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# ENVIRONMENTAL MARKETS EXACERBATE INEQUALITIES

Stefan Ambec\*

July 2020

## Abstract

Environmental markets distribute tradable rights on natural resources that are available for free on the earth such as water, biomass or clean air. In a framework where users differ solely in respect of their access to the resource, I investigate the allocation of rights that are accepted in the sense that, after trading, users obtain at least what they can achieve by sharing the resources they control. I show that, among all accepted rights, the more egalitarian ones do not allow any redistribution among users. Consequently, compared to an efficient allocation of resources, the net trading of rights always increases inequality.

*Keywords: Common-pool resources, environmental externalities, property rights, cooperative game, fairness, tradable quotas, emission permits.*

*JEL codes: C71, D02, D63, Q28, Q38, Q58.*

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# 1 INTRODUCTION

Economic development is driven by the exploitation of natural resources such as water, land, forest, minerals, fossil fuels and fisheries. Economic activities are deteriorating our environment and causing harm to biodiversity, the climate and our health. The uncontrolled extraction of natural and environmental resources leads to the famous “tragedy of the commons” (Gordon, 1954, Hardin, 1968). Resources are wasted, misallocated and potentially exhausted irreversibly.

Economists tend to attribute this tragedy to the lack of clear property rights on open-access natural resources and the environment. It deprives people of any incentive to conserve the resource for future use, or to assign it to those who value it the most either now or in the future. Assigning and enforcing property rights is a solution to the tragedy of the commons for natural resources (Demsetz, 1967) and environmental externalities (Coase, 1960). The trading of property rights in competitive markets ensures that the resource is allocated efficiently among users. Total welfare from resource exploitation is thus improved. The property right approach recommends the implementation of environmental markets (Anderson and Libecap, 2014). Examples include water markets (Grafton et al., 2011), tradable fishing quotas (Hannesson, 2004) or tradable emission schemes for air or water pollution (Shortle, 2012, Schmalensee and Stavins, 2017; see Anderson and Libecap, 2014, for a survey).

Environmental markets privatize resources that used to be free. For this reason, the initial allocation of property rights is controversial. People are reluctant to buy something that used to be available for free. When they do obtain property rights on some of the resource for free, the initial allocation of rights is debatable on the grounds of fairness. For instance, under the principle of equal rights to humans, natural resources should be divided equally. However, if everybody on earth owns an equal share of a resource, the people who used to rely on the resource for their living are now required to buy rights from others.

Examples of such controversies over the initial allocation of rights in environmental markets abound. For air pollution controlled with tradable emission allowances, the way allowances are assigned among polluting firms impacts their profits. Under grandfathering, the most polluting firms enjoy windfall profits from owning and selling allowances they get for free, while new entrants have to buy all their allowances. Similarly, water used for irrigation can be better managed through tradable water rights or quotas. Yet some farmers might experience a loss of welfare by buying rights, compared to unregulated extraction. Others might become far more wealthy by selling their rights to municipalities or industries, rather than irrigating their own land. Setting water markets might exacerbate inequality. Water trade can be a source of conflict among farmers (Libecap, 2009).

This paper investigates the allocation of rights in environmental markets. I examine the initial allocation of rights that would normally be accepted by all the resource users. To do so, I rely on cooperative game theory. I define the welfare that a group of users can secure by sharing the amount of a resource they control collectively under free access. As Ostrom (1990) documented, users are often able to organize themselves to manage natural resources, thereby solving or at least mitigating, the tragedy of the commons. The so-called free-access welfare determines their bargaining power. The allocation of rights is accepted if every group of users is assigned at least its free-access welfare.

To address this problem, I consider a general framework in which resource users enjoy the

same benefit from consuming it. The benefit increases with diminishing return. Users differ solely in respect of their access to the resource. They are connected to one or several pools of a same resource. The ability to draw from some pools and not others can be due to geographical proximity, institutional constraints or technological capability. For instance, the resource pools may be a water reservoir connected to farmers' land by rivers, canals or irrigation ditches; or it may be oil and gas fields in several locations. Users differ in their capability to extract these resources (e.g. being able to drill deeply offshore) or to transform each source of energy into electricity (e.g. running a coal or natural gas power plant). In the case of air pollution and emission allowances, the pool could be the regulated cap on emissions in specific area. Users are firms running production plants in different locations. They can reshuffle their production on the different locations depending on the local price of emission allowances.

In this framework, I set out to examine the extent to which inequalities of access can be ironed out or at least mitigated by trading rights accepted by all users. It turns out that they cannot. Using Dutta and Ray (1989)'s concept of egalitarianism under participation constraints, I show that among all types of allocation of rights accepted by users, the most egalitarian ones entail no net trade. Any trade of accepted rights in environmental markets exacerbates inequalities of access.

The paper is related to the axiomatic analysis of resource division (see Thomson, 2008, for a survey). Free-access welfare corresponds to the core bounds of a cooperative game with externalities: the welfare that a coalition of players can guarantee to itself depends on the behavior of other players (Bloch, 1996). Free-access welfare assumes the worst that could potentially happen for the coalition regarding the pools shared with outsiders, as they are all exhausted. It is as if outsiders were able to exclude the coalition from those shared pools. Under this assumption, the core is not empty and can be quite large. Yet I show that no redistribution of welfare can be achieved within the core. This result is due to a property of the efficient allocation of the resource. The coalition of users who get more than  $x$  units of the resource enjoys exactly their free-access welfare for any  $x$ . Consequently, they block any transfer of welfare to those who get less than  $x$ .

The model shares some features with the river sharing problem introduced by Ambec and Sprumont (2002).<sup>1</sup> However, the spatial structure and the timing differ. In the river sharing problem, access to the resource (water) is sequential from upstream to downstream. In contrast, here, all users who are connected to the same pool have symmetric and simultaneous access.<sup>2</sup>

The structure of the model is similar to that of Bochet, Ilklic and Moulin (2013), who generalize the resource division problem (Sprumont, 1991) with several resource pools and unequal access. They also focus on egalitarian solutions. However, their framework is with non-transferable utility: only one good is consumed, and no trade or compensations are allowed. Their concern is the allocation of the good which has to satisfy several desirable properties. I, on the other hand, am interested in the distribution of the welfare from consuming the good by trading rights in environmental markets.

The rest of the paper proceeds as follows. Section 2 describes the resource-sharing problem: the model in Section 2.1, the efficient allocation of the resource in Section 2.2, and environmental

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<sup>1</sup>The model has been extended by Ambec and Ehlers (2008) and Van der Brink et al. (2011).

<sup>2</sup>Ambec (2008) analyzes a single resource pool shared by several users under symmetric access with similar concave benefit and transferable utility. However, the focus is on the Walrasian allocation with equal endowment which is characterized with fairness principles.

markets in Section 2.3. Section 3 contains the main analysis including the definition on the free-access welfare in Section 3.1, and the main results in Section 3.2. Section 4 concludes with some remarks. All proofs are in Appendix.

## 2 THE RESOURCE SHARING PROBLEM

### 2.1 THE MODEL

A set of agents (consumers, farmers, firms, municipalities, countries)  $N = \{1, \dots, n\}$  called “users” are sharing a homogenous good called “resource” available in different locations called “pools”. Each agent enjoys the same benefit  $b(x)$  from consuming  $x$  units of the resource. The benefit  $b$  is increasing and concave, i.e.,  $b'(x) > 0$  and  $b''(x) < 0$ . It is expressed in terms of money that can be transferred among users at no cost, e.g., through trading of resource rights. We normalize the benefit of zero consumption to zero:  $b(0) = 0$ . We further assume that  $b'_i(0)$  is high enough (e.g.  $b'(0) = +\infty$ ) so that assigning no resource to one agent is never efficient. Let  $M = \{1, \dots, m\}$  be the set of resource pools. Let  $e_j$  denote the amount of resources available at pool  $j$  for every  $j \in M$  with  $e_j > 0$ . Each agent  $i$  has access to some subset  $S_i \subseteq M$  of the sources. Symmetrically, each pool  $j$  can supply a subset  $R_j \subseteq N$  of agents.

A resource-sharing problem  $\mathcal{P} \equiv (N, M, \mathcal{S}, b, \mathbf{e})$  is defined by a set of agents  $N = \{1, \dots, n\}$ , a set of pools  $M = \{1, \dots, m\}$ , the sets of pools  $\mathcal{S} = \{S_i\}_{i \in N}$  that are connected to each user, the benefit function  $b$  and the amount of resource  $\mathbf{e} = (e_1, \dots, e_m)$  available in each pool.<sup>3</sup>

An example of a resource-sharing problem with three users and three pools is represented in Figure 1.

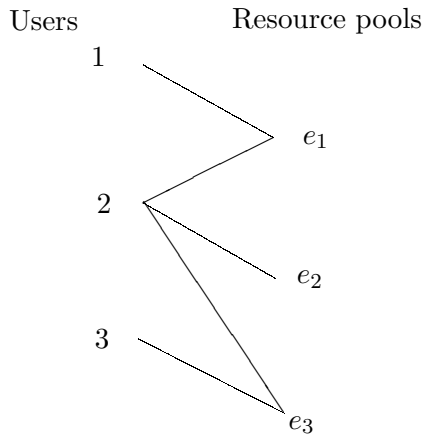


Figure 1: Example of a resource sharing problem with unequal access.

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<sup>3</sup>Note that the model encompasses the extreme cases of equal access to all sources  $S_i = M$  for every  $i \in N$ , as well as exclusive access to pools  $S_i \cap S_j = \emptyset$  for every  $i, j \in N$ .

In the above example, users 1 and 2 share the  $e_1$  units of resources available in pool 1, user 2 has exclusive access to  $e_2$  in pool 2, users 2 and 3 share  $e_3$  in pool 3. Resource access is defined by the sets  $S_1 = \{1\}$ ,  $S_2 = \{1, 2, 3\}$  and  $S_3 = \{3\}$  or, equivalently, by  $R_1 = \{1, 2\}$ ,  $R_2 = \{2\}$  and  $R_3 = \{2, 3\}$ .

An allocation of resources is a matrix  $\mathbf{X} = [x_{ji}]_{j \in M, i \in N}$  where  $x_{ji} \geq 0$  denotes  $i$ 's extraction from pool  $j$  for every  $i \in N$  and  $j \in M$ . The allocation  $\mathbf{X}$  is feasible if it satisfies the following resource constraints for  $j = 1, \dots, m$ :

$$\sum_{i \in R_j} x_{ji} \leq e_j. \quad (1)$$

Let  $\Omega$  denote the set of feasible allocations. The feasible allocation  $\mathbf{X}$  yields the consumption plan  $\mathbf{x} = (x_i)_{i \in N}$  where  $x_i = \sum_{j \in S_i} x_{ji}$  for every  $i \in N$ . User  $i$  enjoys a benefit  $b(x_i)$  from consuming  $x_i$  for  $i = 1, \dots, n$ . The total welfare from consuming  $\mathbf{x}$  is thus  $\sum_{i \in N} b(x_i)$ . I now examine the feasible allocations and consumption plans that maximizes total welfare.

## 2.2 EFFICIENCY

An efficient allocation and consumption plan maximizes total welfare subject to the feasibility constraints.<sup>4</sup> In our framework with concave benefit function, the efficient consumption plan is unique. However, it could be induced by several feasible resource allocations that all lead to the same total welfare. Let us denote the efficient allocations and consumption path with a star as superscript, i.e.,  $\mathbf{X}^*$  and  $\mathbf{x}^*$  respectively.

An efficient allocation solves the following program:

$$\begin{aligned} \max_{\mathbf{X}} \sum_{i \in N} b\left(\sum_{j \in M} x_{ji}\right) \text{ s.t.} \\ x_{ji} = 0 & \quad \forall (j, i) \in M \setminus S_i \times N \\ x_{ji} \geq 0 & \quad \forall (j, i) \in S_i \times N \\ \sum_{i \in R_j} x_{ji} \leq e_j & \quad \forall j \in M \end{aligned} \quad (2)$$

The first set of constraints assigns nothing from pools that users do not have access to. The second one makes sure that extraction is non-negative. The third set consists of the resource constraints that limit extraction to resource availability at each pool.

Denoting  $\mu_j$  and  $\lambda_{ji}$  the Lagrangian multipliers associated with the resource constraint of pool  $j$  and the non-negativity constraint for user  $i$ 's allocation of pool  $j$  for any  $(j, i) \in S_i \times N$ , we obtain the following first-order conditions for every  $(j, i) \in S_i \times N$ :

$$b'(x_i^*) = \mu_j - \lambda_{ji}, \quad (3)$$

plus the complementary slackness conditions derived from the constraints of the program. The first-order conditions (3) equalize each user  $i$ 's marginal benefit to the multiplier of the resource constraint  $\mu_j$  minus the multiplier of the non-negativity constraint  $\lambda_{ji}$  for each pool  $j$  user  $i$  has access to. The first-order conditions have several implications for the solution to the program.

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<sup>4</sup>Note that this definition of efficiency is implied by Pareto efficiency with (costless) transferable utility as assumed here.

Consider two users who have access to the same pool. First, if they both extract some of the resource from this pool, they should consume the same amounts. Technically speaking, if two users  $l$  and  $h$  extract from the same pool  $j$ , it means that  $x_{jl}^* > 0$  and  $x_{jh}^* > 0$ . Therefore the resource constraint associated with pool  $j$  is binding, while the non-negativity constraints are not so for both users. Thus  $\mu_j > 0$  and  $\lambda_{jl} = \lambda_{jh} = 0$ . The first-order conditions implies  $b'(x_l^*) = b'(x_h^*) = \mu_j$ , which implies  $b_l = b_h$  then  $x_l^* = x_h^*$ .

Second, if one of the two users connected to the same pool extracts from the pool and the other does not, then the later consumes at least as much than the former. If, say, user  $l$  extracts from  $j$  but not user  $h$  does not, formally if  $x_{jl}^* > 0$  and  $x_{jh}^* = 0$ , this implies that user  $h$  has the non-negativity constraint binding for pool  $j$ , hence  $\lambda_{jh} = 0$ . The first-order conditions imply  $b'(x_l^*) = \mu_j \geq b'(x_h^*) = \mu_j - \lambda_{jh}$ , which in turn implies  $x_l^* \leq x_h^*$ . Intuitively, if user  $l$  is extracting some resource from pool  $j$  but not from pool  $h$ , it is because user  $h$  has access to other pools which are more “abundant” in the sense that they have lower shadow values defined by the resource constraint multiplier  $\mu_j$ . Therefore user  $h$  should extract from those abundant pools instead of pool  $j$  which is left to  $l$  and to the other users connected to  $j$  who consume less than user  $h$ .

Third, if a user extracts from two pools, those pools should have same shadow value. Formally, if user  $i$  extracts from pools  $h$  and  $l$ , i.e., if  $x_{hi}^* > 0$  and  $x_{li}^* > 0$ , then  $\lambda_{hi} = \lambda_{li} = 0$  in (3), which implies  $b'(x_i^*) = \mu_h = \mu_l$ .

Using the above properties, we can rank users and resource pools according to their shadow values. Formally, an efficient allocation  $\mathbf{X}^*$  solution to (2) defines a partition  $\{N_k\}_{k=1}^K$  of the set of users  $N$  and a partition  $\{M_k\}_{k=1}^K$  of the set of pools  $M$ . Each subset  $M_k$  is the set of pools with same shadow value  $\mu^k$  for  $k = 1, \dots, K$  with  $\mu^1 > \dots > \mu^K$ . Each subset  $N_k$  is the set of users who extract from at least one pool in  $M_k$ . They all enjoy the same marginal benefit equals to  $\mu^k$ . Their consumption is thus the same, equal to  $b'^{-1}(\mu^k)$ . Hence we can rank the efficient consumption plan  $\mathbf{x}^*$  as consumption levels  $\{x^k\}_{k=1}^K$  with  $x^1 < \dots < x^K$  where every agent in  $N_k$  consumes  $x^k$  for  $k = 1, \dots, K$ . Users in  $N_k$  extracts only from the pools in  $M_k$ . They do not extract from pools outside  $M_k$ , either because those pools have a higher shadow value for pools in  $M_1$  to  $M_{k-1}$ , or because they are not connected to those pools for pools in  $M_{k+1}$  to  $M_K$ .

### 2.3 ENVIRONMENTAL MARKETS

The resource pools are divided among users through property rights. An allocation of rights (or “endowment”) is a feasible resource allocation  $\mathbf{W} = [w_{ji}]_{j \in M, i \in N}$ . The column  $i$  of  $\mathbf{W}$  defines the rights that are assigned to user  $i$  on each pool  $j = 1, \dots, m$ . The line  $j$  of  $\mathbf{W}$  describes the rights on pool  $j$  assigned to users  $i = 1, \dots, n$ . Rights be can consumed, sold to other users or bought from other users. Assume complete markets: a market exists for resource located in each pool. User  $i$  can sell part or all the  $w_{ji}$  units she or he owns from pool  $j$  in market  $j$  at a price  $p_j$ . She or he can buy rights on the resource located in pool  $j$  in market  $j$  at price  $p_j$  for  $j = 1, \dots, m$ . I assume that environmental markets are competitive and, therefore, efficient. Users are price-takers, they decide how much to extract, sell or buy from each pool given the equilibrium prices  $\mathbf{p} = (p_j)_{j \in M}$ .

Given the initial allocation of rights  $\mathbf{W}$ , a competitive (Walrasian) equilibrium is defined by a feasible resource allocation  $\mathbf{X}$  and a vector of prices  $\mathbf{p} = (p_j)_{j \in M}$  such that each user  $i \in N$  chooses



the amount of resource  $x_{ji}$  she or he extracts from every pool  $j \in M$  that maximizes

$$b_i \left( \sum_{j \in S_i} x_{ji} \right) + p_j (w_{ji} - x_{ji}), \quad (4)$$

and the following market clearing conditions hold for every  $j \in M$ :

$$\sum_{i \in N} x_{ji} = \sum_{i \in N} w_{ji}. \quad (5)$$

The First Theorem of Welfare applies so that the competitive equilibrium is efficient. The allocations of resource solution to (4) are efficient as described in the previous section. The equilibrium consumption plan is therefore efficient and unique. It is denoted  $\mathbf{x}^*$ . Furthermore, by maximizing (4) with respect to  $(x_{ji})_{j \in M}$  for every user  $i$ , we obtain the following first-order condition that characterize the equilibrium prices :

$$b'(x_i^*) = p_j, \quad (6)$$

for every  $j \in S_i$  and  $i \in N$ . Comparing (3) and (6) shows that equilibrium prices are equal to the shadow value of the resource constraints:  $p_j = \mu_j$  for every  $j \in M$  such that  $x_{ji}^* > 0$  for one  $i \in N$  at least.

To sum up, an allocation of rights  $\mathbf{W}$  might lead to several resource allocations which are all efficient  $\mathbf{X}^*$ . However, the vector of equilibrium prices  $\mathbf{p}$  is unique and equal to the shadow value of resource scarcity at each pool  $\mu_j$  for every pool  $j \in M$ . Similarly, the consumption plan is unique and efficient  $\mathbf{x}^*$ .

Both prices and consumptions depend on the total amount of rights at each pool  $\sum_{i \in N} w_{ji} = e_j$  but not on how rights are divided among users. Yet the allocation of rights determines the distribution the total welfare among users. It allows to transfer welfare among users. The post-trade welfare of an arbitrary user  $i \in N$  is:

$$u_i = b(x_i^*) + \sum_{j \in M} \mu_j (w_{ij} - x_{ji}^*) \quad (7)$$

An allocation of rights  $\mathbf{W}$  yields a distribution  $\mathbf{u} = (u_i)_{i \in N}$  of the total welfare  $\sum_{i \in N} b_i(x_i^*)$  where  $u_i$  is defined by (7) for every  $i \in N$ . Importantly,  $\mathbf{u}$  is unique because, even if  $\mathbf{W}$  induces several efficient resource allocations  $\mathbf{X}^*$ , they all yields the same net trade transfer  $\sum_{j \in M} \mu_j (x_{ji}^* - w_{ij})$  for every  $i \in N$ . Hence  $\mathbf{W}$  yields a unique welfare distribution  $\mathbf{u}$  defined by (7) for  $i = 1, \dots, n$ .

### 3 THE FREE-ACCESS WELFARE AND INEQUALITY

#### 3.1 A COOPERATIVE GAME APPROACH

I now rely on cooperative game theory to investigate which allocations of rights can be accepted by users. An allocation of rights  $\mathbf{W}$  is blocked by a coalition of users if this coalition can secure a higher welfare under free access.  $\mathbf{W}$  is accepted if it is not blocked by a coalition of users. To compute the welfare that a group of users can achieve on its own under free access, I assume that users coordinate extraction efficiently given the resource available to them. This assumption is

conservative on resource availability: a group of users can rely only on the resource pools it fully controls. It is as if users expect to get nothing from pools shared with outsiders. The assumption is the lowest bound on the welfare that a group of users can guarantee to itself providing that the resource is shared efficiently within the group. I now define formally the free-access welfare.

Let us consider an arbitrary group of users or coalition  $T \subseteq N$ . Let  $\xi(T) = \{j \in M | R_j \subseteq T\}$  be the set of pools fully controlled by coalition  $T$ . Denoted  $v(T)$ , coalition  $T$ 's free-access welfare is:

$$\begin{aligned}
v(T) = & \max_{\mathbf{X}_T} \sum_{i \in T} b \left( \sum_{j \in \xi(T)} x_{ji} \right) \\
& \text{s.t.} \\
& x_{ji} = 0 & \forall (j, i) \in M \setminus S_i \times T \\
& x_{ji} \geq 0 & \forall (j, i) \in S_i \cap \xi(T) \times T \\
& \sum_{i \in T} x_{ji} \leq e_j & \forall j \in \xi(T).
\end{aligned} \tag{8}$$

Let us denote by  $\mathbf{X}_T^T$  a resource allocation solution to program (8) for any  $T \subseteq N$ . It is an efficient allocation of the resource-sharing problem  $(T, \xi(T), b, \{S_i\}_{i \in T}, \mathbf{e}_{\xi(T)})$ . Let us denote  $\mathbf{x}_T^T$  the (unique) consumption plan solution to (8).

The allocation of rights  $\mathbf{W}$  is *blocked* by coalition  $T$  if the after-trade welfare distribution  $\mathbf{u}$  is such that  $\sum_{i \in T} u_i < v(T)$ . An allocation of rights  $\mathbf{W}$  is *accepted* if the after-trade welfare distribution  $\mathbf{u}$  is not blocked. Hence, for  $\mathbf{W}$  to be accepted,  $\mathbf{u}$  must satisfy the following free-access welfare bounds for every  $T \subset N$ :

$$\sum_{i \in T} u_i \geq v(T). \tag{9}$$

It is easy to show that the set of accepted rights is not empty. Consider the allocation of rights that assign an efficient allocation of resources. Whenever  $\mathbf{W} = \mathbf{X}^*$ , no net trade occurs among users so that the after-trade welfare of any user  $i$  is simply  $u_i = b(x_i^*)$ , i.e. the benefit of consuming all the rights from the pools she or he has access to. Let us call it the no-trade welfare distribution and denote it as  $\mathbf{u}^0$ . It turns out that the no-trade welfare distribution is not blocked by any coalition. The proof of Lemma 1 is in Appendix A.

**Lemma 1**  $\mathbf{W} = \mathbf{X}^*$  is an accepted initial allocation of rights.

Lemma 1 relies on a simple economic intuition. The allocation of rights  $\mathbf{W} = \mathbf{X}^*$  leads to the no-trade welfare distribution which assigns to any coalition its welfare with the efficient consumption plan. It is also the highest welfare that the coalition can achieve if outsiders extract their efficient consumption levels. The free-access welfare assumes that outsiders extract more than that: they extract all resource from pools they are connected to. Therefore the free-access welfare cannot be higher. It cannot exceed the welfare achieved with the no-trade welfare distribution.

### 3.2 REDUCING INEQUALITY WITH ACCEPTED ALLOCATION OF RIGHTS

Under the same diminishing return from resource consumption as assumed here, the total welfare would be maximal with an equal division of the resource available in every pool. However, some users might not be able to consume it due to unequal access. Indeed, as already mentioned, efficiency assigns the same consumption level for users in each subset  $N_k$  but less for users in  $N_{k-1}$

for  $k = 2, \dots, K$ . I examine whether differences in resource consumption and, therefore, in benefits from resource use, can be mitigated by an allocation of rights that is not blocked by users.

To address this question, I further investigate the free-access welfare. It is often quite low. For instance, as long as a group of users shares all its resource pools with at least one outsider, its free-access welfare bowls down to zero. It therefore does not restrict the set of accepted rights. To identify which coalitions are restricting to the greatest extent the set of accepted rights with its free-access welfare, I need to establish a further property of the efficient resource allocation  $\mathbf{X}^*$ . Consider any threshold consumption level  $x^k$  in  $\mathbf{x}^*$ . Coalition  $\cup_{l=k}^K N_l$  includes all the users who consume at least  $x^k$ . The following Lemma states that users that belong to  $\cup_{l=k}^K N_l$  are extracting only from pools they fully control for  $k = 1, \dots, K$ . The proof is in Appendix B.

**Lemma 2**  $\cup_{l=k}^K M_l = \xi(\cup_{l=k}^K N_l)$  for  $k = 1, \dots, K$ .

Lemma 2 implies that coalition  $\cup_{l=k}^K N_l$ 's free-access welfare coincides with its welfare with the efficient consumption plan. Formally, for every  $l = 1, \dots, K$ , we have:

$$v(\cup_{l=k}^K N_l) = \sum_{i \in \cup_{l=k}^K N_l} b_i(x_i^*) = \sum_{l=k}^K |N_l| b(x^l), \quad (10)$$

where the last equality is due to the decomposition of  $\mathbf{x}^*$  into  $K$  levels of consumption  $(x^k)_{k=1, \dots, K}$ . The group of users consuming at least  $x^k$  obtains their free-access welfare for every threshold level  $x^k$  for  $k = 1, \dots, n$ . This property has a straightforward implication for redistribution through the trading of rights. It precludes any net transfer of wealth through net trade from coalition  $\cup_{l=k}^K N_l$  to coalition  $\cup_{l=1}^{k-1} N_l$  for  $k = 2, \dots, K$ . The set of users with at least  $x^k$  units of resource - each of them is enjoying  $b(x^k)$  or more - blocks any allocation of rights that transfers welfare to those consuming less than  $x^k$  - i.e. those enjoying strictly less than  $b(x^k)$ . They block any allocation of rights that makes them a net buyer of rights. Hence, any allocation of rights that would reduce inequalities of consumption through trade is blocked.

To capture rigorously the idea that inequalities inherent to unequal access cannot be mitigated we rely on the concept of equalitarianism under participation constraints proposed by Dutta and Ray (1989). A welfare distribution  $\mathbf{u}$  is equalitarian under participation constraints if no other welfare distribution that satisfies the participation constraints Lorenz dominates  $\mathbf{u}$ . In our framework, the participation constraints are the free-access welfare bounds. It turns out that the no-trade welfare distribution  $\mathbf{u}^0$  is the Dutta-Ray solution. The proof is in Appendix C.

**Theorem W =  $\mathbf{X}^*$**  leads to the most equalitarian welfare distribution induced by an accepted allocation of rights.

The main result implies that, among all allocation of rights that are accepted by users, the more equalitarian ones do not involve any net trade. Hence any allocation of rights which involves some net trade among users is either blocked by a coalition of users, or leading to a more unequal welfare distribution. The trading of rights that are accepted by users can only exacerbate inequalities due to unequal access.

The reader familiar with the cooperative game literature might wonder whether the game defined by the free-access welfare  $v$  is convex. A cooperative game is convex if the marginal contribution

of each player to any coalition weakly increases when the coalition expands, i.e.,  $v(S) - v(S \setminus i) \geq v(T) - v(T \setminus i)$  for every  $i \in T \subset S$ . It turns out that the cooperative game induced by the resource sharing problem is neither convex nor concave. It is easy to find examples where the marginal contribution of a user weakly increases when the coalition expands. But the reverse can also be true for instance in the resource-sharing problem graphed in Figure 1 with  $(e_1, e_2, e_3) = (3, 1, 7)$ . The marginal contribution of user 1 to  $\{1, 2\}$  is  $v(\{1, 2\}) - v(\{2\}) = 2b(2) - b(1)$ , while her or his marginal contribution to  $\{1, 2, 3\}$  is  $v(\{1, 2, 3\}) - v(\{2, 3\}) = b(3) - 2b(4) - 2b(4) = b(3)$ . By  $b$  being concave,  $b(3) - b(2) < b(2) - b(1)$  which shows  $v(\{1, 2, 3\}) - v(\{2, 3\}) < v(\{1, 2\}) - v(\{2\})$ .

## 4 CONCLUDING COMMENTS

Environmental markets solve the tragedy of the commons. Overexploitation can be avoided by assigning property rights on common-pool resources. Making those rights tradable ensures that the resource is assigned efficiently to those who value it the most. Equity can be addressed by the initial allocation of rights. By trading their rights, users can be compensated for the inequality of access to the resource pools. However, some users may oppose to this redistribution though trading if they are worse off compared to free access. When users are collectively able to manage the resources that they fully control, any allocation of rights that induces some redistribution of welfare by trading rights is opposed by at least one group of users. The most equalitarian allocation of rights that is accepted by all users involves no net trade among them. It requires the resource to be assigned efficiently without trading. Inequality of access cannot be mitigated through the allocation of rights in environmental markets. Worse, any trading of rights that are accepted by users exacerbates inequalities.

I conclude by mentioning further issues that can be addressed within the same framework.

First, another way to mitigate the inequality of access is to connect the users who consume less to more resource pools. This can be done in practice by building canals for surface water, subsidizing tube-wells for groundwater, or merging local permit markets for air pollution. It would reduce the bargaining power of the coalition of users who previously had exclusive access to the resource pools but cannot rely any more on them without including the newcomers. It would make the no-trade welfare distribution less unequal.

Second, the impossibility to compensate for inequality of access to common-pool resources holds not only in environmental markets but also for other regulations. Consider for instance a refunded tax (or price) system in which users have to pay a fee per unit of resource. Efficiency requires the resource to be taxed (or priced) at the shadow value of the pool. Hence, users who consume less would have to pay more per unit of resource extracted since they have access to more expensive pools. However, any redistribution of the revenue collected from taxing the resource that offset the tax bill of poor users is blocked by rich users.

Third, the result relies on the assumption that users can efficiently manage the resource pools they fully control under free access. Conflicts among users would reduce the free-access welfare and therefore their bargaining power. The trading of rights could then allow some redistribution. How much redistribution is accepted, depending on the efficiency loss of collective management, is an open question.

## A PROOF OF LEMMA 1

The no-trade welfare distribution assigns  $\sum_{i \in T} b(x_i^*)$  to an arbitrary coalition  $T$ , which is also the solution to the following program:

$$\begin{aligned}
 \max_{\mathbf{x}_T} \sum_{i \in T} b_i \left( \sum_{j \in M} x_{ji} \right) \text{ s.t.} \\
 x_{ji} = 0 & \quad \forall j \in M \setminus S_i, \forall i \in T \\
 x_{ji} \geq 0 & \quad \forall j \in S_i, \forall i \in T \\
 \sum_{i \in T \cap R_j} x_{ji} \leq e_j - \sum_{i \in R_j \setminus T} x_{ji}^* & \quad \forall j \in S_T
 \end{aligned} \tag{11}$$

Programs (8) and (11) have the same objective and control variables. They differ only on the last set of constraints, which are more stringent in (8) than in (11). Therefore the value of the objective in (8) cannot be higher than the one in (11), which shows that (9) holds for coalition  $T$ .

## B PROOF OF LEMMA 2

First observe that efficiency implies  $\xi(\cup_{l=k}^K N_l) \subset \cup_{l=k}^K M_l$ . Otherwise some pools that belong to  $\xi(\cup_{l=k}^K N_l)$  will be not extracted, because users outside  $\cup_{l=k}^K N_l$  do not have access to them by definition, which contradicts the assumption that  $\mathbf{x}^*$  is efficient. Second we show that  $\xi(\cup_{l=k}^K N_l) \supset \cup_{l=k}^K M_l$ . Suppose that it is not so. Suppose there exists a pool  $j$  such that  $j \in \cup_{l=k}^K M_l$  and  $j \notin \xi(\cup_{l=k}^K N_l)$ . Since  $j \notin \xi(\cup_{l=k}^K N_l)$ , by definition, there exists a user  $f$  outside  $\cup_{l=k}^K N_l$  who has access to pool  $j$ :  $\exists f \in N \setminus \cup_{l=k}^K N_l$  such that  $j \in S_f$ . Pick a user  $h \in \cup_{l=k}^K N_l$  who extracts from pool  $j$ , i.e., such that  $x_{jh}^* > 0$ . Since  $f \in N \setminus \cup_{l=k}^K N_l$  and  $h \in \cup_{l=k}^K N_l$ , then  $b'(x_f) > b'(x_h^*)$ . Therefore  $\exists \epsilon > 0$  such that  $x_{jh}^* - \epsilon > 0$  and  $b'(x_f + \epsilon) > b'(x_h^* - \epsilon)$ . Consider the feasible resource consumption plan  $\mathbf{x}'$  defined by:  $x'_f = x_f + \epsilon$ ,  $x'_h = x_h^* - \epsilon$ ,  $x'_j = x_j^*$  for every  $j \neq f, h$ . It increases total welfare by:

$$\begin{aligned}
 \sum_{i \in N} b(x'_i) - \sum_{i \in N} b(x_i^*) &= b(x'_f) + b(x'_h) - (b(x_f^*) + b(x_h^*)) \\
 &= b(x_f + \epsilon) - b(x_f^*) - [b(x_h^*) - b(x_h^* - \epsilon)] \\
 &> (b'(x_f + \epsilon) - b'(x_h^* - \epsilon))\epsilon \\
 &> 0
 \end{aligned}$$

where the first inequality is due to the concavity of  $b$  while the last one is due to the assumption on  $\epsilon$ . This contradicts that  $\mathbf{x}^*$  is efficient.

## C PROOF OF THEOREM

Let us denote the consumption of users in  $N_k$  as  $x^k$  in the efficient consumption plan  $\mathbf{x}^*$  for  $k = 1, \dots, K$ . We know from Lemma 1 that the no-trade welfare distribution  $u_i^0 = b(x_i^*)$  for  $i = 1, \dots, n$  satisfies the free-access welfare bounds. I show that any welfare distribution with (non-zero) net trade that satisfies the free-access welfare bounds is Lorenz-dominated by the no-trade welfare distribution. Suppose that it is not so. Suppose  $\mathbf{u}^0$  does not dominate a welfare distribution  $\mathbf{u}'$  that satisfies the free-access welfare bounds. Furthermore, assume without loss of generality that

$\mathbf{u}'$  is not dominated by any other welfare distribution which satisfies the free-access welfare bounds. Let us relabel users according to their welfare in  $\mathbf{u}^0$  from the poorest 1 to the richest  $n$ . Then  $\exists j$  such that

$$\sum_{i=1}^j u'_i > \sum_{i=1}^j u_i^0. \quad (12)$$

Since  $\mathbf{u}^0$  and  $\mathbf{u}'$  are welfare distributions, we have

$$\sum_{i=1}^n u'_i = \sum_{i=1}^n u_i^0 = v(N) = \sum_{k=1}^K |N_k| b(x^k). \quad (13)$$

Clearly  $j$  cannot be the richest user  $n$  because then (12) contradicts (13). Suppose  $j = n - 1$ . Equations (12) and (13) imply  $u'_n < u_n^0 = b(x^K)$  where the last equality is due to the definition of  $\mathbf{u}^0$ . The free-access lower bound for coalition  $N_K$  yields:

$$\sum_{i \in N_K} u'_i \geq |N_K| b(x^K)$$

Then  $\exists l \in N_K$  such that  $u'_l \geq b(x^K) > u'_n$  which contradicts the assumption that  $n$  is the wealthier user. Hence  $u'_n \geq u_n^0 = b(x^K)$  and  $j < n - 1$ . Moreover, if  $u'_n > u_n^0$ , then one can define a welfare distribution  $\mathbf{u}''$  with  $u''_n = u_n^0$  and  $u'_n - u_n^0$  transferred to other poor users so as to satisfy the free-access welfare bounds. Then  $\mathbf{u}''$  Lorenz dominates  $\mathbf{u}'$  a contradiction. Therefore  $u'_n = u_n^0 = b(x^K)$ . The same argument shows that  $u'_i = b(x^K) = u_i^0$  for the  $|N_K|$  richest users, i.e. all users  $i \in N_K$ . Hence user  $j$  in (12) is such that  $j \leq n - |N_K|$ . Furthermore, since  $u'_i = b(x^K) = u_i^0$  for every  $i \in N_K$ , (13) implies:

$$\sum_{i=1}^{n-|N_K|} u'_i = \sum_{i=1}^{n-|N_K|} u_i^0 = \sum_{k=1}^{K-1} |N_k| b(x^k)$$

Proceed as before to show that  $u'_i = b(x^{K-1}) = u_i^0$  for all users  $i \in N_{K-1}$ , which implies  $j \leq n - |N_K| - |N_{K-1}|$ . And so forth for  $k = K - 2, \dots, 1$ .

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