Homogenisation: a mathematical technique used to predict the shrinkage of oak as a result of the annual ring morphology and the microscopic properties of tissues.

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As a result of its plant origin, wood is a very complex material, heterogeneous and highly variable. Homogenisation techniques are used in this work to propose a deterministic approach of oak properties. The latter requires to kind of information:

- the morphology that describes the spatial distribution of the components
- the intrinsic properties of each anatomical component.

From these pieces of information, the macroscopic properties are calculated by homogenisation. This paper is devoted to mechanical and shrinkage properties.
The microscopic properties have been measured with the help of specific experimental devices previously developed in our laboratory.

A new numerical X-ray imaging system is presented. It generates numerical images of the annual ring structure. After separation of different components by image processing, a Finite Element mesh is built. Finally, a specifically developed computational tool, *MorphoPore*, solves four homogenisation problems. This software calculates the macroscopic properties of the complete annual ring.

Two examples are proposed, which show how the morphology of the annual ring can be taken into account for the prediction of the mechanical and shrinkage properties.
The first example proves that the proposed approach is capable of quantifying the influence of morphological variations using virtual structures.
The second example deals with two successive actual annual rings: the first one is thin and the second one is large. The influence of morphological changes due to growth rate of the tree becomes obvious.
The most innovative aspect of this tool is that the influences of parameters due to genetic and growth conditions of the tree can be distinguished.

I INTRODUCTION

Oak is a high valued species. Unfortunately, it is also renowned for being very difficult to dry. Among several reasons that can explain this behaviour, shrinkage values in the transverse plane are commonly involved when cracks or warping occur.

As a result of the biologic activity of the tree, wood is a material with high varying properties, both within a tree and from one tree to another. This variability is controlled both by the genetic origin of the tree and by its growth conditions, from which results the anatomical structure of wood. The observation of this structure reveals that wood is a complex composite material. It is generally composed of layers of early wood and late wood made up of highly elongated cells in the longitudinal direction. In case of Oak, the different kinds of cells are grouped and form zones that can be distinguished according to their density, form or orientation in the transverse plane. The diversity of these components, their intrinsic properties and their spatial organisation are parameters that must be taken into consideration to investigate its shrinkage and mechanical properties [Kollman and Coté 1984, Gibson and Ashby 1988, Watanabe 1998]. However, it is very difficult to evaluate quantitatively the structural organisation of wood and microscopic properties of its components.

Different ways are possible to understand the properties of wood. In the present study, we opted for a deterministic approach. Several scales of observation are conceivable. For example, at the cell level, the arrangement of the cells is an important parameter for the understanding of the mechanical properties [Koponen 1991, Farruggia 1998]. In this paper we show how the spatial distribution of the different tissue zones of one annual ring of oak can be taken into account to determine its shrinkage and mechanical properties. The homogenisation techniques are very well appropriate for the study of heterogeneous materials. Figure 1 shows that this deterministic approach requires two types of data:

- the morphology, that is the definition of the different structural elements of the composite and their spatial distribution in the material,

- the microscopic properties : the modelling of shrinkage implicates the mechanical and the shrinkage properties of each constituent of the heterogeneous material to be known.

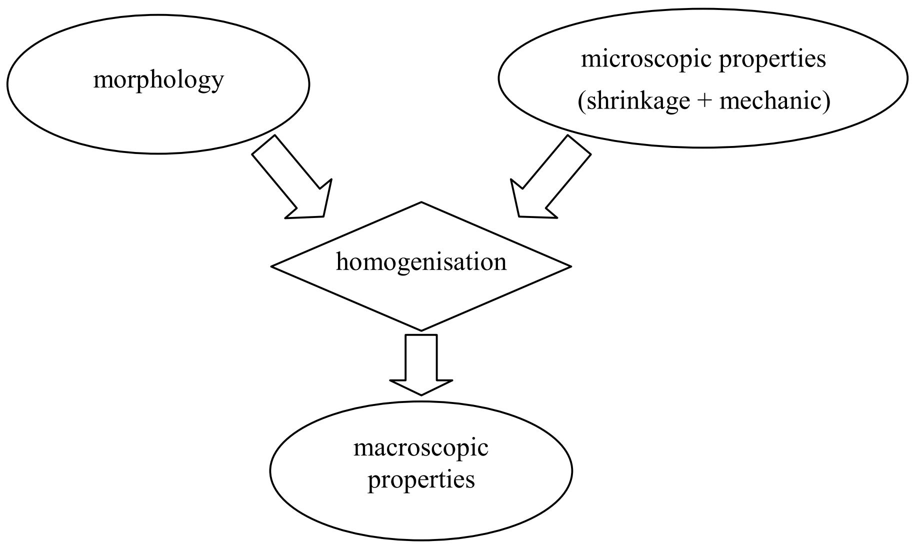


Fig 1 : Principle of the determinist approach. Assumed that the spatial distribution and the microscopic properties of the components are determined, the homogenisation techniques permit to calculate the macroscopic properties of the composite material.

This paper is divided in 3 parts :

- description of the material structure. A new numerical X-ray imaging device has been specially developed in order to describe the wood morphology at the annual ring level,

- construction of a numerical finite element mesh from the previous image,

- calculation of the macroscopic properties with the help of a specific numerical tool, *MorphoPore* [Perré 1998], which uses homogenisation techniques. Two examples are presented. At first, a virtual oak structure is used to modify and control morphology parameters. Then, the method is applied to real structures in order to predict the influence of growth conditions on the morphology changes.

II DESCRIPTION OF THE MATERIAL STRUCTURE

The determinist approach requires morphological information of the actual structure of the material. At the annual ring scale, wood is almost a bi-dimensionnal material. That explains why the principle of X-ray projection is well appropriate for its study [Polge 1966]. Because materials absorb X-ray according to their density, X-ray techniques are commonly used in wood sciences in order to get information on the internal structure. For our description objective, the main idea is that the different tissues of an annual ring absorb differently the X-ray beam.

We developed a new numerical X-ray imaging system based on these principles: attenuation and bidimensional projection. The device is mainly composed of:

- a microfocus X-ray source. Its specificity is the very small size of its spot (∅ ≈ 8 µm). This characteristic allows X-ray magnification without detrimental fuzziness. The voltage can vary from 10 kV to 80 kV and the source intensity from 0 to 100 µA,

- a scintillator. This element is a converter. It absorbs the energy of X-ray beam and re-emits it into visible light,

- a 2D detector. The detector is a coded CCD camera. The low temperature (- 35°C) allows long acquire time with a very low noise. This specific characteristic is required because of the very low photon flux of the microfocus X-ray source. The CCD is made of 1317× 1035 small pixels (6.8×6.8 μm^2 each). A computer drives the camera and records the image.

The physical principle is as follows: the sample absorbs a part of the incident X-ray beam. The scintillator converts the residual beam into visible light, which is deviated by a prism in order to protect the CCD camera from the X-photons. (Figure 2).

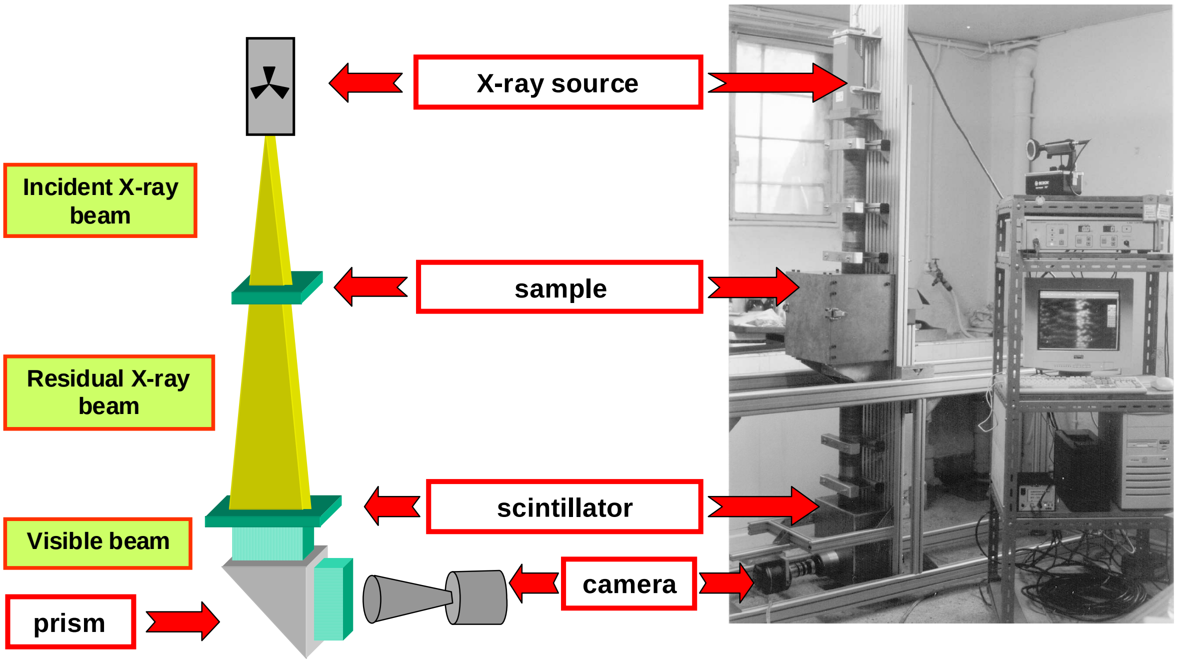


Fig 2 : General setup of the numerical X-ray imaging device. The energy of residual X-ray beam (after the sample) is converted into visible light that is detected by a cooled CCD camera.

The X-ray magnification is determined by the position of the sample support. Figure 2 shows the device with an adjustment for a magnification around ×3. In this configuration, the spatial resolution, determined according to relevant methods [Kaftandjian 1996], corresponds to 25µm.

The grey level recorded on the image (I_R) is proportional to the number of photons transmitted through the sample. A specific image processing permits to obtain the attenuation ratio I/I_0 (I_0: intensity of the incident beam; I: intensity of the residual beam).

At first, a complementary image is acquired without X-ray illumination. This allows the offset of the camera and the mean noise level to be evaluated and to corrected as the real. intensity I_R - I_B . In addition, another image permits the intensity of the incident X-ray beam to be measured. This final image,, calculated pixel by pixel, is free of spatial non-uniformity. This third image is called "Flatfield". Finally, the grey level of each pixel is calculated as follows:

$$G(i,j) = \frac{I_R(i,j) - I_B(i,j)}{I_F(i,j)}$$

i,j : co-ordinates of the pixel on the image
I : intensity of the pixel

G is in a theoretical range from 0 to 1 and can be directly interpreted as the physical ratio I/I_0, which can be calculated (for a polychromatic X-ray beam) as follows:

$$\frac{I}{I_0} = \frac{\int_0^\infty I_0(\lambda) e^{-\mu_m(\lambda)\rho x} d\lambda}{\int_0^\infty I_0(\lambda)\, d\lambda}$$

λ : wavelength
I : intensity
µ : attenuation coefficient
ρ : density

A specific study permitted us to define the best exposure parameters. The best resolution for the ratio Signal/Noise requires high voltage in order to increase the X-ray photon flux. With such conditions (70 kV), an accuracy of 1 % can be reach within 30 min. Figure 3 depicts an example performed with an X-ray magnification ×3. The four main components of an annual ring of oak become evident: the big vessels in the early wood, the ray cells and the late wood that is composed of fiber and parenchyma zones. The grey levels in the vessels are equal to 1 (100 % of the intensity of X-ray beam is transmitted). The darkest part corresponds to dense tissues as fiber (G<1).

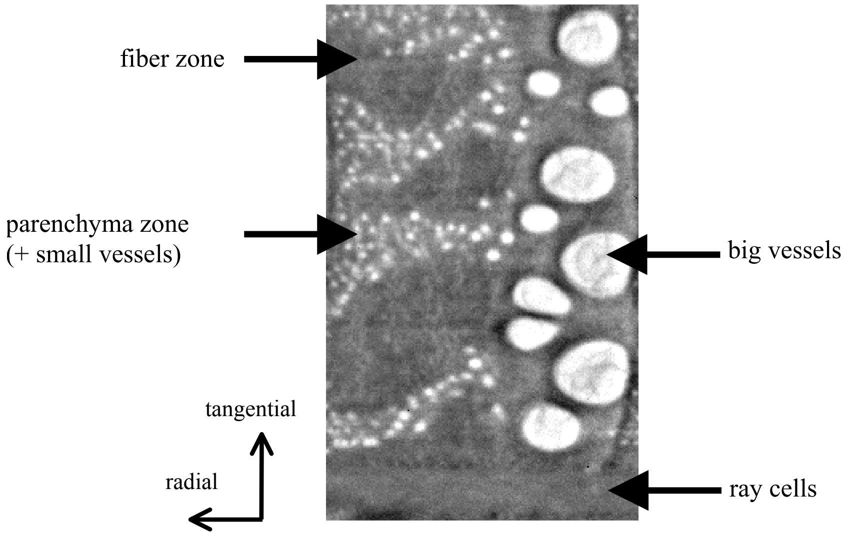


Fig 3 :X-ray image of a cross section of oak. The spatial resolution permits to distinguish details as small vessels in the late wood. The observation shows that the main components are: the big vessels (early wood), the ray cells, the fiber zone and the parenchyma zone.

III FROM THE X-RAY IMAGE TO THE FINITE ELEMENT MESH.

The X-ray image is the basis for the construction of a Finite Element mesh, which represents the actual structure of the material. According to their intrinsic X-ray attenuation properties, the different components of the annual ring are separated. This operation is performed with the help of an image analysis system (VISILOG). Then, the boundaries of the different zones are extracted, smoothed and vectorized. The size of the segments, which define the contours of the different zones, is chosen for each type of tissue. In this study, we distinguish the big vessels, the fiber zone, the parenchyma zone and the ray cells. Finally, specific software, Easymesh [Niceno 1996] generates the F.E. mesh using triangular elements, which permit to fit the boundaries of the complex forms. Figure 4 shows different steps from the image to the final F.E. mesh.

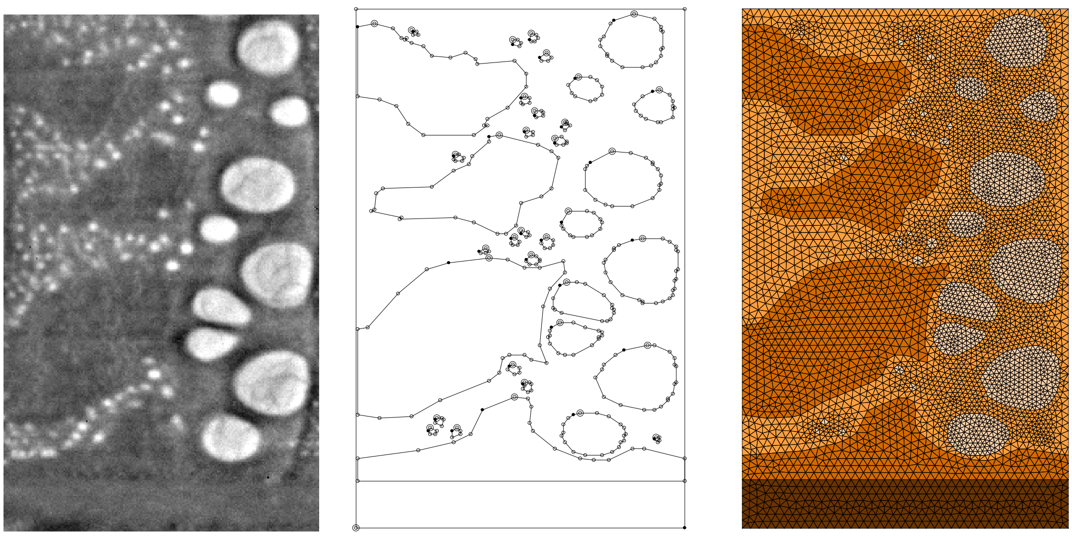


Fig 4 : From the X-ray image to the triangular Finite Element mesh. The different tissues are separated, their boundaries detected , vestorized and the numerical mesh is built.

IV CALCULATION AND EXAMPLES

Homogenisation techniques are well appropriate for the study of heterogeneous material [Suquet 1985]. Assumed that the described annual ring is a representative cell of the periodic material, specific software "MorphoPore" specially developed for this study [Perré 1998] calculates the macroscopic (or homogenised) properties. It solves four different problems (1for shrinkage and 3 for mechanical problems) according to the microscopic properties of the different components. Those come from specific experimental studies [Badel 1999, Badel and Perré 1999]. The main microscopic properties in the transverse plane are:

- rays cells : high rigidity and low shrinkage,

- fiber : high rigidity and high shrinkage (specially in tangential direction),

- parenchyma : medium rigidity (with high anisotropy : $E_R/E_T \sim 2$) and medium shrinkage (with high anisotropy : $\alpha_R/\alpha_T \approx 0.5$).

The first example uses an actual image of oak's structure that has been modified in order to control morphology parameters. The annual ring width has been virtually increased. The early wood (real structure) does not change whereas the late wood has been increased (virtual structure).

Three configurations have been tested:
- late wood without fiber zone
- late wood with rectangular fiber zone
- late wood with triangular fiber zone

The fibre proportion is exactly the same in the two last configurations. Figure 5 represents the evolution of macroscopic properties of these three cases as a function of the annual ring width.

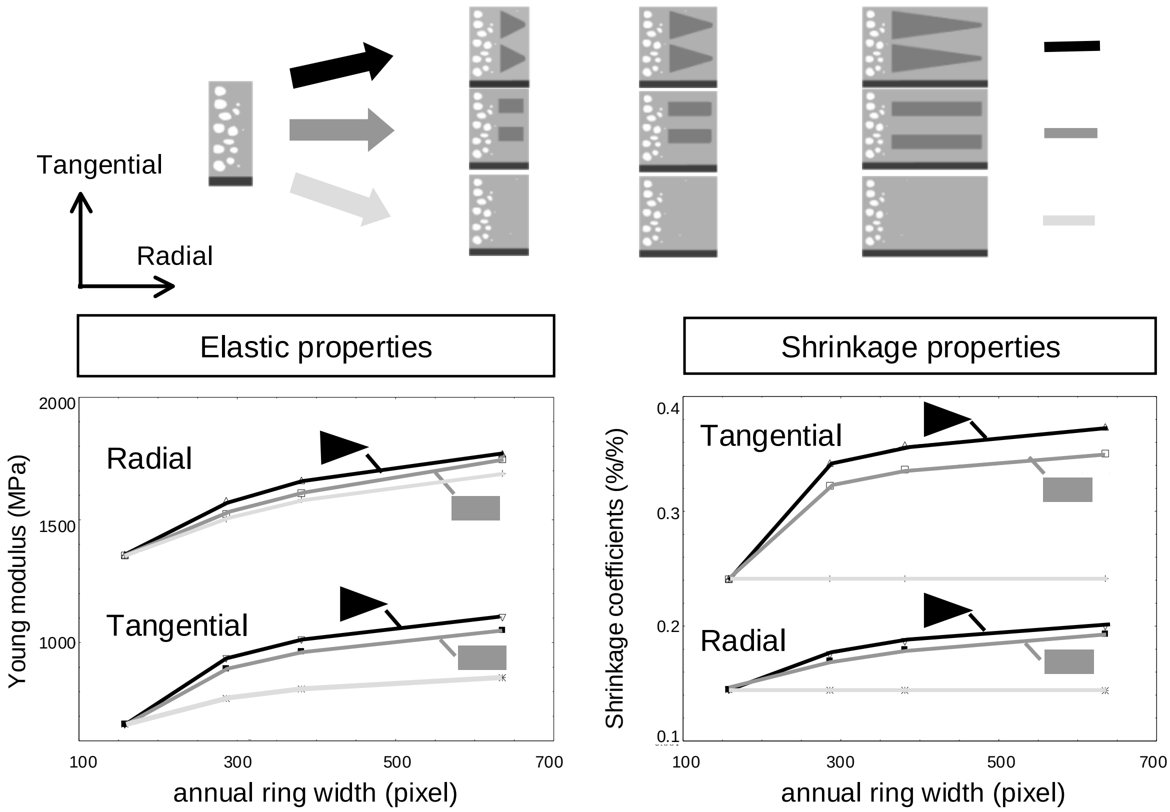


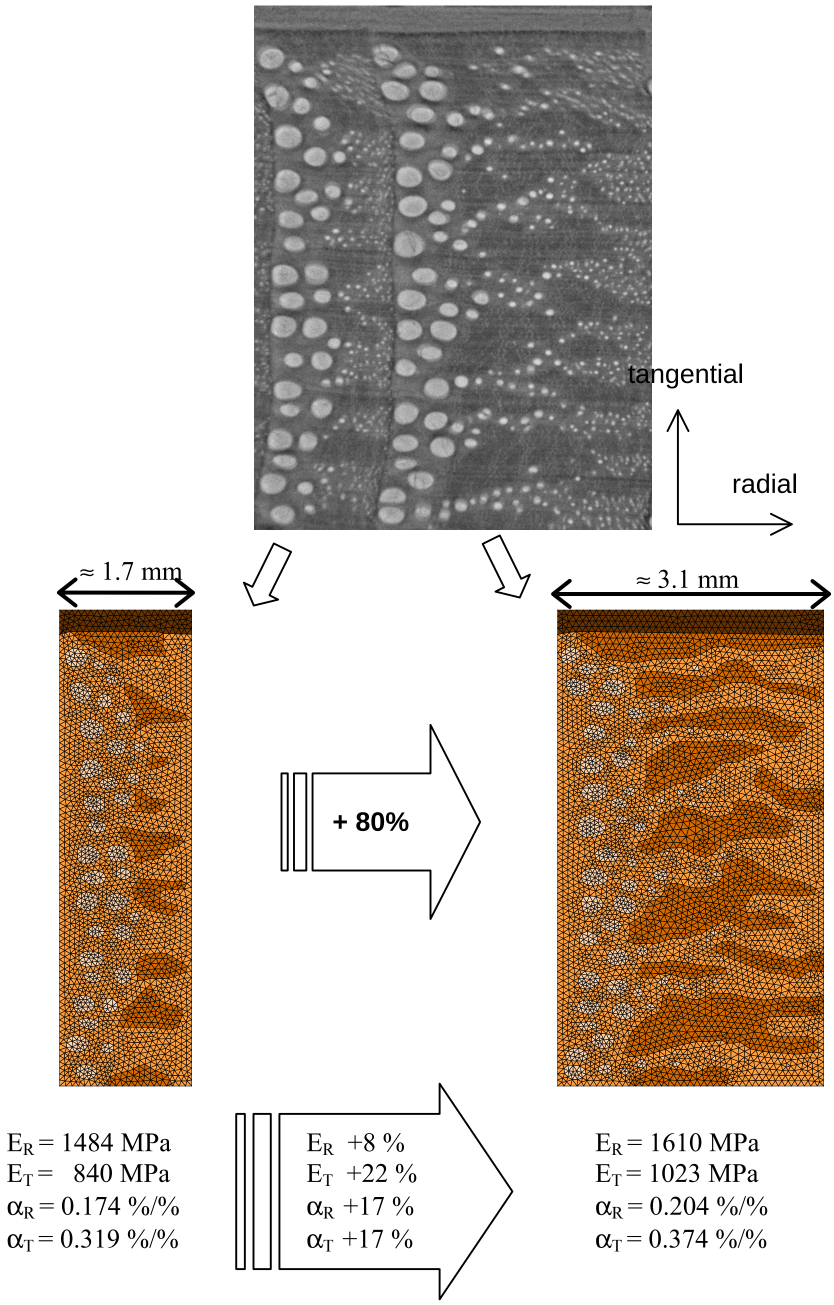
Fig 5 : Effect of annual ring width increase and fiber zones form on elastic and shrinkage properties of Oak in the radial-tangential plane (virtual annual rings)

First, we observe that all properties (shrinkage and Young moduli) increase with the annual ring width. However, the evolution is different according to the direction and/or the late wood structure. The increase is the most important in case of triangular fiber zone and very low in case of late wood without fiber. As the variations are not the same in the two directions, the anisotropy of the properties (well known to be an important factor for drying stresses) is modified. Note that the macroscopic shrinkage values do not vary if fiber is absent in late wood.

This example shows that it is possible to quantify the effect of morphological variations. In particular, we can now separate the influence of genetic parameters from other parameters due to the growth conditions of the tree.

The second example uses only actual morphology. It compares two successive annual rings (Fig. 6). The first has a very low width (1.7 mm) whereas the second has a very large width (3.1 mm). This configuration (very important increase of the annual ring width) is certainly due to a strong thinning resulting from a silvicultural action. As already observed in the previous example, all the properties increase. The shrinkage coefficient increase by the same amount in the two directions (+ 17 %) whereas the Young moduli increase differently inducing a modification of the anisotropy ratio.

This example depicts the possibility to study actual morphology and to quantify the effect of parameter due exclusively to growth conditions of the tree. In this case, the tool gives predictive answers to foresters who want to know the consequences of silvicultural actions.



V CONCLUSION

A comprehensive tools chain is now operational for the determination of the elastic and shrinkage properties in the transverse plan of oak according to its microscopic characteristics. Using microscopic properties and structural description obtained with the help of specific experimental devices developed in the laboratory, the numerical tool *MorphoPore* applied homogenisation techniques in order to calculate the macroscopic properties of the complex structure described by a Finite Element mesh.

Two applications are presented, which work with virtual or actual morphologies. They permit to underscore that effect of genetic and growth parameters can be separated and quantified. This tool can be used in order to understand the role of the different parameters and in the other hand as a predictive tool able to quantify the effects of a change of growth conditions.

Note that a new version of MorphoPore is now available. It computes the microscopic stress field due to the shrinkage. This new output possibility exhibits the zones of the annual ring where the stress is the most important according to the spatial distribution of the anatomical constituents.

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