρ_{13}, ρ_{23} and a non-informative prior for σ_3: $p(\beta, \alpha, \rho_{12}, \rho_{13}, \rho_{23}, \sigma_3) \propto 1/\sigma_3$.[18] To simplify notations we have dropped the dependence of Ω on Σ and the dependence of Σ on $(\rho_{12}, \rho_{13}, \rho_{23}, \sigma_3)'$ when there is no ambiguity.

Step 1. $\mathbf{w_1, w_2 \mid \alpha, \beta, \Sigma, w_3, y, X}$

In the first step, we only need to draw w_1 and w_2 since w_3 is observed. When $y_{1,i} = 0$, we know that $w_{1,i} < 0$, hence for those observations ($i = 1, \ldots, n_1$), we draw $w_{1,i}$ from the standard truncated normal distribution with mean $\mathrm{x}_{1,i}'\beta_1$ and variance 1 truncated on $(-\infty, 0)$. We use the optimal algorithm of Robert (1995) to draw from the truncated normal distribution.[19] For the other observations ($i = n_1+1, \ldots, n$), we know that conditionally on α, β, Σ, y, X, $(w_{1,i}, w_{2,i}, w_{3,i})'$ has a joint normal distribution with mean $(\mathrm{x}_{1,i}'\beta_1, \mathrm{x}_{2,i}'\beta_2, \mathrm{x}_{3,i}'\beta_3)'$ and covariance Σ. Thus,

$$w_{1,i} \mid w_{2,i}, \alpha, \beta, \Sigma, \mathrm{y}, \mathrm{w}_3, \mathrm{X} \sim TN(\mu_{1|23}, \Sigma_{1|23}; B_1)$$

where $TN(a, b; c)$ denotes the normal distribution with mean a, variance b truncated in subspace c and $B_1 = \{z_1 \in \mathrm{R}: z_1 > 0\}$. The conditional moments $\mu_{1|23}$ and $\Sigma_{1|23}$ are given by the standard formulas of the conditional distribution from a multivariate normal distribution. Similarly,

$$w_{2,i} \mid w_{1,i}, \alpha, \beta, \Sigma, \mathrm{y}, \mathrm{w}_3, \mathrm{X} \sim TN(\mu_{2|13}, \Sigma_{2|13}; B_2)$$

[18] The choice of the prior distribution does not matter much when there is a large number of observations, which is usually the case for auction data. Moreover, using the uniform prior distribution provides a direct mean of comparison with the maximum likelihood procedures.
[19] Using the inverse c.d.f. method yielded unreliable results.

where $B_{2j} = \{z \in \mathrm{R}: a_{j-1} < z \le a_j\}$ if $y_{2,i} = j$ (by convention, $\alpha_0 = -\infty$ and $\alpha_J = +\infty$).

Step 2. $\boldsymbol{\alpha \mid \beta, \Sigma, y, w, X}$

It is easy to see that the conditional posterior distribution of α_j is (for $j = 1, ..., J-1$):

$$\alpha_j \mid \beta, \Sigma, \mathrm{y}, \mathrm{w}, \mathrm{X} \sim U(\mathrm{Max}\{w_{2,i}: y_{2,i} = j\}, \mathrm{Min}\{w_{2,i}: y_{2,i} = j+1\})$$

Step 3. $\boldsymbol{\beta \mid \alpha, \Sigma, \mathrm{y}, \mathrm{w}, \mathrm{X}}$

As discussed in the presentation of the (partially) latent model, the conditional distribution of β is readily seen to be:

$$\beta \mid \alpha, \Sigma, \mathrm{y}, \mathrm{w}, \mathrm{X} \sim N((\mathrm{X}'\Omega^{-1}\mathrm{X})^{-1}\,\mathrm{X}'\Omega^{-1}\mathrm{w}, (\mathrm{X}'\Omega^{-1}\mathrm{X})^{-1}).$$

Step 4. $\boldsymbol{\Sigma \mid \alpha, \beta, \mathrm{y}, \mathrm{w}, \mathrm{X}}$

The conditional posterior distribution of Σ is not standard,

$$\Sigma \mid \alpha, \beta, \mathrm{y}, \mathrm{w}, \mathrm{X} \propto |\Sigma|^{-n2/2} \exp(-\varepsilon'\Omega^{-1}\varepsilon/2) / \sigma_{3,}$$

but can be simulated using Metropolis step. Define $\sigma = (\rho_{12}, \rho_{13}, \rho_{23}, \sigma_3)'$. We use a normal jumping distribution $\mathrm{N}(\sigma, \theta'\,\mathrm{I}_{4\times4})$.[20]

[20] We set the elements of θ in the Metropolis-Hastings algorithm to obtain an acceptance rate between 0.2 and 0.25. In general a large step size decreases the speed of convergence of the algorithm but enables to get out of problematic areas of the likelihood function more quickly, while a small value would make the algorithm converge faster at the cost of getting stuck in undesirable areas. The range of values that we have used is standard for the number of parameters used in the application and was found to be a good compromise between the two effects mentioned above. Note that this range of acceptance rates has been shown to be optimal for MCMC algorithms that use a normal jumping distribution. For other sampling schemes, the optimal acceptance rate has to be computed and could be different from the above values. Draws that resulted in values of the correlation coefficients below -1 or above 1, as well as draws not resulting in a positive covariance matrix were rejected. Note also that we used a log transformation of the various probabilities in order to avoid numerical underflows.

Appendix 2: Computation of the partial effects

Let X denotes the matrix of explanatory variables defined in the text. The observed highest bid equation is

$$w_3 = \delta_{21}\, z_{22} + \delta_{22}\, z_{23} + z_3'\gamma_3 + \varepsilon_3 \qquad \text{(A1)}$$

The expected bid conditional on participation is

$$E(w_3 \mid X, y_1{=}1) = \delta_{21}\, E(z_{22} \mid X, y_1{=}1) + \delta_{22}\, E(z_{23} \mid X, y_1{=}1) + z_3'\gamma_3 + \gamma_1\, \lambda(x_1'\beta_1) \qquad \text{(A2)}$$

Where is $\lambda(\cdot)$ is the inverse mill ratio and $\gamma_1 = \rho_{13}/\sigma_3$. The partial effects of the last two terms of (A2) are given from standard computations in the Heckman model. It remains to find the partial effect with respect to the first two terms of the right-hand side of (A2). We can write (dropping the conditioning on X to simplify notations):

$$E(z_{22} \mid y_1{=}1) = p(y_2 = 2 \mid y_1{=}1) = p(\alpha_1 - x_2'\beta_2 < \varepsilon_2 < \alpha_2 - x_2'\beta_2 \mid \varepsilon_1 > -x_1'\beta_1) = P_1 \qquad \text{(A3)}$$

$$E(z_{23} \mid y_1{=}1) = p(y_2 = 3 \mid y_1{=}1) = p(\varepsilon_2 > \alpha_2 - x_2'\beta_2 \mid \varepsilon_1 > -x_1'\beta_1) = P_2$$

We decompose the conditional probability as follows:

$$P_1 = \int_D p(\alpha_1 - x_2'\beta_2 < \varepsilon_2 < \alpha_2 - x_2'\beta_2 \mid \varepsilon_1)\, p(\varepsilon_1 \mid \varepsilon_1 > -x_1'\beta_1)\, d\varepsilon_1,$$

where $D = [-x_1'\beta_1; +\infty]$ is the domain of integration with respect to ε_1. Using the fact that ε_1 is normally distributed, we can write where f(·) is the standard normal density.

$$P_1 = \int_D p(\alpha_1 - x_2'\beta_2 < \varepsilon_2 < \alpha_2 - x_2'\beta_2 \mid \varepsilon_1)\, \phi(\varepsilon_1)\, /(1 - \Phi(-x_1'\beta_1))\, d\varepsilon_1 \qquad \text{(A4)}$$

Using a property of the conditional distribution of a normally distributed random variable, $p(\alpha_1 - x_2'\beta_2 < \varepsilon_2 < \alpha_2 - x_2'\beta_2 \mid \varepsilon_1) = \int_E p(\varepsilon_2 \mid \varepsilon_1)\, d\varepsilon_2 = \int_E \phi(1 - \rho_{12}\varepsilon_1;\, 1 - \rho_{12}^2)\, d\varepsilon_2$, where $E = [\alpha_1 - x_2'\beta_2;\, \alpha_2 - x_2'\beta_2]$ and $\phi(a;\, b)$ denotes the density of a normally distributed variable with mean a and variance b. We use properties of the normal distribution to write

$$p(\alpha_1 - x_2'\beta_2 < \varepsilon_2 < \alpha_2 - x_2'\beta_2 \mid \varepsilon_1) = \int_{E'} \phi(\varepsilon_2)\, d\varepsilon_2$$
$$= \Phi((\alpha_2 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1)) /\sqrt{(1-\rho_{12}^2)}) - \Phi((\alpha_1 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1))/\sqrt{(1-\rho_{12}^2)}), \quad \text{(A5)}$$

where $E' = [(\alpha_2 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1)) /\sqrt{(1-\rho_{12}^2)}; (\alpha_1 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1))/\sqrt{(1-\rho_{12}^2)}]$.

Substituting (A5) in (A4), we have

$$P_1 = \int_D [\Phi((\alpha_2 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1)) /\sqrt{(1-\rho_{12}^2)}) - \Phi((\alpha_1 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1))/\sqrt{(1-\rho_{12}^2)})]$$
$$\phi(\varepsilon_1) /(1-\Phi(-x_1'\beta_1))\, d\varepsilon_1$$

Similarly,

$$P_2 = \int_D [1 - \Phi((\alpha_2 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1))/\sqrt{(1-\rho_{12}^2)})]\, \phi(\varepsilon_1) /(1-\Phi(-x_1'\beta_1))\, d\varepsilon_1$$

We now compute the partial effect of P_1 with respect to x_k (the implicit price of the characteristic) that belongs to the set of variables x_1 and x_2:

$$\partial P_1/\partial x_k = [1-\Phi(-x_1'\beta_1)]^{-1} \{\phi(-x_1'\beta_1)\, (-\beta_{1k})/(1-\Phi(-x_1'\beta_1)) \times$$
$$\int_D [\Phi((\alpha_2 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1))/\sqrt{(1-\rho_{12}^2)}) - \Phi((\alpha_1 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1))/\sqrt{(1-\rho_{12}^2)})]\, \phi(\varepsilon_1)\, d\varepsilon_1$$
$$+ [\Phi((\alpha_2 - x_2'\beta_2 - (1-\rho_{12}\, x_1'\beta_1))/\sqrt{(1-\rho_{12}^2)}) - \Phi((\alpha_1 - x_2'\beta_2 - (1-\rho_{12}\, x_1'\beta_1))/\sqrt{(1-\rho_{12}^2)})] \times$$
$$\phi(-x_1'\beta_1)\, \beta_{1k}$$
$$- \int_D [\phi((\alpha_2 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1))/\sqrt{(1-\rho_{12}^2)}) - \phi((\alpha_1 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1))/\sqrt{(1-\rho_{12}^2)})] \times$$
$$(\beta_{2k}/\sqrt{(1-\rho_{12}^2)})\, \phi(\varepsilon_1)\, d\varepsilon_1\}$$

Integrals in the previous formula can be computed by simulation using the GHK algorithm for instance. Similarly,

$$\partial P_2/\partial x_k = [1-\Phi(-x_1'\beta_1)]^{-1} \{\phi(-x_1'\beta_1)\, (-\beta_{1k})/(1-\Phi(-x_1'\beta_1)) \times$$
$$\int_D [1 - \Phi((\alpha_2 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1))/\sqrt{(1-\rho_{12}^2)})]\, \phi(\varepsilon_1)\, d\varepsilon_1$$
$$+ [1 - \Phi((\alpha_2 - x_2'\beta_2 - (1-\rho_{12}\, x_1'\beta_1))/\sqrt{(1-\rho_{12}^2)})]\, \phi(-x_1'\beta_1)\, \beta_{1k}$$
$$+ \int_D [\phi((\alpha_2 - x_2'\beta_2 - (1-\rho_{12}\varepsilon_1))/\sqrt{(1-\rho_{12}^2)})]\, (\beta_{2k}/\sqrt{(1-\rho_{12}^2)})\, \phi(\varepsilon_1)\, d\varepsilon_1\}$$

Appendix 3: Model 2

The following model is built only with variables available to any buyers since it includes only information that is given in the sale catalogue. In other words, we did not use any private information from the seller to built this alternative model. Thus, this second model is not as good as the first one, nevertheless we present it here to show we observe a selection bias in this model (*cf.* coefficient ρ_{13} or λ in the Heckit procedure Table A3).

Table A1 - Equation (1) Probit regression results of y_1

y_1		Coef.	Std. Dev.
selection cutting & other cutting	***	-0.5215	0.1795
accidental products	***	-1.3852	0.2353
Herfindahl index	***	1.3517	0.2971
number of trees	***	0.2260	0.0827
arranged landing area	**	0.3540	0.1476
State owned-forest	***	0.3220	0.1284
oak volume without crown	***	0.1786	0.0433
crown hardwood volume	**	-0.0782	0.0389
beech volume without crown	***	0.1147	0.0347
first sale	***	-1.0857	0.1408
_cons	***	-1.5618	0.4190

Log-lik = -431.48

Table A2 - Equation (2) ordinal Probit regression results of y_2

y_2		Coef.	Std. Dev.
selection cutting & other cutting	***	-0.5641	0.1907
accidental products	**	-0.7304	0.3237
Herfindahl index	***	1.7760	0.3089
relative order of the auction	***	0.3819	0.1381
no landing area	***	-0.5971	0.2038
State-owned forest	***	0.2578	0.0935
surface	***	-0.2576	0.0815
other hardwood volume without crown	***	0.1532	0.0338
oak volume without crown	***	0.2644	0.0341
beech volume without crown	***	0.1797	0.0303

first sale	*	-0.3145	0.1657
α_1	***	2.0375	0.3675
α_2	***	2.6035	0.3699

Log-lik = -904.78

Table A3 - Equation (3) Heckman regression results of w_3

w_3 = log highest bid		Coef.	Std. Dev.
no restrictions	***	-0.0888	0.0309
accidental products	***	-0.3878	0.1154
regeneration cutting	***	0.1321	0.0318
density	***	0.0050	0.0012
Herfindahl index	***	0.8754	0.1323
number of trees	***	0.3483	0.0389
relative order of the auction	***	0.1391	0.0440
conversion of a stand	***	0.0939	0.0343
coppice forest & coppice with standards	**	0.1356	0.0544
no landing area	***	-0.1912	0.0687
surface (in hectare)	***	0.2175	0.0442
other hardwood volume without crown	***	0.0556	0.0154
oak volume without crown	***	0.1874	0.0160
crown hardwood volume	***	0.0691	0.0105
beech volume without crown	***	0.0901	0.0136
stem volume of the mean-tree	***	0.4486	0.0277
first sale	***	0.2778	0.0632
last sale	***	0.2102	0.0344
y_2 one bid	***	-0.2294	0.0391
y_2 three or more bids	***	0.4120	0.0336
_cons	***	3.6267	0.1626
ρ_{13}	***	-0.6833	0.1264
σ_3	***	0.4188	0.0161
λ	***	-0.2862	0.0622

Table A4 - Bayesian estimation of the 3-equation model

Variable		Coef.	Std. Dev.
Equation (1)			
selection cutting & other cutting	**	-0.4717	0.1896
accidental products	***	-1.3771	0.2379

Herfindahl index	***	1.3253	0.3004
number of trees	***	0.2494	0.0868
arranged landing area	**	0.3551	0.1457
State owned-forest	***	0.3473	0.1294
oak volume without crown	***	0.1636	0.0477
crown hardwood volume	**	-0.0785	0.0377
beech volume without crown	***	0.1012	0.0400
first sale	***	-1.0864	0.1424
_cons	***	-1.5622	0.4191
Equation (2)			
selection cutting & other cutting	***	-0.5490	0.1933
accidental products	**	-0.6837	0.3355
Herfindahl index	***	1.7487	0.3127
relative order of the auction	***	0.3901	0.1394
no landing area	***	-0.5990	0.2038
State owned-forest	***	0.2516	0.0941
surface	***	-0.2595	0.0816
other hardwood volume without crown	***	0.1525	0.0338
oak volume without crown	***	0.2602	0.0347
beech volume without crown	***	0.1782	0.0312
first sale		-0.2825	0.1767
α_1	***	1.9793	0.3880
α_2	***	2.5520	0.0389
Equation (3)			
no restrictions	***	-0.0870	0.0314
accidental products	***	-0.4794	0.1283
regeneration cutting	***	0.1367	0.0323
density	***	0.0051	0.0012
Herfindahl index	***	0.8953	0.1527
number of trees	***	0.3619	0.0399
relative order of the auction	***	0.1308	0.0483
conversion of a stand	***	0.0898	0.0350
coppice forest & coppice with standards	**	0.1371	0.0553
no landing area	**	-0.1811	0.0764
surface (in hectare)	***	0.2289	0.0474
other hardwood volume without crown	***	0.0524	0.0175
oak volume without crown	***	0.1895	0.0193
crown hardwood volume	***	0.0660	0.0106
beech volume without crown	***	0.0911	0.0155
stem volume of the mean-tree	***	0.4518	0.0279

first sale	***	0.2127	0.0743
last sale	***	0.2113	0.0348
y_2 one bid	***	-0.2549	0.0769
y_2 three or more bids	***	0.4422	0.0934
_cons	***	3.4828	0.1897
ρ_{12}		-0.0514	0.1024
ρ_{13}		-0.3038	0.2922
ρ_{23}		-0.0423	0.1865
σ_3	***	0.4098	0.0182

Table A5 – Partial effects

		Partial effects	Std. Dev.
no restrictions	***	-0.0871	0.0314
other cutting		-0.0497	0.0443
accidental products	***	-0.6731	0.1164
regeneration cutting	***	0.1372	0.0322
density	***	0.0051	0.0012
Herfindahl index	***	1.1294	0.1917
number of trees	***	0.3740	0.0428
relative order of the auction	***	0.1708	0.0480
conversion of a stand	***	0.0903	0.0356
coppice forest & coppice with standards	**	0.1369	0.0552
arranged landing area		0.0139	0.0308
no landing area	***	-0.2287	0.0770
State owned-forest		0.0508	0.0319
surface (in hectare)	***	0.2038	0.0492
other hardwood volume without crown	***	0.0677	0.0170
oak volume without crown	***	0.2221	0.0242
crown hardwood volume	***	0.0623	0.0112
beech volume without crown	***	0.1129	0.0175
stem volume of the mean-tree	***	0.4503	0.0278
first sale	*	0.1395	0.0836
last sale	***	0.2125	0.0337

Appendix 4

Figure 1 - Marginal posterior distribution of ρ_{12}

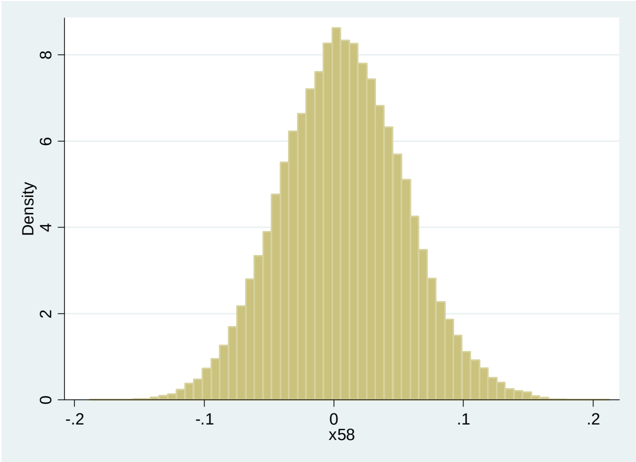

Figure 2 - Marginal posterior distribution of ρ_{13}

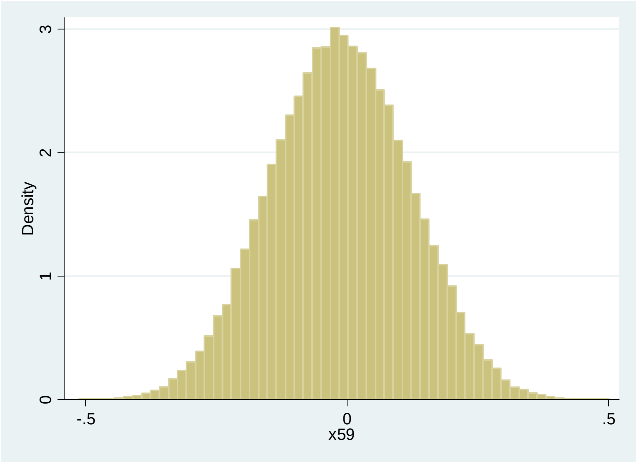

Figure 3 - Marginal posterior distribution of ρ_{23}