Optimal rainfall insurance

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L'assurance contre un aléa climatique : l'exemple de la sécheresse

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Abstract

This article examines the design of optimal rainfall insurance contract where the indemnity is based on the rainfall level observed in an area. We define an actual rainfall insurance contract in which the producer selects a coverage level and a critical rainfall level under which indemnity payments are made. When the drought risk and the other aggregate production risks are independent or when the producer's utility function is quadratic, this insurance scheme is efficient and the optimal coverage level equals the marginal rainfall productivity. When these production risks are correlated, this actual contract yields an inefficient risk sharing. The optimal coverage level is lower or higher than the marginal rainfall productivity, depending on the stochastic dependence between production risks and the producer's prudent behavior.

1. Introduction

The failure of multiple peril crop insurance in which the indemnity is based on the producer's individual yield has promoted the emergence of alternative schemes based on variables exogenous to the individual farm. Such variables are not altered by the producer's behavior and they are perfectly observed by the insurance company. Therefore, the standard problems of moral hazard and adverse selection are substantially eliminated. In addition, administrative costs are reduced because claims are not adjusted individually.

Miranda (1991) proposed area yield crop insurance as a first alternative to individual yield crop insurance. This contract provides the purchasing farmer with an indemnity only when the average yield across all farms in a surrounding area falls below a critical yield.

Another alternative insurance contract consists in basing the indemnity schedule upon weather variables such as rainfall. It was proposed in the past (Sanderson 1943) but has failed to gain acceptance among policy makers. The difficulties of this innovative insurance program was the subject of a debate between Bardsley, Abey and Davenport (1984) and Quiggin (1986). Quiggin concluded by stating that ''there is a need for more research into the most desirable design for an insurance scheme''.

The purpose of this paper is twofold. First, given a stochastic production function affected by a drought risk and another aggregate risk, we derive the design of optimal rainfall insurance contract when the random production variables are independent and when they are correlated. Second, we propose an actual rainfall insurance contract which could be easily implemented. The producer would select a critical rainfall level under which indemnity payments are made and a coverage level. We verify whether such a contract is efficient and we derive the optimal coverage level.

This paper is organized as follows. The model is presented in the next section. The third section is devoted to the determination of the optimal form of rainfall insurance when the drought risk and the aggregate production risk are independent. The fourth section deals with the dependent case. Concluding remarks are given in the final section.

2. The model

A risk-averse farmer is endowed with initial wealth π_0 and a random yield $\tilde{y}$ due to uncertain effects of weather and more specifically uncertain rainfall. We assume that his stochastic production function is affected by two components : drought risk and aggregate production risk. The generalized Just and Pope production function is derived as follows:

$$(1) \qquad \tilde{y} = g(x)\tilde{\omega} + k(x)\tilde{\varepsilon} + h(x)$$

where x is input, $\tilde{\omega}$ a positive random variable which characterizes the rainfall level and $\tilde{\varepsilon}$ is zero-mean aggregate production risk. This function can be viewed as the first order development of a more general production function $f(x,\tilde{\omega},\tilde{\varepsilon})$ around $(E\tilde{\omega}, E\tilde{\varepsilon}=0)$. The functions g, h and k are assumed positive, increasing and concave. Such assumptions about the stochastic production function entails that the output is always increasing with the rainfall level. Therefore we do not take into account the negative effect of flood on the output level and we only study the consequence of drought. The joint cumulative distribution function of the couple of random variables $(\tilde{\omega},\tilde{\varepsilon})$ is denoted $T(\omega,\varepsilon)$ and it is defined on the support $[\omega_{\min},\omega_{\max}]\times[\varepsilon_{\min},\varepsilon_{\max}]$ with $0 \le \omega_{\min} \le \omega_{\max}$ and $\varepsilon_{\min} < 0 < \varepsilon_{\max}$. The marginal distribution function of rainfall variable $\tilde{\omega}$ is denoted Φ.

We only focus on the optimal design of rainfall insurance, so we consider that the level of input selected by the insured producer is fixed. He chooses x after having selected the rainfall insurance contract. This point will be discussed in concluding remarks.

The rainfall insurance contract is described by a couple $\left(I(.), P\right)$ where I is the indemnity schedule and P the premium. In other words, $I(\omega)$ is the indemnity payments received by the insured producer when the insurance company observes the rainfall level ω. A feasible coverage function satisfies:

$$(2) \qquad I(\omega) \geq 0 \text{ for all } \omega \in \left[\omega_{\min}, \omega_{\max}\right]$$

An upper bond is not introduced in (2) because there is no moral hazard problem. The risk-averse producer maximizes the expected utility of his final wealth π. His increasing and concave utility function is denoted u, with $u'(\pi) > 0$ and $u''(\pi) < 0$ for all π. He purchases the insurance contract $\left(I(.), P\right)$ if his expected utility level is greater with this policy than without it:

$$(3) \qquad EEu\left(\pi'_0 + g(x)\tilde{\omega} + I(\tilde{\omega}) - P + k(x)\tilde{\varepsilon}\right) \geq EEu\left(\pi'_0 + g(x)\tilde{\omega} + k(x)\tilde{\varepsilon}\right)$$

with $\pi'_0 = \pi_0 + h(x) - px$ where π_0 is the farmer's initial wealth, p is input price and output price is normalized at unity.

The insurance company maximizes the expected value of his utility function v which is increasing and concave, $v'(w) > 0$ and $v''(w) \leq 0$ for all w. The insurer faces administrative costs. This cost function $c(I)$ is assumed increasing and convex with indemnity payments I. They are divided into fixed and variable components:

$$(4) \qquad c(0) = c_0 \geq 0,\ c'(I) \geq 0 \text{ and } c''(I) \geq 0 \text{ for all } I \geq 0$$

If w_0 denotes his initial wealth, the insurer offers the insurance contract $\left(I(.), P\right)$ if and only if

$$(5) \qquad Ev\left[w_0 + P - I(\tilde{\omega}) - c\left(I(\tilde{\omega})\right)\right] \geq v(w_0)$$

Conditions (3) and (5) define the set of contracts acceptable by both parties. We will assume that this set is not empty, so the rainfall risk is insurable. This hypothesis will be discussed when concluding.

The Pareto optimal rainfall insurance design is the couple $(I(.),P)$ that maximizes the insured producer's expected utility of final wealth under the above-mentioned constraints :

$$(6)\quad \begin{cases} \underset{P,I(.)}{Max} \int_{\omega_{\min}}^{\omega_{\max}} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u[\pi_0' + g(x)\omega + I(\omega) - P + k(x)\varepsilon] dT(\omega,\varepsilon) \\ with \\ I(\omega) \geq 0 \quad \forall \omega \in [\omega_{\min}, \omega_{\max}] \\ \int_{\omega_{\min}}^{\omega_{\max}} v[w_0 + P - I(\omega) - c(I(\omega))] d\Phi(\omega) \geq v(w_0) \end{cases}$$

We define the following actual rainfall insurance contract which can be easily implemented:

$$(7)\qquad I^R(\omega) = \phi \max[\hat{\omega} - \omega, 0]$$

Under this contract, the producer is free to elect a coverage level ϕ and a critical rainfall level which would trigger indemnity payments. It will be

In the next two sections, our main objectives are (i) to verify whether such an actual rainfall contract is optimal by comparing it with the optimal rainfall insurance contract which is solution to this maximization program (6) and (ii) to determine the optimal coverage level, given the relationship between the drought risk and the aggregate production risk.

3. Independence between drought risk and aggregate production risk

The drought risk and the aggregate production risk are assumed independent. This means that the variability of the aggregate production risk is not affected by the rainfall level. The objective function in (6) becomes:

$$(8)\qquad \int_{\omega_{\min}}^{\omega_{\max}} \hat{u}(\pi_0' + g(x)\omega + I(\omega) - P) d\Phi(\omega)$$

where $\hat{u}$ is the indirect utility function defined by:

$$(9) \qquad \hat{u}(\pi) = Eu\left[\pi + k(x)\tilde{\varepsilon}\right] = \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u\left[\pi + k(x)\varepsilon\right] dZ(\varepsilon) \quad \text{for all } \pi$$

and Z is the marginal cumulative distribution function of the aggregate risk $\tilde{\varepsilon}$. The random variable $k(x)\tilde{\varepsilon}$ can be interpreted as an independent and uninsurable background risk. The indirect utility function inherits the properties of the original utility function. It is increasing and concave (Kihlstrom et al. 1981).

The maximization problem (6) with the specific objective function (8) is a calculus variation problem which can be solved in two steps. Given an input level, the insurance premium is first assumed fixed and the optimal insurance design is defined. Then the optimal insurance premium is chosen. Following Raviv (1979) and Kamien and Schwartz (1981), we obtain the first proposition.

Proposition 1 : Assume that the drought risk and the aggregate production risk are independent. There exists a critical rainfall level $\hat{\omega} \in [\omega_{\min}, \omega_{\max}]$ such that the optimal rainfall insurance indemnity, when the premium and the level of input use are fixed, takes the form:

$$(10) \qquad \begin{cases} I^*(\omega) = 0 & \text{if} \quad \omega \geq \hat{\omega} \\ I^*(\omega) > 0 & \text{if} \quad \omega < \hat{\omega} \end{cases}$$

When $I^*(y) > 0$, the optimal indemnity function satisfies:

$$(11) \qquad I^{*\prime}(\omega) = -g(x) \frac{A_{\hat{u}}(\pi)}{A_{\hat{u}}(\pi) + \frac{c''}{1+c'} + (1+c')A_v(w)}$$

where $\pi = \pi_0' + g(x)\omega + I(\omega) - P$, $w = w_0 + P - I(\omega) - c(I(\omega))$, and A_u and A_v are respectively the index of absolute risk aversion associated with the insured producer's indirect utility function and the insurer's utility function.

Proposition 1 is proved in an appendix. The design of rainfall insurance described by Quiggin (1994) turns out to be optimal : an indemnity is paid whenever the rainfall level falls below a critical level $\hat{\omega}$. The optimal form of the indemnity function depends not only on the degree of risk aversion of both agents and the cost function, but also on the marginal rainfall productivity, $g(x)$. The lower the realized level of rainfall is, the greater indemnity payments are.

It is widely admitted in the literature that the insurance premium is proportional to the actuarially fair premium. This is the case when the insurance company is risk-neutral and the administrative cost function is linear. With the initial condition $I^*(\hat{\omega}) = 0$, the optimal indemnity function becomes:

$$(12) \qquad I^*(\omega) = g(x)\max[\hat{\omega} - \omega, 0]$$

We deduce from the comparison of (7) and (12) the following corollary.

Corollary 1: Assume that the administrative cost function of the risk-neutral insurer is linear. When the drought risk and the aggregate production risk are independent, the drought risk sharing generated by the actual rainfall insurance contract I^R is optimal and the producer selects a coverage level which is equal to the marginal rainfall productivity, $\phi^* = g(x)$.

Unlike the coverage level, the critical rainfall level $\hat{\omega}$ is affected by the degree of risk aversion of the insured producer and the value of the insurance premium. It equals the maximum rainfall level ω_{max} if and only if the marginal cost function equals zero, $c'(I) = 0$ for all I (Raviv 1979).

4. Correlation between drought risk and aggregate production risk

We assume now that the variability of the aggregate production risk depends on the realized rainfall level. In other words, $\tilde{\varepsilon}$ is correlated with $\tilde{\omega}$. In a first case, we assume that there exists a negative relationship between the variability of the aggregate production risk and the rainfall level and, in a second one, we assume that this relationship is positive.

4.1 Negative relationship between the variability of the aggregate risk and the rainfall level

We assume that the variability of the aggregate production risk increases as the rainfall level decreases. As an example, a low rainfall level increases the risk of pests infestation. This means that a decrease in rainfall level ω entails a riskier conditional distribution of the aggregate production risk $\tilde{\varepsilon}$, its conditional expectation remaining unchanged. Following Gollier (1996), it is expressed by:

$$(13) \qquad \int_{\varepsilon_{min}}^{\varepsilon} Z_{\omega}(s|\tilde{\omega}=\omega)ds \le 0 \text{ for all } \varepsilon \text{ and } \int_{\varepsilon_{min}}^{\varepsilon_{max}} Z_{\omega}(s|\tilde{\omega}=\omega)ds = 0$$

where $Z(.|\tilde{\omega}=\omega)$ is the cumulative distribution function of $\tilde{\varepsilon}$ conditional to $\tilde{\omega}=\omega$. The objective function of problem (6) is rewritten:

$$(14) \qquad \int_{\omega_{min}}^{\omega_{max}} \int_{\varepsilon_{min}}^{\varepsilon_{max}} u[\pi_0' + g(x)\omega + I(\omega) - P + k(x)\varepsilon] dZ(\varepsilon|\tilde{\omega}=\omega)d\Phi(\omega)$$

Proposition 2 : Assume that a decrease of the rainfall level entails the aggregate production risk to be riskier according to (13). If the farmer exhibits prudence, $u''' > 0$, there exists a critical rainfall level $\hat{\omega} \in [\omega_{min}, \omega_{max}]$ such that the optimal rainfall insurance indemnity, when the premium and the level of input use are fixed, takes the form:

$$(15) \qquad \begin{cases} I^*(\omega) = 0 & if \quad \omega \ge \hat{\omega} \\ I^*(\omega) > 0 & if \quad \omega < \hat{\omega} \end{cases}$$

When $I^*(y) > 0$, the optimal indemnity function satisfies:

$$(16) \qquad I^{*\prime}(\omega) = g(x)\frac{\int_{\varepsilon_{min}}^{\varepsilon_{max}} u''(\pi)dZ(\varepsilon|\tilde{\omega}=\omega)}{D} + k^2(x)\frac{\int_{\varepsilon_{min}}^{\varepsilon_{max}} \left\{ u'''(\pi) \int_{\varepsilon_{min}}^{\varepsilon} Z_{\omega}(s|\tilde{\omega}=\omega)ds \right\} d\varepsilon}{D}$$

with $D \equiv -\int_{\varepsilon_{min}}^{\varepsilon_{max}} u''(\pi)dZ(\varepsilon|\tilde{\omega}=\omega) + \int_{\varepsilon_{min}}^{\varepsilon_{max}} u'(\pi)dZ(\varepsilon|\tilde{\omega}=\omega)\left[\frac{c''}{1+c'} + (1+c')A_v(w)\right] > 0,$

$\pi = \pi_0' + g(x)\omega + k(x)\varepsilon + I(\omega) - P$, $w = w_0 + P - I(\omega) - c(I(\omega))$, A_v is the index of absolute risk aversion of the insurance company.

Proposition 2 is proved in an appendix. When $u''' > 0$, the optimal rainfall insurance contract is such that indemnity payments are made whenever the realized rainfall level falls below a critical level. The convexity of the marginal utility is a well-known condition since Leland (1968). It is a necessary and sufficient condition for an increase in future risk to increase (precautionary) saving. Kimball (1990) defined the term "prudent" to characterize agents who behave in this way.

Consider the marginal indemnity function. The denominator is positive since the producer is risk-averse and since the administrative cost function is increasing and concave. The first term in (16) is thus negative. It reflects the impact of the producer's risk-aversion on the optimal rainfall insurance contract. When a lower rainfall level entails a higher variability of the aggregate production risk expressed by (13), the second term is negative if the producer is prudent. Therefore, this second term reflects the influence of the producer's prudent behavior on the slope of the optimal indemnity function. The optimal indemnity function is thus decreasing with the realized rainfall level.

When the insurer is assumed risk-neutral and the cost function is assumed linear, the optimal indemnity function verifies:

$$(17) \qquad I^{*\prime}(\omega) = -g(x) - k^2(x)\frac{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \left\{ u'''(\pi) \int_{\varepsilon_{\min}}^{\varepsilon} Z_{\omega}(s|\tilde{\omega} = \omega) ds \right\} d\varepsilon}{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi) dZ(\varepsilon|\tilde{\omega} = \omega)}$$

The comparison of expressions (7) and (17) proves that the drought risk sharing induced by the actual rainfall insurance contract I^R is inefficient. We can note that $I^{*\prime}(\omega) < -g(x)$ for all ω if the producer is prudent, so we deduce the following corollary.

Corollary 2 : Assume that the administrative cost function of the risk-neutral insurer is linear and that there exists a correlation between the production risks expressed by (13). If the producer is prudent, the actual rainfall insurance contract I^R is not optimal and, under this contract, the producer selects a coverage level higher than the marginal rainfall productivity, $\phi^* > g(x)$.

It is worth noting that, if the producer's utility function is quadratic, which entails that $u''' = 0$, the rainfall insurance contract I^R is optimal and the producer chooses a coverage level which is equals to $g(x)$, as in the case where the production risks were independent.

4.2 Positive relationship between the variability of the aggregate risk and the rainfall level

We assume now that the variability of the aggregate production risk is increasing with the realized rainfall level. Formally, an increase in rainfall level ω entails a riskier conditional distribution of $\tilde{\varepsilon}$ expressed by :

$$(18) \qquad \int_{\varepsilon_{\min}}^{\varepsilon} Z_{\omega}(s|\tilde{\omega}=\omega)ds \geq 0 \text{ for all } \varepsilon \text{ and } \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} Z_{\omega}(s|\tilde{\omega}=\omega)ds = 0$$

One can verify that, without another assumption, the indemnity schedule can be increasing in some intervals and it can intersect the horizontal line more than once, even if the producer is prudent. This ambiguity is solved when the stochastic production function is restricted as follows:

$$(19) \qquad \tilde{y} = \left(g(x) + k(x)\tilde{\eta}\right)\tilde{\omega} + h(x)$$

where $\tilde{\eta}$ is a zero-mean random variable, $E\tilde{\eta} = 0$, with cumulative distribution function M defined on $\left[\eta_{\min}, \eta_{\max}\right]$, $\eta_{\min} < 0 < \eta_{\max}$. The random rainfall level $\tilde{\omega}$ and the random aggregate risk $\tilde{\eta}$ are assumed independent. The assumptions of the deterministic production functions are maintained and we also assume that:

$$(20) \qquad g(x) + k(x)\eta_{\min} \geq 0$$

This means that output is increasing with the rainfall level for all η. The previous aggregate risk is written $\tilde{\varepsilon} = \tilde{\eta}\tilde{\omega}$. It is straightforward to verify that the cumulative distribution function of $\tilde{\varepsilon} = \tilde{\eta}\tilde{\omega}$ conditional to $\tilde{\omega} = \omega$ becomes more riskier as ω increases according to (18). We thus obtain the following proposition.

Proposition 3 : Given the stochastic production function expressed by (19), there exists a critical rainfall level $\hat{\omega} \in [\omega_{\min}, \omega_{\max}]$ such that the optimal rainfall insurance indemnity, when the premium and the level of input use are fixed, takes the form:

$$(21) \qquad \begin{cases} I^*(\omega) = 0 & \text{if} \quad \omega \geq \hat{\omega} \\ I^*(\omega) > 0 & \text{if} \quad \omega < \hat{\omega} \end{cases}$$

When $I^*(y) > 0$, the optimal indemnity function satisfies

$$(22) \qquad I^{*\prime}(\omega) = \frac{Eu''(\tilde{\pi})}{D} g(x) + k^2(x) \frac{\text{cov}\left[\left(\tilde{\eta}, u''(\tilde{\pi})\right)\right]}{D}$$

with $D \equiv -Eu''(\tilde{\pi}) + Eu'(\tilde{\pi})\left[\frac{c''}{1+c'} + (1+c')A_v(w)\right] > 0$,

$\tilde{\pi} = \pi_0' + [g(x) + k(x)\tilde{\eta}]\omega + I(\omega) - P$, $w = w_0 + P - I(\omega) - c(I(\omega))$, A_v is the index of absolute risk aversion of the insurance company.

Proposition 3 is proved in an appendix. Under the assumptions on the form of the stochastic production function, the producer receives a payout whenever rainfall falls below a critical level. Unlike proposition 2, the existence of a critical rainfall level does not depend on the producer's prudent behavior. However prudence affects the form of the optimal indemnity function. Consider the marginal indemnity function expressed by (22). The covariance $\text{cov}\left[\left(\tilde{\eta}, u''(\tilde{\pi})\right)\right]$ is positive if the producer is prudent, $u''' > 0$, and it is negative if the producer is imprudent, $u''' < 0$. We can also notice that this covariance equals zero if the producer's utility function is quadratic. Therefore, the optimal indemnity function decreases as the realized rainfall level increases when the producer is imprudent. It can be shown (see the appendix) that it is also decreasing when the producer is prudent, even if both terms in (22) have opposite signs.

When the insurer is risk-neutral and the administrative cost function is linear, the comparison of expressions (7) and (22) proves that the drought risk sharing induced by the rainfall insurance contract I^R is inefficient if the producer is prudent or imprudent. This contract is optimal only if the producer's utility function is quadratic. We have also $I^{*}'(\omega) > -g(x)$ for all ω if the producer is prudent, and $I^{*}'(\omega) < -g(x)$ for all ω if the producer is imprudent. From this, we deduce the following corollary.

Corollary 3 : Assume that the administrative cost function of the risk-neutral insurer is linear, and the production function is expressed by (19).

(i) If the producer is prudent, the actual rainfall insurance contract I^R is inefficient and, under this contract, the producer selects a coverage level lower than the marginal rainfall productivity, $\phi^* < g(x)$.

(ii) If the producer is imprudent, the actual rainfall insurance contract I^R is inefficient and, under this contract, he selects a coverage level higher than the marginal rainfall productivity, $\phi^* > g(x)$.

(iii) If the producer's utility function is quadratic, the actual rainfall insurance contract I^R is efficient and, under this contract, he selects a coverage level which is equal to the marginal rainfall productivity, $\phi^* = g(x)$.

The gains due to the increase of the rainfall level offsets the losses caused by the increasing variability of the aggregate production risk when these losses are not to large, as it is assumed in (19). This entails that the prudent producer chooses a coverage level less than the marginal rainfall productivity and the imprudent producer chooses it higher than this one.

5. Concluding remarks

Multiple peril crop insurance has several features which tend to make the problem of insurability particularly acute. Among the alternative insurance scheme which depend on variable exogenous to the individual farm, rainfall insurance is based on the idea that insured producers should receive a payout whenever realized rainfall falls below a critical level. Since the producers can not alter the drought risk, the problem of moral hazard would be eliminated. Similarly, because the information regarding the weather are available, adverse selection problems are reduced. Administrative would also be substantially reduced because claims would not have to be adjusted individually.

This paper has been devoted to the design of optimal rainfall insurance contract. It depends on technological parameters, like the dependent relationship between the drought risk and the aggregate production risk or their marginal productivity, and on producer's behavioral parameters like risk aversion and prudence. The design of the optimal rainfall insurance allows to check whether an actual rainfall insurance, in which the producer would select a rainfall guarantee and a coverage level, yields an efficient risk sharing and define the optimal coverage level under such a contract.

When the production risks are independent, the actual rainfall insurance contract is efficient and the optimal coverage level is equal to the marginal rainfall productivity. This result holds when the farmer's utility function is quadratic, whatever the relationship between the production risks. When the variability of the aggregate production risk increases as the rainfall level decreases, the actual contract is inefficient and the optimal coverage level under this contract is higher than the marginal rainfall productivity if the producer exhibits a prudent behavior. When the variability of the aggregate production risk decreases as the rainfall level decreases, we have had to assume that the production is increasing with the rainfall level, whatever the value of the aggregate production risk. Therefore, the prudent producer selects a coverage level less than the marginal rainfall productivity and the imprudent producer chooses a coverage level higher than this one.

We have assumed that the level of input use was fixed. Consequently, the shape of the indemnity function and the critical rainfall level depend on this decision variable. These results could be therefore instrumental for a further research about supply response to rainfall insurance. This effect was studied when crop insurance is based on individual yield (Ramaswami, 1993). Unlike this traditional insurance contract, moral hazard problem does not arise from the introduction of the rainfall insurance contract and, therefore, the level of input use should be only affected by the risk reduction effect.

The relationship between this rainfall insurance contract and preventive investments in irrigation programs should also be explored. It may allow to save water which is becoming a rare natural resource. Therefore this named-peril crop insurance scheme may be an useful tool for environmental policy.

Our results are based upon the assumption that the drought risk is insurable. This means that there exists a rainfall insurance contract which induces a mutually advantageous risk transfer for the producer and the insurance company. Natural disasters such as drought affect simultaneously a large number of farmers. The high correlation among risks prevent the insurer from using the mutuality principle. Shareholders of the insurance company are not able to diversify this drought risk and therefore they ask for a risk premium which will increase the cost of capital of the insurer. This cost is passed on to the policyholders through a larger premium rate. If the reinsurance chain is not able to spread efficiently this risk because of high transaction costs, it may be transferred through a securitization procedure to the capital markets which offer enormous risk bearing potential. Like area-yield options contracts proposed by the Chicago Board of Trade (Miranda and Glauber 1997), rainfall options contracts based upon an aggregate index of rainfall could be launched. They would be designed to offer protection against yield shortfalls caused by drought. The actual rainfall insurance contract analyzed in this paper can be viewed as a put option: it provides a hedge for low rainfall level and thus low individual yield. It should draw the attention of portfolio managers because it is not correlated with traditional assets, giving them the opportunity to diversify their risks and to increase their returns. This innovative asset would look like catastrophe insurance options based on the aggregate amount of insured losses resulting from catastrophic events. Methodology for pricing drought risk should be developed. Unfortunately, the Black-Scholes option pricing formula does not apply to pricing drought risk. Further research must include an analysis of the willingness of farmers to use such an hedging tool, in order to acquire a deeper understanding of the potential of alternative schemes for named-peril crop insurance.

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Appendix

Proof of proposition 1

Constraint (5) is binding at the optimum. The Hamiltonian of problem (6) with the objective function (8) is given by:

(A1) $$H = \left\{\hat{u}\left(\pi_0' + g(x)\omega + I(\omega) - P\right) + \zeta v\left[w_0 + P - I(\omega) - c(I(\omega))\right]\right\}\varphi(\omega)$$

where φ is the density function of drought risk, $I(\omega)$ is the control variable and ζ is the multiplier function. This latter one is invariant with respect to ω because the state variable doesn't appear in the Hamiltonian. The first order necessary conditions of (A1) are :

(A2) $$\begin{cases} (i)\ I^*(\omega) = 0 \Leftrightarrow J(\omega) \equiv \hat{u}'\left(\pi_0' + g(x)\omega - P\right) - \zeta\left[1 + c'(0)\right]v'\left(w_0 + P - c_0\right) \leq 0 \\ (ii)\ I^*(\omega) > 0 \Leftrightarrow \hat{u}'(\pi) - \zeta\left[1 + c'\left(I^*(\omega)\right)\right]v'(w) = 0 \end{cases}$$

with $\pi = \pi_0' + g(x)\omega + I(\omega) - P$, $w = w_0 + P - I(\omega) - c(I(\omega))$.

We have:

(A3) $$J'(\omega) = \frac{dJ(\omega)}{d\omega} = g(x)\hat{u}''\left(\pi_0' + g(x)\omega - P\right)$$

Since the producer is risk-averse and the deterministic function g is positive, the function J is decreasing with ω. Consequently there exists a unique $\hat{\omega} \in [\omega_{\min}, \omega_{\max}]$ such that $J(\hat{\omega}) = 0$ for all $\omega \in [\omega_{\min}, \omega_{\max}]$. From this we deduce the optimal rainfall insurance contract (10). When $I^*(y) > 0$, the differentiation with respect to ω of (A2.ii) yields expression (11). Proposition 1 is proved.

Proof a proposition 2

The demonstration is analogous to the proof of proposition 1. The Hamiltonian of the maximization problem (6) with the objective function (14) becomes:

(A4)

$$\left\{\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u\left[\pi_0' + g(x)\omega + I(\omega) - P + k(x)\varepsilon\right]dZ\left(\varepsilon \middle| \tilde{\omega} = \omega\right) + \zeta v\left[w_0 + P - I(\omega) - c(I(\omega))\right]\right\}\varphi(\omega)$$

The first order necessary conditions are :

(A5)

(i) $I^{*}(\omega)=0 \Leftrightarrow$

$$J(\omega)=\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'\left(\pi_0'+g(x)\omega-P+k(x)\varepsilon\right) dZ\left(\varepsilon|\widetilde{\omega}=\omega\right)-\zeta\left(1+c'(0)\right)v'\left(w_0+P-c_0\right)\leq 0$$

(ii) $I^{*}(\omega)>0 \Leftrightarrow \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi) dZ\left(\varepsilon|\widetilde{\omega}=\omega\right)-\zeta\left(1+c'\left(I^{*}(\omega)\right)\right)v'(w)=0$

The first derivative of J with respect to ω is:

(A6) $$J'(\omega)=g(x)\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi) dZ\left(\varepsilon|\widetilde{\omega}=\omega\right)+\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'(\pi) dZ_{\omega}\left(\varepsilon|\widetilde{\omega}=\omega\right)$$

which can be rewritten after integrating it by parts twice:

(A7)

$$J'(\omega)=g(x)\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u''(\pi) dZ\left(\varepsilon|\widetilde{\omega}=\omega\right)+k^{2}(x)\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} u'''(\pi)\left\{\int_{\varepsilon_{\min}}^{\varepsilon} Z_{\omega}\left(\varepsilon|\widetilde{\omega}=\omega\right) ds\right\} d\varepsilon$$

with $Z_{\omega}\left(\varepsilon_{\min}|\widetilde{\omega}=\omega\right)=Z_{\omega}\left(\varepsilon_{\max}|\widetilde{\omega}=\omega\right)=0$. This marginal function is negative if the third derivative of the producer's utility function is positive. Therefore there exists a critical rainfall level $\hat{\omega}$ such that $J(\hat{\omega})=0$ for all ω. This leads to the optimal rainfall insurance contract (15). When $I^{*}(y)>0$, the differentiation of the first order condition yield expression (16). The denominator D is positive because the producer and the insurer are risk-averse and because the administrative cost function c is increasing and convex. Proposition 2 is demonstrated.

Proof of proposition 3

The Hamiltonian of the maximization problem (6) becomes:

(A8)

$$H=\left\{Eu\left(\pi_0'+\left(g(x)+k(x)\widetilde{\eta}\right)\omega+I(\omega)-P\right)+\zeta v\left[w_0+P-I(\omega)-c\left(I(\omega)\right)\right]\right\}\varphi(\omega)$$

The first order necessary conditions of (A8) are:

(A9)

(i) $I^*(\omega)=0 \Leftrightarrow J(\omega)=Eu'\big(\pi_0'+\big(g(x)+k(x)\tilde{\eta}\big)\omega-P\big)-\zeta\big(1+c'(0)\big)v'\big(w_0+P-c_0\big)\leq 0$

(ii) $I^*(\omega)>0 \Leftrightarrow Eu'(\tilde{\pi})-\zeta\big(1+c'\big(I^*(\omega)\big)\big)v'(w)=0$

with $\tilde{\pi}=\pi_0'+\left[g(x)+k(x)\tilde{\eta}\right]\omega+I(\omega)-P$, $w=w_0+P-I(\omega)-c\big(I(\omega)\big)$.

We obtain:

(A10) $\quad J'(\omega)=E\left\{\left[g(x)+k(x)\tilde{\eta}\right]u''\left[\pi_0'+g(x)\omega+k(x)\tilde{\eta}-P\right]\right\}$

which is negative because, given the assumptions (19) and (20), we have $g(x)+k(x)\eta\geq 0$ for all $\eta\in\left[\eta_{\min},\eta_{\max}\right]$. We thus obtain the optimal design of the rainfall insurance contract (21). When $I^*(\omega)>0$, the optimal form of the indemnity function expressed by (22) is derived. We conclude by noticing that $E\left[u''(\tilde{\pi})\tilde{\eta}\right]=\operatorname{cov}\left[u''(\tilde{\pi}),\tilde{\eta}\right]$ because $E\tilde{\eta}=0$. Proposition 3 is proved.

Proof of decreasing indemnity function

We want to prove that the indemnity function defined by (A9.ii) is decreasing with ω, for all $\omega<\hat{\omega}$, when the producer is prudent.

Assume that there exist two rainfall levels ω_1 and ω_2 with $\omega_1<\omega_2<\hat{\omega}$ such that the indemnity function is increasing: $I(\omega_1)<I(\omega_2)$. We have for all ε:

(A11)

$$\pi_1\equiv\pi_0'+\left[g(x)+k(x)\eta\right]\omega_1+I(\omega_1)-P<\pi_2\equiv\pi_0'+\left[g(x)+k(x)\eta\right]\omega_2+I(\omega_{21})-P+k(x)\varepsilon$$

and since the producer is risk-averse, this entails:

(A12) $\quad u'(\pi_1)>u'(\pi_2)$ for all ε

Taking the expectation of this inequality with respect $\tilde{\varepsilon}$ yields in turn that:

(A13) $\quad \int_{\eta_{\min}}^{\eta_{\max}} u'(\pi_1)d\mathrm{M}(\eta)>\int_{\eta_{\min}}^{\eta_{\max}} u'(\pi_2)d\mathrm{M}(\eta)$

Since the administrative cost function is increasing and concave and since the insurer is risk-averse or risk-neutral, we have also:

(A14)

$$\left[1+c'\left(I(\omega_1)\right)\right]v'\left[w_0+P-I(\omega_1)-c\left(I(\omega_1)\right)\right]\leq\left[1+c'\left(I(\omega_2)\right)\right]v'\left[w_0+P-I(\omega_2)-c\left(I(\omega_2)\right)\right]$$

This entails that expressions (A13) and (A14) are not compatible with the first order condition (A8.ii) and, therefore, the optimal indemnity function can not be increasing over $\left]\hat{\omega},\omega_{\max}\right]$. This concludes the proof.