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Pollution, children's health and the evolution of human capital inequality

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Pollution, children's health and the evolution of human capital inequality*

Karine Constant[†] and Marion Davin[‡]

November 2020

Abstract

This article examines how pollution and its health effects during childhood can affect the dynamics of inequalities among households. In a model in which children's health is endogenously determined by pollution and the health investments of parents, we show that the economy may exhibit inequality in the long run and be stuck in an inequality trap with steadily increasing disparities, because of pollution. We investigate if an environmental policy, consisting in taxing the polluting production to fund pollution abatement, can address this issue. We find that it can decrease inequality in the long run and enable to escape from the trap if the emission intensity is not too high and if initial disparities are not too wide. Otherwise, we reveal that a policy mix with an additional subsidy to health expenditure may be a better option, at least if parental investment on children's health is sufficiently efficient.

JEL Classification: E60; I14; I24; Q53; Q58

Keywords: Pollution, Health, Human capital, Childhood, Overlapping generations, Inequality.

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1 Introduction

Pollution is one of the most important threat for health. According to the World Health Organization, approximately one-quarter of the global disease burden is due to modifiable environmental factors, representing 13.7 million deaths a year (WHO, 2016). There is considerable evidence that pollution, and in particular air pollution, has a positive and significant effect on morbidity - i.e. the rate of disease in the population - and mortality - i.e. the rate of death.¹ While pollution affects the entire population, children are identified as particularly vulnerable to its damaging health effects (see, e.g., Sacks et al., 2011, Beatty and Shimshack, 2014 or WHO, 2018). Empirical studies identify that these larger effects are due to both a larger vulnerability of children, mainly because their lungs, brains and immune system are not completely developed, and a larger exposure, as they spend more time engaging in physical activity outside - where air pollution levels are usually larger (see e.g. Bateson and Schwartz, 2007).

Such detrimental effect of pollution on children's health is not only a short-term issue but persists later in life (see Currie et al., 2014 for a literature review). Childhood exposure to pollution is found to be associated with poor adult health. Moreover, by increasing school absenteeism (see e.g. Park et al., 2002) and affecting negatively cognitive and learning abilities of children (see e.g. Factor-Litvak et al., 2014), pollution deteriorates also human capital formation. Therefore, it implies long-term negative consequences on human capital and income when adult, representing a persistent threat to the well-being and abilities of individuals.

In addition, the health effects of pollution are characterized by their unequal distribution across the population. Children from households of lower socioeconomic status - in particular in terms of education - are found to be more vulnerable to pollution (see, e.g., Neidell, 2004 or Currie, 2009). Those differences stem from the fact that wealthier and more educated parents are more likely to provide a cleaner environment to their children, but also to invest more in their children's health (see, e.g., Currie et al., 2014). For example, Neidell (2004) shows that carbon monoxide has a significant effect on child hospitalizations for asthma in California and that this effect is greater for children of lower socioeconomic status (even for similar exposure

¹See, e.g., Ostro, 1983 ; Hanna and Olivia, 2015 ; Graff Zivin and Neidell, 2012 on the effect of air pollution on morbidity and Bell and Davis, 2001 ; Pope et al., 2002 ; Bell et al., 2004 ; Evans and Smith, 2005 or Beelen et al., 2014 on its effect on mortality.

to pollution).

All these facts lead us to wonder about the potential role of the health effect of pollution during childhood in the intergenerational transmission of inequality among agents. In this paper, we aim at examining how this mechanism could occur, what would be the consequences, and therefore, whether environmental policies could be a part of the solution to overcome the inequality issue.

We focus on the dynamics of inequality across generations because inequality represents a major challenge for our society. Since 1980, the gap between rich and poor is at its highest level in most developed countries and follows an upward trend (see, e.g., OECD, 2015 or UN, 2020). Such disparities are multidimensional and concern economic, social and health dimensions. Large health inequalities exist in the population according to the socioeconomic status of individuals. According to the OECD (2019), *“across all countries, people in the lowest education category are twice more likely to view their health as poor compared to those with tertiary education”*, and *“people without high-school diploma can expect to live about 6 years less than those with tertiary education”*. These disparities may entail huge costs for society - in terms of well-being, health and social costs, productivity loss, discouraged investments, wasted potential *etc.* Moreover, a growing number of empirical and theoretical studies emphasize the net detrimental effect of inequality on long-term economic growth through its negative effect on human capital accumulation (see, e.g., Galor, 2011, OECD, 2015 or Constant, 2019). For all these reasons, reducing these disparities has become an explicit goal for many governments and, for that, it seems crucial to explore the different channels through which they occur.

To study the potential role of the health effect of pollution during childhood in the transmission of inequality, we formalize an overlapping generations model, with children and parents, in which agents are heterogeneous in terms of human capital. In accordance with the results emphasized by the literature discussed earlier, we take into account the effect of air pollution on children’s health, the possibility for parents to invest in health care to lower this adverse effect, and the role of children’s health in the acquisition of human capital.

Through a theoretical analysis and a numerical illustration, we find that the economy may exhibit several long-term behaviors according to the pollution intensity of production and the initial level of disparities between agents. When they are both sufficiently low, the economy

converges toward a long-term state without inequality. However, if production is highly polluting, inequality will always persist across time - whatever the initial level of inequality - and the economy may even be caught in an inequality trap with steadily rising disparities. The underlying mechanism is the following. Parents choose the level of expenditure aiming at reducing the health effects of pollution. Their human capital being heterogeneous, so are their financial abilities and their investments, which entails an heterogeneous vulnerability to pollution among children. Pollution affects more children from poorer households such that they will be less able to accumulate human capital. Thus, the gap among households increases at each generation due to pollution. Note that we obtain this result despite the fact that we consider no differences in terms of abilities and diminishing marginal returns of human capital accumulation that usually ensure the absence of inequality in the long run. Here, such equality is always found without pollution. But with pollution, depending on its level, the detrimental effect of pollution on children's health may dominate and hence prevent human capital convergence in the long run.

We then explore if specific public interventions focusing on this mechanism are effective to tackle these human capital inequalities. First, pollution being the source of increasing divergence between agents, we examine the consequences of an environmental policy that consists in public maintenance financed by a tax on polluting production. Then, we also study the effects of a combination of an environmental and a health policy, through private health subsidy financed by a production tax. We obtain that an environmental policy is a good option to reduce the inequality issue in the economy but only when pollution intensity and the initial level of disparities among agents are not too high. Otherwise, it is not sufficient and may even reinforce inequality due to the negative income effect of the tax. In this case, we reveal that adding a health policy to the policy package could be an interesting solution. Typically, when health expenditure is sufficiently determining for health with respect to pollution, such a policy mix can prevent the economy to exhibit rising inequalities for a larger set of emission intensity.

This paper contributes to the growing literature on the economic consequences of the health effects of pollution. By focusing on productivity as Aloi and Tournemaine (2011) or van Ewijk and van Wijnbergen (1995), on life expectancy as Pautrel (2009), Mariani et al. (2010) or Raffin and Seegmuller (2014), or on children's learning abilities as Raffin (2012), previous con-

tributions show the importance of considering these effects when examining the macroeconomic consequences and the policy implications of pollution. Among the main results, they identify the risk for the economy to be stuck at a long-term equilibrium with poor health and environment and the major role that an environmental policy may play in avoiding this risk and fostering economic development. Here, we want to go one step further by studying the unequal distribution of the detrimental health effect of environmental degradation. While inequality has received increasing public and academic attention recently, very few studies have examined such an issue for now. Notable exceptions are Aloi and Tournemaine (2013) and Constant (2019). On the one hand, Aloi and Tournemaine (2013) take into account the effect of pollution on human capital accumulation. Assuming that unskilled individuals have a lower ability to learn and are more exposed to pollution, they find that a tighter environmental policy always reduces income inequality. Here, we differ from this paper by assuming no difference in the innate skills of agents nor in their exposure to pollution. Rather, we consider the possibility for parental expenditure on health, capturing all costly activities enabling to reduce the vulnerability of children to pollution (i.e. health care and avoidance behaviors, including larger housing cost to live in a cleaner area for example). In this way, we represent endogenous disparities that depends on the level of pollution and on households' choices. It enables us to represent a more complete set of long-term behaviors of the economy and draw further conclusions as regards the efficiency of an environmental policy. On the other hand, Constant (2019) considers how an endogenous longevity of parents depending on human capital and pollution contributes to the transmission of human capital inequality by modifying the ability and the willingness of parents to invest in education. We differ from this work by studying another kind of health effect that is children's morbidity and by considering health expenditure and health policy. Thus, we highlight other mechanisms and provide a more complete analysis of the policies that can be implemented.²

The rest of this paper is organized as follows. The model is presented in Section 2. Equilibria and dynamics of the economy are examined theoretically and illustrated numerically in Section

²Another exception - but farther given the issue studied - is Schaefer (2020) who consider the effect of pollution on child mortality. He finds that it implies a decrease in the investment of parents in the quality of their children and an increase in their quantity, hence hampering economic development. Note that we focus on children's morbidity rather than their mortality because we are studying developed countries and 99% of under-five deaths occurs in developing countries (UNICEF, 2015).

3. Section 4 is devoted to the policy implications and Section 5 concludes.

2 The model

We consider an overlapping generations economy, with discrete time indexed by $t = 0, 1, 2, \dots, +\infty$. Households live for two periods - childhood and adulthood. At each date t , a new generation of N agents is born. We assume no population growth so that the number of births (N) is normalized to unity. Individuals are indexed by $i = u, s$, corresponding to two groups: unskilled (u) and skilled (s), of size ξ and $1 - \xi$, respectively. The two groups of agents differ in terms of human capital, which is relatively low for unskilled individuals and relatively high for skilled individuals. More precisely, agents born in t differ only in the level of human capital of their parents ($h_{t-1}^u < h_{t-1}^s$).

2.1 Consumer's behavior

An individual of type i born in $t - 1$ cares about her/his consumption levels when an adult c_t , about the future human capital of her/his child h_{t+1} through paternalistic altruism and about the environmental health of her/his child Θ_t . In this paper, we consider separately human capital and health in order to focus on the interactions between health and the accumulation of knowledge of individuals. The preferences of this representative agent are represented by the following utility function:

$$U(c_t^i, h_{t+1}^i, \Theta_t^i) = \ln c_t^i + \gamma \ln h_{t+1}^i + \lambda \ln \Theta_t^i, \quad (1)$$

with γ and $\lambda > 0$.

During childhood, agents devote all of their time to the acquisition of human capital. After reaching adulthood, they are endowed with h_t^i units of human capital, which they use for labor force participation, remunerated at wage w_t per unit of human capital, and they allocate their income between consumption c_t^i , the education of their children e_t^i and expenditure to improve their children's health s_t^i .

Consequently, the budget constraint for an adult of type i born in $t - 1$ is

$$c_t^i + e_t^i + s_t^i = w_t h_t^i. \quad (2)$$

Focusing on the effect of pollution on children's health, the index Θ^i represents the environmental health of a child born in t of type i and depends on pollution P_t and parent's health expenditure s_t^i aiming at reducing the exposition and vulnerability of the child to pollution. Pollution does not act as a pure externality on human capital productivity. Parental behavior can compensate for the adverse effect of pollution on child health. We can appreciate s_t^i as costly parental adaptive or avoidance behaviors allowing to reduce the negative impact of pollution on child development. It can be an investment during childhood under the form of health care expenditures, healthy consumption, or investment to live in a cleaner area (pollution-driven residential sorting).

$$\Theta_t^i = \frac{\bar{\theta} s_t^i - \underline{\theta} P_t}{1 + \eta P_t}, \quad (3)$$

with $\bar{\theta}$, $\underline{\theta}$ and η , three parameters > 0 .

Knowledge accumulation is determined by formal education e_t^i , by the parent's level of human capital h_t^i - representing the transmission of cognitive and social knowledge within the family -, and by the average human capital \bar{h}_t - which represents the quality of the educational system. Moreover, as detailed previously, a consistent body of the empirical literature has demonstrated that health and pollution have key consequences on the abilities of children to learn. Therefore, we assume that the level of human capital of a child born in t h_{t+1}^i depends also on the environmental health of this child Θ_t^i . Note that even if this latter effect is crucial, we assume that parents are not aware of the consequences of their expenditure in terms of children' health on the future human capital of their children to be more realistic.³

$$h_{t+1}^i = \epsilon (\Theta_t^i)^\alpha (e_t^i)^\beta (h_t^i)^\mu (\bar{h}_t)^\delta, \quad (4)$$

³This assumption does not change our results. The opposite assumption would only modify parameters in some equations (typically, λ would be replaced by $\alpha\gamma$ in the first order conditions), without modifying any of our results presented in the following propositions.

where $\epsilon > 0$ is the efficiency of human capital accumulation. The parameters α , β , μ and δ all > 0 capture the respective weights of health, education, intergenerational transmission of human capital within the family and transmission within the society. To focus on cases in which human capital convergence and human capital divergence are both possible, we formulate the following assumption:⁴

Assumption 1

$$\alpha + \beta + \mu < 1.$$

The consumer program is summarized as follows:

$$\max_{e_t^i, s_t^i} U(c_t^i, h_{t+1}^i, \Theta_t^i) = \ln c_t^i + \gamma \ln h_{t+1}^i + \lambda \ln \Theta_t^i \quad (5)$$

$$s.t. \quad c_t^i + e_t^i + s_t^i = w_t h_t^i$$

$$\Theta_t^i = \frac{\bar{\theta} s_t^i - \underline{\theta} P_t}{1 + \eta P_t}$$

$$h_{t+1}^i = \epsilon (\Theta_t^i)^\alpha (e_t^i)^\beta (h_t^i)^\mu (\bar{h}_t)^\delta.$$

An adult maximizes her utility taking into account her budget constraint (2), and her child's health (3) and human capital (4), so that her optimal microeconomic choices are

$$e_t^i = \frac{\gamma\beta}{\bar{\theta}(1 + \gamma\beta + \lambda)} [\bar{\theta} h_t^i w_t - \underline{\theta} P_t] \quad (6)$$

$$s_t^i = \frac{\lambda \bar{\theta} h_t^i w_t + \underline{\theta}(1 + \gamma\beta) P_t}{\bar{\theta}(1 + \gamma\beta + \lambda)}. \quad (7)$$

A parent's health expenditure for her/his child s_t^i depends positively on the level of human capital of the parent and positively on the level of pollution. Indeed, parents' abilities to make such an expenditure are greater when their human capital and hence their wages are larger, while a larger level of pollution represents a larger threat to their children's health, which provides more incentives to protect their children from pollution. This is consistent with empirical evidence that reveals that the vulnerability of children to pollution, and hence

⁴Note that, if $\alpha + \beta + \mu > 1$, human capital convergence would never be possible as it would imply an increasing return of human capital on h_t^i , i.e. that the lower-skilled agents would never be able to catch-up.

their health, is endogenous and depends in particular on parental socioeconomic status. In this regard, Case et al. (2002) present evidence of a gradient between socioeconomic status and health in childhood for the US, according to which relatively richer households have children in better health. Arguments for children's health being positively related to household income rely on parental avoidance behaviors to limit air pollution exposure and investments in health care (see. Currie et al., 2014), both taken into account here with s_t^i representing all costly activities reducing the susceptibility of children.

In the same manner, education expenditure for children e_t^i depends positively on parent's human capital and negatively on pollution. On the one hand, parents with larger h^i have more means to finance their children education. On the other hand, when pollution is high, parents have more incentives to invest in their children's health than in their education. As parents have limited financial means, they make a trade-off between spending for children's education and spending for children's health. Pollution tends to tip this balance in favor of health expenditure.

2.2 Production

The production of the consumption good is performed by a single representative firm. Output of this good is produced according to a constant returns to scale technology:

$$Y_t = AH_t, \tag{8}$$

where H_t is the aggregate stock of the human capital of workers in period t and $A > 0$ measures a technology parameter.

As the size of each generation is normalized to unity, H_t is equal to the average human capital \bar{h}_t . Thus, production corresponds to

$$Y_t = A(\xi h_t^u + (1 - \xi)h_t^s). \tag{9}$$

The firm chooses inputs by maximizing its profit $Y_t - w_t \bar{h}_t$, such that

$$w_t = A. \tag{10}$$

2.3 Pollution

In this paper, we focus on the world's largest single environmental health risk, i.e. air pollution (see WHO, 2014). It is important to note that the health effect of such pollution is due to its level before absorption, deposition or dispersion in the atmosphere, and that the most significant health threats among air pollutants, that are particulate matter and ground-level ozone, remain only for short periods of time in the atmosphere (from hours to weeks). Accordingly, we formalize pollution as the flow currently emitted in the economy. Pollution is a by-product of aggregate production such that

$$P_t = \nu Y_t = \nu A \bar{h}_t, \quad (11)$$

with $\nu > 0$ representing the emission intensity of production, i.e. the emission rate per unit of output.

3 Equilibrium and dynamics of the economy

At the equilibrium, agent's health spending depends positively on family income and on the average level of human capital in the economy (through its impact on pollution). Education choices being affected negatively by pollution, it decreases with the average human capital.

Using the definition of \bar{h}_t corresponding to $\bar{h}_t = \xi h_t^u + (1 - \xi) h_t^s$, we can express the dynamics of the economy through the dynamics of both unskilled and skilled human capital (h_t^u and h_t^s , respectively).

From (4) and agent's optimal choices (6) and (7), we have

$$h_{t+1}^s = \epsilon C_1 \frac{\left(\bar{\theta} h_t^s - \theta \nu (\xi h_t^u + (1 - \xi) h_t^s)\right)^{\alpha + \beta}}{(1 + \eta \nu A (\xi h_t^u + (1 - \xi) h_t^s))^\alpha} (h_t^s)^\mu (\xi h_t^u + (1 - \xi) h_t^s)^\delta, \quad (12)$$

$$h_{t+1}^u = \epsilon C_1 \frac{\left(\bar{\theta} h_t^u - \theta \nu (\xi h_t^u + (1 - \xi) h_t^s)\right)^{\alpha + \beta}}{(1 + \eta \nu A (\xi h_t^u + (1 - \xi) h_t^s))^\alpha} (h_t^u)^\mu (\xi h_t^u + (1 - \xi) h_t^s)^\delta, \quad (13)$$

with

$$C_1 \equiv \frac{\lambda^\alpha}{(1 + \gamma \beta + \lambda)^{\alpha + \beta}} \left(\frac{\gamma \beta}{\theta}\right)^\beta A^{\alpha + \beta}.$$

In order to measure inequality, we define the relative human capital of unskilled individuals with respect to skilled individuals in period t as $x_t \equiv h_t^u/h_t^s$. Initial condition on human capital stocks ($h_0^u < h_0^s$) leads us to focus only on x_t lower than or equal to one, i.e. $\in (0, 1]$. If $x = 1$, there is no inequality among agents. And the lower is x , the wider are disparities.

Using (12), (13) and the definition of x_t , we finally obtain the dynamic equation characterizing equilibrium paths:

Definition 1 *Given the initial condition $x_0 = h_0^u/h_0^s < 1$, the intertemporal equilibrium is the sequence $x_t \in [0, 1]$ which satisfies, at each t , $x_{t+1} = f(x_t)$, with*

$$\begin{cases} f(x_t) = (x_t)^\mu \left[\frac{\bar{\theta}x_t - \underline{\theta}\nu(\xi x_t + (1-\xi))}{\bar{\theta} - \underline{\theta}\nu(\xi x_t + (1-\xi))} \right]^{\alpha+\beta} & \text{for } x_t > \underline{x} \\ f(x_t) = 0 & \text{for } x_t \leq \underline{x}, \end{cases} \quad (14)$$

with $\underline{x} \equiv \frac{\underline{\theta}\nu(1-\xi)}{\bar{\theta} - \underline{\theta}\nu\xi}$.

Using (12) and (13) with the definition of x_t , we can see that the human capital of skilled agents is always positive, as x_t is lower than one. However, for unskilled households, children human capital is positive only if x_t is higher than \underline{x} (given in Definition 1). Otherwise, both the education spending of unskilled households and their children's health are equal to zero, and so does their children human capital, their incomes... It means that this part of the population collapses. Note that to avoid that the economy is always collapsing, \underline{x} need to be lower than 1, which implies that $\bar{\theta} > \underline{\theta}\nu$. In other words, the pollution intensity has to be not too large and health expenditure has to be sufficiently efficient relatively to the health effect of pollution, in order to ensure that unskilled agents may survive.

3.1 Long-term states with and without inequality

From Definition 1, we explore the properties of the dynamic equation $f(x_t)$ and deduce the existence of steady state(s) x corresponding to the solutions of the equation $x = f(x)$. Such a steady state corresponds to a long-term equilibrium in which both skilled and unskilled human capital are stationary. This long-term state is characterized as a state with inequality if $x \neq 1$ and as a state without inequality if $x = 1$, meaning that $h^u = h^s$.

Proposition 1 *Under Assumption 1, there always exists a steady state without inequality $x = 1 \equiv x^E$. According to a critical threshold $\hat{\nu} = \frac{\bar{\theta}(1-\alpha-\beta-\mu)}{\underline{\theta}(1-\mu)}$, we have that*

- *When $\nu < \hat{\nu}$, the steady state without inequality x^E is locally stable and there also exists at least one steady state with inequality $\underline{x} < x < 1$, with the lowest one being unstable.*
- *When $\nu > \hat{\nu}$, the steady state without inequality x^E is locally unstable and there may also exist none or several steady states with inequality $\underline{x} < x < 1$. In the case in which there are several steady states with inequality, the lowest one is unstable while the highest one is stable.*

Proof. See Appendix 6.1 ■

Proposition 1 shows that the economy may converge to a long-term state with or without inequality and illustrates the roles of the emission intensity of production (ν), the human capital accumulation weights (α , β and μ) and the environmental health parameters ($\bar{\theta}$ and $\underline{\theta}$) in achieving one situation or the other.

The role of all these parameters on the long-term behavior of the economy is due to the fact that our model captures different channels through which human capital inequality may widen over time. First, we consider the usual divergent forces in human capital accumulation represented in the literature (see, e.g., Tamura, 1991, Glomm and Ravikumar, 1992 or de la Croix and Doepke, 2003), i.e. forces that perpetuate inequality among agents across generations: the intergenerational transmission of human capital within each family and the parental investment in education. Indeed, skilled parents have more human capital to bequeath to their children (for example, it would be easier for them to help their children with their homework, or to provide informations about graduate schools) and they have also a larger income (as the total wage depends on the level of human capital), enabling them to invest more in education than lower-skilled agents.

Second, we represent an additional divergent force in human capital accumulation that is the environmental health of children.⁵ More precisely, the mechanism occurring in our paper

⁵See Aloi and Tournemaine (2013) for another study taking into account the negative effect of pollution on human capital accumulation. Note that we depart from this work by considering the possibility for parental expenditures on health and by assuming no difference in the innate skills of children nor in their exposure to pollution. We do that in order to adopt a more general framework and to represent an endogenous heterogeneity in terms of vulnerability that is found even for similar exposure.

can be described as follows. When pollution increases (P_t), children's health deteriorates (Θ_t^i). It implies that parents need to spend more money to reduce the exposition and vulnerability of their children (s_t^i) and thus, that they are less able to fund the education of their children. Moreover, a lower health decreases the efficiency of human capital accumulation and hence the return on the education investment. Therefore, pollution entails that parents are less able and less willing to invest in education. Finally, as unskilled agents have a lower level of human capital than skilled agents, they have also a lower total wage. Thus, through both channels, they are more affected by this mechanism than skilled agents. This is the case while we do not assume any difference in exposure to pollution between the two kinds of agents.

This new channel is not only adding a divergent force. It is particularly important because it evolves endogenously with pollution. While the literature on human capital inequality usually find that if the sum of all the divergent forces in the human capital accumulation is lower than one, the convergence of human capital among agents in the long run is ensured, this is not the case in this paper. The weight of these divergent forces (the usual and the new one) are very important for the dynamics of inequality, but such a convergence is not guaranteed even if $\alpha + \beta + \mu < 1$ (i.e. Assumption 1).⁶ This is due to the effect of pollution on the endogenous environmental health of children. Without pollution ($\nu = 0$), the growth of individual human capital along the transitional path would always be larger for unskilled agents under Assumption 1, meaning that the gap between agents would be shrinking and there would always be a human capital convergence among agents.⁷ However, with pollution ($\nu > 0$), the growth of individual human capital along the transitional path may be larger or lower for unskilled agents, thus the economy may achieve a long-term state with or without inequality. Therefore pollution, through its negative effect on the health of children, is essential in our model for explaining the dynamics of the economy.

In order to provide details about the different possible cases, we sum up the economic implications of Proposition 1 in the following corollary:

Corollary 1 *Under Assumption 1*

⁶We focus on the case in which this condition is satisfied because, on the opposite, if $\alpha + \beta + \mu$ were > 1 , it would be impossible to achieve a long-term equilibrium without inequality, and inequality would worsen in all scenarios.

⁷In this case, there is only one steady state, which is without inequality and stable.

- When $\nu < \hat{\nu}$, we observe that
 - if the initial relative human capital of unskilled agents x_0 is sufficiently high, the economy converges toward a long-term state without inequality.
 - if the initial relative human capital of unskilled agents x_0 is too low, the economy will exhibit persistent inequality in the long run. More precisely, the economy
 - * converges toward a steady state with persistent but constant inequality ($x < 1$)
 - * or is caught in an inequality trap with steadily rising disparities (i.e. moving asymptotically toward a situation in which inequality is maximum $x = 0$).
- When $\nu > \hat{\nu}$, initial inequality always persists across time. The economy
 - converges toward a steady state with inequality (relative human capital lower than one)
 - or is stuck in the inequality trap ($x = 0$).

If the pollution intensity and initial inequality are sufficiently low ($\nu < \hat{\nu}$ and x_0 sufficiently high), the economy may converge to a long-term equilibrium without inequality. But in all the other cases, the economy will converge to a long-term state with persistent or increasing inequality.

The emission intensity of production ν favors the transmission of inequality across generations and hence makes the “unequal scenario” more likely. More precisely, with a pollution-intensive production technology (ν too high), the long-term state with equality x^E is unstable, meaning that unskilled agents cannot converge to the same level of human capital as skilled agents. The explanation is twofold. Pollution affects negatively the efficiency of human capital accumulation due to its negative health effect on the children. And pollution prevents parents to invest sufficiently in education as they need to invest in health expenditure to limit the negative effect of pollution. The fact that there is initial inequality makes these effects larger for unskilled households than skilled households. And when the emission intensity of production is high, poor households will never be able to narrow existing disparities - even for very low initial inequality.⁸ Therefore, the economy will converge to a long-term state with inequality.

⁸Under $\nu > \hat{\nu}$, achieving a long-term state without inequality would only be possible for an economy without initial inequality.

In the best scenario, these disparities are persistent but constant in the long run. In the worse scenario, these disparities are steadily increasing over time (inequality trap). In this inequality trap, the living conditions of the poor agents are constantly deteriorating. The economy is moving asymptotically toward the lower bound of the trap in which the unskilled agents would collapse ($x = 0$), meaning that their level of human capital tends to zero, as do their income, their ability to consume *etc.*⁹

With a low-pollution production technology (ν sufficiently low), the long-term state without inequality is stable, meaning that reaching an equal long-term state is possible. Typically, if initial disparities among agents are sufficiently low in this case, the divergent effect of pollution on human capital accumulation is low enough such that the growth of individual human capital of unskilled agents is larger than the one of skilled agents. Therefore, the gap between them reduces at each generation and there is a convergence in the levels of human capital among the population. However, if initial disparities are too large, the negative effects of pollution on human capital is much larger for unskilled households than for skilled households and the gap between them cannot reduce over time. Thus, the economy will exhibit persistent or increasing disparities.

The second item in Proposition 1 emphasizes that there may exist a critical scenario in which the economy can never escape an inequality trap in which the living conditions of poor agents constantly decline (i.e. a unique steady state that is without inequality but unstable). Further investigations allow to identify a specific condition such that this situation occurs.

Proposition 2 *Under Assumption 1 and the following sufficient condition:*

$$(1 - \mu)\bar{\theta}(\bar{\theta} - \underline{\theta}\nu) < (1 - \xi)\xi(\alpha + \beta)(\underline{\theta}\nu)^2, \quad (15)$$

there is a unique steady state, which is without inequality $x = 1$ and unstable. Therefore, the economy is caught in the inequality trap for all initial conditions.

Proof. See Appendix 6.2 ■

Condition (15) holds when ν and/or $\underline{\theta}$ are high enough or when $\bar{\theta}$ is sufficiently low. This implies that health input and education spendings are low because of high pollution damages.

⁹Note that, the economy is moving asymptotically toward this extreme state but cannot be at it, as it would not be bearable.

In this case, the level of unskilled parental investment is never sufficient to observe a more equal distribution of income in the society: human capital of unskilled offspring is too low to catch up the initial unequal distribution of human capital and the situation of unskilled always deteriorates across time relatively to those of skilled agents.

When condition (15) does not hold, there may exist multiple steady states with several configurations, and typically x^E may be stable or not. In the following proposition, we focus on the latter case, meaning that the economy will converge toward a state in which inequalities remain stable or in which they increase over time, depending on its initial conditions.

Proposition 3 *Under Assumption 1 and the following condition*

$$\frac{1 - \mu}{\alpha + \beta} > 1 + \frac{\ln [1 - \mu - (1 - \alpha - \beta - \mu)(1 - 0.5\xi)] - \ln [1 - \mu - 2(1 - \alpha - \beta - \mu)(1 - 0.5\xi)]}{\ln 2}, \quad (16)$$

there exists a critical threshold $\tilde{\nu} > \hat{\nu}$ such that when $\nu \leq \tilde{\nu}$, we have $f(1/2) \geq 1/2$. As a result, for $\hat{\nu} < \nu < \tilde{\nu}$, x^E is unstable and there are also multiple steady states with inequality, among which the highest one is stable.

Proof. See Appendix 6.3 ■

This proposition identifies sufficient conditions to have a long-term state with inequality ($0 < x < 1$) that is stable. It depends on the parameters $\mu + \beta + \alpha$, ξ and ν . In other words, the divergent forces in human capital formation ($\mu + \beta + \alpha$) should be high such that there is inequality in the long run. The share of unskilled agents in the population (ξ) should also be sufficiently high so that the aggregate pollution and its negative health effects are not too detrimental, hence enabling to avoid widening inequalities. And, in the same way, the emission intensity of production (ν) should not be too low, so that inequality exists in the long run, and not too high to limit the size of the pollution effect on health that increases disparities.

3.2 Numerical illustration

In this section, we analyze numerically the model to illustrate the different possible cases emphasized in Propositions 1 to 3. In this way, we provide further insights into the long-term behavior of the economy.

3.2.1 Calibration

For that, we need to assign values to some parameters of the model. We choose values so that the model fits empirical observations for developed countries. They are summarized in Table 1.

Table 1: Description of the model parameters.

Parameter	Description	Calibrated value
α	Weight of environmental health in human capital accumulation	0.2
β	Weight of education in human capital accumulation	0.4
μ	Weight of intergenerational transmission in human capital accumulation	0.3
$\bar{\theta}$	Weight of health expenditure in environmental health	0.6
$\underline{\theta}$	Weight of pollution in environmental health	0.4
ν	Emission rate of production	$[0, 1.5]$
γ	Preference for children's human capital	0.35
λ	Preference for children's environmental health	0.35
ξ	Share of unskilled individuals in each cohort	0.5

In the literature, the return to schooling in developed countries is estimated to be between 8 and 16% (see Ashenfelter and Krueger, 1994, Psacharopoulos, 1994 or Krueger and Lindahl, 2001). These figures only include an opportunity cost in terms of forgone earnings but not education expenditure. Following de la Croix and Doepke (2003), we assume that an additional year of schooling increases such expenditure by 20%. The resulting elasticity of education ranges from 0.4 to 0.8. Thus, we set the sum of the weights concerning education in human capital accumulation - education spendings and environmental health defining learning abilities - to be 0.6. More precisely, we choose α equal to 0.2 and β equal to 0.4. Consequently, the weight of intergenerational transmission of human capital μ should satisfy our assumption that human capital convergence is not impossible, i.e., $\mu \in [0, 1 - \alpha - \beta]$. Thus, we consider $\mu = 0.3$, which matches the values identified in the empirical literature, i.e. between 0.2 and 0.45 (see, e.g., Dearden et al., 1997 or Black et al., 2005).

Concerning children's health parameters, we need to choose values for $\bar{\theta}$ and $\underline{\theta}$. There is no estimation for these parameters in the literature but we need to ensure that $\underline{x} < 1$ so that the economy is not always collapsing. It implies that $\bar{\theta} > \nu \underline{\theta}$. Thus, we assume that $\bar{\theta}$ and $\underline{\theta}$ are equal to 0.6 and 0.4, respectively. And we consider all possible values of ν , which is the key parameter of our model, i.e. $\nu \in [0, 1.5]$.

We assign values to the preference for children's human capital γ and the preference for

children’s environmental health λ to fit the share of education expenditure in GDP at equilibrium in developed countries (i.e., between 4 and 9%)¹⁰. In the model, this share depends on ν , which is let free in the model. We find that the range of possible values for this share of education expenditure in GDP corresponds to the real values for γ and λ equal to 0.35.¹¹

Finally, for simplicity and neutrality purposes, we assume that the two types of agents have equivalent sizes (i.e. $\xi = 0.5$).

3.2.2 Illustration of the long-term behaviors of the economy

The study of the existence and dynamics of the steady states in the calibrated economy gives the following result and is represented in Figures 1 to 3 for different values of $\nu \in [0, 1.5)$.

Numerical result 1 *(i) When $0 < \nu < \hat{\nu}$, there are two steady states: a steady state without inequality that is stable and a steady state with inequality that is unstable. Thus, the economy is stuck in the inequality trap for high initial disparities (x_0 lower than the steady state with inequality), but can converge to a long-term equilibrium without inequality otherwise.*

(ii) When $\nu > \hat{\nu}$, there is a unique steady state, which is without inequality and unstable. Therefore, the economy is stuck in the inequality trap for all levels of initial disparities $x_0 < 1$.

As explained previously, without pollution ($\nu = 0$), the economy would always converge to a long-term equilibrium without inequality, as it is stable and the unique positive equilibrium (left panel of Figure 1). However, in the presence of pollution ($\nu > 0$), inequality may persist in the long run. More precisely, we find that there always exists an inequality trap in which disparities among households persistently grow. In this calibrated economy, the threshold value $\hat{\nu}$ corresponds to 0.2143. As illustrated on the right panel of Figure 1, when $0 < \nu < 0.2143$, the economy is stuck in the trap for high initial disparities but can converge to the long-term state without inequality otherwise. However, when $\nu > 0.2143$, the long-term equilibrium without inequality is the only one left and becomes unstable, meaning that the economy is stuck in an inequality trap whatever its initial condition $x_0 \in (0, 1)$. As represented in Figures 2 and 3, if there is inequality initially - even very few -, the economy will exhibit inequality that

¹⁰See OECD (2020), Education spending (indicator). doi: 10.1787/ca274bac-en (Accessed on 14 May 2020).

¹¹We do not need to calibrate these preferences to represent the long-term behaviors of the economy, but it will be useful to examine the effect of the policy in Section 4.

widens at each generation until the lower-skilled households collapse. Moreover, as ν increases, $f(x_t)$ goes to the right. Thus, the larger the pollution intensity of production ν , the faster the economy collapses ($f(x_t)$ is equal to 0 for a larger set of x).

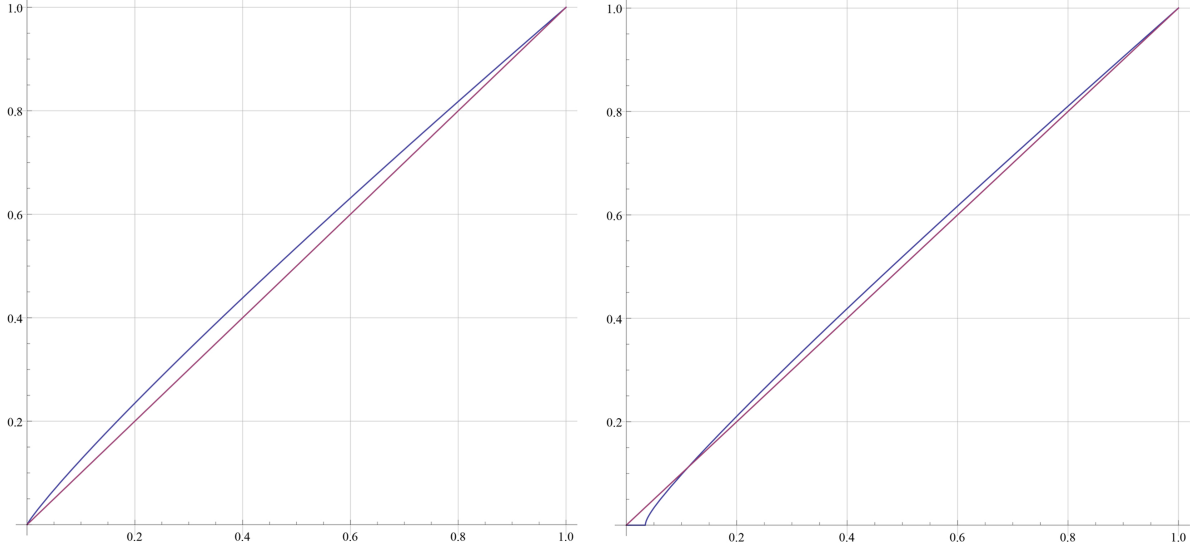


Figure 1: Dynamics for $\nu = 0$ (left panel) and $\nu = 0.1$ (right panel), with x_t on the X-axis and x_{t+1} on the Y-axis. The blue curve is the dynamic equation characterizing equilibrium paths $x_{t+1} = f(x_t)$, while the pink curve is the first bisector $x_{t+1} = x_t$.

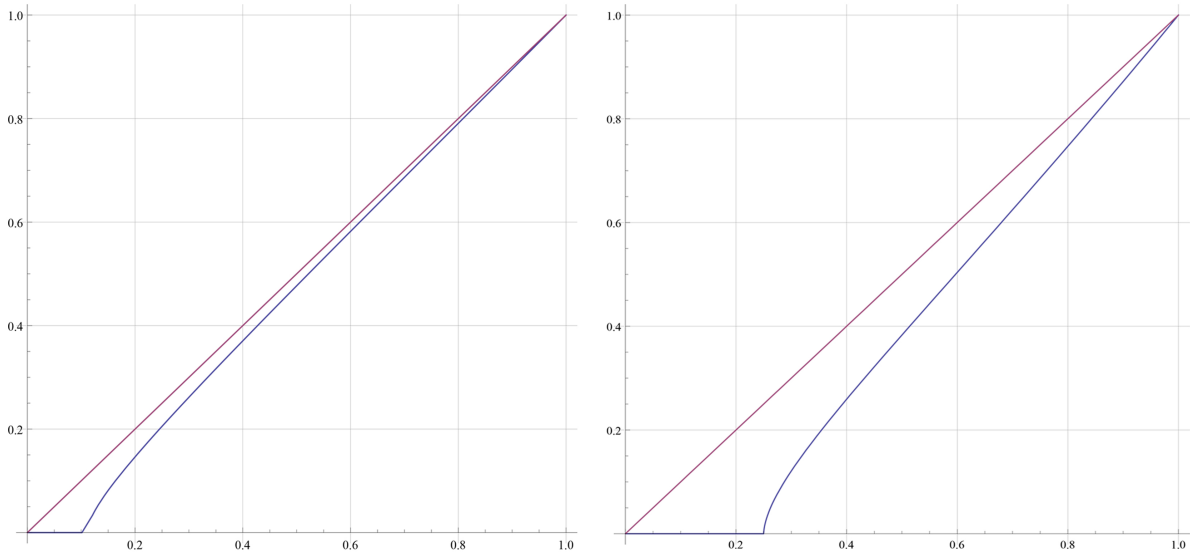


Figure 2: Dynamics for $\nu = 0.3$ (left panel) and $\nu = 0.6$ (right panel), with x_t on the X-axis and x_{t+1} on the Y-axis. The blue curve is the dynamic equation characterizing equilibrium paths $x_{t+1} = f(x_t)$, while the pink curve is the first bisector $x_{t+1} = x_t$.

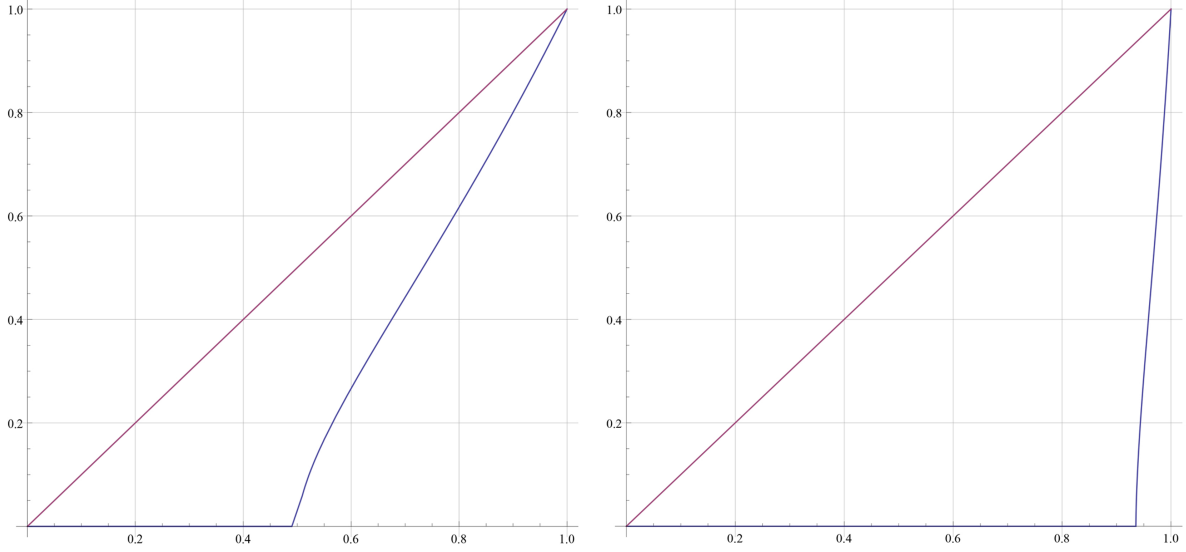


Figure 3: Dynamics for $\nu = 1$ (left panel) and $\nu = 1.45$ (right panel), with x_t on the X-axis and x_{t+1} on the Y-axis. The blue curve is the dynamic equation characterizing equilibrium paths $x_{t+1} = f(x_t)$, while the pink curve is the first bisector $x_{t+1} = x_t$.

For robustness, we test the model for a large set of parameters. In most cases, we find that the economy is either stuck in the inequality trap or converges to the long-term equilibrium without inequality for low value of pollution intensity, as summarized in the Numerical result 1. However, the value of $\hat{\nu}$ varies according to the weights of the divergent forces in human capital accumulation (α , β and μ). The larger they are, the larger the size of the inequality trap, as explained after Proposition 1. This key threshold also depends on the parameters in children's environmental health. The larger the difference between the efficiency of health expenditure $\bar{\theta}$ and the weight of pollution in environmental health $\underline{\theta}$, the lower is the size of the inequality trap. This is because inequality is due to the health effects of pollution and when $\bar{\theta}$ is relatively high with respect to $\underline{\theta}$, it implies that individuals are relatively less vulnerable to pollution and health expenditure enables relatively easily to overcome the negative effect of pollution. For example, for $\bar{\theta} = 0.7$ and $\underline{\theta} = 0.2$ and all other parameters being equal, the threshold $\hat{\nu}$ - above which the long-term equilibrium without inequality is unstable - becomes 0.5. As in the previous calibration, there are two long-term equilibria for small emission intensity, i.e. $\nu \in (0, 0.5)$: the one without inequality is stable, while the equilibrium with inequality is unstable and represents the upper bound of the inequality trap. In this parameter range, the

larger the emission rate, the larger the size of the inequality trap. The economy exhibits the same long-term behaviors as previously but the trap is smaller for a given value of ν . And for $\nu > \hat{\nu}$ (here 0.5), there is only one equilibrium: the long-term state without inequality but it is unstable, meaning that the economy is stuck in the inequality trap for all levels of initial disparities. Thus, the result is the same as previously, but with these parameters, the economy collapses more slowly for a given ν .

For some specific parameters values, we can also find a stable long-term state with inequality, as emphasized in Proposition 3 and illustrated in Figure 4. This equilibrium implies that the economy may achieve a state with persistent but constant inequality in the long run. However, we need to note that when we observe such a steady-state, there are also an equilibrium without inequality that is unstable and a lower unstable equilibrium with inequality. Thus, the inequality trap still exists. The economy may converge to a state with constant inequality in the long run, but only for sufficiently low initial disparities. As commented after Proposition 3, this third long-term state is found under some specific conditions implying that disparities exist in the long run but are not widening across time. In other words, pollution and its health effects need to be intermediary (neither too high nor too small) so that the economy may exhibit constant disparities in the long run when the initial level of inequality is low.

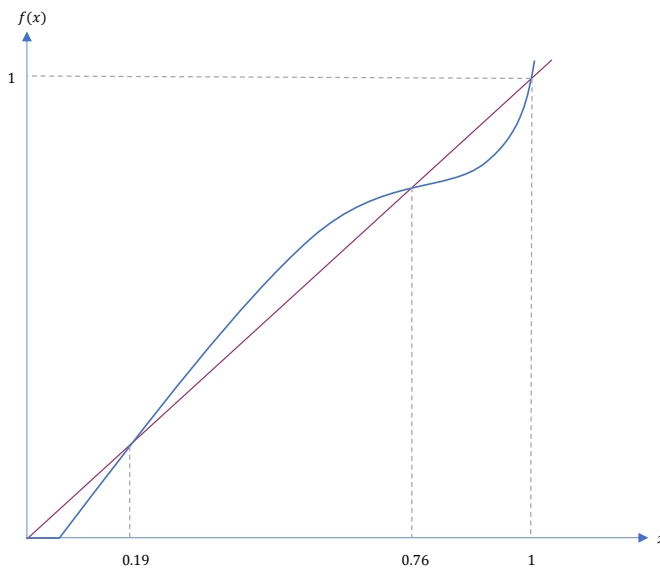


Figure 4: Dynamics for the calibrated values summarized in Table 1 and $\xi = 0.8$ and $\nu = 0.23$.

4 Policy implications

Given that pollution is responsible for the persistence of inequality through its effect on the health of children, we want to examine how policies focusing on environmental health may address this issue. This is the purpose of this section. After providing details about the policy tools considered, we examine the role of an environmental policy in achieving a long-term state without inequality. We also study the effect of a policy mix with an environmental policy - aiming at reducing the source of inequality - and a health policy - to lessen the mechanism through which pollution generates inequality. For each policy, we provide analytical results that we illustrate numerically to provide a more comprehensive overview of the different scenarios.

4.1 The government

We assume that the government implements two taxes on production (which is the source of pollution). An environmental tax τ_p is implemented to finance pollution abatement M_t , while a health tax τ_s is used to provide a subsidy on health spending σ_t .¹² These taxes satisfy that τ_p, τ_s and $\tau_p + \tau_s$ are all $\in [0, 1)$.

The policy is summarized by two instruments, τ_p and τ_s while the amount of maintenance M_t and the rate of subsidy σ_t are endogenously determined to satisfy the public budgets. The government balances its budget such that

$$\begin{cases} M_t = \tau_p Y_t, \\ \sigma_t((1 - \xi)s_t^s + \xi s_t^u) = \tau_s Y_t. \end{cases} \quad (17)$$

With these policies, pollution becomes

$$P_t = \nu Y_t - M_t = (\nu - \tau_p) Y_t = (\nu - \tau_p) A \bar{h}_t, \quad (18)$$

Note that, based on the new law of motion of pollution (18), we assume the following condition to be true in order to ensure that the aggregate economic activity is associated with a net pollution flow that is always positive:¹³

¹²Note that we consider two taxes, as Raffin and Seegmuller (2014), in order to treat the two types of policies independently and to show easily the effect of each instrument.

¹³Otherwise, when $\nu < 1$, there would exist a policy $\tau_p = \nu$ allowing to remove all pollution in the economy

Assumption 2 $\tau_p < \max(\nu, 1)$.

With policy, the budget constraint for an adult of type i is now given by

$$c_t^i + e_t^i + s_t^i(1 - \sigma_t) = w_t h_t^i, \quad (19)$$

and taxes reduce the equilibrium wage w_t which is equal to $A(1 - \tau_p - \tau_s)$. The details of all the new equations for optimal choices and dynamics, integrating the policy tools, are reported in Appendix 6.4.

In presence of policy intervention, Definition 1 that summarizes the dynamic equation characterizing equilibrium paths becomes

Definition 2 *Given the initial condition $x_0 = h_0^u/h_0^s < 1$, the intertemporal equilibrium is the sequence $x_t \in [0, 1]$ which satisfies, at each t , $x_{t+1} = f(x_t)$, with*

$$\begin{cases} f(x_t) = (x_t)^\mu \left[\frac{(1-\tau_p-\tau_s)\bar{\theta}x_t - (1-\sigma(\tau_p, \tau_s))\underline{\theta}(\nu-\tau_p)(\xi x_t + (1-\xi))}{\bar{\theta}(1-\tau_p-\tau_s) - (1-\sigma(\tau_p, \tau_s))\underline{\theta}(\nu-\tau_p)(\xi x_t + (1-\xi))} \right]^{\alpha+\beta} & \text{for } x_t > \underline{x} \\ f(x_t) = 0 & \text{for } x_t \leq \underline{x}, \end{cases} \quad (20)$$

$$\text{with } \underline{x} \equiv \frac{(1-\sigma(\tau_p, \tau_s))\underline{\theta}(\nu-\tau_p)(1-\xi)}{\bar{\theta}(1-\tau_p-\tau_s) - \underline{\theta}(1-\sigma(\tau_p, \tau_s))(\nu-\tau_p)\xi}.$$

Note that the term $\sigma(\tau_p, \tau_s)$ in Definition 2 is the subsidy that balances the public budget at the equilibrium (17). It depends positively on the two taxes because of several effects. First, because both taxes reduce agents' available income and hence their health expenditure. As the government subsidizes a share of this spending, the cost goes down when spending is lower, so the subsidy that balances the budget is higher. In addition, τ_s directly contributes to the budget allocated to health expenditure. These two channels explain the positive effect of τ_s . The tax τ_p has also a second positive effect on σ because it reduces total pollution and hence improves health. Agents decrease their contribution to health input when pollution is lower, which reduces the overall cost of the health policy and allows the government to fix a higher subsidy rate.

To provide some intuitions about the effects of the policy instruments on the dynamics of the economy and on the evolution of inequalities, we present in the following lemma the effect

and a policy $\tau_p > \nu$ such that the net flow of pollution would be negative. Both cases are highly unrealistic, that is why we do not consider them.

of τ_p and τ_s on the dynamic equation $f(x)$.

Lemma 1 *Effect of τ_p and τ_s on the dynamic equation $f(x)$ when $x_t > \underline{x}$:*

$$\text{Sign } df(x_t) = (1 - x_t) \left[\text{Sign} \frac{\partial f(x_t)}{\partial \tau_p} d\tau_p + (\nu - \tau_p) \text{Sign} \frac{\partial f(x_t)}{\partial \tau_s} d\tau_s \right], \quad (21)$$

with

$$\text{Sign} \frac{\partial f(x_t)}{\partial \tau_p} = -\text{Sign} \frac{\partial \underline{x}}{\partial \tau_p} = \text{Sign} \left[(1 - \tau_s - \nu)(1 - \sigma) + \frac{\partial \sigma}{\partial \tau_p} (\nu - \tau_p)(1 - \tau_p - \tau_s) \right]$$

and

$$\text{Sign} \frac{\partial f(x_t)}{\partial \tau_s} = -\text{Sign} \frac{\partial \underline{x}}{\partial \tau_s} = \text{Sign} \left[(1 - \tau_s - \tau_p) \frac{\partial \sigma}{\partial \tau_s} - (1 - \sigma) \right].$$

Proof. Directly obtained by differentiating $f(x)$, given in (20), with respect to τ_p and τ_s . ■

4.2 Environmental policy

As disparities are widening because of pollution in our model, we examine first if the environmental policy alone is sufficient to remove inequalities. Thus, the policy we consider in this subsection only consists in providing public environmental maintenance by taxing production (there is no health policy $\tau_s = \sigma = 0$).

We start by examining how the environmental policy affects the dynamic equation characterizing the long-term state(s) of the economy $f(x)$. As we can see in Lemma 1 when $\tau_s = \sigma = 0$, the environmental tax τ_p has a positive (resp. negative) effect on $f(x_t)$ and a negative (resp. positive) effect on the threshold \underline{x} when $\nu < 1$ (resp. $\nu > 1$).

To have a more accurate view about the effects of the environmental policy on the economy, we pay a particular attention to how the critical threshold in terms of emission intensity $\hat{\nu}$ evolves with the policy. Indeed, examining the position of ν relative to this threshold provides important information about the long-term behaviors of the economy and their properties in terms of inequality. Using details provided in Appendix 6.3, we find that it turns into

$$\hat{\nu}_p \equiv \frac{\bar{\theta}(1 - \tau_p)(1 - \alpha - \beta - \mu)}{\underline{\theta}(1 - \mu)} + \tau_p. \quad (22)$$

Therefore, the long-term state without inequality $x = 1$ is stable (resp. unstable) when $\nu < \hat{\nu}_p$ (resp. $\nu > \hat{\nu}_p$). To show how the policy affects this threshold, we analyze the differential of $\hat{\nu}_p$ with respect to policy instruments τ_p :

$$d\hat{\nu}_p = \underbrace{d\tau_p}_{\text{Environmental effect}} - \underbrace{\frac{\bar{\theta}(1 - \alpha - \beta - \mu)}{\underline{\theta}(1 - \mu)}}_{\text{Income effect}} d\tau_p.$$

The environmental policy can make the stability condition of the equal long-term state more or less restrictive. The tax τ_p generates two competing effects on the critical threshold $\hat{\nu}_p$: a positive environmental effect and a negative income effect. On the one hand, the revenue from the environmental tax is recycled in an investment in environmental maintenance, which reduces pollution. Thus, the tax enables to lower the source of the widening of inequalities, and hence to favor the human capital convergence among households. On the other hand, the tax has a negative effect on agents' available income, which means that parents are less able to invest in their children's health and in their education. As the income of low-skilled agents is lower than the one of high-skilled, the former are even more affected. Thus, the tax also fosters disparities among households through this effect.

A necessary condition to have an environmental policy that favors the stability of the equal steady state is that the threshold $\hat{\nu}_p$ increases with τ_p :

$$\frac{d\hat{\nu}_p}{d\tau_p} > 0 \Leftrightarrow \underline{\theta}(1 - \mu) > \bar{\theta}(1 - \alpha - \beta - \mu) \Leftrightarrow \hat{\nu} < 1 \quad (23)$$

Therefore, the effect of the environmental policy on the stability properties of the equal long-term state x^E depends on the weights of the divergence forces in human capital accumulation (α , β and μ), on the efficiency of health expenditure $\bar{\theta}$ and on the weight of pollution in health $\underline{\theta}$. As we can see, this is directly linked to the threshold in terms of the emission intensity of production without policy $\hat{\nu}$ (see, Proposition 1). The condition (23) implies that $\hat{\nu}$ is lower than 1, which makes long-term inequality true for a large set of emission intensity. In other words, the impact of pollution on disparities should be sufficiently large (relatively high divergent forces and health effect of pollution and relatively low efficiency of health expenditure) so that the environmental effect of the tax is higher than its negative income effect. In this case, the

tax on pollution increases the threshold and makes the convergence toward the equilibrium without inequality more likely. On the contrary, when (23) does not hold, $\hat{\nu}$ is larger than 1, meaning that the set of emission intensity such that the long-term state without inequality is stable is large. In this case, the effect of pollution on disparities is relatively low. Therefore, the positive environmental effect of the environmental policy is relatively low with respect to the negative income effect, and the ability of the policy to decrease inequality is poor.

The effects of the environmental policy on the dynamics of the economy largely depends on the characteristics of the economy before the implementation of the public instruments. In particular, it is important to distinguish between two cases: the case in which, without public intervention, the economy is characterized by a stable long-term state without inequality x^E and the reverse case in which this state is unstable. Based on the elements previously presented, we highlight all the possible scenarios in the following proposition.

Proposition 4 *Under Assumptions 1 and 2,*

1. *If $\nu < \hat{\nu} < 1$, the state x^E is stable without policy and stays stable for all $\tau_p \in (0, \nu)$. Moreover, an increase in τ_p makes $f(x)$ shift upward and lowers \underline{x} . It decreases the value of the lowest state with inequality that is unstable, i.e. reduces the size of the inequality trap.*
2. *If $\hat{\nu} < \nu < 1$, the state x^E is initially unstable and the policy can change this situation. An increase in τ_p makes $f(x)$ move upward and lowers \underline{x} . Thus, if there are multiple steady states, a higher τ_p reduces the size of the inequality trap. Moreover, there exists a threshold $\hat{\tau}_p < \nu$ such that when*

$$\tau_p > \frac{\nu \underline{\theta}(1 - \mu) - \bar{\theta}(1 - \alpha - \beta - \mu)}{\underline{\theta}(1 - \mu) - \bar{\theta}(1 - \alpha - \beta - \mu)} \equiv \hat{\tau}_p,$$

the environmental policy makes x^E stable (i.e., $\hat{\nu}_p > \nu$).

3. *If $\nu < 1 < \hat{\nu}$, the state x^E is initially stable and stays stable for all $\tau_p \in (0, \nu)$ ($\nu < \hat{\nu}_p$). Indeed, $\frac{d\hat{\nu}_p}{d\tau_p} < 0$ but $\hat{\nu}_p > 1 \forall \tau_p \in (0, \nu)$. Moreover, an increase in τ_p makes $f(x)$ shift upward and lowers \underline{x} . It decreases the value of the lowest state with inequality that is unstable, i.e. reduces the size of the inequality trap.*

4. If $1 < \nu < \hat{\nu}$, the state x^E is initially stable. An increase in τ_p makes $f(x)$ move downward and increases \underline{x} . Thus, it increases the value of the lowest state with inequality that is unstable, i.e. increases the size of the inequality trap. Moreover, there exists a threshold $\hat{\tau}_{p2} < 1$ such that when

$$\tau_p > \frac{\bar{\theta}(1 - \alpha - \beta - \mu) - \nu\underline{\theta}(1 - \mu)}{\underline{\theta}(1 - \alpha - \beta - \mu) - \underline{\theta}(1 - \mu)} \equiv \hat{\tau}_{p2},$$

the environmental policy makes x^E unstable (i.e., $\hat{\nu}_p < \nu$).

5. If $\hat{\nu} < 1 < \nu$, the state x^E is initially unstable and stays unstable for all $\tau_p \in (0, 1)$, because even if $\hat{\nu} < 1$, $\hat{\tau}_p$ is always higher than 100%. Moreover, an increase in τ_p moves $f(x)$ downward and increases \underline{x} . Thus, if there are multiple steady states, a higher τ_p increases the size of the inequality trap.

6. If $1 < \hat{\nu} < \nu$, the state x^E is initially unstable and stays unstable for all $\tau_p \in (0, 1)$. Moreover, an increase in τ_p makes $f(x)$ shift downward and increases \underline{x} . Thus, if there are multiple steady states, a higher τ_p increases the size of the inequality trap.

Proposition 4 identifies different situations in which the environmental policy may reduce or increase inequalities. In order to provide further insights on these results, we provide a numerical illustration of them. For that, we use the values reported in Table 1 for the parameters μ , β , α , λ , γ and ξ . Concerning the emission rate of production (ν) and the weights of health expenditure and pollution in the child environmental health ($\bar{\theta}$ and $\underline{\theta}$ respectively), we have seen that they are determining for the policy effects and that there is no empirical counterpart for these parameters. Therefore, we set their values in order to illustrate all the different possible configurations given in Proposition 4. We summarize the cases 1 to 6 of Proposition 4 in the following four scenarios.

Numerical result 2 (a) If $\nu < \min(\hat{\nu}, 1)$, an increase in the environmental tax reduces the size of the inequality trap, while the long-term state without inequality remains stable.

Thus, the environmental policy can enable an economy to escape from the inequality trap.

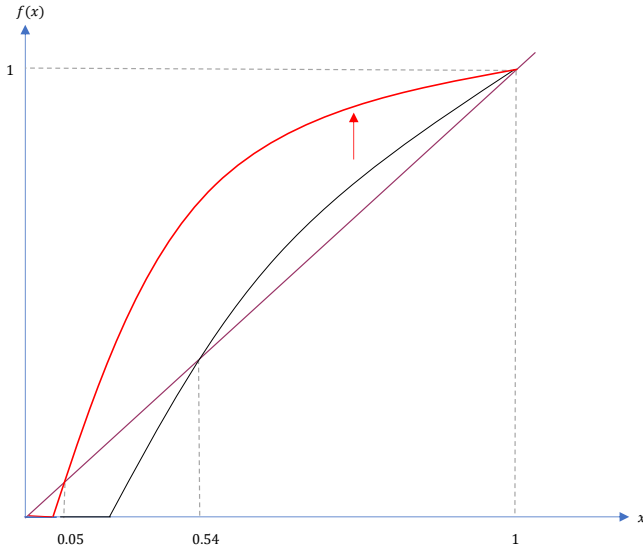
(b) If $\hat{\nu} < \nu < 1$, an increase in the environmental tax is always able to make the long-term

state without inequality stable when $\tau_p > \hat{\tau}_p$. Thus, the policy can allow the economy to converge to this state as long as its initial disparities are sufficiently low.

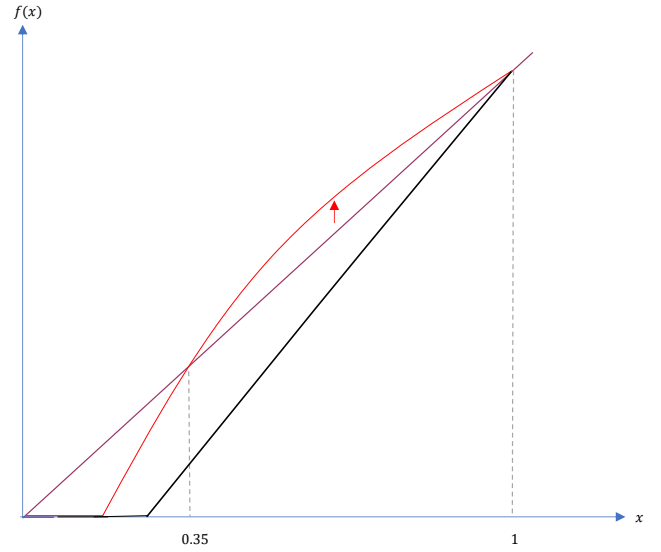
(c) If $1 < \nu < \hat{\nu}$, an increase in the environmental tax increases the size of the inequality trap and when $\tau_p > \hat{\tau}_{p2}$ it even makes the long-term state without inequality unstable. Thus, the policy favors inequality and can make the human capital convergence among agents impossible.

(d) If $\max(\hat{\nu}, 1) < \nu$, the environmental policy is not able to make the long-term state without inequality stable. Moreover, an increase in the environmental tax increases \underline{x} , so that the economy collapses more quickly.

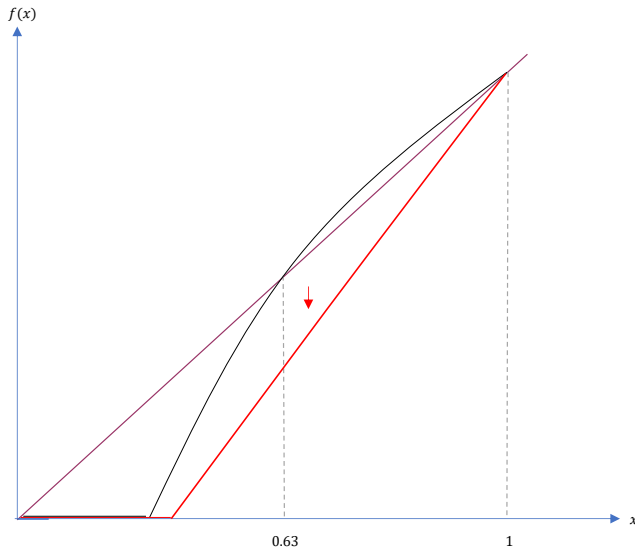
We illustrate the four scenarios of the Numerical result 2 in Figure 5. In (a), while the economy is stuck in the inequality trap for $x_0 \in (0, 0.54]$ without policy (black curve), an environmental policy $\tau_p = 0.15$ (red curve) significantly reduces the size of the trap to $x_0 \in (0, 0.05]$. Thus, the economy stays caught in the inequality trap despite the policy only if initial disparities are very wide. In (b), while there is always inequality in the long run without policy (black curve), an environmental tax $\tau_p = 0.15 > \hat{\tau}_p$ enables the economy to escape from the trap and to converge to a long-term equilibrium without inequality for all $x_0 > 0.35$ (red curve). To illustrate the case (c), we have to consider a spread sufficiently high between $\bar{\theta}$ and $\underline{\theta}$ such that, despite a high value for pollution intensity ($\nu > 1$), the economy is still characterized by a stable long-term state x^E without policy. In this case, the economy converges to a long-term state without inequality for $x_0 > 0.63$ without policy (black curve), but is stuck for all $x_0 < 1$ with an environmental policy $\tau_p = 0.3$ (red curve). Finally, for the last case, the policy does not change the number nor the configuration of the long-term equilibria, but implies that the economy collapses faster.



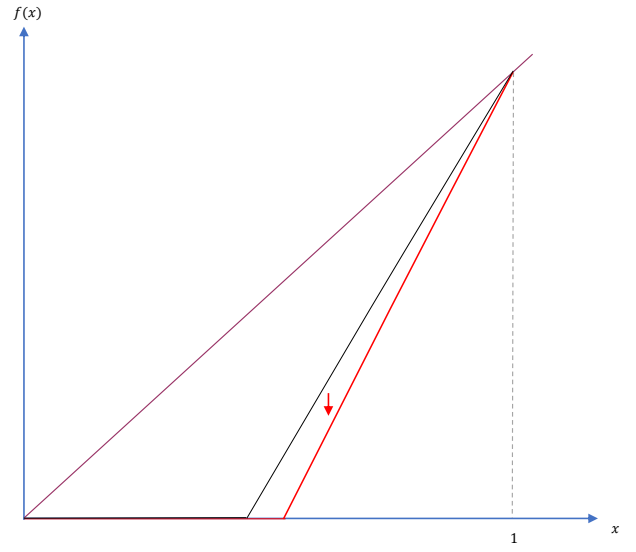
(a) $\nu = 0.2, \bar{\theta} = 0.6, \underline{\theta} = 0.4, \tau_p = 0.15$



(b) $\nu = 0.3, \bar{\theta} = 0.6, \underline{\theta} = 0.4, \tau_p = 0.15$



(c) $\nu = 1.1, \bar{\theta} = 0.8, \underline{\theta} = 0.1, \tau_p = 0.3$



(d) $\nu = 1.2, \bar{\theta} = 0.6, \underline{\theta} = 0.4, \tau_p = 0.15$

Figure 5: Effect of the environmental tax. Black curve: economy without policy, Red curve: economy with policy.

Therefore, the environmental policy can be a useful tool to address the inequality issue (cases (a) and (b)). The policy reduces the size of the inequality trap and can even remove the troublesome situation in which the economy cannot converge to the long-term state without inequality whatever its initial disparities (case (b)). Thus, the environmental policy enables an economy to escape from the trap and to converge to a long-term equilibrium without inequality, but only if the emission intensity of production is sufficiently low ($\nu < 1$) and if the level of inequality is not too high.

If disparities are too wide before the policy is implemented, the environmental policy is not sufficient to compensate the health gap and hence the education gap, meaning that inequalities continue to grow over time despite the policy (cases of an economy below the upper bound of the trap in (a) and (b)).

If the emission intensity of production is too high ($\nu > 1$), the policy is not sufficient and may even make the inequality issue worse (cases (c) and (d)). Indeed, the policy can be really damaging by increasing the size of the inequality trap, accelerating the collapse or making the long-term state without inequality unstable. This is due to the fact that the policy generates two competing effects: the positive effect through the improvement of the environment and hence of health and the negative effect on households' income. If the emission intensity is too high, the gap between emissions and maintenance is huge, meaning that the required improvement in the environment and hence the required tax to compensate are very high. However, the efficiency of the environmental policy is limited ($\tau_p < 1$) and the larger is the tax, the larger is the negative income effect. Therefore, when the emission intensity is too high, the negative income effect dominates the positive environmental effect. Poor households benefit from the improvement in their children's health, but are even more affected by the negative effect of the tax on their income limiting their ability to invest in education and health. Thus, implementing or tightening the environmental policy reinforces inequality in this case.

Given the limits of the environmental policy, we wonder if a policy mix with a preventive action through the reduction of pollution and a curative action through a subsidy to health expenditure could be more efficient. Note that we combine these two instruments rather than studying only the latter because pollution is the source of the problem, but this effect goes

through children's health and hence could be alleviated through health spendings.¹⁴

4.3 Policy Mix

In this section, we consider both instruments. To identify the channels through which the policy mix can affect inequalities, we first examine the critical threshold of ν over which the equal state x^E is unstable. With both instruments, it satisfies the following equality:

$$\nu = \frac{\bar{\theta}(1 - \tau_p - \tau_s)(1 - \alpha - \beta - \mu)}{\underline{\theta}(1 - \sigma(\tau_p, \tau_s, \nu))(1 - \mu)} + \tau_p, \quad (24)$$

with $\sigma(\tau_p, \tau_s, \nu)$, the solution of the following equation:

$$\sigma \lambda \bar{\theta}(1 - \tau_p - \tau_s) = (1 - \sigma) \left[\tau_s \bar{\theta}(1 + \gamma \beta + \lambda) - \sigma \underline{\theta}(1 + \gamma \beta)(\nu - \tau_p) \right]. \quad (25)$$

The term on the right hand side is decreasing in ν meaning that the equilibrium value for σ decreases with ν as well. As $\partial \sigma(\tau_p, \tau_s, \nu) / \partial \nu < 0$, it is clear that there exists a unique value $\hat{\nu}_m$ that is solution of (24). The steady state x^E is stable if we have

$$\nu < \hat{\nu}_m \quad \text{with} \quad \hat{\nu}_m \equiv Sol \left\{ \frac{\bar{\theta}(1 - \tau_p - \tau_s)(1 - \alpha - \beta - \mu)}{\underline{\theta}(1 - \sigma(\tau_p, \tau_s, \nu))(1 - \mu)} + \tau_p - \nu = 0 \right\}. \quad (26)$$

Therefore, a policy mix (τ_p, τ_s) is able to make the long-term state without inequality stable if it satisfies $\nu < \hat{\nu}_m$. In order to analyze more precisely the effects of such a policy, we examine the differential of $\hat{\nu}_m$ with respect to the policy instruments τ_p and τ_s :

$$\begin{aligned} d\hat{\nu}_m \left(1 - (1 - \tau_p - \tau_s) \frac{\partial \sigma}{\partial \nu} \right) = & \underbrace{d\tau_p}_{\text{Environmental effect}} \\ & + \frac{\bar{\theta}(1 - \alpha - \beta - \mu)}{\underline{\theta}(1 - \mu)(1 - \sigma(\tau_p, \tau_s))^2} \left[(1 - \tau_p - \tau_s) \underbrace{\left(\frac{\partial \sigma}{\partial \tau_p} d\tau_p + \frac{\partial \sigma}{\partial \tau_s} d\tau_s \right)}_{\text{Health effect}} - (1 - \sigma(\tau_p, \tau_s)) \underbrace{(d\tau_p + d\tau_s)}_{\text{Income effect}} \right]. \end{aligned} \quad (27)$$

¹⁴We leave aside the cases in which tax on pollution is not used because we consider that preventive action consisting in fighting against the source of inequality issues as the most meaningful. Moreover, omitting this policy instrument seems unreasonable given the numerous other negative effects of pollution on welfare (not all considered here).

As the environmental policy, the health policy can make the stability condition more or less restrictive. Indeed, τ_s generates competing effects on the critical threshold $\hat{\nu}_m$. The tax τ_s leads to a fall in agents' available income that affects even more poor individuals, while the subsidy on health expenditure favors the human capital convergence among households. Moreover, there are interactions between health and environmental instruments. The effects of each policy depend on the other policy. The introduction of a health policy amplifies the negative income effect of the environmental policy. A same variation in the environmental tax $d\tau_p$ generates a larger negative income effect when health is subsidized ($d\tau_p/(1 - \sigma)$). Similarly, the health policy generates a direct positive effect through subsidy, whose magnitude depends on the environmental policy. Indeed, the environmental tax directly contributes to the public budget, which allows to increase the amount of subsidy on health ($\frac{\partial \sigma}{\partial \tau_p} d\tau_p > 0$). In addition, as the environmental tax improves health input (Θ) by reducing the stock of pollution, it leads to a decrease in the amount of private spending allocated to environmental health (s). Public spending for health being a share of the private spending, it relaxes the public budget constraint and allows to increase the rate of subsidy on health spending. When private health spending is subsidized by public authorities, the convergence of agents' human capital in the long run is favored. As a result, the higher the pollution tax, the higher the subsidy and hence the more likely the economy achieves a long-term state without inequality.

The channels through which the policy tools affect inequalities are thus multiple and interact between each others. This is also assessed through equation (21) in Lemma 1, as we see that this policy mix leads to several competing effects on the dynamical equation $f(x_t)$. How does the subsidy respond to tax variations is determining (i.e., the values of $\partial \sigma / \partial \tau_p$ and $\partial \sigma / \partial \tau_s$). Moreover, the equilibrium subsidy being dependent on all the parameters of the model (see (25)), it makes difficult to identify the net consequences of such combination of instruments. Nonetheless, we identify sufficient conditions such that the policy mix can be an efficient option to address the inequality issue. This is the purpose of the following proposition.

Proposition 5 *Under Assumptions 1 and 2 and when $\bar{\theta}/\underline{\theta}$ is sufficiently high, there exists a combination of health and environmental policies (τ_p, τ_s) moving $f(x)$ up and \underline{x} down, hence decreasing the size of the inequality trap in the presence of multiple steady states,*

- *when $\nu < 1$, as the environmental policy does.*

- when $1 < \nu < 1 + \frac{1+\gamma\beta}{\lambda}$, while the environmental policy does not, if with $\tau_s \geq \frac{\lambda(\nu-1)}{1+\gamma\beta}$.

Proof. See Appendix 6.5 ■

The conditions presented in Proposition 5 ($\bar{\theta}/\underline{\theta}$ high enough and $\nu < 1 + \frac{1+\gamma\beta}{\lambda}$) indicate that health expenditure is sufficiently efficient with respect to the detrimental effect of pollution and that the education and health of children are sufficiently valued by parents. They imply that both policy instruments, when implemented together, can have a positive effect on the dynamical equation $f(x_t)$, so that they may reduce inequality. The first condition ($\bar{\theta}/\underline{\theta}$ high enough) ensures that the health policy has a net positive effect on dynamical equation $f(x_t)$ for all possible levels of policy instruments. Combining this condition with the second one ($\nu < 1 + \frac{1+\gamma\beta}{\lambda}$) ensures that the environmental tax can also have a net positive effect on dynamical equation $f(x_t)$. This property is observed for all possible levels of policy instruments when $\nu < 1$, while when $\nu > 1$ this is guaranteed when τ_s is high enough ($\tau_s \geq \frac{\lambda(\nu-1)}{1+\gamma\beta} > 0$) because the environmental policy alone is not efficient (as emphasized in the previous subsection).

Thereafter, we illustrate numerically the potential effects of the policy mix. As previously, we calibrate the parameters μ , β , α , λ , γ and ξ using Table 1 and ν , $\bar{\theta}$ and $\underline{\theta}$ to represent the different relevant scenarios. However, we focus on cases in which the economy cannot achieve a long-term state without inequality (i.e. x^E is unstable) if public authority does not intervene.

In Figure 6, we fix $\nu = 1.2$ and thus illustrate a case in which an environmental tax alone is not sufficient to improve the situation. As pollution intensity is high, a policy mix can be efficient if the health effect of pollution is not too important while the effect of health expenditure is. We thus consider $\bar{\theta} = 0.8$ and $\underline{\theta} = 0.1$. For policy instruments, we fix $\tau_p = 0.1$ and $\tau_s = 0.05$ but the illustration holds for a large set of combinations.^{15,16} As illustrated, the policy mix can prevent the economy to be stuck in a trap in which inequalities constantly widen across generations. While the trap was inevitable without policy and with an environmental policy alone, a policy mix with the same tax burden (15%) enables the economy to converge toward the state without inequality x^E if its initial relative human capital is sufficiently high ($x_0 \in (0.64, 1)$). Thus, the policy mix can be an efficient tool to address the inequality issue as long as initial disparities are not too wide.

¹⁵Given these calibrations, the equilibrium subsidy is equal to $\sigma = 0.15$.

¹⁶Note that to have a policy mix that reduces inequality, this is not necessary to have $\tau_s \geq \frac{\lambda(\nu-1)}{1+\gamma\beta}$ as this is only a sufficient condition.

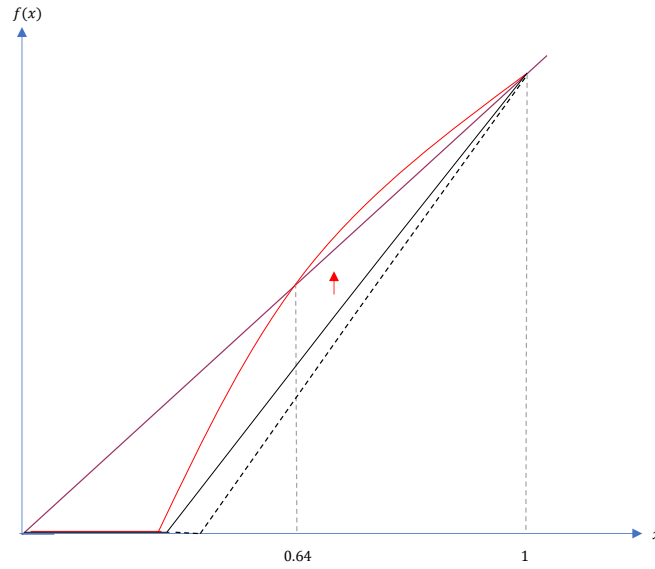


Figure 6: Effect of the policy mix with $\nu = 1.2$, $\bar{\theta} = 0.8$, $\underline{\theta} = 0.1$. Black solid curve: economy without policy, black dotted curve: economy with pollution tax $\tau_p = 0.15$, red curve: economy with policy mix $\tau_p = 0.1$ and $\tau_s = 0.05$.

However, even if a policy mix can be efficient for a larger set of emission intensity than an environmental policy alone, it is not necessarily a better option to address inequality. This is represented in Figure 7. Considering the case illustrated in Figure 2 (with $\nu = 0.3$, $\bar{\theta} = 0.6$, $\underline{\theta} = 0.4$), both policies can be used to reduce disparities among households. Nevertheless, for a given tax burden, long-term inequalities can be avoided when the policy consists only in reducing pollution ($\tau_p = 0.15$ and $\tau_s = 0$), while it is not the case when it is divided between health and environmental policies ($\tau_p = 0.1$ and $\tau_s = 0.05$ for example).

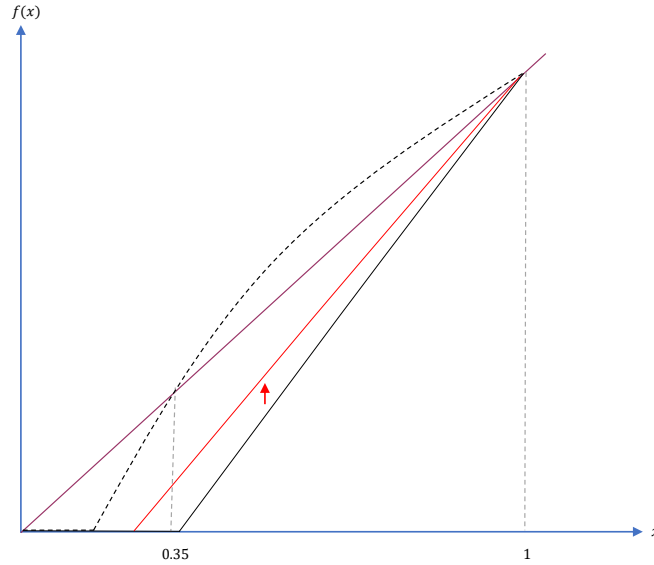


Figure 7: Effect of the policy mix with $\nu = 0.3$, $\bar{\theta} = 0.6$, $\underline{\theta} = 0.4$. Black solid curve: economy without policy, black dotted curve: economy with pollution tax $\tau_p = 0.15$, red curve: economy with policy mix $\tau_p = 0.1$ and $\tau_s = 0.05$.

Therefore, these two policies focusing on the source of the widening of inequalities, i.e. the health effects of pollution on children, represent interesting tools to address the inequality issue. Both an environmental policy alone and a policy mix composed of environmental and health instruments can reduce inequalities in the economy and enable an economy to escape from the inequality trap in which disparities are persistently widening. However, the efficiency of these policies is also limited, especially by the negative income effect of taxes that dominates when the emission intensity is too high and/or if disparities are too wide. Choosing one or the other depends on the sensitivity of children's health with respect to pollution and parental investment in health, on which further data are needed. When none of these policies are sufficient, a redistributive policy needs to be set as a complementary tool to reduce disparities among households.

5 Concluding remarks

Evidence shows that children are highly vulnerable to the health effects of pollution, that these effects cause both short- and long-term damaging economic consequences and that they

are unequally distributed in the population according to the socioeconomic status of parents. However, most prior works abstract from these features to examine the dynamics of inequalities. This paper contributes to the theoretical literature on social mobility and inequality by showing the role of the health effects of pollution during childhood in the intergenerational transmission of inequality and the role that environmental and health policies can play to address this issue.

Considering how children's health may be affected by pollution and by parental expenditure to reduce this detrimental effect, we represent the heterogeneous vulnerability of agents to pollution and how it evolves endogenously across time. In reality, a lot of factors participate in the widening of disparities across time. However, we emphasize that pollution is a key factor explaining human capital divergence among households. Even if we consider a framework in which human capital convergence is usually ensured and if disparities are initially low, we find that the economy always exhibits inequalities in the long run if the emission intensity of production is not small enough. Moreover, through a numerical calibration of the model on developed countries data, we show that in this case the economy is most likely to be stuck in an inequality trap in which disparities are persistently widening across generations. The heterogeneity of agents in terms of human capital entails that parents' abilities to reduce the harmful effects of pollution differ. It follows that children from poor households have more health issues due to pollution and hence more difficulties to acquire human capital than the others, which makes disparities increase across time.

Given these results, we examine if policies focusing on this mechanism can be successfully implemented to reduce the intergenerational transmission of inequality coming from pollution. We show that an environmental policy consisting in taxing the polluting production to fund public abatement can be used to reduce inequality and enable the economy to escape from the trap. Nevertheless, it is not always sufficient and can even be counterproductive, because the negative effect of the improvement in the environment on inequality can be outweighed by the effect of the tax on households' income. This is the case when the emission intensity of production is too high and/or if inequalities are too wide. Finally, we reveal that a policy mix consisting in environmental and health policy tools can be a better solution to reduce inequality, as it is efficient for a larger range of emission intensity if the children's health is sufficiently sensitive to parental health expenditure.

6 Appendix

6.1 Proof of Proposition 1

The function $f(x)$ is increasing, with $f(\underline{x}) = 0$ and $f(1) = 1$, thus $x = 1$ is a steady state. Examining the second derivative of the $f(x)$, we see that the function can be concave or convex, meaning that we can observe one or multiple steady states.

We examine the stability properties of the steady state $x = 1$. As we cannot examine precisely the shape of the function $f(x)$, and hence conduct a global stability analysis, we focus on the local stability. We have

$$f'(x) = x^{\mu-1} \left[\frac{\bar{\theta}x_t - \underline{\theta}\nu(\xi x_t + (1 - \xi))}{\bar{\theta} - \underline{\theta}\nu(\xi x_t + (1 - \xi))} \right]^{\alpha+\beta} \left[\mu + \frac{x(\alpha + \beta)}{\bar{\theta} - \underline{\theta}\nu(\xi x_t + (1 - \xi))} \frac{\bar{\theta}(\bar{\theta} - \underline{\theta}\nu)}{\bar{\theta}x_t - \underline{\theta}\nu(\xi x_t + (1 - \xi))} \right]$$

with

$$f'(1) = \mu + \frac{(\alpha + \beta)\bar{\theta}}{\bar{\theta} - \underline{\theta}\nu}.$$

From this expression, we can define a critical threshold on ν , $\hat{\nu} = \frac{\bar{\theta}(1-\alpha-\beta-\mu)}{\underline{\theta}(1-\mu)}$, to characterize the local stability properties of $x = 1$.

Note that in case of stability, $f(1)$ cuts the bisector by the top. This means that $f(x)$ cuts the bisector at least one time from the bottom between \underline{x} and 1: there is necessarily (at least) another steady state that is with inequality and unstable. Hence, stability condition for $x = 1$ is a sufficient condition to have multiple steady states (as long as $\nu > 0$). \square

6.2 Proof of Proposition 2

Using Definition 1, a steady state satisfies

$$g(x) \equiv x^{1-\mu} \left(\bar{\theta} - \underline{\theta}\nu(\xi x_t + (1 - \xi)) \right)^{\alpha+\beta} = \left(\bar{\theta}x_t - \underline{\theta}\nu(\xi x_t + (1 - \xi)) \right)^{\alpha+\beta} \equiv z(x)$$

with

- $z(x)$ increasing and concave, with $z(\underline{x}) = 0$
- $g(x) > 0$ for all $x \geq \underline{x}$. Moreover, $g(x)$ is increasing and then decreasing, and achieves its

maximal value for $\tilde{x} = \frac{(1-\mu)(\bar{\theta}-\underline{\theta}\nu(1-\xi))}{\underline{\theta}\nu\xi(1-\mu+\alpha+\beta)}$ ($g'(\tilde{x}) = 0$).

- $g(1) = z(1)$.

From these properties, we deduce that if $\underline{x} > \tilde{x}$, $g(x)$ is always decreasing on the interval $x \in [\underline{x}, 1]$, meaning that it crosses $z(x)$ only once, at $x = 1$. Thus, the condition $\underline{x} > \tilde{x}$ is sufficient to have a unique steady state $x = 1$. We have

$$\underline{x} > \tilde{x} \Rightarrow (1 - \mu)\bar{\theta}(\bar{\theta} - \underline{\theta}\nu) < (1 - \xi)\xi(\alpha + \beta)(\underline{\theta}\nu)^2.$$

Note that several steady state are required to have $x = 1$ stable. A necessary (but not sufficient) condition to converge toward a long-term state without inequality is thus to have $\underline{x} < \tilde{x}$. Otherwise, there is only one steady state which is necessarily unstable in our model: the economy cannot converge to a situation in which $x = 1$ and there are inequalities. □

6.3 Proof of Proposition 3

We examine in this Appendix sufficient conditions to have multiple steady states and $x = 1$ unstable.

This requires to have $\nu > \hat{\nu}$ ($x = 1$ unstable) and that there exists at least a $x \in (\underline{x}, 1)$ satisfying $f(x) > x$. This last condition implies

$$x^{\frac{1-\mu}{\alpha+\beta}} < \frac{\bar{\theta}x - \underline{\theta}\nu(\xi x + (1 - \xi))}{\bar{\theta} - \underline{\theta}\nu(\xi x + (1 - \xi))}$$

and can be written as

$$\frac{1 - \mu}{\alpha + \beta} > \frac{\ln [\bar{\theta}x - \underline{\theta}\nu(\xi x + (1 - \xi))] - \ln [\bar{\theta} - \underline{\theta}\nu(\xi x + (1 - \xi))]}{\ln x}.$$

For $x = 1/2$, this inequality (i.e $f(1/2) > 1/2$) becomes

$$\frac{1 - \mu}{\alpha + \beta} > \frac{\ln [\bar{\theta}/2 - \underline{\theta}\nu(1 - 0.5\xi)] - \ln [\bar{\theta} - \underline{\theta}\nu(1 - 0.5\xi)]}{\ln(1/2)}$$

$$\begin{aligned} \frac{1-\mu}{\alpha+\beta} &> \frac{\ln [\bar{\theta} - 2\underline{\theta}\nu(1-0.5\xi)] - \ln [\bar{\theta} - \underline{\theta}\nu(1-0.5\xi)] - \ln 2}{-\ln 2} \\ \frac{1-\mu}{\alpha+\beta} &> 1 + \frac{\ln [\bar{\theta} - \underline{\theta}\nu(1-0.5\xi)] - \ln [\bar{\theta} - 2\underline{\theta}\nu(1-0.5\xi)]}{\ln 2}. \end{aligned} \quad (28)$$

The term on the right hand side is increasing in ν and equal to one for $\nu = 0$. Under Assumption 1, this means that there exists a $\tilde{\nu}$ such that this inequality holds for $\nu < \tilde{\nu}$ and does not hold for $\nu > \tilde{\nu}$.

As we focus on the cases in which $\nu > \hat{\nu}$, the previous inequality can be observed in our context only if $\hat{\nu} < \tilde{\nu}$. Thus, inequality (28) has to be possible when $\nu = \hat{\nu}$. This implies

$$\frac{1-\mu}{\alpha+\beta} > 1 + \frac{\ln [1-\mu - (1-\alpha-\beta-\mu)(1-0.5\xi)] - \ln [1-\mu - 2(1-\alpha-\beta-\mu)(1-0.5\xi)]}{\ln 2}.$$

There exists a set of parameters satisfying this inequality, *i.e* when ξ , and/or $\mu + \beta + \alpha$ sufficiently close to one. In that case, there exists a $\tilde{\nu}$ such that when $\hat{\nu} < \nu < \tilde{\nu}$, the economy is characterized by x^E unstable and at least another steady state $x < 1/2$ which is stable. \square

6.4 The model with the policy instruments

With the policy instruments introduced in Section 4, pollution and the budget constraint of an adult become (18) and (19). Therefore, the following first order conditions of an adult become

$$e_t^i = \frac{\gamma\beta}{\bar{\theta}(1+\gamma\beta+\lambda)} \left[\bar{\theta}h_t^i w_t - (1-\sigma_t)\underline{\theta}P_t \right] \quad (29)$$

and

$$s_t^i = \frac{\lambda\bar{\theta}h_t^i w_t + (1-\sigma_t)\underline{\theta}(1+\gamma\beta)P_t}{(1-\sigma_t)\bar{\theta}(1+\gamma\beta+\lambda)}. \quad (30)$$

The firm maximizes its profit $(1-\tau_p-\tau_s)Y_t - w_t\bar{h}_t$, such that

$$w_t = A(1-\tau_p-\tau_s). \quad (31)$$

The human capital accumulation of the two types of agents can be rewritten as

$$h_{t+1}^s = \epsilon C_1 \frac{\left((1 - \tau_p - \tau_s) \bar{\theta} h_t^s - \underline{\theta} (1 - \sigma_t) (\nu - \tau_p) (\xi h_t^u + (1 - \xi) h_t^s) \right)^{\alpha + \beta}}{(1 + \eta(\nu - \tau_p) A (\xi h_t^u + (1 - \xi) h_t^s))^\alpha (1 - \sigma_t)^\alpha} (h_t^s)^\mu (\xi h_t^u + (1 - \xi) h_t^s)^\delta \quad (32)$$

and

$$h_{t+1}^u = \epsilon C_1 \frac{\left((1 - \tau_p - \tau_s) \bar{\theta} h_t^u - \underline{\theta} (1 - \sigma_t) (\nu - \tau_p) (\xi h_t^u + (1 - \xi) h_t^s) \right)^{\alpha + \beta}}{(1 + \eta(\nu - \tau_p) A (\xi h_t^u + (1 - \xi) h_t^s))^\alpha (1 - \sigma_t)^\alpha} (h_t^u)^\mu (\xi h_t^u + (1 - \xi) h_t^s)^\delta \quad (33)$$

with

$$C_1 \equiv \frac{\lambda^\alpha}{(1 + \gamma\beta + \lambda)^{\alpha + \beta}} \left(\frac{\gamma\beta}{\bar{\theta}} \right)^\beta A^{\alpha + \beta}.$$

From the government budget constraint, we have

$$\sigma_t \sum_{i=u,s} \frac{\lambda \bar{\theta} \xi^i h_t^i (1 - \tau_p - \tau_s) + (1 - \sigma_t) \underline{\theta} (1 + \gamma\beta) (\nu - \tau_p) (\xi h_t^u + (1 - \xi) h_t^s)}{(1 - \sigma_t) \bar{\theta} (1 + \gamma\beta + \lambda)} = \tau_s (\xi h_t^u + (1 - \xi) h_t^s)$$

$$\sigma_t \lambda \bar{\theta} (\xi h_t^u + (1 - \xi) h_t^s) (1 - \tau_p - \tau_s) = (1 - \sigma_t) (\xi h_t^u + (1 - \xi) h_t^s) \left[\tau_s \bar{\theta} (1 + \gamma\beta + \lambda) - \sigma_t \underline{\theta} (1 + \gamma\beta) (\nu - \tau_p) \right].$$

After simplifications, we see that the equilibrium value of σ is time independent and satisfies the following equality:

$$\sigma_t \lambda \bar{\theta} (1 - \tau_p - \tau_s) = (1 - \sigma_t) \left[\tau_s \bar{\theta} (1 + \gamma\beta + \lambda) - \sigma_t \underline{\theta} (1 + \gamma\beta) (\nu - \tau_p) \right]. \quad (34)$$

From this equality, we have a unique equilibrium value for σ that depends on both taxes. It increases with both τ_p and τ_s . It is equal to 0 when $\tau_s = 0$ and tends to 1 when τ_s tends to 1 as well. □

6.5 Proof of Proposition 5

The combined effects of both instruments depend on the response of subsidy rate σ to tax variation, i.e. of the values of $\partial\sigma/\partial\tau_p$ and $\partial\sigma/\partial\tau_s$. The equilibrium value for σ satisfies the

equality (34). This equality can be written as

$$\sigma\lambda(1 - \tau_p - \tau_s) = (1 - \sigma) \left[\tau_s(1 + \gamma\beta + \lambda) - \sigma \frac{\theta}{\bar{\theta}}(1 + \gamma\beta)(\nu - \tau_p) \right].$$

The equilibrium value for σ increases with $\frac{\bar{\theta}}{\theta}$. Moreover, we have that $\lim_{\frac{\bar{\theta}}{\theta} \rightarrow 0} \sigma = 0$ and

$$\lim_{\frac{\bar{\theta}}{\theta} \rightarrow \infty} \sigma = \frac{\tau_s(1 + \gamma\beta + \lambda)}{\tau_s(1 + \gamma\beta + \lambda) + \lambda(1 - \tau_p - \tau_s)}.$$

We pay a particular attention to the effect of the policy mix in the extreme case in which $\frac{\bar{\theta}}{\theta}$ is infinitely high. For that case, we have

$$\partial\sigma/\partial\tau_p = \frac{\tau_s(1 + \gamma\beta + \lambda)\lambda}{(\tau_s(1 + \gamma\beta + \lambda) + \lambda(1 - \tau_p - \tau_s))^2}$$

$$\partial\sigma/\partial\tau_s = \frac{(1 - \tau_p)(1 + \gamma\beta + \lambda)\lambda}{(\tau_s(1 + \gamma\beta + \lambda) + \lambda(1 - \tau_p - \tau_s))^2}$$

and, from Lemma 1,

$$\text{Sign } df(x_t) = (1 - x_t) \left[\text{Sign} \frac{\partial f(x_t)}{\partial\tau_p} d\tau_p + (\nu - \tau_p) \text{Sign} \frac{\partial f(x_t)}{\partial\tau_s} d\tau_s \right]$$

with

$$\text{Sign} \frac{\partial f(x_t)}{\partial\tau_s} = \frac{(1 + \gamma\beta)\lambda(1 - \tau_p - \tau_s)^2}{(\tau_s(1 + \gamma\beta + \lambda) + \lambda(1 - \tau_p - \tau_s))^2} > 0$$

and

$$\text{Sign} \frac{\partial f(x_t)}{\partial\tau_p} = \frac{(\tau_s(1 + \gamma\beta) + \lambda(1 - \nu))\lambda(1 - \tau_p - \tau_s)^2}{(\tau_s(1 + \gamma\beta + \lambda) + \lambda(1 - \tau_p - \tau_s))^2}.$$

We have $\frac{\partial f(x_t)}{\partial\tau_p} \geq 0$ (resp. < 0) for $\tau_s \geq \frac{\lambda(\nu-1)}{1+\gamma\beta}$ (resp. $\tau_s < \frac{\lambda(\nu-1)}{1+\gamma\beta}$). This means that when

$$\nu < 1 + \frac{1 + \gamma\beta}{\lambda}$$

there always exists a value for $\tau_s < 1$ ensuring $\frac{\partial f(x_t)}{\partial\tau_p} > 0$. As we have $\frac{\partial f(x_t)}{\partial\tau_s} > 0$ for all possible taxes satisfying Assumption 2 and $\tau_p + \tau_s < 1$, the condition $\nu < 1 + \frac{1 + \gamma\beta}{\lambda}$ associated with $\frac{\bar{\theta}}{\theta}$ high enough is sufficient to have $df(x_t) > 0$ when both instruments are used. □

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