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Recurrent Event Modeling Based on the Yule Process

Yves Le Gat

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Recurrent Event Modeling Based on the Yule Process

Mathematical Models and Methods in Reliability Set

coordinated by
Nikolaos Limnios and Bo Henry Lindqvist

Volume 2

**Recurrent Event Modeling
Based on the Yule Process**

*Application to Water Network
Asset Management*

Yves Le Gat

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Preface

The research work presented in this book arises from the involvement of the author in engineering studies of the reliability of drinking water pipes. This type of infrastructure is organized as a network of pipelines, and failures, namely leakage or breakage, tend to occur in an aggregative manner on the same network segments. Building relevant strategies of infrastructure asset management requires, therefore, accurate modeling tools of the repeated failures that can affect some pipes, due to the heavy socioeconomic and environmental consequences of leakage and breakage.

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