Why do we guess better in negative feedback situations? An experiment on beauty contest games with negative feedback

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Abstract: *We here introduce a beauty contest game with negative feedback and interior equilibrium in a multi-period experiment. This game is isomorphic to classical BCG but fit economic situations such as crop production or professional investment better. The game is still being analysed from the eductive point of view and with respect to the attempt to establish a typology of players according to their depths of reasoning. Our main contribution to the understanding of this game is the formalization of the process by which the information is processed. Using the Shannon entropy criterion, we evaluated information and made a link between the Sperber analysis of reflective and intuitive beliefs and numerical psychological research (Dehaene, 1993). Information that players take into account in their choices is denoted useful information. As this depends on the exploitation of the strategy interval, it will be higher in BCG- than in BCG+ in the first iterations, because strategies are numbers that are naturally scanned several times. As argued by Sperber (1997), there is a point in the reasoning process starting from which reflective beliefs become intuitive. In order to determine the exact location of the specific point from which players in the BCG- can jump to the REE, we assume that sophisticated reasoning is costly. Therefore, an agent stops calculating at step k which is obtained by the intersection between his marginal cost function and his marginal benefit (information) function. However, there are individuals who are not able to reach that point, because their cognitive constraint is saturated beforehand. There are also individuals for whom the cognitive constraint is saturated for a value higher that k, but who stop at step k because, given the structure of the population, they can win the game at a smaller cost. Therefore, a guess in this game corresponds to the solution of the system comprising these two constraints. For our experiments, we found a depth of reasoning smaller than 3, which can, however, be optimal. Results show that the k-step thinking with k<3 is "a fact of human nature" (Bosch and al., 2000) and not an arbitrary modelling restriction. Even if subjects start with a low degree of sophistication, the final winning numbers are very close to the equilibrium in the BCG-. This is possible, as observed by Guesnerie (1992) on the crop producers market, because situations of negative feedback are stable; therefore, "human nature" is likely to better succeed when confronted with such situations: eductive reasoning is "helped" to stay on the convergence path.*

Keywords: beauty contest – guessing games – interior equilibria – negative feedback – Shannon information – numbers attitude

JEL classification : C72, C91

I. Introduction

Imagine a multicoloured undulating ribbon. Looks familiar? Psychologists proved that we hold this kind of perception of numbers: we locate them on an oriented scale, and each time that we switch from a number to another we naturally scan all intermediary numbers. As numbers that we make use of are not isolated, this mental numerical architecture has an influence on all decisions involving numbers that we take and on our reasoning mechanism. In this paper we make use of such type of psychological results and we link them to beauty contest games in order to establish why we are more likely to reach the equilibrium in a negative feedback environment.

1. Testing for the depth of reasoning

Experiments on guessing games have become popular in the last decade, especially for investigating different learning models (Nagel,1995, Ho et al., 1998), and assumptions about reasoning behaviour (Camerer, 2003). The success of these experiments relies on the fact that they respond to the need to test two closely related issues in economic theory: first, most of the models used to describe market activity rely on the theoretical assumption that agents are substantively rational, and possess the ability to solve almost instantaneously the most complex inference problems to take a decision; second, many models of economic behaviour are based on the hypothesis that, when choosing a strategy, agents maximize their utility under the assumption that all other agents behave in a similar way, i.e. under the assumption of common knowledge of their rationality. These assumptions are used to model expectations formation by rational agents.

The rational expectation hypothesis is considered as the extension of rationality to expectation formation (Muth, 1961). Following Binmore's (1987) terminology, the rational expectations hypothesis relies both on "eductive" and "evolutive" justifications. Evolutive arguments, offered by the repetition of the situation, are inherent to experiments where subjects are repeatedly asked to take analogous decisions. Repetition also provides a basis for observing the success of eductive learning. Eductive learning, which takes place in notional time, is, as emphasized by Guesnerie (1992), a necessary but not sufficient condition for the success of evolutive convergence. That means that the conditions for instantaneous success of eductive learning or asymptotic evolutive learning are the same (both processes lead to the same sequence of results in a game). Eductive learning relies on the mental activity of agents who "forecast the forecast of others", by understanding the logic of the situation, i.e. they use sophisticated reasoning rules to "guess" the equilibrium. Guessing games are a simple tool for testing the validity and the depth of this type of "instantaneous" complex introspection.

The second emphasized issue implies that all agents are equally rational, thus the former type of introspection is collective: all agents believe that all agents believe that…all agents are able to use the same kind of eductive reasoning when "guessing" the equilibrium.

2. Simple games to test depth of reasoning

The basic idea underlying the guessing game was first introduced by Keynes (1936), in his famous metaphor about beauty contests: there are traders who "devote [their] intelligences to anticipate what average opinion expects average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees", exactly like in a game where one is prompted to choose the prettiest girl from one hundred faces; one will not choose the girl one really likes, not even the girl one thinks the others like, but the girl one thinks the others think the others think…is the prettiest.[1]

The rules of the beauty contest game (BCG) are simple (Nagel, 1995). *M* players have to choose simultaneously a number from a closed interval *[l, h]*. A frequently studied case is $l = 0$ and h = 100. The winner is the player whose chosen number is closest to *p* times the mean of all chosen numbers, where *p* is a predetermined number, usually smaller than one. The winner gains a fixed prize, which is eventually shared among all winners if there are several. In an experiment, the game can be repeated several times within the same group, to allow subjects to learn. The parameter *p* captures the idea that in a guessing game, agents do not act exactly as described by Keynes' beauty contest game (where $p = 1$), but that agents want to be a little bit away from the mean. As an example, professional investors are concerned with acting around the average selling time, but just before the others ($p<1$) (Ho and al.,1998).

[1] Thereafter the basic game under scrutiny was indifferently cited as a guessing game, beauty contest game (BCG) or average game (Moulin, 1986, first introduced this game under the latter term). In this paper we will use the two first denominations in equal measure.

3. A variant of the classical BCG

Assume that an investor intends to "sell high" and "buy low". To be successful he must sell shortly before the other investors sell, when the price is at its highest level. This implies a guess about the time when other investors will start selling, to avoid selling during the crash. Similarly, an investor wishes to buy at the lowest price, i.e. just a little before the other investors start buying and pushing the price upwards. Translated into a beauty contest game, this is equivalent to choosing a high number when the mean is expected to be low ("crash" expected), and choosing a low number when the mean is expected to be high ("bubble" expected). In such a game, eductive reasoning implies negative feedback in contrast to the ordinary beauty contest game which involves positive feedback. Positive feedback means that an agent who guesses a high mean announces a (relatively) high number and an agent who guesses a low mean announces a (relatively) low number. With negative feedback, guessing a high mean implies announcing a low number and guessing a low mean leads to an announcement of a high number. The case of crop producers provides a nice illustration: if all producers expect a high price, the market price will be low because a high price expectation will lead to high production levels. Similarly, if producers expect a low price, the market price will be high because of demand shortage.

Introducing negative feedback modifies the basic beauty contest game in two ways: it affects the convergence process to the equilibrium solution, and affects the location of the equilibrium solution. In the positive feedback BCG, noted BCG+ hereafter, both the eductive reasoning process and the evolutionary dynamic process, converge to the rational expectations equilibrium monotonically. For example, in the game for which numbers are chosen between 0 and 100 with $p < 1$, the process begins with a high value and converges monotonically towards 0. In contrast, with negative feedback, the convergence to the equilibrium point is described by a non-monotonic damped oscillating function (that is, a function that approaches the equilibrium solution by oscillating up and down around the equilibrium with decreasing amplitude). This process is of course only possible if there is an interior equilibrium, rather than a boundary equilibrium as in the standard BCG+. Interior equilibria have already been investigated by Camerer and alii.(1988) and by Guth and alii (2002), but under monotonic convergence, i.e. with a positive feedback structure[2]. Thus we will refer to our variant of the beauty contest game as "beauty contest games with negative feedback and interior equilibria", which we note BCG- hereafter.

Besides exploring the issue of possible smaller deviations from the equilibrium in first round choices and hypothetical faster convergence to equilibrium, we address in this paper the question of a cost-benefits analysis of information processing and aim at showing that two-sided elimination of strategies provides "more information" than one-sided reduction because with two-sided reduction the choice interval is "scanned" several times, which makes it computationally easier for subjects to locate the equilibrium solution. More generally, actions generating negative feedback lead to a more predictable outcome.

The assumption of null informational cost is unrealistic. Whenever understanding (by processing) information is costly, an agent endowed with rationality faces the decision problem of whether the expected benefit of acquiring or processing the information is worth the cost of processing. Therefore, the amount of information processed by individuals becomes an element of the decision making process. When full rationality is scarce, the deliberation cost must be taken into account (Conlisk, 1996) because good decisions are costly.

4. Previous literature

Nagel (1999) and Camerer (2003) provide extensive surveys on previous work on the monotonic boundary and interior equilibria BCG (BCG+). Reasoning levels seldom exceed step 2. Other related experimental literature includes a study by Guth and alii. (2002), which introduced not only interior equilibria but also heterogeneous players. They observed faster and closer convergence to the game-theoretic solution with an interior equilibrium and with homogenous players[3]. Weber (2001) analyzed basic boundary equilibria guessing games with no feedback. In his experiments, while there is less learning under no feedback than when outcomes are revealed, there is convergence towards the equilibrium prediction. Kocher and Sutter (2000) who analyzed individual

[2] The winning number in their design was $p \times (c + mean)$).

[3] Their REE is set at 50, which corresponds to a focal point)

versus team behaviour in basic games found that groups learn faster and outperform individuals in terms of payoffs. Other contributions to the study of BCG are theoretical; Lopez (2001) fully characterized the basic beauty contest game from a game theoretical point of view, and Branas and Morales (2002) provided simulations in order to explain the "confusion in unravelling" stressed by classical interpretations of basic beauty contest games results. To our knowledge, non-monotonic convergence in BCG[4] has not yet been studied.

5. Motivation

Our modification to the basic beauty contest game allowed us to explore several issues. Typically, as pointed out by Guth and alii. (2002), interior equilibrium beauty contest games exhibit smaller deviations from the equilibrium even in first round choices. A preliminary question is whether the same result will be observed in games with a negative feedback structure (in which the REE doesn't correspond to a focal point). Furthermore, with an alternating elimination of dominated strategies, convergence to the equilibrium solution might be faster, by reducing the anchoring bias on the previous value (Tversky and Kahneman, 1974) typically observed in standard beauty contest games. An important reason why negative feedback might generate smaller deviations and faster convergence to equilibrium is the stabilization effect. The stabilization effect is due to the fact that any deviation in one direction will be partially offset by a deviation in the other direction. It is well known that negative feedback tends to stabilize the economy because any major change will be offset by the very reactions they generate (Arthur, 1989). The same effect applies to the BCG. Our variant of the beauty contest game generates a convergence process by which intervals are deleted on both sides of the equilibrium point, which allows a more accurate location of the equilibrium, even by individuals who apply only two steps of reasoning. In contrast, after two steps of reasoning in the BCG+, subjects are not able to locate the equilibrium point as accurately. The reason, as we will show, is that two-sided elimination provides “more information” than one-sided reduction because with two-sided reduction the choice interval is "scanned" several times, which makes it easier for subjects to locate

[4] Only complex market games, which are isomorphous to our variant of the beauty contest game (for example cobweb games) have been tested experimentally.

the equilibrium solution by computation. More generally, actions generating negative feedback lead to a more predictable outcome.

6. Paper outline

The paper is organized as follows: section II introduces the theoretical framework of the beauty contest with negative feedback and interior equilibrium, and describes the eductive process which leads to the equilibrium solution by providing a game theoretic characterization of the equilibrium. Section III examines benefits and costs of the sophistication process. Section IV presents the experimental design and results. Section IV concludes.

II. Theoretical framework

1. The beauty contest game with interior equilibrium and negative feedback

A large number (*M)* of players simultaneously have to choose a number from a closed interval *[l,h]*. In the experimental part of the paper we will set the bounds at $l = 0$ and $h = 100$ as in the BCG+. But for this theoretical presentation, we will keep the more general approach. The game might be played repeatedly (in several rounds). The winner of a round is the player whose chosen number is closest to:

$$q - p\ \bar{x}$$

where q is a parameter whose value is equal to 100 (or $q = h$), p is a constant ($p < 1$) and $\bar{x}$ is the *mean* of all chosen numbers within a round, i.e. $\bar{x} = (x_1 + x_2 + \ldots + x_M)/M$. This game is isomorphic to the basic game proposed by Nagel (1995) where the restrictions on the choice space and p are identical, but the target number is $p\bar{x}$ [5] instead of $q - p\ \bar{x}$. At the Nash equilibrium, every player should symmetrically play the

[5] The game has the same structure, but a different mathematical composition.

winning number w such that $w = q - pw$, thus the Nash equilibrium of this game is $w = \frac{q}{1+p}$.

2. Eductive reasoning: iterated elimination of dominated strategies

We will study the associated thought processes, as in the standard game. Here the process of thought under scrutiny is eductive learning (Guesnerie, 1992). Eductive learning takes place in notional time (in people's minds rather than in real time) following several steps of reasoning. Let's call the original choice interval $I_o = [l,h]$.

Step 1: at notional time $t = 0$, each player realizes that the average cannot exceed the value $b_0 = 100$. This results in the elimination of all of the numbers ranging between 0 and $100 - p\times100$. Indeed, the value of the winning number cannot be lower than $b_1 = 100 - p\times100$. This lower limit generates a new interval $I_1 = [b_1, b_0]$, which includes the weakly dominant strategies, after elimination of the strategies lower than b_1.

Step 2: at notional time $t = 1$, each player knows the conclusion of *step* 1, and consequently that the other players will only select numbers higher than b_1. Therefore the winning number cannot be higher than $b_2 = 100 - pb_1$, with $b_2 = 100 \times(1- p + p^2)$). The elimination of the numbers higher than b_2 results in the retainment of numbers in the interval $I_2 = [b_1, b_2]$ only.

.....(the process continues)

Step n : at notional time $t = n - 1$, each player knows the result of the previous step, i.e. b_{n-1}, b_{n-2}, and the interval I_{n-1}, thus the new border is $b_n = 100 - pb_{n-1}$, with $b_n = 100\frac{1-(-1)^n p^n}{1+p} + (-1)^n p^n 100$. For $n\rightarrow\infty$, the corresponding interval I_n approaches a point (by the theorem of convergent series).

In this process I_i is an intermediary interval containing only dominating strategies with respect to the interval identified in the previous step (I_{i-1}). As i becomes larger, the set of dominating strategies narrows down to smaller and smaller intervals through the eductive process. Figure 1 illustrates the iteration process for the first three steps.

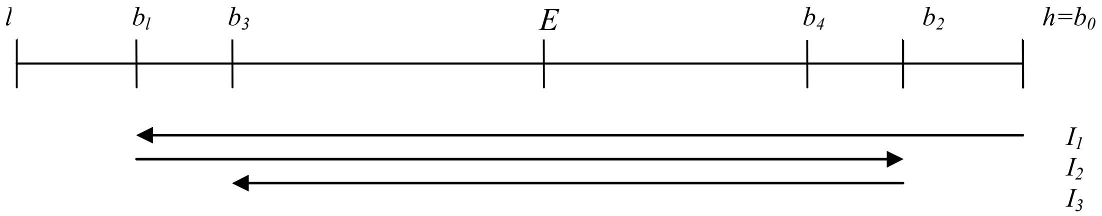

Figure 1: Iterations in beauty contest games with negative feedback

A unique equilibrium, which coincides with the Nash equilibrium, is reached through eductive reasoning. It occurs after an infinite process of elimination of dominated strategies:

$$\lim_{n\to\infty}\left[q\frac{1-(-1)^n p^n}{1+p}+(-1)^n p^n h\right]=\frac{q}{1+p},$$

if $p<1$, which is the stability condition.

3. Characterisation

Our modified BCG is isomorphic to BCG+ but admits an interior solution and the eductive process is characterized by negative feedback.

The sequence of bounds generated by the eductive reasoning in this game is, as described earlier, $h,\ q-ph,\ q-p(q-ph)),\ \ldots,\ q\frac{1-(-1)^n p^n}{1+p}+(-1)^n p^n h$.

In the BCG+, the corresponding sequence is $h, ph, p^2h, \ldots p^n h$. Both games are stable under the condition $p<1$ and have a unique rational expectation equilibrium, which is the limit value of the sequences when $n\to\infty$. Figure 2 gives a representation of the eductive process in the BCG- for different values of $p<1$. The figure shows the winning number for each value of p for iteration steps from 1 to 10.

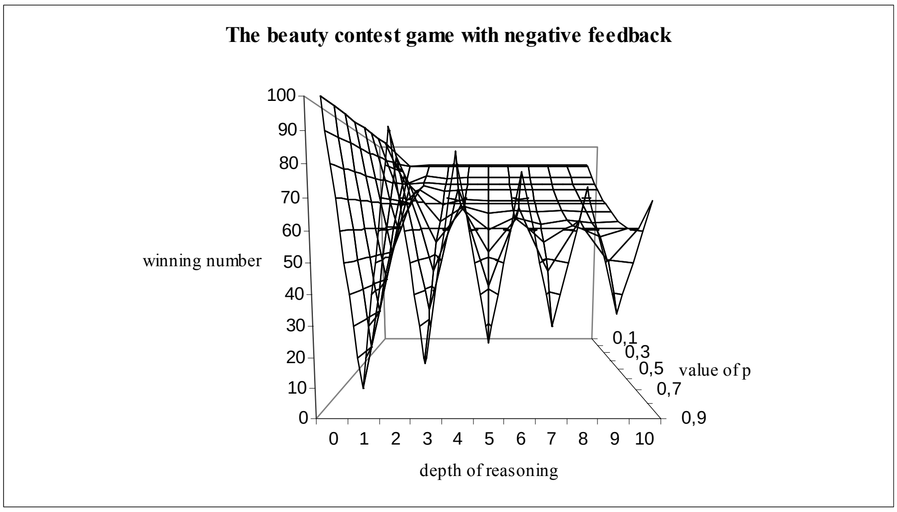

Figure 2 : The beauty contest game with negative feedback
(winning number as a function of the depth of reasoning and the value of p)

In section 2, we calculated successive bounds which correspond to higher levels of eductive reasoning. From these calculations and from figure 1 it was already visible that all odd bounds (b_{2k+1}) are inferior to even bounds (b_{2k}) and, within a category, $b_{2k-1} < b_{2k+1}$ and $b_{2k} > b_{2k+2}$. This characterizes a non-monotonic damped oscillating function (i.e. a function that approaches equilibrium by oscillating up and down around the equilibrium with decreasing amplitude), as visible on figure 2 (non-monotonic left-to-right lines).

The characteristics of this function imply that upper inflexion points correspond to even depths of reasoning, while lower inflexion points are related to an odd depth of reasoning. A BCG has an interior game theoretical solution s_t in period t if $l < s_t < h$, which is the case for our variant[6].

[6] As this solution in the BCG with negative feedback is $\frac{q}{1+p}$, as $p < 1$, then $h > \frac{q}{1+p} > \frac{q}{1+1} = \frac{q}{2} > l$ if $q < 2h$ and $2l < q$. This is true for the particular case when $q = h$ and $l = 0$. This solution is *high* when $p < 1$ (here $\frac{q}{2} = \frac{l+h}{2} < s_t < h$). As $l \leq mean \leq h$, $p\, l \leq pmean \leq ph$, $l \leq q - ph \leq q - pmean \leq q - pl \leq h$ if $0 \leq \frac{q-h}{l} \leq p \leq \frac{q-l}{h} \leq 1$, which is the case for $q = h$ and $l = 0$. Moreover, any empirical solution $s_t = q - pmean_t$ is interior as long as p is a probability.

The greater the number of steps of reasoning in a BCG, the narrower the remaining choice interval. The sequence of narrowing down intervals is $I_0 \supseteq I_1 \supseteq I_2 \supseteq \ldots \supseteq I_n$. Although the process described above arises in notional time, we shall call *speed of convergence* the parameter describing the evolution from interval k to interval $k+1$, for $k \in [0,n]$. The *speed of convergence* measures the percentage of reduction of the interval containing the equilibrium solution, and will be denoted by v_t. v_t is equal to the ratio of the width of interval $k+1$ to the width of the previous interval k, i.e. $v_k = \frac{\|I_{k+1}\|}{\|I_k\|}$.Thus for any type of BCG, the theoretical speed of convergence v_t is a constant[7], and *for* $t>1$, $v_t = p$. Moreover, when $h = q$, the two sequences of bounds calculated in note 7 coincide.

III. Useful information: cost of marginal sophistication and informational benefit

Cognitive psychology has largely documented the fact that humans have limited cognitive abilities (among the last studies, Camerer(2003), Mills and Keil (2004), Todd and Gigerenzel (2003)). Even though cognitive capacities are not binding, standard economic thinking would predict that economically-bounded rational agents will balance costly thinking against the expected rewards of the thinking activity. This means in our context that the number of reasoning steps will be either bounded by the agent's cognitive ability or by his expected net reward of an additional step. We will show that if agents behave is such a manner, their strategies will converge more closely

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[7] The sequences of embedded intervals are respectively :

$$\|I_0\| = h \qquad\qquad \|I_0\| = h$$
$$\|I_1\| = ph \qquad\qquad \|I_1\| = p^0[h(1+p) - q]$$
$$\|I_2\| = p^2h \qquad\qquad \|I_2\| = p^1[h(1+p) - q]$$
$$\ldots \qquad\qquad \ldots$$
$$\|I_n\| = p^nh \qquad\qquad \|I_n\| = p^{n-1}[h(1+p) - q]$$

(classical BCG) *(BCG with negative feedback and interior equilibrium)*

towards the REE in guessing games with negative feedback than that in the BCG+. The reason is that the eductive process in the BCG- generates a larger amount of information in the early steps of reasoning, because the structure of these games allows players to better *localize* the REE through an exploration process over the whole strategy space. Therefore, in this section, we will put forward several conjectures which will constitute our theoretical predictions.

Conjecture 1: **There is more useful information in the first intervals of BCG- than in those of BCG+.**

This statement is based on a calculation of the *useful information* gained in each iteration on the basis of the Shannon entropy criterion. After each step of the eductive process, each player discovers a new guessing interval which contains dominant strategies: in every guessing game, the sequence of narrowing down intervals is $I_0 \supseteq I_1 \supseteq I_2 \supseteq \ldots \supseteq I_n$. Dehaene (1993) showed that humans perceive numbers on a mental logarithmic scale oriented from left to right: the smaller the numbers, the more space they occupy on this scale and the more they approach (by an ordinal position) the left margin (this is called the SNARC effect for Spacial-Numerical Association of Response Codes). When confronted with a number, the human mind has to place it on this scale. For example, the eductive process described earlier starts at b_0. When switching from I_0 to I_1 (and reaching b_1), the brain needs to *scan* all numbers between b_0 and b_1 in order to locate the border b_1. When switching from I_1 to I_2, one needs to scan all numbers between b_1 and b_2 etc... We assume that useful information depends on the exploitation of the guessing interval. Thus useful information for step i is obtained by the intersection of the scanned interval (scanned numbers between b_{i-1} and b_i) with the dominant strategy interval I_i, which is obtained by the elimination of the dominated strategies. More and more eductive steps in the BCG- allow the subject to scan the REE several times, as it is included in all guessing intervals. In contrast, in the BCG+ game the scanned intervals only allow the subject to acquire information on dominated strategies and on one single point corresponding to a border. Indeed, when switching from one border to another, none (except the border point) of the dominant strategies is scanned (because in the BCG+, borders b_i are monotonically ordered, while in the negative feedback game, they alternate. We calculated, for each game, the average available information, according to the information theory formula:

$$H(I) = \sum_{i \in I} prob_i \log_2 (\frac{1}{prob_i})$$

where $prob_i$ is the probability of occurrence of element i, I stands for the information, and the number of possible events is h - l. The probability $prob_i$ of an element in the beauty contest game with negative feedback (BCG-) is $prob_i = \frac{abs(b_i - b_{i+1})}{h-l}$, because all scanned numbers are in the dominant strategies intervals, thus they are useful information. The probability $prob_i$ of an element in the BCG+ is equal to 0.01 because all scanned numbers except one correspond to dominated strategies and as such are not useful information. Thus we calculated the available information for the BCG- as:

$$H(BCG-) = \sum_{i \in I} \frac{abs(b_i - b_{i+1})}{h-l} \log_2 [\frac{h-l}{abs(b_i - b_{i+1})}],$$

where *H(BCG+)* is a sum of constants. The previous equation can be reduced to:

$$H(BCG-) = -\log_2 p \sum_{i \in I} (1+i) p^{1+i}$$

Conjecture 2: **The lower the value of *p*, the higher the relative informative power of the first intervals.**

Marginal useful information measures the increase in total information that a player obtains with an additional step of eductive introspection. Under the assumption of rational behaviour, as the number of steps increases, the probability of guessing the winning number increases. The marginal information curves in the BCG- can exhibit different profiles according to the value of p. For relatively small values of p, the marginal information curve descends, whereas for relatively high values of p the curve is bumped. Figure 4.2 describes the marginal information curves for both guessing games with a high and a small value for parameter p (⅔ and ¼) and for q = 100. The areas under the curves measure the information as calculated before. In this graph, we considered informative intervals of a width that exceeds 0.05, which corresponds to intervals I_1 to I_{20}.

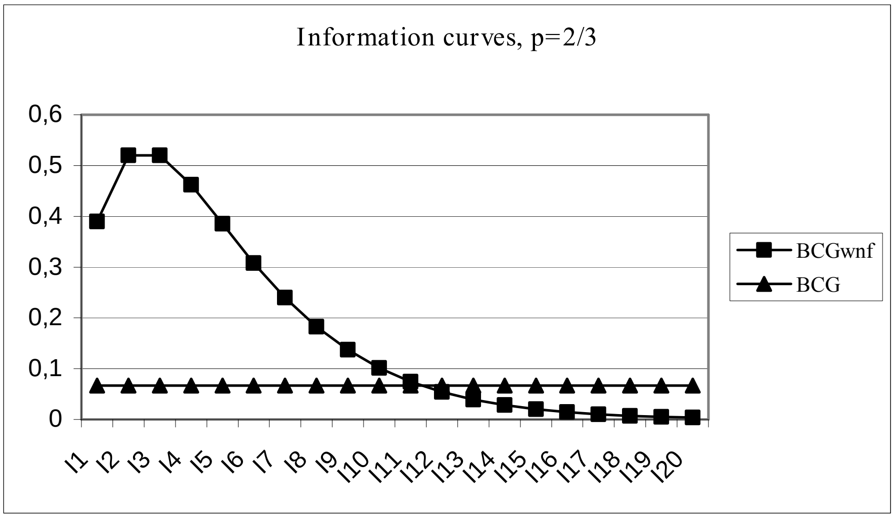

Figure 4.2.(a) Information curves for 20 narrowing down intervals
in the BCG- and BCG+ (p=⅔)

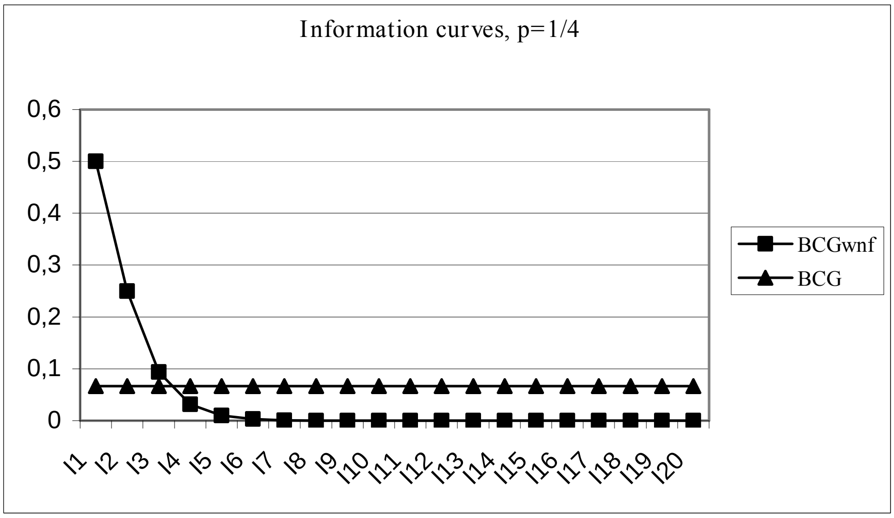

Figure 4.2. (b) Information curves for 20 narrowing down intervals
in the BCG- and BCG+ (p=¼)

The marginal useful information as depicted before measures the additional benefit that an individual can obtain from one more step of eductive introspection (the marginal benefit of the sophistication effort). For a small value of p, the marginal benefit decreases rapidly; the curve corresponds to a fast eductive process: in such a case, discovering the first dominance intervals is enough to "understand" where the REE is located, and calculating more and more eductive steps doesn't add significant additional information. For a larger value of p, the first steps are more informative because the convergence process is slower; it is therefore important to discover several intervals until one's can "jump" to the REE.

Conjecture 3: **For any value of *p*, the initial steps of the eductive process for locating the REE are always more informative in the BCG- than in the BCG+.**

Therefore, for a given level of precision, fewer steps are required, because the marginal information about the location of the REE becomes redundant after several steps. We thus put forward the hypothesis that in BCG-, from a *specific point* onwards, discovering more and more dominance intervals is not necessary (less and less informative). In fact, the figure shows that, for every value of *p*, first useful information intervals in the BCG- contain more informative power than first useful information intervals in the BCG+, which corresponds to good news about guessing the REE: in the BCG- the first intervals are those which give more information about the location of the REE, and discovering a high number of dominant strategies intervals is not necessary, because marginal information about the location of the REE is redundant. In the BCG+, each new interval has the same (low) informative power. It could be said that one has to discover all of the intervals to reach the REE. Therefore, as long as additional information is useful, *H(BCG-)* exceeds *H(BCG+)*.

Conjecture 4: **The stopping rule in the eductive reasoning process is determined either by the cognitive constraint or by a benefit-costs analysis.**

Assuming that sophisticated reasoning is costly, a rational agent will stop calculating at step k for which the marginal cost of reasoning equals the marginal (informational) benefit. Let us denote this condition by $C_m(k) = B_m(k)$ and let k^* be the (unique) solution of this programme. Let us now assume that the agent's cognitive capacity is bounded and that m denotes the maximum number of steps he can achieve. If $m < k^*$, his cognitive constraint is saturated before reaching the optimal number of steps. The number of steps of thinking, $k°$, is therefore defined by $k° = min\ (m, k^*)$. The solution of this system helps determine the exact location of the *specific point* from which in the BCG- one's can jump to the REE.

According to the above arguments, agents will tend to make more steps of reasoning in the BCG- than in the BCG+ because the marginal benefit is always larger in the BCG-. There is a further reason that can explain why subjects get closer to the REE in the BCG-, even if $k°$ (or the distribution of $k°$ in a population) is the same in both games. Bosch and al. (2000) made the assumption that once the first 3 steps of eductive reasoning have been taken, subjects in BCG+ sessions can "jump" to the infinite step of reasoning, because, while calculating the first 3 steps, they learn the direction in which the eductive process should lead them. Our main result is related to the discovery of the k first steps and their informative powers. In the first steps, players in the BCG- collect more information than in BCG+. Starting from interval I_k, each additional interval provides less additional information. Therefore, even if the process of convergence towards the REE is likely to succeed in both games, it will start faster in the BCG-. This analysis leads us to put forward the *hypothesis* that this *specific point* is the point at which reflective beliefs become intuitive; for small values of p the rank of this point will be smaller than the rank of the corresponding point for high values of p.
We conclude that if the structure of an environment is one of negative feedback, the convergence towards the REE is improved because the information is better exploited.

IV. The experiment

1. Experimental design

The experiments were conducted at the X laboratory in May 2004 and at the Y laboratory in October 2004 and April 2005. Participants were students from various disciplines. The software of the computerized experiment was developed within z-Tree (Fischbacher, 1999). A total of 128 subjects participated in the experiment. They were split into 16 independent groups of 8 subjects each and were matched as partners. Each session consisted of 10 rounds, and lasted about 40 minutes. Although our main question is about first round choices, we tested in this paper a multi-period game in order to collect information about the speed of convergence towards the REE when subjects had already had experience with the game. We assumed that a *repetition* factor would probably work from the very first period in the following manner: subjects' choices would be affected by the fact that the situation would repeat itself in exactly the same conditions; a repeated situation is more likely to evolve towards a better outcome, as subjects acquire experience. We assumed that subjects would understand that with repetition they would become experts of the game, and that as experts, their choices would be better. This knowledge would therefore focus their attention on the construction of their strategies: I know that I will become better, so I will try to become better starting from now, and in this way I am likely to be even better and especially better than my opponents.

Subjects received a written questionnaire to check their understanding before the beginning of the session and written instructions. They were required to choose real numbers between 0 and 100. The winner was the subject whose chosen number was closest to 100 - $p \times mean$. We set $p = 2/3$ for 9 groups and $p = 1/4$ for 7 groups, in order to test a small and a high value for p. The REE equilibrium is 60 for the $p = 2/3$ case and 80 for the $p = 1/4$ case. Choosing these two particular values will help us to examine hypotheses on the use of information presented in the previous section. The winner of a round received a prize of 8 euros. In the case of a tie, the prize was shared equally among the winners. Thus a subject could earn a maximum of 80 euros for a session. The maximum amount earned by a subject was 32 euros. Table 1 gives a summary of the experimental design.

Value of p	*REE*	*Number of groups*	*Number of subjects*
2/3	60	9	72
1/4	80	7	56

Table 1. Experimental design

2. Results and discussion

Figure 4 shows the winning numbers for all groups and for the two values of parameter p.

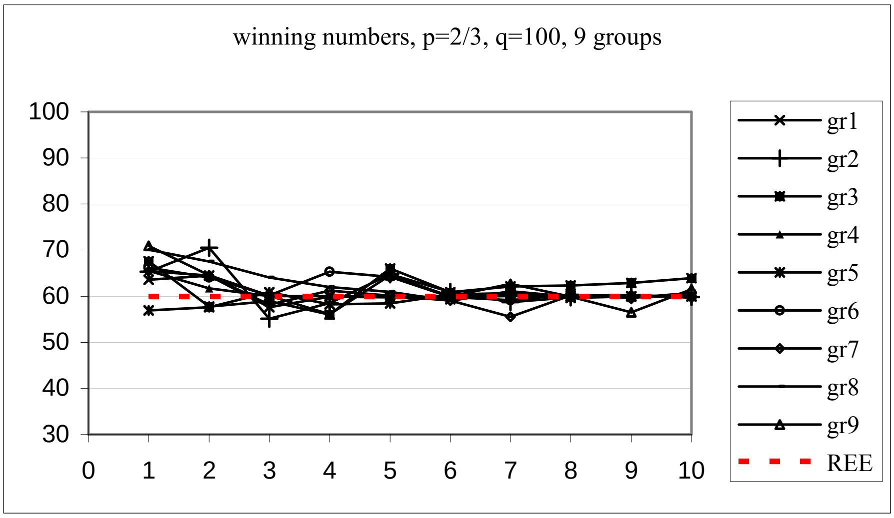

Figure 4 (a): Winning numbers for the BCG-

(groups of 8 subjects)

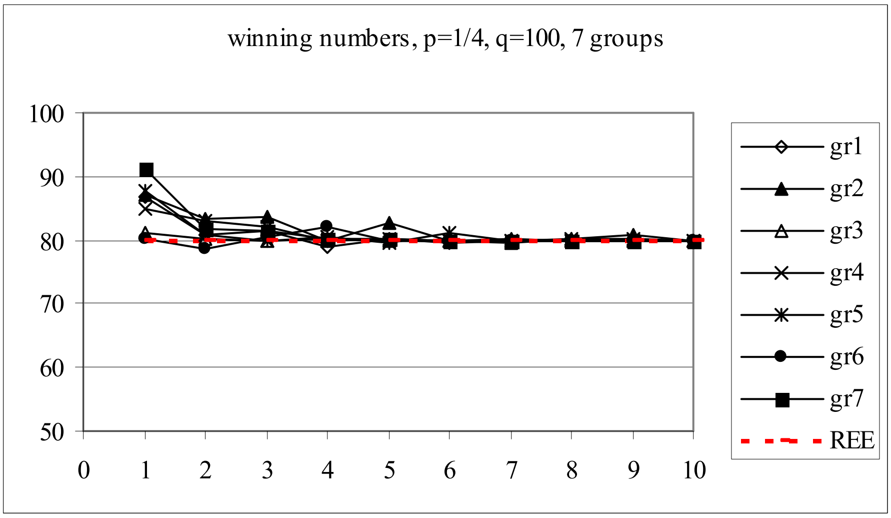

Figure 4 (b): Winning numbers for the BCG-

(groups of 8 subjects)

Result 1: **First period choices correspond to the numbers assigned to steps 0, 1 and 2 of the eductive reasoning process.**

To compute this result we applied the "cognitive hierarchy" model (Camerer, 2003). The model assumes that 0-step players randomize equally across strategies and that k-step players ($k \geq 1$) believe that all other players use only 0 to k-1 steps. The higher the skill of a player (high k), the lower his estimate of the proportion of players of level k-1. We assumed that the beliefs of level k- players about the proportions of level h-players,

$g_k(h)$, was the normalized true distribution ($g_k(h) = f(h) / \sum_{l=0}^{k-1} f(l),$ for $h<k$). Level k-players chose a number which was the best response to the estimated average number chosen by the other players, computed according to their beliefs. Following Camerer (2003), we assumed that the use of more and more reasoning steps would be increasingly rare due to working memory constraints and doubts about the rationality of others. This is captured by letting $f(k)/f(k-1)$ be proportional to $1/k$, which implies that $f(k) = e^{-\tau}\tau^k / k!$, the Poisson distribution, where τ is the mean and variance of the number of reasoning steps. Camerer found that τ lies between 1 and 2, which means that, in the one-shot game, players do not compute more than 2 steps of reasoning.

With our data, we estimated τ = 1.55 , for an average guess of 56.46 (for p = 2/3), and τ = 0.94 ($\tau_{2/3} > \tau_{1/4}$ as in our hypothesis) for an average guess of 78.04 (for p = ¼), which is consistent with Camerer's findings, and is in keeping with the eductive reasoning theory in two ways: first, it shows that players do calculate at least some of the steps of iterated dominance, and second, computing at most 2 steps of reasoning in the first period under negative feedback might be sufficient to make a guess that is very near to the REE. Once the eductive process is implemented, it is self-reinforcing, as we will show with the subsequent results.

Result 2: **Winning numbers exhibit oscillations around equilibrium as in the theoretical design, and numbers are highly concentrated around the REE.**

For the p = 2/3 case, starting from period 5, more than 82% of the numbers lie in a close interval, i.e. [58.7; 61.1]; for the p = ¼ case, the corresponding percentage is 85% of choices in the interval [78;81]. The smallest difference between the chosen number and the winning number was 0.001 in the last period.

Let us compare the above results to those obtained with a BCG+ with interior equilibrium for the case p = ⅔, which also predicted 60 as the equilibrium value. In this game the winner is the player whose choice is the closest to p(*mean*+c), where p is the convergence parameter and c is a positive constant. We ran an experiment with p = ⅔ and c = 30, at the Y laboratory in May 2004, with 32 student subjects split into 4 groups of 8 partners interacting for 10 rounds. Because we only used the results as a benchmark, we considered the aggregate result on convergence (see figure 6). While the process was convergent towards the REE, it always remained below the REE, even in the last period.

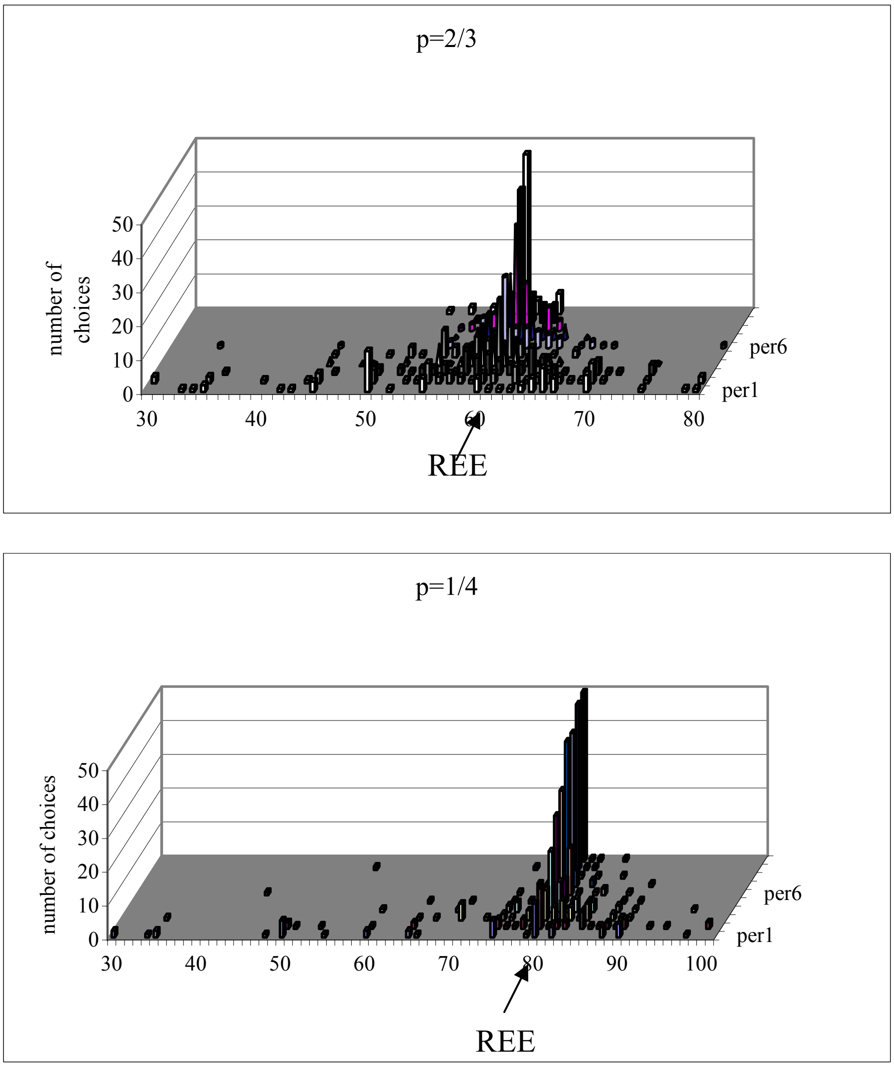

Figure 5. Choices for all players and all periods in the BCG-

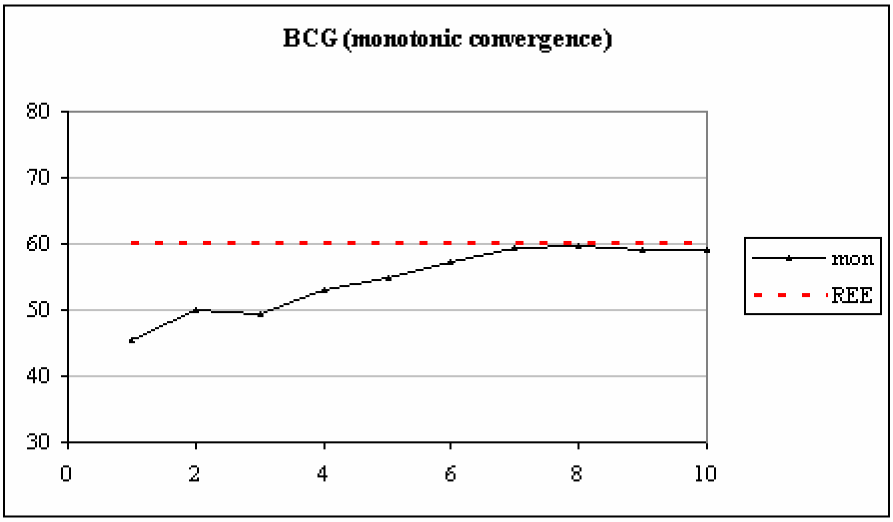

Figure 6. Convergence for 4 groups in the BCG+

in which the winner had to choose ⅔(mean+30)

Result 3: **With negative feedback, the "cognitive hierarchy" model predicts that a guess very close to the REE can be achieved as a winning number in the first period with only 3 steps of reasoning (for the chosen values of p)**

To establish result 3, we constructed a simulation scheme and estimated the proportion of each type of player. We assumed that players would behave as indicated in the cognitive hierarchy model, i.e. they would expect the others to perform fewer iteration steps. We simulated the game using up to 4 steps of iteration. The proportions of other players were also simulated according to the rule explained in the previous section. The following table describes the simulations of these proportions in the case when a 3-step player is able to announce a number in the interval [REE-15%;REE+15%] in the first period. For example, the number in bold print should be read as: proportion of 3-step thinkers according to the expectations of a 4-step thinker. We simulated environments with 1, 2, 3, and 4 types of players, corresponding respectively to 0, 1, 2 or 3 steps, a 3-step player being the smartest, in order to determine the value of τ which could lead to the REE±15% as the winning number in the first round. In environment *i* the "smartest" player implements *i* steps of reasoning, whereas the other players implement *i*-1, *i*-2, …,0 steps. We find that an observed population of 3-step players is enough to lead a 4-step player to announce the REE.

Type of the player→ proportion of opponents↓	*4*	*3*	*2*
3	**0,31**		
2	0,34	0,50	
1	0,25	0,37	0,73
0	0,10	0,13	0,27

Table 2: Average estimated proportion of players when only 3 steps of iteration among the "observed" population are enough to announce the REE for a player who best responds

Stating that the observed environment should be populated with players who hold beliefs of at most step 3, in order to make it possible for an observer to announce the REE in the first period, is realistic. As argued by Sperber (1997), humans have two kinds of beliefs, intuitive beliefs and reflective beliefs. From all of the results on the guessing game it seems that one cannot intuitively hold beliefs with k>3 (high order beliefs) when interaction with a situation is possible only through a game. The 3-step order is the natural order at which reflective beliefs become intuitive because it is the level of beliefs that people hold in order to communicate. The winning player who announces the REE should in this case implement only one additional step over the common intuitive level

V. Conclusion

In this paper we presented the beauty contest game with negative feedback and interior equilibrium in a multi-period experiment. The game is still being analysed from the eductive point of view and with respect to the attempt to establish a typology of players according to their depths of reasoning. Our main contribution to the understanding of this game was the formalization of the process by which the information is processed. Using the Shannon entropy criterion, we evaluated information and made a link between the Sperber analysis of reflective and intuitive beliefs and numerical psychological research (Dehaene, 1993). Information that players take into account in their choices is denoted *useful information*. As this depends on the exploitation of the strategy interval, it will be higher in BCG- than in BCG+ in the first iterations, because strategies are numbers that are naturally scanned several times. As argued by Sperber (1997), there is a point in the reasoning process starting from which reflective beliefs become intuitive. In order to determine the exact location of the *specific point* from which players in the BCG- can jump to the REE, we assumed that sophisticated reasoning is costly. Therefore, an agent stops calculating at step k which is obtained by the intersection between his marginal cost function and his marginal benefit (information) function, i.e. $C_m(k) = B_m(k)$, with usual notations. However, there are individuals who are not able to reach that point, because their cognitive constraint is saturated beforehand (they are able to compute only k-s steps, $s<k$). There are also individuals for whom the cognitive constraint is saturated for a value higher that k, but who stop at step k because, given the structure of the population, they can win the game at a smaller cost. Therefore, a guess in this game corresponds to the solution of the system comprising these two constraints. For our experiments, we found a depth of reasoning smaller than 3, which can, however, be optimal. Results show that the k-step thinking with $k<3$ is "a fact of human nature" (Bosch and al., 2000) and not an arbitrary modelling restriction. Even if subjects start with a low degree of sophistication, the final winning numbers are very close to the equilibrium in the BCG-. This is possible, as observed by Guesnerie (1992) on the crop producers market, because situations of negative feedback are stable; therefore, "human nature" is likely to better succeed when confronted with such situations: eductive reasoning is "helped" to stay on the convergence path.

Appendix

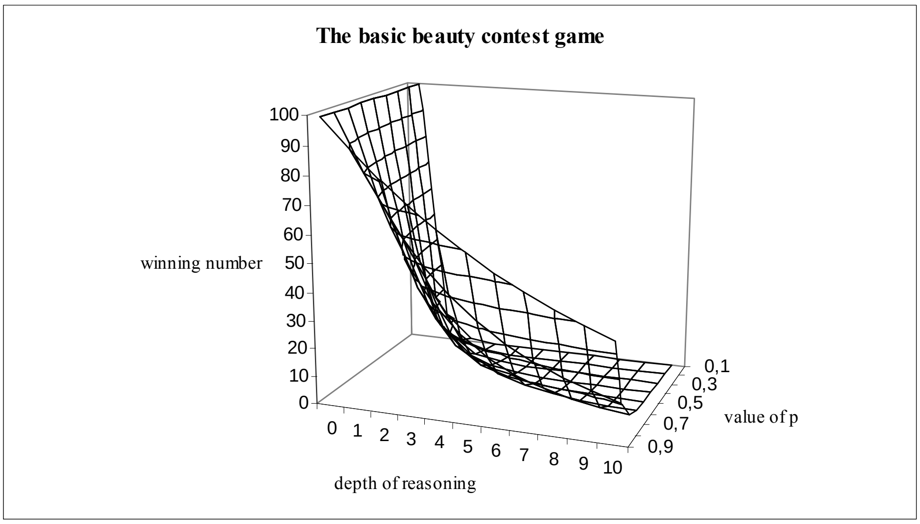

Figure 7: The basic beauty contest game

(winning number as a function of the depth of reasoning and the value of p)

References

ARTHUR, B. (1990), "Positive feedback in the Economy", *Scientific American*, 262, 92-99, February 1990.

BINMORE, K. (1988) "Modeling rational players : Part II", *Economics and Philosophy*,4, pp. 9-55.

BOSCH, A., J. GARCÍA-MONTALVO, R. NAGEL and A. SATORRA, (2000), "One, Two, (Three), Infnity,...: Newspapers and Lab Beauty Contest Games", *American Economic Review*, 92-5, 1687-1701, déc. 2002.

BRANAS, P, MORALES, A. (2002)"Confusing the unraveling : how to interpret experimental BCG results", *mimeo*.

CAMERER, C. F., Ho, T.H. and CHONG J.K. “A Cognitive Hierarchy Theory of One-Shot Games,” *Quarterly Journal of Economics*, August 2004, 119(3), 861-898.

CAMERER, C. (2003) "Behavioural game theory", Princeton Univ. Press.

"Behavioural studies of strategic thinking in games", in *Trends in cognitive sciences.*

DEHAENE, S. (1993) Temporal oscillations in human perception. *Psychological Science*, 1993, 4, 264-270.

DEHAENE, S., BOSSINNI, S., GIRAUX, P (1993) The mental representation of parity and numerical magnitude. *Journal of Experimental Psychology*: General, 1993, 122, 371-396.

DEHAENE, S., CHANGEUX, J.P.(1993) Development of elementary numerical abilities: A neuronal model. *Journal of Cognitive Neuroscience*, 1993, 5, 390-407.

DE GIORGI, E., REIMANN, S.(2003) "A Remark on the Guessing Game", mimeo.

DE GIORGI, E., REIMANN, S.(2004) "The α -Beauty Contest as a Non-Equilibrium Game", *Working Paper* 183, Institute for Empirical Research in Economics University of Zurich

DUFFY, J., NAGEL, R., (1997) "On the robustness of behaviour in experimental 'beauty contest' games", *Economic Journal* 107 (445), 1684–1700.

GUESNERIE, R. (1992) An exploration of the eductive justifications of the rational expectations hypothesis. *American Economic Review*, 82, 1254-1278

GUESNERIE, R. (2004) "When strategic substitutabilities dominate strategic complementarities : towards a standard theory for expectational coordination?", Working Paper, PSE, 2005-07.

GUTH, W., KOCHER, M., SUTTER, M. (2002) "Experimental beauty contests with homogeneous and heterogeneous players and with interior and boundary equilibria", *Economic Letters*, 74, pp. 219-228

HO, T.H., CAMERER C, WEIGELT K. (1998) "Iterated dominance and iterated best response in p-beauty contests", *American Economic Review*, 88-4, pp.947-967

KEYNES, J.M., (1936) "The General Theory of Interest, Employment and Money" Macmillan, London.

KOCHER, M., SUTTER, M., (2000) "When the 'decision maker' matters : Individual versus team behavior in experimental 'beauty-contest' games", *Discussion paper* 2000/ 4, Institute of Public Economics, University of Innsbruck.

LOPEZ, R. (2001) "On p-beauty contest integer games", mimeo, dec. 2001

MILLS, C.M., KEIL F.C. (2004) "Knowing the limits of one's understanding : the development of an awareness of an illusion of explanatory depth", *Journal of Experimental Child Psychology*, 87, 1-32.

NAGEL, R. (1995) "Unravelling in Guessing Games : an Experimental Study", *American Economic Review*, 85-5, pp.1313-1326

NAGEL, R., (1998) "A Survey of Experimental Beauty Contest Games : Bounded Rationality and Learning" , *Games and Human Behaviour, Essays inHonor of Amnon Rapaport*, eds. D. Budescu, I. Erev and R. Zwick. Publisher Lawrence Erlbaum Associates, Inc. New Jersey, p. 105-142.

REIMANN, S. (2004), "Towards understanding the guessing game : a dynamical systems' perspective", *Physica A*, *Volume 339, Issues 3-4, 15 August 2004, Pages 559-573*

SPERBER, D.(1997) "Intuitive and reflective beliefs", *Mind and Language*, 12 (1), pp.67-83

STAHL, D. (1996) "Boundedly rational rule learning in a guessing game", *Games and Economic Behaviour*, 16, 1996, pp. 303-330

STAHL, D.(1998) "Is step-*j* thinking an arbitrary modelling restriction ar a fact of huma nature?", *Journal of Economic Behaviour and Organisation*, 37, pp.33-51

TODD, P.M., GIGERENZEL, G. (2003) "Bounded rationality to the world. *Journal of Economic Psychology*, 24, 143-165.

TVERSKY A, KAHNEMAN D.(1974) Judgment under uncertainty: Heuristics and biases. *Science* 185.

WEBER, R. (2001), "Learning with no feedback in a competitive guessing game", *Games and Economic Behaviour,* 44, 1, 134-144.