On the Axiomatic of Pollution-generating technologies : Non-parametric Production Analysis

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Abstract

This paper analyses the concept of Pollution-generating Technologies (PgT). Following the notion of output congestion, a suitable B-disposable assumption is introduced. This approach aims to reveal any PgT in production processes that are compatible with a minimal set of assumptions. Thus, a more general class of PgT (convex and non-convex) is defined. An empirical illustration is proposed to give an illustrative example of the new B-disposal assumption with respect to convex and non-convex non-parametric technologies.

JEL: C61, D24, Q50.

Keywords: Data Envelopment Analysis, B-disposal Assumption, Convexity/Non-Convexity , Non-Parametric Pollution-generating Technology, Output Congestion.

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1 Introduction

Since the early nineties, researchers strive to model undesirable outputs using non-parametric models (Tyteca, 1996; Zhou et al., 2008; Dakpo et al., 2016). In general, several approaches are distinguished in the literature. Following Scheel (2001), the proposed models can be classified into either direct or indirect approaches. The former consider the original output data and alter the technology assumptions whereas the latter modify the value of the undesirable outputs.

The first approach was to treat bad outputs[1] as inputs (Cropper and Oattes, 1992; Reinhard et al., 2000; Hailu and Veeman, 2001; Sahoo et al., 2011; Mahlberg et al., 2011). Through an illustrative example, Färe and Grosskopf (2003) show that this method is inconsistent with physical laws. Following Pethig (2003, 2006), this approach fails to satisfy the Materials Balance Principle (MBP)[2]. Moreover, considering residual outputs as inputs comes down to model the technology with an unbounded output set (Färe and Grosskopf, 2003; Leleu, 2013). Thus, this model fails to satisfy the standard axioms of production theory. Furthermore, it does not consider interactions among undesirable production and inputs (Førsund, 2009). There also exists approaches that alter the value of undesirable outputs to transform them into desirable outputs. Several authors consider an additive inverse transformation[3] (Koopmans, 1951), and the translation invariance property (Ali and Seidford, 1990; Seidford and Zhu, 2002), while others use a multiplicative inverse alteration (Golany and Roll, 1989). However, as mentioned in Färe and Grosskopf (2004), such approaches are not consistent with physical laws since it considers strong disposal of outputs. Moreover, it is difficult to determine the suitable transformations of the bad outputs (Scheel, 2001).

The second approach introduces additional production axioms to model residual outputs in production theory. Färe et al. (1989) suggest a model based upon the concept of joint-production. This approach relies on the Weak (or ray) Disposability (WD) axiom (Shepard, 1970) and the null jointness assumption. The former means that desirable and undesirable outputs can only be simultaneously decreased by a proportional factor. The latter highlights the pollution problem: desirable production cannot be produced without bad outputs. Nevertheless, models that consider these notions have several limits. First, they consider a single abatement factor that reduces the production set. Hence, it conducts to an artificial high number of efficient Decision Making Units (DMUs). Kuosmanen (2005) proposes to enhance them by introducing a non-uniform abatement

[1]Note that throughout this paper, we use equivalently the terms bad outputs, undesirable outputs and residual outputs.

[2]More precisely, this approach fails to satisfy the first law of thermodynamics. This law can be illustrated through the famous saying: "Nothing is lost, nothing is created, everything is transformed" Antoine Lavoisier (1743-1794).

[3]The additive inverse transformation consists to multiply each undesirable outputs by -1. This approach exhibits the same technology set that considers undesirable outputs as inputs. However, it alters the sign of bad outputs.

factor to capture all feasible production plans.

Second, standard WD model does not exclude positive shadow prices for residual outputs (Hailu and Veeman, 2001; Hailu, 2003). Rødseth (2013) examines this issue, and finds that positive prices may be appropriate in cases where bads are recuperated by good outputs. Third, Kuosmanen and Podinovski (2009) show that conventional WD technologies are not necessary convex. Podinovski and Kuosmanen (2011) suggest to model weak disposability under relaxed convexity assumptions. Finally, Hampf and Rödseth (2015) show that traditional WD model satisfies the MBP only if abatement activities are present. Moreover, these authors show that this model fails to satisfy the second law of thermodynamics[4].

Among the above approaches, the literature in non-parametric environmental efficiency studies shows that WD models are extensively used. Some recent papers assuming WD applied to numerous topics are proposed on leadings journals; see for instance Manello (2017), Shen et al. (2017), Falavigna et al. (2015), Azad and Ancev (2014), Bilsel et al. (2014) or Picazo-Tadeo et al. (2005). Two innovative approaches arose due to the limits associated with the WD model. First, following Ayres and Kneese (1969), Lauwers and Van Huylenbroeck (2003), Coelli et al. (2007) and Lauwers (2009), Rödseth (2017) presents two new axioms of polluting technology based upon the MBP and the entropy law. Second, Murty et al. (2012) suggest an innovative By-Production (BP) technology constructed as an intersection of an intended-production technology and a residual-generation technology. Murty (2015) extends this approach through a set of axioms that corresponds to the properties of polluting technologies. Dakpo et al. (2016) present a critical review of these recent developments.

This paper proposes to model PgT using an innovative B-disposal assumption. This approach considers congested output set with a relaxed disposability assumption (Briec et al., 2016). The new B-disposal assumption is a sort of limited strong disposability. Hence, B-disposal technologies allow to define congestion in the good outputs since the output set does not satisfy the usual disposal assumption. It implies that it is not possible to reduce freely bad outputs; i.e. without any costs. The B-disposal assumption allows to reveal any PgT compatible with a minimal set of assumptions. Moreover, it treats a more general class of PgT satisfying both convex and non-convex assumptions. Indeed, non-convexities often result from negative externalities (eg., pollution). To characterize these technologies, we consider the Shephard output distance function (Shephard, 1953) that is dual to the revenue function (Shepard, 1953; Mc Fadden, 1978). Since these technologies can satisfy both convex and non-convex assumptions then, we define a test of the usual axiom of convexity[5].

This note unfolds as follows. Section 2 presents the traditional technology.

[4]Following the second law of thermodynamics, polluting inputs cannot be totaly transformed into good outputs. Then, the production process necessarily generates a minimal amount of residual outputs if a positive amount of polluting inputs is used.

[5]See Dasgupta and Mäler (2003) for a discussion about economists' convexities and nature's non-convexities.

Furthermore, it introduces the new disposal assumption and the boundaries for the residual outputs. Section 3 highlights the notions of output distance function and revenue function on the new PgT technology. From a dual viewpoint, we establish the duality result between the output distance function and a revenue function allowing for negative prices. Thereafter, we show how to detect B-disposability and we test the consistency with revenue maximization. Section 4 defines both convex and non-convex non-parametric B-disposal PgT. Introducing a generalisation of the B-disposal assumption, we establish relation among the so-called BP approach and the B-disposal model. Section 5 suggests non-parametric procedure to test convexity and disposability of the PgT. An empirical illustration is proposed in section 6. Finally, Section 7 concludes.

2 Technology: Assumptions and Definitions

2.1 Technology Based upon Traditional Assumptions

Let us define the notation used in this paper. $\mathbb{R}^n_+$ is the non-negative Euclidean n-dimensional orthant. For all $y, \nu \in \mathbb{R}^n_+$ we denote $y \leq \nu \iff y_i \leq \nu_i \; \forall i \in [n]$, where $[n]$ is the subset $\{1, ..., n\}$.

A production technology transforms inputs $x = (x_1, ..., x_m) \in \mathbb{R}^m_+$ into outputs $y = (y_1, ..., y_n) \in \mathbb{R}^n_+$. It can be characterized by the output correspondence $P : \mathbb{R}^m_+ \longrightarrow 2^{\mathbb{R}^n_+}$ where $P(x)$ is the set of all outputs vectors that can be produced from x:

$$P(x) = \{y : y \text{ can be produced from } x\}. \tag{2.1}$$

Throughout this paper, we assume that the output correspondence satisfies the following regularity properties (see Hackman, 2008; Jacobsen, 1970; McFadden, 1978):

$P1$: $P(0) = \{0\}$ and $0 \in P(x)$ for all $x \in \mathbb{R}^m_+$.
$P2$: $P(x)$ is bounded above for all $x \in \mathbb{R}^m_+$.
$P3$: $P(x)$ is closed for all $x \in \mathbb{R}^m_+$.

Note that $P1$ imposes that there is no free lunch and that the null output can always be produced. Moreover, $P2$ and $P3$ involve that $P(x)$ is compact. In addition to these axioms, there are three other assumptions that we sometimes invoke on the output correspondence:

$P4 :\; u \geq x \Rightarrow P(x) \subseteq P(u)$.
$P5$: $\forall y \in P(x), 0 \leq v \leq y \Rightarrow v \in P(x)$.
$P6$: $P(x)$ is a convex set for all $x \in \mathbb{R}^m_+$.

Assumption $P6$ postulates convexity of the output correspondence. This is useful to provide a dual interpretation through the revenue function and in empirical applications of, for instance, non-parametric technologies. Notice that under $P1$ and $P6$ if $y \in P(x)$ then $\lambda y \in P(x)$, $\forall \lambda \in [0,1]$. This implies the ray (or weak) disposability of the outputs, while axioms $P4$ and $P5$ impose the more traditional assumption of strong (or free) disposability of inputs and outputs. Note that a convex, ray disposable technology satisfying $P1 - P4$ and $P6$ but failing $P5$ is congested in the sense of Färe and Grosskopf (1983).[6]

Some subsets of the output set $P(x)$ can be defined to measure efficiency. Two subsets denoting production units on the boundary prove useful. For all $x \in \mathbb{R}^m_+$, the efficient subset is defined by:

$$E(x) = \{y \in P(x) : v \geq y \text{ and } v \neq y \Rightarrow v \notin P(x)\}. \tag{2.2}$$

The weak efficient subset is written as:

$$W(x) = \{y \in P(x) : v > y \Rightarrow v \notin P(x)\}. \tag{2.3}$$

2.2 Disposal Assumption for Bad Outputs

Let $B \subset [n]$, indexing the bad outputs of the technology. We introduce the following symbol:

$$y \geq^B v \iff \begin{cases} y_j \leq v_j & \text{if } j \in B \\ y_j \geq v_j & \text{else} \end{cases} \tag{2.4}$$

Moreover:

$$y >^B v \iff \begin{cases} y_j < v_j & \text{if } j \in B \\ y_j > v_j & \text{else} \end{cases} \tag{2.5}$$

Obviously, if $-y \geq^B -v$ then $y \leq^B v$. Notice that if $B = \emptyset$, then we retrieve the standard vector inequality. Indeed, it means that the set of the residual outputs is empty.

We can now define a new disposability assumption for the outputs.

Definition 2.1 *Let P be an output correspondence satisfying P1-P4. For any $y \in \mathbb{R}^n_+$, the output set $P(x)$ satisfies the B-disposal assumption if for all $y^\emptyset$, $y^B \in P(x)$, $y \leq^\emptyset y^\emptyset$ and $y \leq^B y^B$ implies that $y \in P(x)$.*

If $B = \emptyset$, then B-disposal assumption reduces to the standard free disposability assumption.

In this paper, the free disposal assumption is limited through the combination of it with a particular partial reversion of free disposal. The more the output dimensions are subjected to these reversions, the more the free disposability assumption gets limited and thus weakened. Indeed, Definition 2.1 implies that the larger the bad output subset B is, the more difficult one can dispose outputs. In general, these definitions can take account for cases where there is a simultaneous lack of free disposability in all dimensions. However, it is also possible to define this lack independently in several dimensions.

[6] Kuosmanen (2003) shows that this traditional specification fails convexity, but a revised specification is convex.

Let us introduce the following convex cone:

$$K^B = \left\{ y \in \mathbb{R}^n : y \geq^B 0 \right\}. \tag{2.6}$$

This notation implies that $K^\emptyset = \mathbb{R}^n_+$. Definition 2.1 is illustrated in Figure 1 with $B = \{2\}$. For any y, if there is some $y^\emptyset$ that classically dominates y and some y^2 that "$\{2\}$-dominates" y, then $y \in P(x)$. For a given configuration of observations, this allows to construct an output set that presents a lack of disposability in the dimension of the residual outputs. In such a case, there exists a lower bound on bad outputs that reflects cost disposability of the undesirable production (Murty, 2010). For given values of inputs and desirable outputs, the B-disposal production model is characterized by a unique production set with both upper and lower bounds on residual outputs (Figure 1-2).

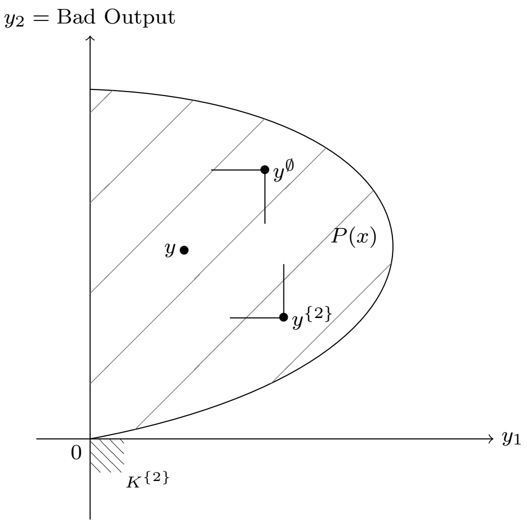

Figure 1: Convex output set with $B = \{2\}$

To study this new disposal assumption from a dual standpoint, we introduce the revenue function $R : \mathbb{R}^n \times \mathbb{R}^m_+ \longrightarrow \mathbb{R} \cup \{-\infty\}$ defined by:

$$R(p, x) = \begin{cases} \sup\limits_y \{p.y : y \in P(x)\} & \text{if } P(x) \neq \emptyset \\ -\infty & \text{if } P(x) = \emptyset \end{cases} \tag{2.7}$$

Notice that this definition allows to take into account negative prices which are specifically linked to PgT.

The following propositions study the properties of the B-disposal assumption.

Proposition 2.2 *Let P be an output correspondence satisfying P1-P4. For all*

$x \in \mathbb{R}^m_+$, $P(x)$ *satisfies the* B*-disposal assumption if and only if:*

$$P(x) = \Big((P(x) - \mathbb{R}^n_+) \cap (P(x) - K^B) \Big) \cap \mathbb{R}^n_+.$$

This proposition characterizes a B-disposal output set in terms of an intersection of convex cones (2.6). Remark that 2.2 is only based on the B-disposal assumption and P1-P4. Therefore, the above proposition holds even if $P(x)$ is not convex (Figure 2).

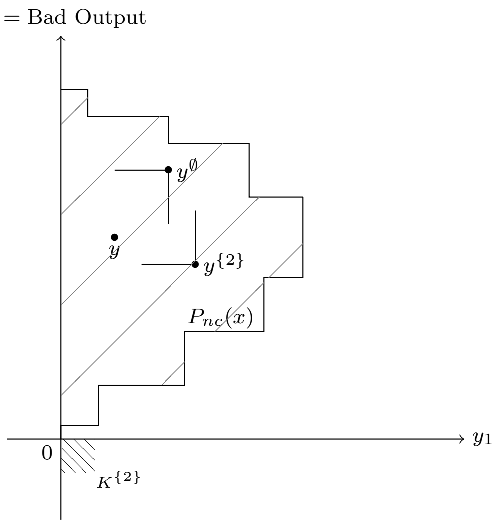

Figure 2: Non-convex output set with $B = \{2\}$

The following proposition extends the results of Proposition 2.2 to a convex output correspondence. A dual characterization of the B-disposability is proposed.

Proposition 2.3 *Let* P *be an output correspondence satisfying P1-P4. Moreover, assume that P6 holds. For all* $x \in \mathbb{R}^m_+$, $P(x)$ *satisfies the* B*-disposal assumption if and only if*

$$P(x) = \Big\{ y \in \mathbb{R}^n_+ : p.y \leq R(p, x), p \in \mathbb{R}^n_+ \cup K^B \Big\}.$$

Intuitively stated, a convex output set satisfying B-disposability can be enveloped by a revenue function for proper prices. This result constitutes the basis for the duality result developed in Section 3.

Now, we define a new notion of congestion in good outputs.

Definition 2.4 *Let* P *be an output correspondence satisfying P1-P4 and let* B *be a subset of* $[n]$*. For all* $x \in \mathbb{R}^m_+$, $P(x)$ *is congested in the desirable outputs if it fails strong disposability assumption but satisfies* B*-disposal assumption.*

This means that:

$$(P(x) - \mathbb{R}^n_+) \cap \mathbb{R}^n_+ \neq \big((P(x) - \mathbb{R}^n_+) \cap (P(x) - K^B)\big) \cap \mathbb{R}^n_+. \tag{2.8}$$

Definition 2.4 provides a strict definition of congestion in good outputs. Recall that in such a case there exists a lower bound on bad outputs for given values of inputs and good outputs. Thus, we have:

$$P(x) \neq (P(x) - \mathbb{R}^n_+) \cap \mathbb{R}^n_+. \tag{2.9}$$

In the following, for all price vector $p \in \mathbb{R}^n$, we say that an output in $P(x)$ is **p-optimal** if it maximizes the revenue $R(\cdot, p)$. An output vector $y \in P(x)$ is **interior**, if $y > 0$. The next result establishes a characterization of the new PgT.

Proposition 2.5 *Let P be an output correspondence that satisfies P1-P4. Assume that P6 holds. $P(x)$ is congested in the good outputs if and only if there exists some interior p^B-optimal output in $P(x)$ with $p^B \in K^B \backslash \mathbb{R}^n_+$.*

2.3 Boundaries for Bad Outputs

It remains an open question: how to detect undesirable outputs from the structure of the output correspondence? To answer this question, it is useful to introduce the concept of bad frontier. Therefore, the following definition identifies a subset that is not efficient, but that is a part of the boundary of a B-disposal output correspondence.

Definition 2.6 *Let P be an output correspondence satisfying P1-P4 and let $B \subset [n]$. For all $x \in \mathbb{R}^m_+$, we call bad output efficient frontier the subset:*

$$E^B(x) = \{y \in P(x) : v \geq^B y \text{ and } v \neq y \Rightarrow v \notin P(x)\}.$$

We call bad output weakly efficient frontier the subset:

$$W^B(x) = \{y \in P(x) : v >^B y \Rightarrow v \notin P(x)\}.$$

It follows that $E^{\emptyset}(x) = E(x)$ is the usual efficient subset of $P(x)$. Moreover, note that $y \in E^B(x)$ if and only if:

$$(P(x) \backslash \{y\}) \cap (y + K^B) = \emptyset. \tag{2.10}$$

Proposition 2.7 *Let P be an output correspondence satisfying P1-P4. Assume that P6 holds.*
(a) The subsets $E^B(x)$ and $W^B(x)$ are closed.
(b) If the output set $P(x)$ is congested in good outputs then, the subset $E^B(x) \backslash E(x)$ is non-empty and contains an interior point.
(c) Suppose that $E^B(x) \backslash E(x)$ is non-empty and contains an interior point. Moreover, assume that $P(x)$ satisfies the B-disposal assumption. Then, $P(x)$ is congested in good outputs.

Remark 2.8 *There exists output sets that are not congested in the good dimension and for which there exists a boundary point in $E^B(x)\backslash E(x)$. For example assume that $P(x)$ is the cube defined by $P(x) = \{(y_1, y_2) \in \mathbb{R}^2_+ : y_1 \leq 1, y_2 \leq 1\}$. Then $y^B = (1, 0) \in E^{\{1\}}\backslash E$. However, $P(x)$ satisfies free disposability of undesirable outputs.*

Note that the bad frontier corresponds to the lower bound of the output set. The bad frontier is of interest for policy makers and researchers to define global (economic and environmental) recommendations.

3 Duality between Technology and Revenue Function Based on B-Disposability

Shephard (1953) introduced the so-called Shephard distance function in production theory. This distance function characterises technology and provides a useful tool in efficiency and productivity measurement.[7] Moreover, it is always feasible under P1-P3 and P6.

3.1 Distance Function and Revenue Function on PgT : A Duality Result

The output distance function $\psi_P : \mathbb{R}^{m+n}_+ \longrightarrow \mathbb{R} \cup \{+\infty\}$ is defined by:

$$\psi_P(x, y) = \begin{cases} \inf\{\lambda > 0 : \frac{1}{\lambda}y \in P(x)\} & \text{if } \frac{1}{\lambda}y \in P(x) \text{ for some } \lambda > 0 \\ +\infty & \text{otherwise} \end{cases} \tag{3.1}$$

The above definition holds for a technology that satisfies the ray disposability assumption.

Traditional duality result (Jacobsen, 1970; McFadden, 1978) allows to state a duality result on an output set $P(x)$ that satisfies the ray disposal assumption. It establishes a connection between the revenue function and the output distance function.

Proposition 3.1 *Let P be an output correspondence satisfying P1-P6. We have the following properties:*
(a) For all $(x, y) \in \mathbb{R}^{m+n}_+$

$$\psi_P(x, y) = \sup_{p \geq 0} \left\{ \frac{p.y}{R(p, x)} : R(p, x) \neq 0 \right\}. \tag{3.2}$$

[7] See Russell (1985, 1987) for an axiomatic approach to the measurement of technical efficiency.

(*b*) *Let* p *be a non-negative output price vector. We have:*

$$R(p,x) = \sup_y \left\{ \frac{p.y}{\psi_P(x,y)} : y \in \mathbb{R}^n_+ \right\}. \tag{3.3}$$

A weaker duality result allows to state duality relationship between the revenue function and the ray (or weak) disposable output distance function (Shephard, 1974). In such a case, some (but not all) prices are allowed to be negative (assumption P5 is dropped).[8]

Now, we extend the properties of the distance function allowing negative orientations. Moreover, we prove that it is compatible with output sets satisfying the B-disposal assumption.

Proposition 3.2 *Let* P *be an output correspondence satisfying P1-P4 and P6. Moreover, assume that* $P(x)$ *satisfies the* B*-disposal assumption. We have the following properties:*
(*a*) *For all* $(x,y) \in \mathbb{R}^{m+n}_+$:

$$\psi_P(x,y) = \sup_{p \in K^B \cup \mathbb{R}^n_+} \left\{ \frac{p.y}{R(p,x)} : R(p,x) \neq 0 \right\}. \tag{3.4}$$

(*b*) *Let* $p \in K^B \cup \mathbb{R}^n_+$ *be an output price vector possibly having some negative components. Then:*

$$R(p,x) = \sup_y \left\{ \frac{p.y}{\psi_P(x,y)} : y \in \mathbb{R}^n_+ \right\}. \tag{3.5}$$

Property (a) extends the results of Shephard (1953) to an output correspondence that may fail both strong and weak disposability assumptions. The converse results expressing the revenue function with respect to the Shephard distance function is stated in (b). This duality result considerably weakens current duality results imposing strong disposability. Otherwise stated, this proposition shows that B-disposal of outputs is a necessary and sufficient condition allowing the output Shephard distance function to characterize technology. Hence, traditional result based upon ray disposability of outputs to characterize technology is substantially weakened.

This new duality result is illustrated in Figure 2. Since the first (good) output is clearly congested, the second (bad) output receives a negative price. Thus, the revenue function presents a positive rather than a negative slope.

In principle it is possible to relax the convexity assumption. Under nonconvexity, the duality result in Proposition 3.2 would only hold locally (similar to the local duality result in, e.g., Briec, Kerstens and Vanden Eeckaut (2004)).

It should be clear by now that when the output set satisfies free disposal, then it contains the output set that satisfies B-disposal assumption. However, the

[8] Also McFadden (1978) anticipates the use of negative prices and maintains that duality results can be preserved under these circumstances.

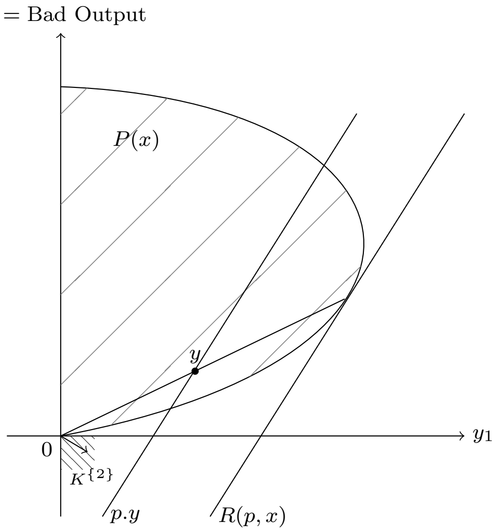

Figure 3: Shephard distance function and duality with $B = \{2\}$

converse is not necessarily true. The same applies to weak disposal assumption: an output set satisfying weak disposability assumption also contains the output set that satisfies the B-disposal assumption. Nevertheless, the converse need not be true.

3.2 Measurement of Bad Disposability

Now, we show relationship between special cases of the output distance function introduced below and the congestion concept. Following Luenberger (1995), to study this relationship from a dual viewpoint we introduce the adjusted price correspondence $p : \mathbb{R}_+^{m+n} \longrightarrow 2^{\mathbb{R}^n}$:

$$p(x,y) = \arg \min_{p \in K^B \cup \mathbb{R}_+^n} \left\{ \frac{p.y}{R(p,x)} : R(p,x) \neq 0 \right\}. \tag{3.6}$$

Notice that if the minimum is not achieved, then $p(x,y) = \emptyset$. At points where $\psi_P(x,\cdot)$ is differentiable and applying the envelop theorem to 3.4 we obtain:

$$\nabla_y \psi_P(x,y) = \frac{p(x,y)}{R(p,x)}. \tag{3.7}$$

Thus,

$$p(x,y) = \nabla_y \psi_P(x,y) R(p,x) \tag{3.8}$$

For the sake of simplicity, we introduce the following notation:

$$P^{\emptyset}(x) = (P(x) - K^{\emptyset}) \cap \mathbb{R}_+^n = (P(x) - \mathbb{R}_+^n) \cap \mathbb{R}_+^n, \tag{3.9}$$

$$P^B(x) = (P(x) - K^B) \cap \mathbb{R}_+^n, \tag{3.10}$$

$$P^{\{\emptyset,B\}}(x) = P^{\emptyset}(x) \cap P^B(x) = \Big((P(x) - \mathbb{R}^n_+) \cap (P(x) - K^B)\Big) \cap \mathbb{R}^n_+. \quad (3.11)$$

In the next proposition, the impact of adding convexity to axioms $P1 - P4$ is analyzed.

Proposition 3.3 *Let P be an output correspondence satisfying P1-P4. Moreover, assume that P6 holds. For all $x \in \mathbb{R}^m_+$, we have the following properties:*
(a) $P(x)$ is congested in good outputs if and only if there exists some $y \in P(x)$ such that $p(x,y) \subset K^B \backslash \mathbb{R}^n_+$.
(b) $P(x)$ is congested in desirable outputs if and only if there exists some $y \in P(x)$ such that $\psi_{P^\emptyset}(x,y) < \psi_{P^{\{\emptyset,B\}}}(x,y)$.

In the following a procedure is proposed to measure congestion in good outputs[9].

Definition 3.4 *Let P be an output correspondence satisfying P1-P4. For all production vector $(x,y) \in T$, we define the following ratio to measure congestion in good outputs:*

$$DC^B(x,y) = \psi_{P^{\{\emptyset,B\}}}(x,y) / \psi_{P^\emptyset}(x,y)$$

.

We can now state the following corollary for our congestion measure.

Corollary 3.5 *Let P be an output correspondence satisfying P1-P4. Assume moreover that for all $x \in \mathbb{R}^m_+$, $P(x)$ satisfies the B-disposal assumption. Then, there exists some $y \in P(x)$ such that $DC^B(x,y) > 1$ if and only if $P(x)$ is congested in desirable outputs.*

This measure $DC^B(x,y)$ evaluates subvector congestion per subset B.

3.3 Testing for Consistency with Revenue Maximization

Suppose given data on input-output vectors (x_z, y_z) and output prices p_z for all $z \in \mathcal{Z}$, where $\mathcal{Z}$ is an index set of natural number. Here we ask whether or not there is a family of output sets $P(x)$ that can make sense of this observed behavior. It is possible to show that the existence of negative prices involves congestion in the general sense defined in this contribution. Following Varian (1984) we say that a family of output sets $P(x)$ rationalizes the data if y_z is a solution of the program:

$$\max_y \{p_z.y : y \in P(x_z)\} \quad (3.12)$$

[9] Since B-congested technologies satisfy the B-disposal assumption but fail strong disposability, the suggested test of congestion allows to measure the loss of good outputs due to a lack of disposability in the residual outputs.

for all $z \in \mathcal{Z}$. Equivalently, a family of output sets $P(x)$ rationalizes the data if for all $z \in \mathcal{Z}$ and all $y \in P(x_z)$:

$$p_z.y_z \geq p_z.y. \tag{3.13}$$

Assume that the output set is one-dimensional ($n = 1$). The main difference with Varian's (1984) Weak Axiom of Profit Maximization (WAPM) is that here prices can be negative. This excludes the strong disposal (or negative monotonic) property of the output set. Following Varian (1984) we assume that the family of output sets is nested by the following assumption:

$$\forall y \in P(x),\ x \leq u \quad \text{implies that} \quad y \in P(u). \tag{3.14}$$

In the following, we suppose that for all z

$$p_{z,j} < 0 \text{ if } j \in B \quad \text{and} \quad p_{z,j} > 0 \text{ if } j \notin B \tag{3.15}$$

The key idea of the following result is that if a collection of output sets $P(x)$ rationalizes the data, then it necessarily satisfies a B-disposal assumption and a congestion assumption in the output dimension.

Proposition 3.6 *The following conditions are equivalent:*
(a) There exists a family of nested output sets $P(x)$ that rationalizes the data.
(b) If $x_k \leq x_z$, then $p_z.y_k \leq p_z.y_z$ for all $z, k \in \mathcal{Z}$.
(c) There exists a family of nontrivial closed, convex and nested output sets that rationalizes the data and that satisfies congestion in the good outputs dimension.

An immediate consequence is that negative prices imply congestion of the technology. Obviously, if all observed prices are nonnegative, then we have $B = \emptyset$ for $z \in \mathcal{Z}$ and, because of $B = \emptyset$, we retrieve the Varian (1984) WAPM result.

Notice that in principle it is possible to relax the convexity assumption (e.g., as in Briec, Kerstens and Vanden Eeckaut (2004)). Obviously, the same remarks as those mentioned at the end of subsection 3.1 apply.

4 Bad Outputs on Non-Parametric Technologies

In this section we focus on convex and non-convex non-parametric technologies. In particular we consider the so-called Data Envelopment Analysis (DEA) model due to Banker, Charnes and Cooper (1984) and the Free Disposal Hull (FDH) non-convex production model (Tulkens, 1993).

4.1 Non-Parametric Convex and Non-Convex Technologies

We consider a set of DMUs $\mathcal{A} = \{(x_z, y_z) : z \in \mathcal{Z}\}$ where $\mathcal{Z}$ is an index set of natural number. We assume that the technology satisfy the Variable Returns to Scale (VRS) assumption (Banker et al., 1984)[10]. In such case the production technology is defined by:

$$T^{\emptyset,DEA} = \Big\{(x,y) : x \geq \sum_{z\in\mathcal{Z}} \mu_z x_z, y \leq \sum_{z\in\mathcal{Z}} \mu_z y_z, \sum_{z\in\mathcal{Z}} \mu_z = 1,\;\; \mu \geq 0\Big\} \tag{4.1}$$

For any observed (x_0, y_0), the output correspondence is:

$$P^{\emptyset,DEA}(x_0) = \Big\{y :\; x_0 \geq \sum_{z\in\mathcal{Z}} \mu_z x_z, y \leq \sum_{z\in\mathcal{Z}} \mu_z y_z, \sum_{z\in\mathcal{Z}} \mu_z = 1, \mu \geq 0\Big\}$$

To establish congestion of the output correspondence, we need to identify the following subset:

$$P^{B,DEA}(x_0) = \Big\{y : x_0 \geq \sum_{z\in\mathcal{Z}} \theta_z x_z, y \leq^B \sum_{z\in\mathcal{Z}} \theta_z y_z, \sum_{z\in\mathcal{Z}} \theta_z = 1, \theta \geq 0\Big\} \tag{4.2}$$

We now have $P^{\{\emptyset,B\}}(x_0) = P^{\emptyset}(x_0) \cap P^B(x_0) = \Big(\big(P(x_0) - \mathbb{R}^n_+\big) \cap \big(P(x_0) - K^B\big)\Big) \cap \mathbb{R}^n_+$. Equivalently, we have:

$$P^{\{\emptyset,B\},DEA}(x_0) = P^{\emptyset,DEA}(x_0) \cap P^{B,DEA}(x_0) \tag{4.3}$$

The subset (4.2) allows to define the bad frontier of the PgT. The latter corresponds to the lower bound of the output set. The upper bound is established by the frontier of the subset $P^{\emptyset,DEA}(x_0)$. Thus, we have

[10]Notice that if we assume that $\mathcal{A}$ contains the null input-output vector $(0,0)$ then axiom P1 holds true. Equivalently, one can suppose a non-increasing returns to scale assumption (Färe et al., 1983).

$$P^{\{\emptyset,B\},DEA}(x_0) = \Big\{ y : x_0 \geq \sum_{z \in \mathcal{Z}} \theta_z x_z,\ x_0 \geq \sum_{z \in \mathcal{Z}} \mu_z x_z$$

$$y \leq^B \sum_{z \in \mathcal{Z}} \theta_z y_z,\ y \leq \sum_{z \in \mathcal{Z}} \mu_z y_z$$

$$\sum_{z \in \mathcal{Z}} \theta_z = \sum_{z \in \mathcal{Z}} \mu_z = 1,\ \ \theta, \mu \geq 0 \Big\} \tag{4.4}$$

$P^{\{\emptyset,B\},DEA}(x_0)$ characterizes an overall convex PgT with both upper and lower bounds on bad outputs for given values of desirable outputs and inputs. We can now state the following result:

Proposition 4.1 *The non-parametric convex output correspondence satisfies the following properties.*
(a) $P^{\{\emptyset,B\},DEA}$ *is convex;*
(b) $P^{\{\emptyset,B\},DEA}$ *satisfies the B-disposal assumption;*
(c) $P^{\{\emptyset,B\},DEA}$ *is a closed subset of* $\mathbb{R}^n_+$*.*

The above system of linear inequations (4.4) can be formulated:

$$P^{\{\emptyset,B\},DEA}(x_0) = \Big\{ y : x_{0,i} \geq \sum_{z \in \mathcal{Z}} \theta_z x_{z,i},\ \ i = 1, ..., m$$

$$x_{0,i} \geq \sum_{z \in \mathcal{Z}} \mu_z x_{z,i},\ \ i = 1, ..., m$$

$$y_j \geq \sum_{z \in \mathcal{Z}} \theta_z y_{z,j},\ \ j \in B$$

$$y_j \leq \sum_{z \in \mathcal{Z}} \theta_z y_{z,j},\ \ j \notin B$$

$$y_j \leq \sum_{z \in \mathcal{Z}} \mu_z y_{z,j},\ \ j = 1, ..., n$$

$$\sum_{z \in \mathcal{Z}} \theta_z = \sum_{z \in \mathcal{Z}} \mu_z = 1,\ \ \theta, \mu \geq 0 \Big\} \tag{4.5}$$

Remark that, if $\theta = \mu$ then, (4.5) shows non-disposability of undesirable outputs (Kuosmanen, 2005). Following Leleu (2013), this representation is an incorrect modeling of VRS assumption in traditional Shepard's weakly disposable technology. Nevertheless, this modeling has been widely implemented in the literature (see for instance Picazo-Tadeo et al., 2005; Bilsel et al., 2014). This contribution provides an innovative axiomatic characterization of the incorrect modeling of VRS assumption in traditional Shepard's weakly disposable technology.

Furthermore, notice that if we consider a set of DMUs $\mathcal{A}' = \{(x_z, y_z), (x_z, 0) : z \in \mathcal{Z}\}$[11] then, we find the correct way to linearize VRS Shepard's weakly disposable technology suggested in Kuosmanen (2005). Kuosmanen and Podinovski (2009), show that this technology is the smallest convex extension of Shepard's weakly disposable technology. Following the initial work of Podinovski (2004), they highlight that Kuosmanen's technology is the correct minimum extrapolation technology that verified the stated axioms. This modeling permits to consider proper abatement factor for each observed activity. Through this approach, a dual interpretation of weak disposability is proposed in Kuosmanen and Matin (2011)[12]. This paper provides an axiomatic characterization of the Kuosmanen's technology.

In the same vein, if we consider $\mathcal{A}'_0 = \{(x_z, y_z), (x_0, 0) : z \in \mathcal{Z}\}$[11] then, we are getting the correct way to linearize VRS Shepard's weakly disposable technology suggested in Leleu (2013). This modeling allows to define dual interpretation of weak disposability assumption.

As mentioned previously (subsection 2.2), it is possible to introduce non-convex B-disposal technologies. Let us consider the following individual production possibility sets:

$$S^{\emptyset}(x_z, y_z) = \Big\{(x, y) \in \mathbb{R}^{n+m}_+ :\ x \geq x_z,\ y \leq y_z\Big\} \tag{4.6}$$

and

$$S^{B}(x_z, y_z) = \Big\{(x, y) \in \mathbb{R}^{n+m}_+ :\ x \geq x_z,\ y \leq^{B} y_z\Big\}. \tag{4.7}$$

Intersection of the non-convex unions of (4.6) and (4.7) allows to define FDH non-convex PgT.

$$P^{\{\emptyset,B\},DEA}_{nc}(x) = \Big\{y : (x, y) \in \big(\cup_{z\in\mathcal{Z}} S^{\emptyset}(x_z, y_z)\big) \cap \big(\cup_{z\in\mathcal{Z}} S^{B}(x_z, y_z)\big)\Big\}. \tag{4.8}$$

Note that VRS assumption is imposed in the above FDH non-convex production model. Other returns to scale assumption can be introduced adding specific scaling parameter in (4.6) and (4.7). More precisely,

$$S^{\emptyset,\delta}(x_z, y_z) = \Big\{(x, y) \in \mathbb{R}^{n+m}_+ :\ x \geq \delta x_z,\ y \leq \delta y_z\Big\} \tag{4.9}$$

and

$$S^{B,\delta}(x_z, y_z) = \Big\{(x, y) \in \mathbb{R}^{n+m}_+ :\ x \geq \delta x_z,\ y \leq^{B} \delta y_z\Big\}. \tag{4.10}$$

With $\delta \geq 0$ (CRS assumption), $\delta \in [0, 1]$ (NIRS assumption) or $\delta \geq 1$ (NDRS assumption).

[11] In such a case axiom P1 holds true.

[12] Kuosmanen and Matin (2011) introduce the concept of "limited liability condition" to provide a dual interpretation of the weak disposability. If the maximum profit is not positive and smaller than the sunk costs of inputs the "limited liability condition" is not satisfied. In such a case, it is optimal to stop the production activity.

Following Briec et al. (2004), we can define a consolidated B-disposal PgT as follows:

$$
\begin{aligned}
P^{\{\emptyset,B\},DEA}_{\Omega,\Delta}(x) = \Big\{ y : x \geq \delta \sum_{z\in\mathcal{Z}} \theta_z x_z,\ & x \geq \delta \sum_{z\in\mathcal{Z}} \mu_z x_z \\
y \leq^B \delta \sum_{z\in\mathcal{Z}} \theta_z y_z,\ & y \leq \delta \sum_{z\in\mathcal{Z}} \mu_z y_z \\
\theta, \mu \in \Omega, \delta \in \Delta \Big\}. &
\end{aligned}
$$

Where $\Omega \in \{\Omega_c, \Omega_{nc}\}$, with $\Omega_c = \big\{(\theta,\mu) : \sum_{z\in\mathcal{Z}} \theta_z = \sum_{z\in\mathcal{Z}} \mu_z = 1,\ \ \theta,\mu \geq 0\big\}$ and $\Omega_{nc} = \big\{(\theta,\mu) : \sum_{z\in\mathcal{Z}} \theta_z = \sum_{z\in\mathcal{Z}} \mu_z = 1,\ \ \theta,\mu \in \{0,1\}\big\}$. Moreover, $\Delta \in \big\{\{\delta : \delta = 1\}; \{\delta : \delta \geq 0\}; \{\delta : \delta \in [0,1]\}; \{\delta : \delta \geq 1\}\big\}$ allows to consider several returns to scale assumptions (ie, VRS, CRS, NIRS and NDRS).

Furthermore, since the B-disposal technology is defined as an intersection of sub-technologies, we can introduce a hybrid (convex and non convex) PgT as follows:

$$
\begin{aligned}
P^{\{\emptyset,B\},DEA}_{h^-,\Delta}(x) = \Big\{ y : x \geq \delta \sum_{z\in\mathcal{Z}} \theta_z x_z,\ & x \geq \delta \sum_{z\in\mathcal{Z}} \mu_z x_z \\
y \leq^B \delta \sum_{z\in\mathcal{Z}} \theta_z y_z,\ & y \leq \delta \sum_{z\in\mathcal{Z}} \mu_z y_z \\
\theta \in \Omega_{nc}, \mu \in \Omega_c, \delta \in \Delta \Big\}. &
\end{aligned} \tag{4.11}
$$

In (4.11), upper bound of the output set presents convexity whereas non-convexity applies to the bad frontier (Figure 4). In such a case, the hybrid PgT corresponds to the intersection of the convex union of (4.9) and the non-convex union of (4.10).

$$
P^{\{\emptyset,B\},DEA}_{h^-,\Delta}(x) = \Big\{ y : (x,y) \in Co\big(\cup_{z\in\mathcal{Z}} S^{\emptyset,\delta}(x_z,y_z)\big) \cap \big(\cup_{z\in\mathcal{Z}} S^{B,\delta}(x_z,y_z)\big) \Big\}. \tag{4.12}
$$

4.2 By-production technology and generalised B-disposal assumption

In the previous section, the B-disposability corresponds to the cost disposability assumption of Murty et al. (2012). However, the B-disposal assumption is applied to the costly disposal hull of the technology rather than to the technology itself. The Murty et al.'s (2012) model is more general because it does not assume the strong disposability assumption on the input side. This approach considers a partition of inputs into polluting and no-polluting ones such that, the former

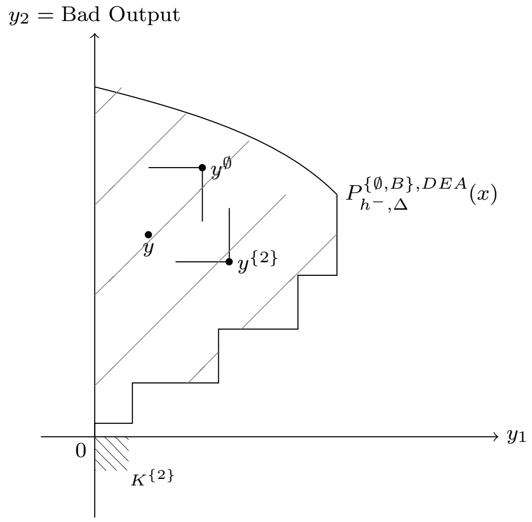

Figure 4: A hybrid output set with $B = \{2\}$

satisfies costly disposability assumption. In the previous sections, we focus on the output side and do not fix an a priori input partition (i.e., polluting versus no-polluting). However, all this framework could be extended to a general case considering the B-disposal assumption both in inputs and outputs sides.

We first present the notation used to define a generalised version of the B-disposal assumption. Let $B = \{B_{\text{in}}, B_{\text{out}}\} \subset [m] \times [n]$, indexing the inputs generating pollution and the bad outputs of the technology. We assume that there are polluting and no-polluting inputs such that $x = (x^p, x^{np})$. Let T a production technology satisfying the following regularity properties:

$T1$: $(0, 0) \in T$, $(0, y) \in T \Rightarrow y = 0$.
$T2$: $T(y) = \{(u, v) \in T : v \leq y\}$ is bounded for all $y \in \mathbb{R}^n_+$.
$T3$: T is closed.
$T4$: $\forall (x, y) \in T \quad \wedge \quad \forall (u, v) \in \mathbb{R}^m_+ \times \mathbb{R}^n_+$ if $(x, -y) \leq (u, -v)$ then $(u, v) \in T$.

The assumptions $T1 - T3$ are equivalent to $P1 - P3$. T4 imposes traditional assumption of strong disposability of inputs and outputs.

Definition 4.2 *Let T a production technology satisfying T1-T3. For any $(x, y) \in \mathbb{R}^m_+ \times \mathbb{R}^n_+$, the technology T satisfies the generalised B-disposal assumption if for all $(x^{\emptyset}, y^{\emptyset})$, $(x^B, y^B) \in T$, $(-x, y) \leq^{\emptyset} (-x^{\emptyset}, y^{\emptyset})$ and $(-x, y) \leq^B (-x^B, y^B)$ implies that $(x, y) \in T$.*

Where, $B = \{B_{\text{in}}, B_{\text{out}}\} \subset [m] \times [n]$ indexes the inputs generating pollution and the bad outputs of the technology. If $B = \emptyset$, then the generalised B-disposal assumption reduces to the standard free disposability assumption ($T4$).

Proposition 4.3 *Let T be a technology satisfying T1-T3. For all $(x, y) \in \mathbb{R}^m_+ \times \mathbb{R}^n_+$, T satisfies the generalised B-disposal assumption if and only if:*

$$T = \Big(\big(T + (\mathbb{R}^m_+ \times (-\mathbb{R}^n_+))\big) \cap \big(T + (K^{B_{\text{in}}} \times (-K^{B_{\text{out}}}))\big) \Big) \cap (\mathbb{R}^m_+ \times \mathbb{R}^n_+).$$

Note that proposition 4.3 holds true even if the technology is not convex. For simplicity, we introduce the following notations:

$$T^{\emptyset} = \big(T + (\mathbb{R}^m_+ \times (-\mathbb{R}^n_+))\big) \cap (\mathbb{R}^m_+ \times \mathbb{R}^n_+), \tag{4.13}$$

$$T^{B} = \big(T + (K^{B_{\text{in}}} \times (-K^{B_{\text{out}}}))\big) \cap (\mathbb{R}^m_+ \times \mathbb{R}^n_+), \tag{4.14}$$

$$T^{\{\emptyset,B\}} = T^{\emptyset} \cap T^{B} = \Big(\big(T + (\mathbb{R}^m_+ \times (-\mathbb{R}^n_+))\big) \cap \big(T + (K^{B_{\text{in}}} \times (-K^{B_{\text{out}}}))\big) \Big) \cap (\mathbb{R}^m_+ \times \mathbb{R}^n_+). \tag{4.15}$$

We assume that the technology satisfies Variable Returns to Scale (VRS) assumption (Banker et al., 1984). To establish generalised congestion of polluting inputs and desirable outputs, we need to identify the following subset:

$$T^{B,DEA} = \Big\{ (x, y) : x \geq^{B_{\text{in}}} \sum_{z \in \mathcal{Z}} \theta_z x_z, y \leq^{B_{\text{out}}} \sum_{z \in \mathcal{Z}} \theta_z y_z, \sum_{z \in \mathcal{Z}} \theta_z = 1, \ \ \theta \geq 0 \Big\} \tag{4.16}$$

Now, we can state:

$$T^{\{\emptyset,B\},DEA} = T^{\emptyset,DEA} \cap T^{B,DEA} \tag{4.17}$$

Thus, we have

$$\begin{aligned} T^{\{\emptyset,B\},DEA} = \Big\{ (x, y) : x \geq \sum_{z \in \mathcal{Z}} \mu_z x_z, x \geq^{B_{\text{in}}} \sum_{z \in \mathcal{Z}} \theta_z x_z \\ y \leq \sum_{z \in \mathcal{Z}} \mu_z y_z, y \leq^{B_{\text{out}}} \sum_{z \in \mathcal{Z}} \theta_z y_z \\ \sum_{z \in \mathcal{Z}} \theta_z = \sum_{z \in \mathcal{Z}} \mu_z = 1, \ \ \theta, \mu \geq 0 \Big\} \end{aligned}$$

$T^{\{\emptyset,B\},DEA}$ defines an overall PgT with a lower bound on bad outputs and an upper bound on polluting inputs. This kind of PgT allows to identify simultaneously congestion in the dimensions of polluting input and good output (see Appendix B).

Following the Murty et al.'s (2012) words, $T^{B,DEA}$ reflects nature's residual generation. $T^{\emptyset,DEA}$ allows to capture the intended-production activities of firms. The intersection of $T^{B,DEA}$ and $T^{\emptyset,DEA}$ defines a generalised B-disposal PgT. Note that the subsets $T^{B,DEA}$ and $T^{\emptyset,DEA}$ consider both polluting and no-polluting inputs. Then, intended and unintended outputs depend on the same set of inputs (Försund, 2016).

The above system of linear inequations can be rewritten as follows:

$$
\begin{aligned}
T^{\{\emptyset,B\},DEA} = \Big\{(x,y) : & \; x_i \leq \sum_{z\in\mathcal{Z}} \theta_z x_{z,i}, \;\; i \in B_{\text{in}} \\
& x_i \geq \sum_{z\in\mathcal{Z}} \theta_z x_{z,i}, \;\; i \notin B_{\text{in}} \\
& x_i \geq \sum_{z\in\mathcal{Z}} \mu_z x_{z,i}, \;\; i = 1,...,m \\
& y_j \geq \sum_{z\in\mathcal{Z}} \theta_z y_{z,j}, \;\; j \in B_{\text{out}} \\
& y_j \leq \sum_{z\in\mathcal{Z}} \theta_z y_{z,j}, \;\; j \notin B_{\text{out}} \\
& y_j \leq \sum_{z\in\mathcal{Z}} \mu_z y_{z,j}, \;\; j = 1,...,n \\
& \sum_{z\in\mathcal{Z}} \mu_z = 1, \sum_{z\in\mathcal{Z}} \theta_z = 1, \;\; \mu \geq 0, \;\; \theta \geq 0\Big\}
\end{aligned}
\tag{4.18}
$$

The above PgT (4.18) does not consider abatement outputs, but obviously it is straightforward to introduce such outputs. We just have to insert the following constraint: $y_j \geq \sum_{z\in\mathcal{Z}} \theta_z y_{z,j}, \;\; j \in B'_{\text{out}}$. Where, $B = \{B_{\text{in}}, B_{\text{out}}, B'_{\text{out}}\} \subset [m] \times [n]$ indexes the inputs generating pollution, the bad outputs and the abatement outputs of the technology. Now, consider the following constraints:

$$
\sum_{z\in\mathcal{Z}} \theta_z x_{z,i} = \sum_{z\in\mathcal{Z}} \mu_z x_{z,i}, \;\; i \notin B_{\text{in}} \tag{4.19}
$$

and

$$
\sum_{z\in\mathcal{Z}} \theta_z y_{z,j} = \sum_{z\in\mathcal{Z}} \mu_z y_{z,j}, \;\; j \notin B_{\text{out}}, \tag{4.20}
$$

Note that if we suppose the independence of $T^{\emptyset,DEA}$ from $\{y_j\}_{j\in B_{\text{out}}}$ and if we add the constraints (4.19) and (4.20) to (4.18) then, the PgT corresponds to the by-production technology of Murty et al. (2012)[13].

Let us consider the following individual production possibility set:

$$
S^{B_{in},B_{out}}(x_z, y_z) = \Big\{(x,y) \in \mathbb{R}^{n+m}_+ : \; x \geq^{B_{in}} x_z, \; y \leq^{B_{out}} y_z\Big\}. \tag{4.21}
$$

[13]If dependence contraints are considered then, we retrieve the extension of the by-production technology suggested by Dakpo (2016).

Following axioms $T1 - T3$, we can introduce FDH non-convex PgT as an intersection of the non-convex unions of (4.6) and (4.21):

$$T_{nc}^{\{\emptyset,B\},DEA} = \Big\{(x,y) : (x,y) \in \big(\cup_{z\in\mathcal{Z}} S^{\emptyset}(x_z,y_z)\big) \cap \big(\cup_{z\in\mathcal{Z}} S^{B_{in},B_{out}}(x_z,y_z)\big)\Big\}. \tag{4.22}$$

Then, a consolidated generalised B-disposal technologies can be suggested as follows:

$$\begin{aligned} T_{\Omega,\Delta}^{\{\emptyset,B\},DEA} = \Big\{(x,y) : x_i &\leq \delta \sum_{z\in\mathcal{Z}} \theta_z x_{z,i}, \;\; i \in B_{\text{in}} \\ x_i &\geq \delta \sum_{z\in\mathcal{Z}} \theta_z x_{z,i}, \;\; i \notin B_{\text{in}} \\ x_i &\geq \delta \sum_{z\in\mathcal{Z}} \mu_z x_{z,i}, \;\; i = 1,...,m \\ y_j &\geq \delta \sum_{z\in\mathcal{Z}} \theta_z y_{z,j}, \;\; j \in B_{\text{out}} \\ y_j &\leq \delta \sum_{z\in\mathcal{Z}} \theta_z y_{z,j}, \;\; j \notin B_{\text{out}} \\ y_j &\leq \delta \sum_{z\in\mathcal{Z}} \mu_z y_{z,j}, \;\; j = 1,...,n \\ &\theta, \mu \in \Omega, \;\; \delta \in \Delta\Big\} \end{aligned} \tag{4.23}$$

Adding the constraints (4.19), (4.20) and the independence of $T_{nc}^{\emptyset,DEA}$ from $\{y_j\}_{j\in B_{\text{out}}}$ in (4.23), we can introduce a non-convex version of the by-production technology. A hybrid (convex and non-convex) version of (4.23) is defined as,

$$\begin{aligned} T_{h^-,\Delta}^{\{\emptyset,B\},DEA} = \Big\{(x,y) : x_i &\leq \delta \sum_{z\in\mathcal{Z}} \theta_z x_{z,i}, \;\; i \in B_{\text{in}} \\ x_i &\geq \delta \sum_{z\in\mathcal{Z}} \theta_z x_{z,i}, \;\; i \notin B_{\text{in}} \\ x_i &\geq \delta \sum_{z\in\mathcal{Z}} \mu_z x_{z,i}, \;\; i = 1,...,m \\ y_j &\geq \delta \sum_{z\in\mathcal{Z}} \theta_z y_{z,j}, \;\; j \in B_{\text{out}} \\ y_j &\leq \delta \sum_{z\in\mathcal{Z}} \theta_z y_{z,j}, \;\; j \notin B_{\text{out}} \\ y_j &\leq \delta \sum_{z\in\mathcal{Z}} \mu_z y_{z,j}, \;\; j = 1,...,n \\ &\theta \in \Omega_{nc}, \mu \in \Omega_c, \;\; \delta \in \Delta\Big\} \end{aligned} \tag{4.24}$$

$T_{h^-,\Delta}^{\{\emptyset,B\},DEA}$ presents non convexity on the lower bound of undesirable outputs and on the upper bound of polluting inputs (see Figure 11 and 12 in Appendix B). Thus,

$$T_{h^-,\Delta}^{\{\emptyset,B\},DEA} = \Big\{(x,y) : (x,y) \in Co\big(\cup_{z\in\mathcal{Z}} S^{\emptyset,\delta}(x_z,y_z)\big) \cap \big(\cup_{z\in\mathcal{Z}} S^{B_{in},B_{out},\delta}(x_z,y_z)\big)\Big\}. \tag{4.25}$$

Where $S^{B_{in},B_{out},\delta}(x_z,y_z) = \Big\{(x,y) \in \mathbb{R}_+^{n+m} : \ x \geq^{B_{in}} \delta x_z, \ y \leq^{B_{out}} \delta y_z\Big\}$ *with* $\delta \in \Delta$. This class of PgT allows to consider possible non-convexity in the nature's residual sub-technology (Dasgupta and Mäler, 2003).

5 Non-Parametric Test: Disposability and Convexity

5.1 Non-Parametric Test of Congestion in Good Outputs

To test congestion in good outputs we need to be able to compute a distance function with respect to the output set. In Figure 5, congestion in desirable outputs can be detected at points A, C and D.

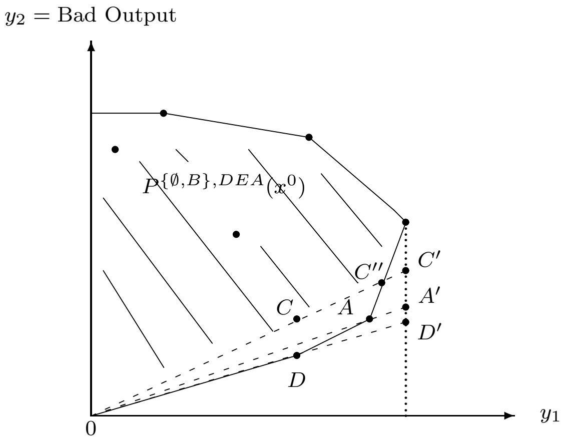

Figure 5: A non-parametric test of disposability with $B = \{2\}$.

From the specification of convex non-parametric technologies, it is quite straightforward to derive the following mathematical program[14]:

[14]Remark that, if $\theta = \mu$ then $\psi_{P^{\{\emptyset,B\},DEA}}(x_0, y_0)$ can be implemented based on the set of DMUs $\mathcal{A}$, $\mathcal{A}'$ or $\mathcal{A}'_0$.

$$\begin{aligned}
\psi_{P\{\emptyset,B\},DEA}(x_0,y_0) = \inf \lambda & \\
s.t.\ \ x_{0,i} &\geq \sum_{z\in\mathcal{Z}} \theta_z x_{z,i},\ \ i=1,...,m \\
x_{0,i} &\geq \sum_{z\in\mathcal{Z}} \mu_z x_{z,i},\ \ i=1,...,m \\
\frac{1}{\lambda} y_{0,j} &\geq \sum_{z\in\mathcal{Z}} \theta_z y_{z,j},\ \ j\in B \\
\frac{1}{\lambda} y_{0,j} &\leq \sum_{z\in\mathcal{Z}} \theta_z y_{z,j},\ \ j\notin B \\
\frac{1}{\lambda} y_{0,j} &\leq \sum_{z\in\mathcal{Z}} \mu_z y_{z,j},\ \ j=1,...,n \\
\sum_{z\in\mathcal{Z}} \theta_z &= \sum_{z\in\mathcal{Z}} \mu_z = 1,\ \ \theta,\mu\geq 0
\end{aligned}$$

The above program has $2(m+n)+1+\mathrm{Card}(B)$ constraints, where $\mathrm{Card}(B)$ is the number of B elements. When the technology is DEA convex, then the solution is obtained by solving a linear program. To measure congestion in good outputs we need to compute $\psi_{P\{\emptyset,B\},DEA}(x^0,y^0)/\psi_{P\emptyset,DEA}(x^0,y^0)$[15]. In the same way $\psi_{P\emptyset,DEA}(x^0,y^0)$ can be computed as follows:

$$\begin{aligned}
\psi_{P\emptyset,DEA}(x_0,y_0) = \inf \lambda & \\
s.t.\ \ x_{0,i} &\geq \sum_{z\in\mathcal{Z}} \theta_z x_{z,i},\ \ i=1,...,m \\
\frac{1}{\lambda} y_{0,j} &\leq \sum_{z\in\mathcal{Z}} \theta_z y_{z,j},\ \ j=1,...,n \\
\sum_{z\in\mathcal{Z}} \theta_z &= 1,\ \ \theta\geq 0
\end{aligned}$$

Remark that following (4.3) we have :

$$\psi_{P\{\emptyset,B\},DEA}(x_0,y_0) = \max\left\{\psi_{P\emptyset,DEA}(x_0,y_0);\psi_{PB,DEA}(x_0,y_0)\right\}. \tag{5.1}$$

[15] Replacing the VRS DEA technologies by CRS technologies and assuming that $\theta=\mu$ then, the test of congestion in good outputs is equivalent to that in Färe et al. (1989) (not paying attention to the choice of distance function).

Where,

$$\psi_{PB,DEA}(x_0,y_0) = \inf \lambda$$
$$s.t.\ \ x_{0,i} \geq \sum_{z\in\mathcal{Z}} \theta_z x_{z,i},\ \ i=1,...,m$$
$$\frac{1}{\lambda} y_{0,j} \geq \sum_{z\in\mathcal{Z}} \theta_z y_{z,j},\ \ j\in B$$
$$\frac{1}{\lambda} y_{0,j} \leq \sum_{z\in\mathcal{Z}} \theta_z y_{z,j},\ \ j\notin B$$
$$\sum_{z\in\mathcal{Z}} \theta_z = 1,\ \ \theta \geq 0$$

5.2 Non-Parametric Test of Convexity

In this subsection, we suggest to test convexity of the new B-disposal technologies. First, we propose a global test of convexity (Figure 6).

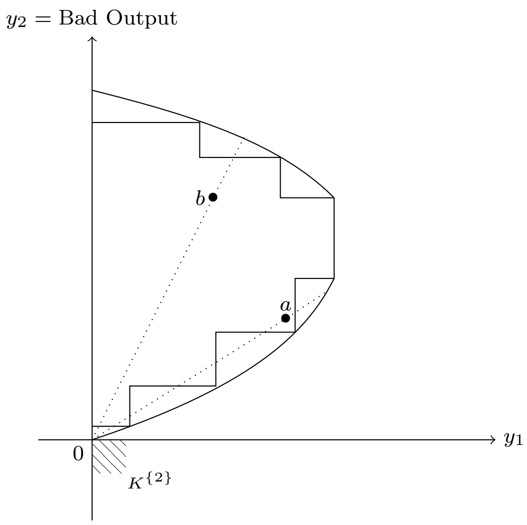

Figure 6: A global non-parametric test of convexity with $B = \{2\}$.

Figure 6 shows non-convexity of the B-disposal technology at points a and b. Point a allows to test convexity on the bad frontier of the PgT. Reversely, point b permits to compute a test of convexity on the upper bound of B-disposal technology. In order to implement this test, we have to compute an output distance function with respect to the non-convex output set.

$$\psi_{P_{nc}^{\{\emptyset,B\},DEA}}(x,y) = \begin{cases} \inf\{\lambda > 0 : \frac{1}{\lambda}y \in P_{nc}^{\{\emptyset,B\},DEA}(x)\} & \text{if } \frac{1}{\lambda}y \in P_{nc}^{\{\emptyset,B\},DEA}(x) \text{ for some } \lambda > 0 \\ +\infty & \text{otherwise} \end{cases} \tag{5.2}$$

Following (4.8), we propose to use the enumerative principle to compute $\psi_{P_{nc}^{\{\emptyset,B\},DEA}}(x,y)$ (Briec et al., 2004).

Proposition 5.1 *$\psi_{P_{nc}^{\{\emptyset,B\},DEA}}(x,y)$ on non-convex B-disposal technologies is defined as follows:*

$$\psi_{P_{nc}^{\{\emptyset,B\},DEA}}(x,y|_{S^{\emptyset}(x_z,y_z)\cap S^{B}(x_z,y_z)})^{-1} = \begin{cases} \min(\frac{y_z}{y}) & \text{if } \max_{j\in B}(\frac{y_{z,j}}{y_j}) \leq \min(\frac{y_z}{y}) \\ 1 & \text{else} \end{cases}$$

Note that VRS assumption is imposed in proposition 5.1. Other returns to scale assumption can be introduced adding specific scaling parameter (see (4.9) and (4.10)). Conventional CRS assumption can be considered as follows:

Proposition 5.2 *$\psi_{P_{nc}^{\{\emptyset,B\},DEA}}(x,y)$ on CRS non-convex B-disposal technologies is as follows:*

$$\psi_{P_{nc,crs}^{\{\emptyset,B\},DEA}}(x,y|_{S^{\emptyset,\delta}(x_z,y_z)\cap S^{B,\delta}(x_z,y_z)})^{-1} = \begin{cases} \min(\frac{x}{x_z})\min(\frac{y_z}{y}) & \text{if } \max_{j\in B}(\frac{y_{z,j}}{y_j}) \leq \min(\frac{y_z}{y}) \\ 1 & \text{else} \end{cases}$$

Notice that it is possible to compute a specific test of convexity (Figure 7). This test is of particular interest when we want to test separately convexity of the upper or of the lower bound of the PgT. Recall that this test is an immediate consequence of the B-disposal assumption definition (an intersection of sub-technologies).

We compute the following ratios to test global (CT_g^B) and specific (CT_h^B) convexity of the B-disposal PgT:

$$\mathrm{CT}_g^B = \frac{\psi_{P_{nc}^{\{\emptyset,B\},DEA}}}{\psi_{P^{\{\emptyset,B\},DEA}}}$$

and

$$\mathrm{CT}_h^B = \frac{\psi_{P_h^{\{\emptyset,B\},DEA}}}{\psi_{P^{\{\emptyset,B\},DEA}}}$$

.

When $\mathrm{CT}_g^B > 1$ then, B-disposal PgT presents global non-convexity. Remark that, if $\mathrm{CT}_g^B > 1$ and $\psi_{P^{\{\emptyset,B\},DEA}} = \psi_{P^{\emptyset},DEA}$ (ie., $DC^B = 1$) then, the upper bound of the B-disposal PgT presents non-convexity (ie., $\mathrm{CT}_{h^+}^B > 1$). Reversely, if $\mathrm{CT}_g^B > 1$ and $\psi_{P^{\{\emptyset,B\},DEA}} > \psi_{P^{\emptyset},DEA}$ (ie., $DC^B > 1$) then, the lower bound of B-disposal PgT shows non-convexity (ie., $\mathrm{CT}_{h^-}^B > 1$).

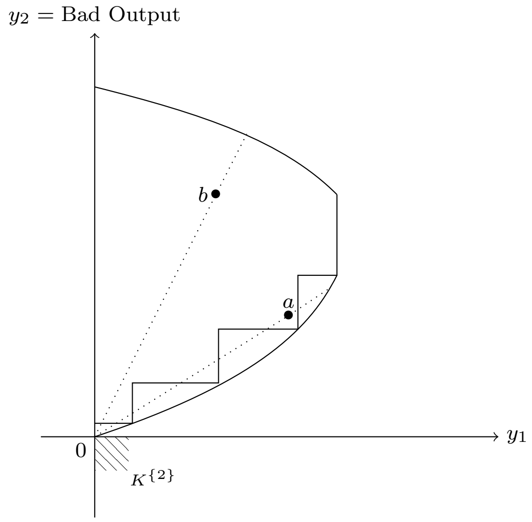

Figure 7: A specific non-parametric test of convexity with $B = \{2\}$.

6 Empirical illustration

This empirical part gives an illustrative example of the B-disposability. In such a case, the efficiency measures are estimated under B-disposal, weak disposal and strong disposal assumptions[16].

6.1 Data

A sample of 13 representative French airports is considered over the period 2007-2011. We implement the new B-disposal assumption on both convex and non-convex non-parametric technologies. The dataset comes from several reports and documents of the Ministère de l'écologie, du Développement durable et de l'Énergie (http://www.developpement-durable.gouv.fr). Two inputs are selected: (i) number of employees of each airport and (ii) operational costs of each airport. These inputs permit to produce different outputs. Thus, we consider one desirable output, (iii) number of passengers from all airlines ; and one undesirable output represented by (iv) CO_2 emissions evaluated at each airport. This bad output is measured by using the TARMAAC (Traitements et Analyses des Rejets éMis dans l'Atmosphère par l'Aviation Civile) tool of the Direction générale de l'Aviation civile (DGAC)[17].

Table 1 presents the descriptive statistics of the variables used in this study.

[16]Some articles explore the consistency between the selected model and the variables (see for instance, Halkos and Polemis (2018)). Thus, it could be investigated in further research for the case of the B-disposability assumption.

[17]Abad and Ravelojaona (2017) use a part of this panel data.

Table 1: Characteristics of inputs and outputs

Variables	Min	Max	Mean	St. Dev.
	Inputs			
Employees (quantity)	67	3813	738	1166
Operational costs (Keuros)	15614	1112248	187521	329679
	Good Output			
Passengers (quantity)	1014704	60970551	10328725	15646444
	Bad Output			
CO_2 emissions (millions of tons)	13	896	136	222

6.2 Results

Table 2 presents the weak disposable and the B-disposable Shephard distance functions. It is shown that the B-disposal efficiency scores relies on the estimation of efficiency measures with regard to both lower and upper bounds of the output set. The maximum between the efficiency scores with respect to both lower and upper bounds yields the B-disposable efficiency scores. This is not surprisingly since the B-disposal output set is defined as an intersection of two subsets. Hence, this gives indications on the projection path of inefficient DMUs. It appears that the weak disposal efficiency scores are equivalent to the B-disposal ones. This strengthens the statement that a production technology satisfying a weak disposal assumption satisfies a B-disposal assumption. However, recall that the converse is not necessarily true.

Table 3 presents Shephard output distance function projected respectively upon a convex strong disposal technology (column 2), a convex B-disposal technology (column 3) and a non-convex B-disposal technology (column 4). Columns 5 and 6 propose respectively a measure of congestion in good outputs and a test of global convexity. Column 7 identifies the part of the technology where outputs are projected through the Shephard distance function (ie., lower or upper bound of the technology). Furthermore we precise if the projection concerns a convex (C) or a non-convex (NC) part of the technology. Column 8 indicates if good outputs are congested (Cong) or not congested (N Cong). Readers can see that the B-disposal model allows to identify congestion in good outputs ($DC^B > 1$). In such a case, the production technology is bounded from below. Moreover, this model permits to identify possible non-convexity of the technology ($CT_g^B > 1$). This table shows that for each time period, more DMUs are efficient with respect to a B-disposable convex technology rather than under a strong disposable technology. Besides, we can note that the B-disposable non-convex technology presents more efficient DMUs than the B-disposable convex technology. Indeed, non-convexity restrict the production set. Then, the following embedding holds:

$$P_{nc}^{\{\emptyset,B\},DEA}(x) \subseteq P_h^{\{\emptyset,B\},DEA}(x) \subseteq P^{\{\emptyset,B\},DEA}(x) \subseteq P^{\emptyset,DEA}(x).$$

Remark that this is an empirical example. However, following Pérez et al. (2017) it could be interesting to compare and to explore the limitation of each model through an empirical application.

Airport	Shep. WD	Shep. BD		
		Upper bound	Lower bound	Max
		2007		
Ble-Mulhouse	0.7804	0.7754	0.7804	0.7804
Beauvais	1	0.6569	1	1
Bordeaux-Mrignac	1	1	1	1
Lille	0.6419	0.6419	0.5881	0.6419
Lyon-Saint Exupry	0.8818	0.8818	0.7640	0.8818
Marseille-Provence	0.9506	0.9491	0.9506	0.9506
Montpellier-Mditerrane	0.8927	0.7503	0.8927	0.8927
Nantes-Atlantique	0.8713	0.8713	0.8379	0.8713
Nice-Cte d'azur	0.9930	0.9930	0.8941	0.9930
Paris CDG	0.7627	0.7627	0.6463	0.7627
Paris ORY	0.5023	0.3397	0.5023	0.5023
Strasbourg-Entzheim	1	1	0.7744	1
Toulouse-Blagnac	1	1	1	1
		2008		
Ble-Mulhouse	0.7581	0.7579	0.7581	0.7581
Beauvais	1	0.6112	1	1
Bordeaux-Mrignac	1	1	1	1
Lille	0.6944	0.6944	0.6257	0.6944
Lyon-Saint Exupry	0.9086	0.9086	0.8115	0.9086
Marseille-Provence	0.9237	0.9237	0.8814	0.9237
Montpellier-Mditerrane	0.8793	0.7354	0.8793	0.8793
Nantes-Atlantique	0.9433	0.9433	0.8869	0.9433
Nice-Cte d'azur	0.9749	0.9749	0.8681	0.9749
Paris CDG	0.8213	0.8213	0.6946	0.8213
Paris ORY	0.4767	0.3563	0.4767	0.4767
Strasbourg-Entzheim	0.8689	0.8689	0.5817	0.8689
Toulouse-Blagnac	1	1	1	1
		2009		
Ble-Mulhouse	0.8369	0.8369	0.7363	0.8369
Beauvais	1	0.8037	1	1
Bordeaux-Mrignac	1	1	1	1
Lille	0.7731	0.7731	0.7712	0.7731
Lyon-Saint Exupry	0.9226	0.9226	0.8325	0.9226
Marseille-Provence	1	1	0.9875	1
Montpellier-Mditerrane	0.7980	0.7548	0.7980	0.7980
Nantes-Atlantique	0.9987	0.9987	0.9117	0.9987
Nice-Cte d'azur	0.9685	0.9685	0.9047	0.9685
Paris CDG	0.8302	0.8302	0.7144	0.8302
Paris ORY	0.4697	0.3751	0.4697	0.4697
Strasbourg-Entzheim	0.8738	0.8738	0.5790	0.8738
Toulouse-Blagnac	1	1	1	1
		2010		
Ble-Mulhouse	0.8353	0.8353	0.7746	0.8353
Beauvais	1	0.7980	1	1
Bordeaux-Mrignac	1	1	1	1
Lille	0.8812	0.7758	0.8812	0.8812
Lyon-Saint Exupry	0.8663	0.8663	0.8274	0.8663
Marseille-Provence	1	1	1	1
Montpellier-Mditerrane	0.7597	0.7200	0.7597	0.7597
Nantes-Atlantique	0.9497	0.9497	0.8960	0.9497
Nice-Cte d'azur	0.9176	0.9176	0.8451	0.9176
Paris CDG	0.8011	0.8011	0.7003	0.8011
Paris ORY	0.4439	0.3713	0.4439	0.4439
Strasbourg-Entzheim	0.8097	0.8097	0.5815	0.8097
Toulouse-Blagnac	0.9793	0.9793	0.9708	0.9793
		2011		
Ble-Mulhouse	0.8839	0.8210	0.8839	0.8839
Beauvais	1	0.7718	1	1
Bordeaux-Mrignac	1	1	1	1
Lille	0.7707	0.7034	0.7707	0.7707
Lyon-Saint Exupry	0.8215	0.8070	0.8215	0.8215
Marseille-Provence	0.9534	0.9485	0.9534	0.9534
Montpellier-Mditerrane	0.7151	0.6885	0.7151	0.7151
Nantes-Atlantique	0.9300	0.9300	0.8738	0.9300
Nice-Cte d'azur	0.9234	0.9234	0.8254	0.9234
Paris CDG	0.8012	0.8012	0.7082	0.8012
Paris ORY	0.4640	0.3880	0.4640	0.4640
Strasbourg-Entzheim	0.7448	0.7448	0.5887	0.7448
Toulouse-Blagnac	1	1	1	1

Table 2: Weak disposable and B-disposable Shephard distance functions

Airport	Shep. SD_c	Shep. BD_c	Shep. BD_{nc}	DC^B	CT_g^B	Bound		Good
						Upper	Lower	
			2007					
Bâle-Mulhouse	0.7754	0.7804	0.8075	1.0064	1.0348	-	NC	Cong
Beauvais	0.6569	1	1	1.5224	1	-	C	Cong
Bordeaux-Mérignac	1	1	1	1	1	C	-	N Cong
Lille	0.6419	0.6419	0.6419	1	1	C	-	N Cong
Lyon-Saint Exupéry	0.8818	0.8818	0.9127	1	1.0350	NC	-	N Cong
Marseille-Provence	0.9491	0.9506	0.9993	1.0016	1.0512	-	NC	Cong
Montpellier-Méditerranée	0.7503	0.8927	1	1.1898	1.1202	-	NC	Cong
Nantes-Atlantique	0.8713	0.8713	0.8753	1	1.0047	NC	-	N Cong
Nice-Côte d'azur	0.9930	0.9930	1	1	1.0071	NC	-	N Cong
Paris CDG	0.7627	0.7627	0.7627	1	1	C	-	N Cong
Paris ORY	0.3397	0.5023	1	1.4788	1.9908	-	NC	Cong
Strasbourg-Entzheim	1	1	1	1	1	C	-	N Cong
Toulouse-Blagnac	1	1	1	1	1	C	-	N Cong
			2008					
Bâle-Mulhouse	0.7579	0.7581	0.8178	1.0002	1.0787	-	NC	Cong
Beauvais	0.6112	1	1	1.6360	1	-	C	Cong
Bordeaux-Mérignac	1	1	1	1	1	C	-	N Cong
Lille	0.6944	0.6944	0.6944	1	1	C	-	N Cong
Lyon-Saint Exupéry	0.9086	0.9086	0.9267	1	1.0200	NC	-	N Cong
Marseille-Provence	0.9237	0.9237	0.9237	1	1	C	-	N Cong
Montpellier-Méditerranée	0.7354	0.8793	1	1.1957	1.1372	-	NC	Cong
Nantes-Atlantique	0.9433	0.9433	0.9433	1	1	C	-	N Cong
Nice-Côte d'azur	0.9749	0.9749	0.9785	1	1.0037	NC	-	N Cong
Paris CDG	0.8213	0.8213	0.8213	1	1	C	-	N Cong
Paris ORY	0.3563	0.4767	1	1.3379	2.0979	-	NC	Cong
Strasbourg-Entzheim	0.8689	0.8689	0.8741	1	1.0060	NC	-	N Cong
Toulouse-Blagnac	1	1	1	1	1	C	-	N Cong
			2009					
Bâle-Mulhouse	0.8369	0.8369	0.8457	1	1.0105	NC	-	N Cong
Beauvais	0.8037	1	1	1.2442	1	-	C	Cong
Bordeaux-Mérignac	1	1	1	1	1	C	-	N Cong
Lille	0.7731	0.7731	0.7754	1	1.0030	NC	-	N Cong
Lyon-Saint Exupéry	0.9226	0.9226	0.9354	1	1.0140	NC	-	N Cong
Marseille-Provence	1	1	1	1	1	C	-	N Cong
Montpellier-Méditerranée	0.7548	0.7980	1	1.0573	1.2531	-	NC	Cong
Nantes-Atlantique	0.9987	0.9987	0.9987	1	1	C	-	N Cong
Nice-Côte d'azur	0.9685	0.9685	0.9750	1	1.0067	NC	-	N Cong
Paris CDG	0.8302	0.8302	0.8302	1	1	C	-	N Cong
Paris ORY	0.3751	0.4697	1	1.2522	2.1290	-	NC	Cong
Strasbourg-Entzheim	0.8738	0.8738	0.8808	1	1.0080	NC	-	N Cong
Toulouse-Blagnac	1	1	1	1	1	C	-	N Cong
			2010					
Bâle-Mulhouse	0.8353	0.8353	0.8415	1	1.0074	NC	-	N Cong
Beauvais	0.7980	1	1	1.2531	1	-	C	Cong
Bordeaux-Mérignac	1	1	1	1	1	C	-	N Cong
Lille	0.7758	0.8812	1	1.1358	1.1348	-	NC	Cong
Lyon-Saint Exupéry	0.8663	0.8663	0.8708	1	1.0052	NC	-	N Cong
Marseille-Provence	1	1	1	1	1	C	-	N Cong
Montpellier-Méditerranée	0.7200	0.7597	1	1.0552	1.3163	-	NC	Cong
Nantes-Atlantique	0.9497	0.9497	0.9497	1	1	C	-	N Cong
Nice-Côte d'azur	0.9176	0.9176	0.9236	1	1.0066	NC	-	N Cong
Paris CDG	0.8011	0.8011	0.8011	1	1	C	-	N Cong
Paris ORY	0.3713	0.4439	0.8764	1.1954	1.9743	-	NC	Cong
Strasbourg-Entzheim	0.8097	0.8097	0.8097	1	1	C	-	N Cong
Toulouse-Blagnac	0.9793	0.9793	0.9793	1	1	C	-	N Cong
			2011					
Bâle-Mulhouse	0.8210	0.8839	1	1.0767	1.1313	-	NC	Cong
Beauvais	0.7718	1	1	1.2956	1	-	C	Cong
Bordeaux-Mérignac	1	1	1	1	1	C	-	N Cong
Lille	0.7034	0.7707	1	1.0957	1.2974	-	NC	Cong
Lyon-Saint Exupéry	0.8070	0.8215	1	1.0180	1.2172	-	NC	Cong
Marseille-Provence	0.9485	0.9534	1	1.0052	1.0488	-	NC	Cong
Montpellier-Méditerranée	0.6885	0.7151	0.9118	1.0387	1.2752	-	NC	Cong
Nantes-Atlantique	0.9300	0.9300	0.9300	1	1	C	-	N Cong
Nice-Côte d'azur	0.9234	0.9234	0.9741	1	1.0549	NC	-	N Cong
Paris CDG	0.8012	0.8012	0.8012	1	1	C	-	N Cong
Paris ORY	0.3880	0.4640	0.8639	1.1960	1.8619	-	NC	Cong
Strasbourg-Entzheim	0.7448	0.7448	0.7870	1	1.0567	NC	-	N Cong
Toulouse-Blagnac	1	1	1	1	1	C	-	N Cong

Table 3: Tests of congestion in good outputs and of convexity

The combination of the measures of congestion and convexity offer informations about the structure of the technology. Table 3 summarizes the conditions of technology characterization.

	$\mathbf{DC}^B = 1$	$\mathbf{DC}^B > 1$
$\mathbf{CT}_g^B = 1$	Upper bound (C), Good outputs (N Cong)	Lower bound (C), Good outputs (Cong)
$\mathbf{CT}_g^B > 1$	Upper bound (NC), Good outputs (N Cong)	Lower bound (NC), Good outputs (Cong)

Table 4: Characterization of the technology

7 Concluding Comments

This paper introduces the new B-disposal assumption that is a limited strong disposability. Along this line, a class of PgT satisfying both convex and non-convex axioms, is proposed. Moreover, we characterize these technologies with the Shephard output distance function. The duality result between the distance function and the revenue function allows to consider possible ngative shadow prices. In such a case, assuming weak disposability of outputs is not necessary to provide duality results. Through a non-parametric approach, we provide an innovative axiomatic characterization of the incorrect modeling of the VRS assumption in traditional Shephard's weakly disposable technology. Thus, we retrieve the linearization proposed by Kuosmanen (2005) and Leleu (2013). Furthermore, we show that a B-disposable technology can be rewritten as the by-production technology of Murty et al. (2012) and we extend it to non-convex case. Finally, we propose to test congestion in good outputs and the convexity of B-disposal PgT which allows to suggest a procedure to characterize the structure of the technology.

We can highlight one limitation of this paper. Indeed, we focus on the output distance function and its dual relation with the revenue function. The duality result, the new measure of congestion in good outputs and the test of convexity of the B-disposal technology can also be defined using the so-called directional distance function (Luenberger, 1992, 1995; Chambers et al., 1996; Chambers and Pope, 1996). Moreover, input and/or graph orientation could be explored. Futhermore, following Pedraja-Chaparro et al. (1999), the quality of the model under B-disposal assumption could be analyzed. These could be the topic of future research.

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