Updating Choquet Valuation

and Discounting Information Arrivals

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- And the Spirit of Alain Chateauneuf!

Origins

- Modeling Bid and Ask asset prices

•As Choquet integrals of asset payoffs

X: (S, F, μ) → R, $\int_S X d\nu$ and $-\int_S -X d\nu$

•Bid Ask Spread → non linearity

•Additivity on comonotonic sets only :

no hegding.

Time, Information, Valuation

- Dynamics = Information arrivals = time
- Economic measure of time: discount factors:
- Payoffs in 2: $\rho(2)$,
- Information in 1/ Value in 1: $\rho(1)$
- Payoffs in 2 valued with information 1: $\rho(1,2)$
- Past time (no uncertain payoffs) Consistency:
- $\rho(2) = \rho(1)\ \rho(1,2)$
- Future time and uncertain payoffs valuation
- $V_0(X_2) = V_0\ [V_1(X/\text{Info in } 1)]$
- $\rho(2)V(X) = \rho(1)V[\rho(1,2)\ V^Y(X)]$

Time consistent valuation formula:

- $\rho(2)V(X) = \rho(1)V[\rho(1,2)\ V^Y(X)]$
- If V and V^Y are Choquet integrals:

$$\rho(2)\int_S X d\nu \quad = \ \rho(1)\int_S \rho(1,2)[\int_S X d\nu^Y] d\nu$$

1) X and $V^Y(X)$ comonotonic: OK, same proba μ_+ in Core(ν)

2) X and $V^Y(X)$ antimonotonic: OK same proba μ_- in Core(ν)

Conditioning: OK, two Lebesgue μ_+^Y and μ_-^Y

Two updating formulas: Dempster-Shafer and Bayes

Good news or opposite of bad news enforce valuation

Individual valuation version

- Chateauneuf Me and Lapied (01) (Venice):
- CEU Decision maker is model consistent:
- $V(X)$ and $V^Y(X)$ are CEI_d (Chateauneuf 91)
- And we're back to the previous story and formulas.
- Questions: Denneberg, Epstein, Jaffray, Ourselves ! etc.

Questions

- Dynamic consistency and CEU ???
- E.g.: Epstein and Le Breton (93): No!
- Border and Segal (94): quasi no!
- But:Karni and Schmeidler (91),
- Machina (98)
- Sarin and Wakker (98): gave a list of axioms
- Other updating formulas and definitions:

Denneberg's, other cases?

This paper

- Other formulas and extensions: Tableau
- Dynamic cash flows valuation: NPV and discount factors
- Assume model consistency (Chateauneuf 91) and Time and States hierarchy:
- Preferences on certain cash flows: $\rho(t)$
- Preferences on uncertain payoffs: $V(X_t)$
- Time separability (no aversion to variability in time):

•$W(X_1, \ldots, X_T) = \sum_{t=1}^{t=T} \rho(t)\, V(X_t)$

Dynamic consistency, model consistency, consequentialism : Which one is the violated axiom ?

Definitions

- The usual definition for conditional expectations:

$$\forall C \in \sigma(Y) \int_C X d\mu = \int_C E^Y(X) d\mu$$

- is equivalent to:

$$\int_C [X - E^Y(X)] d\mu = 0$$

And to:

$$\int_C [E^Y(X) - X] d\mu = 0$$

Def. Cont.

- They are not equivalent if integrals are not additive (capacity ν instead of proba μ)
- Furthermore : $\int_C X d\nu = \int_S 1_C X d\nu$

Is not equivalent to:

$$\int_C X d\nu = \int_{-\infty}^{0} (1 - \nu[X > x] \cap C) dx + \int_{0}^{+\infty} \nu[X > x] \cap C) dx$$

Conditioning and updating rules

- For each of the six definitions and
- For $X = 1_A$ and $Y = 1_B$
- And for each set B, B^C and S, we compute:

 $\nu^B(A) = \nu^{[1_B = 1]}(1_A)$ and

 $\nu^{B^C}(A) = \nu^{[1_B = 0]}(1_A)$

Tableau

- Bayes' rule
- Dempster-Shafer's rule (D-S)
- Full Bayesian updating rule (FUBU)
- FUBU on conjugate capacity (FUBU/C)
- D-S on conjugate capacity (D-S/C)
- And no conditions nor updating formulas
- in the general case (neither comonotononicity nor « antimonotonicity ») when C=S.

Dynamics and NPV

- Assume X is a payoff at date T,
- information arrives at date t and value is NPV, where V is some mean value with:

$NPV(X_T) = \rho(T)\ V(X_T)$ and $NPV^{Yt}(X_T) = \rho^{Yt}(T)\ V^{Yt}(X_T)$

then definitions become:

- $NPV(X_T) = NPV[\ NPV^{Yt}(X_T)\ ]$
- $NPV[\ X_T - NPV^{Yt}(X_T)\ ] = 0$
- $NPV[\ NPV^{Yt}(X_T) - X_T\ ] = 0$
- The first one is: $\rho(T)\ V(X_T) = \rho(t)\ V[\ \rho^{Yt}(T)\ V^{Yt}(X_T)\ ]$
- The other ones present a difficulty: payoffs X_T and $V^{Yt}(X_T)$ are not at the same date and one should be discounted.

NPV Time Consistency

- The first equation for riskless payoffs yields :
  $\rho(T) = \rho(t)\ V[\ \rho^{Yt}(T)\ ]$
- If, furthermore: $\rho^{Yt}(T) = \rho^{t}(T)$ for any date t and information Y_t, then:
  $\rho(T) = \rho(t)\ \rho^{t}(T)$ is the accountants' time consistency.

In this case, the first equation is:

$V(X_T) = V[\ V^{Yt}(X_T)\ ]$

(CKL 2001 definition of time consistent conditional Choquet integral)

Atemporal NPV Conditioning

- The second equation could be interpreted as:

  $NPV[\rho^{Yt}(T)\, X_T - NPV^{Yt}(X_T)\,] = 0$
- Which is:

$$\rho(T)\, V[\rho^{Yt}(T)\, X_T - \rho^{Yt}(T)\, V^{Yt}(X_T)\,] = 0$$

Same problem and if $\rho^{Yt}(T) = \rho^{t}(T)$:

- $V[\, X_T - V^{Yt}(X_T)\,] = 0$

An atemporal definition of conditional Choquet integral

cannot integrate time consistency considerations

Time consistent valuation

- CKL is time consistent
- Which is implied by Dynamic consistency
- It assumes model consistency
- It yields two updating rules depending on comonotonicity of information and of payoffs
- **It violates Consequentialism** :
- Comonotonicity is checked on outcomes that will not be reached under information

What makes an information « good news »?
Is that it is positively correlated with future payofffs

Conclusions

- Only: $V(X_T) = V[\ V^{yt}(X_T)]$ (CKL)
  is consitent with information arrivals
- It implies whether Bayes rule
  (information is comonotonic with payoffs)
- or Dempster-Shafer rule
  (information is antimonotonic with payoffs).
- It doesn't yield a simple way to value uncertain cash flows because of the sum of products:
  $$V[\rho^{Yt}(t+1)\ V^{Yt}(X_{t+1}) + \ldots + \rho^{Yt}(T)\ V^{Yt}(X_T)]$$
- It doesn't say how to condition payoffs not comonotonic with information.

For future work: Valuing the Future

- Use consequentialism when not como? → FUBU
- How can we update for approx. como? (Kendall < 1)
- Most works neglected time as a component of the future. However:
- Gilboa (1989), De Waegenaere and Wakker (2001) , Chateauneuf and Rebillé (2003): valuation can violate separability (time variability hedging)
- but **without uncertainty**.
- The challenge is to value **time** together with **uncertainty** and satisfy time consistency:

$$W(X) = W(X_1, \ldots, X_{t-1}, [X_t + W^{yt}(X_{t+1}, \ldots, X_T)], 0, \ldots, 0)$$