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## **APPLYING DUALITY THEORY IN AGRICULTURAL PRODUCTION ECONOMICS AS A BASIS OF POLICY DECISION MAKING**

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### **1. Introduction**

Economics is useful in evaluating producer responses to potential policy changes. As specific provisions of policy change, so will the incentives faced by agricultural producers. Changes in economic incentives will in turn influence resource allocation decisions, as producers adapt to a new economic environment in a manner that is consistent with the highest perceived benefit.

This paper surveys some recent methodological advances in the measurement of producer responses to a changing economic environment. The techniques surveyed are especially designed to provide both quantitative and qualitative responses to counterfactual questions commonly encountered by policy makers. For example, will producers use more or less labour if the price of wheat increases? By how much will corn production rise if the corn price increases? How will capital and labour be reallocated between different enterprises and across different regions under CAP reform? Will CAP reform result in a flight of industries across nations as relative economic competitiveness is disturbed? ...

Information provided by economic models of producer behaviour can help policy makers to evaluate potential consequences of changing an established policy. However, this is easier said than done because even if a policy question is properly framed, there is no unique way to proceed with providing a solution. Rather, the modeller is forced to confront an array of complex questions. Constructing an economic model which gives accurate and reliable responses is more an art than a science. There are inevitable trade-offs between modelling objectives, model specification, econometric estimation techniques and data availability.

The present survey seeks to inform at many levels. First, some methodological issues in

applying duality theory to measure producer response are discussed. This discussion is designed to reveal some of the hard choices confronted by a modeller in deriving reasonable estimates. Secondly, evidence obtained by the growing body of literature in the EC is summarised. Finally, potential new research that may improve understanding of producer response is highlighted.

## **2. Illustrating how duality can be used to estimate models of primary production : the case of the profit function**

It is common to encounter situations, at both the macro-economic level and the farm level, where agriculture production takes place in a multioutput-multiinput framework. The technical production possibilities that farms face can be equivalently described by alternative primal representations, i.e., by a production possibility set  $T$  giving all feasible netput combinations, ii) by an input requirement set  $X(y)$ , where  $y$  is a vector of  $m$  positive outputs, iii) by a producible output set  $Y(x)$ , where  $x$  is a vector of  $n$  positive inputs, iv) by output  $D(x,y)$  and input  $G(y,x)$  distance functions, and v) by a transformation function  $F(y,x)=0$ .

A significant development in production economics has been the recognition that dual functions, i.e., profit, cost and revenue functions, can completely characterise the economic properties of the technologies : economies of scale, elasticities of substitution, technical change, separability and jointness, ... Conditional on a maintained behavioural hypothesis, observations on resource allocation can be utilised to estimate parameters of a dual function. The empirical analysis of agricultural technologies has been greatly enhanced by the development of flexible functional forms which, unlike to Cobb-Douglas or Constant Elasticity of Substitution (CES) functions, impose few a priori restrictions. Consequently, the function, its first derivatives, and its second derivatives can attain any arbitrary value.

The aim of this section is to briefly present duality theory on the example of the profit function and to show how this modelling framework is relevant to the estimation of supply and demand response functions. We first define the restricted profit function and derive its properties from the assumed properties of the production possibility set  $T$ . The second paragraph illustrates the theoretical model of this section two particular cases (i.e., translog and quadratic) and discusses the problems of choosing a particular flexible functional form and of testing or imposing some specific restrictions (curvature conditions in particular).

### ***2.1. The profit function : definition and properties***

Following Chambers (1988 ; 1989), we assume that the production possibility set  $T$ , i.e., the set of all input-output bundles  $(x,y)$  compatible with the available technology, is :

Ti) non empty, with  $(0,y) \in T \Leftrightarrow y = 0$ ,

Tii) closed,

Tiii) convex,

Tiv) bounded from above (for finite  $x$ ), and

Tv) permits free disposal of outputs and inputs.

The multioutput-multiinput unrestricted profit function for a price-taking producer is then defined by the following alternative identities :

$$\begin{aligned}\pi(p, w) &= \max_{y,x} [py - wx; (x, y) \in T] & [1] \\ &= \max_y [py - C(w, y)] \\ &= \max_x [R(p, x) - wx]\end{aligned}$$

where  $p$  is the output price vector ( $p \in \mathfrak{R}_+^m$ ),  $w$  the input price vector ( $w \in \mathfrak{R}_+^n$ ),  $\pi(p,w)$  the unrestricted profit function,  $C(w,y)$  the unrestricted cost function and  $R(p,x)$  the unrestricted revenue function. We assume that prices are given, i.e., that farmers have no control on output and input prices. This is a reasonable assumption, particularly for analysis of micro-level data.

Regularity conditions on the production possibility set, when combined with the maintained behavioural hypothesis of profit maximisation, imply restrictions on the profit function as well. This is in essence the central idea of duality theory : the indirect objective function inherits properties from the technology. Specifically, if  $T$  satisfies the properties Ti) to Tv), then the unrestricted profit function  $\pi(p,w)$  is<sup>1</sup> :

Pi) non negative,

Pii) non decreasing in  $p$  and non increasing in  $w$ ,

Piii) continuous and linear homogenous in prices, and

Piv) convex in prices.

Using Hotelling's lemma, we can then derive unrestricted Marshallian output supply and input demand functions, i.e.,

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<sup>1</sup> We assume that behavioural functions are twice differentiable.

$$\partial\pi(p, w) / \partial p_i = y_i(p, w) \quad \forall i = 1, \dots, m \quad [2]$$

$$-\partial\pi(p, w) / \partial w_j = x_j(p, w) \quad \forall j = 1, \dots, n \quad [3]$$

Together, equations [2] and [3] provide a convenient way of recovery optimal policy functions which are defined in terms of first derivatives of the profit function.

But, the quasi-fixity of some factor stocks (land, capital, labour, cattle) makes the estimation of unrestricted or full equilibrium profit functions inappropriate and requires a modelling strategy that explicitly recognises the implications of short-run fixity of these inputs. Two basic approaches can be followed to model input fixity. The first method, based on dynamic optimisation theory, incorporates adjustment costs for the quasi-fixed inputs. Epstein (1981) suggests various flexible functional forms that meet the required conditions for an intertemporal profit function. The second method assumes that the farm is in static equilibrium with respect to some outputs and inputs, conditional on the levels of the remaining inputs that are known to be fixed<sup>23</sup>. In that case, we define a restricted profit function as :

$$\pi R(p, w^o, x^1) = \max_{y, x^o} [py - w^o x^o; (y, x^o, x^1) \in T] \quad [4]$$

where the input vector  $x$  is partitioned into a sub-vector  $x^o$  of variable inputs and a sub-vector  $x^1$  of quasi-fixed inputs, i.e.,  $x = (x^o, x^1)$ ,  $x^o \in \mathfrak{R}_+^{n^o}$ ,  $x^1 \in \mathfrak{R}_+^{n^1}$ ,  $n^o + n^1 = n$ . A similar partition applies to the input price vector ( $w = (w^o, w^1)$ ).

The restricted profit function is non negative, non decreasing in  $p$  and non increasing in  $w$ , linear homogeneous in prices, convex and continuous in prices, non decreasing and concave in fixed input quantities (see, for example, Diewert, 1974). By applying Hotelling's lemma, restricted output supply and input demand functions may be derived :

$$\partial\pi R(p, w^o, x^1) / \partial p_i = y_i^r(p, w^o, x^1) \quad \forall i = 1, \dots, m \quad [5]$$

$$\partial\pi R(p, w^o, x^1) / \partial w_j^o = -x_j^{o,r}(p, w^o, x^1) \quad \forall j = 1, \dots, n^o \quad [6]$$

Note that restricted supply and input demand functions [5] and [6] are conditional, in particular, on the fixed values of inputs  $x^1$ , unlike the unrestricted counterparts in equations [2] and [3]. Changing the level of fixed inputs will alter both restricted supply and variable input demand.

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<sup>2</sup> While this second approach accounts for input fixity, it fails to explain why inputs are fixed at a specific level at each point in time.

<sup>3</sup> The approach can easily be generalised to the case where some outputs are constrained, by production quotas for example.

Furthermore, if the restricted profit function is differentiable with respect to quasi-fixed input quantities, then the shadow price functions for these quasi-fixed inputs may be obtained by :

$$\partial \pi R(p, w^o, x^1) / \partial x_j^1 = w_j^{1r}(p, w^o, x^1) \quad \forall j = 1, \dots, n^1 \quad [7]$$

These shadow prices are a measure of scarcity in the same way as market prices. They reflect the relative valuation a producer might place to utilise an additional unit of the resource.

Restricted output supply, input demand and quasi-fixed input shadow price functions verify some properties to be consistent with production theory, i.e., properties of the production possibility set T, and profit maximisation behaviour : i) restricted output supply and input demand functions are continuous and homogeneous of degree zero in variable netput prices<sup>4</sup>, ii) output supply functions are upward sloping, iii) input demand functions are downward sloping, iv) quasi-fixed input shadow price functions are continuous, homogeneous of degree one in variable netput prices and downward sloping, and iv) cross effects are symmetric (Young's theorem<sup>5</sup>).

The monotonicity restriction requires that profits rise with an increase in the price of an output and fall as the price of an input increases. Requiring the profit function to be linearly homogeneous in prices implies that resource allocations decisions are influenced by relative prices only, but not by absolute prices. A proportional increase in all prices will not affect resource allocation decisions. The constraint that a profit function be convex in prices has the following interpretation. If the price of, say, output increases and the producer continue to use the same resource combination to produce the same level of output, then profit will increase linearly. However, rational producers will not exhibit inertial behaviour and will change the resource combination to produce a different output level. By ruling out such inertial behaviour, economic theory predicts that a price increase will cause profits to increase by more than the amount associated with inertial behaviour. This justifies non linearity, i.e., convexity, of the profit function. Another restriction meriting discussion is concavity of the restricted profit function in fixed inputs. This restriction suggests that the shadow price of the fixed input rises as the fixity constraint is tightened.

## 2.2. Empirical implementation

Ideally, functional forms used in applied econometrics should impose as few restrictions on the theoretical properties of the underlying technology and be empirically tractable.

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<sup>4</sup> If a function is homogeneous of degree k, then its first derivative are homogeneous of degree k-1.

<sup>5</sup> By Young's theorem, a cross-partial derivative is invariant with respect to the order of differentiation.



Although the popular Cobb-Douglas production function has the virtue of simplicity, it imposes strong a priori restrictions on the technology (unitary elasticities of substitution between inputs, for example). Flexible functional forms were introduced to circumvent this problem so that prior choice of a functional form imposes few a priori restrictions on the technology.

Following Fuss et al. (1978), consider the case of a single output-multiinput technology. In that case,  $(n+1)(n+2)/2$  distinct economic effects can be measured in terms of the production function itself (one effect corresponding to the output level) and its first and second partial derivatives (returns to scale, distributive shares and own- and cross-price elasticities corresponding to 1,  $n$  and  $n(n+1)/2$  distinct effects, respectively). A flexible functional form is then defined as a form which can reproduce these  $(n+1)(n+2)/2$  distinct effects. A necessary and sufficient condition for a functional form to reproduce these  $(n+1)(n+2)/2$  distinct comparative static effects at a particular point is to have  $(n+1)(n+2)/2$  distinct parameters<sup>6</sup>, as would be provided by a second-order Taylor series expansion<sup>7</sup> (Fuss et al., p. 231).

Most flexible functional forms developed in the applied econometrics literature are linear in parameters expansions of the unknown true function. Linearity is particularly convenient because it allows for easy econometric estimation. The parameters of the functional form are chosen so that the values of the approximating function and of its first and second partial derivatives equal those of the underlying true function at the point of approximation. This definition of flexibility (Diewert, 1971, 1974) raises the problem of accuracy of the approximation, i.e., the use of the estimated parameters for policy analysis outside the neighbourhood of the approximation point where the form may be a poor approximation of the true function and does not necessarily verify its theoretical properties. In other words, the global approximation properties are generally unknown.

Numerous flexible functional forms have been proposed in the literature, including the translog, the quadratic, the generalised Leontief, the generalised Cobb-Douglas and the generalised Box-Cox. By definition, all these forms have the same attractive local property of being consistent with the underlying technology at a base point. But, some concern has arisen

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<sup>6</sup> Obviously, the Cobb-Douglas production function which has only  $n+1$  distinct parameters cannot represent these  $(n+1)(n+2)/2$  distinct economic effects without imposing cross restrictions.

<sup>7</sup> Most functional forms can be interpreted as Taylor's expansions. But, it is also possible to use other series expansions such as the Laurent series expansion (Barnett, 1985) or the Fourier series method (Gallant, 1984). As for the Taylor series expansion, there is no guarantee the Laurent series can approximate globally the true function. However, a Laurent series expansion will always provide a superior approximation because it entails a smaller remainder term. The Fourier series form has the capability to globally approximate the true function in a broader statistical sense, but its use is controversial (necessity of using truncated series for empirical work, instability of estimated elasticities which have, furthermore, often the wrong signs, ...).

about the behaviour of these forms not only at the approximation point, but also over a finite range of data points. This suggests that global, and not only local, properties of functional forms must be analysed in policy oriented empirical work. This is particularly true in the case of the analysis of the CAP reform of May 1992 where models estimated with historical data and designed in an environment of price support policy instruments must now be used for future years in the context of new instruments (set-aside, compensatory payments, ...).

### *2.2.1. Criteria for functional form selection*

Duality theory provides a convenient basis for output supply and input demand modelling, but the choice of the most appropriate flexible functional form for a particular data set is more art than science.

Several criteria can be used to discriminate among competing flexible functional forms. Some of these are purely statistical, such as Bayesian analysis, non-nested hypothesis tests and parametric tests of nested models (Thompson, 1988). The natural and easiest approach seems to be the latter. Most applications of flexible functional forms have used either the translog, the generalised square root quadratic or the generalised Leontief. The generalised Box-Cox developed by Berndt and Khaled (1979) includes these three functional forms as limiting cases and therefore allows the practitioner to carry out parametric tests to discriminate among these three forms (see, for example, Appelbaum, 1979). Unfortunately, the Box Cox is highly non linear in parameters and estimation is often difficult and impractical.

The true technology is unknown. Consequently, evaluating the performance of flexible functional forms on the basis of how well they fit the data is a necessary but not a sufficient condition. One possible solution uses Monte Carlo techniques in order to determine the range of data points where a particular functional form provides acceptable approximations of the technology which is now known (Wales, 1977, Guilkey et al., 1983). Unfortunately, these studies cannot determine whether a particular functional form is unequivocally superior.

Statistical criteria are useful for discriminating among flexible functional forms. However, economic performance of flexible functional forms should also be taken into account. A convenient property in empirical analyses is that local curvature properties imply global properties. The translog function does not satisfy this condition but the quadratic does. Nevertheless, Lopez (1985) has shown that the quadratic imposes more restrictions on the technology than the translog (see paragraph 2.2.2 below). Finally, miscellaneous criteria (goodness of fit, parameter significance, tests on the structure of the technology, ...) are also useful.

In sum, researchers have not succeeded in determining whether one particular

functional form outperforms another form in terms of statistical and/or economic criteria. The most simple and natural approach should be to estimate each model with various alternative functional forms (Berndt et al., 1977 ; Baffes et Vasavada, 1989, ...) and to retain the form which seems the best suitable for the particular policy problem to be solved. The criteria of choice will vary according to data, behavioural assumptions and the problem at hand.

### *2.2.2. Two examples : the translog and normalised quadratic unrestricted profit functions*

In order to simplify analytical expressions of profit, supply and input demand functions, we adopt a more parsimonious notation than in paragraph 2.1. Let  $z$  the netput (i.e., outputs and inputs) vector and  $v$  the corresponding price vector. Output quantities are non negative, input quantities are non positive and prices are positive. We consider only the unrestricted case. The translog profit function is detailed in Table 1, while the normalised quadratic profit function is presented in Table 2.

Consider first the translog flexible functional form. In this case, the profit function and  $(n+m-1)$  share equations are simultaneously estimated, using either iterative seemingly unrelated regression (ITSUR) or maximum likelihood methods<sup>8</sup>. Because of the adding-up restriction, the sum of shares is equal to one and one share must be omitted from the system to avoid redundancy problems<sup>9</sup>. Symmetry and homogeneity in prices can easily be tested through linear restrictions on parameters (see Table 1). Homogeneity in prices is equivalent to a normalisation by the price of one numéraire netput. Furthermore, it is worthwhile to note that the adding-up restriction, together with the symmetry restrictions, implies the homogeneity restrictions. Monotonicity and convexity in prices can neither be tested nor imposed by a set of linear restrictions on the parameters. Monotonicity is generally only checked ex-post by examining estimated shares, which should be positive for outputs and negative for inputs. The problem of checking, testing or imposing convexity in prices is more difficult to solve. Lau (1978) presented an appealing method for parametrically imposing second order derivative restrictions exploiting the Cholesky decomposition of a positive semidefinite matrix.

The Cholesky representation of a real symmetric square matrix  $A$  is the factorisation  $LDL'$ , where  $L$  is an unit lower triangular matrix and  $D$  is a diagonal matrix whose elements are the Cholesky values. The matrix  $A$  is positive (negative) semidefinite if and only if all Cholesky values are non negative (non positive). The first step for testing convexity of the translog profit function is then to estimate the parameters of the model in terms of the parameters of the Cholesky decomposition. This approach allows the practitioner to either test or impose the

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<sup>8</sup> The two estimation procedures are asymptotically equivalent.

<sup>9</sup> Due to the adding-up property of shares, the variance-covariance matrix of the complete system of shares is not of full rank.

derivative restrictions at the point of approximation. Unfortunately, this reparameterization renders the initial model highly non-linear. Another solution proposed by Hazilla and Kopp (1986) reverses the Lau's procedure by reparameterizing the Cholesky decomposition in terms of the original parameters and restrict a function of these parameters. This approach has the advantage of retaining linearity on parameters while imposing convexity in prices for the complete data set used for estimation. Nevertheless, this approach suffers from the disadvantage that it cannot test for convexity.

While the Cholesky decomposition method can be utilised to impose curvature, this method is often criticised for its use of "brute force" to ensure consistency with economic theory. A second approach, which uses inequality restrictions as priors, can be applied to impose curvature (Geweke, 1986, 1989). Convexity of the profit function can be imposed by the following procedure. In the first instance, parameters of the profit function are estimated without imposing the convexity restrictions. Random samples are then drawn from a multivariate normal distribution and the eigenvalues of the Hessian matrix computed. All draws yielding negative values of the Hessian matrix are excluded from consideration. Only draws that are consistent with convexity are retained in the computation of the mean value of the parameter vector. Numerical values for standard errors can be computed as well, when draws from a multivariate t distribution are used. The latter technique is termed "importance sampling" (Chalfant et al., 1991).

Now consider the problem of local versus global convexity. The normalised quadratic profit function has the desirable feature of allowing for testing or imposing curvature conditions globally because the sub-Hessian is a matrix of constants. In that case, convexity in prices will be globally satisfied if the matrix  $\pi_w = [a_{ij}]$  is positive semidefinite, i.e., if the estimated coefficients  $d_{ij}$  of the Cholesky matrix D are all non negative. Nevertheless, for this functional form, linear homogeneity in prices cannot be tested since the associated parametric restrictions render this functional form inflexible. Rather, homogeneity can be imposed by normalising prices and profits by the price of a numéraire netput. Finally, it is worth noting that the normalised quadratic profit function imposes some prior restrictions on the structure of the technology, i.e., weak separability between inputs and outputs<sup>10</sup> and quasi-homotheticity<sup>11</sup> (Lopez, 1985).

An additional disadvantage of the translog with respect to the quadratic is that zero values are inadmissible. Therefore, in a translog multioutput-multiinput cost function, firms

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<sup>10</sup> Under weak separability between inputs and outputs, the marginal rates of output transformation are independent of factor intensities or factor prices.

<sup>11</sup> Under quasi-homotheticity, the expansion path is linear which implies that the marginal rate of input substitution is independent of outputs levels.

which do not produce some of each output would have to be excluded from the sample.

*Table 1. The translog unrestricted profit function*

i) profit

$$\log \pi = a_o + \sum_{i=1}^{m+n} a_i \log v_i + 0.5 \sum_i^{m+n} \sum_j^{m+n} a_{ij} \log v_i \log v_j \quad [\text{T1}]$$

ii) output supply and input demand

$$\begin{aligned} M_i &= \partial \log \pi / \partial \log v_i \\ &= a_i + 0.5 \sum_j^{m+n} (a_{ij} + a_{ji}) \log v_j \end{aligned} \quad [\text{T2}]$$

where  $M_i$  is the share of netput  $i$  in total profit ( $M_i > 0$  if  $i$  is an output,  $M_i < 0$  if  $i$  is an input).

iii) restrictions

a) symmetry :  $a_{ij} = a_{ji}$ .

b) linear homogeneity in prices :  $\sum_i a_i = 1$  and  $\sum_j a_{ij} = \sum_i a_{ji} = 0$ .

c) monotonicity :  $M_i(v) \geq 0$  for an output,  $M_i(v) \leq 0$  for an input.

d) convexity in prices : sub-Hessian  $\pi_{vv}$  positive semidefinite.

*Table 2. The normalised quadratic unrestricted profit function*

i) profit

$$\pi = a_o + \sum_{i=1}^{m+n-1} a_i v_i^* + 0.5 \sum_i^{m+n-1} \sum_j^{m+n-1} a_{ij} v_i^* v_j^*$$

where  $v_i^*$  is the price of netput  $i$  ( $i=1, \dots, m+n-1$ ) normalised by the price of netput  $n$ .

ii) output supply and input demand

$$z_i = a_i + 0.5 \sum_{j=1}^{m+n-1} (a_{ij} + a_{ji}) v_j^*$$

iii) restrictions

- a) symmetry :  $a_{ij} = a_{ji}$  .
- b) linear homogeneity in prices : imposed by the price normalisation.
- c) monotonicity :  $z_i(v) \geq 0$  for an output,  $z_i(v) \leq 0$  for an input.
- d) convexity in prices : sub-Hessian  $\pi_{vv}$  positive semidefinite.

### 3. Applying duality to agricultural economics : characteristics of the technology

The behaviour of a firm may be first summarised by price and quantity elasticities, which can easily be computed from second partial derivatives of profit functions. Additional properties, namely, scale, separability, jointness and technical change characteristics, may also be defined and tested with restrictions on first and second partial derivatives of profit functions. These characteristics may be used in order to reduce the number of parameters in econometric estimation. While an exhaustive survey of these characteristics is beyond the scope of this survey, we present a brief synthesis of the concepts and discuss specific implications for agricultural policy analysis.

#### 3.1. *The choice among behavioural assumptions*<sup>12</sup>

The choice of a behavioural assumption first depends on the context in which production occurs. Take the case of the EC dairy sector which is characterised by a supply management programme whereby a producer has a fixed quota. In this instance, output is predetermined and the choice of cost minimisation is appropriate. On the other hand, whenever production takes place according to marginal cost pricing and the level of output is chosen by the producer, then a profit function approach is relevant.

Lau (1976) has characterised the conditions under which supply and demand responses without quantity constraints can be derived from the responses estimated under some input and/or output fixity, and has shown how the Le Chatelier's principle applies. This problem, generally known as the problem of moving from short-run to long-run responses, has been the subject of numerous empirical studies based on the seminal exposition of Brown and Christensen (1981) who derive specific procedures for estimating "full (i.e., unrestricted) static

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<sup>12</sup> We do not consider the choice between primal and dual approaches (see, for example, Chambers, 1988). Nevertheless, it is worthwhile noting that structural characteristics of the technology can be obtained from both primal and dual models. The choice of a primal or dual approach depends essentially on the purpose of the research and on the ease with which characteristics of the technology can be derived (Capalbo, 1985).

equilibrium substitution possibilities" from "partial (i.e., restricted) static equilibrium substitution possibilities" in the case of a restricted translog cost function<sup>13</sup>. The long-run technology is inferred by optimally adjusting the quasi-fixed inputs until total costs are minimised. If the observed technology is in long run equilibrium, then long-run and observed levels of quasi-fixed inputs should be equal. This equality property can then be used to test the fixity of inputs by comparing optimal and observed quantities. Alternatively, the test can be conducted in the price space by comparing observed and shadow prices of quasi-fixed inputs (Kulatilaka, 1985). The same approach can also be used for outputs in order, for example, to evaluate the quasi-rent, i.e., the difference between observed and shadow prices, associated with an output quota. More generally, the statistical significance of the departure between observed price and marginal cost of an output can be used as a basis for a profit maximisation test.

Long-run responses can then be deduced solely from the estimated parameters of the short-run function, and vice-versa. There is an evident symmetry between relaxing fixity on one hand, and introducing input fixity and/or output rationing on the other hand. The implications of this property are important, particularly in the context of agricultural policies where numerous policy instruments are used simultaneously and change over time. Take again the example of milk quotas in the EC. The previous analysis suggests that it is possible to use an empirical sector model that has been estimated according to profit maximising behaviour in the new context of a quota policy regime under the condition that output and input responses are adjusted accordingly. In effect, when some outputs are pegged at a given level by production quotas, the remaining variable netputs will exhibit constrained response to exogenous variables. Output quantities which can still be freely adjusted will not behave in the same way with respect to prices of netputs which remain variable. Furthermore, they will not depend on prices of outputs under quota, but on quantities of these rationed outputs. In sum, we have two alternative representations of the same technology, i.e., either by deriving the rationed equilibrium from its unrationed counterpart or by directly defining a rationed profit function. These alternatives can be used indifferently depending on the available information.

Second, and perhaps more important, the choice of a behavioural assumption depends on the purpose of the research. It is easier to directly obtain output responses to price changes using a profit function approach than by using a cost function. But, Hicksian technical change biases or scale characteristics are easier to deduce from a cost function than from a profit function.

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<sup>13</sup> The basic tool to be employed in such analyses is the Hessian identity relating short-run and long-run functions.

Finally, the choice of a behavioural assumption depends also on the data availability. For many cases, the researcher may not have access to primary data or may use data collected for a different purpose. This constrains the ability of a researcher to define a model when the data are inconsistent with modelling assumptions.

### 3.2. *Substitutability of inputs and outputs*

When the price of an output decreases, it is reasonable to suppose that less of that output will be produced. Similarly, when an input is cheaper, more of that input may be used. The degree of output and input responses to such price changes will depend crucially on substitution possibilities.

In the multioutput-multinput case, there is no unique way to measure input and output substitution. The most popular indicator, defined in the single output-multiinput case, is the Allen partial elasticity of substitution. A generalisation to the multioutput regime was proposed by Diewert. The Diewert elasticity of transformation is defined in terms of the second partial derivatives of the unrestricted profit function. Diewert extends this definition to the case of a restricted profit function and the "restricted" elasticity of transformation is then defined as :

$$\theta_{l,k} = \pi R \cdot \pi R_{lk} / \pi R_l \cdot \pi R_k$$

where l and k refer to output prices, variable input prices or quasi-fixed input quantities. The economic interpretation of  $\theta_{lk}$  is not straightforward. Its main interest lies in the fact that price and quantity elasticities can easily be derived from the restricted elasticities of transformation as :

$$\varepsilon'_{ik} = \partial \log y_i^r / \partial \log v_k = \theta_{ik} / S_i \quad [9]$$

$$\varepsilon'_{jk} = \partial \log x_j^{or} / \partial \log v_k = \theta_{jk} / S_j \quad [10]$$

$$\varepsilon'_{hk} = \partial \log w_h^{lr} / \partial \log v_k = \theta_{hk} / S_h \quad [11]$$

where i refers to an output, j to a variable input and h to a quasi-fixed input ;  $v_k$  refers either to an output price (in that case,  $v_k = p_k$ ), to a variable input price ( $v_k = w_k^o$ ) or to a quasi-fixed input quantity ( $v_k = x_k^l$ ) ; and  $S_k$  is the share of commodity k in restricted profits, i.e.,  $p_i y_i^r / \pi R$  in the case of an output,  $w_j^o x_j^{or} / \pi R$  in the case of a variable input and  $w_h^{lr} x_h^l / \pi R$  in the case of a quasi-fixed input.

$\varepsilon'_{ik}$ ,  $\varepsilon'_{jh}$  and  $\varepsilon'_{hk}$  are restricted price and quantity elasticities of supply, demand and shadow price functions, respectively. Convexity of the restricted profit function in prices implies that own-price elasticities of supply and demand are positive and negative, respectively.



Concavity of the restricted profit function in fixed input quantities implies that own-quantity elasticities of shadow prices are negative. Homogeneity conditions imply that the following restriction hold :

$$\sum_{k=1}^{m+n^o} \varepsilon'_{ik} = 0 \quad \forall i = 1, \dots, m$$

$$\sum_{k=1}^{m+n^o} \varepsilon'_{jk} = 0 \quad \forall j = 1, \dots, n^o$$

Production theory does not allow us to sign cross-price elasticities which can be either positive or negative. The "normal" case (Sakai, 1974) rules out substitution among outputs and variable inputs, and regressive or inferiority relations between outputs and variable inputs<sup>14</sup>.

More precisely, Sakai has defined four propositions which govern the market conduct of the multiproduct-multifactor firm in the "normal" case : the total marginal cost of an output decreases with increases in other output quantities (cost complementarities) and does not decrease with increases in input prices (input normality), and the total marginal revenue of an input does not decrease with increases in other input quantities (cooperant inputs) and output prices (output normality), i.e.,

$$\partial^2 C / \partial y_i \partial y_k \leq 0 \quad \forall i, \forall k, i \neq k \quad [P_1]$$

$$\partial^2 C / \partial y_i \partial w_j \geq 0 \quad \forall i, \forall j, \quad [P_2]$$

$$\partial^2 R / \partial x_j \partial x_k \geq 0 \quad \forall j, \forall k, j \neq k \quad [P_3]$$

$$\partial^2 R / \partial x_j \partial p_i \geq 0 \quad \forall j, \forall i, \quad [P_4]$$

Propositions [1] to [4] imply some restrictions on partial second derivatives of the profit function and therefore on cross-price elasticities, i.e.,

$$\partial^2 \pi / \partial p_i \partial p_k \geq 0 \quad \forall i, \forall k, \quad [C_1]$$

$$\partial^2 \pi / \partial w_j \partial w_k \geq 0 \quad \forall j, \forall k, \quad [C_2]$$

$$\partial^2 \pi / \partial p_i \partial w_j = \partial^2 \pi / \partial w_j \partial p_i \leq 0 \quad \forall i, \forall j, \quad [C_3]$$

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<sup>14</sup> Two outputs will be said restricted Marshallian substitutes if  $\varepsilon'_{ik} \leq 0$  ; two variable inputs will be said restricted Marshallian substitutes if  $\varepsilon'_{jk} \geq 0$  ; and an output will be said Marshallian regressif or inferior with respect to a variable input if  $\varepsilon'_{ik} \geq 0$ . If elasticities are defined in an unrestricted Marshallian equilibrium, we will speak of unrestricted Marshallian substitution and inferiority.

Hertel (1984, 1987) and Moschini (1988, 1989) have analysed these four propositions at some length, particularly in order to evaluate their validity in the context of agriculture at both the firm and the sector levels. In evaluating the validity of these propositions and of their implications, the most important point to note is that they are defined in the context of total or unrestricted behavioural functions, where all inputs for the cost function and all outputs for the revenue function are assumed to be variable. [P2] and [P4] appear to be plausible for agriculture (Hertel, 1984). [P1] is a generalisation to the multiproduct case of Rader's (1968) concept of co-operant factors of production. But, as underlined by Moschini (1989), [P4] has no equivalent in the single output case. [P4] means, for example, that the marginal cost of producing, say, wheat will not increase as the quantities of, say, corn increases. It may even decrease. Baumol et al. (1982) have shown that the unrestricted cost function exhibits cost complementarities when the source of jointness in input quantities arises uniquely from an input whose use in one production process does not diminish its use in another<sup>15</sup> (in this sense, it is a public input). On the other hand, Moschini has shown that cost complementarities are not verified when jointness in input quantities is due to normal allocatable fixed inputs. In other words, property [P1] and consequence [C2] are inversed under allocatable fixed inputs<sup>16</sup>, i.e.,

$$\partial^2 CR / \partial y_i \partial y_k \geq 0 \quad \forall i, \forall k, i \neq k$$

$$\partial^2 \pi R / \partial p_i \partial p_k \leq 0 \quad \forall i, \forall k, i = k$$

To sum up this short discussion of comparative statics of supply and demand functions, it is worthwhile noting that similar restrictions can be tested on quasi-fixed input shadow price functions.

### 3.3. Economies of scale

It is often interesting to know whether a technology imposes restrictions on how output responds to changes in input levels. For instance, a producer may acquire more land and utilise more of all inputs to cultivate a larger farm. Will this increase in utilisation of all inputs increase output? If so, by how much? To answer these questions, a class of technologies, which are termed homothetic, prove to be useful.

A technology is homothetic when two conditions hold. First, a single aggregate input

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<sup>15</sup> The model of Baumol et al. assumes that there is only one public input. The restricted cost function they consider is non joint in input quantities, i.e.,  $\partial^2 CR / \partial y_i \partial y_k = 0 \quad \forall i, \forall k, i \neq k$ , where  $CR(\cdot)$  is the restricted cost function.

<sup>16</sup> In the model of Moschini, the unrestricted cost function is non joint in input quantities, i.e.,  $\partial^2 C / \partial y_i \partial y_k = 0 \quad \forall i, \forall k, i \neq k$ , but the restricted cost function is joint in input quantities, i.e.,  $\partial^2 CR / \partial y_i \partial y_k \neq 0, \quad \forall i, \forall k, i \neq k$ .

mimics the behaviour of the comprehensive set of inputs. Second, output increases when more of the single aggregate input is applied by a producer. The first condition implies that proportional changes in all inputs can be represented by proportional changes in a single aggregate input. This property proves to be useful because changes in the single aggregate input can be viewed as a proxy for changes in the scale of a farm operation. The second requirement ensures that as the scale of the operation expands, more output is produced.

Several interesting implications follow when a technology is homothetic. Perhaps the most interesting is that a homothetic technology restricts the marginal rate of substitution between inputs to be constant along a ray from the origin. This suggests, for example, that the isoquant map will consist of a series of parallel isoquants. For a homothetic technology, relative input utilisation will depend only on relative input prices. If all input prices change such that relative input prices are constant, then use of all inputs change in a manner such that relative input use is the same as before the price change.

Before mentioning another implication of homotheticity, it is useful to briefly review the concept of elasticity of scale. Elasticity of scale is a measure of output responsiveness to changes in the scale of input utilisation. Specifically, given an input bundles, it measures output response to a proportional change in the level of all inputs in the bundle. Homotheticity restricts the elasticity of scale so that it depends only on the level of output. Knowledge about the initial level of output is sufficient to identify the responsiveness of output to changes in the scale of a farm operation.

For the class of homothetic technologies, economists are often interested in technologies which imply a constant value for the elasticity of scale. In general, the elasticity of scale for a homothetic technology is not a constant and is functionally related to the level of output. However, it is possible to further restrict homothetic technologies to yield a constant elasticity of scale.

Three pathological cases for technologies with a constant elasticity of scale have been the focus of economist's attention. These are i) constant returns to scale, ii) increasing returns to scale, and iii) decreasing returns to scale. Under constant returns to scale, the elasticity of scale equals one and a proportional change in all inputs will change output proportionally. Increasing returns to scale suggests that the output change will be proportionally greater than the input change. Finally, decreasing returns to scale imply that, when all inputs change, the output change will be proportionally smaller than the input change.

The dual approach to production economics is useful for understanding the nature of scale economies in agricultural production. Information about scale economies may be important as policy makers assess the impact of agricultural policy on the structure of farm

production. For instance, policy makers may wish to know how small, medium, and large farms will be affected by a policy change. An important piece of information for answering this question is the nature of economies of scale in farm production.

Another reason why economies of scale may be of interest to policy makers is that it has a bearing on the nature of competition in agricultural markets. A technology that exhibits increasing returns to scale is incompatible with perfect competition in agricultural markets. This is because a farm that is subject to increasing returns to scale can expand indefinitely and increase profits by doing so. This will eventually result in a monopolistic market structure.

By adopting the dual approach, information about economies of scale can be recovered by a sequence of hypothesis tests. In the first instance, based on a behavioural hypothesis (e.g., cost minimisation), parameters of a dual cost function for a nonhomothetic technology can be estimated. Once this is accomplished, the hypothesis of homotheticity can be statistically tested by imposing appropriate parametric restrictions. In the final stage of the analysis, the hypothesis of a constant elasticity of scale can be tested. Such a sequence of hypothesis tests can yield information that is useful to policy-makers.

It is worthwhile noting that economies of scale are difficult to measure when firms produce multiple outputs. The difficult results because economies of scale are typically defined as the change in a scalar valued function with respect to some measure of quantity. With multiple outputs, a decision must be made as to the status of the different outputs (Hallam, 1991). In the multiproduct case, the concept of economies of scope which measures the cost advantages to firms of providing a large number of diversified products as opposed to specializing in the production of a single output is also interesting. When there are economies of scope, it is less expensive to produce the various commodities in a single firm than it is to produce each commodity in a distinct firm.

### ***3.4. Separability***

Empirical analysis of agricultural technologies can be greatly simplified if it is assumed that production occurs in more than one stage, i.e., if it is possible to assume that producers follow a sequential multi stage optimising process. Separability assumptions justify such a multi-stage optimisation which, in turn, permits consistent aggregation between netputs.

Perhaps one of the most interesting application of the separability concept to agricultural economics is the so-called Armington model. According to the Armington assumption, a country first chooses the total amount of a product to be imported and then divides total imports among several importing sources. This is an useful abstraction which allows import demands to be modelled in a framework where only a few parameters are to be

estimated. The Armington model has been fruitfully employed in trade policy analysis for evaluating the response of inputs to price changes and to the total amount imported. In addition to the trade sector, separability has useful applications to the production agricultural sector as well. For example, different forms of capital are used in agricultural production such as buildings, machinery, real estate, etc. Building an econometric model which incorporates these different input categories would be virtually impossible because such a high level of disaggregation would exhaust available degrees of freedom and cause severe multicollinearity problems. In essence, separability allows the modeller to lump these different forms of capital into a single category, aggregate capital. Aggregation of outputs and inputs is thus frequently a necessary step in empirical studies. Separability assumptions can also be used to build a comprehensive system of derived demand for feed ingredients. Since the main technical requirements in feed are related to energy and protein contents, Mahé (1987) has built a model based on the aggregation of individual items in groups on the basis of their energy-protein ratio while assuming that these aggregates are separable.

To better understand the theoretical basis for the use of aggregator functions in the separability concept, consider a partition of the set of outputs and inputs. The transformation function is weakly separable with respect to this partition if the marginal rates of substitution within a netput group, say capital, are invariant to changes in netput levels outside the group, say labour. An interesting result established by Lau (1978) is that when the transformation function is separable in a certain partition, then the total profit function is separable in this partition too. In that case, optimal levels of netputs within a group (capital) are invariant to changes in prices of netputs outside the group (e.g., family labour, hired labour, ...).

While the concept of separability is intuitively appealing, its widespread application can mainly be attributed to the simplification offered by a multi-stage optimisation procedure. Oddly enough, availability of too much data can be a curse because existing modelling techniques do not allow these detailed database to be meaningfully organised for policy analysis.

### 3.5. Jointness

The hypothesis of non-jointness in input quantities also plays an important role in applied agricultural economics. This assumption implies that cost and profit functions can be expressed as separable functions of levels and prices of outputs, respectively, i.e.,

$$C(y, w) = \sum_{j=1}^m C_j(y_j, w)$$

where  $C_j(\cdot)$  is the cost function associated with output  $j$ .

$$\pi(p, w) = \sum_{j=1}^m \pi_j(p_j, w)$$

where  $\pi_j$  is the profit function associated with output  $j$ .

These equations reveal that total profits (costs) for the agricultural sector can be broken up into the sum of individual sectoral profit (cost) functions such as grains, livestock or fruits. A profit (cost) function for each of these sectors can individually be estimated if data are available.

Non jointness in input prices therefore implies that the sub-Hessians  $C_{yy}$  and  $\pi_{pp}$  are null matrices (Hall, 1973). These restrictions can be tested easily via restrictions on estimated parameters. In practice, non-jointness in input quantities implies that each output can be produced by a separate production function, factors being allocated to the different productions. Another powerful implication of nonjointness is that the supply function of a particular output does not depend on quantities or prices of other outputs, i.e., on decisions about other outputs. In other words, the supply function of, say corn, will not depend on the production of, say wheat. In much the same way as separability, imposing nonjointness can free up valuable degrees of freedom. There are eminently reasonable situations where maintaining the nonjointness assumption as a prior will be appropriate. For instance, specialised fruit and vegetable farms may not produce grains, and apple production will typically be invariant to wheat production.

Three other non-jointness concepts have been defined in the literature. Non-jointness in output quantities is also well known and arises when an activity uses one input only which is divided into a number of outputs (sheep for producing wool and mutton, dairy cattle for producing milk and beef, for example). Kohli (1983) proposes also two additional concepts of non-jointness, in input prices and output prices respectively, but they are most likely less relevant for agriculture.

If the underlying technology is joint, then policy analysis based on a sectoral model will provide misleading results. Such a partial analysis will fail to take account of constraints embedded in the technology. Also, jointness has powerful implications to enterprise budgeting for measuring farm-level profitability. It is common to develop enterprise budgets for, say, a typical livestock producer and evaluate policy response based on such budgets. But when technology is joint, the approach to be followed is whole farm budgeting as opposed to single enterprise budgeting.

### ***3.6. Technical change and total factor productivity***

Observed changes in the quantities of outputs and inputs can be attributed to fluctuations in relative prices as well as due to the effect of technical change (TC). TC can reduce overall profitability or increase costs in which case it is termed regressive. On the other hand, it may increase profitability or reduce costs. The latter is the more typical case.

Policy makers may be interested in understanding how agricultural policies have influenced both the rate and the bias of technical change. For instance, there has been a slowdown in productivity growth within the United States. In this case, it would be useful to understand how policy variables can be used to accelerate the rate of productivity growth and the competitiveness of the US economy or, for that matter, any other economy.

Another interesting policy issue pertains to the bias of technical change. TC may have the effect of reducing the quantity utilised of some inputs. Equally, TC may increase utilisation of other inputs. From a policy perspective, it may be interesting to know whether TC has a tendency to reduce labour use in agriculture. This information may be helpful to plan intersectoral shifts in the labour force into other sectors and to develop retraining programmes to ensure that the labour force is gainfully employed in other occupations.

Based on Hick's original definition and assuming a two input - one output linearly homogeneous technology, technical change is said to be neutral if it leaves unchanged the rate of substitution between input pairs. However, as noted by Blackorby et al. (1976), "to compare situations before and after technical change, something must be held constant. Exactly what is to be held constant has been the subject of some debate and constitutes the crux of the issue at hand". If factor endowments are held constant, technical change is measured along a ray where factor production remains the same. For agricultural technologies, at both the firm and the farm levels, it seems more useful to define neutrality holding factor price ratios constant (Binswanger, 1974). The dual measure of technical change biases he proposes is :

$$B_{ii} = \partial \log S_i / \partial \left. \frac{w_i}{w_j} \right| \begin{array}{l} > 0 \text{ if TC is input } i \text{ using} \\ = 0 \text{ if TC is input } i \text{ neutral} \\ < 0 \text{ if TC is input } i \text{ saving} \end{array}$$

where  $S_i$  is the share of input  $i$  in total costs. Technical change is said neutral if all biases equal zero.

In order to use time series to characterise a technology, an identifying assumption must be made about the nature of technical change over the period. Although the identification of technical change with a time trend must be viewed as being more a measure of our ignorance than anything else (Chambers, 1988), this specification constitutes the general rule of applied analyses. In that case, all coefficients of behavioural functions are implicitly assumed to be

constant over time. Nevertheless, the use of a flexible functional form, and therefore the introduction of quadratic terms on time and interactions of the time trend with explanatory variables allows TC to vary at a non constant rate and to change with explanatory variable variations (input prices, for example). A first alternative is to specify a model such that parameters may change over time. So, in an unrestricted cost function, all coefficients are then assumed to vary linearly or log linearly with time. This model has the advantage of directly testing for price induced technical change and innovations<sup>17</sup> (Binswanger and Ruttan, 1978).

The importance of the induced innovation assumption for policy makers is clear and may be illustrated by the animal feed sector in the EC. The May 1992 CAP reform would result in a substantial increase in feed demand for grains due to a better price competitiveness of EC grains with respect to other feed ingredients, oil cakes and grain substitutes, which enter into the Community without tariffs. In the pre-reform CAP, the price wedge between EC grains and substitutes induced a strong demand for the latter. Following the induced innovation assumption, this demand was further enhanced by TC biases, favourable to grain substitutes at the expense of domestic grains. Under the CAP reform, new price ratios, more favourable to EC grains, would induce technical change which would be more grain using and imported ingredient saving. This effect would reinforce the substitution or price effect between domestic grains and other feed ingredients in EC animal rations. More research is needed to evaluate the potential magnitude of this technical change effect.

#### **4. Duality theory and measurement of production response in the EC**

The previous two sections illustrated how duality theory could be gainfully employed to derive potentially useful information on agricultural producer response to changes in a policy environment. Like any methodology, the ability of a technique to provide accurate and reliable estimates depends very much on how well the modeller can graft the essentials of a policy problem into a dual economic framework. Much of the appeal of using duality theory lies in its capacity to handle a wide spectrum of behavioural assumptions and to recover flexible representations of the technological constraints faced by an agricultural producer. More work is needed however to link the wide ranging policy scenarios encountered in production agriculture with dual economic models.

Given the rapidly changing policy environment in European agriculture, it is meaningful to assess the potential of applying duality theory to the task of answering a chockfull of questions. Specifically, in the changing policy environment, will all sectors be affected equally

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<sup>17</sup> In the spirit of Hicks, TC requires time. Consequently, any index of technology should depend on past values of the variables relevant to the investment technical process, and not on current values.



(e.g., crops versus livestock) ? How will different countries within the EC react to policy changes ? What impact will CAP reform have on utilisation of polluting inputs ? Will CAP reform induce extensification of production techniques ?

Fortunately, a significant body of evidence has accumulated to address some of these questions within a consistent economic framework. While numerous efforts to model European production response have been made, only research that utilises duality theory will be reviewed. This is deeping in line with our focus on the coherence of models with economic theory. Such a survey will reveal the widespread applicability of duality theory in different contexts as well as serve as a litmus test on whether this technique has provided policy relevant information (Table 3).

A direct comparison of structural characteristics of the EC agricultural production process from existing econometric studies would require adjustments to account for differences in behavioural assumptions, functional forms, other maintained assumptions and data used. In the particular case of the EC, this comparison is rendered even more difficult because applications span a wide spectrum of sectors in many EC Member States.

Given that the evidence from a cross section of countries pertains to different time periods, different behavioural assumptions, and different production sectors within a country, only broad qualitative comparisons are feasible. In most instances, specific quantitative estimates cannot be directly compared. For this reason, this available fragmented evidence for comparable issues was collated to derive some general conclusions.

Keeping in mind that our objective is merely to give a flavour of the wide variety of empirical work, consider Table 3 which summarises the results of various studies. Some general conclusions emerge from this table.

i) The focus of virtually all studies is narrow and limited in scope to the production sector of a specific Member State in the EC.

ii) The translog specification is the form the most commonly used, while the generalised Leontief or more recent functional forms are seldom used. This contrasts with the trend observed in North American empirical work where the quadratic and the Leontief are often employed.

iii) Almost, all reported studies assume that some inputs are fixed or quasi-fixed, although they are not explicitly modelled using dynamic economic theory.

iv) Symmetry and homogeneity are very often maintained hypotheses. In the case of symmetry, this may not be an important limitation because, when tested, it is generally

accepted. Noticeable exceptions are Higgins and Burrell. However, the homogeneity property is commonly rejected.

v) Generally, curvature restrictions have not been tested or imposed in estimation. No general conclusions can be drawn regarding the failure or success with imposing or testing curvature properties. Most studies do not mention convexity and, therefore, the reported results are not informative about this topic. This is a glaring omission and needs to be addressed. It would appear that the modelling community is quick to embrace restrictions that can be imposed or tested in a straightforward manner. Surprisingly, the enthusiasm to investigate curvature appears to be lukewarm despite availability of numerical algorithms to impose this restriction.

vi) Separability and jointness characteristics are rarely tested. Mergos and Yotopoulos use a separable production structure, but do not test it. Surry assumes livestock outputs to be separable from feed ingredients, but this assumption is not tested.

vii) Table 4 summarises own-price elasticity estimates of input demand derived from several studies reported in table 3. One should be cautious in comparing these parameters first because they concern various EC member countries and second because they are obtained from either cost or profit function models. From table 4, the following conclusions can be made : (a) all studies (cost and profit function models) report own-price elasticities of labour demand which are less than 1 in absolute value. Thus, this result seems to indicate that the demand for labour from the agricultural sector within the EC is inelastic; (b) to contrast, feed and fertilizer input demands appear more elastic since their respective own-price elasticity estimates are often near equal to or greater than 1; (c) in almost all the reported studies in table 4, labour and feed are complements on the demand side while other inputs substitute for each other (except for Tiffin where feed complements to fertiliser). Hence, it seems that input substitution patterns are more common in the agricultural production process in the EC.

Table 3. Overview of empirical studies in agricultural economics based on the duality approach for EC countries

Source	Main objective	Country	Data type	Coverage	Form
1. Glass and McKillop (1989)	Investigating the structure of the agriculture	Northern Ireland	time series 1955-85	Sector	Translog
2. Guyomard and Vermersch (1988)	Inferring various equilibria for the firm	France	micro-data 1981	Grains	Translog
3. Burrell (1989)	Analysing the demand of fertiliser in the UK	United Kingdom	time series 1950-80	Sector	Translog
4. Grings (1986)	Modelling the agricultural sector	Germany	time series 1950-82	Sector	Generalised Leontief
5. Boyle and Guyomard (1989)	Inferring long-run supply elasticities	Ireland	time series 1960-82	Sector	Translog
6. Bouchet et al. (1988)	Analysing the sources of growth	France	time series 1959-84	Sector	Quadratic
7. Becker and Frenz (1989)	Characterizing the technical change	Germany	panel data 1977-88	Sector	Translog
8. Boyle and O' Neill (1989)	Modelling the agr. sector	Ireland	time series 1960-82	Sector	Translog
9. Tiffin (1991)	Modelling the production decisions	England and Wales	time series 1964-84	Dairy	Translog
10. Thijssen (1992a)	Modelling dairy farms behaviour	Netherlands	panel data 1970-82	Dairy	Quadratic
11. Thijssen (1992b)	Modelling dairy farms behaviour	Netherlands	panel data 1970-82	Dairy	Quadratic
11. Fernandez-Cornejo et al. (1992)	Analysing the economic behaviour of dairy producers	Germany	panel data 1981-85	Dairy	Quadratic
12. Luijt and Hillebrand (1992)	Analysing the relation availability of fixed inputs - continuity of farms	Netherlands	micro-data average 1975-79	Dairy	Translog
13. Stefanou et al. (1992)	Analysing the economic behaviour of dairy producers	Germany	panel data 1981-85	Dairy	Quadratic
14. Higgins (1986)	Modelling input demand-output supply on farms	Ireland	micro-data 1982	Sector	Translog
15. Elhorst (1993)	Analysing investment	Netherlands	micro-data	Dairy	Quadratic
16. Mergos and Yotopoulos (1988)	Modelling the livestock sector	Greece	time series 1960-81	Livestock	Translog
17. Peeters and Surry (1991)	Estimating feed utilisation matrices	Belgium	time series 1962-88	Compound feed sector	S. G. Mc Fadden
18. Surry (1993)	Modelling the comp. feed sector	EC	time series 1964-84	Compound feed sector	CDE

Source	Restricted - Unrestricted	Static - Dynamic	Behaviour	Constraints	Specification Tests*	Curvature conditions
1. Glass and McKillop (1989)	Unrestricted	Static	Cost	Symmetry Homogeneity	Homotheticity R Const returns to scale R	Satisfied
2. Guyomard and Vermersch (1988)	Restricted	Static	Cost	Symmetry Homogeneity		Satisfied
3. Burrel (1989)	Restricted	Static	Profit		Homogeneity R Symmetry R	Violated
4. Grings (1986)	Restricted	Static	Profit	Symmetry Homogeneity CRS** convexity		
5. Boyle and Guyomard (1989)	Restricted	Static	Revenue	Symmetry Homogeneity Convexity		
6. Bouchet et al. (1988)	Restricted	Static	Profit	Homogeneity	Symmetry A	Violated
7. Becker and Frenz (1989)	Restricted	Static	Profit	Symmetry Homogeneity		Violated
8. Boyle and O' Neill (1989)	Restricted	Static	Profit	Symmetry Homogeneity Separability (3-stage optim.)	Parameter stability	Violated
9. Tiffin (1991)	Restricted	Static	Profit		Symmetry A Homogeneity R	Violated
10. Thijssen (1992a)	Restricted	Static	Profit	Homogeneity	Symmetry A Convexity A	
11. Thijssen (1992b)	Restricted	Dynamic	Profit	Homogeneity	Convexity A	
11. Fernandez-Cornejo et al. (1992)	Restricted	Dynamic	Cost	Homogeneity		
12. Luijt and Hillebrand (1992)	Restricted	Static	Profit	Convexity	Homogeneity R	
13. Stefanou et al. (1992)	Restricted	Dynamic	Cost	Quasi-homotheticity		
14. Higgins (1986)	Restricted	Static	Profit		Homogeneity A Symmetry R	Violated
15. Elhorst (1993)	Restricted	Dynamic	Profit			
16. Mergos and Yotopoulos (1988)	Unrestricted	Static	Cost	Symmetry Homogeneity Separability (2-stage optim.) CRS		
17. Peeters and Surry (1991)	Unrestricted	Static	Cost	Homogeneity Nonjointness in feed quantities CRS		Satisfied
18. Surry (1993)	Unrestricted	Static	Cost	Separability livestock outputs- feed ingredients		Violated (for 2 obs)

\* R = rejected; A = accepted.

\*\* CRS means Constant returns to scale.

*Table 4 : Own-price elasticity estimates of input demand obtained by several authors*

Source	Own-price elasticity of demand of				Detected complementarity
	Capital	Labour	Feed	Fertiliser	
Glass & McKillop (1989)	-2.02	-0.93	-0.37(a)	-0.66	Labour/Feed
Burrel (1989)	-	-	-	-0.42	
Bouchet et al. (1988)	-	-0.68	-1.90	-0.88	Labour/Feed
Boyle and O'Neill (1989)	-	-0.37	-1.07	-1.06(b)	Labour/Feed
Tiffin (1991)	-	-0.58	-0.22	-0.95	Feed/Fertiliser
Higgins (1986)	-	-0.38	-0.95	-1.38	Labour/Feed

(a) Feed, seed and livestock

(b) Nitrogen

## 5. Concluding comments

Research surveyed in this paper has noted impressive gains in our understanding of the EC agricultural production sector's response. Individual contributions to this strain of the literature must be commended both for paying attention to economic theory and for blending economic theory with econometric practice.

Despite our optimism about this accomplishments, some obvious shortcomings can be identified. Specifically, we find the evidence to be sketchy and fragmented. To fill remaining gaps in the literature, a coordinated research strategy needs to be pursued. Such a research strategy would emphasise :

- i) different functional forms should be tested for the same data set and for the same set of countries,
- ii) different industries within the same country should be studied,
- iii) different countries should be studied over the same time period using the same functional form,
- iv) the dynamic adjustment of quasi-fixed inputs should be analysed, ...

While these recommendations are ambitious, the current survey logically supports a more detailed analysis than has been previously attempted. The road to a better understanding of the impact of CAP reform on the production sector of individual EC Member States is paved with potholes. Unless these potholes are filled, our understanding will remain sketchy and incomplete.

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## Outline

1. Introduction.....	1
2. Illustrating how duality can be used to estimate models of primary production : the case of the profit function .....	2
2.1. The profit function : definition and properties.....	2
2.2. Empirical implementation.....	5
2.2.1. Criteria for functional form selection.....	7
2.2.2. Two examples : the translog and normalised quadratic unrestricted profit functions .....	8
3. Applying duality to agricultural economics : characteristics of the technology .....	11
3.1. The choice among behavioural assumptions .....	11
3.2. Substitutability of inputs and outputs .....	13
3.3. Economies of scale .....	15
3.4. Separability.....	17
3.5. Jointness .....	18
3.6. Technical change and total factor productivity .....	19
4. Duality theory and measurement of production response in the EC.....	21
5. Concluding comments.....	26
References .....	27