How to use Rosen's normalised equilibrium to enforce a socially desirable Pareto efficient solution

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How to use Rosen's normalised equilibrium to enforce a socially desirable Pareto efficient solution

Jacek B. Krawczyk* Mabel Tidball†

Abstract

We consider a situation, in which a regulator believes that constraining a complex good created jointly by competitive agents, is socially desirable. Individual levels of outputs that generate the constrained amount of the externality can be computed as a Pareto efficient solution of the agents' joint utility maximisation problem. However, generically, a Pareto efficient solution is not an equilibrium. We suggest the regulator calculates a Nash-Rosen coupled-constraint equilibrium (or a "generalised" Nash equilibrium) and uses the coupled-constraint Lagrange multiplier to formulate a threat, under which the agents will play a *decoupled* Nash game. An equilibrium of this game will possibly coincide with the Pareto efficient solution. We focus on situations when the constraints are saturated and examine, under which conditions a match between an equilibrium and a Pareto solution is possible. We illustrate our findings using a model for a *coordination* problem, in which firms' outputs depend on each other and where the output levels are important for the regulator.

Keywords: Coupled constraints; generalised Nash equilibrium; Pareto efficient solution
JEL: C6, C7, D7

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1 Introduction

The aim of this paper is twofold. First, we want to formulate sufficient conditions, under which a two-player concave game with coupled constraints has an equilibrium à la Rosen (see [13]) that is also a Pareto-optimal solution of a centralised problem. Secondly, we want to illustrate this result by analysing a stylised real-life game where two competitive players contribute to a public good.

We are interested in situations, in which a regulator wants to control competitive agents so that their jointly created externality satisfies constraints. The constraints can be imposed on pollution (*e.g.*, emitted by a cluster of pulp mills, see *e.g.*, [5] or [8] or by thermal generators, see *e.g.*, [1]), or on the amount of some public good like transportation capacity or hospital beds, available to a local population. The former concern a *negative* externality, which needs to be restricted; the latter are imposed on a *positive* externality that has to satisfy a level deemed necessary.

Also, problems involving competition for a scarce resource by independent operators that depend on some legislation (like private fishermen operating on a fishery or internet users logging in to a server, see *e.g.*, [6]) can be analysed using the same framework as for the externality problems and thus are of interest to us.

In all those cases the joint restrictions are likely to be saturated. This might be because agents' individually optimal solutions limit the amount of the positive externality, because not emitting negative externality is costly, or because the contested resources are scarce. Notice that if there was a slack on the constraints, there would be "no problem" for the regulator. In this paper, we will assume that the regulator is dealing with the "interesting" case of saturated constrains and wishes to know how to apportion the responsibility for the constraints' satisfaction among the agents.

Individual levels of outputs (and inputs) that generate the desired amount of the externality can be computed as a Pareto efficient solution to the agents' joint utility maximisation problem. The regulator might use an arbitrary weight $\alpha \in (0, 1)$ to balance the agents' utility functions or seek α that maximises the sum.

However, generically, a Pareto efficient solution is not an equilibrium hence not self-enforcing, *hence* of problematic use in a competitive environment. We suggest the regulator calculates a Rosen coupled-constraint equilibrium (Nash normalised), see *e.g.*, [13] or [8], (or a "generalised" Nash equilibrium as this type of equilibrium is called in *e.g.*, [12]) and uses the coupled-constraint Lagrange multiplier to formulate a threat, under which the agents will play a *decoupled* Nash game. An equilibrium of this game will possibly coincide with the desired Pareto-efficient solution. If so, the Pareto solution will be achieved as a Nash equilibrium hence self-enforcing.

In the paper, we examine, under which conditions a match between those two solutions is possible. This is the line of research started in [14]. Here, we generalise the results obtained in the above paper for an environmental problem. We illustrate our findings using a model for a *coordination problem* in which firms' outputs depend on each other and where the output levels are important for the regulator.

The model considered in this paper is deterministic and information is "symmetric". We notice that should any of these assumptions be not satisfied, the regulator might assign penalty functions that will prompt the agents to produce externality amounts that are different from expected by the regulator and in general not optimal for the agents. If so, they will start trading out the excess amounts until they become individually optimal. However, such trading problem surpasses the scope of this paper.

What follows is a brief outline of what this paper contains. In Section 2, we describe a model in which two-firm outputs are coordinated by a regulator. This motives our search for a map between Patero-efficient solutions and competitive equilibria. In Sections 3 and 4, we develop the solution concepts for the coordination problem and revise the mathematics needed for the uniqueness of equilibrium. We develop the sufficient conditions for the map's existence in Section 5. We apply these conditions to the motivating example in Section 6. The concluding remarks summarise our findings, which include a socio-economic interpretation of the results.

2 A "public" good delivery

The mathematics of the model described below is taken from [13]. The interpretations and intuitions are ours.

2.1 A model

Consider two competitive agents whose outputs are $x_1 \geq 0$ and $x_2 \geq 0$, respectively. Maintaining the outputs is expensive; the cost function of the first agent is $\frac{x_1^2}{2}$ and x_2^2 of the second.

The revenue of the first agent can only be created using the second agent's output (*positive* externality) and, in this case, it equals $x_1 x_2$. However, the goods produced by the second agent "suffer" when are utilised by the first agent (*negative* externality) so, the revenue of the second agent is $-x_1 x_2$.

In absence of regulation, the unique equilibrium of the Nash-Cournot game

$$\left.\begin{array}{rcl} \max\limits_{x_1}\Big(\phi_1(x) & = & -\frac{1}{2}x_1^2 + x_1 x_2\Big) \\ \max\limits_{x_2}\Big(\phi_2(x) & = & -x_2^2 - x_1 x_2\Big) \\ g_1(x) & = & x_1 \geq 0 \\ g_2(x) & = & x_2 \geq 0 \end{array}\right\} \qquad (1)$$

"played" among the agents is (0,0).

However, maintaining some positive combination of levels x_1 and x_2 may be important for the regulator. Mathematically, the regulator may want the outputs to satisfy

$$h(x) = x_1 + x_2 - 1 \geq 0\,. \qquad (2)$$

Throughout this paper, when $\phi_i(x)$, $i = 1, 2$ are not explicitly defined as in (1) they will be assumed continuous in all arguments and concave in x_i. The common constraint $h(\cdot)$ will then be assumed such that the constraint set

$$x \in X \equiv \{(x_1, x_2) : x_1 \geq 0, x_2(x) \geq 0, h(x) \geq 0\} \tag{3}$$

is convex, closed and bounded subset of $\mathbb{R}^2$.

The problem of how to entice the agents to satisfy constraint (2) boils down into two subproblems:

a. What should be the levels of x_1 and x_2?

b. What should the regulator do to induce the players to choose these levels?

Briefly, the levels x_1 and x_2 can be established as a Pareto efficient solution and implemented as Rosen (Nash normalised) equilibrium. In the rest of this paper we study the mathematical conditions that $\phi_1(\cdot)$, $\phi_2(\cdot)$ and $h(\cdot)$ need to satisfy for the Pareto and Rosen solutions to exist and coincide.

2.2 Interpretations and intuitions

To focus attention we suggest that the above mathematical problem can have the following socio-economic origin.

Consider a rail network owned by a public firm and a private firm responsible for rolling stock and transportation.

Let x_1 be the tonnage of the goods transported through the network; let x_2 be the length of the tracks owned by the tracks' owner. The revenue of the transportation firm is proportional to the tonnage and to the tracks' length $\beta_1 x_1 x_2$.

In absence of a discount price for super-large trains, perhaps due to an imperfect state of the tracks, a reasonable approximation of the cost function to the transportation firm may be $-\frac{\alpha_1 x_1^2}{2}$: the more goods to transport, the more hardware needs to be maintained.

In brief, the operator of the rolling stock has variable revenue $\beta_1 x_1 x_2$ and costs $-\frac{\alpha_1 x_1^2}{2}$ whose combination approximates the firm's profit $\phi_1(x)$.

The public firm operating the tracks is paid a fixed amount, which is normalised to zero. The costs of maintaining the tracks at level x_2 is $\beta_2 x_1 x_2 + \alpha_2 x_2^2$ (where the first term is motivated by the destruction caused by tonnage x_1). Hence $\phi_2(x) = -\alpha_2 x_2^2 - \beta_2 x_1 x_2$.

For social reasons, the government wants transportation activity $\gamma_1 x_1 + \gamma_2 x_2$ to be above level M. This can be written as $\gamma_1 x_1 + \gamma_2 x_2 - M \geq 0$.

The above provides motivation for model (1), (2), in which $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = \gamma_1 = \gamma_2 = M = 1$.

3 Solution concepts

Here, we propose that a Pareto efficient solution can be realised as a coupled constraints equilibrium.

3.1 Pareto efficient solutions

We will establish a solution to subproblem (a.), page 4 *i.e.*, compute what output levels the regulator may want the agents to produce.

Consider model (1), (2).

The regulator is typically interested in a Pareto efficient solution $\hat{x} = \big(\hat{x}_1, \hat{x}_2\big)$ *i.e.*, such that

$$\text{if } \phi_i(x_1, \hat{x}_2) > \phi_i(\hat{x}_1, \hat{x}_2) \text{ then } \phi_{-i}(x_1, \hat{x}_2) < \phi_{-i}(\hat{x}_1, \hat{x}_2) \;\; i = 1, 2 \; . \tag{4}$$

If $\alpha \in (0,1)$ and $\phi_1(x_1, \cdot)$ and $\phi_1(\cdot, x_2)$ are concave, differentiable and $x \in X$ then

$$\hat{x} = \big(\hat{x}_1, \hat{x}_2\big) = \arg\max_{x \in X} \{\alpha\phi_1(x) + (1-\alpha)\phi_2(x)\} \; . \tag{5}$$

To stress that $\hat{x}$ depends on α we will write $\hat{x}(\alpha) = \big(\hat{x}_1(\alpha), \hat{x}_2(\alpha)\big)$. Notice that the larger α, the larger the payoff of the first player. We can say that α is a marginal rate of substitution between the two players' payoffs.

If there is no particular reason for the regulator to prefer one specific value of α, solving the following problem will deliver the "best" $\hat{\alpha}$ and the best output pair $\big(\hat{x}_1(\hat{\alpha}), \hat{x}_2(\hat{\alpha})\big)$

$$\hat{\alpha} = \arg\max_{\alpha \in (0,1)} \{\alpha\phi(\hat{x}_1(\alpha), \hat{x}_2(\alpha)) + (1-\alpha)\phi(\hat{x}_1(\alpha), \hat{x}_2(\alpha))\} \; . \tag{6}$$

In this paper we assume that the regulator is interested not only in $\hat{\alpha}$ and the corresponding outputs and payoffs but also in the full array of Pareto efficient solutions $\big(\hat{x}_1(\alpha), \hat{x}_2(\alpha)\big)$ and the corresponding payoffs $\phi_i(\hat{x}_1(\alpha), \hat{x}_2(\alpha))$, $i = 1, 2$.

3.2 Coupled constraints equilibria

Rosen [13] introduced *coupled constraints equilibrium* (CCE), also known as *generalised Nash equilibrium* (see *e.g.*, [3] or [12]) for games with constraints in the combined strategy space of all agents. In such games, the regulator seeks a solution that can be adopted by competitive players and guarantees completion of constraints that depend on the actions undertaken by all agents. For history of this solution concept and examples of use see [13], [4], [5], [9] and [2].

We will exploit the CCE politico-economic appeal. Once the regulator establishes a desired CCE (explained below), the equilibrium implementation is straightforward. The equilibrium Karush-Kuhn-Tucker multipliers associated with the joint constraints need to be used as penalty tax rates for the constraints' violation and the players have to allow for these penalties in their payoffs. Then the players will "play" a *decoupled* game whose solution is the desired equilibrium.

Knowledge that a CCE exists and is *unique* is crucial for the above enticement mechanism. It suffices to say that without the equilibrium uniqueness, the tax effectiveness could not be established. However, in general, there is a plethora of equilibria when *joint* actions of the players are restricted.

Rosen [13] allows for a discriminatory treatment of players through the introduction of *weights* $r_i > 0$, $i = 1, 2$, with which the regulator can appraise each agent's payoff (*e.g.*, from a view point of the community). On the other hand, the weights help control which equilibrium is established. This is so because, given sufficient concavity of the payoffs, an equilibrium that corresponds to a particular $r = [r_1, r_2]$ is unique. (We notice that one of them may be the Pareto efficient solution $(\hat{x}_1(\alpha), \hat{x}_2(\alpha))$.)

The main role of the weighs in controlling the agents' behaviour is that they can modify the Karush-Kuhn-Tucker multipliers and adjust the tax rates among players to entice them to choose actions that lead to a desired equilibrium outcome.

Below we will review the mathematics of CCE and its implementation, including the uniqueness conditions; for details see [13], [4] or [9].

4 Existence and uniqueness of coupled constraints equilibrium

We will adopt the literature results to the two-person game (1) with one joint constraint (2). For the proofs see [13].

4.1 Introductory remarks

The solution to game (1) with the joint constraint (2) can be written as

$$x^* = \text{equil}_{y_i|x^*_{-i} \in X} \{\phi_1(x), \phi_2(x)\} \;, \tag{7}$$

which means that $\phi_i(x^*), i = 1, 2$ satisfy

$$\phi_i(x^*) = \max_{y_i|x^*_{-i} \in X} \phi_i(y_i|x^*_{-i}), \;\; i = 1, 2 \tag{8}$$

where $y_i|x_{-i} \equiv (y_i, x_{-i})$ denotes a collection of actions when the i-th agent "tries" y_i while the other agent is playing $x_{-i}, i = 1, 2$.

At x^* no player can improve their own payoff by a unilateral change in his (or her) strategy, which keeps the combined vector in $X \subset \mathbb{R}^2$. In general, the strategy set X is assumed a convex, closed and bounded subset of $\mathbb{R}^2$ and $X \equiv \{(x_1, x_2) : x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \geq M\}$, as in (3).

Game (7) shall be called a *coupled constraints game* (à la Rosen, see [13]). The *coupling* refers to the fact that one player's action affects what the other players' actions can be. In the special case where $X = X_1 \times X_2$ *i.e.*, each player's action is individually constrained, the game is said to have *uncoupled* constraints.

If each payoff function ϕ_i is multiplied by weight $r_i > 0$

$$x^*(r) = \text{equil}_{y_i|\mathbf{x}^*_{-\mathbf{i}} \in X} \{r_1\phi_1(x), r_2\phi_2(x)\} \;, \tag{9}$$

where $r = [r_1, r_2] \in \mathbb{R}^2_+$. Our aim is to examine when $x^*(r)$ can match $\hat{x}(\alpha)$ for a given $\alpha \in (0, 1)$.

4.2 Definition

We know from [13][1] that an equilibrium exists and is unique if the game is *diagonally strictly concave* (*DSC*).

The economic interpretation of *DSC* is quite simple. A game that is *DSC*, is one in which each player has more control over his own payoff than the other players have over it. This is a desired, and rather common, feature of many economic models. Mathematically, a ("smooth") game is *DSC* if the so-called pseudo-Hessian (*i.e.* Jacobian of pseudo-gradient, see *e.g.*, [13] or [10]) for this given game is negative definite.

Theorem 4.1. *In a game with uncoupled constraints, if the joint payoff function* $f(x,r)$ *is* DSC *for some* $r > 0$*, then there exists a unique Nash equilibrium.*

When the constraints are *coupled*, there are no such guarantees, and a special type of equilibrium must be defined.

For that purpose, assume that the constraint set X is defined through (2). (In general, X needs to be defined by a collection of functions concave functions, see [13] or [9] and such that the constraint qualification conditions are satisfied.)

Denote the constraint shadow price vector for player i by $\lambda_i^* \geq 0$. Then, $x^* \in X$, is a *coupled constraint equilibrium* point if and only if it satisfies the following Karush-Kuhn-Tucker conditions:

$$h(x^*) \geq 0 \tag{10}$$

$$\lambda_i^* h(x^*) = 0 \tag{11}$$

$$\phi_i(x^*) \geq \phi_i(y_i|x^*) + \lambda_i^* h(y_i|x^*) \tag{12}$$

for all $i = 1, \ldots n$ and where $y_i|x$ was defined in (7).

The above conditions establish a solution to (7) under the adopted differentiability and qualification assumptions. We notice that conditions (10)-(12) define x^* as a vector of non improvable strategies if $x^* \in X$, hence Nash.

In general, the multipliers[2] λ_1 and λ_2 will not be related to each other. However, we shall consider a special kind of equilibrium, which can reflect the different levels of agent responsibility for the constraint satisfaction (expressed by the vector r) and is unique.

Definition 4.1. *An equilibrium point* x^* *is a Rosen (Nash* <u>*normalised*</u>[3]*) equilibrium point if, for some vectors* $r > 0$ *and* $\lambda^* \geq 0$*, conditions (10)-(12) determine* x^* *and are satisfied for*

$$\lambda_i^* = \frac{\lambda^*}{r_i} \tag{13}$$

for each i*.* [4]

[1] Also see [10] or [9] for some applications.

[2] They will be vectors if there were more coupled constraints to satisfy.

[3] "Normalisation" means in this context that both players face the same constraint shadow price λ if $r = 1$. For $r > 1$, the first player's constraint shadow price is $\frac{\lambda}{r}$.

[4] We could say that λ^* are the "objective" shadow prices while λ_i^* are the "subjective" ones.

For shortness we have dropped *coupled constraints* from the equilibrium definition.

Now, we can better understand the role of the weights r_i. If an agent's weight r_i (see (9)) is greater than those of his (or her) competitors, then his (or her) Lagrange multipliers are lessened, relative to the competitors'. This means that the marginal cost of the constraint's violation is lower for this agent than for their competitor. Paraphrasing, the vector r tells us of how the regulator has distributed the burden of the constraints' satisfaction among the agents.

The wording of the following theorem crucial for coupled-constraint games is a bit stronger than in [13], see [7].

Theorem 4.2. *Let the weighting $\bar{r} \in Q$ be given where Q is a convex subset of $I\!R^2_+$. Let $r_1\phi_1(x,\bar{r}) + r_1\phi_2(x,\bar{r})$ be diagonally strictly concave on the convex set X and such that the Karush-Kuhn-Tucker multipliers exist. Then, for the weighting $\bar{r}$, there is a unique Rosen (Nash normalised) equilibrium point.*

In other words, if a game is *DSC* for a feasible distribution of the constraint satisfaction responsibilities, then the game possesses a unique coupled constraint equilibrium for each such distribution.

4.3 Enforcement through taxation

Here we establish a solution to subproblem (b.), page 4. In particular, we explain how specific output levels, including those desired by the regulator, can be made optimal for the agents.

In Section 3.2 we mentioned a *decoupled* game that the agents play after the regulator has modified their payoffs. The decoupling means that the players decide upon their actions without the explicit knowledge on the common constraint. Instead, their payoffs contain penalty functions for the common constraint violation.

The penalty functions $T_i(\lambda^*, r_i, x)$, $i = 1, 2$ contain the tax rate determined by the Lagrange multiplier obtained as CCE of the constrained game and the constraint violation term.

$$T_i(\lambda^*, r_i, x) = \frac{\lambda^*}{r_i}\max(0, -h(x)) \tag{14}$$

where λ^* is the Lagrange multiplier associated with the constraint and r_i is player i's weight[5] that defines their responsibility for the constraints' satisfaction.

Hence, if the weight for player i is, for example $r_i > 1$ and the weight for the other player is 1, then the responsibility of player i for the constraints' satisfaction is lessened.

The players' payoff functions, so modified, will be

$$\underline{\phi}_i(x) = \phi_i(x) - T_i(\lambda^*, r_i, x)\,. \tag{16}$$

[5]If the weights r were identical $[1\,, 1\,, \ldots 1]$ then the penalty term for constraint ℓ is the same for each player f

$$T_i(\lambda^*, 1, x) = \lambda^* \max(0, x_1 + x_2 - 1)\,. \tag{15}$$

.

Notice that under this taxation scheme the penalties remain "nominal" (*i.e.*, zero) if all constraints are satisfied.

The Nash equilibrium of the new unconstrained game with payoff functions $\underline{\phi}$ is implicitly defined by the equation

$$\underline{\phi}(x^{**}) = \max_{y_i \in \mathbb{R}^+} \underline{\phi}(y_i | x^{**}) \qquad \forall\, i, \tag{17}$$

(compare with equation (8)).

We can easily see that the equilibrium conditions for x^{**} are equivalent to (10)-(12) and conclude that $x^{**} = x^*$.(See [11], [8] or [9] for a discussion and examples).

5 A relationship between Pareto efficient solutions and Rosen's equilibria

5.1 Pareto efficiency first order conditions

Consider the regulator problem (5) of dealing with two economic agents whose outputs need to be controlled for social reasons. We repeat the mathematical model for this problem, which is:

$$\max_{x \in X} \{\alpha \phi_1(x_1, x_2) + (1-\alpha)\phi_2(x_1, x_2)\} \;. \tag{18}$$

We will use $P(\cdot)$ or simply P to refer to the contents of the curly brackets above.

As in Section 2, $\phi_i(\cdot,\cdot)$, $i = 1, 2$ are differentiable payoff functions concave in the player's own decision variable and X is a convex set of output combinations that the optimal solutions need to satisfy.

We will assume that the regulator is interested in optimal solutions that saturate the constraint $h(x_1, x_2) = 0$. (*e.g.*, because of the good's scarcity, or abundance of pollution).

The Lagrangean is:

$$L^P = \alpha \phi_1(x_1, x_2) + (1-\alpha)\phi_2(x_1, x_2) + \mu h(x_1, x_2). \tag{19}$$

The first order conditions for a Pareto optimal solution (when $h(x_1, x_2) = 0$) are:

$$\left.\begin{aligned} \frac{\partial L^P}{\partial x_1} &= \alpha \frac{\partial \phi_1(x_1, x_2)}{\partial x_1} + (1-\alpha)\frac{\partial \phi_2(x_1, x_2)}{\partial x_1} + \mu \frac{\partial h(x_1, x_2)}{\partial x_1} = 0, \\ \frac{\partial L^P}{\partial x_2} &= \alpha \frac{\partial \phi_1(x_1, x_2)}{\partial x_2} + (1-\alpha)\frac{\partial \phi_2(x_1, x_2)}{\partial x_2} + \mu \frac{\partial h(x_1, x_2)}{\partial x_2} = 0, \end{aligned}\right\} \tag{20}$$

Given concavity of the payoff functions and convexity of the constraint set, the above conditions are also sufficient for a solution $\hat{x}_1(\alpha), \hat{x}_2(\alpha)$, to (20) to be a Pareto optimal solution to (18).

5.2 Rosen's equilibrium first order conditions

It is well known that a Pareto optimal (efficient) solution *i.e.*, the pair $x_1(\alpha), x_2(\alpha)$ that solves problem (18) is not a generic Nash equilibrium. Consequently, it does not have the self-enforcing properties that the latter solution concept enjoys.

On the other hand the regulator knows from Section 4.3 (also, see [5], [8] or [8]) that it is possible to control competitive agents, who share a common constraint, to satisfy this constraint. This is achieved through a threat function (14), which results from a CCE. This equilibrium does possess the features of Nash equilibrium.

Mathematically, the regulator may then seek $x_1(r), x_2(r)$ that satisfy:

$$\left.\begin{array}{c} \max_{x_1} r\,\phi_1(x_1,x_2) \\ \max_{x_2} \phi_2(x_1,x_2) \\ h(x_1,x_2)=0. \end{array}\right\} \tag{21}$$

where $r \geq 1$ is a weight, which the regulator attaches to the first player's payoff relative[6] to the second player's payoff.

The player Lagrangeans are:

$$L_1^R = r\phi_1(x_1,x_2) + \lambda h(x_1,x_2), \quad L_2^R = \phi_2(x_1,x_2) + \lambda h(x_1,x_2) \tag{22}$$

Following (10)-(12), (13) and when $h(x_1,x_2)=0$, a pair $x_1(r), x_2(r)$ is a *normalised equilibrium*, called Rosen's, of game (21) if it satisfies the following first order conditions (KKT):

$$\left.\begin{array}{l} \dfrac{\partial L_1^R}{\partial x_1} = r\dfrac{\partial \phi_1(x_1,x_2)}{\partial x_1} + \lambda\,\dfrac{\partial h(x_1,x_2)}{\partial x_1} = 0, \\ \\ \dfrac{\partial L^P}{\partial x_2} = \dfrac{\partial \phi_2(x_1,x_2)}{\partial x_2} + \lambda\,\dfrac{\partial h(x_1,x_2)}{\partial x_2} = 0, \end{array}\right\} \tag{23}$$

If the payoff functions are jointly *diagonally strictly concave* then the pair $x_1(r), x_2(r)$, which satisfies (23), is the unique normalised (Rosen) equilibrium of game (21), see Theorem 4.2 .

5.3 Relations between α and r

We want to find a relationship between α and r such that the solutions for the two problems (Pareto and Rosen) are identical *i.e.*, $x^*(r) = \hat{x}(\alpha)$.

Assume that

$$\mu = K\lambda. \tag{24}$$

If we find K that satisfies this equation then the regulator will be able to use a Rosen's equilibrium to enforce a Pareto optimal solution.

[6] See Appendix A for a proof that Theorem 4.2 is true when the regulator uses just one r to apprise the second player's payoff relative to the first player's instead of using two "absolute" weights r_1 and r_2.

If solutions $x_1(\alpha), x_2(\alpha)$ and $x_1(r), x_2(r)$ are to be the same, then (20) and (23) imply:

$$Kr\frac{\partial\phi_1(x_1,x_2)}{\partial x_1} = \alpha\frac{\partial\phi_1(x_1,x_2)}{\partial x_1} + (1-\alpha)\frac{\partial\phi_2(x_1,x_2)}{\partial x_1}, \tag{25}$$

$$K\frac{\partial\phi_2(x_1,x_2)}{\partial x_2} = \alpha\frac{\partial\phi_1(x_1,x_2)}{\partial x_2} + (1-\alpha)\frac{\partial\phi_2(x_1,x_2)}{\partial x_2}. \tag{26}$$

Conditions (25) and (26) give two equations for two unknown K and r. Solving these equations yields

$$r(\alpha) = \frac{\frac{\partial\phi_2}{\partial x_2}}{\frac{\partial\phi_1}{\partial x_1}}\;\frac{\alpha\frac{\partial\phi_1}{\partial x_1} + (1-\alpha)\frac{\partial\phi_2}{\partial x_1}}{\alpha\frac{\partial\phi_1}{\partial x_2} + (1-\alpha)\frac{\partial\phi_2}{\partial x_2}} \tag{27}$$

and

$$K(\alpha) = \frac{\alpha\frac{\partial\phi_1}{\partial x_2} + (1-\alpha)\frac{\partial\phi_2}{\partial x_2}}{\frac{\partial\phi_2}{\partial x_2}}. \tag{28}$$

The derivatives in equations (27) and (28) are evaluated at $x_1(\alpha), x_2(\alpha)$ hence, $r = r(\alpha)$, $K = K(\alpha)$ (are functions of α).

Note that the numerator $\alpha\frac{\partial\phi_1}{\partial x_1} + (1-\alpha)\frac{\partial\phi_2}{\partial x_1}$ in the expression for r (27) can be negative or zero if $\phi_2(x_1, x_2)$ decreases in x_1 (*i.e.*, if x_1 is a negative externality in the problem) and if α is small (*i.e.*, the second player's payoff is somehow preferred).

Also, the denominator $\alpha\frac{\partial\phi_1}{\partial x_2} + (1-\alpha)\frac{\partial\phi_2}{\partial x_2}$ can be negative or zero if $\phi_1(x_1, x_2)$ decreases in x_2 (*i.e.*, if x_2 is a negative externality in a problem) and if α is large (*i.e.*, the first player's payoff is somehow preferred). It follows from the above that if there are negative externalities, then $r(\alpha)$ can have breaks in domain and attain negative values that preclude the existence of a Nash equilibrium, which could implement the desired Pareto solution. We can say that:

Theorem 5.1.
(a) For $\alpha \in (0, 1)$ such that a solution to (18) exists with $\lambda > 0$ and if $0 < r(\alpha) < \infty$ the regulator can implement a desired Pareto-efficient solution as a Rosen (Nash normalised) equilibrium. In particular, formula (27) determines the level of responsibility of the first player for the constraint satisfaction relative to the level of the second player, for a specific level of α.
(b) Moreover, if $\frac{\partial\phi_i}{\partial x_j} > 0$ i.e., if there are no negative externalities then $0 < r(\alpha) < \infty$. Hence, for a given value of α, the corresponding Pareto-efficient solution can be made optimal for individual agents.

In the next section we are going to solve a specific example in which we will establish the values of α that verify $0 < r(\alpha) < \infty$.

6 Realisation of a public good delivery

We now analyse the public good's delivery model (1), under the delivery condition (2).

6.1 Does status quo need be modified?

The regulator needs to establish whether the solution $\overline{x}$ to the *unconstrained* game, presumably "played" at present (hence "status quo"),

$$\overline{x} = \text{equil}_{y_i|\overline{x}_{-i}\in \mathbb{R}^2} \{\phi_1(x), \phi_2(x)\} \ , \tag{29}$$

generates

$$\overline{x}_1 + \overline{x}_2 - 1 < 0 \tag{30}$$

(scarcity). If $\overline{x}_1 + \overline{x}_2 - 1 \geq 0$ (abundance), then there is "no problem"[7] for the regulator to solve because the constraint is satisfied in a Nash equilibrium. Condition (30) implies that, in a *constrained* equilibrium, $\lambda > 0$, if such an equilibrium exists.

As we said in Section 2.1, a solution to unconstrained game (1) is $\overline{x}_1 = 0$, $\overline{x}_2 = 0$. This clearly satisfies (30) hence, the regulator's problem of how to assure satisfaction of the constraint is real.

6.2 Which Pareto-efficiency programmes can be enforced?

The regulator has to verify that the Pareto-efficiency programme (18) without the constraint (2) also generates a "scarce" solution *i.e.*, that $\hat{x}_1(\alpha) + \hat{x}_2(\alpha) - 1 < 0$. This will imply that a *constrained* Pareto-solution will be saturated hence $\mu > 0$ in (19). Consequently, $K > 0$ in (24).

6.3 Necessary conditions

Resolving the Karush-Kuhn-Tucker conditions formulated for programme (18) with $h(x) = x_1 + x_2 - 1 \geq 0$ and $x_i \geq 0, i = 1, 2$

$$-\alpha x_1 + (2\alpha - 1)x_2 + \mu \leq 0 \tag{31}$$
$$x_1(-\alpha x_1 + (2\alpha - 1)x_2 + \mu) = 0 \tag{32}$$
$$\alpha x_1 - (1 - \alpha)(2x_2 + x_1) + \mu \leq 0 \tag{33}$$
$$x_2(\alpha x_1 - (1 - \alpha)(2x_2 + x_1) + \mu) = 0 \tag{34}$$
$$x_1 + x_2 - 1 \geq 0 \tag{35}$$
$$\mu(x_1 + x_2 - 1) = 0 \tag{36}$$

results in several threads of solutions.

- For $\mu > 0$

[7] Unless the regulator would like to improve welfare. In this paper, we assume that the regulator's main concern is the constraint satisfaction.

a. $x_1 > 0$, $x_2 > 0$
From (32), (34), (36)

$$\mu(\alpha) = \frac{6\,\alpha - 6\,\alpha^2 - 1}{3\alpha} > 0 \quad \text{for } \alpha \in \left(\frac{1}{2} - \frac{\sqrt{3}}{6}, \frac{1}{2} + \frac{\sqrt{3}}{6}\right), \tag{37}$$

or, approximately, for $\alpha \in (0.211, 0.789)$. The dash-dotted blue line in Figure 1 shows the values of μ, for which a constrained non-corner solution exists.

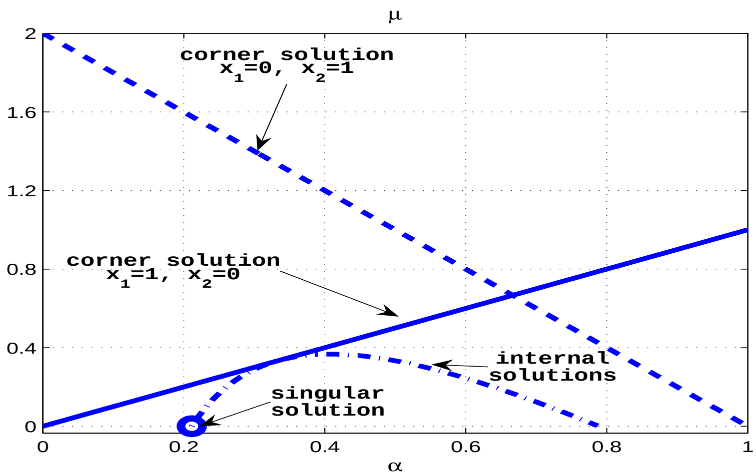

Figure 1: Multiple $\mu > 0$

.

The output values that maximise the Pareto program (18), under the delivery condition (2), are:

$$\hat{x}_1(\alpha) = \frac{1}{3\alpha}, \qquad \hat{x}_2(\alpha) = \frac{3\alpha - 1}{3\alpha}. \tag{38}$$

We observe that only for $\alpha > \frac{1}{3}$ both outputs are positive, see Figure 2 first panel, blue lines; the corresponding payoffs and the regulator's goal are shown in the second and third panels, respectively (blue lines).
no entiendo la figura 2, dibujas las ganancias de los dos jugadores separadas?

b. $x_1 = 0$, $x_2 > 0$
From (36) $x_2 = 1$ and from (34)

$$(1 - \alpha)(-2 \cdot 1) + \mu = 0 \tag{39}$$

thus $\mu = 2(1 - \alpha)$. This combination of x_1, x_2, μ does not satisfy (31), hence there is no "corner" solution of the regulator's programme at $x_1 = 0$, $x_2 = 1$.

c. $x_2 = 0$, $x_1 > 0$
From (36) $x_1 = 1$ and from (32)

$$-\alpha + \mu = 0 \tag{40}$$

hence $\mu = \alpha > 0$ for any $\alpha \in (0, 1)$.
The KKT conditions with $\mu > 0$ are satisfied for $\alpha < \frac{1}{3}$ and we conclude that the corner solution $x_1 = 1$, $x_2 = 0$ exists for $0 < \mu = \alpha < \frac{1}{3}$. We notice the non-uniqueness of μ for $\alpha \in (0.211, 0.333)$. However, the blue dash-dotted line implies the outputs (38) that can be realised for $\alpha > \frac{1}{3}$ only. Hence, from the regulator's point of view, the solution is unique: for $\alpha \in (0, 0.333)$, $x_1 = 1, x_2 = 0$; for $\alpha \in (0.333, 0.789)$ (38) and there appears to be no solution for $\alpha \in (0.789, 1)$.

The payoffs to the players are displayed in Figure 2 second panel as the red lines. The lines are at the levels $\phi_1(1, 0) = -0.5$ (dashed line) and $\phi_1(1, 0) = 0$ (solid line). todo este parrafo yo lo sacaria y dejaria solamnete que el caso $x1 = 1$, $x2 = 0$, $\mu = \alpha$ es una solucion posible para todo alpha esntre 0 y 1. Sigo sin entender la figura 2 primer y segundo dibujo. Por ahora tampoco sabemos porque para alphas grandes no hay solucion yo dejaria esa frase para la conclusion

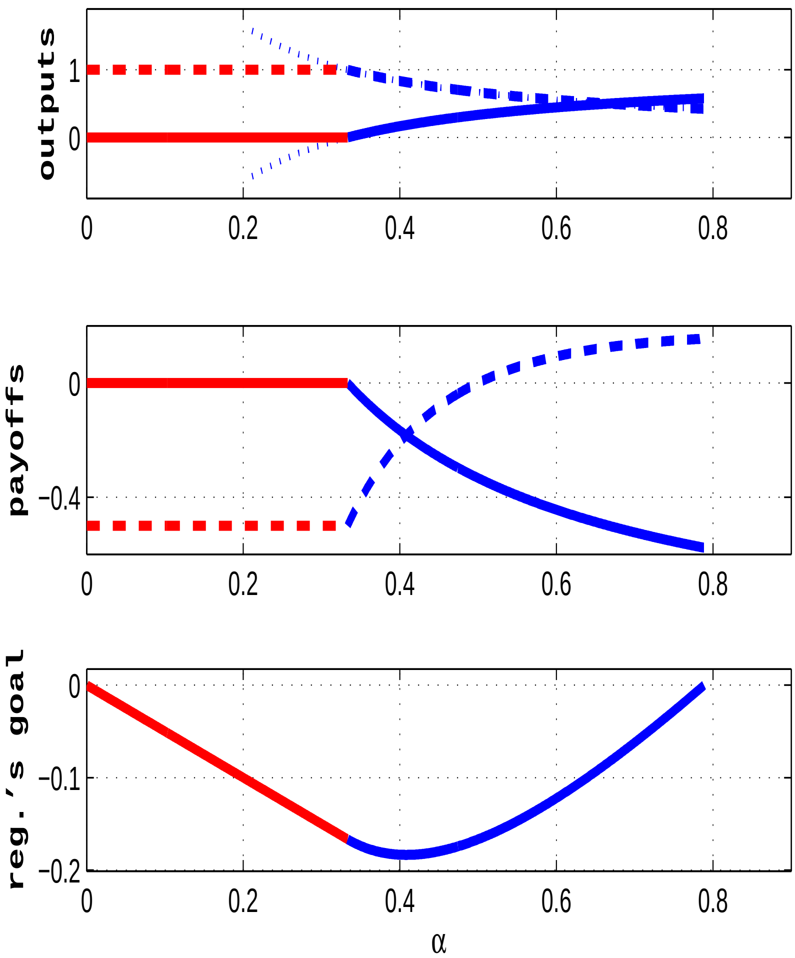

Figure 2: Regulator's solutions as a function of α.

- For $\mu = 0$

  a. The cases of $x_1 = 0$, $x_2 = 1$ and $x_1 = 1$, $x_2 = 0$ coincide with items (b.) and (c.) above, for $\alpha = 1$ and $\alpha = 0$, respectively.

  b. Assuming $x_1 > 0$, $x_2 > 0$ and using (32), (34) yields

$$-\alpha x_1 + (2\alpha - 1)x_2 = 0 \tag{41}$$

$$(2\alpha - 1)x_1 - 2(1 - \alpha)x_2 = 0\,. \tag{42}$$

This system has the zero solution that contradicts (35).

However, if $\alpha_s = \dfrac{1}{2} + \dfrac{\sqrt{3}}{6}$ then

$$\Delta = 6\alpha - 6\alpha^2 - 1 = 0 \tag{43}$$

and we could have a singular solution

$$x_1^s = \frac{2\sqrt{3}}{3+\sqrt{3}} x_2^s \tag{44}$$

From (35),

$$\frac{2\sqrt{3}}{3+\sqrt{3}} x_2^s + x_2^s = \frac{\sqrt{3}+3}{\sqrt{3}+1} x_2^s \geq 1 \tag{45}$$

so $x_2^s \geq \dfrac{\sqrt{3}+1}{\sqrt{3}+3}$.

If we use α_s in programme (18) and allow for $x_1 = \dfrac{2\sqrt{3}}{3+\sqrt{3}} x_2$ then we obtain $P = 0$.

However, we do not expect the regulator to have their preferences between firms set at *exactly* $\alpha = \alpha_s$ and will not delve into the singular solution.

6.3.1 Sufficient conditions

Yo aca pondria el analisis de la concavidad que te mande de la funcion sin restricciones y entonces despues podes poner todas las conclusiones More importantly, for the function $P(\cdot)$ ceases to have a finite maximum for $\alpha \in (0.789, 1)$. Therefore the regulator's choices from this interval will not be implementable as coupled constraints equilibria.

Overall, we observe multiple positive μ in Figure 1. However, the regulator resolves this non-uniqueness because only the corner solutions are available for $\alpha < 0.333$. Hence, the upper envelope in Figure 1 determines the unique map between α and μ.

We notice that the solutions on the interval (0,1; 1,0) cannot be obtained when some "extreme" values of α (*i.e.*, α from outside of $\left(\dfrac{1}{3}, \dfrac{1}{2} + \dfrac{\sqrt{3}}{6}\right) \approx (0.333, 0.789)$) are employed, which would have favoured the income generated by one of the firms.

We conclude that Pareto optimal solutions for $\alpha \in (0., 0.789)$ exist and are unique. Hence, there are non trivial values of marginal rates of substitution between the two players' payoffs and the corresponding positive Lagrange multipliers ($\mu > 0$), for which the regulator's problem can be solved.

6.4 Which coupled constraints equilibria are available

Subsequently, existence and uniqueness of a coupled constraints' equilibrium for $r > 0$ needs to be established. For that purpose we compute pseudo-Hessian (see Section 4.2):

$$\mathcal{H} = \begin{bmatrix} -r & -\frac{1}{2} + \frac{1}{2}r \\ -\frac{1}{2} + \frac{1}{2}r & -2 \end{bmatrix} \tag{46}$$

that is negative definite for

$$\frac{5}{2}r - \frac{1}{4} - \frac{1}{4}r^2 = -r^2 + 10r - 1 > 0 \tag{47}$$

$$\textit{i.e.,} \quad 5 - 2\sqrt{6} < r < 5 + 2\sqrt{6} \quad \text{or, approximately,} \quad 0.101 < r < 9.899\,. \tag{48}$$

So, we know there exists an interval for r, for which the CCE exists and is unique.

We compute the mapping $\alpha \rightsquigarrow r$ from (27)

$$r(\alpha) = \frac{1 - 6\,\alpha}{-2 + 3\,\alpha} \tag{49}$$

and plot it in Figure 3.

Map (49) enables us to compute $\bar{\alpha}$ that corresponds to the upper end of interval (48). This is $\bar{\alpha} \approx 0.583$, the largest value of α, for which a unique equilibrium is guaranteed[8]. The lower end of this interval corresponds to $\underline{\alpha} \approx 0.191$. We notice that $\underline{\alpha} > 0$ *i.e.*, $\underline{\alpha}$ is above the smallest α, for which a Pareto solution exists. On the other hand, $\bar{\alpha} < 0.789$ *i.e.*, $\bar{\alpha}$ is below the largest α, for which the outputs are non-negative.

We observe that the interval $(\underline{\alpha}, \bar{\alpha})$ is included in $\left(\frac{1}{6}, \frac{2}{3}\right)$, for which $r(\alpha) > 0$. The intersection[9] is

$$\alpha \in (\underline{\alpha}, \bar{\alpha}) \approx (0.191,\, 0.583) \tag{50}$$

that defines the interval of α for which $r > 0$ and such that the uniqueness of equilibria is guaranteed.

We can see that as the regulator attaches more weight to the first firm's payoff *i.e.*, if α grows from $\underline{\alpha}$, to $\bar{\alpha}$ (*i.e.*, from 0.191 to 0.583), the preferential treatment as measured by r, which diminishes marginal cost of the constraint's violation (see (14)), becomes increasingly stronger. This appears logical: the more income from firm 1 the regulator "wants", the smaller the marginal cost this firm should face.

This enables us to see the *dual* function of $r > 1$. On a one hand it stimulates the first firm's production by diminishing its marginal cost; on the other hand, it motivates firm 2 to produce because of the fear of punishment.

In summary, if $\alpha \in (0.191, 0.583)$, then $0.101 < r < 9.899$ and (49) defines a relationship between a Pareto solution and CCE.

Recall, a value of α "close" to 0 signifies that the second firm's payoff is of more value to the regulator than the first firm's; an α "close" to 1 means more importance attached to first firm's payoff. It becomes clear that because of the constraint (2), the regulator *cannot* prioritise the second firms' payoffs in some "extreme" fashion.

[8] We notice that $\mathcal{H} > 0$ is a sufficient condition for uniqueness and cannot exclude that uniqueness may be achieved for $r > 0$ from outside the above interval.

[9] Notice that $\alpha = \frac{2}{3}$ is special in that $x_1\left(\frac{2}{3}\right) = x_2\left(\frac{2}{3}\right) = \frac{1}{2}$; furthermore, for $\alpha > \frac{2}{3}$ the contribution of the second firm toward the constraint satisfaction is greater than of the first firm. However, $\alpha = \frac{2}{3} > \bar{\alpha}$ that is outside the interval, for which unique equilibria are guaranteed.

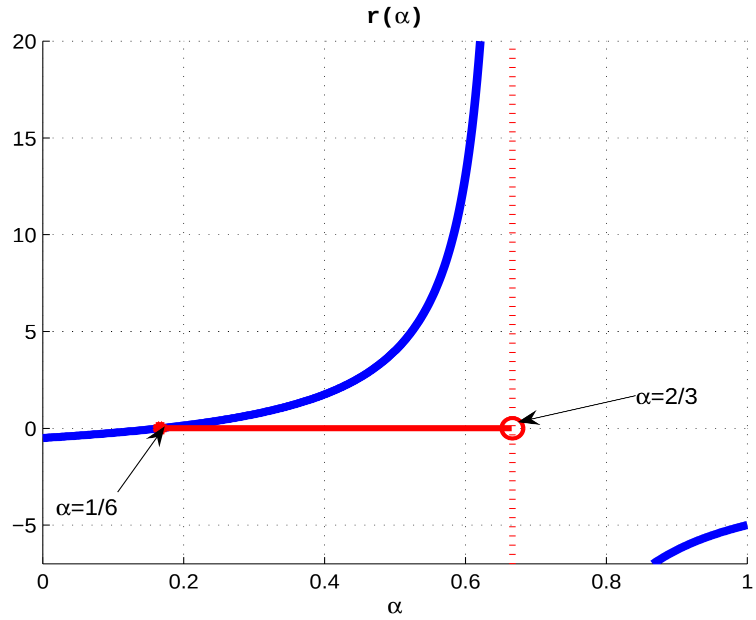

Figure 3: The map between α and r.

However, this does not preclude existence of the "corner" solution $x_1 = 1$, $x_2 = 0$, that exists for any $\alpha \in (0.191\, 0.333)$.

We have also computed $\hat{\alpha} = \dfrac{\sqrt{6}}{6} \approx 0.4082 \in (0.191, \bar{\alpha})$ that *minimises* the regulator's programme (6), which is a convex function of α, see Figure 2 third panel. This suggests that the regulator might seek to implement an equilibrium that corresponds to $\bar{\alpha}$.

7 Concluding remarks

We have proved Theorem 5.1, which formulates the necessary conditions, under which a constrained Pareto-efficient solution can be supported by a coupled constraints equilibrium *à* la Rosen.

We have also proposed a novel approach to solution of a politico-economic coordination problem. We have used a game theoretic framework based on the concept of coupled constraints equilibrium that has allowed us to formulate this problem *naturally*.

We have used Theorem 5.1 to solve the problem. We have concluded that if agents interact through positive and negative externalities then the regulator's choices for his (or her) preferred solutions may exclude some extreme values of the marginal rate of substitution between the firms' payoffs.

Furthermore, the array of unique equilibria that can support the regulator's choices is non symmetrical with the respect to $\alpha = 0.5$ and exclude solutions, in which the "public" firm's payoff would have contributed more than 58% toward the regulator's programme. On the other hand, heavy preferences of the "private" firm's payoff (*i.e.*, small α) are also excluded.

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Appendix

A Rosen's relative weights in $\mathbb{R}_+^2$

Consider a game with payoffs $\Pi_1(e), \Pi_2(e)$ that satisfy know that this game has a unique equilibrium for a choice of r_1, r_2. The equilibrium fist order conditions are

$$\left.\begin{aligned} \frac{\partial \Pi_1(e)}{\partial e_1} &= -\frac{\lambda(r_1, r_2)}{r_1} \\ \frac{\partial \Pi_2(e)}{\partial e_2} &= -\frac{\lambda(r_1, r_2)}{r_2} \end{aligned}\right\} \tag{51}$$

where $\lambda \geq 0$ is the shadow price of the common constraint of type (2).

Let us choose $r_1 = r, r \in (0, \infty)$ and $r_2 = 1$. The first order conditions (51) become now

$$\left.\begin{aligned} \frac{\partial \Pi_1(e)}{\partial e_1} &= -\frac{\lambda'(r, 1)}{r} \\ \frac{\partial \Pi_2(e)}{\partial e_2} &= -\lambda'(r, 1) \end{aligned}\right\} \tag{52}$$

where $\lambda' > 0$ is the Lagrange multiplier that corresponds to this choice or r.

We notice that conditions (51) are equivalent to (52) if

$$\left.\begin{aligned} \frac{\lambda(r_1, r_2)}{r_1} &= \frac{\lambda'(r, 1)}{r} \\ \frac{\lambda(r_1, r_2)}{r_2} &= \lambda'(r, 1) \end{aligned}\right\}. \tag{53}$$

The above is true if and only if

$$r \equiv \frac{r_1}{r_2}. \tag{54}$$

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