Supply behavior with production quotas and quasi-fixed factors

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SUPPLY BEHAVIOR WITH PRODUCTION QUOTAS AND QUASI-FIXED FACTORS

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1. Introduction

In the recent period policy makers have become reluctant to cut price support in order to reduce the widespread excessive capacity of the farm sector in developed countries. Intervention schemes which do not disturb the vested interests but prevent budget costs from increasing further have become more popular.

As making the transfer as obscure as possible is also a target of farm pressure groups, it does not come as a surprise that production quotas, land set aside, fertilizer rationing and all socalled supply management policies play an increasing role in farm programs.

There is a growing literature on production quotas and supply control policies, particularly in Canada where these types of instruments have been implemented for some time (Arcus, 1978 ; Barichello, 1981, 1984 ; Veeman, 1982 ; Schmitz, 1983 ; Gouin, 1988). In Europe, the dairy quota has attracted an extensive work of analysis (Harvey and Hubbard, 1984 ; Perraud ed., 1988).

Most studies deal with the evaluation of the rent or of the transfers and welfare loss in a partial equilibrium framework approach in the context of markets for quotas. But quotas have also become a challenge to model builders, since models estimated and designed in an environment of price support policy instruments have to be used now in the context of a supply management scheme. The natural response of the economist in that context is to find out the "price cut equivalent to the quota constraint". This has been done by some authors (Ionnidis, 1981 ; Bingley, Burton, Strak, 1985). But a real endogeneisation of the shadow price equivalent to the quota, which traces the cross effects in the model, is rarely done. This is however important in multioutput multiinput commodity models. Munk (1985) has achieved an actual endogeneisation of the milk and sugar shadow prices in a EC supply model closely based on modern production theory. Mahé, Tavéra, Trochet (1988) have also fully endogeneised the dairy and sugar shadow prices in an international agricultural trade model focussing on EC and US.

Lau (1976) has characterized the conditions under which supply and demand parameters without quantity constraints can be deduced from the parameters estimated under some input or output fixity and has shown how Le Chatelier's principle applies. Brown and Christensen (1981) ; Kulatilaka (1985) ; Squires (1987) ; Guyomard and Vermersch (1989) have implemented practically the procedure proposed by Lau in order to derive long-run input demand elasticities from short-run parameters estimated under input and/or output fixity.

Recently Moschini (1988) has, for the first time apparently, used the dual approach to estimate a multioutput - multiinput supply model in a context of supply management policies. He raises the issue of analysing the comparative statics of producer's response under supply management and, after studying two sources of jointness in input quantities (Kohli, 1981 ; Shumway, Pope, Nash, 1984), he states that *without a general guideline, whether or not supply restricting policies will increase or decrease the supply of unrestricted output and the demand of variable inputs will be pursued at the empirical level.*

The purpose of the present paper is to explore various cases under which the comparative statics of supply behavior under input and output rationing can be characterized in more details. We treat jointly the cases of a quota and a factor fixity as they really are mirror images of the same problem. In order to do that, the theoretical basis of the approach is laid down in section 2 where the use of shadow prices and disequilibrium gaps is introduced as a simple way to carry the analysis. In section 3, we develop the essentials of the comparative statics of supply behavior with respect to the level of the quota and the fixed factor. Emphasis is put on the comparison of the supply behavior of unconstrained quantities while some other inputs or outputs are rationed, with respect to the same behavior without rationing. Attention is centered on cross price effects. Section 4 deals with a dynamic approach of the same problem, and simple analytical relations between short-run and long-run responses are derived. They lead to an estimable model of the demand for quasi-fixed inputs which is less general but simpler to specify than the one of Epstein (1981). A numerical illustration of the gradual evolution of supply and derived demand elasticities over the time lags after the initial price changes, is finally provided.

2. A theoretical framework to analyse supply behavior under rationing

In order to characterize the behavior under constraints, we make use the knowledge of supply response without or before the implementation of the rationing. This is made easy under fairly general conditions as the comparative statics of unconstrained supply behavior is better known (e. g. Sakai, 1974). As there is an evident symmetry between introducing constraints on the one hand and relaxing them on the other, the comparison with the problem of moving from short-run to long-run response is of some interest.

2.1. The effect of rationing on producer's behavior

When all prices are given and producers are free to adjust immediately, the familiar producer problem is,

$$(2.1)\quad \max_{q} \{ v'q \; ; \; q \in T \} = \Pi^u (v)$$

where q is the vector of $(n+m)$ netput quantities, v' the (transposed) vector of corresponding prices and $\Pi^u(v)$ the (unconstrained) profit function. The feasible set T is assumed strictly convex so that optimal quantities are uniquely determined and well behaved function of prices. The vector q is partitionned into two subvectors of quantities q_1 always variable and quantities q_0 susceptible of being constrained. A similar subdivision applies to the vector of prices v. Problem (2.1) can then be written as, (2.2) $\underset{q_1, q_0}{\text{Max}} \{v'_1 q_1 + v'_1 q_0 ; (q_1, q_0) \in T\} = \Pi^u (v_1, v_0)$

Supply functions of netputs are obtained from Hotelling's Lemma,

(2.3) a) $\Pi^u_{v_1} = q^u_1 (v_1, v_0)$

(2.3) b) $\Pi^u_{v_0} = q^u_0 (v_1, v_0)$

where $\Pi^u_{v_1}$ and $\Pi^u_{v_0}$ are the gradients of the profit function with respect to v_1 and v_0. When quantitites are pegged at say $\bar{q}_0$ by policy instruments (quotas, set aside...) or market rigidities and adjustment cost (factor fixity in short-run), variable quantities q_1 do not behave in the same way with respect to exogenous prices v_1, since they are also a function of $\bar{q}_0$. Following Rorbarth (1941), define ν_0 the vector of virtual prices which ensure that the unconstrained quantities q^u_0 as functions of prices will stay at level $\bar{q}_0$, by :

(2.4) $q^u_0 (v_1, \nu_0) = \bar{q}_0$,

Then from the knowledge of (2.3) we can infer how supply managed goods will modify the behavior of the other supplied and demanded goods and factors. Following Sakai's (1974) decomposition rule, and if (2.4) can be solved for the virtual prices ν_0 as function of v_1 and $\bar{q}_0$, we can define the relation between the restricted behavioral functions q^R_1 and the unconstrained functions defined as in (2.3),

$$(2.5) \quad q_1^R(v_1, \bar{q}_0) = q_1^u[v_1, \nu_0(v_1, \bar{q}_0)]$$

with ν_0 as defined by (2.4). Restricted supply and demand functions q_1^R can then be analysed, as they are equivalent to the solution of the system (2.3 a) and (2.4). In particular the comparative statics of (2.5) can be easily derived from the Hessian of the unrestricted profit function $\Pi^u(v_1, \nu_0)$, when twice differentiability is assumed.

$$(2.6) \quad \partial q_1^R(v_1, \bar{q}_0)/\partial v_1 = \partial q_1^u(v_1, \nu_0)/\partial v_1 + (\partial q_1^u(v_1, \nu_0)/\partial v_0).(\partial \nu_0/\partial v_1)$$

or,

$$(2.7) \quad \partial q_1^R(v_1, \bar{q}_0)/\partial v_1 = \Pi^u_{v_1 v_1}(v_1, \nu_0) + \Pi^u_{v_1 v_0}(v_1, \nu_0).(\partial \nu_0/\partial v_1)$$

The gradient of virtual price vector w.r.t. v_1 is derived from differentiating (2.4), taking (2.3) b) into account,

$$(2.8) \quad \Pi^u_{v_0 v_1}(v_1, \nu_0)\, d\, v_1 + \Pi^u_{v_0 v_0}(v_1, \nu_0)\, d\nu_0 = d\bar{q}_0$$

An explicit solution of (2.7) and (2.8) for dq_1^R in terms of dv_1 and $d\bar{q}_0$ around the point (v_1, ν_0) can be derived. The same result can be obtained by total differentiation of (2.5) with respect to v_1 and ν_0 where $d\nu_0$ and $d\bar{q}_0$ are related by equation (2.8) which defines ν_0 implicitely. The complete system of supply response can then be written in terms of the Jacobian of the unconstrained profit function.

$$(2.9)\quad \begin{bmatrix} dq_1 \\ d\bar{q}_0 \end{bmatrix} = \begin{bmatrix} \Pi^u_{v_1 v_1}(v_1, \nu_0) & \Pi^u_{v_1 v_0}(v_1, \nu_0) \\ \Pi^u_{v_0 v_1}(v_1, \nu_0) & \Pi^u_{v_0 v_0}(v_1, \nu_0) \end{bmatrix} \begin{bmatrix} dv_1 \\ d\nu_0 \end{bmatrix}$$

It should be emphasized that the cross partial derivatives of Π^u are evaluated at the point $(v_1, \bar{q}_0)$ i.e. (v_1, ν_0). Then (2.9) can be solved for the actual endogenous variables i.e. $(dq_1, d\nu_0)$ with respect to the new set of exogenous ones which are $(dv_1, d\bar{q}_0)$, that is, dropping the arguments in the profit function,

$$(2.10)\quad \begin{bmatrix} dq_1 \\ d\nu_0 \end{bmatrix} = \begin{bmatrix} \Pi^u_{v_1 v_1} - \Pi^u_{v_1 v_0} (\Pi^u_{v_0 v_0})^{-1} \Pi^u_{v_0 v_1} & \Pi^u_{v_1 v_0} (\Pi^u_{v_0 v_0})^{-1} \\ -(\Pi^u_{v_0 v_0})^{-1} \Pi^u_{v_0 v_1} & (\Pi^u_{v_0 v_0})^{-1} \end{bmatrix} \begin{bmatrix} dv_1 \\ d\bar{q}_0 \end{bmatrix}$$

The strict convexity of the production set ensures that $\Pi^u_{v_0 v_0}$ is positive definite and invertible (1). System (2.10) allows to completely characterize the comparative statics under constraints, on the basis of the information of the unconstrained supply and derived demand responses, evaluated at the relevant contrained equilibrium $(\bar{q}_0, \nu_0)$.

There is a similarity between the system of equations (2.10) which allows to describe the constrained behavior on the basis of the unconstrained one, and the problem of deriving long run behavior from the short-run responses. To be more specific, the same information as in (2.10) could be obtained directly if the Jacobian of the restricted profit function $\Pi^R(v_1, \bar{q}_0)$ is known. $\Pi^R(v_1, \bar{q}_0)$ is defined by the producer's problem :

$$(2.11)\quad \underset{q_1}{\mathrm{Max}} \left[v_1' q_1 : (q_1, \bar{q}_0) \in T \right] = \Pi^R (v_1, \bar{q}_0)$$

Applying the enveloppe theorem to (2.11) provides the supply functions of the unrestricted goods q_1 when goods q_0 are constrained at level $\bar{q}_0$. The same procedure also provides the shadow price ν_0 as a function of v_1 and $\bar{q}_0$.

$$(2.12)\ \mathrm{a)}\quad \Pi^R_{v_1} (v_1, \bar{q}_0) = q_1^R (v_1, \bar{q}_0)$$

$$(2.12)\ \mathrm{b)}\quad \Pi^R_{q_0} (v_1, \bar{q}_0) = - \nu_0 (v_1, \bar{q}_0)$$

An equation similar to (2.10) can be obtained by total differentiation of system (2.12),

$$(2.13)\quad \begin{bmatrix} dq_1 \\ -d\nu_0 \end{bmatrix} = \begin{bmatrix} \Pi^R_{v_1 v_1} (v_1, \bar{q}_0) & \Pi^R_{v_1 q_0} (v_1, \bar{q}_0) \\ \Pi^R_{q_0 v_1} (v_1, \bar{q}_0) & \Pi^R_{q_0 q_0} (v_1, \bar{q}_0) \end{bmatrix} \begin{bmatrix} dv_1 \\ d\bar{q}_0 \end{bmatrix}$$

The equivalence between (2.13) and (2.10) can be further explored by using a procedure similar to Lau or using an appropriate version of Sakai's decomposition. It can be shown that the "contraction effect" $-\Pi^u_{v_1 v_0} (\Pi^u_{v_0 v_0})^{-1} \Pi^u_{v_0 v_1}$ is just the negative of the expansion effect of the problem of moving from a short-run to a long-run equilibrium (Mahé, Guyomard , 1989).

In particular it appears in equation (2.10) that supply and derived demand response of goods q_1 with respect to the price vector v_1 is now modified. Moreover, another application of Le Chatelier's principle is apparent as the diagonal terms of the matrix $\Pi^u_{v_1 v_0} (\Pi^u_{v_0 v_0})^{-1} \Pi^u_{v_0 v_1}$ are quadratic forms around the positive definite matrix $(\Pi^u_{v_0 v_0})^{-1}$. Therefore output supply responses of unconstrained quantities to their own price become smaller with constraints put on quantities q_0. A similar result applies to the magnitude of input demand responses to their own prices. This is the exact counterpart of Lau's result derived from the restricted profit function.

As noted by Moschini (1988) what happens to cross price effects is much less obvious particularly when one uses the restricted profit function, where cross effects between quantities of the type $\partial q^R_{1,i}(v_1, \bar{q}_0) / \partial \bar{q}_{0,j}$, are not easy to interpret. The situation is much easier when one starts with $\Pi^u(v_1, \nu_0)$ since cross effects are just cross price elasticities and are more familiar parameters.

2.2. *The notion of "pairwise similarity"*

In order to characterize the behavior under rationing, it is convenient to introduce the definition of "pairwise similarity" : a pair of netputs q_r and q_s will be said to be similar with respect to a third netput q_0 if the cross-price elasticities of q^u_r and q^u_s with respect to v_0 have the same sign, that is, with usual notations if :

$$(2.14)\quad \varepsilon^u_{q_r v_0} . \varepsilon^u_{q_s v_0} = (v_0/q^u_r . \partial q^u_r/\partial v_0).(v_0/q^u_s . \partial q^u_s/\partial v_0) \geq 0$$

$r = \{i, h\}$; $s = \{j, k\}$; $i, j = 1, ..., n$; $h, k = 1, ..., m$.

For purpose of clarity, outputs $y_i = q_i$; $i = 1, \ldots, n$; with prices p_i and inputs $x_h = - q_h$; $h = 1, \ldots, m$; with prices w_h are now identified. Inequality (2.14) may then be written in terms of the second partial derivatives of the profit function $\Pi^u(v)$ and in terms of the cross price derivatives of the factor demand or output supply equations (2). Table 2.1 summarizes these different expressions by distinguishing the nature, output or input, of the netputs taken into account.

[insert table 2.1]

As an example, two inputs x_h and x_k are said to be similar with respect to input x_0 if both are substitutes for x_0 or if both are complements to x_0. Two outputs y_i and y_j are said to be similar with respect to x_0 if this factor is superior or inferior in the production of both outputs. In the same way, output y_i and input x_h are said to be similar with respect to input x_0 if $\varepsilon^u_{q_i v_0}$. $\varepsilon^u_{q_h v_0}$ is non negative, that is if input x_0 is superior (inferior) in the production of y_i and inputs x_h and x_0 are complement (substituable). Similar interpretations of inequality (2.25) can be extended to all the possible cases.

A pairwise similarity of two netputs with respect to a third netput is defined at a given equilibrium point. As an example, given price vector v, similarity is defined on the basis of the unconstrained profit function. Of course similarity of q_r and q_s with respect to q_0 at this point does not imply similarity at an other equilibrium point, and more generally, does not imply similarity globally. As a consequence, two netputs q_r and q_s can be similar with respect to q_0 at the point E corresponding to $\Pi^u(v)$ and not at the point E^* corresponding to $\Pi^u(v^*)$. It can be readily verified that a technology which is normal in the sense of Sakai (1974) has all pairs of inputs and outputs similar. It may be called strongly similar. This condition of similarity, which does not seem overly restrictive for a fairly aggregated definition of goods, will allow to characterize how the cross price effects between unconstrained goods are altered by constraining some quantities.

3. The modified cross effects of the comparative statics under rationing

The main results of comparative statics will be carried basically on system (2.10) which assumes (2.9) known, but in order to make the results more transparent and easier to interpret (i) inputs and outputs are separated again, and (ii), to keep notations hopefully simple and to provide convenient formulae for empirical use, the basic equation corresponding to (2.9) is written in terms of price elasticities. With a self evident partition of the matrix one gets :

$$(3.1)\quad \underset{(n+m)\times 1}{\begin{bmatrix} \hat{y}^1 \\ \hat{y}^0 \\ \hat{x}^1 \\ \hat{x}^0 \end{bmatrix}} = \underset{(n+m)\times(n+m)}{\begin{bmatrix} E^{11} & E^{10} & F^{11} & F^{10} \\ E^{01} & E^{00} & F^{01} & F^{00} \\ G^{11} & G^{10} & H^{11} & H^{10} \\ G^{01} & G^{00} & H^{01} & H^{00} \end{bmatrix}} \underset{(n+m)\times 1}{\begin{bmatrix} \hat{p}^1 \\ \hat{\rho}^0 \\ \hat{w}^1 \\ \hat{\omega}^0 \end{bmatrix}} + \underset{(n+m)\times 1}{\begin{bmatrix} \hat{a}^1 \\ \hat{a}^0 \\ \hat{b}^1 \\ \hat{b}^0 \end{bmatrix}}$$

where $\hat{y}^1$, $\hat{x}^1$ are the vectors of percentage changes in unconstrained output and input quantities with corresponding vectors of price changes $\hat{p}^1$ and $\hat{w}^1$ and the $\hat{a}^1$'s and $\hat{b}^1$'s the technical progress biases expressed in percentage terms as well. To keep the analytics simple only one output quota and one fixed factor are maintained. $\hat{y}^0$ is the percentage change of the quota with shadow price ρ^0 and $\hat{x}^0$, ω^0 are the equivalent notations for fixed inputs. $\hat{a}^0$ and $\hat{b}^0$ are the rate of technical change bias on fixed quantities. (3.5) has to be solved for the vector $[\hat{y}^1, \hat{\rho}^0, \hat{x}^1, \hat{\omega}^0]$ in terms of the vector $[\hat{p}^1, \hat{y}^0, \hat{w}^1, \hat{x}^0]$ and of the biases a's and b's. (3.1) is first solved for the virtual price changes ρ^0, ω^0,

$$(3.2) \quad \begin{bmatrix} \hat{y}^0 \\ \hat{x}^0 \end{bmatrix} = \begin{bmatrix} E^{01} & E^{00} & F^{01} & F^{00} \\ G^{01} & G^{00} & H^{01} & H^{00} \end{bmatrix} \begin{bmatrix} \hat{p}^1 \\ \hat{\rho}^0 \\ \hat{w}^1 \\ \hat{\omega}^0 \end{bmatrix} + \begin{bmatrix} \hat{a}^0 \\ \hat{b}^0 \end{bmatrix}$$

or after collecting unrationed and rationed outputs and factors together :

$$(3.3) \quad \begin{bmatrix} \hat{y}^0 \\ \hat{x}^0 \end{bmatrix} = \begin{bmatrix} E^{00} & F^{00} \\ G^{00} & H^{00} \end{bmatrix} \begin{bmatrix} \hat{\rho}^0 \\ \hat{\omega}^0 \end{bmatrix} + \begin{bmatrix} E^{01} & F^{01} \\ G^{01} & H^{01} \end{bmatrix} \begin{bmatrix} \hat{p}^1 \\ \hat{w}^1 \end{bmatrix} + \begin{bmatrix} \hat{a}^0 \\ \hat{b}^0 \end{bmatrix}$$

From the strict convexity of the production set, we know that the matrix $\begin{bmatrix} E^{00} & F^{00} \\ G^{00} & H^{00} \end{bmatrix}$ is non singular and its determinant

$D = E^{00} H^{00} - G^{00} F^{00}$ is negative (3).

The full comparative statics of endogenous variables i.e. shadow prices, quota rent, fixity loss and unconstrained supply and demand, can be investigated by solving equation (3.1). This has been done elsewhere (Mahé and Guyomard, 1989) and will not be pursued here (4). The impact of quotas and factor fixity on cross price elasticities will be explored under the condition of pairwise similarity introduced above.

In order to investigate only the cross price effects, we assume now $\hat{y}^0 = \hat{x}^0 = \hat{a}^0 = \hat{b}^0 = \hat{a}^1 = \hat{b}^1 = 0$. Then the percentage changes in unconstrained quantities can be expressed as follows :

(3.4)
$$\hat{y}^1 = [E^{11} + E^{10}(-H^{00}E^{01} + F^{00}G^{01})D^{-1} + F^{10}(-E^{00}G^{01} + G^{00}E^{01})D^{-1}]\hat{p}^1 + [F^{11} + E^{10}(-H^{00}F^{01} + F^{00}H^{01})D^{-1} + F^{10}(-E^{00}H^{01} + G^{00}F^{01})D^{-1}]\hat{w}^1$$

(3.5)
$$\hat{x}^1 = [G^{11} + G^{10}(-H^{00}E^{01} + F^{00}G^{01})D^{-1} + H^{10}(-E^{00}G^{01} + G^{00}E^{01})D^{-1}]\hat{p}^1 + [H^{11} + G^{10}(-H^{00}F^{01} + F^{00}H^{01})D^{-1} + H^{10}(-E^{00}H^{01} + G^{00}F^{01})D^{-1}]\hat{w}^1$$

Properties (5) concerning the cross price effects among unconstrained quantities can now be derived, under the assumption of similarity of each pair of goods with respect to y^0 and x^0 ; we first examine the one quota or one fixed factor case, which is unambiguous.

property 1 : under the similarity condition a binding constraint on an output or an input makes the unrestricted inputs and outputs more substitutable.

property 2 : under the similarity condition, a binding constraint on an output or an input makes the unrestricted outputs more regressive and consequently the unrestricted inputs more inferior.

If only one quota is implemented, which amounts to assuming that the system (3.1) is written without the last row, expressions (3.4) and (3.5) simplify into

$$(3.4a)\qquad \hat{y}^1 = \left[E^{11} - E^{10}\,(E^{00})^{-1}\,E^{01}\right]\hat{p}^1 + \left[F^{11} - E^{10}\,(E^{00})^{-1}\,F^{01}\right]\hat{w}^1$$

$$(3.5a)\qquad \hat{x}^1 = \left[G^{11} - G^{10}\,(E^{00})^{-1}\,E^{01}\right]\hat{p}^1 + \left[H^{11} - G^{10}\,(E^{00})^{-1}\,F^{01}\right]\hat{w}^1$$

A typical element of the matrix of cross price elasticities between outputs i and j in (3.4a) is E^{11R}_{ij}

$$(3.6)\qquad E^{11R}_{ij} = E^{11}_{ij} - E^{00})^{-1}\,E^{10}_{i0}\,E^{01}_{0j}$$

under the similarity condition, we have $E^{10}_{i0}\,E^{01}_{0j} \geq 0$ and since E^{00} is positive,

$$(3.7)\qquad E^{11R}_{ij} \leq E^{11}_{ij}$$

And the two outputs tend to become more substitutable under the output quota.

In a similar fashion a typical rationed cross elasticity between inputs in (3.5a) is given by :

$$(3.8) \quad H^{11R}_{hk} = H^{11}_{hk} - (E^{00})^{-1} G^{10}_{h0} F^{01}_{0k}$$

In the case of similarity $G^{10}_{h0} G^{10}_{k0} \geq 0$, and by symmetry $G^{10}_{h0} F^{01}_{0k} \leq 0$, therefore $H^{11R}_{hk} \geq H^{11}_{hk}$, hence inputs h and k tend to become less complements or more substitutes.

By a similar argument it can be seen that cross elasticity of input demands w.r.t. output prices become smaller algebraicly, hence the tendency toward inferiority,

$$(3.9) \quad G^{11R}_{hj} = G^{11}_{hj} - (E^{00})^{-1} G^{10}_{h0} E^{01}_{0j},$$

Similarity of input h and output j w.r.t. the output under quota q_0 implies $G^{10}_{h0} E^{01}_{0j} \geq 0$ and therefore $G^{11R}_{hj} \leq G^{11}_{hj}$. Similar results can be derived for one fixed factor.

This is as far as similarity can bring unambiguous results. As mentioned before, when one starts with the long-run normal technology of Sakai, where similarity is verified, implementing quotas or fixed factors one at a time will induce more substitution and more input inferiority. Hence we get away from the normal technology and at some point there will be pairs of inputs or outputs which may no longer be similar. It would be convenient to have more general results when several outputs or inputs are constrained in the same time as in expression (3.1) which leads to the following relation between E^{11R}_{ij} and E^{11}_{ij} :

$$(3.10) \quad E^{11R}_{ij} = E^{11}_{ij} + D^{-1} \begin{bmatrix} E^{10}_{i0} & F^{10}_{i0} \end{bmatrix} \begin{bmatrix} -H^{00} & -F^{00} \\ G^{00} & E^{00} \end{bmatrix} \begin{bmatrix} E^{01}_{0j} \\ -G^{01}_{0j} \end{bmatrix}$$

It is convenient to rewrite (3.10) so that submatrix $\pi_{\nu_0 \nu_0}$ with known properties appears,

$$\text{(3.6a)} \quad E_{ij}^{11R} = E_{ij}^{11} + D^{-1} \left(\frac{y^0 \cdot w^0 \cdot p_j}{y_i \cdot x^0 \cdot y^0} \right) Q, \text{ with}$$

$$\text{(3.6b)} \quad Q = (\partial y_i / \partial p^0, \ \partial y_j / \partial w^0) \begin{pmatrix} -\dfrac{\partial x^0}{\partial w^0} & \dfrac{\partial y^0}{\partial w^0} \\ \dfrac{\partial x^0}{\partial p^0} & \dfrac{\partial y^0}{\partial p^0} \end{pmatrix} \begin{pmatrix} \partial y^0 / \partial p_j \\ -\partial x^0 / \partial p_j \end{pmatrix}$$

As D is negative, if Q is non negative, then the tendency toward substitution would follow. It is worth noting first that when i = j, Q is now a quadratic form around a symmetric positive definite matrix, hence the Le Chatelier's effect is verified. When i $\neq$ j, the similarity condition can be used to write Q as a quasi-definite form (e.g. Murata, 1977, p. 67),

$$\frac{\partial y^0}{\partial p_j} = \frac{\partial y_j}{\partial p^0} = \alpha \frac{\partial y_i}{\partial p^0}, \text{ with } \alpha > 0$$

$$-\frac{\partial x^0}{\partial p_j} = \frac{\partial y_j}{\partial w^0} = \beta \frac{\partial y_i}{\partial w^0}, \text{ with } \beta > 0$$

Then Q can be written as a quadratic form around a non symmetric matrix,

$$Q = \begin{pmatrix} \dfrac{\partial y_i}{\partial p^0} & \dfrac{\partial y_i}{\partial w^0} \end{pmatrix} \begin{pmatrix} -\alpha \dfrac{\partial x^0}{\partial w^0} & -\beta \dfrac{\partial y^0}{\partial w^0} \\ \alpha \dfrac{\partial x^0}{\partial p^0} & \beta \dfrac{\partial y^0}{\partial w^0} \end{pmatrix} \begin{pmatrix} \partial y_i / \partial p^0 \\ -\partial y_i / \partial w^0 \end{pmatrix}$$

Let A be the non symmetric matrix in Q, from the positive definiteness of $\pi_{\nu_0\nu_0}$ the diagonal elements of A are positive. Then if A + A', where A' is the tranpose of A, has a column diagonal dominance, then A + A' is positive definite and Q is positive (Murata, p. 61). Column diagonal dominance of A + A' would be ensured by the existence of positive numbers d_1 and d_2 such that :

$$(3.7) \quad d_1 \, 2\alpha \left| - \frac{\partial x^0}{\partial w^0} \right| > d_2 \, . \, (\alpha+\beta) \left| \frac{\partial x^0}{\partial p^0} \right|$$

If one considers that in most cases the own price effect is larger than the cross effects, then $d_1 = 1/\alpha$ and $d_2 = 1/(\alpha+\beta)$ exist so that $\left| - \frac{\partial x^0}{\partial w^0} \right| > \left| \frac{\partial x^0}{\partial p^0} \right|$ ensures that (3.7) is verified. It is worth noting that if α and β are equal, i.e., if the two goods are such that the ratio of their supply responses with respect to the price of the good under quota is equal to the ratio of their supply response to the fixed factor, then the result of more substitution or less complementarity follows. Therefore while the result does not seem to be completely general with the condition of similarity, the tendency toward substitution when constraints are imposed seems to be valid under fairly general conditions.

4. The dynamics of adjustment of fixed quantities and the observable technology

In the previous section the nature of cross effects in the behavioral equations was shown to depend on the extent of binding contraints on producer profit maximizing decisions. The dual technology which is underlying is also clearly dependent on those contraints of fixity. As the producer cannot adjust immediately to price changes, the firm is never observed in an equilibrium either of short run or of long run but somewhere in between on a transitory path toward it. Then the speed of adjustment is the key factor in interpreting the observed technology as for example in econometric work where the adjustment process is not formally included but implicitely assumed. This section attempts to built on the disequilibrium framework used above to derive a simple dynamic estimable model of supply behavior in the presence of quasi-fixed factors.

Given the prices (v_{1t}, v_{2t}) of unconstrained and quasi-fixed netputs two behavioral models are relevant. The first is long-run equilibrium model which corresponds to costless and immediate adjustment to new prices. Assuming linearity, we obtain,

$$(4.1) \quad \begin{bmatrix} q^u_{1t} \\ q^u_{2t} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} + \begin{bmatrix} \alpha_1(t) \\ \alpha_2(t) \end{bmatrix}$$

where $\alpha_1(t)$, $\alpha_2(t)$ are function of t standing for the intercept, the technical change and a random error term. The partitionned matrix in (4.1) is the Hessian of the unconstrained profit function i.e. $A_{rs} = \Pi^u_{v_r v_s}$; r, s $\in$ (1, 2) : 1 refers to a variable netput, 2 to a quasi-fixed netput.

Now, if in fact q_2 cannot adjust immediately to the optimal level, then the observed quantities q_{1t}, q_{2t} are produced by the same model where virtual prices ν_{2t} are substituted for observed price v_{2t}.

$$(4.2) \begin{cases} a) \\ b) \end{cases} \begin{bmatrix} q_{1t} \\ q_{2t} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} v_{1t} \\ \nu_{2t} \end{bmatrix} + \begin{bmatrix} \alpha_1(t) \\ \alpha_2(t) \end{bmatrix}$$

We can assume to start with, that a partial adjustment process describes the movement of the firm toward the optimal target.

$$(4.3) \left[q_{2t} - q_{2t-1} \right] = \Lambda \left[q^{u}_{2t} - q_{2t-1} \right]$$

It is assumed that outputs can adjust freely in absence of output quotas and that only inputs lag behind optimal levels, as they are the actual instruments to reach desired output levels q^{u}_{1t} (v_{1t}, v_{2t}). Equation (4.3) could be made more complex to account for errors in meeting the quota level with the subsequent penalty fine which raises a specific problem of risk behavior in the presence of quotas, but this is left aside here.

By combining (4.1), (4.2) and (4.3) a system of observable equations obtains. For q_{2t} we get,

$$(4.4) \; q_{2t} = \Lambda \left[A_{21} v_{1t} + A_{22} v_{2t} + \alpha_2(t) \right] + (I - \Lambda) \, q_{2t-1}$$

This generalized adjustment scheme permits disequilibrium in one quasi-fixed factor market to affect the demand for another quasi-fixed input. The actual levels of quasi-fixed inputs are a weighted average of the optimal levels at time t and past observed levels at time t-1, where the adjustment matrices Λ and $(I-\Lambda)$ serve as weights. Through substitution, we can see that equation (4.4) can be rewritten as :

$$q_{2t} = \sum_{i=0}^{\infty} (I - \Lambda)^{i} \Lambda q^{u}_{2t-i} + (I - \Lambda)^{\infty} q_{2t-\infty}$$

$$(4.5) \quad q_{2t} = \sum_{i=0}^{\infty} (I-\Lambda)^{i} \Lambda \left[A_{21} v_{1t-i} + A_{22} v_{2t-i} + \alpha_2(t-i) \right] + (I-\Lambda)^{\infty} q_{2t-\infty}$$

In equilibrium $q^{u}_{2t-i} = q^{u}_{2}$. Therefore in the long-run,

$$q_{2t} = \sum_{i=0}^{\infty} (I - \Lambda)^{i} \Lambda q^{u}_{2} + (I - \Lambda)^{\infty} q_{2t-\infty}$$

Stability of the adjustment scheme requires that the characteristic roots of the matrix $(I - \Lambda)$ be within the unit circle. Furthermore the adjustment path is monotonic if the characteristic roots are real positive numbers and oscillates otherwise (Nadiri and Rosen, 1969). From (4.5) short-run and interim price elasticities of q_2 can be derived. (6)

For observed q_{1t}, there is a virtual price vector ν_{2t} which corresponds to the observed vector q_{2t} and which explains the optimal level of unconstrained netputs given the quasi fixity of factors. First, solving (4.2 b) for ν_{2t} and using (4.4), we obtain :

$$(4.6) \quad \nu_{2t} = A_{22}^{-1} (\Lambda - I) A_{21} v_{1t} + A_{22}^{-1} \Lambda A_{22} v_{2t} + A_{22}^{-1} (\Lambda - I) \alpha_2 (t) + A_{22}^{-1} (I - \Lambda) q_{2t-1}$$

The observed sequence for q_{1t} may now be derived,

$$q_{1t} = A_{11} v_{1t} + A_{12} \nu_{2t} + \alpha_1 (t)$$

$$= A_{11} v_{1t} + A_{12}\Big[A_{22}^{-1}(\Lambda - I) A_{21}v_{1t}+ A_{22}^{-1} \Lambda A_{22}v_{2t}+ A_{22}^{-1} (\Lambda - I) \alpha_2(t)$$

$$+ A_{22}^{-1} (I - \Lambda) q_{2t-1}\Big] + \alpha_1(t)$$

Finally,

$$(4.7) \quad q_{1t}=\Big[A_{11}+ A_{12} A_{22}^{-1} (\Lambda - I) A_{21}\Big] v_{1t}$$

$$+ A_{12} A_{22}^{-1} \Lambda A_{22} v_{2t} + A_{12} A_{22}^{-1} (\Lambda - I) \alpha_2 (t) + \alpha_1 (t)$$

$$- A_{12} A_{22}^{-1} (\Lambda - I) q_{2t-1}$$

In (4.7) it can be checked that if $\Lambda = I$ as in absence of input constraints the first part of equation (4.1) is retrieved and actual matches optimal path. Short-run, interim and long-run price elasticities of unconstrained good q_1 can be derived from (4.7) in a similar way to (4.5).

Equations (4.4) and (4.7) provide analytical forms which are estimable with proper specification of the error terms. It can be seen that in (4.7) two variables appear in simultaneously, namely the observed quantities of the quasi-fixed inputs and their market rental prices, while in most econometric work of estimation of production technology and supply response we find either prices or quantities of goods but not both. This is necessary here as it appears clearly that observed quantities are neither in short-run nor long-run equilibrium but converging toward the latter.

Clearly (4.7) shows that by failing to specify the netput interactions created by the quasi-fixities in the estimated model, there is hardly a chance to obtain consistent estimates of short or long-run response or of technical change biases. These parameters depend genuinely on the speed of adjustment and there is no way to speak about magnitude of elasticities without a time frame in mind as it is well known. As for the technology and particularly for input-output substitution, complementarity, normality relationships, the speed of adjustment and therefore the time frame is also a genuine element of information to be specified clearly.

When there are strictly exogenous quantities $\bar{q}_{0t}$ (output and/or input quotas) the model structure has to be changed in the following way. In a first step, the Hessian matrix in (4.1) is modified using equation (2.10) in order to take into account the new constraints, i.e. the optimal q^*_{1t}, q^*_{2t} are conditional on the level of production quotas $\bar{q}_{0t}$.

$$(4.8) \quad \begin{bmatrix} q^*_{1t} \\ q^*_{2t} \\ \bar{q}_{0t} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{10} \\ A_{21} & A_{22} & A_{20} \\ A_{01} & A_{02} & A_{00} \end{bmatrix} \begin{bmatrix} v_{1t} \\ v_{2t} \\ \nu_{0t} \end{bmatrix} + \begin{bmatrix} \alpha_1(t) \\ \alpha_2(t) \\ \alpha_0(t) \end{bmatrix}$$

An equivalent expression to (4.8) is :

$$(4.9) \quad \begin{bmatrix} q^*_{1t} \\ q^*_{2t} \\ \nu_{0t} \end{bmatrix} = \begin{bmatrix} A_{11} - A_{10} A_{00}^{-1} A_{01} & A_{12} - A_{10} A_{00}^{-1} A_{02} & A_{10} A_{00}^{-1} \\ A_{21} - A_{20} A_{00}^{-1} A_{01} & A_{22} - A_{20} A_{00}^{-1} A_{02} & A_{20} A_{00}^{-1} \\ - A_{00}^{-1} A_{01} & - A_{00}^{-1} A_{02} & A_{00}^{-1} \end{bmatrix} \begin{bmatrix} v_{1t} \\ v_{2t} \\ \bar{q}_{0t} \end{bmatrix}$$

$$+ \begin{bmatrix} \alpha_1(t) - A_{10} A_{00}^{-1} \alpha_0(t) \\ \alpha_2(t) - A_{20} A_{00}^{-1} \alpha_0(t) \\ - A_{00}^{-1} \alpha_0(t) \end{bmatrix}$$

In a second step equations similar to (4.4) and (4.7) are derived for observed q_{2t}, using the relevant partial derivatives modified in such a way that strict quotas are now accounted for.

$$(4.10)\quad q_{2t} = \Lambda \Big[(A_{21} - A_{20} A_{00}^{-1} A_{01})\, v_{1t} + (A_{22} - A_{20} A_{00}^{-1} A_{02})\, v_{2t} + A_{20} A_{00}^{-1} \bar{q}_{0t} + \alpha_2(t) - A_{20} A_{00}^{-1} \alpha_0(t) \Big] + (I - \Lambda)\, q_{2t-1}$$

Observed q_{1t} are now the result of a process generated by v_{1t}, ν_{2t}, $\bar{q}_{0t}$ as in expression (4.2). Solving (4.10) and the second row of the matrix equation (4.8) for ν_{2t}, using (4.10) and plugging into (4.9) gives the general equation for observed q_{1t} in the context of quotas $\bar{q}_{0t}$ and quasi-fixed factors q_{2t}.

$$(4.11)\quad q_{1t} = \Big[(A_{11} - A_{10}A_{00}^{-1}A_{01}) + (A_{12} - A_{10}A_{00}^{-1}A_{02})\,(A_{22} - A_{20}A_{00}^{-1} A_{02})^{-1} (\Lambda - I)\,(A_{21} - A_{20} A_{00}^{-1} A_{01})\Big]\, v_{1t}$$

$$+ \Big[(A_{12} - A_{10}A_{00}^{-1}A_{02})\,(A_{22} - A_{20}A_{00}^{-1}A_{02})^{-1}\, \Lambda\, (A_{22} - A_{20}A_{00}^{-1} A_{02})\Big] v_{2t}$$

$$+ \Big[A_{10} A_{00}^{-1} + (A_{12} - A_{10}A_{00}^{-1} A_{02})\,(A_{22} - A_{20}A_{00}^{-1} A_{02})^{-1} (\Lambda - I)\, A_{20} A_{00}^{-1}\Big] \bar{q}_{0t}$$

$$+ \Big[\alpha_1(t) - A_{10} A_{00}^{-1} \alpha_0(t) + (A_{12} - A_{10} A_{00}^{-1} A_{02})\,(A_{22} - A_{20} A_{00}^{-1} A_{02})^{-1}$$

$$(\Lambda - I)\ (\alpha_2(t) - A_{20}\ A_{00}^{-1}\ \alpha_0(t))\Big]$$

$$- \Big[(A_{12} - A_{10}\ A_{00}^{-1}\ A_{02})\ (A_{22} - A_{20}\ A_{00}^{-1}\ A_{02})^{-1}\ (\Lambda - I)\Big]\ q_{2t-1}$$

This expression can be highly simplified in the case where Λ is scalar due to the symmetry of several matrices in (4.11).

5. A numerical illustration

The production structure of the EEC agricultural sector is first summarized by a price elasticity matrix, presented in table 5.1. In order to illustrate the qualitative and quantitiative relevance of the foregoing analysis, consider the following numerical example. These elasticities, correspond to the long-run Marshallian equilibrium where all choice variables, except land, are permitted to adjust optimally. Seven outputs, five variable inputs and two quasi-fixed inputs are included : the elasticities are derived form the literature as far as magnitude is concerned but theoretical constraints (homogeneity of degree zero with respect to prices, symmetry of the Hessian matrix) are imposed to improve consistency (Mahé, Tavéra, Trochet, 1988) (7). Gross complementarity between outputs and between inputs prevail except for certain pairs of netputs : grains-GSU ; grains-sugar ; cakes-sugar and cakes (used as an input) - other inputs. Furthermore regressive relationships between inputs and outputs are ruled out. Table 5.2 a) shows the compensated output supply elasticities which are derived from table (5.1) by using expression (3.4) where all inputs are fixed and all output variable. As expected, and because each pair of outputs is similar with respect to all inputs, in the long run unconstrained case, these Hicksian supply elasticities are smaller than their long-run Marshallian counterparts : the contraction effect is always negative and large enough so that outputs are now net substitutes (see table 5.2 b)). Similarly, compensated input demand elasticities are presented in table 5.3 a)) : they correspond to the case where all outputs are constrained by production quotas. Similarity properties of each pair of inputs with respect to all outputs imply that the Hicksian demand elasticities are larger in algebraic value than their Marshallian counterparts. Again, note that the magnitude of the contraction effect (table 5.2 b)) is large enough so that all inputs are now net substitutes.

[insert tables 5.1, 5.2 and 5.3]

The short-run Marshallian price elasticities are provided in table 5.4. Thus table 5.4 provides information about substitutions and complementarities among netput pairs at the short-run Marshallian equilibrium, that is for a year. Examining the on-diagonal elements of table 5.4 shows that the Le Chatelier's principle applies but short-run elasticities are not very different from their long-run Marshallian counterparts in table 5.1. It is also worth noting that some regressive relationships now appear, particularly between grains, cakes and ceral substitutes considered as outputs or as inputs. Another notable point is that some outputs are now substitutable : as an example beef, pork and poultry and milk are now gross short-run substitutes with grains. Of course farm capital and labor appear to be the less elastic factors with uncompensated short-run price elasticities of -0,104 and -0,083 respectively.

[insert table 5.4]

The simplified dynamic framework proposed in section 4 allows to construct this Marshallian price elasticity matrix not only for one year but also for two, three, ..., n lagged years. This dynamics is illustrated by figure 5.1 which show that the expansion effect due to the gradual adjustment of the quasi-fixed factors to their long-run equilibrium levels is more important for cross effects than for own effects. Own price elasticities, in absolute value, are increasing functions of time as a consequence of the Le Chatelier's principle but the magnitude of the effect is small. It is worth mentionning that this fairly unnoticed result is due to the pattern of evolution of cross effects over the time lag structure. For own supply price elasticities as an example, cross price elasticities between outputs increase a lot under similarity, but the cross effects of input prices on output is cut drastically as well. Therefore Le Chatelier's effect on own price elasticities which is the net effect of the two due to homogeneity of degree zero, has to be fairly small compared to the effect on cross elasticities.

[insert figure 5.1]

CONCLUSION

The main implications of strict fixities like production quotas or input rationing is to alter the behavior of unconstrained supply and derived demand functions to prices, in a fairly predictable way. Under the similarity condition which is weaker than Sakai's normality condition but implied by it, inputs and outputs will tend to become more substitutable when constraints of fixity or quasi-fixity are imposed on some inputs or outputs.

The cross commodity relationships i.e. complementarity - substitutability, which are often considered as basic features of the underlying technology, can therefore be seen to depend highly on the economic environment and on the time perspective where they are observed. It was shown that under the similarity hypothesis, the firm under quantity constraints and/or in the short-run, will tend to exhibit more substitutability both among inputs and among outputs. When time is allowed for response and when quantity rationing is relaxed, goods tend to become more complements.

The observable technology is therefore always in a temporary stage between short and long-run and cannot be characterized without making a clear reference to existing strict or quasi fixities or to the time lag after the shock affecting exogenous variables of the firm's environment.

In that perspective, it is clear that concept of jointness, technical progress biases, economies of scope must also the looked at with a reference to a time frame and to a degree of constraining environment. The widespread use of flexible forms for the technology in empirical studies where estimation is performed without dynamics, is therefore quite exposed to capturing an undeterminate concept of technology with respect to time frame, with an undefined mix of technological constraints and speed of adjustment.

The condition of similarity which is useful in order to avoid ambiguous effects of rationing, does not seem too demanding at least when it is defined on the long-run unconstrained supply system. It should be verified in many situations where the aggregation structure over outputs and inputs is carried with a fairly homogenous degree of detail. Clearly, when very closely related outputs are kept in parallel with highly aggregated groups, then substitutability and complementarity among inputs or outputs may coexist even in the long-run and the similarity condition be violated.

From a policy point of view the preceeding analysis suggests that the efficiency of public intervention will be more likely to run into pitfalls in the presence of strict or quasi-fixity of quantities. The effect of constraints is to reduce the response of the system to the traditional price incentives. By the Le Chatelier's effect and the tendency toward substitutability and inferiority, unconstrained outputs react less to their own prices, less negatively to the prices of inputs but less positively to the prices of other outputs. The supply system could then be broadly characterized by a smaller reaction to its environnement and a higher degree of internal interaction. Policy instruments which apply to, say, only one output or one input, are more likely to induce spill-over effects on other goods. The shift of the budget problem of the EC from the grain to the oilseed sector in the recent years is a case in point. It is the same for the implication of a fast technical progress on a constrained output (as the beef industry in EC is likely to show due to the quota on dairy production).

Policy reforms will tend to shift problems rather than to solve them, if they do not tackle the whole sector at once. In agriculture when existing distorsions are large and widespread, problems of second best limitations of partial policy reforms are more likely to occur.

The above analysis also suggests that the issue of policy reform, in a sector like agriculture with declining outlets and steady pace of technical change, adjusting downward production capabilities raises specific problems, because internal cross relations are exacerbated by the quasi-fixities of factors and increasingly of some products.

NOTES

(1) Under strict convexity the matrix Π^{u}_{vv} is of rank n+m-1 (Guesnerie, 1980).

(2) Similarity is defined on the basis of the elasticities because only one expression is enough to uncompass inputs and outputs. This is not the case if one would start with the cross derivatives of $\Pi^{u}(v)$. Naturally an expression equivalent to (2.14) works as well with the slopes rather than elasticities, but it is the slopes of the positive quantities of outputs and inputs rather than of the netputs which are to be used.

(3) by writing

$$\begin{bmatrix} E^{00} & F^{00} \\ G^{00} & H^{00} \end{bmatrix} = \begin{bmatrix} (y^{0})^{-1} & 0 \\ 0 & (-x^{0})^{-1} \end{bmatrix} \begin{bmatrix} \Pi_{v_0 v_0} \end{bmatrix} \begin{bmatrix} \rho^{0} & 0 \\ 0 & \omega^{0} \end{bmatrix}$$

which has a negative determinant as being the product of one negative definite and two positive definite matrices.

(4) This comparative statics also shows that when constraints on some inputs and outputs are effective, the apparent technical progress bias on the unconstrained quantities is affected by the bias on the constrained inputs and outputs. A sign reversal is even possible as the dairy quota in EC has done already on the demand for dairy feed.

(5) These properties are also valid when there is only one output quota or one fixed input.

(6) Such a case is however unlikely if the model is derived from a flexible accelerator issued from dynamic optimization (Treadway, 1971).

(7) The original matrix did not include labor, and was chosen to be relevant to a 3-5 year projection. Its long run counterpart has been calculated by making use of formulae (4.4) and (4.7) and assuming that the adjustment matrix is diagonal and that the coefficients are equal to 0,10 for capital and 0,05 for labor.

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Table 2.1. Similarity of two netputs q_r and q_s with respect to a third q_0 : alternative formulations.

alternative formulations of the pairwise similarity of q_r and q_s w.r.t. q_0		
q_r and q_s are two outputs y_i and y_j	q_r is an output y_i and q_s is an input $-x_h$	q_r and q_s are two inputs $-x_h$ and $-x_k$
$\varepsilon^u_{y_i v_0} \cdot \varepsilon^u_{y_j v_0} \geq 0$	$\varepsilon^u_{y_i v_0} \cdot \varepsilon^u_{x_h v_0} \geq 0$	$\varepsilon^u_{x_h v_0} \cdot \varepsilon^u_{x_k v_0} \geq 0$
$\partial y^u_i / \partial v_0 \cdot \partial y^u_j / \partial v_0 \geq 0$	$\partial y^u_i / \partial v_0 \cdot \partial x^u_h / \partial v_0 \geq 0$	$\partial x^u_h / \partial v_0 \cdot \partial x^u_k / \partial v_0 \geq 0$
$\Pi^u_{v_i v_0} \cdot \Pi^u_{v_j v_0} \geq 0$	$(\Pi^u_{v_i v_0} \cdot (-\Pi^u_{v_h v_0}) \geq 0)$ $\Pi^u_{v_i v_0} \cdot \Pi^u_{v_h v_0} \leq 0$	$((-\Pi^u_{v_h v_0})(-\Pi^u_{v_k v_0}) \geq 0)$ $\Pi^u_{v_h v_0} \cdot \Pi^u_{v_k v_0} \geq 0$

Table 5.1. Long-run Marshallian price elasticities for EEC

	GRA	CAK	GSU	BEE	P&P	MIK	SUG	ROA	CER	CAK	GSU	OTH	OIC	CAP	WRK
GRA	0.877	0.001	-0.002	0.147	0.123	0.223	-0.010	0.139	-0.078	-0.021	-0.025	-0.032	-0.330	-0.268	-0.719
CAK	0.035	0.894	0.000	0.179	0.145	0.246	-0.005	0.244	-0.102	-0.028	-0.033	-0.037	-0.364	-0.279	-0.863
GSU	-0.075	0.000	0.075	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BEE	0.185	0.006	0.000	1.072	0.202	0.555	0.051	0.368	-0.215	-0.072	-0.081	-0.125	-0.418	-0.415	-1.079
P&P	0.145	0.004	0.000	0.189	1.870	0.271	0.037	0.273	-0.807	-0.132	-0.199	-0.139	-0.393	-0.388	-0.719
MIK	0.215	0.006	0.000	0.421	0.221	1.473	0.059	0.303	-0.240	-0.091	-0.100	-0.097	-0.513	-0.558	-1.079
SUG	-0.051	-0.001	0.000	0.194	0.151	0.294	0.956	0.267	-0.107	-0.029	-0.034	-0.039	-0.379	-0.291	-0.899
ROA	0.077	0.003	0.000	0.160	0.126	0.171	0.031	1.027	-0.096	-0.026	-0.030	-0.035	-0.332	-0.339	-0.719
CER	0.118	0.004	0.000	0.255	1.030	0.375	0.034	0.267	-0.900	-0.008	-0.011	-0.044	-0.266	-0.306	-0.539
CAK	0.118	0.004	0.000	0.312	0.620	0.520	0.034	0.266	-0.030	-0.652	-0.085	0.011	-0.268	-0.302	-0.539
GSU	0.138	0.005	0.000	0.349	0.931	0.569	0.039	0.302	-0.039	-0.085	-0.846	-0.110	-0.287	-0.302	-0.647
OTH	0.134	0.004	0.000	0.403	0.489	0.417	0.034	0.266	-0.121	-0.009	-0.083	-0.434	-0.268	-0.302	-0.539
OIC	0.228	0.006	0.000	0.231	0.234	0.374	0.055	0.426	-0.124	-0.034	-0.037	-0.045	-0.464	-0.302	-0.539
CAP	0.275	0.007	0.000	0.340	0.349	0.613	0.062	0.652	-0.216	-0.058	-0.058	-0.078	-0.457	-1.044	-0.388
WRK	0.442	0.013	0.000	0.529	0.388	0.710	0.115	0.829	-0.228	-0.062	-0.075	-0.083	-0.489	-0.431	-1.659

Table 5.2. Long-run input compensated, supply price elasticities and corresponding contraction effects

Table 5.3. Long-run output compensated, demand price elasticities and corresponding contraction effects

Table 5.2 a
substitution matrix

	GRA	CAK	GSU	BEE	P&P	MIK	SUG	ROA
GRA	0.643	-0.006	-0.002	-0.097	-0.014	-0.134	-0.068	-0.300
CAK	-0.231	0.886	0.000	-0.106	-0.032	-0.163	-0.072	-0.252
GSU	-0.075	0.000	0.075	0.000	0.000	0.000	0.000	0.000
BEE	-0.123	-0.003	0.000	0.654	-0.226	-0.015	-0.028	-0.226
P&P	-0.015	-0.001	0.000	-0.206	0.610	-0.301	-0.008	-0.078
MIK	-0.131	-0.004	0.000	-0.018	-0.248	0.826	-0.028	-0.380
SUG	-0.328	-0.009	0.000	-0.103	-0.034	-0.132	0.886	-0.250
ROA	-0.163	-0.004	0.000	-0.099	-0.036	-0.215	-0.029	0.563

Table 5.3 a
substitution matrix

	CER	CAK	GSU	OTH	OIC	CAP	WRK
CER	-0.425	0.080	0.114	0.053	0.055	0.018	0.100
CAK	0.293	-0.581	0.009	0.093	0.045	0.020	0.116
GSU	0.418	0.008	-0.719	-0.004	0.089	0.084	0.125
OTH	0.147	0.053	-0.003	-0.355	0.026	-0.006	0.113
OIC	0.026	0.006	0.011	0.005	-0.163	-0.007	0.115
CAP	0.011	0.003	0.016	-0.003	-0.016	-0.601	0.564
WRK	0.040	0.013	0.014	0.016	0.098	0.141	-0.358

Table 5.2 b
contraction matrix

	GRA	CAK	GSU	BEE	P&P	MIK	SUG	ROA
GRA	-0.234	-0.007	0.000	-0.244	-0.137	-0.357	-0.058	-0.439
CAK	-0.266	-0.008	0.000	-0.285	-0.177	-0.409	-0.067	-0.496
GSU	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BEE	-0.308	-0.009	0.000	-0.418	-0.428	-0.570	-0.079	-0.594
P&P	-0.160	-0.005	0.000	-0.395	-1.260	-0.572	-0.045	-0.351
MIK	-0.346	-0.010	0.000	-0.439	-0.469	-0.647	-0.087	-0.683
SUG	-0.277	-0.008	0.000	-0.297	-0.185	-0.426	-0.070	-0.517
ROA	-0.240	-0.007	0.000	-0.259	-0.162	-0.386	-0.060	-0.464

Table 5.3 b
contraction matrix

	CER	CAK	GSU	OTH	OIC	CAP	WRK
CER	0.475	0.088	0.125	0.097	0.321	0.324	0.639
CAK	0.323	0.071	0.094	0.082	0.313	0.322	0.655
GSU	0.457	0.093	0.127	0.106	0.376	0.386	0.772
OTH	0.268	0.062	0.080	0.079	0.294	0.296	0.652
OIC	0.150	0.040	0.048	0.050	0.301	0.295	0.654
CAP	0.227	0.061	0.074	0.075	0.441	0.443	0.952
WRK	0.268	0.075	0.089	0.099	0.587	0.572	1.301

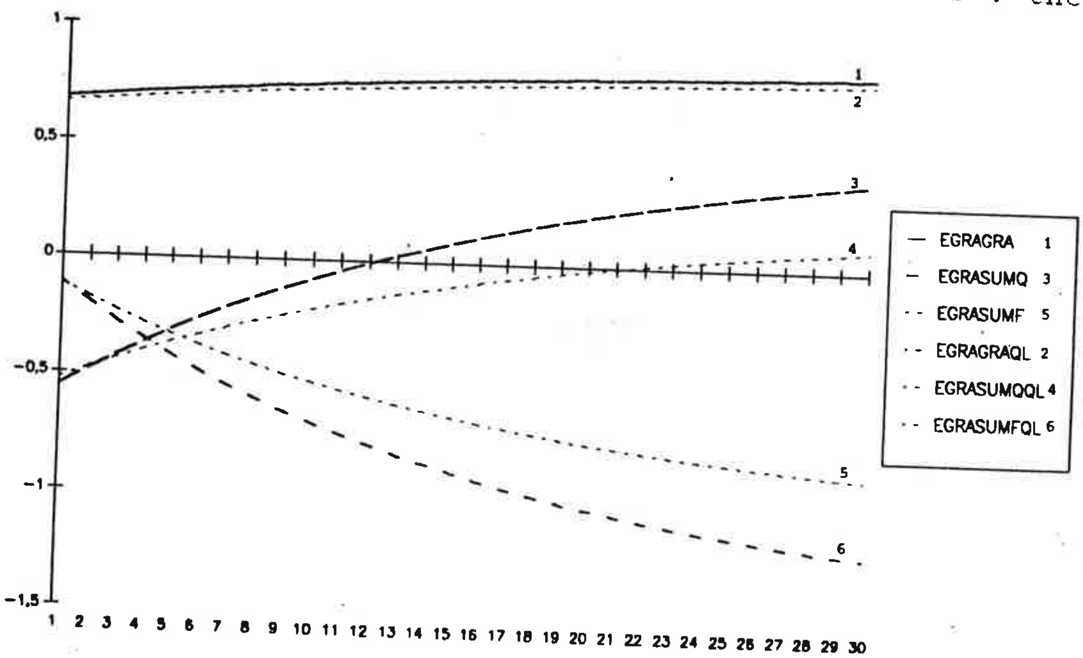
GRA = Grains
CAK = Vegetal Proteins
GSU = Grains Substituts
BEE = Beef
P&P = Pork and Poultry
MIK = Milk
SUG = Sugar
ROA = Rest of Agriculture

CER = Grains
CAK = Vegetal Proteins
GSU = Grains Substituts
OTH = Other Animal Feed
OIC = Other Raw Materials
CAP = Capital
WRK = Work

Table 5.4. Short-run Marshallian price elasticities for EEC (one year)

	GRA	CAK	GSU	BEE	P&P	MIK	SUG	ROA	CER	CAK	GSU	OTH	OIC	CAP	WRK
GRA	0.682	-0.005	-0.002	-0.087	-0.057	-0.103	-0.060	-0.238	0.028	0.008	0.008	0.006	-0.102	-0.027	-0.036
CAK	-0.192	0.887	0.000	-0.093	-0.060	-0.128	-0.063	-0.189	0.019	0.005	0.006	0.007	-0.105	-0.028	-0.043
GSU	-0.075	0.000	0.075	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BEE	-0.110	-0.003	0.000	0.718	-0.070	0.060	-0.024	-0.202	-0.054	-0.028	-0.030	-0.066	-0.073	-0.042	-0.054
P&P	-0.069	-0.002	0.000	-0.069	1.661	-0.105	-0.016	-0.154	-0.682	-0.099	-0.161	-0.094	-0.127	-0.039	-0.036
MIK	-0.102	-0.003	0.000	0.039	-0.086	0.918	-0.020	-0.328	-0.057	-0.041	-0.043	-0.031	-0.122	-0.056	-0.054
SUG	-0.288	-0.008	0.000	-0.090	-0.063	-0.095	0.895	-0.184	0.019	0.005	0.006	0.007	-0.109	-0.029	-0.045
ROA	-0.129	-0.003	0.000	-0.088	-0.071	-0.185	-0.021	0.620	0.021	0.006	0.006	0.008	-0.082	-0.034	-0.036
CER	-0.045	-0.001	0.000	0.059	0.870	0.087	-0.007	-0.060	-0.804	0.018	0.018	-0.009	-0.062	-0.031	-0.027
CAK	-0.044	-0.001	0.000	0.116	0.461	0.233	-0.007	-0.059	0.065	-0.626	-0.056	0.046	-0.065	-0.030	-0.027
GSU	-0.047	-0.001	0.000	0.126	0.755	0.250	-0.008	-0.063	0.066	-0.057	-0.813	-0.072	-0.063	-0.030	-0.032
OTH	-0.029	-0.001	0.000	0.208	0.330	0.130	-0.007	-0.059	-0.026	0.034	-0.054	-0.399	-0.065	-0.030	-0.027
OIC	0.066	0.002	0.000	0.035	0.075	0.087	0.014	0.101	-0.029	-0.008	-0.008	-0.011	-0.262	-0.030	-0.027
CAP	0.028	0.001	0.000	0.034	0.035	0.061	0.006	0.065	-0.022	-0.006	-0.006	-0.008	-0.046	-0.104	-0.039
WRK	0.022	0.001	0.000	0.026	0.019	0.035	0.006	0.041	-0.011	-0.003	-0.004	-0.004	-0.024	-0.022	-0.083

Figure 5.1. The time structure of price elasticities : the case of grains



EGRAGRA : own price elasticity of grains ; EGRASUMO : sum of cross price elasticities of grains w.r.t. other outputs ; EGRASUMF : sum of cross price elasticities of grains w.r.t. inputs. A suffix QL indicates that the relevant elasticities are calculated in the case of milk quotas.