Control of the crop-production in a network of agricultural plots

Céline Casenave, Anne Bisson

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Céline Casenave
UMR MISTEA
INRAE, Montpellier SupAgro, Univ Montpellier,
Montpellier, France
celine.casenave@inrae.fr; ORCID: 0000-0003-0489-4665

Anne Bisson
UMR MISTEA
INRAE, Montpellier SupAgro, Univ Montpellier,
Montpellier, France
anne.bisson@inrae.fr

***Abstract*—In the present paper, we propose a strategy to control the crop-production of a network of agricultural plots. The network is composed of rangeland and cropland subsystems that are connected to each other through the livestock that is a vector of nutrient from rangeland to cropland. The objective is to reach a given crop production (at the scale of the whole network) while ensuring a minimal production in each of the cropland plot. To take into account the saturation constraints on the control inputs (that are the rangeland removal rates and the manure distribution coefficients), we use a method based on a time-scale transformation. After designing the control law, we apply it on a simple numerical example to highlight the results.**

***Index Terms*—nonlinear control, input constraints, time-scale transformation, agroecosystems**

I. Introduction

In traditional mixed crop-livestock farming systems, that are still present in some parts of the world such as in West-Africa, it is well-known that livestock plays a key role [1]; it is a vector of nutrient from rangeland to cropland that benefits to the production of the whole agro-ecosystem [2]. In occidental countries, the livestock farms are often separated from the crop fields and the livestock has thus lost its role as nutrient carrier. At the same time, the management of livestock dejections that contributes to the pollution of water bodies and soils is a serious ecological issue [3]. Yet, livestock dejections are good fertilizers for the crops. It is therefore important to reconnect the livestock farms to the crop fields by using the manure produced by the livestock as fertilizer for the crop. It necessitates the implementation of a smart management at the scale of the whole network of agricultural plots, which is the objective of the present paper.

In [4], we studied a mixed farming system that we represented as a meta-ecosystem composed of a cropland subsystem connected to a rangeland subsystem through the livestock-mediated nutrient transfer T. In this farming system, the livestock grazes on the rangeland during the day and is parked on the cropland during the night (night coralling): it is a vector of nutrient from rangeland to cropland through the feces and urine. The model introduced in [4] can also represent a farming system where the livestock farm is separated from the fields. In that case, one part of the rangeland biomass is used to feed the livestock. And the manure produced by the livestock is then reused as fertilizer and spread on both the rangeland and crop fields.

In the present paper, on the basis of the meta-ecosystem structure and equations introduced in [4], we consider a meta-ecosystem composed of M_r rangeland subsystems and M_c cropland subsystems that are connected to each other through a livestock mediated nutrient transfer as in [4] (see figure 1). Here again, one part of the plant biomass of the rangeland subystems is used to feed the livestock, and the manure produced by the livestock is spread on the cropland subsystems. We thus consider here a network of interconnected rangeland and cropland subsystems and the livestock farms are assumed to be separated from the fields as it is the case in occidental countries.

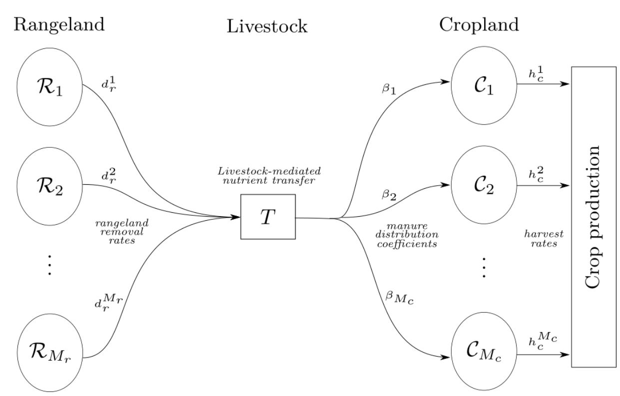

Fig. 1. Scheme of the network of agricultural plots.

The objective of the control problem considered in this paper is to reach a given crop production (at the scale of the network of agricultural plots) while ensuring a minimal production in each of the cropland subsystems. The control inputs we consider are the rangeland removal rates (to feed the livestock), and the coefficients of manure distribution between the different cropland subsystems. These control inputs are subject to saturation constraints. To take into account these constraints, we propose to use the method presented in [5] that is based on a time-scale transformation, that is on a nonlocal

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change of time-variable. This method enables to transform a constraint control problem into and unconstrained problem that can be solved by classical control methods.

The paper is organized as follows. In section II, we introduce the model of the agricultural plots network and then the associated control problem. In section III, we give the main result of the method proposed in [5] for the control of nonlinear systems with input positive constraints. Then, the design of the control law is presented in section IV. Finally, a numerical example is presented in section V.

II. Problem under consideration

A. Model of the agricultural plots network

As explained previously, we consider here a network of cropland and rangeland subsystem that are connected to each other through livestock-mediated nutrient transfer (see figure 1). For each subsystem (rangeland or cropland), the plant and the nutrient compartments are represented in the model by the variables P_z^i and N_z^i (in kgN.ha^{-1}) with $z = c$ for the cropland, and $z = r$ for the rangeland, i being the subsystem number. The plants P_z^i grow on the nutrient at a rate denoted $G_z^i(P_z^i, N_z^i)$ and have a mortality rate m_z^i: the dead plants are then recycled in the nutrient compartment. Nutrient enters the subsystems through dry depositions (flux e_z^i). Losses of nutrient are mainly due to erosion, leaching, volatilization and denitrification (loss rate o_z^i).

In the j^{th} rangeland subsystem, one part of the plant compartment is taken at a rate d_r^j to feed the livestock. The flux of nutrient that is taken is divided into two parts. One part (percentage $\alpha_j \in [0,1]$) is directly recycled on the rangeland subsystem on which it has been taken. The other part (percentage $1-\alpha_j \in [0,1]$) is ingested by the livestock and then recovered as manure to be spread as fertilizer on the cropland subsystems.

The equations of the rangeland subsystems are therefore given by: $\forall j = 1 : M_r$,

$$(\mathcal{R}_j)\begin{cases} \frac{dP_r^j}{dt} &= G_r^j(P_r^j, N_r^j) - m_r^j P_r^j - d_r^j P_r^j \\ \frac{dN_r^j}{dt} &= -G_r^j(P_r^j, N_r^j) + m_r^j P_r^j - o_r^j N_r^j \\ & \quad +e_r^j + \alpha_j d_r^j P_r^j \end{cases} \quad (1)$$

where $G_r^j(P,N) = u_r^j PN(1 - \frac{P}{K_r^j})$ is the growth rate of the plants cultivated in the j^{th} rangeland subsystem.
The flux of nutrients that is transferred from the rangeland subsystems to the cropland subsystems through the livestock is given by:

$$T = \sum_{j=1}^{M_r} (1-\alpha_j) s_r^j d_r^j P_r^j \quad (2)$$

where s_r^j is the surface of the j^{th} rangeland subsystem.
The nutrient flux T is divided into M_c parts to be spread on the M_c cropland subsystems. By denoting $\beta_k \in [0,1]$ the fraction of T that is spread on the k^{th} cropland subsystem, we get the following system of equations for the cropland subsystems: $\forall k = 1 : M_c$,

$$(\mathcal{C}_k)\begin{cases} \frac{dP_c^k}{dt} &= G_c^k(P_c^k, N_c^k) - m_c^k P_c^k \\ \frac{dN_c^k}{dt} &= -G_c^k(P_c^k, N_c^k) - o_c^k N_c^k + m_c^k P_c^k \\ & \quad +e_c^k + \frac{\beta_k T}{s_c^k} \end{cases} \quad (3)$$

where $G_c^k(P,N) = u_c^k PN(1 - \frac{P}{K_c^k})$ is the growth rate of the plants cultivated in the k^{th} cropland subsystem and s_c^k is the surface of the k^{th} cropland subsystem. Note that we necessary have:

$$\sum_{k=1}^{M_c} \beta_k = 1. \quad (4)$$

B. Control problem under consideration

Let denote by C the crop production at the scale of the agricultural plots network which is given by:

$$C = \sum_{k=1}^{M_c} s_c^k h_c^k P_c^k \quad (5)$$

where h_c^k is the percentage of plants that are harvested in the k^{th} cropland subsystem at harvest time t_h. The objective is to make the crop production C reach a given value C^* at the harvest time t_h. We consider as control inputs the rangeland removal rates d_r^j, $j = 1 : M_r$ and the manure distribution coefficients $\beta_k \in [0,1]$, $k = 1 : M_c$. According to the analysis presented in [4], d_r^j (that is necessarily positive) has to be smaller than $\frac{u_r^j e_r^j}{o_r^j} - m_r^j$ in order to maintain a plant biomass in the rangeland subsystem; we thus have the following input constraint:

$$\forall t > 0,\ d_{min}^j := 0 \leqslant d_r^j(t) \leqslant \frac{u_r^j e_r^j}{o_r^j} - m_r^j =: d_{max}^j \quad (6)$$

We also want to garantuee a minimal production C_{min}^k for each cropland subsystem $(\mathcal{C}_k)$, which can be expressed as follows:

$$\forall k = 1 : M_c,\ C_{min}^k \leqslant s_c^k h_c^k P_c^k(t_h). \quad (7)$$

III. Preliminary result

In the control problem considered in this paper, we have to take into account some contraints on the control input values (condition (6)). The control strategy we have chosen to apply relies on a method that is presented in [5] and that is dedicated to control problem with input positive constraints. The main result (proposition 8 in [5]) is given here after.

Proposition 3.1: Consider the system:

$$\begin{cases} \frac{dx}{dt} &= f(x,u) \\ y &= h(x) \end{cases} \quad (8)$$

where $\forall t \geqslant 0$, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$, with $n, m, p \in \mathbb{N}$ and $f : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ and $h : \mathbb{R}^n \mapsto \mathbb{R}^p$ are two continuously differentiable functions.

Assume that the control input u is subject to L input positive constraints:

$$k_l(u(t)) \geqslant 0,\ l = 1 : L, \forall t > 0, \quad (9)$$

where $k_l : \mathbb{R}^m \to \mathbb{R}$, $l = 1 : L$ are continuously differentiable functions, and denote Ω_c the subspace of $\mathbb{R}^m$ defined by:

$$\Omega_c := \{u \in \mathbb{R}^m, \text{ such that } k_l(u) \geqslant 0, \forall l = 1 : L\}. \quad (10)$$

Consider $y^* \in \mathbb{R}^p$ and assume there exists a unique $x^* \in \mathbb{R}^n$ such that $h(x^*) = y^*$. Given L positive functions $K_l : \Omega_c \mapsto \mathbb{R}^+$, $l = 1 : L$ such that, $\forall l$, $k_l(u) = 0 \Leftrightarrow K_l(u) = 0$, let's denote τ the time variable defined by:

$$\partial_t \tau = \prod_{l=1}^{L} K_l(u) \text{ and } \partial_\tau t = \frac{1}{\prod_{l=1}^{L} K_l(u)}. \quad (11)$$

For any variable z, $\tilde{z}$ represents the same variable but expressed in the time τ; we thus have $\tilde{z}(\tau) = z(t(\tau))$.

Suppose that there exists a dynamic control law $\tilde{u} = (\tilde{u}_1, \dots \tilde{u}_m)$ defined by:

$$\frac{d\tilde{u}_i}{d\tau} = \gamma_i(\tilde{x}, \tilde{u}), \, i = 1 : m, \quad (12)$$

with $\gamma_i : (\tilde{x}, \tilde{u}) \in \mathbb{R}^n \times \Omega_c \mapsto \gamma_i(\tilde{x}, \tilde{u})$, $i = 1 : m$ some bounded functions, such that:

(i) x^* is a globally asymptotically stable equilibrium point of:

$$\frac{d\tilde{x}}{d\tau} = \frac{f(\tilde{x}, \tilde{u})}{\prod_{l=1}^{L} K_l(\tilde{u})} \left(=: \overline{f}(\tilde{x}, \tilde{u})\right) \quad (13)$$

with $\tilde{x}(0) = x_0 \in \mathbb{R}^n$,

(ii) the trajectories $(\tilde{x}, \tilde{u})$ solution of system (13,12) are bounded.

Then the control law defined by:

$$\frac{du_i}{dt} = \left(\prod_{l=1}^{L} K_l(u)\right) \gamma_i(x, u), \, i = 1 : m, \quad (14)$$

stabilizes the system (8) at x^*, while ensuring that the constraints (9) are fulfilled for all $t > 0$ (provided that $u(0)$ fulfills the constraints (9)).

IV. Control strategy

To solve the considered problem, we will use a two-loops control strategy: (i) one loop that will control the crop-production C with livestock-mediated nutrient transfer T and the manure distribution coefficients β_k, $k = 1 : M_c$, (ii) and a second loop that will control T with the rangeland removal rates d_r^j, $j = 1 : M_r$. For both control loops, we will apply the result of proposition 3.1. But first we will see how to handle the constraint (7).

A. Constraint on the minimal production

For each cropland subsystem $(\mathcal{C}_k)$, consider a value P_*^k that is not necessarily constant over time. P_*^k is the setpoint value for the variable P_c^k. We want the value of P_*^k to fulfill the constraint (7) for all time $t \geqslant 0$, that is:

$$C_{min}^k \leqslant s_c^k h_c^k P_*^k(t). \quad (15)$$

To garantee that, we will look at a dynamic equation for P_*^k of the form:

$$\frac{dP_*^k}{dt} = H_k(P_*^k) \eta_k(P_c^k, N_c^k, P_*^k) \quad (16)$$

with:

$$H_k(P) = \left(P - \frac{C_{min}^k}{s_c^k h_c^k}\right) \quad (17)$$

Provided that $H_k(P_*^k(t = 0)) > 0$, this equation will ensure that, for all $t > 0$, $H_k(P_*^k(t)) > 0 \Leftrightarrow C_{min}^k \leqslant s_c^k h_c^k P_*^k(t)$. The function $\eta_k(P_*^1, \dots, P_*^{M_c})$ will be chosen to ensure the convergence of P_c^k towards P_*^k.

Let's denote τ_k the time variable defined by:

$$\partial_t \tau_k = H_k(P_*^k) \text{ and } \partial_{\tau_k} t = \frac{1}{H_k(P_*^k)}. \quad (18)$$

In the new time τ_k, the equation (16) rewrites as follows:

$$\frac{d\tilde{P}_*^k}{d\tau_k} = \eta_k(\tilde{P}_c^k, \tilde{N}_c^k, \tilde{P}_*^k). \quad (19)$$

After simple computations, we can show that with:

$$\eta_k(\tilde{P}_c^k, \tilde{N}_c^k, \tilde{P}_*^k) = \frac{\varphi_k(\tilde{P}_c^k, \tilde{N}_c^k)}{H_k(\tilde{P}_*^k)} - k_k\left(\tilde{P}_*^k - \tilde{P}_c^k\right) \quad (20)$$

where $\varphi_k(P, N) := G_c^k(P, N) - m_c^k P$ and $k_k > 0$, the closed loop dynamic of $\tilde{P}_c^k$ (in time τ_k) is a first order one:

$$\frac{d\left(\tilde{P}_c^k - \tilde{P}_*^k\right)}{d\tau_k} = k_k\left(\tilde{P}_*^k - \tilde{P}_c^k\right), \quad (21)$$

that will ensure the convergence of $\tilde{P}_c^k$ towards $\tilde{P}_*^k$.

B. Control of C with β_k and T

A direct consequence of (21) is that the crop production C (defined by (5)) will converge towards $\sum_{k=1}^{M_c} s_c^k h_c^k P_*^k$. Let's now show how to make $\sum_{k=1}^{M_c} s_c^k h_c^k P_*^k$ converge towards C^*, by using $T_k := \beta_k T$, $k = 1 : M_c$ as control inputs.

To be physically acceptable, note that each control input T_k has to remain positive and below a maximum value T_k^{max} that depends on the production capacity of the rangeland subsystems:

$$\forall t > 0, \, T_k^{min} := 0 \leqslant T_k(t) \leqslant T_k^{max}. \quad (22)$$

In order to guarantee that the condition (22) will be fulfilled, we will look at a dynamic control law of the form:

$$\frac{dT_k}{dt} = K_k(T_k) \gamma_k(P_c^k, N_c^k, T_k, P_*^k) \quad (23)$$

with:

$$K_k(T_k) = \frac{T_k - T_k^{min}}{K_{m,k} + T_k - T_k^{min}} \times \frac{T_k^{max} - T_k}{K_{M,k} + T_k^{max} - T_k}. \quad (24)$$

The function γ_k will be chosen to ensure the convergence of $\sum_{k=1}^{M_c} s_c^k h_c^k P_*^k$ towards C^*.

We will proceed in two steps. First, after simple computations, we can show that if $N_c^k = N_*^k$ with:

$$N_*^k = \frac{H_k(P_*^k) k_k (P_*^k - P_c^k) + m_c^k P_c^k + b\left(\frac{C^*}{M_c s_c^k h_c^k} - P_*^k\right)}{u_c^k P_c^k (1 - \frac{P_c^k}{K_c})}, \quad (25)$$

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then:

$$\frac{d}{dt}\left(\sum_{k=1}^{M_c} s_c^k h_c^k P_*^k\right) = b\left(C^* - \sum_{k=1}^{M_c} s_c^k h_c^k P_*^k\right). \quad (26)$$

The objective is now to find a function γ_k that will make the variable N_c^k follow the time-varying setpoint N_*^k. Let's denote σ_k the time variable defined by:

$$\partial_t \sigma_k = K_k(T_k) \text{ and } \partial_{\sigma_k} t = \frac{1}{K_k(T_k)}. \quad (27)$$

The system composed of equations (3,23) can be rewritten in the new time σ_k as follows:

$$\begin{cases} \frac{d\tilde{P}_c^k}{d\sigma_k} &= \frac{\varphi_k(\tilde{P}_c^k,\tilde{N}_c^k)}{K_k(\tilde{T}_k)} \\ \frac{d\tilde{N}_c^k}{d\sigma_k} &= \frac{\rho_k(\tilde{P}_c^k,\tilde{N}_c^k)}{K_k(\tilde{T}_k)} + \frac{\tilde{T}_k}{K_k(\tilde{T}_k)s_c^k} \\ \frac{d\tilde{T}_k}{d\sigma_k} &= \gamma_k(\tilde{P}_c^k, \tilde{N}_c^k, \tilde{T}_k, \tilde{P}_*^k) \end{cases} \quad (28)$$

with $\varphi_k(P,N) := G_c^k(P,N) - m_c^k P$, and $\rho_k(P,N) := -G_c^k(P,N) - o_c^k N + m_c^k P + e_c^k$.

After simple computation, we can then show that, if[1]:

$$\frac{d\tilde{T}_k}{d\sigma_k} = \gamma_k(\tilde{P}_c^k, \tilde{N}_c^k, \tilde{T}_k, \tilde{P}_*^k) \quad (29)$$

with $\gamma_k(\tilde{P}_c^k, \tilde{N}_c^k, \tilde{T}_k, \tilde{P}_*^k) = \frac{1}{\frac{K_k}{s_c^k} - K_k'\left(\rho_k + \frac{\tilde{T}_k}{s_c^k}\right) - K_k^2 B_k}$

$$\times \left[K_k^2\left(A_k + 2\xi_k\omega_k\left(\frac{d\tilde{N}_*^k}{d\sigma_k} - \frac{d\tilde{N}_c^k}{d\sigma_k}\right) + \omega_k^2\left(\tilde{N}_*^k - \tilde{N}_c^k\right)\right) -\varphi_k\partial_P\rho_k - \left(\rho_k + \frac{\tilde{T}_k}{s_c^k}\right)\partial_N\rho_k\right] \quad (30)$$

where A_k and B_k are defined as the quantities such that:

$$\frac{d^2 N_*^k}{d\sigma_k^2} = A_k + \gamma_k B_k \quad (31)$$

then, the dynamic of $\tilde{N}_c^k$ will be a second-order dynamic (in time σ_k), that is:

$$\frac{d^2\left(\tilde{N}_*^k - \tilde{N}_c^k\right)}{d\sigma_k^2} + 2\xi_k\omega_k\frac{d\left(\tilde{N}_*^k - \tilde{N}_c^k\right)}{d\sigma_k} + \omega_k^2\left(\tilde{N}_*^k - \tilde{N}_c^k\right) = 0. \quad (32)$$

From proposition 3.1, we can then conclude that the control law (23) with γ_k defined by (30) will ensure the convergence of $N_c^k - N_*^k$ towards 0 in time t and the fulfillment of constraint (22).

From the values of T_k computed from (23), we can easily deduce the values of β_k and the one of the setpoint value T^* of T as follows:

$$T^* = \sum_{k=1}^{M_c} T_k \text{ and } \beta_k = \frac{T_k}{T^*}, \quad (33)$$

since we have $\sum_{k=1}^{M_c} \beta_k = 1$.

β_k can be applied directly. The second loop of the control strategy will be used to make the variable T follow the time-varying setpoint T^* defined above.

C. Control of T with d_r^j

The objective of the second loop of the control strategy is to control the variable T with the control inputs d_r^j, $j = 1 : M_r$. As already said in section II-B, the control inputs d_r^j are subject to the saturation contraints (6). We will therefore again use the result of proposition 3.1 to design the control law.

In order to guarantee that the condition (6) will be fulfilled, we will look at a dynamic control law of the form:

$$\frac{dd_r^j}{dt} = J(d_r^1, \ldots, d_r^{M_r})\delta_j(P_r^j, N_r^j, T, T^*) \quad (34)$$

where $J(d_r^1, \ldots, d_r^{M_r}) :=$

$$\prod_{l=1}^{M_r}\left(\frac{d_r^l - d_{min}^l}{D_{m,l} + d_r^l - d_{min}^l} \times \frac{d_{max}^l - d_r^l}{D_{M,l} + d_{max}^l - d_r^l}\right). \quad (35)$$

Let's denote ξ the time variable defined by:

$$\partial_t\xi = J(d_r^1, \ldots, d_r^{M_r}) \text{ and } \partial_\xi t = \frac{1}{J(d_r^1, \ldots, d_r^{M_r})}. \quad (36)$$

In the new time ξ, the equation (34) rewrites as follows:

$$\frac{d\tilde{d}_r^j}{d\xi} = \delta_j(\tilde{P}_r^j, \tilde{N}_r^j, \tilde{T}, \tilde{T}^*). \quad (37)$$

After simple computations, we can show that with:

$$\delta_j(P_r^j, N_r^j, T, T^*) = \frac{1}{P_r^j}\left(\theta_j\frac{a(T^* - T) + \frac{dT^*}{d\xi}}{(1-\alpha_j)s_r^j} - d_r^j\frac{\psi_j}{J}\right) \quad (38)$$

where $\psi_j(P,N) := G_r^j(P,N) - m_r^j P - d_r^j P$, $a > 0$ and $\sum_{j=1}^{M_r}\theta_j = 1$, the closed loop dynamic of $\tilde{T} - \tilde{T}^*$ (in time ξ) is a first order one:

$$\frac{d(\tilde{T} - \tilde{T}^*)}{d\xi} = a(\tilde{T}^* - \tilde{T}). \quad (39)$$

From proposition 3.1, we then conclude that the control law (34) with δ_j defined by (38) will ensure the convergence of $T - T^*$ towards 0 in time t and the fulfillment of constraint (6).

D. Summary of the control strategy

Finally, the control law can be written as follows.
For the cropland subsystems, the manure distribution coefficients are given by: $\forall k = 1 : M_c$,

$$\beta_k = \frac{T_k}{\sum_{l=1}^{M_c} T_l} \quad (40)$$

with:

$$\frac{dP_*^k}{dt} = H_k(P_*^k)\eta_k(P_c^k, N_c^k, P_*^k) \quad (41)$$

$$\frac{dT_k}{dt} = K_k(T_k)\gamma_k(P_c^k, N_c^k, T_k, P_*^k) \quad (42)$$

[1]For simplicity, the variables of the functions are not written in the formula. We will for example denote φ_k instead of $\varphi_k\left(\tilde{P}_c^k, \tilde{N}_c^k\right)$.

rangeland			
j	1	2	3
u_r^j	0.1	0.4	0.2
K_r^j	80	40	60
m_r^j	0.08	0.06	0.07
o_r^j	0.08	0.5	0.3
e_r^j	2.0	1.6	2.0
s_r^j	10	20	15
α_r^j	0.5	0.4	0.5

cropland		
k	1	2
u_c^k	0.01	0.02
K_c^k	40	30
m_c^k	0.05	0.06
o_c^k	0.4	0.5
e_c^k	0.05	0.1
s_c^k	5.24	10
h_c^k	0.8	0.8

TABLE I
TABLE OF PARAMETERS VALUES OF THE AGRICULTURAL PLOTS NETWORK CONSIDERED IN THE NUMERICAL EXAMPLE.

where H_k, η_k, K_k and γ_k are respectively given by (17), (20), (24) and (30).
For the rangeland subsystems, the rangeland removal rates are given by: $\forall j = 1 : M_r$:

$$\frac{dd_r^j}{dt} = J(d_r^1, \ldots, d_r^{M_r})\delta_j(P_r^j, N_r^j, T, T^*) \tag{43}$$

where $T^* := \sum_{l=1}^{M_c} T_l$, with J and δ_j respectively given by (35) and (38).

For the numerical implementation of the control law, note that the modification used in [6] has been applied, which consists in applying the following practical control law:

$$\frac{dT_k}{dt} = \begin{cases} \max(0, K_k\gamma_k) & \text{if } T_k \in [T_k^{min}, T_k^{min} + \varepsilon_T[\\ K_k\gamma_k & \text{if } T_k \in [T_k^{min} + \varepsilon_T, T_k^{max} - \varepsilon_T] \\ \min(0, K_k\gamma_k) & \text{if } T_k \in]T_k^{max} - \varepsilon_T, T_k^{max}] \end{cases} \tag{44}$$

with $\varepsilon_T > 0$ instead of (42). The same modification (with $\varepsilon_d > 0$) has been applied to equation (43).

V. NUMERICAL EXAMPLE

In this section we consider a meta-ecosystem composed of $M_c = 2$ cropland subsystems and $M_r = 3$ rangeland subsytems. The parameters of each cropland and rangeland subsystem are given in table I. We applied the control law described in section IV on this meta-ecosystem, with the following parameter values (where $P_{min}^k := \frac{C_{min}^k}{s_c^k h_c^k}$):

$$\begin{aligned}
&P_{min}^1 = 7,\ P_{min}^2 = 2,\ k_1 = k_2 = 0.04,\\
&T_1^{min} = T_2^{min} = 0,\ T_1^{max} = 16,\ T_2^{max} = 25,\\
&K_{m,1} = K_{M,1} = K_{m,2} = K_{M,2} = 0.01,\ b = 0.04,\\
&\xi_1 = \xi_2 = 0.9,\ \omega_1 = \omega_2 = 0.6,\ \varepsilon_T = 0.5,\\
&d_{min}^1 = d_{min}^2 = d_{min}^3 = 0,\\
&d_{max}^1 = 0.035,\ d_{max}^2 = 0.04,\ d_{max}^3 = 0.03,\ \varepsilon_d = 0.001,\\
&D_{m,1} = D_{M,1} = D_{m,2} = D_{M,2} = D_{m,3} = D_{M,3} = 0.002,\\
&\theta_1 = \theta_2 = \theta_3 = 1/M_r = 1/3,\ a = M_r 2^{2M_r} = 192.
\end{aligned}$$

Note that the maximal value d_{max}^k considered here is not equal to $\frac{u_r^k e_r^k}{o_r^k} - m_r^k$ as defined in (6). We took a smaller value, in order to force the value d_r^k to reach the upper bound.

The objective was to make the crop production C reach the value:

$$C^* = 70 \text{ kgN} \tag{45}$$

at the end of the year. The results are shown in figures 2 and 3.

In figure 2 (top), the evolution of the crop production at the scale of the whole agricultural plots network (variable C) and the one of $\sum_{k=1}^{M_c} s_c^k h_c^k P_*^k$ are plotted. We see that C and $\sum_{k=1}^{M_c} s_c^k h_c^k P_*^k$ both converge towards C^* as it was expected. If we now look at each cropland subsytem (second subfigure in 2), we see that each variable P_c^k follows its time-varying setpoint P_*^k that finally stabilizes at a value that is greater than the minimal plant biomass value $P_{min}^k = \frac{C_{min}^k}{s_c^k h_c^k}$ that we have chosen. For each cropland subsystem, the quantity of manure $\beta_k T$ that is spread on the subsystem as fertilizer is plotted: it well follows the setpoint T_k while never exceeding the value $T_k^{max} = 30$ (third subfigure in 2). Finally, the value of the manure distribution coefficients β_k are shown on figure 2 (bottom). We see that the distribution between the two cropland subsystems varies with time. At the beginning of the year, most of the manure is spread on one of the cropland subsystem whereas it is more balanced at the end of the year.

In figure 3 (top), the flux of nutrients removed from the rangeland subsystems to feed the livestock (variable T) is shown with its setpoint value T^*. In figure 3 (bottom), the rangeland removal rates that are applied are shown. We see that the removal has to be greater at the beginning of the year, when the plants in cropland subsystems are still small. Note here again that saturation constraints on the control inputs d_r^k are respected. The difference between the control input values d_r^k and the upper bounds d_{max}^k at the beginning of the year are due to the introduction of parameter ε_d in the control law.

VI. CONCLUSION

In this paper, we propose a strategy to control the crop-production in a network of agricultural plots. This network is composed of two types of subsystems: some rangeland subsystems, and some cropland subsystems. The rangeland subsystems are used to feed the livestock which is parked separately from the rangeland and cropland fields. The manure produced by the livestock is then used as fertilizer on the cropland subsystems. The objective was to reach a given crop production (at the scale of the agricultural plots network) while ensuring a minimal production in each of the cropland subsystems. The control inputs being subject to saturation constraints, we proposed to use a method base on a time-scale transformation in order to design the control law. The control law was composed of two loops: one that control the crop production with the flux of manure coming from the livestock farms and the second one to control the flux of manure with the rate at which the plant biomass is removed from rangeland subsystems to feed the livestock. We applied this control strategy on a simple example of network composed of three rangeland subsystems, and two cropland susbsytems. The simulations give satisfactory results. The next step will consist in considering bigger networks that will be more realistic to see if the control strategy is sufficiently robust.

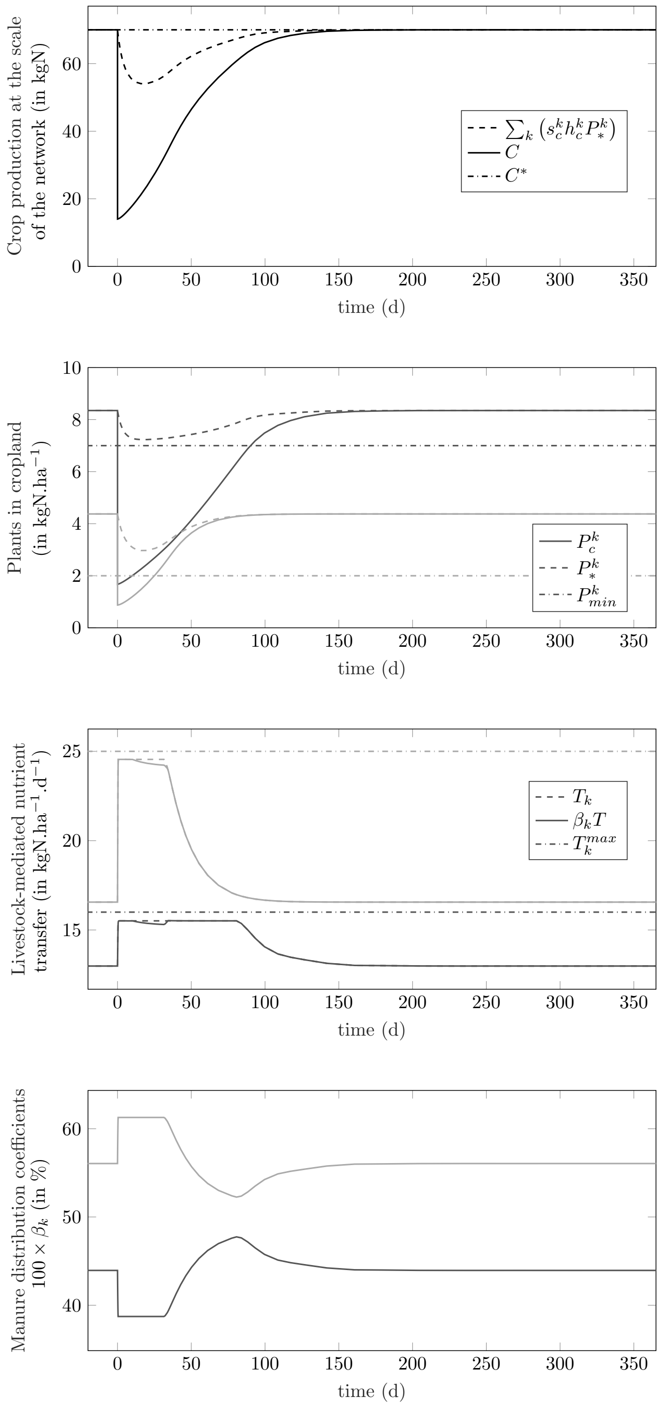

Fig. 2. Time evolution of the variables related to the control of C with T and β_k. From top to bottom: (1) Crop production at the scale of the network C, crop production setpoint C^* and intermediate variable $\sum_{k=1}^{M_c} s_c^k h_c^k P_*^k$. (2) Plants biomass in each cropland subsystem P_c^k, plants biomass setpoint P_*^k and minimum plant biomass $P_{min}^k := \frac{C_{min}^k}{s_c^k h_c^k}$. (3) Livestock-mediated nutrient transfer in each cropland subsystem $\beta_k T$, nutrient transfer setpoint T_k, and maximum nutrient transfer T_k^{max}. (4) Manure distribution coefficients β_k, expressed in percentage.

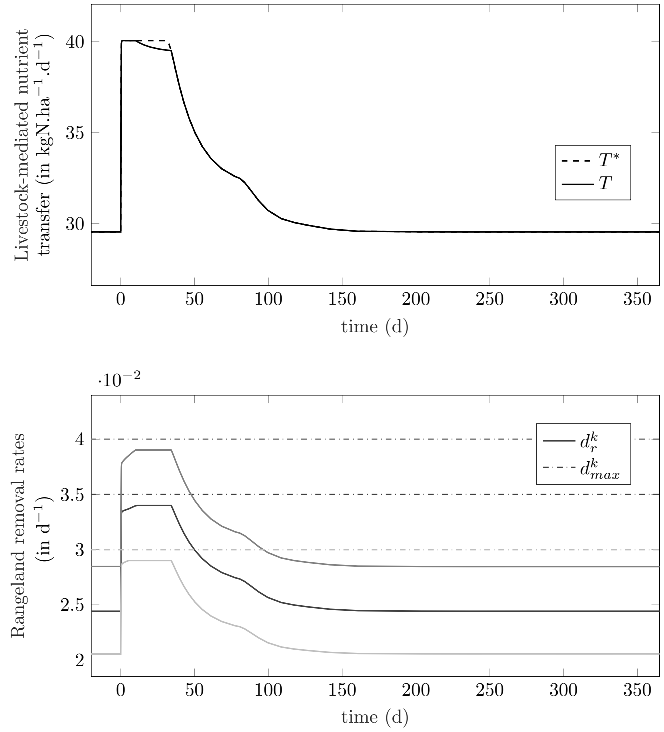

Fig. 3. Time evolution of the variables related to the control of T with d_r^k (second control loop). Top: Livestock-mediated nutrient transfer from rangeland subsystem T and its setpoint T^*. Bottom: Rangeland removal rates d_r^k, $k = 1:3$.

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