water.surf.transfer-su.hayami

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Name : *Water transfer on surface units using hayami propagation method*
Version : *19.06*
Domain : *hydrology*
Description : *Calculation of discharge routing through the channel network using diffusive wave equation resolved with Hayami method*

Parameters

`maxsteps`	used	Maximum hayami kernel steps	$-$
`meancel`	used	Wave mean celerity on SUs	m/s
`meansigma`	used	Mean diffusivity on SUs	$m2/s$

Attributes

`area`	required	SU	Area of SU	$m2$
`flowdist`	required	SU	Flow distance between the SU and the downstream unit (SU or RS)	m
`nmanning`	required	SU	Manning roughness coefficient of SU	$s/m(-1/3)$
`slope`	required	SU	Mean slope of SU	m/m

Variables

`water.surf.H.runoff`	required	SU	Runoff height on surface of SU	m
`water.sz-surf.H.exfiltration`	used	GU	Water exfiltration height from saturated zone of GU to surface of SU	m
`water.uz-surf.H.exfiltration`	used	GU	Water exfiltration height from unsaturated zone of GU to surface of SU	m
`water.uz-surf.Q.exfiltration`	used	SU	Water exfiltration output volume from the soil reservoir to surface of SU	$m3/s$
`water.surf.Q.downstream-su`	produced	SU	Output volume at the outlet of SU	$m3/s$

Abstract

The simulator "Water transfer on surface units using Hayami propagation method" produces water discharge at the outlet of SU from runoff. Firstly, the simulator calculates the water height to propagate, which is the sum of the water runoff generated by a production function and the exfiltration volume if it exists. Then, the simulator computes the water discharge between connected units using diffusive wave model resolved with the Hayami method. The main parameters of this model are wave celerity and diffusivity.

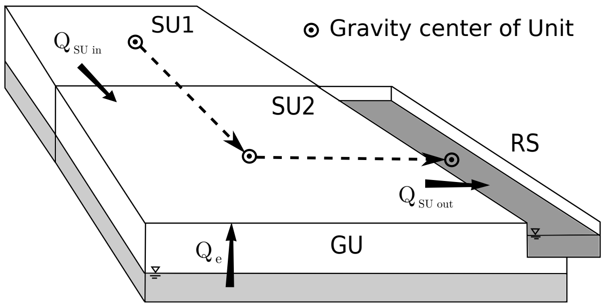
1 Scientific concepts

1.1 Calculation of water height to propagate

The first step consists in computing the water quantity at the top of the SU. This value is calculated using the following equation if the variable exfiltration is previously produced.

$$H = R + \left(\frac{Q_e \times \Delta t}{A} \right) \tag{1}$$

where H is the water height at the soil surface (m), R is the runoff water height (m) generated by the production function, Q_e is the exfiltration discharge (m^3/s), A is the surface of the SU (m^2) and Δt is the simulation time step (s).



The exfiltration discharge Q_e could be produced by the simulator "Interflow".

1.2 Propagation using Hayami kernel

Then, the water height previously calculated is propagated between the gravity center of the main SU and the downstream unit. The downstream surface unit could be an other SU or a reach segment RS according to the topology. Thus, the calculation is done following the Straler order, from upper SU to downstream SU and RS.

The propagation is done using the diffusive wave model :

$$\frac{\delta Q}{\delta t} = -C \times \frac{\delta Q}{\delta x} + D\delta\frac{\delta^2 Q}{\delta x^2} \tag{2}$$

Celerity and diffusivity are considered as time constants to simplify the equation and resolved it using the Hayami analytical method [?]. The model consists to convolute the input water height by using the following equation :

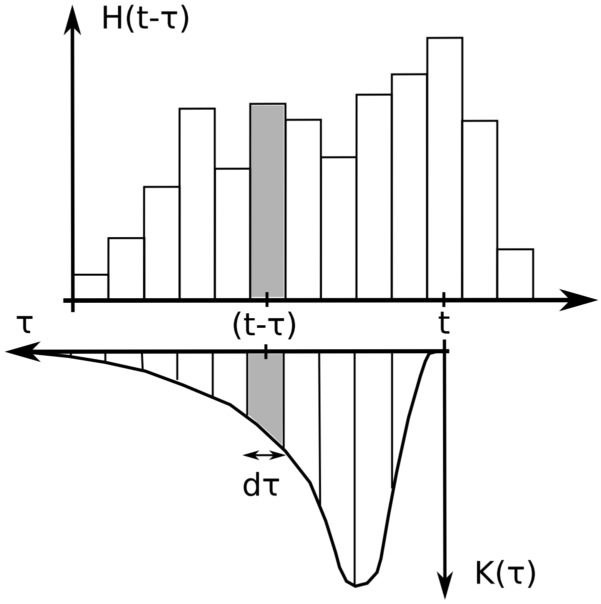
$$\begin{aligned} Q_{SU}(t) = & \tfrac{d}{2\times(\pi\times D)^{1/2}} \times exp^{\frac{C\times d}{2\times D}} \\ & \times \int_0^t H(t-\tau) \times A \times \tfrac{exp^{\frac{C\times d\times}{4\times D}\times\left(\frac{d}{C\times\tau}+\frac{C\times\tau}{d}\right)}}{\tau^{3/2}} \delta\tau \end{aligned} \tag{3}$$

where $Q(t)$ is the produced water discharge at time t (m^3/s), d is the distance between the gravity center of the two connected units (m), D is the wave diffusivity (m^2/s), C is the wave celerity (m/s), and A is the surface of the SU (m^2).

The term $H(t-\tau)$ is the water height at the soil surface calculated in equation 1. This water height is convoluted with the "Hayami kernel" $K(t)$ which is given as :

$$K(t) = \frac{d}{2\times(\pi D)^{\frac{1}{2}}} \times \frac{exp^{\frac{Cd}{4D}\times\left(2-\frac{d}{Ct}-\frac{Ct}{d}\right)}}{t^{\frac{3}{2}}} \tag{4}$$

The convolution operation is schematized below :



τ is an operator of the convolution product which is internal to the simulator. To do this, the simulator temporarily sweeps the Hayami kernel, at the simulation time step $\delta\tau$, up to a maximum step defined as $MaxSteps$ parameter $(-)$. So, the duration of Hayami kernel is equal to $MaxSteps.\Delta t$. Consequently, this parameter should be ajusted according to the kernel shape and the simulation time step. The higher this parameter is, the better the difinition of Hayami kernel is, but the longer the simlation is. Moreover, the smaller the simulation time step is, the better the Hayami kernel resolution is. To optimize the parametrization for a given simulation time step, the better way is to set a high value of $MaxSteps$ and decreases it while the results doesn't change.

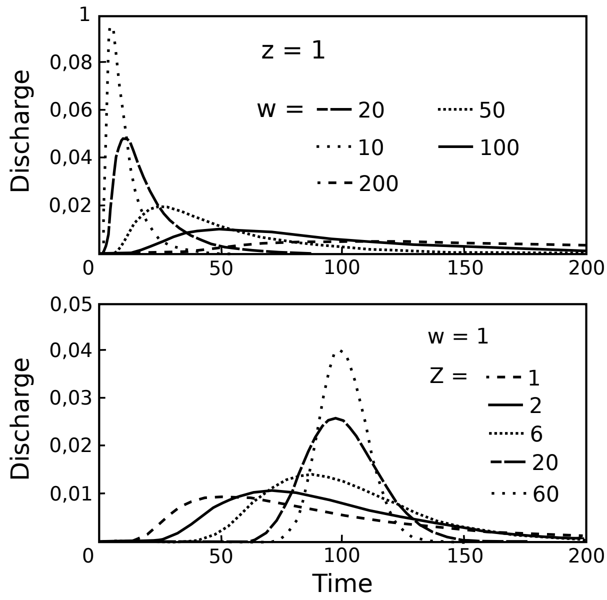
The two main parameters C and D of the diffusive model could be related to the slope and the rugosity of the surface unit using Manning-Strickler relation.

$$C = C_u \times \sqrt{\frac{\beta}{\beta_m}} \times \frac{n_m}{n} \quad et \quad D = D_u \times \frac{\beta}{\beta_m} \times \frac{n_m}{n} \tag{5}$$

where C_u is the mean celerity of SUs (m/s), β is the slope of the SU on which the calculation is done (m/m), β_m is the mean slope of SUs (m/m), n is the rugosity coefficient of the SU $(s/m^{1/3})$, n_m is the mean rugosity coefficient of SUs $(s/m^{1/3})$ and D_u is the mean diffusivity of SUs (m^2/s). This calculation is done only once at the beginning of the simulation.

Both parameters determine the Hayami unit hydrogram shape. The following figure shows different shape of hydrogram as a function of w and z parameters which are expressed hereafter :

$$w = \frac{L}{D} \qquad z = \frac{C.L}{4.D} \tag{6}$$



Examples of use and parametrization are available in the thesis of Chahinian [?] and the paper of Moussa and al. [?].

2 Functional description

2.1 Simulator name

The ID of the simulator is `water.surf.transfer-su.hayami`.

2.2 Simulator parameters

The simulator "Water transfer on surface units using Hayami propagation method" must be used with the following parameters :

Symbol	*Name*	*Value range*	*Unit*
$MaxSteps$	`maxsteps`	> 0	$-$
C	`meancel`	> 0	m/s
D	`meansigma`	> 0	m^2/s

Thus, the correct syntax to use in the `model.xml` file is illustrated hereafter.

```
<simulator ID="water.surf.
           transfer-su.hayami">
    <param name="maxsteps" value="100" />
    <param name="meancel" value="0.05" />
    <param name="meansigma" value="500" />
</simulator>
```

2.3 Unit properties required

This simulator requires some geometric properties and soil characteristics. These are described in the following table.

Symbol	*Name*	*Value range*	*Unit*
n	`nmanning`	> 0	$s/m^{-1/3}$
A	`area`	> 0	m^2
β	`slope`	> 0	m/m
d	`flowdist`	> 0	m

2.4 Variables

Variables produced, required and updated by this simulator are listed hereafter.

Symbol	*Name*	*Unit*
R	`water.surf.H.runoff`	m
Q_e	`water.uz-surf.Q.exfiltration`	m^3/s
Q_{SU}	`water.surf.Q.downstream-su`	m^3/s

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