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ECONOMY AND TECHNOLOGY OF THE FRENCH CEREAL SECTOR : A DUAL APPROACH

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INTRODUCTION

Empirical analysis of agricultural technology is too often carried out within a long run static equilibrium framework : most of the studies assume a putty-putty production technology in which all inputs (and outputs if the objective function is the profit maximization) fully adjust to their optimal level. However, such an assumption seems restrictive and inappropriate, especially for the agricultural sector since certain inputs cannot be freely varied within the single period of observation. It is convenient to refer to this semiputty-putty model as one of short run static equilibrium : only the variable inputs adjust instantaneously to a change in relative prices ; the remaining inputs are fixed at observed levels (Lau, 1976 ; Kulatilaka, 1985). Although the putty-putty and semiputty-putty refer to as two different equilibria, both can achieve the same long run equilibrium.

Consequently the first objective of this paper is to provide an exhaustive characterization of a short run equilibrium. This one is derived from a restricted cost function. The second goal of this research is to show that this short run equilibrium framework allows the characterization of the different equilibria (short run and long run Hicksian equilibria ; short run and long run Marshallian equilibria), under sufficient conditions of curvature restrictions.

The paper is divided into six parts. Section 1 establishes the existence and properties of the restricted cost function, as well as the duality results between technology and restricted cost minimization economic behavior. Section 2 develops the model of the firm's choice of technique in this optimization framework. Section 3 shows how the different possible equilibria can be derived from the knowledge of the restricted cost function. Section 4 proposes different possible measures of the disequilibrium. The model is then applied to the french cereal sector in section 5. The conclusions are outlined in section 6 : they must be seen as permitting a better unterstanding of previous studies.

1. Duality between technology and restricted cost minimization economic behavior.

Let us consider a firm which uses M + N inputs $(z_1, \ldots z_M, x_1, \ldots x_N) = (z, x)$, $x \geq 0$, $z \geq o$ at prices (p_x, p_z) to produce R outputs $(y_1, \ldots y_R) = y$. The corresponding production possibilities set, T, is supposed to have the following properties :

(i) T is closed and non-empty.

(ii) if $y \neq 0$ then $x \neq 0$ or $z \neq 0$.

(iii) for all $(x, y, z) \in T$, if $x < \infty$ and $z < \infty$ then $y < \infty$

(iv) There is free disposal of inputs and outputs ; i.e., for all $(x, y, z) \in T$, the production plan (x', y', z') such that : $(x' \geq x ; z' \geq z ; y' \leq y)$ is possible, i.e $(x', y', z') \in T$ (1)

(v) The set $X(z, y) = [x ; (x, y, z) \in T]$ is strictly convex.

A few comments can be made upon these assumptions. Properties (i), (ii) and (iii) can be viewed as regularity hypotheses upon T ; (i) alone ensures the existence of solutions to the cost minimization program ; (ii) states that there is no free production (if $x = 0$ and $z = 0 \Rightarrow y = 0$) ; (iii) is a boundedness hypothesis : it ensures the existence of the corresponding production function for every finite value of x and z.

Assumption (iv) is a monotonicity assumption : if (x, z) can produce y, then (x', z'), with $x' \geq x$ and $z' \geq y$ can also produce y ; furthermore, if (x, z) can produce y, it can produce also every y' such that $y' \leq y$. Until now, the free disposal hypothesis is quiet plausible in the cereal sector.

(1) $x' \geq x \Leftrightarrow x'_n \geq x_n \quad n = 1 \ldots N$

At last, assumption (v) is a basic hypothesis to ensure the dual approach in production theory.

The restricted cost function is then defined by :

$$CR\ (p_x,\ y,\ z) = \underset{x}{\text{Min}}\ [p'_x\ .\ x\ ;\ x \in X(y,\ z)] \qquad (1)$$

With a strictly positive input prices vector p_x, hypothesis (i) ensures the existence of CR $(p_x,\ y,\ z)$: furthermore, CR (p_x,y,z) is non-negative, positive when y is non-zero, non-decreasing, positively linear homogeneous, concave and continuous in p_x [Mac Fadden, 1978, p.11]. Program (1) considers the inputs $(z_1,\ \ldots\ z_M)$ as fixed factors and these are also arguments of the CR function : we show now that property (iv) implies that :

$CR(p_x,\ y,z)$ is non-decreasing in y
$CR(p_x,\ y,z)$ is non-increasing in z.

Consider y_o and y_1, two producible outputs such that $y_o \leq y_1$. By free disposal of outputs, we have : $X(y_1,z) \subset X(y_o,\ z)$ and this implies :

$$CR\ (p_x,\ y_o,\ z) = \underset{x}{\text{Min}}\ [p'_x.\ x\ ;\ x \in X(y_o,z)]$$
$$\leq \underset{x}{\text{Min}}\ [p'_x.x\ ;\ x \in X(y_1,z)].$$

or

$$\underset{x}{\text{Min}}\ [p'_x.x\ ;\ x \in X(y_1,\ z)\ ] = CR\ (p_x,\ y_1,\ z)$$

By the same way, let us consider, two vectors , z_1 and z_2, of fixed inputs such that $z_1 \leq z_2$. By free disposal of inputs, we have :

$X(y, z_1) \subset X(y, z_2)$

So, by the same argument as before, $CR(p_x, y, z_2)$ is obtained by a minimization upon a set $X(y, z_2)$ containing $X(y, z_1)$ and that implies :

$$CR(p_x, y, z_2) = \underset{x}{\text{Min}} [p'_x .x \; ; \; x \in X(y, z_2)]$$

$$\leq \underset{x}{\text{Min}} [p'_x .x \; ; \; x \in X(y, z_1)]$$

and $\underset{x}{\text{Min}} [p'_x . x \; ; \; x \in X(y, z_1) = CR(p_x, y, z_1)$

Subsequently, we will go back on that result. In addition, under the convexity hypothesis (v) of the section $X(y,z)$ of T, the knowledge of the restricted cost function $CR(p_x, y, z)$ is sufficient to describe, in an exhaustive manner, the short run Hicksian technology which is employed (2). Finally, duality results state that :

$$X^*(y,z) = [x \geq 0 \; ; \; p'_x .x \geq CR(p_x, y, z), \; \forall \, p_x > 0 \,] = X(y,z)$$

In order to establish this statement, Mac Fadden (Mac Fadden, 1978, p.8) uses, with the property of free disposal (iv), a less restrictive hypothesis of convexity upon $X(y,z)$: the convexity from below. However, we can see that these two last assumptions imply necessarily the convexity of $X(y,z)$:

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(2) The Hicksian short run technology, at level z_o, can be defined by : $T^{z_o} = [(x, y) : (x, y, z_o) \in T]$

Let be $X(y,z)$ convex from below ; so, we have :

$\forall$ x, $x' \in X(y,z)$; $\exists$ $x^* \in X(y,z)$ such that $x^* \leq x" = \lambda x + (1 - \lambda)x'$.

But, by free disposal of inputs, this equation implies :

$x" \in X(y,z) \Rightarrow X(y, z)$ is convex.

So, the two assumptions of convexity from below of the section $X(y,z)$ and free disposal imply that the section $X(y,z)$ is convex.

We end this characterization of the restricted cost function with a discussion about the differentiability assumption of CR (p_x, y, z). Indeed, in many economic applications, particularly comparative statics, it is convenient to know that the objective function has a differential in input prices for all positive input prices vectors ; for that reason, we imposed the strict convexity of $X(y, z)$ which implies that CR (p_x, y, z) is continuously differentiable in input prices ; furthermore, recent works show that this last involvement is an equivalence under certain assumptions upon the technology (see, for example, Mac Fadden, 1978 ; Saijo, 1983).

Differentiability of the cost fuction is intimately related to the well-known Shephard's lemma :

$$\partial CR / \partial p_{x_n} = \bar{x}_n (p_x, y, z) \qquad n = 1 \dots N \qquad (2)$$

In addition, as it has been noted by Mac Fadden (1978, I, p. 15), the cost function has first and second derivatives with respect to input prices for almost all positive input prices. This implies that the matrix of partial derivatives of inputs with respect to input prices (when they exist) is negative semi-definite, symmetric and of rank N-1. So, the following Hessian matrix :

$$\Sigma_{p_x p_x} = [\partial^2 CR / \partial p_{x_n} \partial p_{x_{n'}}] \qquad n = 1 \ldots N, \quad n' = 1 \ldots N$$

is negative semi-definite, symmetric and of rank N-1.

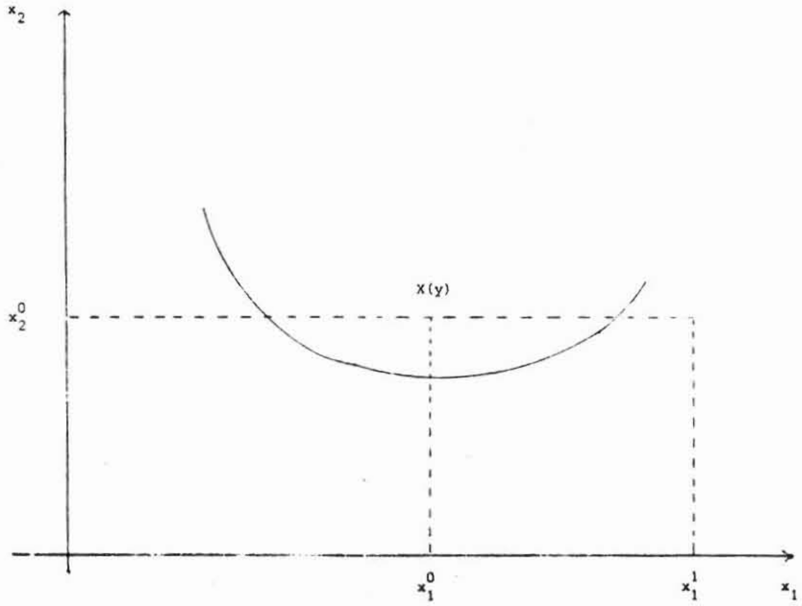
Finally, under assumptions of differentiability of CR (p_x, y, z) with respect to the fixed factors z_m and to the outputs y_r, it is convenient to define the total Hessian matrix :

$$\Sigma = \begin{bmatrix} \Sigma_{p_x p_x} & \Sigma_{p_x z} & \Sigma_{p_x y} \\ \Sigma_{z p_x} & \Sigma_{zz} & \Sigma_{zy} \\ \Sigma_{y p_x} & \Sigma_{yz} & \Sigma_{yy} \end{bmatrix}$$

At short run Hicksian equilibrium it is not necessary to impose curvature restrictions on the submatrices Σ_{zz}, Σ_{yy} and

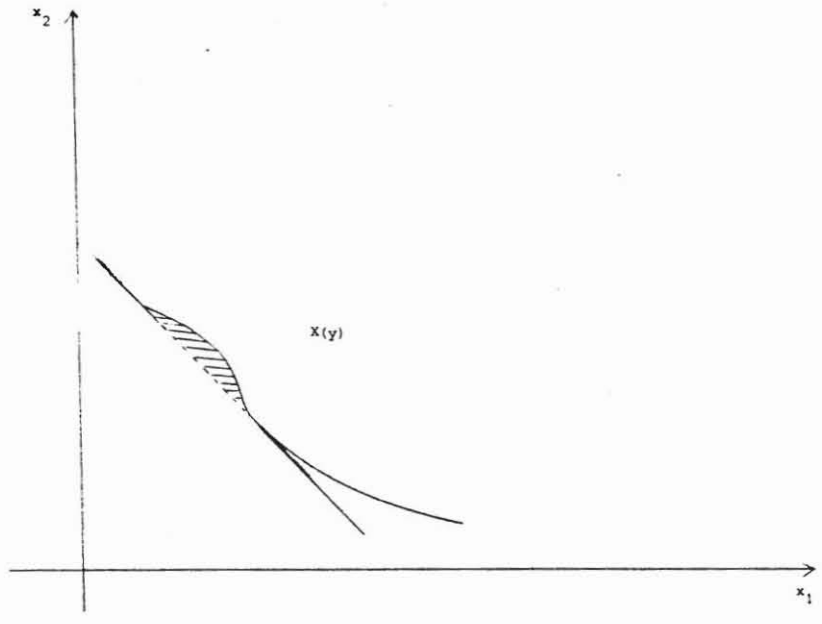
$$\begin{bmatrix} \Sigma_{zz} & \Sigma_{zy} \\ \Sigma_{yz} & \Sigma_{yy} \end{bmatrix}.$$

Figure 1. Lack of free disposal



$(x_1^0,\ x_2^0,\ y) \in T$ et $(x_1^1,\ x_2^0,\ y) \notin T$ with $x_1^1 > x_1^0$.

Figure 2. Non-convexity of X(y)



The support function C(y, p) generates the convex hull of X(y) : $X^c(y)$.

2. The restricted cost function : characterization of the technology at short run Hicksian equilibrium.

The restricted (or variable) cost function, corresponding to the programm of minimizing the cost of some subset of the input set, other factors being fixed, provides a functional characterization of the technology at short run Hicksian equilibrium. By Shephard's lemma, the short run compensated variable input demand is defined as the partial derivative of the restricted cost function with respect to price :

$$\partial \, CR \; (p \; , \; y, \; z) \; / \; \partial \, p_{x_n} \; = \bar{x} \; (p_n, \; y, \; z)$$

The constant outputs and quasi-fixed factors price elasticities of demand can be computed from the variable cost function : $\bar{\varepsilon}_{nn'}$, is the logarithmic derivative $\partial \log \bar{x}_n \,/\, \partial \log p_{x_n}$, where y and z are held constant, all x_n (n = 1, ... N) adjust to their short run compensated optimal levels :

$$\bar{\varepsilon}_{n.n'} = \partial \operatorname{Log} \bar{x}_n \;/\; \partial \operatorname{Log} p_{x_{n'}} \Bigg|_{p_{x_i}, z, y \qquad i \neq n'} \qquad (3)$$

A short run measure of partial returns to scale, i.e. for each commodity y_r, can be defined as the increase in output y_r resulting from a proportional increase in all variable inputs, with the z_m and $y_{r'}$, ($r' \neq r$) held fixed (at the observed levels). RTS_r^{SR} is the reciprocal of the elasticity variable cost with respect to this output along a local expansion path (or short run global expansion path).

$$RTS_r^{SR} = 1 / (\partial \log CR / \partial \log y_r) \quad \forall r = 1, \ldots, R \qquad (4a)$$

The overall short run returns to scale are therefore obtained as :

$$RTS^{SR} = 1 / \sum_{r=1}^{R} (\partial \log CR / \partial \log y_r) \qquad (4b)$$

Following Caves, Christensen et Swanson (1981), a different measure of returns to scale can be defined as the percentage increase in short run total cost resulting from a proportional increase in each output. This last measure, RTS^{MR}, is not calculated along the global expansion path, relative to all inputs, in a sense where the quasi-fixed factors are not necessarily initially at their optimal levels (long run Hicksian optimal levels). So, it generally differs from the traditional long run measure derived from a total cost, function (Vermersch, 1988) : RTS^{LR}

$$RTS^{MR} = (1 - \sum_{m=1}^{M} (\partial \log CR / \partial \log z_m)) / \sum_{r=1}^{R} (\partial \log CR/\partial \log y_r) \qquad (5)$$

$$RTS^{LR} = 1 / \sum_{r=1}^{R} (\partial \log CT / \partial \log y_r) \qquad (6)$$

The short run production technology exhibits increasing, decreasing and constant short run (respectively medium run) returns to scale for $RTS^{SR} > 1$, < 1 and $= 1$ (respectively for $RTS^{MR} > 1$, < 1 and $= 1$).

Under the new assumption of marginal variable cost pricing for outputs, RTS_r^{SR} is the inverse of the revenue share of output y_r.

if $MVC_r = \partial CR / \partial y_r = p_{y_r}$, then :

$$RTS_r^{SR} = (\partial \log CR / \partial \log y_r)^{-1} = 1 / (p_{y_r} . y_r / CR) = 1 / S_r^{CV}$$

The restricted cost function allows also to test the validity of separability (between inputs, outputs, inputs and outputs), jointness, homogeneity of degree μ in all inputs or in variable inputs only, ..., hypotheses of the short run technology. For example, by applying the theoretical results established by Hall (1973) in the case of a static total cost function, it is easy to show the following statements :

- If the production frontier is separable into a function of outputs and a function of inputs (sufficient condition for the existence of a consistent output aggregator function), the restricted cost function may be written as :

$$CR\ (p_x, y, z) = g\ [\ h(y), p_x, z\ ] \qquad (7)$$

- The short run technology is nonjoint in input quantities if the restricted cost function may be written as the sum of independent cost functions for each kind of output :

$$CR\ (p_x, y, z) = \sum_{r=1}^{R} CR^{(r)}(y_r, p_x, z) \qquad (8)$$

The change in single short run variable factor demand can be decomposed into four effects as shown in the following equations obtained by total derivation of input $\bar{x}_n$ demand equation : (figure n° 3)

$$\bar{x}_n \, (p_x, \, z, \, y, \, A)$$

$$\Rightarrow \quad d\,\bar{x}_n = \sum_{r=1}^{R} (\partial\,\bar{x}_n / \partial\,y_r) \,.\, d\,y_r + \sum_{m=1}^{M} (\partial\,\bar{x}_n / \partial\,z_m) \,.\, d\,z_m$$

$$+ \sum_{n'=1}^{N} (\partial\,\bar{x}_n / \partial\,p_{x_{n'}}) \,.\, d\,p_{x_{n'}} + (\partial\,\bar{x}_n / \partial\,A) \,.\, d\,A \qquad (9a)$$

$$\Rightarrow d\log\bar{x}_n = \sum_{r=1}^{R} (\partial\log\bar{x}_n / \partial\log y_r)\, d\log y_r \qquad E_1$$

$$+ \sum_{m=1}^{M} (\partial\log\bar{x}_n / \partial\log z_m)\, d\log z_m \qquad E_2$$

$$+ \sum_{n=1}^{N} (\partial\log\bar{x}_n / \partial\log p_{x_{n'}})\, d\log p_{x_{n'}} \qquad E_3$$

$$+ (\partial\log\bar{x}_n / \partial\log A)\;\; d\log A \qquad E_4 \qquad (9b)$$

$$\Rightarrow \quad d\log(x_i \,/\, x_j) = \sum_{r=1}^{R} (\partial\log(x_i/x_j) \,/\, \partial\log y_r) \,.\, d\log y_r \qquad E_1$$

$$+ \sum_{m=1}^{M} (\partial \log (x_i/x_j) / \partial \log z_m) . d \log z_m \qquad E_2$$

$$+ \sum_{n=1}^{N} (\partial \log (x_i/x_j) / \partial \log p_{x_n}) . d \log p_{x_n} \qquad E_3$$

$$+ (\partial \log (x_i/x_j) / \partial \log A) . d \log A \qquad E_4$$

(9c)

with :

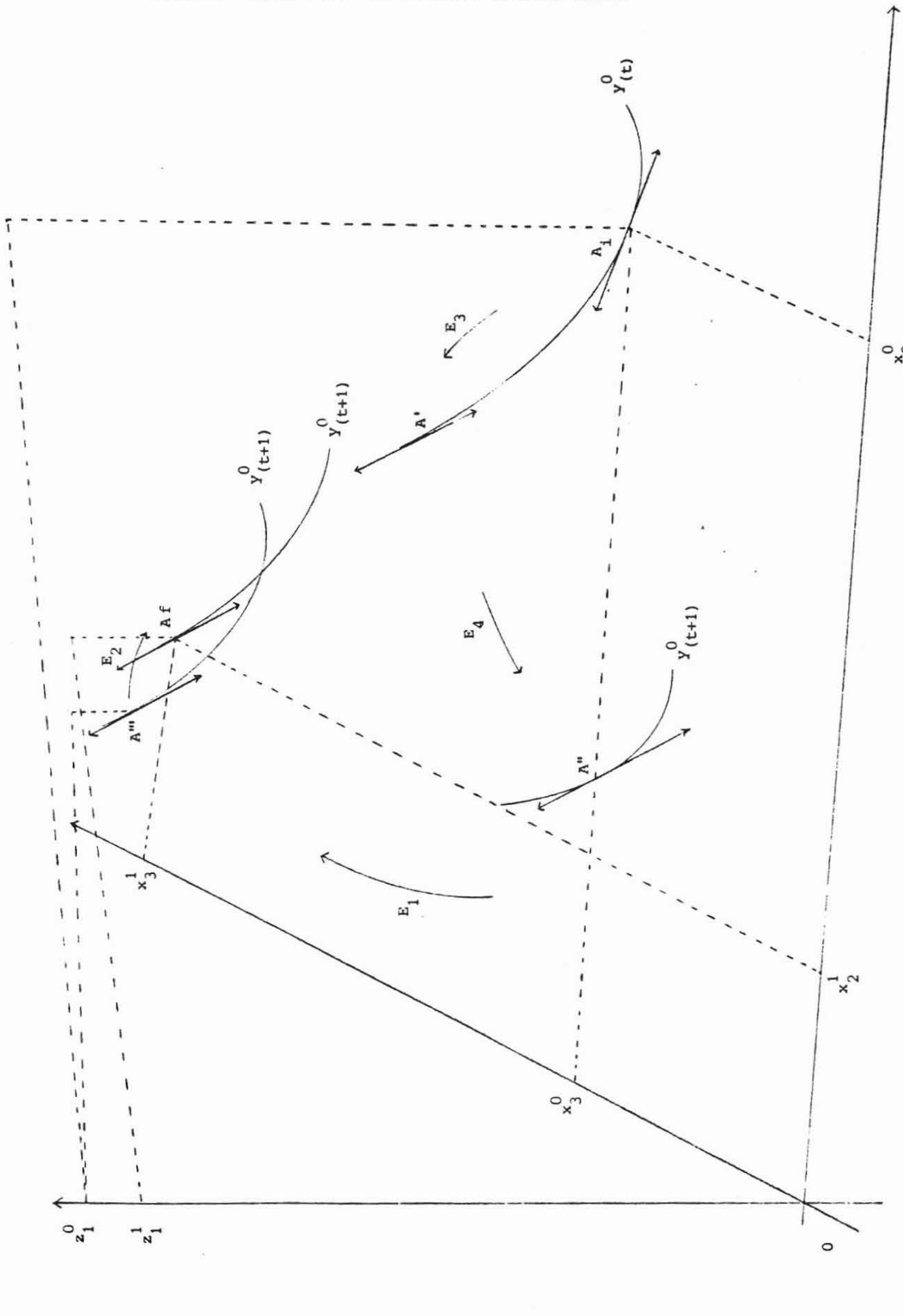
E_1 = outputs effect(s)

E_2 = quasi fixed factors effect(s)

E_3 = total short run substitution effect(s)

E_4 = technical change, research or development expenditures, ..., effect(s).

Figure 3. Decomposition of relative short run variable factor demand : illustration in the one output, one quasi-fixed factor and two variable input case



3. Inferring theoretical possible equilibria from the restricted cost function.

The model further allows the calculation of shadow price for each quasi fixed factor (Lau, 1976) :

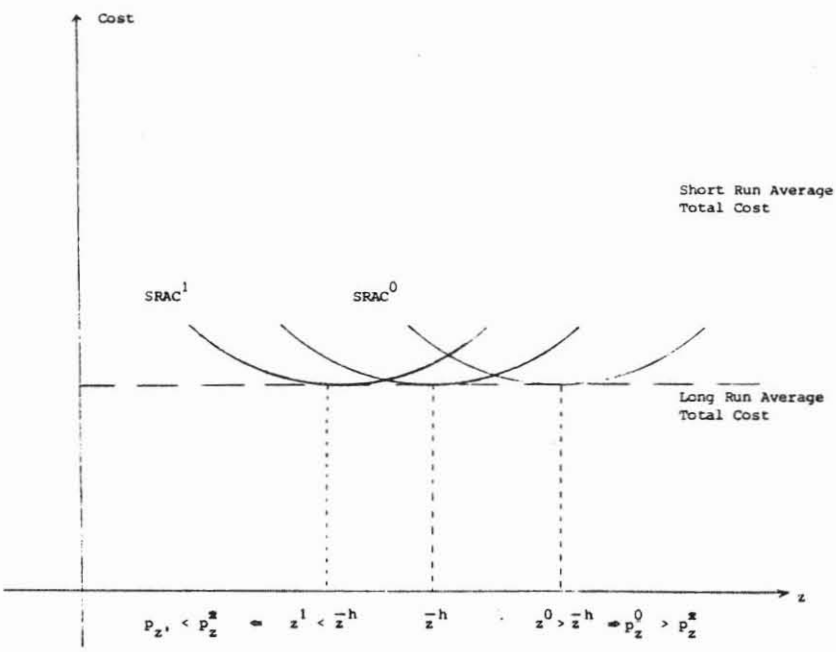
$$- \partial \ CR \ (p_x, \ z, \ y) \ / \ \partial \ z_m = p^*_{z_m} \ (p_x, \ z, \ y) \qquad (10)$$

The term ∂ CR / ∂ z_m is the reduction in variable costs for a unit increase in the level of the quasi-fixed factor z_m. As intuition suggests and as demonstrated by Lau (1976) and Diewert (1982), at the long run cost minimizing level of a quasi-fixed input, the shadow price $p^*_{z_m}$ is equal the observed market price p_{z_m}. If the ex-post shadow price and the ex-ante observed price for each quasi-fixed factor are equal, then the actual vector z corresponds to the long run hicksian vector $\bar{z}^h$; and the short run and long run compensated equilibria coincide (Kulatilaka, 1985). Indeed, assuming that there exists only one quasi-fixed input and that the restricted cost function is strictly convex in z_m (in a domain which includes the observed and optimal long run hicksian level of z_m : sufficient condition) ; it is easy to show (Guyomard et Vermersch, 1987) : (figure 4)

$$p^*_{z_m} \leq p_{z_m} \Leftrightarrow \bar{z}^h_m \leq z_m \qquad (11a)$$

$$p^*_{z_m} \geq p_{z_m} \Leftrightarrow \bar{z}^h_m \geq z_m \qquad (11b)$$

Figure 4. Graphical comparison between $\bar{z}^h_m$ and z_m, $p^*_{z_m}$ and p_{z_m} in a simplified case



Under the assumptions of one output y, one quasi-fixed factor and long run returns to scale, at the $\bar{z}^h$ point :

$$\partial\, SRAC \,/\, \partial z = 0$$

where SRAC is the short run total average cost.

$$\Leftrightarrow \quad 1/y\ (\partial\, CR \,/\, \partial z + p_z) = 0$$

$$\Leftrightarrow \quad - p^*_z + p_z = 0$$

$$\Leftrightarrow \quad p^*_z = p_z$$

if $z > \bar{z}^h$ (figure 4), then the slope of the tangent to the SRAC curve in z is positive and so $p_z > p^*_z$. In a symmetrical way, if $z < \bar{z}^h$, then the slope of the tangent to the SRAC curve in z is negative and so $p_z < p^*_z$.

In a dual way, if the firm optimizes not only with respect to the variable inputs but also with respect to the outputs and is in short run Marshallian equilibrium, then, by Hotelling's lemma, the shadow prices (respectively the short run uncompensated levels) equal the markets prices (respectively the observed levels) of the outputs.

$$\partial \, CR \, [p_x, \, z, \, y] \, / \partial \, y_r = p^*_{y_r} (p_x \, , \, z, \, p_y) \tag{12a}$$

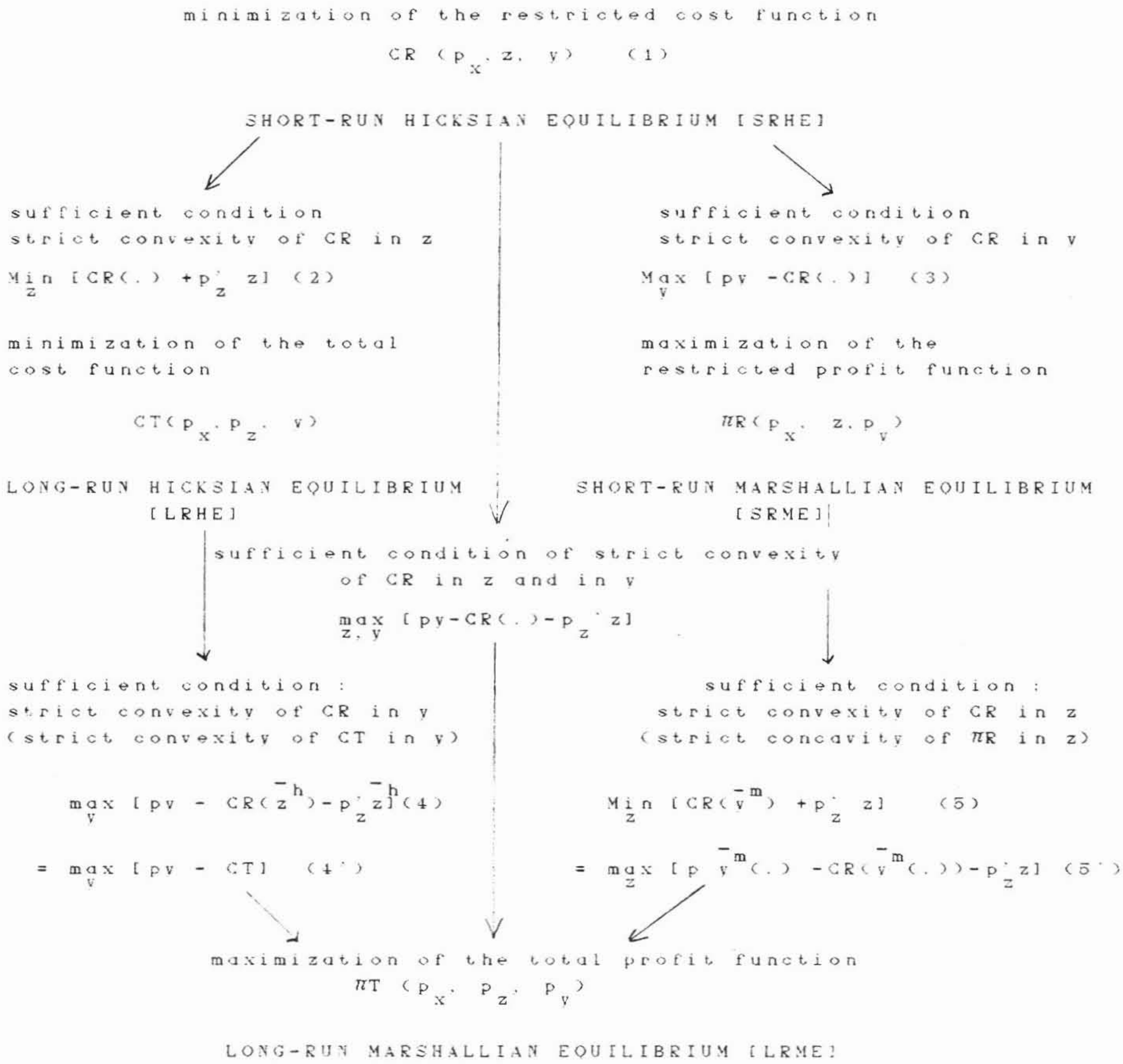
$$\partial \, CR \, [p, \, z, \, \bar{y}^m \, (p_x, \, z, \, p_y) \,] \, / \, \partial \, y_r = p_{y_r} \tag{12b}$$

Finally, if the firm optimizes with respect to the variable inputs, the quasi-fixed factors and the outputs and is in long run Marshallian equilibrium, then the shadow prices (of the quasi-fixed factors and the outputs) equal the observed prices. Therefore, the long run Marshallian situation, not necessarily achieved by the observed technology, can be approached by three equivalent ways : either by profit maximization with respect to z and to y, while holding the variable inputs at their cost minimizing levels ; or by profit maximization with respect to y given z and then by cost minimization with respect to z, holding the variable inputs at their cost minimizing levels ; or by cost minimization with respect to z given y and then by profit maximization with respect to y, the variable inputs held at their cost minimizing levels :

$$
\begin{aligned}
x^m_n(p_x,p_z,p_y) &= \bar{x}_n \, (p_x, \, z^m \, (p_x, \, p_z, \, p_y), \, y^m \, (p_x, \, p_z, \, p_y) \,) \\
&= \bar{x}_n \, (p_x \, , \, \bar{z}^h \, (p_x \, , \, p_z \, , \, y^m \, (p_x \, , \, p_z \, , \, p_y \,) \,), \, y^m \, (p_x \, , \, p_z \, , \, p_y \,) \\
&= \bar{x}_n \, (p_x, \, z^m \, (p_x, \, p_z, \, p_y) \, , \, y \, (p_x, \, z^m \, (p_x, \, p_z, \, p_y), \, p_y) \,)
\end{aligned} \tag{13}
$$

Therefore, it is possible to infer the different theoretical possible equilibria (short run and long run Hicksian equilibria, short run and long run Marshallian equilibria), given knowledge of short run environment, i. e. from the restricted cost function (cf. table 1).

Table 1. Characterization of the different theoretical equilibriums from the knowledge of the only restricted cost function $CR\ (p_x,\ z,\ y)$.



Sufficient conditions to use the framework presented in table 1 are the following :

Σ_{zz} must be positive definite (in a domain which includes the observed and long run Hicksian levels of z_m ; m = 1, ..., M) to infer the compensated demand functions from the short run compensated demand functions.

Σ_{yy} must be positive definite (in a domain which includes the observed and short run Marshallian levels of y_r ; r = 1, ... , R) to infer the uncompensated short run demand and supply equations from the short run Hicksian equilibrium.

$$\begin{bmatrix} \Sigma_{zz} & \Sigma_{zy} \\ \Sigma_{yz} & \Sigma_{yy} \end{bmatrix}$$ must be positive definite (in a domain which includes the observed and long run Marshallian levels of z_m and y_r ; m = 1, ..., M and r = 1, ..., R) to infer the uncompensated long run equilibrium from the compensated short run equilibrium.

4. Quasi-fixed factors desequilibrium : different possible measures

Assume that the firm's technology can be represented by a restricted cost function with only one output and one quasi-fixed factor : the generalization to the R outputs and M quasi-fixed factors case is straightforward. If the observed level of the quasi-fixed input is consistent with long run optimization behavior, i. e. minimization of the total cost function, then short run and long run Hicksian equilibria coïncide. In a dual way, in the price space, significant departures between the actual and the shadow prices of the quasi-fixed factor can characterize the disequilibrium.

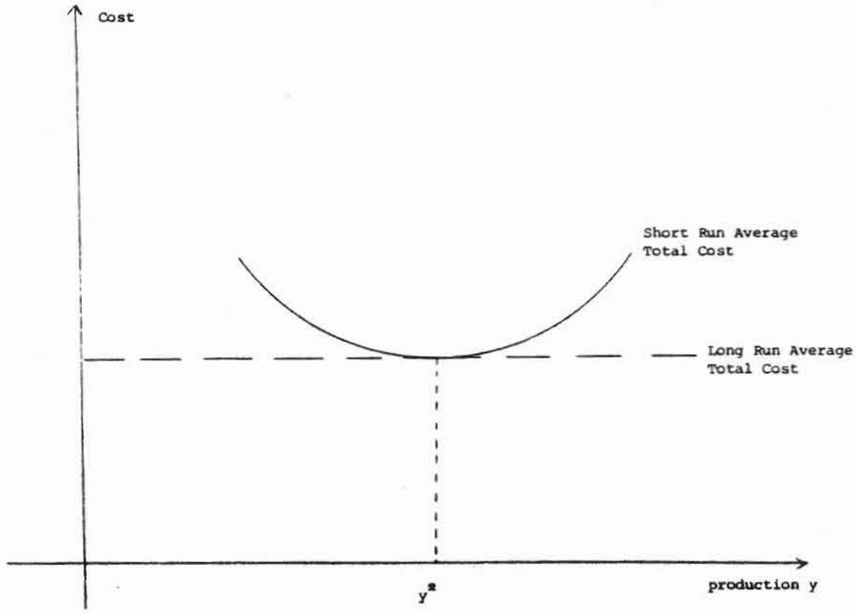
In the one output, one quasi-fixed factor case, three other measures of the possible disequilibrium can be proposed.

The first-one is the primal capacity utilization rate, DUC^P, defined as actual output y over capacity (or potential) output :

$$DUC^P = y / y^*$$

This capacity output y^* is simply defined as the output at which the short and long run average total cost curves are tangent to one another (Berndt and Morrison, 1981 ; Morrison, 1985, 1986) (figure 5 in the case of constant long run returns to scale, one output and one quasi-fixed factor). The ratio DUC^P captures, the difference between short run and long run Hicksian equilibria in the output quantity space. DUC^P is greater than, equal to, or less than unity only if the shadow price (respectively the optimal level) of the quasi-fixed factor is greater than, equal to, or less than the market price (respectively the observed level).

Figure 5. Capacity output y^*



Under the assumptions of one output y, one quasi-fixed factor z and constant long run returns to scale, the capacity output y^* is defined by :

$$\partial/\partial y \ [\ CR(y^*) \ / \ y^* + p_z \ z \ / \ y^* \ ] = 0$$

$$\Leftrightarrow \quad \partial \ CR(y^*) \ / \ \partial y \ . \ y^* - CR(y^*) - p_z \ z = 0$$

$$\text{or } \partial \ CR(y^*)/\partial y = [1 - (\partial \ CR(y^*)/\partial z) \ (z/y^*) \ ] \ [CR(y^*)/y^* \ ]$$

then

$$CR(y^*) - (\partial \ CR(y^*) \ / \ \partial z) \ z - CR(y^*) - p_z \ z = 0$$

$$\Leftrightarrow (\partial \ CR(y^*) \ / \ \partial z) \ z + p_z \ z = 0$$

$$\Leftrightarrow [(\partial \ CR(y^*) \ / \ \partial z) + p_z] \ z = 0$$

In the m quasi-fixed factors case, y^* is defined by :

$$\sum_{m=1}^{M} [\ (\partial \ CR(y^*) \ / \ \partial \ z_m) + p_{z_m} \ ] \ z_m = 0$$

The two last measures embody identical information, i.e. the difference between the long run Hicksian equilibrium and short run Hicksian equilibrium in the cost space. The first measure is equal to the ratio of long run returns to scale (RTS^{LR}) to medium run returns to scale, as defined in section 2 (RTS^{MR}). The second measure is the dual capacity utilization rate DUC^{D}. Morrison (1985) shows that many equivalent definitions of this utilization degree are possible. For simplicity we choose the measure defined as the ratio of shadow costs on total costs.

5. A case study for the french cereal sector

Our purpose, in this section, is simply to illustrate some previous theoretical issues in an econometric model in order to characterize the structure of the french cereal sector. Data concern a cross-section of farms and are detailed in Guyomard and Vermersch (1988). We specify a translog functional form for the restricted cost function with an additive error term ε_{CT} :

$$\text{Ln } CR\ (y, p_i, z_j) = a_o + a_1\ (\text{Ln } y) + \frac{1}{2} a_2\ (\text{Ln } y)^2$$

$$+ \sum_{i=1}^{4} c_i\ (\text{Ln } p_i) + \frac{1}{2} \sum_{i=1}^{4} \sum_{j=1}^{4} d_{ij}\ (\text{Ln } p_i)\ (\text{Ln } p_j)$$

$$+ \sum_{h=1}^{2} f_h\ (\text{Ln } z_h) + \frac{1}{2} \sum_{h=1}^{2} \sum_{k=1}^{2} g_{hk}\ (\text{Ln } z_h)\ (\text{Ln } z_k) +$$

$$\sum_{i=1}^{4} \sum_{k=1}^{2} k_{ih}\ (\text{Ln } p_i)\ (\text{Ln } z_h) + \varepsilon_{CT} \qquad (14)$$

where CR is restricted cost ; y is aggregate output ; p_i are variable input prices, as for fuel and oil, fertilizer, capital, hired labor ; z_j are the quasi-fixed inputs : family labor and land.

Without loss of generality, symmetry is imposed on the parameters d_{ij} and g_{hk}. Shephard's lemma gives the cost share equations, on which we add a disturbance term, ε_i, to reflect errors in optimization :

$$M_i = p_i\ x_i / CR = c_i + \sum_{j=1}^{4} d_{ij}\ (\text{Ln } pj) + \sum_{h=1}^{2} k_{ih}\ (\text{Ln } z_h) + \varepsilon_i \qquad (15)$$

$i = 1, 2, 3, 4$

Symmetry and additivity constraints ensure the theoretical restriction of homogeneity of degree one in inputs prices.

The set of equations (14) and (15) will be used to estimate the parameters of CR, from which the short run Hicksian price elasticities of demand will be derived :

$$E_{ij}^{CT} = (d_{ij} + M_i M_j) / M_i$$

$$E_{ii}^{CT} = (d_{ii} + M_i^2 - M_i) / M_i \qquad \forall i,\ \forall j,\ j \neq i$$

By using the procedure initially developped by Brown and Christensen [1981, p. 212-214]; we can compute the long run Hicksian price elasticities of demand, not only for a variable factor but also for a quasi-fixed factor. By the same way, a long run measure of returns to scale can be derived. We obtain the following expressions for the long run Hicksian price-elasticities :

$$E_{ij}^{LT(h)} = E_{ij}^{CT} - (1/M_i)(A(i,j)/B(h,k))$$

$$E_{ih}^{LT(h)} = (M_k/M_i).(C(i,\ h,\ k)/\ B(h,k))$$

$$E_{hi}^{LT(h)} = -\ C\ (i,\ h,\ k)/\ B\ (h,k)$$

$$E_{hk}^{LT(h)} = -\ M_k(g_{kh} + M_h .\ M_k)\ /\ B\ (h,k)$$

$$E_{hh}^{LT(h)} = M_h\ (g_{kk} + M_k^2 - M_k)\ /\ B\ (h,k)$$

$$M_h = -\ p_h .\ \bar{z}_h^h\ /\ CR\ (y,\ p_x,\ \bar{z}^h)$$

$$M_k = -\ p_k\ .\ \bar{z}_k^h\ /CR\ (y,\ p_x,\ \bar{z}^h)$$

$$A(i,j) = (M_i\ M_k + k_{ik})[(g_{hh} + M_h^2 - M_h)(M_j\ .\ M_k + k_{jk}) -$$

$$(g_{hk} + M_k .\ M_h)(M_j\ M_h + k_{jh})] +$$

$$(M_i\ M_h + k_{ih})\ [(g_{kk} + M_k^2 - M_k)(M_j\ .M_h + k_{jh}) -$$

$$(g_{hk} + M_k\ M_h)(M_j\ M_k + k_{jk})]$$

$$B(h,k) = (g_{kk} + M_k^2 - M_k)\ (g_{hh} + M_h^2 - M_h) - (g_{hk} + M_h\ .M_k)^2$$

$$C\ (i,\ h,k) = (g_{kk} + M_k^2 - M_k)(M_i\ M_h + k_{ih}) - (g_{hk} + M_h\ M_k)(M_i M_k + k_{ik})$$

i and j refer to a variable factor

h and k refer to a quasi-fixed factor.

The parameter estimates for the final form of the model are shown in table 2 together with their estimated standard errors. For each point, the estimated cost shares are positive and the concavity of the restricted cost function in inputs prices is verified at the sample average. If z_h and z_k represent respectively the levels of family labor and land, at the sample average, the fitted cost function is convex in z_h and concave in z_k: the wrong sign of parameter f_k is probably derived from multicollinearity between z_k and y, the level of output. However, f_k is reestimated by a production function model relying y (the level of output) to z_k (the level of the quasi-fixed factor), in order to solve the optimal level $\bar{z}_k^h$: in this case, the multicollinearity problems are replaced by simultaneity problems.

Table 2. Parameter estimates

Parameter	Estimate	Standard error
a_0	12.146	0.035
a_1	0.379	0.121
a_2	0.055	0.117
c_1	0.109	0.004
c_2	0.306	0.011
c_3	0.139	0.013
d_{11}	0.027	0.015
d_{12}	- 0.026	0.007
d_{13}	- 0.022	0.011
d_{22}	0.107	0.016
d_{23}	0.002	0.018
d_{24}	- 0.084	0.018
d_{34}	- 0.081	0.031
d_{33}	0.101	0.033
d_{44}	0.145	0.039
f_1	- 0.137	0.086
f_2	0.730	0.144
g_{11}	- 0.080	0.136
g_{12}	0.011	0.204
g_{22}	- 0.228	0.311
k_{11}	0.004	0.100
k_{12}	- 0.002	0.009
k_{21}	0.035	0.029
k_{22}	0.034	0.025
k_{31}	- 0.095	0.033
k_{32}	0.009	0.030

Estimates of short-run Hicksian price elasticities of demand, evaluated at sample average, are shown in table 3 with their estimated t ratios. Except for hired labor, all the estimated own-price elasticities are significant at the 5 % level. Demands for energy and capital are the most price-responsive ; significant substitution possibilities between capital and hired labor, capital and fertilizers are found. At the sample average, ECH^{CT} = 2,63 : this figure may be viewed as too high but multicollinearity problems or omitted factors can distort this result. Nevertheless, the result is consistent with other measures of returns to scale based on a total cost function where all inputs are variable [Boutitie and al., 1987] : both studies show that there are scale economies.

Table 3. Short-run Hicksian price elasticities of demand, evaluated at sample average, z_h and z_k (h: family labor, k : land)

price / quantity	fuel and oil	capital	hired labor	fertilizers
fuel and oil	- 0.638 (4.63)	- 0.075 (1.24)	- 0.065 (0.62)	0.628 (4.53)
capital	- 0.026 (1.24)	- 0.344 (6.68)	0.145 (3.91)	0.173 (2.98)
hired labor	- 0.051 (0.62)	0.321 (3.91)	- 0.136 (0.56)	- 0.135 (0.6)
fertilizers	0.153 (4.53)	- 0.118 (2.98)	- 0.042 (0.6)	- 0.23 (2.65)

Assuming that observed levels of quasi-fixed factors, are long run Hicksian levels, estimates of long run price elasticities of demand are shown in table 4 : these are consistent with Le Chatelier principle. Substitution possibilities appear between family labor and hired labor, family labor and capital, land and energy, land and capital, land and fertilizers.

Table 4. Long-run Hicksian price elasticities of demand, evaluated at sample average z_h and z_k (h : family labor, k : capital)

price / quantity	fuel and oil	capital	hired labor	fertilizers	family labor	land
fuel and oil	- 0.706	-0.047	-0.16	0.307	0.119	0.492
capital	- 0.0168	-0.425	-0.065	-0.022	0.130	0.268
hired labor	- 0.129	-0.144	0.496	-0.337	-0.961	-0.143
fertilizers	0.075	-0.015	-0.105	-0.634	-0.039	0.718
family labor	0.034	0.106	0.355	0.047	-1.09	0.064
land	0.170	0.262	-0.063	1.025	0.763	-2.16

Table 5 shows the long run Hicksian price elasticities in the case where only fixity of land is relaxed : the previous relations of substituability with land also appear ; the last elasticities can't be compared, in the view of the Le Chatelier principle, with the previous short run Hicksian price elasticities because, in each case, the point of approximation is different. Finally, in the case where fixity of land is relaxed, the measure of returns to scale give :

$$ECH^{LT}\ (z_h,\ \bar{z}_k\ (z_h)) = 3.69$$

where h corresponds to family labor and k to land.

Table 5. Long-run Hicksian price elasticities of demand, evaluated at sample average, z_h and $\bar{z}_k^h$ (h : family labor, k :land)

price / quantity	fuel and oil	capital	hired labor	fertilizers	land
fuel and oil	- 0.734	- 0.192	- 0.205	0.228	0.89
capital	- 0.07	- 0.606	0.012	0.22	0.87
hired labor	- 0.156	0.030	- 0.283	- 0.575	0.966
fertilizers	0.06	- 0.15	- 0.18	- 0.63	0.895
land	0.013	0.036	0.018	0.054	-0.119

6. Concluding remarks

In this last section, we point out that the explanatory statements, provided in the preceding sections, allow a better understanding of some previous empirical works.

First, let us consider the case of a negative shadow price for a quasi-fixed factor (see, for example, Huy, Elterich and Gempesaw, 1987). This dual price is simply obtained by derivation of $CR(p_x, y, z)$ with respect to z_m.

$$\partial\, CR\ (p_x,\ y,\ z)\ /\ \partial\ z_m = -\ p^*_{z_m}$$

This equation implies that a negative dual price $p^*_{z_m}$ is equivalent to impose that the restricted cost function is increasing with respect to z_m, in a neighbourhood of z_m. However, as it was demonstrated in the first section, property (IV) of free disposal implies that CR (p_x, y, z) is non-decreasing in z_m. Consequently, a negative shadow price involves a lack of free disposal. This last statement is untenable regarding some usual inputs as land or capital for instance. So, except the lack of free disposal, an economic interpretation of a negative shadow price is risky.

Our point of view is that this issue is provided by wrong signs of parameters which are probably due to multicollinearity problems.

Secondly, most applied studies take only one quasi-fixed factor into account. We have developed the analytical necessary expressions for derivating the long run Hicksian price elasticities within the specific case of a translog restricted function and two quasi-fixed factors. However, the generalization to the m quasi-fixed factors case is not straightforward. Thus, the formulas established by Squires (1987) are inappropriate, particularly because the optimal cost share of each quasi-fixed input are not explicitely and analytically defined.

Finally our paper shows the importance of taking into account the quasi-fixity of some inputs. The research also illustrates the sensitivity of findings to the specification of equilibria. So, if the farmers are short run profit maximisers (model of Higgins, 1986) the calculated substitution effects represent a movement along the old isoquant and the expansion effect. But the uncompensated elasticities do not correspond to the total effects of a price change due to the quasi fixity of some inputs.

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