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Abstract

We explore the interplay between price signaling and independent monitoring for communicating information about the credence attribute of a good, such as environmental quality. We augment the standard model of price signaling allowing consumers to use the results of noisy monitoring as a complementary source of information. We show that monitoring restores the credibility of price signaling by saving partly or fully the signaling cost borne by green firms to deter cheating. A key reason for this is that monitoring compensates for the lack of information resulting from arbitrary beliefs based on surprising prices. The more accurate monitoring, the cheaper price signaling. The signaling behavior of green firms also depends on their number. We determine which proportion of firms choose to improve environmental quality.

Keywords: credence good, fraud, monitoring, signaling.

JEL Code: D8, H4, L15, L31, Q5.

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1 Introduction

Fraudulent behaviors, in particular those called adverse selection[1], have always been a challenge in the markets for credence goods. Unlike experience goods the quality of which can be evaluated after purchase (Nelson 1974), the quality of credence goods includes components based on social concerns, e. g., environmental, ethical or political, that are difficult if not impossible to verify by consumption or use (Darby and Karni, 1973). Thus, buyers' uncertainty about quality persists after the purchase of credence goods unless sellers find a way of signaling good quality that cannot be faked by sellers of bad quality. However, the signaling devices used to solve adverse selection problems for experience goods— i. e., price, advertising, reputation, brand name or warranty —do not always work for credence goods.

Two real-word examples illustrate the failure of signaling strategies as a result of fraudulent behaviors.

Before the law of 29 June 1907 that protects natural wine against tampered wine in France, the wine market was partly supplied by piquette. Natural wine can roughly be considered as the former name for organic wine in France. Piquette was made from the pomace resulting from pressing, diluted with water, or by grape juice of poor quality whose color and taste were chemically improved. Although it was drinkable, piquette was not really wine. However, it was sold under the name of wine to mislead consumers and trick them into buying an inferior good. Vintners and wine merchants who contributed to stimulate this unfair competition represented around five percent of the market (Le Roy Ladurie, 2007). The production of fake wines together with strong local production and wine imports caused prices to fall precipitously after 1900 (Massé, 1984).[2] As fraud was widely practised in the Midi of France, wine growers generally felt that illicit industry in adulterated wines was the main reason for the wine glut. The small wine growers in Languedoc and the Pyrénées-Orientales were ruined after the 1906 harvest did not sell. The wine slump led to a spontaneous movement of revolt in 1907, which was bloodily suppressed by the government (Smith, 1978). However, the law of 29th June and related decrees organized the prevention and control of fraud. It assigned the task of monitoring compliance with legal standards to an entity called Confédération Générale des Vignerons (CGV) that represented the wine growers' economic interests. As a result, fraud became almost impossible.[3]

A more recent example of fraudulent behaviors in the market for a credence good relates to false claims made by some car makers in commercials promoting their "Clean Diesel" ve-

[1] Adverse selection is one type of fraudulent behavior characterized by people taking advantage of their private information about some hidden attributes. Adverse selection was identified by Akerlof (1970) in the markets for used cars with hidden flaws ("lemons").

[2] After averaging 16 francs from 1890-9, the price for a hectolitre of wine in the Midi fell to 5 francs in 1901, recovered to reach 25 francs in 1903, before falling to 6 francs in 1904-6 (Warner, 1960, p. 20).

[3] The decree of 3 September 1907 stated that: "No drink may be owned or transported for sale or sold under the name of wine unless it comes exclusively from the alcoholic fermentation of fresh grapes or grape juice". On 21 October 1907 another decree established the "Fraud Repression Service" and defined its functions, authority and resources.

hicles as environmentally friendly. On 18 September 2015, the United States Environmental Protection Agency accused German automaker Volkswagen Group to breach the Clean Air Act.[4] This charge was based on a study on nitrogen oxide (NOx) emissions discrepancies between European and US models of vehicles commissioned in 2014 by the International Council on Clean Transportation (ICCT). This independent nonprofit organization found that Volkswagen had equipped diesel cars with software designed to cheat on emissions tests. The fraud allowed the vehicles' NOx output to meet US standards during regulatory testing but emit up to 40 times more NOx in real-world driving conditions. Thereafter, independent tests carried out by the German car club ADAC, the world's second-biggest motoring organisation, proved that, under normal driving conditions, diesel vehicles including the Volvo S60, Renault's Espace Energy and the Jeep Renegade, exceeded legal European emission limits for NOx by more than 10 times.[5] In January 2017, Volkswagen pleaded guilty to criminal charges.

In both of these examples, an intriguing question is why, despite spurious claims, the standard mechanism of price signaling failed in revealing the true quality of products. Obviously, the market for French wine in the early 20th century was perfectly competitive. As wine growers were price takers, they could not use price as signal. Rather, an effective signal about wine quality was the peaceful organization of mass meetings to protest again the disastrous slump in wine (Smith, 1978). In contrast, European car makers nowadays have enough market power to signal environmental quality through price. Nonetheless, price signaling has proved unsuccessful to prevent misleading advertising about clean diesel engines. Interestingly enough, it turns out that the monitoring of the credence attribute was essential to solve the adverse selection problem in both cases. From 1907 on, the monitoring role performed by the CGV halted fraud and helped restore confidence in the market prices for French wine. Similarly, the ICCT and ADAC have played a significant role in monitoring diesel engines and detecting fraudulent behaviors.

The purpose of this paper is to explore the interplay between price signaling and monitoring for communicating information about the quality of a credence good. We augment the standard model of price signaling allowing consumers to use the results of noisy monitoring as a complementary source of information. Monitoring is performed by a third-party auditor that tests the green products of high environmental quality and succeeds in learning the truth with imperfect accuracy. Reading the auditor's report, consumers learn exactly what monitoring has revealed and they perfectly know its level of accuracy. This piece of information supplements that based on firms' price signaling. Firms and the auditor simultaneously display information and we assume that both sources of information are independent.

In our setting, monitoring improves the green firms' incentives to signal high environmental quality through price. When price signaling alone could not prevent brown firms of low

[4] https://www.ftc.gov/news-events/press-releases/2016/03/ftc-charges-volkswagen-deceived-consumers-its-clean-diesel.

[5] https://www.independent.co.uk/news/business/news/volkswagen-emissions-scandal-more-carmakers-implicated-tests-reveal-pollution-levels-popular-diesels-a6674386.html.

quality from cheating, additional monitoring proves useful to make the price signaling mechanism work. We first show that independent (albeit noisy) monitoring restores the existence of separating price equilibria. One reason for this is that monitoring reduces the signaling cost incurred by green firms and this reduction is all the more significant since the level of monitoring accuracy is high. Hence, green firms can attract consumers with separating prices that neutralize the brown firms' threat of mimicking. Another reason is that consumers use monitoring to correct arbitrary beliefs based on prices observed off the equilibrium path. When firms deviate from an equilibrium price, Bayes' rule cannot apply, which makes beliefs about quality arbitrary. However, monitoring compensates for the lack of information off the equilibrium path. Thanks to monitoring, consumers can still differentiate (at least slightly) the truly green product from the spurious one after observing "surprising" prices.

We also find that the signaling behavior of green firms depends not only on the accuracy level of monitoring but also on the proportion of green firms. As monitoring restores the credibility of price signaling, switching to green production become worthwhile for a number of firms. We further determine the proportion of firms that invest in high environmental quality. On the basis of our findings, we can argue that, even though not perfect, monitoring is essential in order for green firms to honor their commitment to improve quality.

The present work is closely related to the previous literature on price signaling quality in that we extend the model under imperfect competition to include the effects of exogenous independent monitoring on the signaling mechanism. Building on Spence's (1973) canonical model, the economic literature has long investigated whether and how firms use prices to signal quality in the markets for experience goods. Many authors including Wolinsky (1983), Bagwell and Riordan (1991), Judd and Riordan (1994), Daughety and Reinganum (2008) and Janssen and Roy (2010) show that firms signal experience products of high quality with high prices. Furthermore, the efficient way of signaling high quality may be to charge prices at levels so high that they exceed the profit-maximizing price under full information. The reason for this is that high-quality firms must deter fraudulent firms of lower quality from mimicking high prices. Bayesian consumers rationally infer from the distorted price that the signal of high quality cannot be faked. Hence, an additional signaling cost may arise from the need to prevent cheating.

The same logic applies in markets for credence goods. Mahenc (2007 and 2008) and Sengupta (2012) find that a monopolist may need to distort price upward in order to signal high environmental quality unless it is cheaper to produce than low environmental quality (in which case the price distortion is downward). Sengupta (2015) analyzes the impact of price competition on the signaling cost incurred by a clean firm in an imperfectly competitive industry made of two firms. In this model, the success of price signaling depends on consumers' perception of firms' environmental practices as well as the liability payment imposed by the environmental regulation. Price signaling in turn strengthens firms' incentives to invest in better environmental practices. In the present framework, markets are not subject to environmental regulation and firms' investment in cleaner technology is not publicly observable.

Rather, our focus is on independent monitoring. We show that price signaling is successful depending on the accuracy level of monitoring and we endogenously determine the proportion of firms that invest in cleaner technology.

As firms' investment is not observable in our setting, there is a moral hazard issue that stems from the problem of adverse selection. Dynamic models of quality premia (Klein and Leffler, 1981; Shapiro, 1983) highlight the role played by repeat purchases and reputation in mitigating the moral hazard problem occurring in markets for experience goods. Dealing with markets for credence goods, we abstract from the reputation mechanism and show how monitoring alleviates the moral hazard problem. Another difference from Klein and Leffler (1981) is that, in our analysis, competition between firms is imperfect, whereas it is perfect in their analysis.

Misleading certification is central to the present analysis, as is the case for a number of articles in environmental economics. Feddersen and Gilligan (2001) question the honesty of third-party certification in a model where a certifier biased toward environmental protection is responsible for sending misleading messages to consumers. Hamilton and Zilberman (2006) show that markets for environmentally-friendly products suffer from fraudulent labeling. Baksi and Bose (2007) argue that third-party labeling may be too expensive to be socially desirable when some firms make false claims or display spurious labels. In Mahenc (2017), the certifier transmits information on environmental quality through market prices. Third-party certification turns out to be misleading when the certifier is driven more by profit than by social welfare and certification cannot be credible unless market prices truthfully reveal information. Unlike the certifier, firms cannot send price signals since they are perfectly competitive. In contrast with these papers, the present analysis allows firms to use prices as quality signals to prevent misleading certification. Monitoring proves helpful in restoring the credibility of price signaling, and consequently the reliability of certification.

The rest of this article is organized as follows. Section 2 introduces the model in which consumers have two sources of information: independent monitoring and firms' price signaling. Section 3 investigates the existence of separating equilibria in the price signaling game with and without monitoring. We show that there exists no separating equilibrium without monitoring and we fully characterize the least costly separating equilibrium with monitoring. In Section 4, we analyze firms' choice of the product type. Our concluding remarks appear in Section 5.

2 The model

The protagonists in the economy are n identical firms, consumers and an independent third-party auditor. Firms are price-takers supplying a conventional product. This product is a bundle of observable characteristics that can be properly verified by inspection and therefore are perfectly known to consumers. Every firm can invest in credence attributes that provide a quality level e higher than that of the conventional product, such that:

(i) everything else being equal, all consumers agree that the product is more valuable with than without the credence attributes;

(ii) investment in quality e involves some fixed setup cost $F > 0$ and an additional marginal cost of production $c(e)$ that is a function of e;

(iii) consumers cannot directly observe whether the product has or has not the credence attributes, either prior to or subsequent to consumption;

(iv) a specific label is intended to guarantee product quality e but this certification may not be credible.

Although the spectrum of externalities associated with quality e encompasses all kinds of social concerns, political, ethical and environmental, we will take environmental quality as a paradigmatic example to illustrate our point. Hence, we refer to the product of quality e as being the "green" type (indexed by $t = g$) and to the conventional product as being the "brown" type (indexed by $t = b$). Green certification is not trustworthy due to "greenwash" or "launder" trafficking in illegal products.

The market for the brown product represents business as usual. Conventional firms have access to a technology for producing the brown good with marginal cost and average cost normalized to zero. Consumers have identical valuation for the brown product. Thus, Bertrand price competition between conventional firms drives profits down to zero.[6]

A firm can command a price premium over its product by switching to green production. However, the decision of whether to switch is unobservable and a firm uses price to signal green quality to consumers. A firm incurs the setup cost F one period before engaging in green production; F includes at least the fee paid to have the product certified.[7] We normalize quality so that $c(e) = e$. The assumption of constant return to scale implicitly means that firms have access to perfectly elastic supplies of all the factors needed for their activity.

The third-party auditor conducts monitoring in an imperfect but honest way. The auditor tests the green product and succeeds in learning the correct type of a firm with probability $\alpha, 0 \leq \alpha < 1$, which represents the level of accuracy in monitoring ($\alpha = 1$ would be the limit case of perfect monitoring, in which consumers get full information about the green product). For example, the auditor measures the polluting emissions generated by the product of a firm

[6]Alternatively, we could use the model of monopolistic competition developed by Salop (1979) to formalize business as usual in our setting. Under this framework, the market for the brown product is represented by a circle of unit length on which consumers are uniformly distributed. Each point on the circle corresponds to one consumer's most preferred variant of the brown product. A large number of firms are symmetrically located around the circle. Each firm produces a single variant of the brown product with an identical linear technology. There is free entry into the brown market so that firms continue to enter until profits are driven to zero. If t is the transport cost per unit of distance from an ideal location, the constant unit product cost is zero and f is the fixed cost of entry, the market price for the brown product in the symmetric equilibrium is $p_b = \sqrt{tf}$. These additional variables would make calculations more cumbersome and risk clouding the issue.

[7]Besides the certification fee, F includes unsalvageable expenditures in developing quality attributes and specific entrepreneurial skills, sunk investments in specialized machinery for green production, fuel-saving equipment or long-term rental contracts that cannot be resold.

claiming that it is green, and compares them with the standard required to be green: if the polluting emissions are observed to be lower than the standard, then the product passes the test, which indicates that the product is green with probability α.

The auditor's monitoring takes place over the sale period simultaneously with firms' pricing. The auditor's report is publicly available. Reading this report, consumers learn exactly what monitoring has revealed and they perfectly know its level of accuracy. This piece of information supplements that based on firms' price signaling. We assume that the two sources of information are independent. Consumers use both of them to make their purchase decision.

The firm $i \in N = \{1, 2, ..., n\}$ of type $t \in T = \{b, g\}$ sells its product at price p_{it}. Given that firms are completely symmetric at the beginning of the whole game, we assume that firms employ the same pricing rule in equilibrium, and so we can write $p_{it} = p_t$ for all $i \in N$.

The total number of consumers is normalized to unity. Each consumer has exogenous wealth of w and purchases at most one unit of either type of the product. Consumers have heterogeneous preferences similar to those in Mussa and Rosen (1978) : they differ according to a taste parameter x for the green quality, which is assumed to be continuously and uniformly distributed over the interval $[0, l]$. Thus, the indirect utility of a consumer with taste x for quality e_t, who purchases one unit of the type-t product at price p, is given by

$$V_t\left(p, e, x\right) = w + xe_t - p, \tag{1}$$

where $e_g = e$ and $e_b = 0$.

Market demands are determined from a critical value of the taste distribution. The preference level of the consumer who is indifferent between purchasing the green product and its brown substitute is found by setting

$$w + xe - p_g = w - p_b \tag{2}$$

and solving for x. Doing so yields

$$X = \min\{\frac{p_g - p_b}{e}, l\}. \tag{3}$$

All consumers with values of x that satisfy $x \geq X$ purchase the green product and the remaining consumers purchase the brown product. This implies linear demand functions for both products. One possibility in the green market equilibrium is that the cost of providing the green quality is so high that consumers have zero demand for the green product, even when it is sold at marginal cost. The following inequality ensures that demand for the green product is strictly positive when it is sold at marginal cost

$$l > 1. \tag{4}$$

The timing.—The whole game is a three-stage game that proceeds as follows:

(i) In the first stage, firms simultaneously choose their types. If a firm decides to switch to green production, it pays the setup cost F.

(ii) In the second stage, firms simultaneously set prices for the green product and the auditor releases its report. Consumers observe these actions. The market for the brown product clears at marginal cost.

(iii) In the third stage, consumers update their beliefs about the firms' types upon seeing prices, supplement this information with that conveyed by the auditor's report and, finally, decide from which firm to buy. Consumers use Bayes' rule whenever possible to form posterior beliefs from the observed prices and the auditor's report.

In stage 1, each firm $i \in N$ selects a type t from the set $T = \{b, g\}$ and may randomize over these pure strategies. Firms are committed to their technology decisions until the end of the game: they will not change their types in the next stages because the appropriate production facilities have a second-hand value lower than their initial value. The technology decision is private information to each firm: its type is unknown to every other protagonist (rival firms, consumers and the auditor). This implies that the setup cost F specific to the green production is not observable either. A mixed strategy for firm i, $\sigma_i : T \to [0, 1]$, assigns probability $\sigma_i(t)$ that it chooses the type t, where $\Sigma_{t \in T} \sigma_i(t) = 1$. Given that firms are completely symmetric and share the same information at the beginning of the game, we can simplify the notation $\sigma_i(g)$ and write σ instead; that is, the fraction of firms that switch to green production.

In stage 2, the setup cost F is sunk, if ever. The probability distribution σ is the only statistics for past play, and hence represents the prior beliefs of the uninformed protagonists at the beginning of stage 2. The sequential order between the decisions regarding technology and price captures the notion that a price can in practice be varied at will, unlike the commitment to a type. The subgame starting at stage 2 works like a standard signaling game in the spirit of Spence (1973).

In stage 3, consumers draw inferences about the firm's types and cross-check this information with that released by the auditor's report. Finally, consumers make their purchase decisions to maximize their expected payoffs, given their posterior beliefs. The payoff to a consumer is her expected net surplus if she buys, and zero otherwise. The payoff to each firm is its expected profits.

We can focus on firms' strategic interplay in the green market due to the assumption of zero profits (perfect competition) in the brown market. We require that strategies in the green market form a Perfect Bayesian Equilibrium (PBE); that is, strategies must yield a Bayesian equilibrium not only for the whole game, but also for every subgame, including that starting after any possible choice of a type made by firms. In addition, we will require that equilibrium beliefs off the equilibrium path satisfy the Cho-Kreps intuitive criterion (Cho and Kreps, 1987).

3 The price signaling game with and without monitoring

To address the issue of price signaling in stage 2, we focus on pure-strategy separating equilibria $(p_t^*)_{t \in T}$ such that $p_b^* \neq p_g^*$ and consumers' perception of the firms' types is correct after observing prices. The separating pricing rule used by every firm in equilibrium predicts that a firm will charge the equilibrium price p_b^* with probability $1-\sigma$ and the equilibrium price price p_g^* with probability σ. Furthermore, if a brown firm is perceived as such, its best response is to sell at marginal cost $p_b^* = 0$.

The focus on pure-strategy separating prices will ensure that green certification is credible in equilibrium. Following Mahenc (2017), the credibility of green certification requires separating prices in the green market, and conversely, pooling prices (in which the firm's price is independent of its true type) undermine the reliability of certification. Were prices to pool in the green market, information would be concealed by market prices, which would make the information disclosed by the green label inconsistent.

Consumers' perception of the firms' types builds on information from two sources: firms' prices and the auditor's report. We assume that the information released by firms through price is *independent* from that released by the auditor's monitoring. Therefore, we treat the two events {the firm is green conditional on observing price} and {the firm is green conditional on reading the auditor's report} as being independent (see Appendix 1 for a formal definition).

To formalize consumers' perception of a firm's type based on price, we define a posterior belief function $\mu(t|p) : T \times R^+ \to [0,1]$, that specifies the probability assigned to either firm of being type t after observing the price p in the market for the green product. This belief function is the same function for all firms. Along the equilibrium path, consumers use Bayes' rule to update beliefs from the prior distribution σ. When consumers observe a price off the equilibrium path, they update beliefs with an arbitrary rule instead of Bayes' rule. We simplify the notation $\mu(g|p)$ and write $\mu(p)$ instead, so that $\mu(b|p) = 1 - \mu(p)$.

If consumers' inference process yields posterior beliefs $\mu = \mu(p)$, the expected value consumers infer from price p is denoted $e(\mu) = \mu e$. Consumers supplement their information about a firm's type with the auditor's report. From Appendix 1, the quality expected by consumers in the market for the green product is given by the equation

$$\widetilde{e}_t(\mu) = \mu e + \alpha(e_t - \mu e) = \begin{cases} \mu e + \alpha(1-\mu)e \text{ if } t = g, \\ \mu e - \alpha \mu e \text{ if } t = b. \end{cases} \tag{5}$$

Equation (5) shows how consumers combine the information inferred from price with that provided by monitoring to form their perception of environmental quality. This combination depends on whether consumers use Bayes' rule or an arbitrary rule to update beliefs from prices. Recall that prices reveal the truth with Bayes' rule while they may be misleading with an arbitrary updating. In the latter case, monitoring cannot be misleading too if the auditor is assumed to be independent from firms. In the present setting, monitoring supports the beliefs updated after observing prices on the equilibrium path and corrects the beliefs updated off the equilibrium path. If the reading of the auditor's report provides no information ($\alpha = 0$), then consumers' perception relies only on the inferences from prices. If the auditor's monitoring is perfectly accurate ($\alpha = 1$), then consumers fully learn the correct type. On the equilibrium path, prices reveal the truth since consumers use Bayes' rule to update beliefs from prices. From equation (5), monitoring does not change these beliefs and hence consumers' perception of the firms' type is correct; that is, $\widetilde{e}_g(1) = e$ and $\widetilde{e}_b(0) = 0$. Off the equilibrium path, any price is a zero probability event. Observing such a "surprising" price, consumers use an arbitrary rule to update beliefs. In that case, monitoring provides information by correcting consumers' arbitrary perception according to the formula (5).

In any separating equilibrium, consumers have correct perception of the firms' types after observing p_b^* and p_g^*; that is, $\widetilde{e}_b(\mu(p_b^*)) = 0$ and $\widetilde{e}_g(\mu(p_g^*)) = e$. In the market for the green product, firms maximize expected profits with respect to price, given the auditor's report and the consumers' beliefs function. We define $\Pi_{it}(p, \mu)$ (resp. $D_{it}(p, \mu)$) as the expected profits (resp. demand) for firm i of the actual type t, perceived to be green with probability μ after observing the price p. In any pure-strategy separating PBE, there is a fraction σ of green firms that simultaneously set price. Henceforth, we consider, without loss of generality, that firm 1 represents the group of firms that have switched to green production green and firm 2 will represent every other firm that sticks to brown production.

With this convention, consumers' expected valuation for firm 2's product is $\widetilde{e}_b(\mu(p_b^*)) = 0$ in equilibrium, which yields the expected profit $\Pi_{2b}(p_b^*, 0) = 0$. As firms' type is unobservable, firm 2 might be tempted to pass off as green by selling its product at price p_g^* in the market for the green product. This will not happen in a separating equilibrium where, by definition, the price signal cannot be faked.

3.1 Firm 2's temptation to cheat

Asymmetric information provides the fraction $1 - \sigma$ of firm 2 with an incentive to cheat consumers by passing off as green. Claiming to offer the green product, firm 2 can charge the price p_g^* upon which consumers infer $\mu(p_g^*) = 1$. From (5), consumers' perception of firm 2's product is $\widetilde{e}_b(\mu(p_g^*)) = (1 - \alpha)e$. Firm 2 deals with the contingencies stemming from the separating pricing rule. On one hand, firm 1 may charge the price p_b^* with probability $1 - \sigma$, and on the other hand, firm 1 may charges the price p_g^* with probability σ. In the former event, the rival firm is perceived as being brown for sure and firm 2 gains market power with a differentiated product. Then, demand for the spurious product is equally divided between $(1 - \sigma) n$ brown firms. However, the cheating strategy fails in the event where firm 1 charges the price p_g^*. Indeed, consumers correctly value the product sold by firm 1, inferring that $\widetilde{e}_g(\mu(p_g^*)) = e$, which strictly exceeds $\widetilde{e}_b(\mu(p_g^*))$ for $\alpha > 0$. It turns out that the auditor's monitoring in the green market helps consumers differentiate products correctly. In contrast, with no monitoring ($\alpha = 0$), the two products sold in the green market look the same in consumers' eyes when firm 2 is cheating. As no product differentiation can be detected, demand is equally divided between n firms.

Firm 2's expected demand from cheating is given by

$$D_{2b}\left(p_g^*,1\right)=\begin{cases}(1-\sigma)\,\frac{1}{(1-\sigma)n}\frac{l(1-\alpha)e+p_b^*-p_g^*}{l(1-\alpha)e} & \text{if } \alpha>0,\\ (1-\sigma)\,\frac{1}{(1-\sigma)n}\frac{le+p_b^*-p_g^*}{le}+\sigma\frac{1}{n}\frac{le-p_g^*}{le} & \text{if } \alpha=0.\end{cases} \tag{6}$$

Cheating gives firm 2 the following expected profits:

$$\Pi_{2b}\left(p_g^*,1\right)=\begin{cases}p_g^*\frac{l(1-\alpha)e-p_g^*}{nl(1-\alpha)e} & \text{if } \alpha>0,\\ (\sigma+1)\,p_g^*\frac{le-p_g^*}{nle} & \text{if } \alpha=0.\end{cases} \tag{7}$$

Everything else being equal, the cheating profit decreases in α[8]. Hence, the more accurate the auditor's report, the lower is the temptation to cheat for the brown type. To prevent firm 2 from passing off as green, the separating price p_g^* must satisfy the following credibility constraint:

$$\Pi_{2b}\left(p_b^*,0\right)\geq\Pi_{2b}\left(p_g^*,1\right). \tag{8}$$

This condition guarantees that the equilibrium price in the green market is high enough to deter mimicry by firm 2, which then reverts to p_b^*. As the left-hand side of (8) is zero, firm 2 has nothing to lose from tricking consumers into buying at a positive price. The only way to deter cheating is to set p_g^* at a level that reduces firm 2's expected demand to zero. As a result, the credibility constraint (8) determines a lower bound for the equilibrium price, $\overline{p}_b$, which must satisfy

$$p_g^*\geq\overline{p}_b, \tag{9}$$

where

$$\overline{p}_b=l(1-\alpha)e. \tag{10}$$

Lemma 1: *In any separating equilibrium, green firms must signal their product with prices p_g^* above the threshold $\overline{p}_b$.*

Several candidates for p_g^* may satisfy (9). In order to avoid this familiar problem of multiplicity, we will focus on the least costly separating equilibrium, which is the only PBE robust to the Cho-Kreps intuitive criterion (Cho and Kreps, 1987). This outcome seems reasonable because it is supported by the price that maximizes the profit of green firms among all separating equilibria.[9]

[8]One can check that $\frac{\partial\Pi_{2b}\left(p_g^*,1\right)}{\partial\alpha}=-\frac{1}{n}\frac{{p_g^*}^2}{le(1-\alpha)^2}<0$.

[9]In the present setting, one can check that any separating equilibrium other than the least costly one requires to signal the green type at a higher price level. At such a price, green firms find it more profitable to deviate to the least costly separating price because their brown counterpart cannot profit from this deviation for any belief. Therefore, any separating equilibrium other than the least costly one fails the Cho-Kreps intuitive criterion.

3.2 Firm 1's incentive to signal the green product

In a separating equilibrium, firm 1 charges p_g^* and consumers' perception of quality based on this price is correct; that is, $\widetilde{e}_g(\mu(p_g^*)) = e$. From the separating pricing rule, firm 1 predicts that firm 2 sells the brown product at price p_b^* with probability $1-\sigma$. In this event, product differentiation confers market power on firm 1 and demand for the green product is divided between σn firms. In the event that firm 2 sets the price p_g^*, which occurs with probability σ, consumers correctly infer that firm 1's product is more valuable than firm 2's product sold at the same price, provided that $\alpha > 0$; indeed, consumers' perception of firm 2's product is $\widetilde{e}_b(\mu(p_g^*)) = (1-\alpha)e < e$. The auditor's monitoring allows firm 1 to enjoy market power. This cannot happen in the absence of monitoring because firm 2's cheating removes differentiation between products.

When firm 1 charges p_g^*, firm 1's expected demand is

$$D_{1g}\left(p_g^*, 1\right) = \begin{cases} (1-\sigma)\frac{1}{\sigma n}\frac{le+p_b^*-p_g^*}{le} + \sigma\frac{1}{\sigma n}\frac{le-p_g^*}{le} & \text{if } \alpha > 0, \\ (1-\sigma)\frac{1}{\sigma n}\frac{le+p_b^*-p_g^*}{le} + \sigma\frac{1}{n}\frac{le-p_g^*}{le} & \text{if } \alpha = 0. \end{cases} \tag{11}$$

From this demand, we can write firm 1's expected profits as follows

$$\Pi_{1g}\left(p_g^*, 1\right) = \begin{cases} \frac{\left(p_g^*-e\right)\left(le-p_g^*\right)}{\sigma nle} & \text{if } \alpha > 0, \\ \frac{1-\sigma+\sigma^2}{\sigma nle}\left(p_g^*-e\right)\left(le-p_g^*\right) & \text{if } \alpha = 0. \end{cases} \tag{12}$$

Without the threat of cheating by firm 2, firm 1 would maximize profits (12) by setting the price $p_{1g}(1) = \frac{e(1+l)}{2}$ and the resulting profits would be

$$\Pi_{1g}\left(p_{1g}(1), 1\right) = \begin{cases} \frac{1}{\sigma n}\frac{e(l-1)^2}{4l} & \text{if } \alpha > 0, \\ \frac{e\left(1-\sigma+\sigma^2\right)(l-1)^2}{4\sigma nl} & \text{if } \alpha = 0. \end{cases} \tag{13}$$

Given that $p_b^* = 0$, the price $p_{1g}(1)$ coincides with the monopoly price that would be charged for the green product under full information.[10] Observe that $p_{1g}(1)$ may be too low to satisfy the credibility constraint (9), which occurs when $p_{1g}(1) < \overline{p}_b$. In this case, signaling the green type involves an upward distortion in price relative to the full-information monopoly price, which includes firm 2's forgone profit from cheating. Hence, firm 1 must bear an additional strategic cost arising from the necessity to deter firm 2 from mimicking the price of the green product. This signaling cost has been previously identified by Mahenc (2007 and 2008) and Sengupta (2012) in the monopoly market for a credence good. Both authors show that an upward-distorted price may signal high environmental quality as long as the good of low quality is cheaper to produce than that of high quality. The same logic applies to price signaling quality in markets for experience goods. In Bagwell and Riordan (1991), a monopolist signals high quality by raising price up to the level where the loss of

[10] This would not be the case if p_b^* were strictly positive; then firm 1's best response would be $p_{1g}(1) = \begin{cases} \frac{e(1+l)}{2} + p_b^*\frac{(1-\sigma)}{2} & \text{if } \alpha > 0, \\ \frac{e(1+l)}{2} + p_b^*\frac{1-\sigma}{2(1-\sigma+\sigma^2)} & \text{if } \alpha = 0. \end{cases}$.

sales for the low quality type is not worth the rent from cheating. Thus, the cost of signaling high quality is determined by the forgone rent from cheating.

If $p_{1g}(1) < \overline{p}_b$, the least costly separation is achieved by setting the price $\overline{p}_b$ for the green product, yielding the following expected profits:

$$\Pi_{1g}(\overline{p}_b, 1) = \frac{\alpha e(l(1-\alpha)-1)}{\sigma n}. \tag{14}$$

Straightforward calculations give $\frac{\partial \Pi_{1g}(\overline{p}_b,1)}{\partial \alpha} = \frac{e[(l-1)-2l\alpha]}{\sigma n}$, which is positive for all $\alpha < \overline{\alpha}$. Hence, more accurate monitoring strengthens the green firm's incentive to signal its type with an upward distortion in price.

The minimum signaling cost is measured by the price differential

$$\overline{p}_b - p_{1g}(1) = \frac{e}{2}(l-1) - el\alpha. \tag{15}$$

Observe that the presence of monitoring moderates the distortionary effects of signaling. It is unsurprising that the signaling cost decreases as the auditor's monitoring accuracy increases because the cheating profit $\Pi_{2b}\left(p_g^*, 1\right)$ decreases in α, as previously mentioned. Let $\overline{\alpha} = \frac{l-1}{2l}$ be the critical accuracy level of monitoring at which $\overline{p}_b = p_{1g}(1)$. When α falls short of $\overline{\alpha}$, green firms must incur a signaling cost to prevent brown firms from cheating. For the sake of clarity, we henceforth refer to $\alpha > \frac{1}{2}$, $\alpha \in \left[\overline{\alpha}, \frac{1}{2}\right]$ and $\alpha < \overline{\alpha}$, respectively, as "high" "intermediate" and "low"accuracy for monitoring.

Lemma 2: *In any separating equilibrium, the least costly way of signaling the green type is to distort price upward relative to what a monopolist would do under full information when the accuracy for monitoring is low, and otherwise to set the full information monopoly price; that is,*

$$p_g^* = \begin{cases} \overline{p}_b \text{ if } \alpha < \overline{\alpha}, \\ p_{1g}(1) \text{ otherwise.} \end{cases} \tag{16}$$

When $\alpha < \overline{\alpha}$, the threat of cheating posed by brown firms entails a positive signaling cost for green firms. The price distortion allows green firms to prove that they are less reluctant than their brown rivals to restrict sales volume.

3.3 Price signaling without monitoring: the case $\alpha = 0$

As a benchmark, let us now consider price signaling without monitoring; that is, $\alpha = 0$. Lemma 2 states that firm 1 must signal the green product with a price p_g^* no lower than $\overline{p}_b = le$ in order to deter firm 2 from cheating consumers. Looking at firm 1's expected profits given by(12) when $\alpha = 0$, we see that $p_g^* = \overline{p}_b$ is the minimum price above which firm 1's expected sales volume falls down to zero. Thus, signaling costs are so high that firm 1 has no incentive to deter firm 2 from mimicking p_g^*. In a nutshell, credible price signaling is too demanding to be attractive with no monitoring. As a result, no firm will pay the setup cost F to switch to green production.

Corollary 1: *In the absence of auditor's monitoring, there exists no separating equilibrium in which the green firm can credibly signal its type. As green certification cannot be credible, no firm has an incentive to switch to green production.*

The problem of adverse selection suggested by Akerlof (1970) arises without monitoring. There is a market breakdown for the green product because green firms fail to signal their type. The reason is that the cost of signaling with prices higher than $\overline{p}_b$ is the same for both types of firms. Above the threshold $\overline{p}_b$, there is no way to separate brown and green firms because the usual single-crossing condition on profits does not hold, which reflects here that the cost of signaling is inversely related to firms' production costs.

Corollary 1 is closely related to the moral hazard problem initially pointed out by Klein and Leffler (1981): a firm may refrain from producing high quality products because it would lose all consumers at the minimum price needed to signal high quality. Klein and Leffler show that the threat of losing future business is likely to prevent a firm from reneging on its promise to enhance product quality. Unlike their analysis, the present model highlights the role played by monitoring rather than reputation in solving the moral hazard problem.

3.4 Price signaling with monitoring: the case $\alpha > 0$

We now introduce the auditor to the previous benchmark and we allow for the possibility of imperfect but honest monitoring. A key difference with the benchmark is that the auditor's monitoring allows consumers to differentiate products when the brown firm tricks them into buying at price p_g^*. Besides the credibility constraint (8), separation of the types places the further requirement on green firms that they should prefer to signal their true type than to pass off as the brown type.

Suppose that consumers are confident that firm 1 is brown after observing a price p off the equilibrium path; that is, $\mu(p) = 0$, which yields consumers' perception of its product $\widetilde{e}_g(0) = \alpha e$. Faced with so pessimistic beliefs, firm 1 expects profits $\Pi_{1g}(p, 0)$ calculated on the basis of the separating rule. In the event that firm 2 charges the price p_b^* with probability $1 - \sigma$, consumers' perception of firm 2's type is correct, i. e., $\widetilde{e}_b(0) = 0$. Even though firm 1's product is mistaken for the brown product, monitoring helps consumers differentiate, at least slightly, the rival products. Alternatively, firm 2 charges the price p_g^* with probability σ, which entails consumers' perception of its product $\widetilde{e}_b(1) = (1 - \alpha)e$. Indeed, consumers infer from p_g^* that firm 2's type is green with probability 1 and monitoring supports this belief with probability $1 - \alpha$. Then, in consumers' eyes, whether firm 1's product is more or less valuable than firm 2's product depends on whether α is higher or lower than $\frac{1}{2}$, respectively. When consumers' perception is wrong, firm 1's expected demand is given by

$$D_{1g}(p, 0) = \begin{cases} (1-\sigma)\frac{1}{\sigma n}\frac{l\alpha e - p}{l\alpha e} + \sigma\frac{1}{\sigma n}\frac{l(2\alpha-1)e - p + p_g^*}{l(2\alpha-1)e} \text{ if } \alpha \geq \frac{1}{2}, \\ (1-\sigma)\frac{1}{\sigma n}\frac{l\alpha e - p}{l\alpha e} + \sigma\frac{1}{\sigma n}\frac{p_g^* - p}{l(1-2\alpha)e} \text{ if } \alpha \in \left(0, \frac{1}{2}\right). \end{cases} \tag{17}$$

As previously mentioned, monitoring corrects consumers' misperceptions of firm 1's product after observing a price off the equilibrium path. The expression of demand above shows that monitoring provides firm 1 with market power on the market for the brown product when firm 2 is correctly identified. Moreover, the effect of monitoring depends on the accuracy level α when firm 2 is cheating on the market for the green product. Within the range $\left[0, \frac{1}{2}\right]$, consumers find firm 1's product less valuable than the rival product and product differentiation decreases with the accuracy level of monitoring. In contrast, when monitoring accuracy is high, i. e., $\alpha \geq \frac{1}{2}$, firm 1's product becomes more valuable than the rival product and product differentiation increases with α.

Let $p_{1g}(0)$ be firm 1's best response when it is mistaken for brown off the equilibrium path on the basis of price alone. Since firms' prices are strategic complements, an increase in p_g^* encourages firm 1 to increase $p_{1g}(0)$.[11] Thus, an upward distortion in p_g^* will soften price competition off the equilibrium path between green firms and their fraudulent rivals. Moreover, $p_{1g}(0)$ should increase with the product differentiation entailed by monitoring. In particular, when monitoring accuracy is high ($\alpha \geq \frac{1}{2}$), firm 1 will respond to firm 2's cheating by setting $p_{1g}(0)$ far above the price p_g^*. We denote the resulting profits by the reduced-form function $\widetilde{\Pi}_{1g}(\sigma) = \Pi_{1g}(p_{1g}(0), 0)$: it represents the spectrum of best worst-outcomes obtained by firm 1 when it defects.

Firm 1 may be discouraged from signaling its true type with p_g^* if it earns more by deviating to the price $p_{1g}(0)$. To prevent firm 1's defection, the price p_g^* should satisfy the following constraint

$$\Pi_{1g}\left(p_g^*, 1\right) \geq \widetilde{\Pi}_{1g}(\sigma). \tag{18}$$

This condition guarantees that it is worthwhile for firm 1 to use the signal p_g^* rather than to pass off as brown on the basis of price alone. The right-hand side of (18) can be interpreted as firm 1's opportunity cost of signaling its true type. Depending on product differentiation, monitoring accuracy may raise this cost, which makes it harder for firm 1 to reveal the truth about its type.

Condition (18) is necessary and sufficient for the existence of a least costly separating equilibrium with $p_g^* \geq \overline{p}_b$, supported by the beliefs $\mu^*(p) = 0$ when $p < p_g^*$, and $\mu^*(p) = 1$ when $p \geq p_g^*$. We thus have the following existence proposition:

Proposition 1: *There exists a least costly separating equilibrium consisting of* $p_g^* = \max\{p_{1g}(1), \overline{p}_b\}$ *if and only if* p_g^* *satisfies* (18).

In order to illustrate the existence result by way of an example, consider the case $\sigma = \frac{1}{2}$. Figure 1 maps out the best-worst profit $\widetilde{\Pi}_{1g}\left(\frac{1}{2}\right)$ obtained off the equilibrium path, as a function (in blue) of α, given $l = 3$. Knowing *a priori* that half of firms have chosen to switch to green production, firm 1 expects firm 2 either to truthfully signal its brown type or to

[11] One can check that $\frac{\partial^2 \Pi_{1g}(p,0)}{\partial p \partial p_g^*} = \begin{cases} \frac{1}{nl(2\alpha-1)e} \text{ if } \alpha \geq \frac{1}{2}, \\ \frac{1}{nl(1-2\alpha)e} \text{ if } \alpha \in \left(0, \frac{1}{2}\right), \end{cases}$ which is positive in both cases.

cheat consumers by charging p_g^* with the same probability $\frac{1}{2}$. Figure 1 also depicts firm 1's expected profits $\Pi_{1g}\left(p_g^*, 1\right)$ along the equilibrium path, as a function (in black) of α. The level of monitoring accuracy $\overline{\alpha} = \frac{1}{3}$ is the threshold below which there is a cost to signaling the green type.

Appendix 2 presents the calculation results for firm 1's outcomes.

Figure 1 shows that there is an interval of α within which requirement (18) for the existence of the least cost separating equilibrium is met.

Insert Figure 1 here: *Best-worst and least-costly profits when* $\sigma = \frac{1}{2}$.

When $\alpha < \overline{\alpha}$, monitoring accuracy is low and firm 1 must distort the separating price upward to signal the green product, thereby losing sales volume. Charging $p_g^* = 3(1-\alpha)e$, firm 1 makes profit $\pi_{1g}\left(p_g^*, 1\right)$ which increases with α because a better accuracy in monitoring reduces the signaling cost. If firm 1 deviates from p_g^*, consumers believe it to be brown off the equilibrium path on the basis of price alone, and then firm 1 is better off attracting consumers with the price $p_{1g}(0) = \frac{e\left(1+5\alpha-9\alpha^2\right)}{2(1-\alpha)}$ far below p_g^*. Figure 1 shows that firm 1's opportunity cost of revealing the truth $\widetilde{\Pi}_{1g}\left(\frac{1}{2}\right)$ is a convex function of α over the interval $\left[0, \frac{1}{2}\right]$, with a minimum at $\alpha = 0,19$ lower than $\overline{\alpha}$. Firm 1's profit $\Pi_{1g}\left(p_g^*, 1\right)$ exceeds $\widetilde{\Pi}_{1g}\left(\frac{1}{2}\right)$ at some level $\alpha = 0,08$ above which it becomes worthwhile to signal the green product. Thus, price signaling cannot be effective unless a minimum level of monitoring accuracy is met.

When $\overline{\alpha} \leq \alpha \leq \frac{1}{2}$, monitoring accuracy is intermediate. In that case, there is no longer a need to distort price for signaling the green type and firm 1 charges $p_g^* = 2e$. With zero cost of signaling, monitoring accuracy has no impact on firm 1's profit. In contrast, the best-worst profit $\widetilde{\Pi}_{1g}\left(\frac{1}{2}\right)$ is increasing with α to a level $0,4$ above which signaling the green type is no longer worthwhile. Again, when consumers falsely perceive firm 1 to be brown off the equilibrium path on the basis of price alone, it sells its product at price $p_{1g}(0) = \frac{e\left(1+4\alpha-6\alpha^2\right)}{2(1-\alpha)}$ below p_g^*, while firm 2 cheats consumers with the price p_g^*. As monitoring slightly corrects consumers' misperceptions off the equilibrium path, firm 1 can earn positive profits even though it passes off as brown. However, after reading the auditor's report, consumers find the truly green product less valuable than the spurious one sold at p_g^*. When α approaches $\frac{1}{2}$, the rival products become close substitutes in consumers' eyes, and firm 1 takes the lion's share of the market by attracting consumers with a lower price off the equilibrium path.

When $\alpha > \frac{1}{2}$, monitoring accuracy is high. Figure 1 shows that $\pi_{1g}\left(p_g^*, 1\right)$ falls short of $\widetilde{\Pi}_{1g}\left(\frac{1}{2}\right)$ over the interval $\left[\frac{1}{2}, 1\right]$. Thus, firm 1 fails in signaling the green product because it earns more profit when consumers falsely perceive its product to be brown off the equilibrium path. Nevertheless, monitoring helps consumers differentiate products by correcting their misperceptions off the equilibrium path. Furthermore, consumers now find the truly green product more valuable than the spurious one. Being taken for a brown firm off the equilibrium path on the basis of price alone, firm 1 charges $p_{1g}(0) = \frac{e}{2}(1+4\alpha)$ while firm 2 charges $p_g^* = 2e$. Product differentiation increases with α, which softens price competition. Above some threshold level of monitoring accuracy, $p_{1g}(0)$ exceeds p_g^*. In some sense, monitoring induces firm 1 to respond less aggressively off the equilibrium path to firm 2's cheating, which is detrimental to price signaling.

Corollary 2: *Assume that $l = 3$ and $\sigma = \frac{1}{2}$. With imperfect monitoring, green firms credibly signal their type with $p_g^* = \max\{p_{1g}(1), \overline{p}_b\}$ if and only if $\alpha \in [0.08, 0.4]$.*

To sum up the lessons drawn from the case $\sigma = \frac{1}{2}$, monitoring helps consumers to correct misperceptions off the equilibrium path when brown firms are cheating. Reading the auditor's report, consumers can differentiate the truly green product from the spurious one. However, they value more the latter than the former when $\alpha < \frac{1}{2}$. Then, price competition between green firms and fraudulent brown firms reduces the profits green firms would make if they were mistaken for brown firms on the basis of price alone. When $\alpha \in [0.08, 0.4]$, green firms find it more profitable to correctly signal their type than to pass off as brown off the equilibrium path. Outside this interval of accuracy levels in monitoring, price signaling is not worth it, either because the price distortion is too costly, and/or the brown firms' mimicking strategy puts too much pressure on green firms to deter cheating.

Obviously, the signaling behavior of green firms depends not only on the accuracy level of monitoring but also on the probability σ which represents both the proportion of green firms and the probability of cheating. The next proposition establishes that there is some critical probability $\overline{\sigma}$ below which green firms can successfully signal their type. The best-worst profit $\widetilde{\Pi}_{1g}(\sigma)$ reaches a maximum at $p_{1g}(0, \sigma)$, where it takes the following values, depending on whether α is higher or lower than $\frac{1}{2}$,

$$\widetilde{\Pi}_{1g}(\sigma) = \begin{cases} \frac{\left(p_g^* \alpha\sigma + e\left(\sigma - 1 + \alpha(2 + l - \sigma(l+3)) + 2l\alpha^2(\sigma - 1)\right)\right)^2}{4le\alpha(2\alpha - 1)(1 - 2\alpha + \sigma(3\alpha - 1))\sigma n} & \text{if } \alpha \leq \frac{1}{2}, \\ \frac{\left(p_g^* \alpha\sigma - e\left(1 - \sigma + \alpha(\sigma - l - 2) + 2l\alpha^2\right)\right)^2}{4el\alpha(1 - 2\alpha)(1 - \sigma + \alpha\sigma - 2\alpha)\sigma n} & \text{otherwise.} \end{cases} \tag{19}$$

We know from Lemma 2 that the least costly price separating price is $p_g^* = p_{1g}(1)$ if $\alpha \geq \overline{\alpha}$, and $p_g^* = \overline{p}_b$ otherwise. Hence, we get different expressions of the best-worst profit $\widetilde{\Pi}_{1g}(\sigma)$ by substituting $p_{1g}(1)$ and $\overline{p}_b$ for p_g^* in (19). Due to the complexity of this algebra, we leave these details to the Appendix.[12] Setting $\widetilde{\Pi}_{1g}(\sigma) = 0$ defines a threshold $\underline{\sigma}$ for σ, above which the profit margin is positive for firm 1. It turns out that $\underline{\sigma} < 1$ for all $\alpha \in [0, 1]$. We then consider the set of σ such that $\Pi_{1g}\left(p_g^*, 1\right) = \widetilde{\Pi}_{1g}(\sigma)$; this equation has an upper root σ^+ higher than $\underline{\sigma}$, which may exceed 1 too. Let $\overline{\sigma} = \min\{\sigma^+, 1\}$ be the critical probability at which green firms are indifferent between signaling their type and passing off as brown. In Appendix 3, we show that the least costly separating equilibrium exists for any value of α provided that σ is sufficiently small.

Proposition 2: *Assume that $l \geq 2$. The least costly separating equilibrium exists if and*

[12] All the calculations and proofs for the scenario with monitoring can be found in Appendix 3 where we assume that $l \geq 2$ to reduce the number of cases to review.

only if $\sigma \leq \overline{\sigma}$. This equilibrium involves the following:

(i) *when monitoring accuracy is high, green firms incur no signaling cost and $\overline{\sigma} < 1$;*

(ii) *when monitoring accuracy is intermediate, green firms incur no signaling cost; $\overline{\sigma} < 1$ if $\alpha > \dfrac{3}{8}$ and $\overline{\sigma} = 1$ otherwise;*

(iii) *when monitoring accuracy is low, green firms incur a signaling cost to prevent brown firms from cheating; $\overline{\sigma} = 1$ if $\alpha \in [\alpha_0(l), \alpha_1(l)]$, where $\alpha_0(l) = \frac{5l-\sqrt{l(32-7l)}}{16l}$ and $\alpha_1(l) = \min\{\overline{\alpha}, \frac{5l+\sqrt{l(32-7l)}}{16l}\}$; $\overline{\sigma} < 1$ for any α outside this interval.*

Proof: see Appendix 3.

We find that monitoring solves the problem of adverse selection by restoring the credibility of price signaling. The information disclosed by the auditor helps consumers differentiate, at least slightly, between the truly green product and its spurious substitute. As a result, brown firms get penalized for cheating consumers by setting high prices. As previously mentioned, more accurate monitoring reduces the signaling cost until it is fully eliminated when accuracy is moderate or high.

Insert Figure 2 here: *Existence of separating equilibria.*

Figure 2 divides the (l, α) space into three regions, in which the least costly separating equilibrium exists.

In Region 1, monitoring accuracy is high. There is no need for green firms to distort price upward in order to deter brown firms from cheating. Thus, the full-information monopoly price $p_{1g}(1)$ is the least costly way of signaling the green type. Separation can occur only if the proportion of green firms does not exceed the threshold $\overline{\sigma}$. When σ exceeds $\overline{\sigma}$, green firms prefer to pass off as brown instead of signaling their type because they benefit from soft competition against their fraudulent rivals.

In Region 2, monitoring accuracy is moderate but sufficient to reduce the signaling cost to zero for green firms. When $\alpha > \max\{\overline{\alpha}, \dfrac{3}{8}\}$, the existence conditions of the least cost separating equilibrium are similar to those in Region 1: green firms signal their type with $p_g^* = p_{1g}(1)$ provided that $\sigma \leq \overline{\sigma}$. In this parameter configuration, the best-worst profit $\widetilde{\Pi}_{1g}(\sigma)$ may be very high when α is close to $\frac{1}{2}$. Green firms respond aggressively to brown firms' cheating by charging the price $p_{1g}(0)$ off the equilibrium path price far below p_g^* to steal business. When the proportion of green firms exceeds the threshold $\overline{\sigma}$, they find it more profitable to be mistaken for their brown counterpart than signaling their type. When α falls below $\dfrac{3}{8}$ (green area), $\widetilde{\Pi}_{1g}(\sigma)$ decreases because green firms respond less aggressively to their fraudulent rivals and lose sales volume. This weakens the incentives for green firms to defect from equilibrium, even though σ is close to 1, and hence green firms are almost sure that brown firms are cheating consumers. As a result, the least costly separating equilibrium exists regardless of σ.

In Region 3, monitoring accuracy is low. Green firms need to incur a signaling cost to prevent cheating. The most efficient way to signal their type is to charge the upward-distorted price $\overline{p}_b > p_{1g}(1)$. This is a credible strategy because green firms are less affected than their brown rivals by the consequent loss of sales volume. Moreover, as long as α lies inside $[\alpha_0(l), \alpha_1(l)]$ (green area), green firms are better off signaling their true type than passing off as brown off the equilibrium path regardless of σ. When α is outside the green area, the cheating strategy is less aggressive since the price to mimic is higher. Consequently, the best-worst profit $\widetilde{\Pi}_{1g}(1)$ increases to the point that green firms are worse off with the price $\overline{p}_b$ than with the lower price $p_{1g}(0)$ off the equilibrium path. Given α, the probability σ must fall short of $\overline{\sigma}$ to reduce $\widetilde{\Pi}_{1g}(\sigma)$ enough for green firms to separate with $\overline{p}_b$.

4 Firms' choice of the product type

We now examine how firms choose their type at the first stage of the whole game. In this stage, firms simultaneously decide wether or not to switch to green production. We assume that there are no barriers to switching to green production other than fixed costs. Consequently, the equilibrium profit of switching is zero.

In the price signaling game starting at the second stage, we have characterized the least costly separating equilibrium among all the symmetric PBE based on the prior probability σ that a firm is of the green type. Firms use the information displayed by σ to make subsequent predictions about their rivals' pricing. This information is derived from that publicly available at the end of the first stage, in which each firm chooses a mixed strategy from the set of probability distributions over T. The randomization over types summarizes public uncertainty about what each firm does at the first stage. As there is nothing that can alter beliefs between the end of stage 1 and the beginning of stage 2, we assume that the information summarized by the mixed strategies over types is the same as that displayed by σ.

In stage 1, firm i's mixed strategy is the belief that a rival $j \neq i$ will play the pure strategies $\{b, g\}$ with the probabilities $(1-\sigma_j, \sigma_j)$. As previously shown, positive profits can only be expected by switching to green production and successfully signaling this choice with the least costly pair of separating equilibrium prices $\left(p_b^*, p_g^*\right)$. From Proposition 2, we know that firm i can only resort to such a credible signaling if $\sigma_j \leq \overline{\sigma}$.

Firm i's expected profits from playing a mixed strategy are the weighted sum of the expected profits for each of the pure strategies $\{b, g\}$, where the weights are the probabilities $(1-\sigma_i, \sigma_i)$:

$$\Pi_i(\sigma_i, \sigma_j) = \begin{cases} (1-\sigma_i)\,\Pi_{ib}\left(p_b^*, 0\right) + \sigma_i\left[\Pi_{ig}\left(p_g^*, 1\right) - F\right] \text{ if } \sigma_j \leq \overline{\sigma}, \\ 0 \text{ otherwise.} \end{cases} \quad (20)$$

Our goal is now to characterize firm i's mixed strategies given by the probabilities $(1-\sigma_i^*, \sigma_i^*)$ of choosing a type from T, and separating equilibrium prices $\left(p_b^*, p_g^*\right)$ in the signaling game. In the signaling stage, no firm must want to change its decision about its type given the decision made by the other firms about their type. This requirement must be met when firms anticipate that prices will subsequently reveal their true type. From Proposition 2, we know that credible price signaling occurs provided that $\sigma_j \leq \overline{\sigma}$.

Given that firms are completely symmetric at the beginning of the game, we want to characterize a symmetric Nash equilibrium, wherein all firms choose the same mixed strategy $(1-\sigma, \sigma), \sigma \in [0,1]$, and are free to switch to green production. The mixed-strategy profile σ^* is a Nash equilibrium in which firm i chooses to switch to green production with probability σ^* if and only if σ^* is a best response to the other firms' equilibrium mixed strategies σ^*. For every $\sigma_i \in (0,1)$,

$$\Pi_i\left(\sigma_i^*, \sigma^*\right) \geq \Pi_i\left(\sigma_i, \sigma^*\right). \tag{21}$$

Whatever firm i might think about a rival's choice in the first stage, firm i's expected profits from sticking to brown production is zero, hence $\Pi_{ib}\left(p_b^*, 0\right) = 0$.

If firm i now chooses to switch to green production, it pays the setup cost F and earns expected profits $\Pi_{ig}\left(p_g^*, 1\right) - F$, where $\Pi_{1g}\left(p_g^*, 1\right)$ is given by (14). Thus, if $\sigma \leq \overline{\sigma}$, we can write

$$\Pi_i\left(\sigma_i, \sigma\right) = \begin{cases} \frac{e(l-1)^2}{4ln} - F \text{ if } \alpha \geq \overline{\alpha}, \\ \frac{\alpha e(l(1-\alpha)-1)}{n} - F \text{ if } 0 < \alpha < \overline{\alpha}. \end{cases} \tag{22}$$

We can now construct firm i's best response function to any $\sigma \in [0,1]$. If $\sigma \leq \overline{\sigma}$ then firm i's unique best response is $t = g$, whereas if $\sigma > \overline{\sigma}$, then firm i's unique best response is $t = b$ because it would incur the loss of F by changing its decision to switch to green production.

Insert Figure 3 here: *Best response functions and Nash equilibrium.*

The best response functions of two firms are depicted in Figure 3. The Nash equilibria correspond to the points at which the best response functions intersect. There are three Nash equilibria: two pure in which $(1-\sigma^*, \sigma^*) = (0,1)$ and $(1,0)$, and one mixed in which $(1-\sigma^*, \sigma^*) = (\overline{\sigma}, \overline{\sigma})$. In the first pure equilibria, firms do not randomize and all of them choose the same type of production.

The mixed-strategy Nash equilibrium is the most interesting one. From Proposition 2, we must consider two cases depending on whether $\overline{\sigma}$ is lower or equal to 1.

First, if $\alpha \notin [\alpha_0(l), \alpha_1(l)]$, then $\overline{\sigma} < 1$. In this case, a firm's best response function is

$$\sigma^*(\sigma) = \text{any } \sigma \in [0,1] \text{ if } \sigma = \overline{\sigma}. \tag{23a}$$

Second, if $\alpha \in [\alpha_0(l), \alpha_1(l)]$, then $\overline{\sigma} = 1$, in which case equilibrium strategies are pure and all the firms switch to green production.

As firms are free to switch to green production, the number of firms n adjusts at the beginning of the game until there are no excess profits for green firms; that is, $\Pi_i\left(\sigma_i, \sigma\right) =$

$0, i \in N$. This zero-profit condition together with (20) determines the equilibrium number of firms. Ignoring the integer constraint, we obtain:

$$n^* = \begin{cases} \frac{e(l-1)^2}{4lF} \text{ if } p_g^* = p_{1g}(1), \\ \frac{\alpha e(l(1-\alpha)-1)}{F} \text{ if } p_g^* = \overline{p}_b. \end{cases} \tag{24}$$

Straightforward calculations yield $\frac{\partial n^*}{\partial \alpha} = \frac{l(1-2\alpha)-1}{F} > 0$ for all $\alpha < \overline{\alpha}$, such that $p_g^* = \overline{p}_b$. Hence, more accurate monitoring raises the number of firms in the economy.

We summarize our findings in

Proposition 3: *Assume that $l \geq 2$. With imperfect monitoring, there exists a symmetric Nash equilibrium with free switching to green production, wherein firms choose to switch with probability σ^*such that:*

$$\sigma^* = \begin{cases} \overline{\sigma} \textit{ if and only if } \alpha \notin [\alpha_0(l), \alpha_1(l)], \\ 1 \text{ otherwise.} \end{cases} \tag{25}$$

The equilibrium number of firms n^* in the economy is given by (24)*.*

When $\alpha \notin [\alpha_0(l), \alpha_1(l)]$, all the firms choose to switch to green production with the same probability $\overline{\sigma}$. In these equilibria, the market is segmented between the variants brown and green of the product, which are perfectly identified by all the protagonists of the game, including consumers. The probability $\overline{\sigma}$ turns into the probability of cheating in the signaling subgame. Intuitively, cheating is as likely to occur as the switch to green production because the fraudulent strategy boils down to mimicking the green behavior as often as possible. The probability $\overline{\sigma}$ is sufficiently low in equilibrium to meet the two requirements for curtailing cheating and revealing the truth. Another interpretation of the mixed-strategy equilibrium is that the proportion of firms that choose to switch to green production in the economy must remain reasonably low for price signaling to be credible. If too high a proportion of firms turn into green firms, the likelihood that brown firms will mimic is just as high, and this threat may be too strong to afford the cost of signaling the green product.

We have seen that price signaling entails no further cost for green firms when monitoring accuracy is high or intermediate ($\alpha \geq \overline{\alpha}$). In those cases, the problem of cheating is less a matter of concern than that of revealing the truth for green firms. These firms must forego the off-the-equilibrium-path profit earned in the best worst-outcome in which consumers mistake them for the brown type. The fraction $\overline{\sigma}$ of firms switching to green production (or equivalently, the probability of cheating) must offer green firms a balanced compromise between revealing the truth on the equilibrium path and passing off as the brown type off the equilibrium path.

In the case where monitoring is low ($\alpha < \overline{\alpha}$), the problem of cheating is more severe because price signaling turns to be costly for green firms. In equilibrium, $\overline{\sigma}$ is sufficiently low to make brown firms indifferent between signaling their true type and using the cheating strategy. Increased accuracy in monitoring mitigates the signaling cost incurred by green firms, thereby alleviating the threat of cheating.

When $\alpha \in [\alpha_0(l), \alpha_1(l)]$, all the firms choose to switch to green production with probability 1. In these equilibria, all the protagonists of the game, including consumers, become certain that the green product is the only variant available in the market. However, this certainty entails a signaling cost for firms when $\alpha < \overline{\alpha}$ because the threat of cheating persists (with probability 1), even though it is not carried out in equilibrium. This is the reason why firms charge the upward-distorted price $\overline{p}_b$ instead of $p_{1g}(1)$ to signal the green product. The price $\overline{p}_b$ maximizes the profit of green firms among all separating equilibria. This is the least costly and credible way to support the green label. The auditor's monitoring appears to be essential to ensure the credibility of price signaling. If ever consumers observed a deviation from price equilibrium, monitoring would partly correct consumers' misperception by allowing them to differentiate the truly green product from the spurious one. Nevertheless, monitoring would neutralize the cheating strategy by making the off-the-equilibrium-path profit unattractive compared to the equilibrium profit. Thanks to the auditor's report, everyone is fully convinced by price signaling that cheating is worthless.

5 Conclusion

The development of greenwashing practices over recent years shows that prices may fail to truthfully signal firms' environmental quality. However, monitoring can help green firms to transmit information about environmental quality in a more credible manner than employing price signaling alone.

In a context of asymmetric information where imperfectly competitive firms have private information about environmental quality, we permit Bayesian consumers to supplement the information from price signals with the information displayed by independent monitoring. Independent means here that monitoring both supports separating prices on the equilibrium path and corrects, at least slightly, price signaling when Bayesian updating cannot apply off the equilibrium path. Propositions 1 and 2 show that monitoring, although imperfect, allows green firms to separate from brown firms while price signaling alone could not prevent fraudulent firms from cheating consumers. Knowing that consumers can correctly infer environmental quality from prices, together with the monitoring report, firms can commit resources to improve environmental quality. Hence, monitoring is key to solve both problems of adverse selection and moral hazard.

Our findings provide insight on how monitoring interplays with price signaling. When price signaling turns to be costly due to the threat of mimicking prices posed by brown firms, monitoring alleviates the pressure that green firms must withstand to deter cheating. As a result, the signaling cost decreases with enhanced accuracy in monitoring because the cheating profit is lower. Beyond a certain level of accuracy, monitoring saves the full cost of price signaling. Finally, the prospect of positive profits at the production stage allows a fraction of firms, if not all, to recover the fixed cost of switching to green production. Proposition 3 determines the proportion of firms that switch to green production in accordance with the level of monitoring accuracy. This proportion exactly coincides with the critical probability at which green firms are indifferent between signaling their type and passing off as brown.

In the present world, monitoring is increasingly conducted by non-governmental organizations (NGOs). Among various activities, they operate as watchdogs for public and private certification of credence goods by verifying the industry's compliance with certification standards. Their monitoring is constantly improving with advances in information and communications technology. In various recent cases, NGOs have demonstrated skill and ability in disclosing accurate information on fraudulent practices in industry.[13] As previously mentioned, the tests performed by the NGO ICCT showed evidence that Volkswagen diesel vehicles defeated the NOx emissions control systems. Greenpeace and the Environmental Investigation Agency have produced evidence that the Forest Stewardship Council certification has been granted to logging companies operating with little regard for sustainability or even legality.[14] Although some firms see NGOs' monitoring as a threat, others may be willing to invite them to strengthen their commitment to improve quality. This suggests that firms and independent auditors somehow interact to release information to the public, in a way that can be cooperative or not. In order to investigate this strategic interaction, it would be worthwhile to endogenize the monitoring behavior in the present model.

[13]Several examples can be found in Delmas and Burbano (2011) and Heyes and Martin (2017).

[14]See 20/02/2018 Greenwashed Timber: How Sustainable Forest Certification Has Failed.

6 Appendix

6.1 Appendix 1: Consumers' perception of quality on the market for the green product

Let $\{g/p\}$ be the event that the firm is green after observing the price p, and $\{g/m\}$ the event that the firm is green based on the auditor's monitoring. The set of these events is collectively exhaustive, meaning that at least one of the events must occur. The probabilities of $\{g/p\}$ and $\{g/m\}$ are $\mu = P(g/p)$ and $\alpha = P(g/m)$, respectively. If $\{g/p\}$ and $\{g/m\}$ are *independent* events, then the joint probability of both occurring is

$$P(\{g/p\} \cap \{g/m\}) = P(g/p)\, P(g/m)\,. \tag{26}$$

The probability of one or both events occurring is denoted

$$P(\{g/p\} \cup \{g/m\}) = P(g/p) + P(g/m) - P(\{g/p\} \cap \{g/m\})\,. \tag{27}$$

Thus, if $\{g/p\}$ and $\{g/m\}$ are independent, the probability that the firm is green after observing either prices or monitoring is

$$P(\{g/p\} \cup \{g/m\}) = \mu + \alpha - \mu\alpha = \mu + (1-\mu)\alpha.$$

Assuming that price signaling and monitoring are independent sources of information, the quality expected by consumers on the market for the green product is given by the equation

$$\widetilde{e}_t(\mu) = e(\mu) + \alpha e_t - \alpha e(\mu) = \mu e + \alpha(e_t - \mu e). \tag{28}$$

When Bayes' rule applies in equilibrium to beliefs formed from pure-strategy prices (p_b, p_g), posterior beliefs are $\mu(g|p_t) = 1 - \mu(b|p_t) = 1$ for $t \in T$, giving consumers correct perceptions whatever the type, since

$$\widetilde{e}_t(\mu(t|p_t)) = \mu(t|p_t)e + \alpha(e_t - \mu(t|p_t)e) = \begin{cases} e \text{ if } t = g, \\ 0 \text{ if } t = b. \end{cases} \tag{29}$$

Hence, consumers align their readings of the auditor's report with updated beliefs when prices reveal the truth. Otherwise, consumers observe a deviation from price equilibrium. Then, given an arbitrary belief μ based on a probability-0 price, consumers' perception is given by (28). In the extreme case where beliefs based on prices are pessimistic ($\mu = 0$), even if the actual type is green, monitoring reduces the leeway in specifying off-the-equilibrium-path beliefs. In that case, the perception consumers have about the type, i. e., $\widetilde{e}_g(0) = \alpha e$, improves with monitoring accuracy. In some sense, monitoring helps consumers refine the multiplicity of equilibria supported by unrestricted beliefs based on unexpected prices.

The way monitoring restricts beliefs in our setting clearly differs from how beliefs update based on grades in Daley and Green (2014). These authors assume that how beliefs update after observing a signal and grades follows a two-stage process. In the first stage, receivers observe a signal through the investment in education, and they use Bayes' rule to update beliefs, as consumers do after observing prices in the present article. This first updating results in "interim beliefs", based on the history h^1 of the game in the first stage. In the continuation game starting at the second stage, receivers observe grades and again update beliefs from their interim beliefs *via* Bayes' rule. This requirement is stronger than simply using Bayes' rule in the usual fashion since it applies to updating from the first stage to the second stage, whether or not h^1 has probability 0, and whether or not the signal sent in the first stage has probability 0. Hence, the likelihood of grades implicitly depends on the observation of prices *via* the second Bayesian updating. Applying Bayes' rule twice captures a form of redundancy in the information transmission, which is not desirable in the context of our article. The sender influences the decision to give a grade by sending a costly signal in Daley and Green (2014). Rather, we assume that the information released by monitoring is independent from that transmitted through price signaling.

6.2 Appendix 2: Calculation results for firm 1's outcomes

The following table presents the calculation results for firm 1's outcomes.

Firm 1's profits for $l = 3$ and $\sigma = \frac{1}{2}$	Expressions
Firm 1's best-worst profit	$\widetilde{\Pi}_{1g}\left(\frac{1}{2}\right) = \begin{cases} \frac{e(1-4\alpha)^2(1-3\alpha)}{12\alpha(1-2\alpha)n} \text{ if } \alpha \geq \frac{1}{2}, \\ \frac{e\left(1-6\alpha+6\alpha^2\right)^2}{12\alpha(1-\alpha)(1-2\alpha)n} \text{ if } \alpha \in \left(\overline{\alpha}, \frac{1}{2}\right), \\ \frac{e\left(1-7\alpha+9\alpha^2\right)^2}{12\alpha(1-\alpha)(1-2\alpha)n} \text{ if } \alpha \leq \overline{\alpha} = \frac{1}{3}. \end{cases}$
Firm 1's best response when it is mistaken for the brown type	$p_{1g}(0) = \begin{cases} \frac{e}{2}(1+4\alpha) \text{ if } \alpha \geq \frac{1}{2}, \\ \frac{e\left(1+4\alpha-6\alpha^2\right)}{2(1-\alpha)} \text{ if } \alpha \in \left(\overline{\alpha}, \frac{1}{2}\right), \\ \frac{e\left(1+5\alpha-9\alpha^2\right)}{2(1-\alpha)} \text{ if } \alpha \leq \overline{\alpha}. \end{cases}$
Firm 1's sales volume when it is mistaken for the brown type	$D_{1g}\left(p_{1g}(0), 0\right) = \begin{cases} \frac{e(4\alpha-1)(3\alpha-1)}{6\alpha(2\alpha-1)n} \text{ if } \alpha \geq \frac{1}{2}, \\ \frac{e\left(6\alpha-1-6\alpha^2\right)}{6\alpha(1-2\alpha)n} \text{ if } \alpha \in \left(\overline{\alpha}, \frac{1}{2}\right), \\ \frac{e\left(7\alpha-1-9\alpha^2\right)}{6\alpha(1-2\alpha)n} \text{ if } \alpha \leq \overline{\alpha}. \end{cases}$
Firm 1's least-costly signaling profit	$\Pi_{1g}\left(p_g^*, 1\right) = \begin{cases} \frac{2e}{3n} \text{ if } \alpha \geq \overline{\alpha}, \\ \frac{2e\alpha(2-3\alpha)}{n} \text{ otherwise.} \end{cases}$
Firm 1's least-costly signaling price	$p_g^* = \begin{cases} p_{1g}(1) = 2e \text{ if } \alpha \geq \overline{\alpha}, \\ \overline{p}_b = 3(1-\alpha)e \text{ otherwise.} \end{cases}$
Firm 1's sales volume with p_g^*	$D_{1g}\left(p_g^*, 1\right) = \begin{cases} \frac{1}{3n} \text{ if } \alpha \geq \overline{\alpha}, \\ \frac{2\alpha}{n} \text{ otherwise.} \end{cases}$

6.3 Appendix 3: Proof of Proposition 2

We distinguish between three cases.

1. Monitoring accuracy is high: $\alpha > \frac{1}{2}$

For all $\sigma > \underline{\sigma}$, the best-worst profit is

$$\widetilde{\Pi}_{1g}(\sigma) = \frac{e\left(2 + 4l\alpha^2 - 2\sigma + \alpha\left(3\sigma - 4 + \sigma l - 2l\right)\right)^2}{16l\alpha\left(1 - 2\alpha\right)\left(1 - \sigma + \alpha\sigma - 2\alpha\right)\sigma n}, \tag{30}$$

and the margin profit is

$$p_{1g}(0,\sigma) - e = \frac{e\left(2 + 4l\alpha^2 - 2\sigma + \alpha\left(3\sigma - 4 + \sigma l - 2l\right)\right)}{4\left(1 - \sigma + \alpha\sigma - 2\alpha\right)}. \tag{31}$$

In order that margin and demand be positive at the same time for the green firm falsely perceived to be brown on the basis of price alone, a necessary condition is $\sigma > \underline{\sigma} = \frac{2(1-2\alpha)(l\alpha-1)}{3\alpha+l\alpha-2}$. Note that $1 - \sigma + \alpha\sigma - 2\alpha < 0$ and $3\alpha + l\alpha - 2 > 0$ when $\alpha \geq \frac{1}{2}$, and hence $\underline{\sigma} > 0$ for all $\frac{1}{2} < \alpha < \frac{1}{l}$.

Let us consider

$$\Delta(\sigma) = 0 \tag{32}$$

This is a quadratic equation in σ with at most two real roots, σ^- and σ^+, such that $\sigma^- < \sigma^+$ whenever they exist. Furthermore, $\Delta(\sigma)$ is a concave function of σ because its second derivative with respect to σ is negative. Thus, $\Delta(\sigma) > 0$ if the discriminant $D(l)$ of (32) is positive and σ lies inside $[\sigma^-, \sigma^+]$. It turns out that $D(l) > 0$ for all $l > \frac{1-\alpha-\alpha^2}{1-3\alpha+3\alpha^2}$, which is satisfied for any $l > 1$ because $1 - \frac{1-\alpha-\alpha^2}{1-3\alpha+3\alpha^2} = \frac{2\alpha(2\alpha-1)}{1-3\alpha+3\alpha^2} > 0$.

The calculations done by Mathematica produce the following expressions

$$\sigma^+ \text{ (resp. } \sigma^-\text{)} = 2\frac{\begin{array}{c}2 - \left(8 + l + l^2\right)\alpha + \left(9 + 3l + 4l^2\right)\alpha^2 - 2\left(1 + l + 2l^2\right)\alpha^3 \\ + \text{(resp. } -\text{)}\sqrt{(l-1)^2(l+1)(1-2\alpha)^2\alpha^2\left(l - 1 + (1-3l)\alpha + (1+3l)\alpha^2\right)}\end{array}}{((3+l)\alpha-2)^2}.$$

From these expressions, further calculations show that $\sigma^- < \underline{\sigma} < \sigma^+ < 1$ for all $\alpha > \frac{1}{2}$. We can conclude that $\sigma^+ = \overline{\sigma}$ is the critical σ below which separation is possible.

2. Monitoring accuracy is intermediate: $\alpha \in \left[\overline{\alpha}, \frac{1}{2}\right]$

For all $\sigma > \underline{\sigma}$, the best-worst profit is

$$\widetilde{\Pi}_{1g}(\sigma) = \frac{e\left(\left((l+5)\alpha - 2 - 4l\alpha^2\right)\sigma + 2\left(2\alpha - 1\right)\left(l\alpha - 1\right)\right)^2}{16le\alpha\left(1 - 2\alpha\right)\left(\sigma\left(3\alpha - 1\right) + 1 - 2\alpha\right)\sigma n}, \tag{33}$$

and the margin profit is

$$p_{1g}(0,\sigma) - e = \frac{e\left(\left(2 - (5+l)\alpha + 4l\alpha^2\right)\sigma - 2\left(2\alpha - 1\right)\left(l\alpha - 1\right)\right)}{4(\sigma\left(3\alpha - 1\right) + 1 - 2\alpha)}. \tag{34}$$

In order that both the margin and demand be positive for the green firm falsely perceived to be brown on the basis of price alone, a necessary condition is $\sigma > \underline{\sigma} =$

$\frac{2(2\alpha-1)(l\alpha-1)}{2-(5+l)\alpha+4l\alpha^2}$. Note that $2-(5+l)\,\alpha+4l\alpha^2>0$ and so $\underline{\sigma}>0$ for all $\alpha<\min\{\frac{1}{2},\frac{1}{l}\}$ and $\underline{\sigma}<1$.

Equation (32) is quadratic in σ with at most two real roots, σ^- and σ^+, such that $\sigma^-<\sigma^+$ whenever they exist. Furthermore, $\Delta(\sigma)$ is a concave function of σ because its second derivative with respect to σ is negative. Thus, $\Delta(\sigma)>0$ if the discriminant $D(l)$ of (32) is positive and σ lies inside $[\sigma^-,\sigma^+]$. $D(l)$ is a quadratic and convex function of l, such that $D(l)>0$ when $l\in\left(1,\frac{1}{1-2\alpha}\right)$. The highest root in l of equation $D(l)=0$ is given by $\frac{\alpha(1-2\alpha)+\sqrt{(8\alpha-3)(1-4\alpha+3\alpha^2)^2}}{-1+7\alpha-15\alpha^2+8\alpha^3}$: it does exist for any $\alpha>\frac{3}{8}$ and falls short of 1. Otherwise, $D(l)>0$ when $\alpha\leq\frac{3}{8}$. Finally, $D(l)>0$ when $\frac{(l-1)}{2l}<\alpha\leq\frac{1}{2}$ and so both roots σ^- and σ^+ exist.

The calculations done by Mathematica produce the following expressions

$$\sigma^+\text{ (resp. }\sigma^-)$$

$$=2\frac{\begin{array}{c}2-\left(10+l+l^2\right)\alpha+\left(15+5l+6l^2\right)\alpha^2-6\left(1+l+2l^2\right)\alpha^3+8l^2\alpha^4\\ +\text{ (resp. }-)\sqrt{(l-1)^2\,(1-2\alpha)^2\alpha^2\left(3-11\alpha+9\alpha^2+2l\alpha(1-2\alpha)+l^2\left(1-7\alpha+15\alpha^2-8\alpha^3\right)\right)}\end{array}}{(2-(5+l)\alpha+4l\alpha^2)^2}.$$

From these expressions, further calculations show that, for all $\alpha\in\left[\overline{\alpha},\frac{1}{2}\right]$, $\sigma^-<\underline{\sigma}<\sigma^+$, and, moreover, $0<\sigma^+<1$ when $\alpha>\max\left\{\overline{\alpha},\frac{3}{8}\right\}$, in which case $\sigma^+=\overline{\sigma}$. We can conclude that $\min\{\sigma^+,1\}$ is the critical σ below which separation is possible.

3. Monitoring accuracy is low: $\alpha<\overline{\alpha}$

For all $\sigma>\underline{\sigma}$, the best-worst profit is

$$\widetilde{\Pi}_{1g}(\sigma)=\frac{e\left(\left(1-3\alpha+l\alpha^2\right)\sigma-(2\alpha-1)\,(l\alpha-1)\right)^2}{4l\alpha\,(1-2\alpha)\,(\sigma\,(3\alpha-1)+1-2\alpha)\,\sigma n},\tag{35}$$

and the margin profit is

$$p_{1g}(0,\sigma)-e=\frac{e\left(\left(1-3\alpha+l\alpha^2\right)\sigma-(2\alpha-1)\,(l\alpha-1)\right)}{2\,(\sigma\,(3\alpha-1)+1-2\alpha)}.\tag{36}$$

In order that both the margin and demand be positive for the green firm falsely perceived to be brown on the basis of price alone, a necessary condition is $\sigma>\underline{\sigma}=\frac{(2\alpha-1)(l\alpha-1)}{1-3\alpha+l\alpha^2}$. Note that $1-3\alpha+l\alpha^2>0$ and hence $\underline{\sigma}>0$ for all $\alpha<\min\{\frac{1}{2},\frac{1}{l}\}$.

Equation (32) is quadratic in σ with at most two real roots, σ^- and σ^+, such that $\sigma^-<\sigma^+$ whenever they exist. Furthermore, $\Delta(\sigma)$ is a concave function of σ because its second derivative with respect to σ is negative. Thus, $\Delta(\sigma)>0$ provided that the discriminant $D(l)$ of (32) is positive. This turns out to be true when $\alpha<\frac{(l-1)}{2l}$, or, equivalently, $l>\frac{1}{1-2\alpha}$, because, first, $D(l)$ is a quadratic and convex function of l, and second, $\frac{1}{1-2\alpha}$ exceeds $\frac{1}{1-\alpha}$, which turns out to be the highest root in l of equation $D(l)=0$. Thus, both roots σ^- and σ^+ do exist.

The calculations done by Mathematica produce the following expressions

$$\sigma^{+}\,(\text{resp. }\sigma^{-}) = \frac{\begin{array}{c} 1-(5+l)\,\alpha+\left(6+8l-2l^{2}\right)\alpha^{2}+l\,(11l-18)\,\alpha^{3}-4l\,(5l-3)\,\alpha^{4}+12l^{2}\alpha^{5} \\ +\,(\text{resp. }-)\,2\sqrt{\alpha^{3}\,(l-2l\alpha)^{2}\left(\begin{array}{c} -1+6\alpha-12\alpha^{2}+9\alpha^{3}+l(1-8\alpha+24\alpha^{2}-34\alpha^{3}+18\alpha^{4}) \\ +l^{2}\alpha\left(1-7\alpha+19\alpha^{2}-22\alpha^{3}+9\alpha^{4}\right)\end{array}\right)}\end{array}}{(1-3\alpha+l\alpha^{2})^{2}}.$$

From these expressions, further calculations show that, for all $\alpha < \overline{\alpha}$, $\sigma^{-} < \underline{\sigma} < \sigma^{+}$, and $0 < \sigma^{+} \leq 1$ when α lies outside $\left[\frac{5l-\sqrt{l(32-7l)}}{16l}, \min\left\{\overline{\alpha}, \frac{5l+\sqrt{l(32-7l)}}{16l}\right\}\right]$. Moreover, σ^{+} reaches a minimum of 0 at $\alpha = \frac{3l-\sqrt{l(9l-16)}}{8l} < \frac{1}{4}$ for all $l \geq 2$. We can conclude that $\min\{\sigma^{+}, 1\}$ is the critical σ below which separation is possible.

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Figures. Price Signaling and Quality Monitoring in Markets for Credence Goods

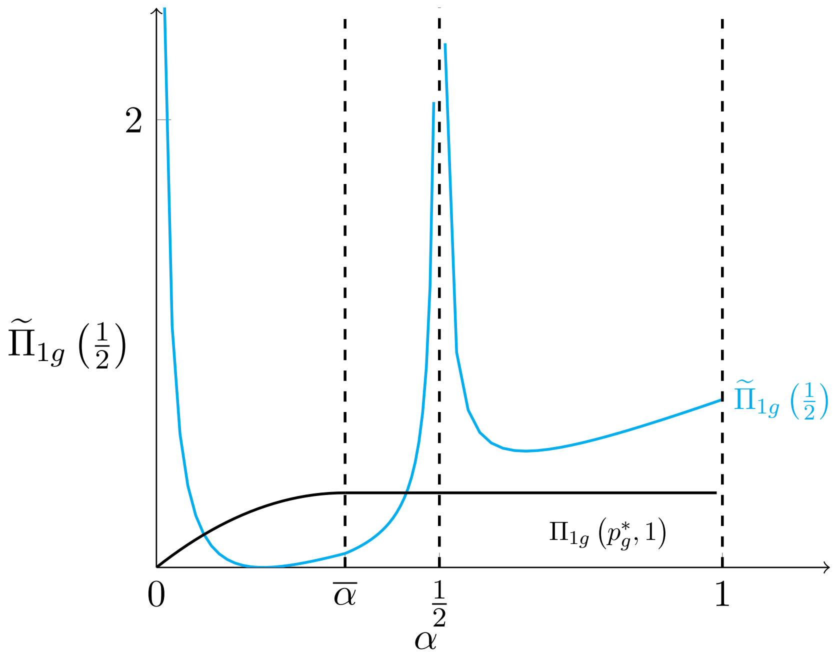

Figure 1: *Best-worst and least-costly profits when* $\sigma = 1/2$

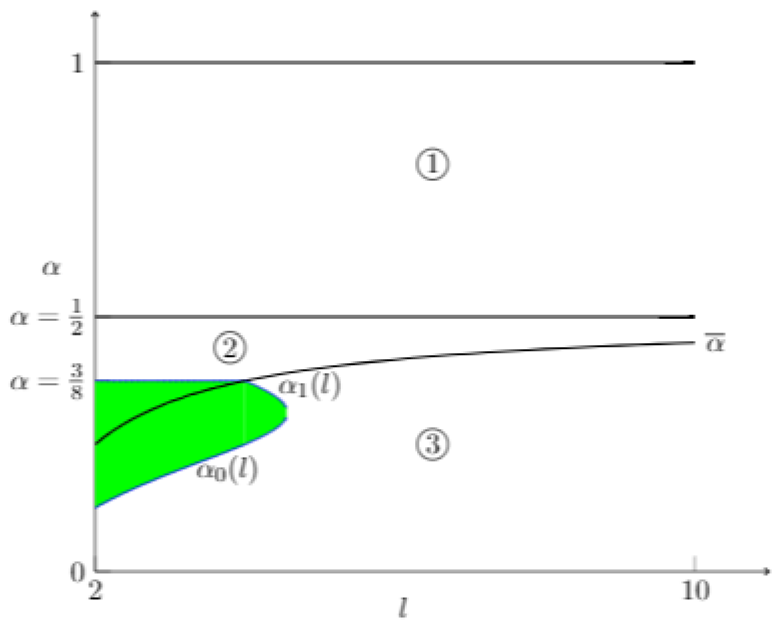

Figure 2: *Existence of separating equilibria*

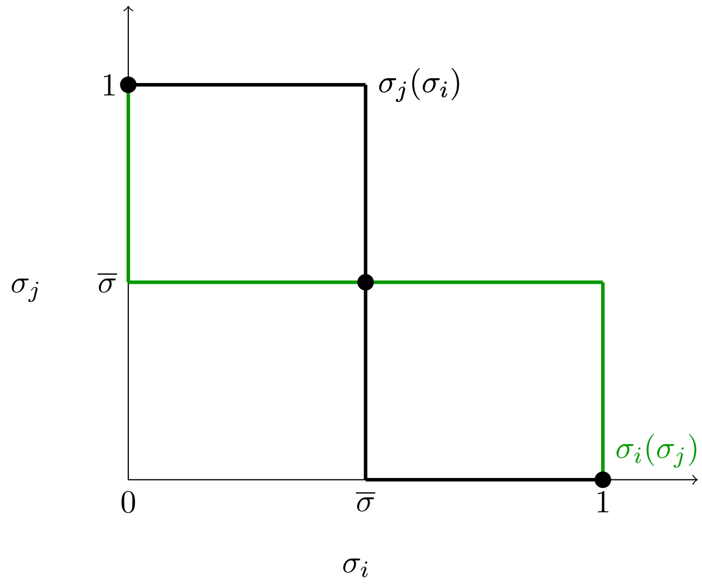

Figure 3: *Best response functions and Nash equilibrium*

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