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S. Thoyer, A. Hailu. Multi-unit auctions to allocate water scarcity simulating bidding behaviour with agent based models. 2005. hal-02827522

HAL Id: hal-02827522
https://hal.inrae.fr/hal-02827522
Preprint submitted on 7 Jun 2020

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LAMETA Working Paper
2005-01

Multi-Unit Auctions to Allocate Water Scarcity
Simulating bidding behaviour with agent based models

Sophie Thoyer* and Atakelty Hailu**

Résumé: Les enchères multi-unitaires sont un outil potentiellement intéressant de réallocation des droits d'usage de l'eau car elles évitent le problème du « lumpy bid ». En revanche, la théorie sur les enchères multi-unitaires ne permet pas d'identifier les stratégies optimales des enchérisseurs (car il existe une multiplicité des équilibres) et ne fournit donc que des indications partielles sur les formats d'enchère les plus performants. Les études empiriques et expérimentales complètent notre connaissance mais elles sont relativement rares, limitées à des cas simples (deux enchérisseurs, deux unités) et démontrent que les enchérisseurs ont une rationalité limitée et apprennent à améliorer leur stratégie par l'expérience et la répétition. Ce papier construit un modèle de simulation multi-agents pour comparer la performance de différents formats d'enchères inversées (prix uniforme, discriminant et Vickrey généralisé) lorsque les enchérisseurs soumettent des fonctions d'offre continues et apprennent à ajuster leur stratégie pour améliorer leurs gains nets (algorithme d'apprentissage par renforcement de Roth et Erev). Nous démontrons que, conformément à la théorie, le format Vickrey généralisé implique les stratégies les plus sincères et nous confirmons la réduction de demande dans le cas des enchères uniformes et discriminantes. Notre étude démontre les contributions potentielles à la recherche dans ce domaine d'une approche de simulation multi-agents et nous permet de formuler des recommandations sur la mise en place d'enchères multi-unitaires.

Mots clé : enchères multi-unitaires, apprentissage, simulation multi-agent, allocation de l'eau

Abstract : Multi-unit auctions are promising mechanisms for the reallocation of water. The main advantage of such auctions is to avoid the lumpy bid issue. However, there is great uncertainty about the best auction formats when multi-unit auctions are used. The theory can only supply the structural properties of equilibrium strategies and the multiplicity of equilibria makes comparisons across auction formats difficult. Empirical studies and experiments have improved our knowledge of multi-unit auctions but they remain scarce and most experiments are restricted to two bidders and two units. Moreover, they demonstrate that bidders have limited rationality and learn through experience. This paper constructs an agent-based model of bidders to compare the performance of alternative auction formats under circumstances where bidders submit continuous bid supply functions and learn over time to adjust their bids to improve their net incomes (reinforcement learning of Roth and Erev). We demonstrate that under the generalized Vickrey, simulated bids converge towards truthful bids as predicted by the theory and that bid shading is the rule for the uniform auction. Our study allows us to assess the potential gains from agent-based modelling approaches in the assessment of the dynamic performance of multi-unit procurement auctions. Some recommendations on the desirable format of water auctions are drawn.

Key words : multi-unit auctions, learning, multi-agent models, water allocation

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Introduction

Preserving minimal instream flows in rivers for environmental and recreational purposes may entail the permanent (when river water is over-allocated) or transitory (in the case of droughts) reduction of water allocation to farmers. It can take the form of compulsory curtailments with a negotiated compensation or it can be voluntarily traded between irrigators and the public agency in which environmental interests are vested. The second option has been implemented in various contexts in the US through auctions to acquire water rights, mostly on a short-term basis (Cummings et al, 2002).

This article analyses the use of auctions to allocate water scarcity between farmers and explores the scope for multi-unit auctions as an alternative to other existing irrigation auction designs. Multi-unit or multiple unit auctions are auctions where bidders bid with their demand or supply curves or schedules. The main advantage of multi-unit auctions is that they allow us to avoid the lumpy bid issue (Tenorio, 1993). However, there is great uncertainty about the best auction formats when multi-unit auctions are employed (Laury, 2002). The theory can only supply the structural properties of equilibrium strategies; however, the multiplicity of equilibria does not allow for comparisons across auction formats (see Martimort 2002 for a review). Empirical studies (Tenorio, 1999; Wolfram, 1998, List and Lucking-Reiley, 2000) and experiments (Kagel and Levin, 2001; Engelmann and Grimm, 2003) have improved our knowledge of multi-unit auctions but they remain scarce and most experiments are restricted to two bidders and two units. Moreover, they demonstrate that bidders have limited calculation capacities and learn from experience through repeated play instead of landing on the equilibrium strategies at the outset of the game.

This study builds an agent-based model of a multi-unit auction to study the performance of alternative auction formats. Agent-based modeling (ABM) or agent-based computational economics (ACE) is a tool that is increasingly being used to complement theoretical and experimental studies in economics. Tesfatsion (2002) surveys the economics research areas in which ACE has been applied while Duffy (2004) examines the relationship between ACE and human-subject experiments in economics and provides an overview of studies using ACE to examine findings from human subject experiments. ACE is the study of artificial societies of interacting autonomous agents that directly emulate the behaviours of individuals, institutions and environmental components that make up the system being studied (Epstein and Axtell 1996 and Tesfatsion 2002). Unlike conventional or deductive approaches, the starting point in ACE is the specification of agent attributes and behaviours rather than equations or equilibrium conditions describing the system under study. Therefore, ACE is suited to the study of systems where modelling outcomes can be gainfully enriched through the explicit incorporation of phenomena like agent heterogeneity, local interactions and networking, inductive learning, as well as through the relaxation of other restrictive assumptions that are normally imposed under conventional analysis for tractability purposes (Tesfatsion 2002). Studies applying ACE to the study of auctions include Andreoni and Miller (1995), Nicolaisen, Petrov and Tesfatsion (2001), Bower and Bunn (2001), Bunn and Oliveria (2001), Hailu and Schilizzi (2004). In our study, competing bidders submit continuous bid supply functions in the auction and employ reinforcement learning algorithms to update their bidding strategies.

The paper is organized as follows. In the first part, we describe the main issues associated with water auctions which have already taken place and we discuss the advantages of multi-unit auctions. We also review the various auction formats and the structural properties of equilibrium strategies. In the second part, we develop an agent-based model (ABM) where bidders are represented by software agents mimicking their attributes and behaviours (Hailu, Schilizzi and Thoyer, 2004): they have a boundedly rational and learn to improve their bidding strategy through repeated play using the Erev and Roth (1998) reinforcement learning algorithm. In the third part, we conduct a number of computer-based experiments in order to compare equilibrium strategies, costs and efficiency, in the following three alternative sealed-bid auction formats: the uniform-price multi-unit auction; the discriminatory multi-unit auction; and the generalized Vickrey auction (Ausubel, 1997).

We demonstrate that under the Ausubel format, simulated bids converge towards truthful bids as predicted by the theory and that bid shading is the rule for the uniform auction. However, the simulated bidding behavior under discriminatory pricing is different from what the theory expects, although it is in accordance with experimental results obtained by other authors. Our study allows us to assess the potential gains from agent-based modeling approaches in the assessment of the dynamic performance of multi-unit procurement auctions. We were able to assess changes in bidding behaviour accompanying changes in competition levels, and to compare the differences in bidding strategies among bidders with different opportunity cost (true supply curve) structures. We then draw some recommendations for the desirable format of water auctions.

1. Reverse multiple-bid auctions to re-allocate water entitlements

1.1 Water auctions

Increasing concerns regarding the environmental consequences of instream water scarcity have led a number of policy-makers to set-up water license buyback schemes. These schemes aim to reduce the consumptive use of water and increase environmental flows in times of drought. In the United States, auctions have been trialed as an alternative to other voluntary schemes such as posted prices. Auctions are expected to improve the efficiency of water curtailment and reduce budgetary expeditures through the minimization of information rents paid to farmers.

The Georgia irrigation auction is one of the most interesting examples. The Flint River Drought Protection Act was enacted in April 2000. It requires that state authorities use an “auction-like process” to pay farmers to suspend irrigation in declared drought years. The Environmental Protection Division is required to assess the risks of an upcoming drought and to determine consequently the number of hectares, which must be taken out of irrigation to maintain acceptable instream flows. An auction is organized whereby farmers may offer to voluntarily forego irrigation of all land covered by a specific water-use permit for the remainder of the cultivation year in exchange for a lump sum payment. There is one single buyer, the State, and multiple sellers, the farmers. Pilot sessions were conducted within an experimental setting to assess and compare different auction designs: a discriminative-price auction, a uniform-price auction, an iterative uniform price auction as well as posted price offers (Cummings and al, 2002; Laury, 2002). The target acreage is not public knowledge. A real case auction was finally conducted through the Internet in March 2001. It was set up as an iterative discriminative auction in which farmers were allowed to revise their offers after preliminary results were announced. The process continued until no one wished to submit a revised offer. Although the rate of participation was high, it was found that the iterative process had led to a substantial increase of payments to farmers and to inefficiencies, although the contrary was expected. Consequently, a revision of the auction format was planned.

The San-Antonio auction (Texas) is a programme which seeks to acquire additional aquifer pumping rights by purchase or lease from irrigators West of san Antonio. The auction was held in April 2003, with the San Antonio Water Systems being the buyer and seeking to relieve short term scarcity issues. It was an ascending multi-unit auction. The auction was conducted in real-time through the Internet, and bidders could re-evaluate and adjust a bid in response to other bidder’s offerings during the auction.

It is also worth mentioning here the Deschutes water exchange in Oregon. The Deschutes Resources Conservancy (DRC) is a private nonprofit organization dedicated to stream flow restoration and water quality improvement in the Deschutes Basin. The water exchange programme was created in 2001 to foster an active market for water rights in order to promote efficient water use and restore depleted stream flows. The DRC started a payment scheme to provide incentives for the right holders to forego the use of all or a portion of their rights on an annual basis (used subsequently for instream use). The 2002 policy was based on a fixed price per acre-foot: participation was secured among small acreage landowners. The exchange considered using a reverse sealed-bid auction mechanism as part of its 2003 program.

These examples highlight two important features about real world auction trials. First, that there is little guidance for policy-makers about the best auction formats. Formats so far employed vary from open format to sealed bid, from ascending uniform price auctions to iterative auctions with discriminatory payments. Second, all the trials share one common feature. All auctions were multi-unit, single-bid auctions: bidders were allowed to submit only one bid for a pre-defined quantity (either an acre of land or a price per cubic meter). They were not offered the option to submit a bid supply curve (i.e. a schedule of incremental quantity-price offers) . This raises the issue of lumpy bids, mentioned by Tenorio (1999) but also illustrated in the case of the auctioning of conservation contracts (Chan et al, 2003). If a landholder cannot submit several bids in order to obtain different rental fees for different parts of his/her land, and if marginal conservation costs tend to increase with the land area conserved, then landowners will avoid bidding for large areas of land. This is because the landholder would have to increase the bid with the size of the land offered (in order to properly reflect the average opportunity cost of the land), thereby reducing his/her chance of winning. The same problem is likely to arise with water buyback auctions: it is likely that farmers could choose to adopt water saving practices or technologies which could allow them to forego part of their entitlement at a reasonable cost, but that they would be more reluctant to give up irrigation altogether on large pieces of land since it would have a much higher cost in terms of foregone benefits. Therefore, if, say for monitoring reasons, bidders are only allowed to offer only one piece of land for withdrawal from irrigation, then water right holders are unlikely to offer large amounts of land. This phenomenon leads to lower participation, higher cost per unit of water recovered and lower efficiency because the auction exploits only differences in the average costs of individual right holders, but not the differences in the marginal costs of alternative techniques or levels of water curtailment that can be undertaken by a particular right holder.

One way to get around this issue of lumpy bid problem is to organize multi-unit auctions whereby farmers can actually submit a supply schedule rather than a single bid. The focus of this study is to examine the performance of alternative auction formats for auctions where sellers submit supply schedules represented as bid functions.

1.2 Multi-unit auctions

The theoretical literature on multi-unit auctions usually distinguishes between sequential and simultaneous auctions of several objects with bidders wanting more than one unit. In the case of water auctions, we concentrate on simultaneous procurement auctions, for multiple identical units, within a symmetric independent private value model. We therefore assume that the whole issue of affiliated or common values is not relevant in the case of water right buybacks since what counts for the farmer is the private cost associated with a reduction of his water entitlements.[1]

We also assume that the number of units that the auctioneer wishes to buy is fixed (as opposed to a budget constrained auctioneer or an auctioneer with a downward-sloping demand curve). Since most of the literature on multi-object multiple-bid auctions describes selling auctions, here we describe and summarize briefly what would be the equivalent predictions for a procurement auction in the case of a continuous rather than discrete bid specification. The continuous bid specification can be interpreted as one relating to the purchase of perfectly divisible units and was initially developed by Wilson (1979) who called it “auctions of shares”.

There are two main options for buying multiple units simultaneously: open format auctions or sealed bid auctions. In the sealed bid auction, payments can be discriminatory, uniform or follow the generalized Vickrey rule. In the open format, we can have ascending auctions (discriminatory), descending auctions (uniform) or Ausubel (1997) auctions. The relation

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[1] Landholders or farmers will be different in their productivity levels and, therefore, in their values for water. This may not be the case in the rare circumstance where there is scope for alternative resale on a water market between private right-holders or when farmers have heterogeneous expectations concerning the severity of the upcoming drought and the resulting scarcity.

between sealed bid formats and corresponding open formats is theoretically one of outcome-equivalence (Krishna, 2002).

Table 1: Outcome equivalence for various formats of procurement multi-unit auctions and structural properties of equilibrium strategies

Sealed-bid format	**Open format**	**Structural property of equilibrium strategies and efficiency**[2]
Discriminatory	Ascending clock	Scope for flat supply and for supply inflation Inefficient allocation
Uniform-Price	Descending clock	Supply "inflation"[a]: increasing marginal bias with quantity supplied[b]. Inefficient allocation Coordination at a high price equilibrium
Generalized Vickrey	Descending clock with clinched purchase (Ausubel, 1997)	Truthful bidding is a dominant strategy Efficient allocation

[a] Equivalent to demand reduction (or bid shading) in a selling auction

[b] It is a weakly dominant strategy to bid one's true value on the first unit then to start overbidding at an increasing rate. It therefore implies differential overbidding. Bidders with the same opportunity cost per unit increase their bids by different amounts according to the quantity they are offering to provide: the larger participants bid more than their smaller competitors for units with comparable unit costs.

In sealed bid auctions, each bidder is asked to submit multiple bids indicating the price he is willing to accept for different quantities sold. In effect, these multiple bids are equivalent to an inverse supply function. In certain settings, bidders can be asked to submit a continuous supply schedule rather than discrete bids. To describe the allocation procedures in the different auction formats, we need to define the concept of residual demand. Let's define the supply schedule of bidder i as $Q^i = S^i(b)$ with b the per-unit bids. We can then define the residual demand facing bidder i, $D^{-i}(b)$, as the total demand Q_d less the sum of the amounts offered by other bidders for each level of bid.

$$D^{-i}(b)=\max\left\{0, Q_d-\sum_{j\neq i} S^j(b)\right\}$$

Each bidder is awarded the quantity Q^{*i} at which his supply schedule intersects his residual demand.

$$Q^{*i}=D^{-i}(b^*)=S^i(b^*)$$

However, the three formats differ in the calculation of the payments for the winners (see figure 1 in appendix):

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[2] Efficiency here refers to the social opportunity cost of reallocated resources or water. An efficient reallocation is obtained when water is obtained from sources with the least opportunity cost for water.

- in a discriminatory auction, each bidder is paid an amount equal to the sum of his actual winning bids (or the area under his supply schedule up to Q^{*i} in the continuous case).
- in a uniform-price auction, all units sold are paid at the market-clearing price equating aggregate supply to demand. Therefore, infra-marginal winners receive payments that are higher than the production costs implied by their bid supply.
- in a generalized Vickrey auction, each bidder is paid for each unit sold the highest value of the corresponding losing bid. If he has won the first unit, he is paid the highest losing bid of his competitors on this first unit. For the second unit, he is paid the highest losing bid of his competitors on this second unit. In a continuous case, where bidders would submit supply schedules rather than discrete bids, each successful bidder is paid the entire area under the residual demand up to Q^{*i}.

Equilibrium strategies, efficiency, and revenue

For multiple-bid auctions, no closed form expressions of the bidding strategies are available in the general case and most authors have therefore focused on the structural properties of the equilibrium strategies. Wilson (1979), Back and Zender (1993), Englebrecht-Wiggans and Kuhn (1998), Tenorio (1999) and Ausubel and Cramton (2002), have analysed the outcomes of different multi-unit auction formats and shown that the revenue equivalence theorem does not extend to the case of multiple-bid auctions.

They demonstrate[3], using different but simplified auctions, the issue of bid shading (or demand reduction) associated with a uniform-price multiple-bid selling auction. Their findings agree: although it is a dominant strategy to bid truthfully for the first unit (or, in the continuous case, when quantity tends to zero), it is efficient for the bidders to shade their bids for additional units or quantities. Moreover, the amount of bid shading increases with quantities offered. "The reason for this differential shading is that the incentive to win units at any price below marginal value is offset by the incentive to reduce the price paid on infra-marginal units that are won anyway" (Ausubel and Cramton, 2002, p23). The latter becomes increasingly important when quantities increase, which explains the increasing bid shading. The consequence is that bids no longer

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[3] All demonstrations are made for a model where bidders' values are private and independently distributed, and ex-ante symmetric (the distribution of information is uniform across bidders in the pre-auction situation). Since we only consider a private values environment, any equilibrium in the sealed bid format is outcome equivalent to an equilibrium of the corresponding open auction

correlate with opportunity costs, leading to efficiency losses. All demonstrations can be carried over with no restriction to the procurement case. This phenomenon has been verified empirically by Wolfram (1998) who conducted an econometric analysis of "supply inflation" in the electricity procurement auction in England and Whales.

It is also demonstrated that there is an incentive, in a discriminatory format, to submit flatter supply curves than in a uniform price auction. If bidders are risk neutral, submitting entirely flat supply curves is a possible equilibrium (Back and Zender, 1993), although drastic demand reduction is also a possible outcome, especially when the difference in marginal values is high (Englebrecht-Wiggans and Kahn, 1998; Krishna, 2002) . Since there are different classes of equilibrium strategies, it is difficult to analyse how bidders coordinate or even compare the efficiency of the two formats. Tenorio (1993) compares the discriminatory and uniform price auction formats in the Zambian foreign exchange market, which successively implemented the two. He demonstrates that the uniform price auction yields higher average revenue to the auctioneer. His case study includes some form of affiliated values, however.

Only in the generalized Vickrey payment or its counterpart in the open format (i.e. the ascending auction with "clinched" quantities designed by Ausubel (1997)) is truthful bidding a weakly dominant strategy, resulting in efficient allocation. On the other hand, for uniform and discriminatory formats, only increased competition can lead to the reduction of strategic behaviour and to more truthful bidding (Ausubel and Cramton, 1996; Swinkels 1999). The generalized Vickrey is more complex and has not been employed in practice. It would be very useful to how the familiar formats compare with it and the extent to which the relative performance of the different formats depends on the competition level and other details about the auctions.

Experiments

One way to circumvent the unavailability of predictive analytical results is to turn to experimental methods. Experiments can be designed to confirm theoretical results but their function can also be to carry a problem beyond the analytical capabilities of theoretical analysis or to fill a void when theory is incomplete or not available (Hailu and Schilizzi, 2003).

Given both the weakness of the theory on multiple-bid auctions and the increasing use of such auctions in economic life, a growing number of researchers have tried to investigate bidding behaviour with laboratory experiments. Alemgeest, Noussair and Olson (1998) demonstrate that, in the two unit case, an ascending clock auction generates less revenue than the uniform sealed-bid auction, due to strategic bid shading. Kagel and Levin (2001) also compare uniform-price sealed bid and open ascending auctions, with a real player with flat demand for two units playing against a robot with unit demand. Their findings also highlight the issue of demand reduction and show that in the open format, bids converge towards equilibrium values more rapidly than in a sealed bid format, as if "clock could enhance learning". They confirm that the Ausubel format leads to truthful bidding.

Engelmann and Grimm (2003) compare bidding behaviour under five auctions formats and for a case where two bidders with flat demand curves compete to buy two units. Their experiments demonstrate that demand reduction is more acute in uniform open than in uniform sealed bid auctions, and that the Ausubel format eliminates bid shading. They also find that, in clear contrast to the theoretical prediction, bidders in discriminatory auctions place substantially different bids on the first and second unit, even when their valuations for the two units are close. They suspect that it might be due to myopic zero profit aversion of the bidders but do not prove it.

All these experiments are conducted in extremely simplified settings. Human experiments can also be extremely costly and complicated to run when exploring issues such as competition or heterogeneity amongst bidders. One way to deal with these difficulties is to employ computational or *in silico* experiments, conducted with autonomous “artificial” agents interacting in artificial societies (Tesfatsion, 2002). The starting point of such agent-based models is the specification of agent attributes and behaviors rather than equations relating system level variables to describe the dynamics of the system. In the field of auctions, such simulations have already been conducted to study electricity markets (Nicolaisen et al., 2001; Dunn and Oliveira, 2001; Bower and Dunn, 2001) and to compare learnt and optimal bids in the context of single bid auctions (Andreoni and Miller 1995).

2. The modelling of bidding strategies

2.1 Structure of agent based model

Our auction model has a population of agents selling water in a sealed-bid auction to a single buyer, the government agent. The government agent has a target or demand level for water purchases. Each seller is characterized by a (true) water supply function and a capacity indicating the maximum amount of water it has for sale. The water supply function represents the marginal value or opportunity cost of water to the seller and we assume that it is a non decreasing function of the amount of water given up. In the case of water, this maximum capacity would be equal to the bidders current water entitlement. The government agent does not know the true water supply functions of the different bidders and makes selection based on submitted or declared water supply bid functions. Over time, sellers learn to choose supply bid functions that maximize their expected net incomes.

Each auction round involves two stages. In the first stage, the government collects bid functions from the sellers and calculates the residual demand for each bidder and determines the equilibrium quantities bought from each of them at the intersection of their bid supply and their residual demand. In the second stage, payments to individual bidders are determined according to the auction format in use. See section 1.2 and Figure 1 in appendix for a description of how payments are determined in the different auction types. Sellers use the results of the auction to compute their net incomes and to update the probabilities with which they choose their bid strategies for the next round. The strategy choice probabilities of a bidder therefore depend on his opportunity costs (true water supply function) as well as on the history of choices he has made and rewards obtained for those choices.

For the sake of simplicity, it is assumed that the true water supply function of all a sellers i is linear and can be written as:

$P_i = a_i^0 + b_i^0 Q_i$ with $0 \le Q_i \le ms_i$, where ms_i is the maximum water capacity of bidder I (equivalent to his water entitlement)

The model is built for a heterogeneous population of eight sellers with the water supply capacity, intercept and slope parameters indicated in Table 2. Capacity takes either of two values, namely, 0.25 and 0.75. The intercept parameter a_i^0 can be either 0.25 or 0.75 while the slope parameter b_i^0 can be 0.25 or 0.75. No two bidders are identical. The intercept parameter is the entry price, and reflects the fixed cost of giving up water. The maximum price (Max Price) is the price required by the seller when selling his whole entitlement ms_i .

Table 2: Composition of water seller population

Sellers (Bidders)	Maximum capacity (*ms*)	True intercept parameter (a^0) (true entry price)	True slope parameter(b^0)	Max price
Seller 1	0.25	0.25	0.25	0.3125
Seller 2	0.75	0.25	0.25	0,4375
Seller 3	0.25	0.25	0.75	0,4375
Seller 4	0.75	0.25	0.75	0,8125
Seller 5	0.25	0.75	0.25	0,8125
Seller 6	0.75	0.75	0.25	0,9375
Seller 7	0.25	0.75	0.75	0,9375
Seller 8	0.75	0.75	0.75	1,3125

From table 2, we can see that sellers 7 and 8 have the highest marginal costs of supply for all quantities whereas players 1 and 2 have the lowest marginal costs and will therefore have many more opportunities to be selected in a competitive bidding process.

2.2 Seller choice strategies

We make the assumption that the learnt bidding curve is also linear and can therefore be written:

$$P_i \quad = \quad a_i + b_i Q_i \text{ with } \beta_i(Q_i) \text{ the strategic bid of player i}$$

There are therefore two dimensions to the seller's choice strategy: intercept choice (a_i) and slope choice (b_i). The learning algorithm described in 2.3 will allow bidders to progressively explore different combinations of *a* and *b* and to retain the best values based on the performance of past bids. Intercept and slope choices are discretized into ten steps. For the intercept, for example, the seller has ten choices 0, 0.1, 0.2, … ,1.0. For the slope parameter, there is a choice of ten values equally spaced between 0 and the maximum slope value implied by the constraint discussed below. The true intercept and slope parameters are included in the choice sets to allow for truthful revelation of supply function parameters.

A constraint is imposed on the choice of strategies so that the chosen bid function does not have any section falling below the true cost function. This is guaranteed by restricting the allowed or feasible parameter choices (*a* and *b*) as shown in Figure 1. The constraints imposed are: 1) that the learnt supply curve should not fall below the true supply curve; and 2) that the relevant marginal price for any bidder is not more than a certain proportion (maxPF, or maximum price factor) of the marginal cost of supply of the most expensive unit by the most expensive supplier (maxP). In this case, max PF is set to 3 (with maxP the marginal price of the last unit supplied by seller 8, or 1.325) The first constraint was included to ensure that the learnt supply curve does not lead to losses. And the second prevents the relevant strategy space from becoming unnecessarily big.

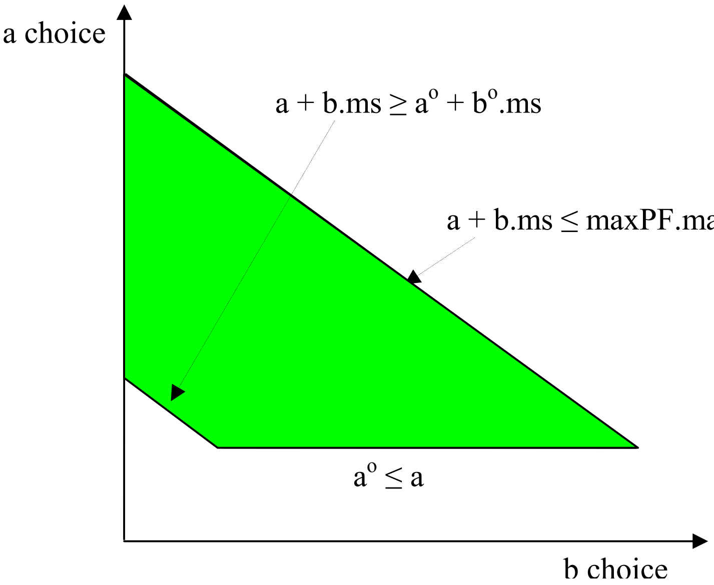

Figure 1: Seller's feasible strategies of bid function choice.

2.3 The learning algorithm

Different learning models have been developed over the last several decades. A typology of learning models presented by Camerer (2003) shows the relationship between these learning algorithms and how certain variants can be obtained as special cases of others. The models differ in terms of their information requirements or assumptions. The reinforcement-learning algorithm is chosen for this study as it is particularly suitable for modelling bidding behaviour without requiring that players be knowledgeable about forgone payoffs associated with strategies that they did not select.

The reinforcement-learning algorithm was developed by Roth and Erev (1995) based on the reinforcement principle that is widely accepted in the psychology literature. Erev and Roth (1998) extend and use this learning algorithm to model behaviour from 12 experimental studies[4] of repeated games with unique nontrivial equilibria in mixed strategies. They find that the reinforcement learning model's predictions of the choices of experimental subjects generally outperform equilibrium predictions. The Roth-Erev algorithm or modified versions of it have been used in agent-based studies of electricity auction markets (e.g. Nicolaisen *et al* 2001; Bunn and Oliveira, 2001).

The Roth-Erev learning algorithm is based on the following four principles rooted in the psychology of learning (Erev and Roth 1998): *the law of effect, the power law of practice, experimentation* and *recency. The law of effect* asserts that the tendency to choose an action is strengthened (*reinforced*) or weakened depending upon whether the action produces favourable results or not. This principle implies that choice behaviour is probabilistic. *The power law of practice* refers to the fact that learning curves tend to be initially steep. *Experimentation (*or *generalization)* implies that strategies that are similar to previously chosen successful ones will be employed more often. Experimentation prevents players from quickly being locked into particular choices. *Recency* (or *forgetting*) requires that recent experience has more impact on behaviour than past experience.

The main features of the algorithm can be described using the following three equations. If the propensity of player *i* to choose strategy (a,b)*)* at time *t* is denoted by $q_i^{ab}(t)$, the propensity updating function can be written as (Erev and Roth 1998, p. 863):

$$q_i^{ab}(t+1)=(1-\phi)q_i^{ab}(t)+E_{cd}(a,b,R) \qquad (1)$$

where: ϕ is the recency parameter, *R* is the reward or reinforcement from previous choice of strategy *(c,d)and is the payment above true supply function obtained by the seller*, while $E_{cd}(a,b,R)$ is the following three step generalization function[5]:

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[4] Eleven of these games were conducted by different researchers in the period between 1960 and 1995.

[5] For strategy sets without linear order, the generalization function should be specified as a two-step function. See Erev and Roth (1998, p. 863).

$$E_{cd}(a, b, R) = \begin{cases} R(1-\varepsilon) & \text{if } a = c \text{ and } b = d \\ R.\,(\varepsilon/2) & \text{if } (a, b) \text{ is neighboring strategy of } (c,d) \\ 0 & \text{otherwise} \end{cases}$$

where ε is an experimentation parameter (see Figure 2).

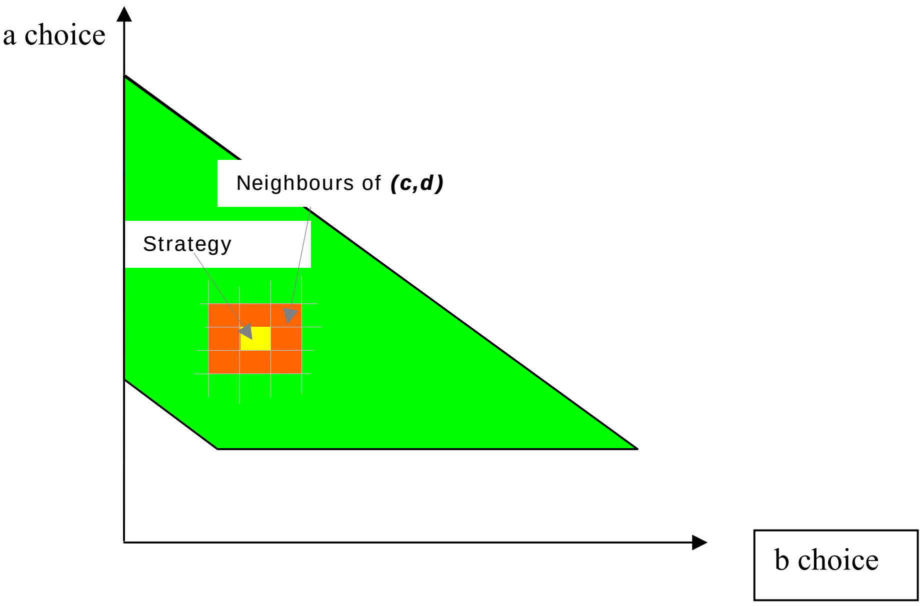

Figure 2: Generalization (experimentation) with Moore Neighbours.

The probability that player *i* uses his *(c,d)* strategy is then given by:

$$p_i^{cd}(t) = \frac{q_i^{cd}(t)}{\sum\sum q_i^{cd}(t)} \qquad (3)$$

Therefore, this learning algorithm has three parameters, namely, the recency parameter, the experimentation parameter as well as a scale parameter (*s*) that determines the initial propensities[6]. The values of these parameters that provided the best data for the 12 games studied

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[6] The initial propensity for any given strategy is set as the product of the scale parameter and an expected profit from bidding. In our case, the latter is set at 10% of the cost of supplying an amount equal to the maximum capacity.

in Erev and Roth (1998) were used in this study. These values are 0.1, 0.2 and 9, respectively. Our experiments with alternative values for these parameters show that the convergence between theoretical and learnt bids is insensitive to the values of these parameters.

3. Simulation results, comparative statics and discussion

The model was used to conduct experiments at five different levels of demand, namely, 0.1, 0.5, 1, 2, and 3 units, and for the three auction formats (generalized Vickrey, discriminatory and uniform price). Compared to the aggregate or total capacity of 4.0 units offered by the 8 bidders, these demand levels cover a wide range of competition levels (from 2.5% to 75%). The relative performance of the different auctions depends on the level of this competition as discussed below. Difference in auction results becomes more important as the demand level increases.

In all the experiments, bidder agents start with randomly selected bids and adjust their bids over the auction rounds. To ensure convergence, 5000 rounds were conducted for each auction. The results converge to their final values much earlier that the 5000th round. However, discussions below are based on the results form the 5000th round. For each auction case (demand level and auction format), 100 replication with different random seeds were run. Results discussed here are based on the averages of these replications.

Expected results

As discussed in Section 1.2, the theory helps us characterize the structural properties of equilibrium strategies. These characterizations are summarized in figure 3 below.

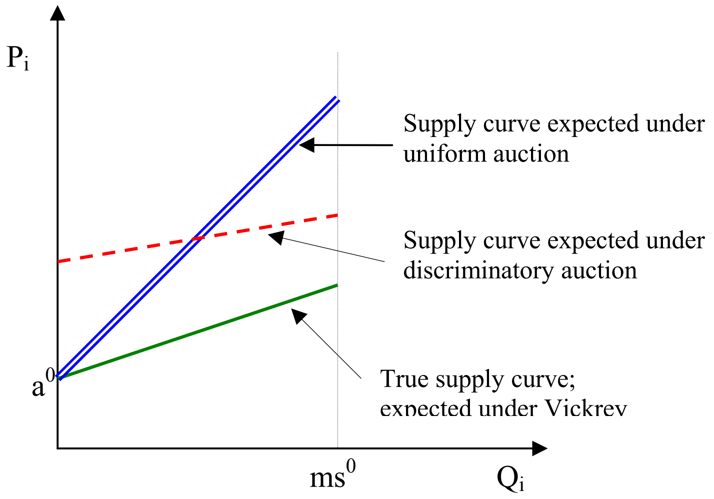

Figure 3: structural properties of equilibrium strategies under discriminatory and uniform-price formats.

Individual learning

Since learning involves two parameters, a good measure of the deviation of the learnt values from the true values is the Euclidean distance between the two pairs. The deviation of learnt from true supply parameter values is smallest in the case of the Vickrey auction, at all demand levels (See Table 3), which confirms the truthful revelation mechanism nature of this format. This deviation is highest under the discriminatory auction where bidders learn to alter their entry prices and supply slopes. Uniform auctions lead to strategic biases but they are less dramatic than in the case of discriminatory auctions.

It is useful to note that for low levels of demand, corresponding to high competition, only the lowest marginal cost bidders (bidders 1 and 2) are selected and are therefore offered an opportunity to learn. Even for a level of demand of 2 (auction presented in table 4), sellers 5 and 6 are rarely selected in the successive bidding processes. This is reflected in table 4d which highlights that highest opportunity cost sellers (sellers 8, 7, 6 and 5) rarely succeed in selling their water in the auction.

Table 3a: Average Euclidian distance of learnt intercept and slope parameter pair from true parameter values when demand is 2 (50% of capacity).

Sellers	Generalized Vickrey	Discriminatory auction	Uniform auction
Seller 1	0.83	0.98	1.07
Seller 2	0.47	0.84	0.62
Seller 3	0.83	1.35	0.82
Seller 4	0.42	0.81	0.66
Seller 5	0.65	1.51	0.62
Seller 6	0.27	0.73	0.93
Seller 7	0.92	1.67	0.88
Seller 8	0.74	1.43	0.97
Average	**0.64**	**1.17**	**0.82**

Table 3b: Average Euclidian distance of learnt intercept parameter from true intercept value when demand is 2

Sellers	Vickrey auction	Discriminatory auction	Uniform auction
Seller 1	0.11	0.46	0.19
Seller 2	0.10	0.42	0.17
Seller 3	0.10	0.46	0.18
Seller 4	0.11	0.49	0.16
Seller 5	0.03	0.27	0.06
Seller 6	0.03	0.31	0.04
Seller 7	0.02	0.26	0.04
Seller 8	0.04	0.04	0.04
Average	**0.07**	**0.34**	**0.11**

Table 3c: Average of learnt slope parameter from true slope value when demand is 2

Sellers	Vickrey auction	Discriminatory auction	Uniform auction
Seller 1	3.51	2.02	4.34
Seller 2	0.50	0.50	0.65
Seller 3	3.20	3.48	2.66
Seller 4	0.39	0.28	0.70
Seller 5	3.00	6.20	2.84
Seller 6	0.40	0.38	1.78
Seller 7	4.17	5.89	3.68
Seller 8	1.20	2.40	1.54
Average	**2.05**	**2.64**	**2.27**

Table 3d: Average quantities supplied by bidders as % of their own capacity when demand is 2 .

Sellers	Vickrey auction	Discriminatory auction	Uniform auction
Seller 1	76	88	72
Seller 2	73.3	84	70.7
Seller 3	76	80	80
Seller 4	69.3	62.7	61.3
Seller 5	60	28	68
Seller 6	32	24	29.3
Seller 7	36	28	52
Seller 8	9.3	26.7	13.3
Average	**54**	**52.7**	**55.8**

The range of learnt parameters does not exactly match the theoretical predictions but confirm a number of hypotheses (See Tables 4 and 5). First of all, the Vickrey auction and the uniform price auctions lead to comparable results, although on average the generalized Vickrey leads to slightly more sincere bidding than the uniform format. It is interesting to note that in both formats, bidding on the first unit is almost truthful, which is consistent with theory, but that sellers with low capacity display very aggressive overbidding strategies on the following units whereas sellers with a high capacity display almost sincere bidding on all units. This is not predicted by theory and deserves further investigation.

For discriminatory auctions, the average learnt entry price is clearly above the true entry price; however, the phenomenon of the “flatter supply” is not verified. They all display overbidding strategies where both the entry prices and payments required for additional units are inflated. This confirms the results by Englebrecht-Wiggans and Kahn (1998).

These tendencies are more clearly marked when the level of competition declines. Bid shading increases slightly for Vickrey and uniform auctions, although strategies under the Vickrey auction remain closer to sincere bidding than strategies under uniform price. However, the discriminatory auction displays unexpected results: learnt entry prices are very high whereas the overbidding on following units is reduced.

Table 4: Learnt versus true supply parameters when demand is 2 (50% of capacity).

Sellers	Vickrey auction		Discriminatory auction		Uniform auction	
	Ratio of learnt to true entry price (a/a^0)	Ratio of learnt to true supply slope b/b^0	Ratio of learnt to true entry price (a/a^0)	Ratio of learnt to true supply slope b/b^0	Ratio of learnt to true entry price (a/a^0)	Ratio of learnt to true supply slope B/b^0
Seller 1	1.54	4.14	3.49	2.71	1.92	4.05
Seller 2	1.65	2.05	3.48	1.56	2.03	2.34
Seller 3	1.57	1.94	3.47	1.56	1.95	1.83
Seller 4	1.58	1.32	3.52	1.00	1.86	1.50
Seller 5	1.11	3.30	1.63	4.35	1.14	3.39
Seller 6	1.12	2.07	1.67	1.07	1.14	4.67
Seller 7	1.11	1.99	1.62	1.65	1.12	2.24
Seller 8	1.10	1.94	1.11	2.88	1.11	2.28
Average	**1.35**	**2.34**	**2.50**	**2.10**	**1.53**	**2.79**
Expected [1]	=1	=1	>1	?	=1	>1

[1] see Table 3.

Table 5: **Learnt versus true supply parameters when demand is 3 (75% of capacity).**

Sellers	Vickrey auction		Discriminatory auction		Uniform auction	
	Ratio of learnt to true entry price	Ratio of learnt to true supply slope	Ratio of learnt to true entry price	Ratio of learnt to true supply slope	Ratio of learnt to true entry price	Ratio of learnt to true supply slope
Seller 1	2.80	5.28	5.69	3.09	2.88	6.10
Seller 2	3.05	2.09	5.85	1.49	3.11	2.43
Seller 3	2.66	2.50	5.71	1.46	2.87	2.65
Seller 4	2.63	1.26	5.79	0.67	2.92	1.33
Seller 5	1.37	3.50	2.24	1.20	1.47	3.93
Seller 6	1.43	1.98	2.30	0.65	1.54	2.56
Seller 7	1.33	1.67	2.24	0.67	1.42	1.72
Seller 8	1.23	1.67	2.28	0.57	1.31	2.04
Average	**2.06**	**2.49**	**4.01**	**1.22**	**2.19**	**2.85**
Expected [1]	=1	=1	>1	?	=1	>1

Auction outcomes

The overall outcomes of the auction can be judged using different criteria. From the policy maker's point of view, both budgetary costs and allocative efficiency are important. Table 6 shows that for very high levels of competition, the three auctions perform almost identically. However, when competition is less acute, we observe larger discrepancies between auctions, with discriminatory auctions displaying bad performance indicators and uniform auctions. When competition is really low (demand = 75% of aggregate supply), the three formats perform equally bad, with high budgetary outlays paid by unit of water..

Table 7 shows the performance of auctions relative to the full information case. The Vickrey auction exhibits least distortion from outcomes obtained under truthful bidding, especially for low levels of competition whereas the uniform and discriminatory auctions might involve substantial misrepresentation of bid functions.

We observe as well that if budgetary outlay is the decisive criterion, then discriminatory auctions perform well for high levels of competition, although it looses its budgetary savings advantage at higher levels of demand. At the highest demand level considered here, even the uniform auction entails payments per unit higher than those required under the Vickery auction.

Table 6: Performance of alternative auction formats under different demand levels: Multi-unit auctions with 8 bidders learning over bid function parameters.

Demand level		Auction	% winners	Price from auction	Total program outlay ($)	Average program outlay ($/unit)	Average opportunity cost ($/unit)	Average net income, selected farmers ($/bidder)
Demand	Demand relative to aggregate capacity							
0.1	2.50%	Vickrey	41.88%	0.269	0.029	0.007	0.258	0.037
		Discrim.	44.38%	0.370	0.031	0.008	0.260	0.051
		Uniform	44.13%	0.350	0.035	0.009	0.261	0.089
0.5	12.50%	Vickrey	43.13%	0.374	0.227	0.057	0.291	0.163
		Discrim.	66.75%	0.728	0.261	0.065	0.332	0.190
		Uniform	50.63%	0.561	0.281	0.070	0.305	0.256
1	25.00%	Vickrey	50.00%	0.516	0.634	0.159	0.351	0.283
		Discrim.	86.63%	0.874	0.794	0.198	0.413	0.380
		Uniform	75.38%	0.748	0.748	0.187	0.391	0.356
2	50.00%	Vickrey	89.25%	0.855	1.802	0.450	0.502	0.399
		Discrim.	78.63%	1.225	1.982	0.496	0.485	0.484
		Uniform	90.00%	0.925	1.850	0.463	0.512	0.413
3	75.00%	Vickrey	94.75%	1.350	4.540	1.135	0.603	0.902
		Discrim.	91.63%	1.912	5.036	1.259	0.621	0.967
		Uniform	95.38%	1.498	4.535	1.134	0.602	0.896

Table 7: Performance of alternative auction formats relative to full information case (honest bidding): Multi-unit auctions with 8 bidders learning over bid function values.

Demand level		Auction	% winners	Price from auction	Total program outlay ($)	Average program outlay ($/unit)	Average opportunity cost ($/unit)	Average net income, selected farmers ($/bidder)
Demand	Demand relative to aggregate capacity							
0.1	2.50%	Vickrey	83.75%	1.036	1.125	1.125	1.012	5.282
		Discrim.	88.75%	1.425	1.220	1.220	1.022	NA
		Uniform	88.25%	1.350	1.350	1.350	1.026	18.959
0.5	12.50%	Vickrey	86.25%	1.261	1.465	1.465	1.063	4.481
		Discrim.	133.50%	2.452	1.910	1.910	1.214	NA
		Uniform	101.25%	1.891	1.891	1.891	1.117	10.916
1	25.00%	Vickrey	100.00%	1.424	1.506	1.506	1.172	2.334
		Discrim.	173.25%	2.411	2.646	2.646	1.378	NA
		Uniform	150.75%	2.063	2.063	2.063	1.305	5.702
2	50.00%	Vickrey	89.25%	1.129	1.152	1.152	1.239	1.058
		Discrim.	78.63%	1.619	2.447	2.447	1.198	NA
		Uniform	90.00%	1.222	1.222	1.222	1.263	1.175
3	75.00%	Vickrey	94.75%	1.566	1.635	1.635	1.122	2.323
		Discrim.	91.63%	2.217	3.123	3.123	1.156	NA
		Uniform	95.38%	1.737	1.753	1.753	1.120	2.757

Note: the auction outcome under perfect information is given in Table 8 in the appendix.

Conclusions

As management agencies around the world look for better ways of reallocating or allocating water, interest in mechanisms like procurement multiple-bid auctions is likely to grow. There have already been some applications of this mechanism. The potential benefits of alternative auction designs need to be studied.

This paper considered three alternative auction types that could be employed in cases where sellers' bids are in the form of supply schedules or bid functions rather than single quantity-bid pairs. Theoretical predictions regarding three multi-unit auctions, namely, generalized Vickrey, discriminatory and uniform prices, are first discussed. An agent-based model with eight sellers bidding to supply water is then used to study the performance of the three auctions under circumstances where bidders learn to change their bid supplies over time. The bidders learn over a strategy space with dimensions relating to the intercept and slope parameters of bid function. The comparison is undertaken for five different demand levels, ranging in magnitude from 2.5% to 75% of aggregate supplier capacity.

Learnt bid supply functions are found to be closest to true ones for the Vickrey auction. The uniform auction comes second by this criterion and discriminatory auctions lead to extreme overbidding, especially for low levels of competition.

Misrepresentations in declared (or submitted) bid supply functions mean that water purchase targets are not necessarily met using least cost sources of water. The Vickrey auction leads to reallocation of water with minimum opportunity cost. However, it is important to note that for very high levels of competition or very low levels of competition, the performance of the three auctions are in fact similar and if budget expenditures carry more weight than gains in allocative efficiency, then uniform auctions can be a better choice. Therefore, a careful assessment of the tradeoffs implied by the choice of auction type is necessary. The Vickrey auction is unfamiliar and more complicated. But the potential benefits it offers in minimizing social cost of reallocated water for all demand levels and budgetary outlay makes it an attractive format, especially when competition is not acute. This could happen in the case of very high demand, as would occur, for example, if there is a severe drought or when water can only be bought in a restricted area from a small number of potential suppliers. However, in most cases, auctions are organized to respond optimally to a forecasted deficit of water during the summer months: demand by the policy-maker is usually much lower than the aggregate potential supply of all farmers. In such case, the choice of an auction format is less important. However, agencies in charge of water procurement might need to have standard auction formats in place. The results from our study point to the importance of tailoring the format to the intensity of demand that is likely to be most frequent.

Finally, the model used here has focused on learnt supply curves that are linear. This simplified the modelling process and has allowed us to say more about the performance of alternative auctions than we could be based on the theory alone. However, it would be interesting to investigate in further research the implications of more general or learnt supply curves that are nonlinear.

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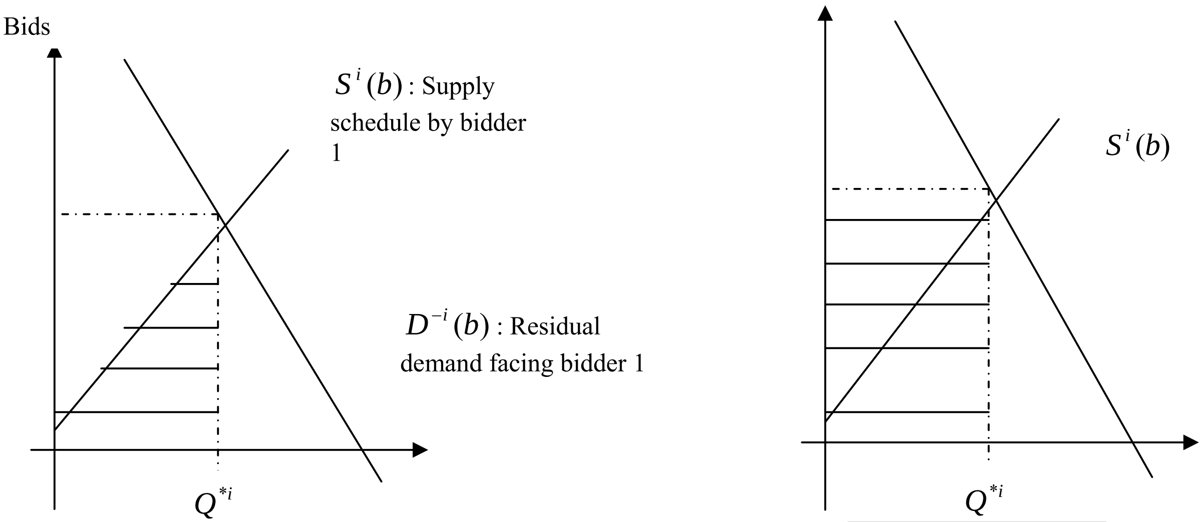
APPENDIX

Table 9: Performance of alternative auction formats under full information case (honest bidding): Multi-unit auctions with 8 bidders**.**

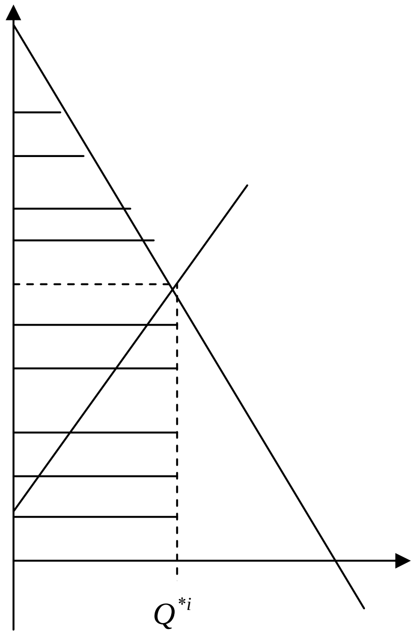
Demand level			Performance measure values					
Demand	Demand relative to aggregate capacity	Auction	% winners	Price from auction	Total program outlay ($)	Average program outlay ($/unit)	Average opportunity cost ($/unit)	Average net income, selected farmers ($/bidder)
0.1	2.50%	Vickrey	50.00%	0.260	0.026	0.006	0.255	0.007
		Discrim.	50.00%	0.260	0.025	0.007	0.254	0.000
		Uniform	50.00%	0.260	0.026	0.007	0.254	0.005
0.5	12.50%	Vickrey	50.01%	0.297	0.155	0.039	0.274	0.036
		Discrim.	50.00%	0.297	0.137	0.034	0.273	0.000
		Uniform	50.00%	0.297	0.149	0.037	0.273	0.023
1	25.00%	Vickrey	50.00%	0.362	0.421	0.106	0.300	0.121
		Discrim.	50.00%	0.362	0.300	0.075	0.300	0.000
		Uniform	50.00%	0.362	0.363	0.091	0.300	0.062
2	50.00%	Vickrey	100.00%	0.757	1.565	0.391	0.405	0.377
		Discrim.	100.01%	0.757	0.810	0.203	0.405	0.000
		Uniform	100.00%	0.757	1.514	0.379	0.405	0.351
3	75.00%	Vickrey	100.00%	0.862	2.777	0.694	0.538	0.388
		Discrim.	100.01%	0.862	1.612	0.403	0.537	0.000
		Uniform	100.01%	0.862	2.587	0.647	0.538	0.325

Figure 1: Total payments under different auction formats



Discriminatory

Uniform-Price



Vickrey