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The impact of income inequality on public environmental expenditure with green consumerism

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Abstract

This article analyzes the impact of income inequality on environmental policy in the presence of green consumers. We first develop a model with two main ingredients: citizens, with different income capacities, have access to two commodities whose consumption differs in terms of price and environmental impact, and they vote on the environmental policy. In this setting, there exists a unique political equilibrium in which the population is split into two groups, that differ in the type of good, conventional vs. green, they consume. The analysis shows that a change in the level of inequality induces variations in both the size and composition of these two groups of citizens. This in turn determines whether or not more inequality stimulates the public policy. We then conduct an empirical investigation on a panel of European countries over the period 1996-2019. We find the existence of an inverted J-shape relationship between inequality and public environmental spending. This outcome can be explained by the combination of a composition effect, affecting the green group, and a substitution effect between private green consumption and public environmental spending.

Keywords: income distribution, inequality, green consumption, environmental public expenditure, political equilibrium.

JEL classification: Q58, H23, D31, D72

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1 Introduction

This paper addresses the old and important question of the link between the income distribution, especially income inequality, and the public policy. Since the early 2000s, and even more in the recent period, the issue has regained attention as it has become necessary to think about the design of policies capable of responding to the many environmental challenges modern societies face, while being socially acceptable.

As perfectly noticed by Stiglitz (2014), there is a two-way relationship between the environmental policy and the income distribution. So scholars’ contributions on this topic are naturally divided in two distinct literatures. Some examine the distributional impact of the environmental policy (Aubert and Chiroleu-Assouline, 2019, and Jacobs and van der Ploeg, 2019), while others try to understand how the income distribution shapes the environmental policy (Boyce, 1994, Magnani, 2000, and subsequent contributions).

This paper falls within the second category and has been motivated by the following observations, based on recent data collected in Europe. First, we observe a positive correlation between GDP per capita and general government expenditure in Environmental Protection Per Capita (EPPC) (see Figure 1).\(^1\) This is in line with the intuition and supported by most of the arguments put forward to explain the decreasing part of the so-called Environmental Kuznets curve (EKC).\(^2\) In particular, as people get richer, one expects that the demand for environmental protection rises.

Second, and more interestingly, the relationship between income inequality and environmental policy does not seem to be monotone (see Figure 2). Precisely, a higher level of inequality seems to impair environmental public spending where/when it is already low.

\(^1\)Hereafter we focus on public environmental expenditure as an indicator of environmental policy. Information about data collection is explained in details in Section 5.

\(^2\)The EKC is the inverted U-shaped relationship linking income per capita to some measures of pollution. It was detected in the early 90s, and has been a subject of lively debate since then.
to moderate, while the opposite conclusion seems to hold true where/when environmental expenditure is higher. To understand this mixed picture, we decided to turn to the existing literature and came to two conclusions. First, though it blew its twenty candles recently, this literature is relatively sparse both on theoretical (Magnani, 2000, Eriksson and Persson, 2003, Kempf and Rossignol (2007) and empirical (Magnani, 2000, Grunewald et al., 2017, Martínez-Zarzoso and Phillips, 2020) grounds. Second, existing theories all conclude to the existence of a negative relationship between inequality and environmental policy, which tends to be validated empirically.\(^3\) So, the literature proves unable to explain the stylized facts presented above.

This paper aims at filling this gap not only by providing a theoretical explanation to these stylized facts, but also by examining their empirical validity. Our approach is novel in that it relies on the interaction between private green consumption and the collective decision on environmental public expenditure. Green consumerism – the fact that some people display a preference for the green version of some good (food, cars etc.) – has

\(^3\)See Section 2 for a literature review on this topic and the others connected to our work.
not been considered by the above literature so far, as it is a relatively new phenomenon. Its impacts on policy making is just beginning to be analyzed (Ambec and Donder, 2020). Moreover, taking account of green consumption allows us to conduct an analysis that is very much in line with another literature assessing the link between the income distribution and the collective choice on the provision of public good, like the public funding of education (de la Croix and Doepke, 2009, Arcalean and Schiopu, 2016, Melindi-Ghidi, 2018).

To investigate how this original mechanism shapes the relationship between inequality and environmental public spending, we first develop an original political economy model that combines the following ingredients. The economy is composed of citizens who display an heterogeneity in income, take consumption decisions, and care about the environment. Citizens exhibit green consumerism, that is, a preference for green goods.\(^4\) Green consumption does not find its origin in their ability to perfectly internalize its environmental impact. It is better explained by other private motives like being more healthy and the existence of warm-glow effect (Andreoni, 1990). In other words, there exists an environmental externality of consumption. Preferences are also defined over environmental quality that is determined by private green consumption and public expenditure. Indeed, the public

---

\(^4\)This includes organic food, energy saving household appliances, hybrid and electric vehicles etc.
policy consists in taxing citizens’ income and using the resulting revenue to finance environmental public expenditure. This public policy is the outcome of a voting procedure. Finally, we account for the existence of a price premium: green goods are more expensive than their conventional/neutral counterpart.

In the political equilibrium, the income tax is associated with a critical income level that splits the population into two groups, those who consume the green good, and those who do not. We then assess how the features of the income distribution shape environmental spending. In line with the intuition and stylized fact reported in Figure 1, we find that public environmental expenditure unconditionally increases with the average income, keeping the standard deviation constant. Considering a mean preserving spread (MPS), conclusions are less clear-cut as we get that an increase in inequality induces a decrease in environmental expenditure if and only if the equilibrium tax is lower than a critical threshold. Green consumerism is the key mechanism underlying this outcome. A variation in the level of inequality changes the size and composition of both groups. This in turn affects both the marginal benefit and each group’s marginal cost of the policy, and consequently the outcome of the electoral process. We carefully dissect the mechanisms at stake and interpret our results that remarkably reproduce stylized facts (Figure 2). Last but not least, we identify a sufficient condition, depending on the environmental concern and price premium, for having a negative impact of income inequality on environmental policy.

To get more insight into the link between the income distribution and environmental spending, we then perform an empirical analysis using a sample covering 31 European countries over the period 1996-2019. Through the adoption of a fixed-effect model with robust standard errors, we analyze the impact of mean income and inequality, as measured by the Gini index, on both general and local government expenditure in environmental protection. As expected, we find that GDP per capita is positively correlated with public
environmental spending. In addition, empirical results show the existence of an inverted J-shape relationship between inequality and environmental policy, thereby confirming the non-monotone nature of that relation, first highlighted in Figure 2 and then pointed out by the theoretical analysis.\(^5\) The intuition of the result draws on our theory: a MPS is associated with thicker tails and finer middle of the income distribution. More rich people means more green consumption, which is good for the environment and calls for less public spending. Less middle income people implies just the opposite provided that part of the middle class buys green. When inequality is already high, and average income is low, the threshold income is low as well, which means that many middle income people consume the green good, and the negative effect on green consumption prevails. People then ask for more public environmental expenditure, as a compensation.

The paper is organized as follows. Section 2 reviews the related literatures. Section 3 presents the model. Section 4 is devoted to the equilibrium analysis and assesses the impact of the income distribution on the public policy. Section 5 is dedicated to the empirical analysis. Section 6 concludes.

## 2 Related literature

The link going from environmental policy to income distribution has been examined recently (see among others, Aubert and Chiroleu-Assouline, 2019, and Jacobs and van der Ploeg, 2019). This strand of literature deals with the impact of an environmental tax reform on the different income groups that compose society. It also addresses the optimal design of environmental taxes when distributional effects are taken into account. It is finally interested in the efficiency of the economic and fiscal system.\(^6\)

\(^5\)Results are robust to the use of “environmental taxation” as an alternative dependent variable.

\(^6\)Papers on environmental taxation generally conduct their analyses in microeconomic second best frameworks. They assume that the population is heterogenous in terms of income capacities, and sometimes
In the coming analysis, we look at the problem the other way around by asking how income inequality can shape the environmental policy. This question has long been debated by economists. Dating back to the seminal paper of Boyce (1994), the literature provides a series of arguments explaining why more inequality is bad for the environment, or the reverse. In a recent survey, Berthe and Elie (2015) classify these arguments into two categories depending on whether they involve individual behaviors, and how they relate to environmental pressure, or emphasize collective decision making. Central to all this discussion is the idea that there exist potential conflicts in societies among social and income groups – typically the poor vs. the rich – especially regarding the demand for environmental protection, and that these conflicts are exacerbated by inequality. The authors do however note that there is no theoretical nor empirical consensus on this topic.

There also exist a few formal studies of the impact of inequality on environmental policy. The most prominent contribution to this line of research is due to Magnani (2000). The author develops a simple political economy model where individuals’ preferences are defined over consumption and environmental quality. The government enhances environmental quality thanks to public expenditure that are financed by an income tax (accounting for the marginal cost of public funds). People vote on the tax rate. Focusing on majority voting, Magnani (2000) shows the existence of a negative relationship between inequality and environmental policy. In her model, the key mechanism that explains this negative link is the dependence of individuals’ environmental preferences on the relative income. Subsequent contributions (Eriksson and Persson, 2003, Kempf and Rossignol, 2007) also build majority voting models and reach the same unambiguous conclusion, even though
they consider different mechanisms.\footnote{Eriksson and Persson (2003) consider a uniform distribution of individuals who care about consumption and pollution in Stokey (1998)'s static model. Individual consumption is equal to the product of a collectively chosen pollution standard and production, which is an increasing and convex transformation of the individual type. They capture an increase in inequality by an increase in the gap between the median voter's production and the average one, and show that when this gap increases, the median voter asks for a less stringent standard. Kempf and Rossignol (2007) build an endogenous growth model \textit{a la Barro} (1990) in which pollution arises from production. The government levies a tax on income that is used to finance both environmental (abatement) and productive (infrastructure) spending. The median voter has to choose how to share the fiscal revenue between these two types of expenditure. More inequality induces the median voter to support growth at the expense of the environment.}

This literature is thus unable to explain the situation depicted in Figure 2. This is where the first contribution of our paper lies. We propose a new theory based on the substitution between public environmental spending and private green consumption. Considering green consumption echoes the observation that nowadays a growing number of people display a willingness-to-pay for green goods, and a willingness-to-accept a price premium compared to their neutral counterparts (McFadden and Huffman, 2017, Poder and He, 2017). Ambec and Donder (2020) are the first to analyze the impact of green consumerism on environmental policy. Our approach differs from theirs as they assume that the proportion of green consumers in the economy is exogenous, and do not deal with the heterogeneity of the income distribution. Compared to the above mentioned literature, our approach is also more general because we pay a great deal of attention to the interplay between individual and collective decisions.\footnote{Papers in the literature deal with the collective decision dimension only.} Finally, we depart from the literature by using a probabilistic voting model. Thus, our paper has also a connection with the political economy literature on public goods provision, especially with recent contributions on private education vs. public schooling (de la Croix and Doepke, 2009, Arcalean and Schiopu, 2016, Melindi-Ghidi, 2018). Compared to majority voting, probabilistic voting shifts the political power from the poorer to the wealthier people, that are also those who consume green, in the determination of the political outcome.\footnote{See the discussion on probabilistic voting models in Section 4.1.}
On empirical grounds, the literature on the link between income inequality and environmental policy is sparse.\textsuperscript{10} Its main contribution lies in the validation of the negative link between inequality and indicators of environmental policy, though there seems to be a dependence of the results on the level of income. Magnani (2000) measures inequality by the Gini index while environmental policy is captured by public R&D expenditure to protect the physical environment. Working with a panel dataset for OECD countries over the period 1980-1991, the author shows the existence of a negative correlation between income inequality and environmental policy.\textsuperscript{11} Vona and Patriarca (2011) follow the leads of Magnani (2000). They consider a more recent and longer period (1985-2005, also on OECD countries) and focus on environmental innovations like green R&D and the production of environmental patents, especially by the public sector. Their empirical results highlight that inequality negatively influences the diffusion of innovations in countries with high per-capita income. The dependance over GDP result can be explained by the methodology used in these papers. The regressions include a second order polynomial in the GDP and an interaction term between GPD and the Gini index. This typically falls within the EKC empirical literature tradition.\textsuperscript{12}

Our contribution to the empirical literature is two-fold. First, we use a more recent and broader dataset, focusing on European countries, and a more exhaustive variable to capture environmental public expenditure.\textsuperscript{13} Second, based on stylized facts, we adopt

\begin{footnotesize}
\begin{enumerate}
\item The literature examining the link between inequality and pollution, or environmental degradation indicators, is more substantial (see among others, Torras and Boyce, 1998, Heerink et al., 2001 and Baek and Gweisah, 2013). But it is also more distant from our problematic. Indeed, people vote to choose public policies (public spending, taxation), not the level of pollution, for many reasons, observability and measurement issues being the most important ones.
\item These results seem valid for high-income countries only. In addition, the small number of observations of the empirical analysis allows the author to derive qualitative empirical conclusions only.
\item Note that such a dependence also shows up in recent papers assessing the relationship between income inequality and environmental degradation, for the very same reason. For instance, Grunewald et al. (2017) find that the relationship between income inequality and CO\textsubscript{2} emissions depends on income levels: at higher (lower) levels of income, higher income inequality increases (decreases) CO\textsubscript{2} emissions.
\item This variable includes all public expenditure related to the environment, such as waste management, water management, pollution abatement, protection of biodiversity, and also R&D environmental protec-
\end{enumerate}
\end{footnotesize}
(and justify) an empirical strategy that accounts for the potential non-monotonicity of the impact of inequality on environmental policy.\textsuperscript{14} We estimate an equation that includes a second order polynomial in the Gini index, not in GDP. This is in sharp contrast with the literature (but similar to Martínez-Zarzoso and Phillips, 2020).

3 Model

As in the literatures on environmental taxation and voting on public funding of education, the fundamental ingredient of our model is the (income) heterogeneity of the population. Our work is closely connected to the former series of papers because of its problematic even if we adopt a different (yet complementary) perspective. In the modeling approach, we share with them the general shape of preferences. In particular, we work with a non-homothetic utility function, account for the environmental impact of consumptions, and assume the existence of a consumption externality. And that’s it, since in the main, we adapt and extend de la Croix and Doepke (2009)’s framework.

We consider two types of commodities that differ in terms of their environmental impact. We work with an index of environmental quality, $Q$, with reference level normalized to 0.\textsuperscript{15} The first commodity, whose consumption is denoted by $c$, is environmentally-neutral while the second, $d$, is environmental-friendly. Consuming good $d$ has a positive side-effect on the environment. Typical examples of consumptions that improve environmental quality along some – possibly different – dimensions are organic food (quality of soils etc.) and electric vehicles (atmospheric pollution). Beside the consumption channel, environmental quality can be increased thanks to environmental expenditure by the government. Overall,

\textsuperscript{14}Figure 1 reveals that European countries are all located on the increasing part of the EKC: a linear term should be enough to get insight into the relation between GDP and environmental policy. Figure 2 gives us a hint as to the nature of the relation between inequality and environmental policy. \textsuperscript{15}It is defined in relation to a business-as-usual level of pollution taken as given.
environmental quality is taken as given by the citizens, which means that there exists a positive consumption externality.\textsuperscript{16}

The population is constant with its size normalized to 1. There is a continuum of individuals who differ with respect to the wage rate. Wages are distributed on the support $[w_m, \infty)$, with $w_m > 0$, according to density and cumulative distribution functions $f(w)$ and $F(w)$. In the analysis to come, we make use of a Pareto distribution:\textsuperscript{17}

\[
F(w) = 1 - \left(\frac{w_m}{w}\right)^k, \quad f(w) = kw_m^k w^{-(1+k)} \text{ with } k > 2,
\]

and pay attention to its two main features, the average, $\mu$, and standard deviation, $\sigma$.

Following the discussion conducted in the Introduction, people exhibit a willingness-to-pay (or willingness-to-accept a price premium) for green goods. At the same time however, it seems difficult to assign this WTP(A) to some sort of environmental awareness whereby they would be able to evaluate the impact of their (consumption) decisions on the environment. From a modeling viewpoint, this leads us to represent preferences by a utility function with three components: the two consumptions and the level of environmental quality, taken as given. For the sake of the analysis, we choose a quasilinear representation of the non-environmental utility that is combined with a linear environmental benefit. We

\textsuperscript{16}We then adopt a symmetric yet similar approach as the literature on environmental taxation.

\textsuperscript{17}Uniform and Pareto distributions are the most commonly used in the literature because of their tractability. \textit{de la Croix and Doepke (2009)} deal with a uniform distribution while \textit{Arcalean and Schiopu (2016)} extend their analysis to a Pareto distribution.
also assume that people display the same preferences, with utility function:\(^{18}\)

\[ U(c, s, Q) = u(c, d) + \beta Q = \frac{\gamma}{\alpha} (c)^{\alpha} + d + \beta Q \]  

(1)

with \(\alpha \in (0, 1)\) and \(\gamma, \beta > 0\), the relative weight of respectively non green (or environmentally neutral) consumption and the environment in the preferences. Consumption decisions are subject to the budget constraint:

\[ (1 - t)w = c + \pi d \]  

(2)

with \(t \geq 0\) the (linear) income tax, and \(\pi\) the (relative) price of the green good. Hereafter we impose \(\pi > 1\), which sounds like a very reasonable assumption for categories of goods of interest.\(^{19}\) Indeed, focusing on organic products, the United States Department of Agriculture (USDA, see Coleman-Jensen et al. (2017)) gets an estimate for the price premium that ranges from 7\% to 82\%.\(^{20}\) Liu (2014) also measures a differential of about 17\% between the mean price of hybrid cars and of conventional cars sold in the US.

In the same vein as de la Croix and Doepke (2009), we consider a generic income tax whose purpose is to finance the public provision of environmental quality, or environmental public spending, \(G\). In addition, the government follows a balanced budget rule:

\[ G = \int_{w_{min}}^{\infty} twf(w)dw = t\mu. \]

\(^{18}\)Our results would remain qualitatively the same with Stone-Geary preferences in consumptions (like in Aubert and Chiroleu-Assouline, 2019), and a (strictly) concave function for the environmental benefit. However the resolution and comparative statics would require unnecessary complicated algebra. Moreover, de la Croix and Doepke (2009) use a log-additive utility defined over consumption, the quantity and quality of children. Utility derived from the latter depends on a discrete choice to educate children in the private vs. public schooling system. In our setting, our “discrete choice” is whether or not to consume the green good. As we want a smooth representation of preferences, we cannot use the log form. Finally, their problem does not include any externality.

\(^{19}\)It is used for instance by Nyborg et al. (2006).

\(^{20}\)That is, the price of organic products relative to that of conventional alternatives.
Public spending adds up to private consumption of the green good to determine the realized level of environmental quality:

\[ Q = G + \int_{w_m}^{\infty} df(w) dw. \]

The timing of events is the following: citizens first elect a government that pre-commits to a policy platform \( \{t, G\} \). Once elected, the government sets the tax rate. People then choose their consumption levels, which finally results in a level of environmental spending and quality. We assume perfect foresight which especially means that when political parties choose their strategy in the electoral competition, they perfectly anticipate people’s reaction to the public policy. This a typical Stackelberg game that can be solved backwards by first determining individual decisions as a function of policies and then choosing policies taking this dependency into account.\(^{21}\)

This baseline model serves as a vehicle for the coming analysis where our main is first to establish that the problem above has a solution – a political equilibrium – and next to examine how the equilibrium features, especially the public policy, change when the main characteristics of the income distribution, average and standard deviation, vary.

\(^{21}\)Note that de la Croix and Doepke (2009) consider the other timing where individuals “move first,” before the policy is chosen. They provide the argument that contrary to the public policy, decisions on fertility and education can not be revised frequently. In our setting, we can support the timing considered by providing the exact opposite argument because we are dealing with consumption decisions. Moreover, this timing is similar to the one arising in second best analyses of environmental taxation (Jacobs and van der Ploeg, 2019).
4 Theoretical investigation

4.1 Political equilibrium

Let us start with individual decisions. Environmental quality enters utility as a pure externality: each consumer takes the quality of the environment as given when she/he maximizes (1) subject to (2), and \( c, d \geq 0 \). Solving for this program, we identify a (unique) critical income level

\[
\tilde{w}(t) = \frac{(\gamma \pi)^{\frac{1}{1-\alpha}}}{1-t}, \text{ with } \tilde{w}'(t) = \frac{\tilde{w}(t)}{1-t} > 0,
\]

that determines whether or not a consumer purchases the green good. In fact, a consumer devotes a positive amount of resources to green consumption if and only if she earns enough money, i.e., \( w > \tilde{w}(t) \). For an interesting problem, this threshold must belong to \((w_m, \infty)\).

This leads us to identify two boundaries, \( t_m \) and \( t_M \) with \( t_m = 1 - w_m^{-\frac{1}{\alpha}}(\gamma \pi)^{\frac{1}{1-\alpha}} < 1 = t_M \).

The lower bound \( t_m \) can be positive or negative, which does not matter for the analysis.

Whatever the tax rate \( t \in (\max\{0, t_m\}, t_M) \), the population can be split into two groups, respectively labeled by \( N \) and \( G \) (for “non green” or ‘neutral” vs. “green” consumers). Membership to a particular group is determined by the individual’s income. It is a member of group \( N \) whenever \( w \in (w_m, \tilde{w}(t)) \), otherwise she belongs to \( G \) (for \( w \in (\tilde{w}(t), \infty) \)). So wealthier people form group \( G \) while poorer folks are part of group \( N \). This dichotomy is in line for instance with descriptive statics provided by Liu (2014) that illustrate that demand for hybrid cars essentially arises from people that belong to the upper income classes. Decisions made by individuals within each group are summarized by the following
equations (for decisions we use a superscript letter):

Group $N$: \[ d^n = 0, \quad c^n(w,t) = (1 - t)w, \quad (4) \]

Group $G$: \[ d^g(w,t) = \pi^{-1} \left( (1 - t)w - \left( \gamma \pi \right)^{1\alpha} \right), \quad c^g = \left( \gamma \pi \right)^{1\alpha}. \quad (5) \]

A member of group $N$ can not afford the green good and thus devotes her entire income to purchasing the environmentally neutral and cheaper good. By contrast, a green consumer spends a constant amount of money on the neutral good and the extra money goes to the green good.\(^{22}\) Hereafter, we will make use of the indirect utility functions:

\[ v^g(t,w) = \pi^{-1} \left( (1 - t)w - \left( \gamma \pi \right)^{1\alpha} \right) + \frac{\gamma}{\alpha} \left( \gamma \pi \right)^{\alpha}, \]

\[ v^n(t,w) = \frac{\gamma}{\alpha}((1 - t)w)^\alpha. \]

Both are decreasing in the tax rate, and the larger the income, the larger the marginal disutility from taxation.

The level of environmental quality is obtained by adding environmental public expenditure, which is financed by the income tax (under the balanced budget rule), and aggregate private consumption of the green good:

\[ Q(t) = t\mu + \int_{\tilde{w}(t)}^{\infty} d^g(w,t)f(w)dw. \]

Environmental quality is increasing and convex in the tax rate. In fact, increasing the tax has two opposite effects on $Q$:

\[ Q'(t) = \mu + \int_{\tilde{w}(t)}^{\infty} \frac{\partial d^g(w,t)}{\partial t} dw. \quad (6) \]

A higher tax means more public expenditure for a given tax base, which is good for the

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\(^{22}\)The quasilinear utility explains why $c^g$ is constant. This is innocuous for the analysis.
environment. However, it diverts consumers away from the green good, which negatively affects $Q$. The overall effect remains positive though: $Q'(t) > 0$.

With all this information in hand, we can move back to the analysis of the electoral competition. To deal with this issue, we consider a probabilistic voting model as in de la Croix and Doepke (2009), Arcalean and Schiopu (2016), and Melindi-Ghidi (2018). Probabilistic voting, by “smoothing” the payoffs of parties involved in the political game, generally ensures the existence of a Nash equilibrium in situations where the majority voting rule does not. The key point with probabilistic voting is that it introduces “noise” in the outcome of the electoral process. Indeed, it is assumed that besides the policy platforms the different – generally two – candidates offer, voters’ preferences also depend on a non-policy outcome of the election. In the literature, this additional concern is typically associated with the ideology. In the end, for any policy platform, a party does not know the exact number of voters who will support it. Indeed, contrary to standard (majority) voting models, individuals belonging to the same economic group do not have the same ideological preferences. So the best a party can do is to evaluate its vote share that is defined as the sum of probabilities that people in each group vote for it multiplied by the relative group size. A party’s objective is then to choose the platform that maximizes its vote share. As in a two-party electoral competition, parties’ decision problems are symmetric, one generally focus on the symmetric Nash equilibrium in pure strategies of the zero-sum game. It is then pretty easy to show that parties’ equilibrium policies maximize the following utilitarian social welfare function:

$$
\int_{w_{\min}}^{\tilde{w}(t)} (v^n(t, w) + \beta Q(t))\theta(w)f(w)dw + \int_{\tilde{w}(t)}^{\infty} (v^g(w, t) + \beta Q(t))\theta(w)f(w)dw,
$$

23See the above mentioned papers and references therein for details.

24If there are two parties $A$ and $B$, then the probability that an individual votes for party $A$ is an increasing function of the difference of utility levels brought by each party once elected. This function is a cumulative distribution function that captures how ideology is spread in society.
where $\theta(w)$ represents the political power of a voter with income $w$. For simplicity, we assume away this particular dimension of the problem by considering that citizens share the same political power, i.e., $\theta(w) = 1$ for all $w$. This implies that the only weights that matter in the objective function, denoted by $W(t)$, are given by the relative size of each group, and this function reduces to:

$$W(t) = \int_{\bar{w}(t)}^{\bar{w}(t)} v^n(t, w) f(w) dw + \int_{\bar{w}(t)}^{\infty} v^g(w, t) f(w) dw + \beta Q(t).$$

On may note that its first derivative,

$$W'(t) = \beta Q'(t) + \int_{w}^{\bar{w}(t)} \frac{\partial v^n(w, t)}{\partial t} f(w) dw + \int_{\bar{w}(t)}^{\infty} \frac{\partial v^g(w, t)}{\partial t} f(w) dw, \quad (7)$$

illustrates the simple trade-off faced by the economy when collectively deciding upon the public policy. Increasing the tax rate induces a marginal environmental benefit, hereafter $MB$ (first term). But it also comes with marginal costs because of the decrease in the indirect utility of both groups, resulting from the decrease in consumptions, denoted respectively by $MC^n$ and $MC^g$ (last two terms).

Overall, solving for the political equilibrium boils down to searching for the tax that maximizes $W(t)$. Assessing its properties, we first identify a threshold $t^c$, with

$$t^c = 1 - \frac{1}{w_m} \left( \frac{1}{1 - \frac{\beta(k - \alpha)}{1 - \alpha}} \right)^{\frac{1}{\alpha - k}},$$

such that $W''(t) < 0 \iff t > t^c$. To ease the discussion, we assume that this critical tax rate is positive. This boils down to imposing

$$w_m > \left( \frac{1}{1 - \frac{\beta(k - \alpha)}{1 - \alpha}} \right)^{\frac{1}{\alpha - k}}, \quad (8)$$

This is not the aim of the paper to account for this additional source of heterogeneity. It would be an interesting extension of the present work though.
and denote the corresponding critical income level as \( \tilde{w}(t^c) = \tilde{w}^c \). Then, we can establish the following existence result (see the Appendix A.1):

\textbf{Proposition 1.} A necessary and sufficient condition for the existence of a unique political equilibrium associated with policy platform \((t^*, G^*)\) is \( W'(t^c) > 0 \). This is equivalent to imposing

\[ \pi > \bar{\pi}(\beta) \equiv \beta^{-1} \left( 1 + \frac{\beta(k - \alpha)}{1 - \alpha} \right)^{\frac{k - \alpha}{k - \alpha}}. \tag{9} \]

The existence condition (9) can be interpreted in terms of two critical parameters of the current analysis, the environmental concern \( \beta \) and the relative price of green goods, \( \pi \). Indeed, the threshold \( \bar{\pi}(\beta) \) is decreasing in \( \beta \). It is difficult to get a precise estimate of the environmental concern. Following a tradition that finds its origin in Sociology, scholars run surveys that include a series of questions to elicit respondents’ WTP for environmental protection, knowledge about environmental issues, and so on and so forth. In the end, they build an environmental concern index with the purpose of identifying its main drivers.\(^{26}\) They all reach a consensus regarding the most important determinant of environmental concern, that is the level of wealth. When it comes to the representation of the utility function, one may argue that as individuals prioritize consumptions, the relative weight of the environment should be lower than one. From a more aggregate perspective, findings of this literature strongly suggest that on average, environmental concern should be the highest in the richest countries. Taking \( \beta \in (0, 1) \), we obtain that \( \bar{\pi}(\beta) > 1 \). Intuitively, not only people should care enough about the environment but also the price of the green good should be high enough (but the higher \( \beta \) the less stringent the condition on \( \pi \)) for them to be willing to incur the cost of the public provision of environmental quality. A political equilibrium of this sort is then more likely to arise in relatively rich countries, like OECD and European countries.

The equilibrium tax \( t^* \) is defined implicitly only. But it is quite easy to check that \( t^* \)

\(^{26}\)For an interesting work representative of this line of research, see \textit{Franzen and Meyer (2009)}.
is increasing in both $\beta$ and $\pi$. On the one hand, a larger $\beta$ means that the population cares more about the environment, which raises the incentive to tax incomes in order to finance public expenditure on the environment. On the other hand, a larger $\pi$ makes green consumption more costly, thereby lowering it. Thus taxation and public provision of the environment should increase as a compensation.

The next Section is devoted to a comprehensive comparative statics exercise.

4.2 Impact of a change in the income distribution

We want to explain how public policy changes as a response to the two important statistics of the income distribution, which are the average and standard deviation. Intuition suggests that taxation and environmental public expenditure should be higher, the larger the average income. Things may not be so obvious when it comes to the impact of $\sigma$. The main purpose of this Section is then to understand what is the impact of inequalities on the public provision of environmental quality.

In order to address these issues, we proceed in two separate steps. We assess the change in the equilibrium tax resulting from 1/ a variation in the average income, taking the standard deviation as given, and 2/ a variation of the standard deviation taking the average income as given. Our analysis, summarized in the Appendix A.2, leads to the following results:

**Proposition 2.**

- An increase in the average income translates into an increase in the equilibrium tax, $t^*$, for given standard deviation.

- There exists a critical tax rate $t^* \in (t^c, 1)$ such that an increase in the standard deviation induces a decrease in the equilibrium tax, $t^*$, for given average income, if and only if $t^* < t^c$. 
Not surprisingly, we find that rich countries – that is, countries where the average income is high – levy a larger fiscal revenue to finance environmental quality than poor countries. This outcome is very much in line with stylized facts reported in Figure 1. A change in $\mu$ has repercussions on all the components of marginal welfare (7), especially on the $MB$ through a tax base effect (see the first terms in (6) and (7)). Dissecting the various channels through which $\mu$ impacts $W'(t^*)$ would be an interesting yet unnecessary exercise. Indeed, the comparative statics result is unambiguous, and the analysis would share many similarities (and then be redundant) with what comes next, that is the most important study of the impact of $\sigma$ on public policy.

Hereafter we focus on the interpretation of the impact of inequalities, captured by the standard deviation.\footnote{In fact, working with a constant mean, a variation of $\sigma$ exactly corresponds to a variation of the coefficient of variation, that is a measure of the level of inequality. It is different yet positively correlated to the Gini index. See also the discussion in Section 5.} A change in the standard deviation affects marginal welfare by changing not only the distribution of the population between the two groups (size effect), but also the composition of each group (composition effect).

Denote the size of group $I = G, N$ as $N^i$ with $i = g, n$, given that $N^g = 1 - N^n$, and $\bar{w}^* = \bar{w}(t^*), \bar{w}^s = \bar{w}(t^s)$.

The overall impact of a marginal change in $\sigma$ can be decomposed into three terms:

$$\frac{\partial^2 W(t^*)}{\partial t \partial \sigma} = \frac{\partial MB}{\partial \sigma} - \left[ \frac{\partial MC^g}{\partial \sigma} + \frac{\partial MC^n}{\partial \sigma} \right],$$

with

$$\frac{\partial MC^g}{\partial \sigma} = \frac{\pi^{-1} \bar{w}^*}{k-1} \left( \frac{N^g}{N^g} \frac{\partial k}{\partial \sigma} + k \frac{\partial N^g}{\partial \sigma} \right),$$

$$\frac{\partial MB}{\partial \sigma} = -\beta \frac{\partial MC^s}{\partial \sigma},$$

$$\frac{\partial MC^n}{\partial \sigma} = \frac{\pi^{-1} \bar{w}^*}{k-\alpha} \left[ -\alpha \frac{\partial k}{\partial \sigma} \left( \left( \frac{w_m}{\bar{w}^s} \right)^{\alpha} - 1 + N^n \right) + k \frac{\alpha}{\bar{w}^s} \left( \frac{w_m}{\bar{w}^s} \right)^{\alpha-1} \frac{\partial w_m}{\partial \sigma} + k \frac{\partial N^n}{\partial \sigma} \right].$$
and $\frac{\partial w_m}{\partial \sigma}, \frac{\partial k}{\partial \sigma} < 0$.

We observe that $MB$ and $MC^g$ move in opposite direction, in a proportional way. It means that these two components add up to change marginal welfare. So let us focus on the marginal cost components, starting with $MC^g$. The sign of $\frac{\partial MC^g}{\partial \sigma}$ is determined by the aggregation of the composition effect (first term between the parentheses, positive) and the size effect (second term). The composition effect works as follows. The interval of incomes associated with the green group is invariant but the density of people at each income levels within this interval is affected by the variation of $\sigma$. Now it turns out that the density increases for the highest income levels. Given that the disutility of taxation (resulting from the decrease in green consumption) is larger, the larger the income, this composition effect tends to increase $MC^g$. The size effect may add or, on the contrary, alleviate the composition effect depending on whether or not the size of group $G$ increases as a result of the increase in the level of inequalities. As to group $N$, the same two effects are at play but the composition effect features another component. Indeed, the lower bound of the interval of incomes corresponding to that group, $w_m$, decreases as a result of the increase in $\sigma$. According to this additional part (second term between the parentheses in the expression of $MC^n$, negative), and other things equal, there are more people located around the lower income levels as $\sigma$ increases. This pushes toward a lower $MC^n$. Let us call it the dispersion effect, which is part of the composition effect.

Given the expressions above, a particular comparative statics result can be obtained \emph{via} different combinations of the sign of $\frac{\partial MC^g}{\partial \sigma}$ and $\frac{\partial MC^n}{\partial \sigma}$. And it is unclear a priori what

\footnote{It may or may not decrease for income levels closed to the threshold, $\tilde{w}^*$, that determines the division of the population into groups $N$ and $G$ at the equilibrium.}
is the relevant case to consider.\textsuperscript{29} We can however ease the discussion simply by imposing:

\[ \pi < \exp^{\frac{k-1}{k}}. \]

So we set an upper bound on the price of the green good. For example, taking \( k = 3 \), \( \exp^{\frac{k-1}{k}} \approx 1.95 \): we ask the price of the green goods to be less than twice the price of the other non-green goods. This seems acceptable given the figures we provided earlier for organic food and hybrid vehicles.

In this situation, \( \frac{\partial MC^g}{\partial \sigma} < 0 \), which implies \( \frac{\partial N^g}{\partial \sigma} < 0 \). Green people become less numerous and the size effect dominates the composition one. As a result, their (positive) contribution to the marginal cost of taxation – remind that green consumption, \( d^g \), is decreasing in the tax – shrinks. And so does their (negative) contribution to the marginal benefit for the very same reason. Measuring the impact of a change in \( \sigma \) on the cost borne by group \( N \) is less simple. The relative size of group \( N \) increases. So the size effect is positive. In addition, their density at any income level in the interval \((w_m, \tilde{w}^*)\) decreases \( (\frac{\partial f(w)}{\partial \sigma} < 0) \), while this interval expands thanks to the dispersion effect. Overall, it is unclear whether the composition effect is positive or negative. This is where the ranking between the critical levels \( \tilde{w}^* \) and \( \tilde{w}^s \) comes into the picture.

In Proposition 2, we prove that when the equilibrium tax is pretty high so that \( \tilde{w}^* > \tilde{w}^s \), the composition effect is negative and strong enough to offset – partly or totally – the size effect on group \( N \): either \( MC^n \) decreases, or it increases but overall we have:

\[ \left| \frac{\partial MC^n}{\partial \sigma} \right| < \left| \frac{\partial MC^g}{\partial \sigma} - \frac{\partial MB}{\partial \sigma} \right|. \]

This is all driven by the dispersion effect that finds full expression when the threshold

\textsuperscript{29}The problem is that we do not have the explicit form of \( t^* \) and \( t^s \). As a result, \( \frac{\partial^2 W(t^*)}{\partial t \partial \sigma} > 0 \) can be the outcome of \( MC^g \) increasing and \( MC^n \) decreasing, the latter effect prevailing over the former etc. This leads to several (four) possibilities.
$\tilde{w}^*$ is high enough (compared to $w_m$). So we can conclude that $\frac{\partial W(t^*)}{\partial \sigma} > 0$: $t^*$ should increase when the standard deviation goes up.

In the opposite situation, $\tilde{w}^* < \tilde{w}^s$, $MC^n$ necessarily increases. This time, the composition effect remains positive but is dominated by the size effect as:

$$\left| \frac{\partial MC^n}{\partial \sigma} \right| > \left| \frac{\partial MC^g}{\partial \sigma} - \frac{\partial MB}{\partial \sigma} \right|,$$

which gives $\frac{\partial W(t^*)}{\partial \sigma} < 0$, calling for a decrease in $t^*$.

The statement in Proposition 2 is interesting because it highlights that the impact of inequalities on the public policy varies depending on a country's characteristics. This finding quite remarkably echoes the stylized fact reported in Figure 2. Indeed splitting roughly the observations into two groups, we observed that the impact of inequalities on environmental spending was seemingly opposite for these two groups. However, it is fair to say that in its current version, Proposition 2 does not allow us to draw more insightful conclusions essentially because $t^*$ and $t_s$ are both endogenous variables whose ranking is a priori undetermined. This means that we need to dig deeper into the analysis to identify some condition that provides us with a clear-cut policy message. This is precisely the purpose of the next corollary that brings together the results of Proposition 1 and 2.

**Corollary 1.** From the existence condition (9), we get:

$$\lim_{\pi \to \pi(\beta)} t^* = t^c.$$

By construction, we have $t^c < t^s$. In that situation, an increase in inequality induces a reduction of the equilibrium tax, $t^*$, and of the resulting public provision of environmental quality, $G^*$, for given average income. By a continuity argument, the same conclusion holds true for $\pi$ higher than, but close enough to, the threshold $\pi(\beta)$.

Remind that, from the discussion following Proposition 1, the threshold $\pi(\beta)$ is larger...
than 1 and decreasing in $\beta$.\footnote{The former property holds for $\beta$ low enough, and is always satisfied for $\beta \in (0, 1)$.} In addition, the critical level $t^*$ identified in Proposition 2 does not depend on $\pi$. This all points to the following conclusion. In countries where people display enough concerns for the environment ($\beta$ high enough, which in turns implies $\pi(\beta)$ low enough), and where the relative price of green goods is above 1 but remains mild, we expect that a higher level of inequality negatively impairs the public provision of environmental quality.

The analysis conducted up to now unveils the important forces that drive environmental policy changes, as a result of a variation of the level of inequality. Though the overall impact of inequality on taxation and environmental spending can clearly be characterized in the special case of Corollary 1, the general impact remains ambiguous (Proposition 2 and following discussion). Besides, the sufficient condition of Corollary 1 involves parameters whose values in the different countries are poorly known. To uncover how inequality affects environmental policy, we then need to perform an econometric analysis. Of course, we should ultimately check that our simple theory provides a meaningful explanation of the obtained empirical results.

5 Empirical analysis

This Section aims at examining the general link between the income distribution and the environmental policy. In our model, public environmental expenditure are monotone increasing in the tax rate. So we can work with either variables and select the former due to data availability and modeling options. We first provide a short description of the panel dataset we use in the empirical investigation. We then describe our econometric models and go over the main results.
5.1 Data description

We build our dataset using data from Eurostat, the statistical office of the European Union. Values for environmental protection expenditures include a large set of items and ensure a high degree of international comparability. The database captures all government expenditures in terms of waste management, waste water management, pollution abatement, protection of biodiversity and landscape, R&D environmental protection, and others.\footnote{According to the European System of Accounts (ESA), examples of environmental protection expenditures are: “investments in clean technologies, restoring the environment after it has been polluted, recycling, the production of environmental goods and services, conservation and the management of natural assets and resources”.
}

In the coming analysis, we consider two main dependent variables that are extracted from the category government expenditure in environmental protection.\footnote{This dataset provides the total government expenditure by functions and by type of institution. In this dataset there are data for 31 European countries over the period 1996-2019. List of countries: Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, UK. Variables are extracted from gov\_10a\_exp.} As first dependent variable, we take general government environmental protection expenditure. To account for different types of political systems, we also deal with a second dependent variable: environmental protection expenditure by local governments.\footnote{According to the ESA, general governments are “institutional units which are non-market producers whose output is intended for individual and collective consumption, and are financed by compulsory payments made by units belonging to other sectors, and institutional units principally engaged in the redistribution of national income and wealth”. Local government are: “public administration whose competence extends to only a local part of the economic territory, apart from local agencies of social security funds”.} In Section 5.4, we finally consider a third dependent variable as robustness test: total environmental taxation.\footnote{Even though the theoretical model does not consider environmental taxation, we look at this outcome variable because we expect that taxation and public environmental spending move in the same directions. There are 34 countries in the latter dataset, with Liechtenstein, Serbia and Turkey in the sample.}

We express the dependent variable in per-capita terms since the variable $Q$, introduced in the model, also represents the average environmental quality when population size is normalized to 1. The main explanatory variables are the GDP per capita in current euros
and the Gini index, taken from the European Union Statistics on income and living conditions survey (EU-SILC). In Section 4.2, we assessed the impact of income inequality on environmental policy by considering a mean preserving spread – that is a change in the standard deviation for given average income – which boils down to measuring inequality via the coefficient of variation (CV). Here we use the Gini index because it is strongly (positively) correlated with the CV, and this is the measure of inequality the most commonly used in the literature. We also consider some demographic indicators by adding the variables *density* – measured by habitants by km – and *population growth* to the database. We include both variables because they are potential determinants of environmental pressure.\(^{35}\)

Summary statistics, for the main variables are displayed in Table 1 while the distribution of the environmental expenditure levels are shown in Figure 3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N. Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gen Gov Exp</td>
<td>551</td>
<td>209.05</td>
<td>178.52</td>
<td>3.58</td>
<td>963.32</td>
</tr>
<tr>
<td>Loc Gov Exp</td>
<td>553</td>
<td>134.82</td>
<td>133.41</td>
<td>0</td>
<td>630.56</td>
</tr>
<tr>
<td>GDP</td>
<td>621</td>
<td>26507.49</td>
<td>18446.82</td>
<td>1758.88</td>
<td>103464.6</td>
</tr>
<tr>
<td>Population (millions)</td>
<td>621</td>
<td>18.8</td>
<td>23.9</td>
<td>0.29</td>
<td>83.0</td>
</tr>
<tr>
<td>Gini Index</td>
<td>621</td>
<td>29.94</td>
<td>4.66</td>
<td>20</td>
<td>46</td>
</tr>
</tbody>
</table>

Source: Authors from the Eurostat dataset over the period 1996 - 2019.

5.2 Specification tests and Econometric model

Our basic empirical model is given by the following equation:

\[
y_{it} = \alpha + \beta_1 Gini_{i,t-k} + \beta_2 Gini^2_{i,t-k} + \beta_3 GDP_{i,t} + \beta_j X'_{i,t} + u_{i,t} \tag{10}\]

\(^{35}\)Focusing on US cities, Ribeiro et al. (2019) find a positive correlation between population size changes and CO\(_2\) emissions, while the correlation turns negative for population density.
where \( i \) denotes the cross-sectional unit (country) and \( t \) the time period (year). The variable \( y_{it} \) is the log of the GDP per capita, \( \text{Gini}_{i,t-k} \) is the \( k \)-year lagged Gini index, and \( X_{it} \) is a vector of controls, \( x_{ijt} \). We take a five-year lag for the Gini index \( (k = 5) \) to account for causality between income inequality and environmental policy.\(^{36}\) Gini observations are average values over 5 years. We introduce the lagged Gini-squared in the model to account for possible non-monotonicity, as suggested the stylized facts described in Figure 2 and later confirmed by our equilibrium analysis. As mentioned above, the vector \( X_{it} \) controls for density and population growth. Parameter \( \alpha \) is a common intercept, \( \beta_j \) are coefficients associated with the independent variables, and \( u_{i,t} \) is the error term. Table 2 summarizes the transformations we made in the coming regressions, for each country \( i \) and period \( t \).

There are different approaches to deal with cross-country panel data. Fixed-effect and random-effect models are the most common.\(^{37}\) We use the following decomposition:

\[
u_{it} = \mu_i + \epsilon_{i,t},\]

where \( \mu_i \) is an unobserved individual specific effect, while \( \epsilon_{i,t} \) refers to an idiosyncratic error term. Whether \( \mu_i \) is treated as a random or fixed effect determines the estimation method. To decide which model better fits with our panel dataset, we run

\(^{36}\)In Section 5.4, we do provide the estimation results with the current \( \text{Gini}_{i,t} \), showing that results are not affected. However, the use of lagged regressor reduce the number of observations from 621 to 548.

\(^{37}\)We exclude pooled OLS since F-tests reject at 1% level equal fixed effects across units for all dependent variables.
Table 2: Data description

<table>
<thead>
<tr>
<th>Variable</th>
<th>Original Eurostat variable</th>
<th>Transformed variable</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gen Gov Exp</td>
<td>Government expenditure in environmental protection by general government in million current euros</td>
<td>( \log \left( \frac{\text{Gov}<em>{10a \exp</em>{1,i,t}} \times 10^6}{\text{Population}_{i,t}} \right) )</td>
</tr>
<tr>
<td>Loc Gov Exp</td>
<td>Government expenditure in environmental protection by local government in million current euros</td>
<td>( \log \left( \frac{\text{Gov}<em>{10a \exp</em>{2,i,t}} \times 10^6}{\text{Population}_{i,t}} \right) )</td>
</tr>
<tr>
<td><strong>Independent variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP per capita</td>
<td>GDP in million current euros</td>
<td>( \log \left( \frac{\text{nama}<em>{10 \exp</em>{1_t}} \times 10^6}{\text{Population}_{i,t}} \right) )</td>
</tr>
<tr>
<td>Gini index lagged</td>
<td>Gini coefficient of equivalised disposable income</td>
<td>( \log \text{gini}<em>{i,t} = \frac{\sum</em>{j=5}^{10} \text{gini}_{i,t-j}}{5} )</td>
</tr>
<tr>
<td>Population density</td>
<td>Population over land cover in total</td>
<td>( \frac{\text{Population}<em>{i,t}}{\text{Total Landcover}</em>{i}} )</td>
</tr>
<tr>
<td>Population growth</td>
<td>Population change on 1 January</td>
<td>( \frac{\text{Population}<em>{i,t} - \text{Population}</em>{i,t-1}}{\text{Population}_{i,t}} )</td>
</tr>
</tbody>
</table>

Source: Authors from the Eurostat dataset.

First, we run the Hausman specification test (Hausman, 1978). The test rejects the null hypothesis (preferred model is random effects), which suggests that unobserved country-specific effects are better modeled by a fixed-effect model.\(^{38}\) Even though the Hausman test is valid under restrictive assumptions and does not support robust standard errors, it clearly indicates the existence of a correlation between the individual errors and the regressors in the model that should be analyzed with a fixed-effects model.

Next, we check if time dummies among the regressors should be included in the regression. Precisely, we test if the dummies for all years are equal to 0 and reject this assumption for both dependent variables at 1% significance level. Inclusion of time dummies is important here, given that environmental policies have been influenced by European and international treaties for the last 25 years, and cannot be fully explained by variations in observed socio-economic variables at country level. Moreover, introducing both time and

\(^{38}\)Note that we do not include to our regression other control variables of the database because models with fixed-effects do not allow to estimate the coefficients of time-invariant regressors, such as gender, land, education, etc.
country invariant fixed-effects might adjust for potential omitted-variable bias.

Before moving to the analysis, we should also consider tests for heteroskedasticity, autocorrelation and cross-sectional dependence. The modified Wald test for groupwise heteroskedasticity in fixed-effect models and the Wooldridge test for autocorrelation show that parameters can be consistently estimated using robust or clustered standard errors, that is, by treating each country as a cluster (Wooldridge, 2010). Since the Hausman test does not support robust standard errors, we implement a test of overidentifying restrictions (Sargan-Hansen test) robust to arbitrary heteroskedasticity and within-group correlation (Schaffer and Stillman, 2006). This test again rejects the null hypothesis (preferred model is random effects) and clearly suggests we implement fixed-effect models at 1% significance level for both outcome variables. We then verify the presence of cross-sectional independence within the residuals using the test of Pesaran (Pesaran, 2020). This test of cross sectional dependence provides no evidence for rejecting the null hypothesis of no cross-sectional dependence at 5% level. However, since the average absolute correlation of the residuals is quite high for both outcome variables, we decide to perform robustness estimations using Driscoll and Kraay standard errors, usually implemented in presence of cross-sectional dependence (Hoechle, 2007).

The final check has to do with the model specification. More precisely, we implement the test developed by Lind and Mehlum (2010) to check the existence of a non-monotone/linear relationship between our dependent variables and the independent variable measuring income inequality, i.e., the Gini index. The null hypothesis is either a monotone or inverted $U-$shaped relationship. The test rejects the null hypothesis for both dependent variables, showing the existence of either $U-$shaped or a $J-$shaped relationship between public expenditure on the environment and the Gini index. Moreover, the Ramsey Reset Test (Ramsey, 1969) suggests no evidence of functional form misspecification, confirming that

\[39\text{Results are robust with 5-years lagged and non-lagged Gini index.}\]
the model seems well specified.

<table>
<thead>
<tr>
<th>Test</th>
<th>Gen Gov Exp</th>
<th>Loc Gov Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hausman Test</td>
<td>$\chi^2(5) = 13.88$</td>
<td>$\chi^2(5) = 75.24$</td>
</tr>
<tr>
<td>H0: Random vs. Fixed</td>
<td>$Pr &gt; \chi^2 = 0.0164$</td>
<td>$Pr &gt; \chi^2 = 0.0000$</td>
</tr>
<tr>
<td>Time-fixed Effects Test</td>
<td>$F(18,383) = 3.69$</td>
<td>$F(18,385) = 2.66$</td>
</tr>
<tr>
<td>H0: No Time Dummies</td>
<td>$Pr &gt; F = 0.0000$</td>
<td>$Pr &gt; F = 0.0003$</td>
</tr>
<tr>
<td>Modified Wald Test</td>
<td>$\chi^2(28) = 4593.75$</td>
<td>$\chi^2(28) = 6809.86$</td>
</tr>
<tr>
<td>H0: $\sigma(i)^2 = \sigma^2 \forall i$</td>
<td>$Pr &gt; \chi^2 = 0.0000$</td>
<td>$Pr &gt; \chi^2 = 0.0000$</td>
</tr>
<tr>
<td>Wooldridge Test</td>
<td>$F(1,27) = 33.626$</td>
<td>$F(1,27) = 13.660$</td>
</tr>
<tr>
<td>H0: No First-order Autocorrelation</td>
<td>$Pr &gt; F = 0.0000$</td>
<td>$Pr &gt; F = 0.0010$</td>
</tr>
<tr>
<td>Sargan-Hansen test</td>
<td>$\chi^2(15) = 1700.827$</td>
<td>$\chi^2(14) = 7106.747$</td>
</tr>
<tr>
<td>H0: Random vs. Fixed (robust)</td>
<td>P-value = 0.0000</td>
<td>P-value = 0.0000</td>
</tr>
<tr>
<td>Av. abs. value diagonal elements</td>
<td>0.341</td>
<td>0.365</td>
</tr>
<tr>
<td>H0: Cross Sectional Independence</td>
<td>Pr = 0.316</td>
<td>Pr = 0.092</td>
</tr>
<tr>
<td>Test of presence of a U-shape</td>
<td>t-value=2.70</td>
<td>t-value=2.09</td>
</tr>
<tr>
<td>H0: Monotone or Inverse U-shape</td>
<td>$P &gt;</td>
<td>t</td>
</tr>
<tr>
<td>Ramsey Reset Test</td>
<td>$F(2,27)=1.35$</td>
<td>$F(2,27)=2.09$</td>
</tr>
<tr>
<td>H0: Functional Form Misspecification</td>
<td>$Pr &gt; F = 0.2764$</td>
<td>$Pr &gt; F = 0.1437$</td>
</tr>
</tbody>
</table>

Note: to perform the Hausman test, we have scaled the variable population growth (x10) to obtain coefficients on a similar scale.

### 5.3 Empirical results

Following the recommendations obtained from the above tests, we estimate the equation (10) considering country fixed-effects ($\mu_i$), time dummies affecting all countries uniformly ($\lambda_t$) and robust standard errors clustered by country.

Table 4 summarizes the results when general government expenditure on environmental protection per capita (EPPC) is the dependent variable, while Table 5 considers local government EPPC as outcome. In both Tables, column (1) does not incorporate the time effects and the control variables, thereby focusing only on the effect of GDP per capita and the Gini index. In column (2), we add the time dummies to the regression. Column
Looking at the effect of the GDP per capita on environmental expenditure, coefficients are positive for both dependent variables. This positive effect is consistent with both the theoretical results and conclusions drawn in the related literature.

The effect of income inequality on environmental policy is captured by the coefficients $Gini$ and $Gini^2$. All specifications exhibit a negative effect of the Gini index on the variables describing the EPPC. A higher Gini index is associated with the following first order effect: it makes the environmental policy less stringent. On average, the coefficients captured by general or local spending are the same for $Gini$: -0.22. If we consider general government expenditures in EPPC, the effect seems to be more volatile, because the coefficients vary
Table 5: Local Government Expenditures in EPPC

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
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<tbody>
<tr>
<td>Gini</td>
<td>-0.2060**</td>
<td>-0.2315**</td>
<td>-0.2064**</td>
<td>-0.2360**</td>
<td>-0.2055*</td>
</tr>
<tr>
<td></td>
<td>(0.076)</td>
<td>(0.085)</td>
<td>(0.094)</td>
<td>(0.088)</td>
<td>(0.111)</td>
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<tr>
<td>Gini²</td>
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<td>0.0041***</td>
<td>0.0038**</td>
<td>0.0042***</td>
<td>0.0037*</td>
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<tr>
<td></td>
<td>(0.012)</td>
<td>(0.014)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
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<tr>
<td>GDP</td>
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<td>0.6367***</td>
<td>0.6597***</td>
<td>0.5524*</td>
<td>0.6398**</td>
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<td></td>
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<td>(0.205)</td>
<td>(0.236)</td>
<td>(0.287)</td>
<td>(0.260)</td>
</tr>
<tr>
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<td>-0.0015*</td>
<td>-0.0016*</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔPop</td>
<td></td>
<td></td>
<td></td>
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<td>0.5203</td>
</tr>
<tr>
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<td></td>
<td>(0.538)</td>
<td>(0.333)</td>
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<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>R² (within)</td>
<td>0.20</td>
<td>0.28</td>
<td>0.28</td>
<td>0.29</td>
<td>0.20</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ_u</td>
<td>0.65</td>
<td>0.66</td>
<td>0.67</td>
<td>0.75</td>
<td>N/A</td>
</tr>
<tr>
<td>σ_e</td>
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<td>0.22</td>
<td>N/A</td>
</tr>
<tr>
<td>ρ</td>
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<td>0.90</td>
<td>0.90</td>
<td>0.92</td>
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</tr>
<tr>
<td>Observations</td>
<td>461</td>
<td>461</td>
<td>436</td>
<td>436</td>
<td>436</td>
</tr>
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</table>

Notes: ***p<0.01  **p<0.05  *p<0.10. Cluster-robust (1-4) and Drisc/Kraay standard errors are in parentheses. All regressions include country fixed effects. Year represents the time fixed effect. The dependent variable and GDP per capita are expressed in log.

More compared to the specification with expenditures by local government.\textsuperscript{40} Considering the $R^2$ (within), it appears that the most significant specification, i.e. columns (3) and (4) – as also indicated by the different tests conducted earlier – displays consistent coefficients on average equal to -0.25.

As of second order effects, we find that coefficients of $Gini^2$ are positive but very small for all specifications, and both for general and local government environmental spending. By conducting the simple exercise of computing the ratio $\beta_1/(2\beta_2)$, we find that the minimum is reached, on average, for $Gini = 30$. It turns out that almost 75% of our observations are below this threshold, even if it is exactly the mean of our sample, the median being at 29. When $Gini < 30$, our results clearly highlight the existence of a decreasing and convex relationship between inequality and environmental expenditure. Considering

\textsuperscript{40}For the first variable, the coefficients of specifications (1) to (5) are respectively: -0.1781,-0.2415, -0.2345, -0.2705, -0.1741.
all the observations, the interaction between outcome variables and the Gini index can be interpreted as an inverted $J$-shaped rather than an $U$-shaped relationship. This means that an increase in the Gini index is first associated with a decrease in environmental expenditure at a decreasing rate, and from $Gini = 30$ onwards, the effect turns positive though the connection is weaker.\footnote{We check this claim with some robustness tests run in Section 5.4.}

We can provide an intuition of this result that is perfectly consistent with our theory.\footnote{Even though our theoretical analysis was somehow inconclusive on the impact of inequality, in the general case.} The basic idea is to highlight how the starting point, i.e., the equilibrium situation “before” a change occurs, affects the analysis of the impact of a mean preserving spread (MPS) on public spending. For that purpose, we first need to acknowledge the existence of a strong negative correlation between per capita income and inequality, that is between $\mu$ and $\sigma$, in European countries (Pearson correlation coefficient of $-0.421$, $p-value = 0.000$). Then there are two cases to consider: an initially high $\sigma$/low $\mu$ vs. the opposite. Hereafter, we discuss the first case, the analysis of the second case follows by symmetry. A low $\mu$ means a low equilibrium tax and a low level of public spending. In turn, the income threshold is low as well. Thus, rich but also middle income people consume the green good, which tends to “compensate” for the low level of public spending. So, there is a substitution effect at work: green consumption supplants environmental public spending. In this situation, following a MPS, the middle income class shrinks whereas the mass of people located at both tails of the income distribution rises (there are more rich and more poor people at the same time). This affects green consumption as well as environmental expenditure, by the substitution effect. Thus, a MPS indeed involves two opposing effects: the decrease in the middle class impairs green consumption, while the increase in the mass of rich tends to stimulates it. But in this case, because a large part of green consumption comes from the middle income class, the negative effect should be larger than the positive one. This means that when
inequality is already high, following a further increase in inequality, higher public spending is needed to compensate for the lower green consumption.

Finally, the control variables – density and population growth – effects on governmental expenditure in EPPC are given by the coefficients $Density$ and $\Delta Pop$, respectively. The introduction of both controls increases slightly the significance of the model, the $R^2$ (within) for general (local) government spending is equal to 0.35 (0.29) with both controls while it is equal to 0.34 (0.28) without them. However, the most important features of the specifications come from the inclusion of time and country fixed effects with the use of clustered standard errors. They allow us to control for most of the unobservable interactions which are not captured by the model and might account for potential omitted-variable bias.

### 5.4 Robustness checks

In this section, we implement three robustness tests to confirm our empirical results. We first study the non-linear vs. non-monotone relationship between inequality and public expenditure in environmental protection. As previously discussed, the test of Lind and Mehlum (2010) rejects the null hypothesis of either a monotone or inverted $U$-shaped relationship. Looking at Figure 2, the data seems to be better fitted by an inverted $J$-shaped rather than a $U$-shaped relationship. According to Haans et al. (2016), several conditions have to be met in order to confirm the existence of a $U$-shaped curve. First, the coefficients associated with both the linear and quadratic variables must be significant. Second, the turning-point should not be “extreme”: it has to lie strictly inside the sample. Finally, the slope on both sides of the $U$-shaped curve must be steep enough. While the first two conditions are verified in our analysis, the third condition is not, because the coefficients associated with Gini$^2$ are very small, ranging from 0.0030 to 0.0045.

To test the shape of the relationship between Gini index and public expenditure, we
conduct two more regressions on two sub-samples, as suggested by Qian et al. (2010). The lower (upper) sub-sample comprises the observations with a Gini smaller (greater) than or equal to the average threshold value, that is $Gini = 30$. We choose this value because it corresponds to the average of all the minima found in the regressions, and it is also the mean of the entire sample. According to this test, if the $U-$shaped relationship were to be confirmed, we would observe a negative and significant coefficient for the lower sub-sample, and positive and significant coefficient for the Gini in the upper sub-sample. Table 6 describes the results for the lower and upper samples.

As expected, for the lower sub-sample, coefficients of the linear Gini are significant whatever the specification. We also find that coefficients are greater than – but close enough to – those obtained when considering the entire sample. The $Gini^2$ coefficient is not always significant. For the upper sample, the coefficients of Gini index (linear and quadratic term) are not significant whatever the outcome variable considered (see Table 6). Therefore, considering all the sample, the interaction between inequalities and per capita environmental public policy is indeed described by an inverted $J-$shaped curve.

In a second robustness test, we consider a third dependent variable, which is the total environmental taxation per inhabitant. Overall, we get the same qualitative results as those found with environmental expenditure. Proceeding as before, the results of the tests are similar to those we obtained with environmental protection expenditure. However, the Pesaran’s test indicates cross-sectional dependence at 1% level. Therefore, the accurate model for this outcome variable is the regression with the Driscoll and Kraay standard errors. As for government environmental spending, coefficients for the $Gini$ and the $Gini^2$

---

43 This is true whether we analyze the cluster-robust (4) or the Driscoll and Kraay standard error (5) regressions.

44 For the first dependent variable, the coefficient for the lagged Gini is -0.31 (-0.18 for (5)) instead of -0.27 (-0.17 for (5)) in the original regression. For the second variable, the coefficient obtained on the sub-sample is equal to -0.30 (-0.20 for (5)), while it was -0.23 (-0.21) in the complete analysis.

45 Total environmental taxation is extracted from $env\_ac\_tax$. It is calculated as: $\log \left( \frac{env\_ac\_tax_{i,t} \times 10^6}{Population_{i,t}} \right)$
display a significant convex and negative correlation between inequalities and environmental tax. Also, the computed turning point – close to 30 – is consistent with the previous regressions.

We report in Table 7 the estimated coefficients considering total environmental taxation a outcome variable. In all specifications, the GDP per capita has a positive and significant effect on the taxation, as also suggested by our theoretical results. The coefficients for this outcome variable are ranked from 0.66 to 0.90. The introduction of controls improves the results even if their coefficients are not significant in specifications (3) and (4). Surprisingly, the sign of the coefficient for the population growth given by $\Delta Pop$ in column 1 is negative and significant for specification (5), while it was positive for central government expenditures in environmental protection. A plausible explanation is the following: as the regressions show, omitted controls are already captured by the fixed effects (country and
time). Therefore, the residual mechanisms involved by the population density, as well as the population growth, are quite intricate and differ depending on whether taxation or public expenditure are under scrutiny.

The third robustness test performs the analysis considering the Gini index without the 5-year lag. The regression results in Table 8 are very similar to those for the Cluster-robust model (column 4). Using Drisc/Kraay standard errors (column 5), the coefficients of the Gini index for the total environmental taxation are strongly significant, while they are not significant for government expenditure variables. However, a simple comparison between $R^2$ (within) clearly show that our preferred model for environmental expenditure variables (Cluster-robust model) is also robust to time lags. Therefore, we are quite confident that the regressions exhibit the negative and convex effect of inequalities on the environmental policy.
Table 8: Robustness check with Gini not lagged

<table>
<thead>
<tr>
<th></th>
<th>Exp gen gov</th>
<th>Exp loc gov</th>
<th>Tot env tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(4)</td>
<td>(5)</td>
<td>(4)</td>
</tr>
<tr>
<td><strong>Gini</strong></td>
<td>-0.2169**</td>
<td>-0.1191</td>
<td>-0.1206*</td>
</tr>
<tr>
<td></td>
<td>(0.088)</td>
<td>(0.115)</td>
<td>(0.065)</td>
</tr>
<tr>
<td><strong>Gini^2</strong></td>
<td>0.0034**</td>
<td>0.0019</td>
<td>0.0019*</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
<tr>
<td><strong>GDP</strong></td>
<td>1.0741***</td>
<td>1.2161***</td>
<td>0.9487***</td>
</tr>
<tr>
<td></td>
<td>(0.256)</td>
<td>(0.189)</td>
<td>(0.238)</td>
</tr>
<tr>
<td><strong>Density</strong></td>
<td>-0.0009</td>
<td>-0.0013*</td>
<td>-0.0011</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td><strong>ΔPop</strong></td>
<td>-0.0755</td>
<td>-0.1320</td>
<td>0.1803</td>
</tr>
<tr>
<td></td>
<td>(5.18)</td>
<td>(3.417)</td>
<td>(5.790)</td>
</tr>
<tr>
<td><strong>Year</strong></td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
</tr>
</tbody>
</table>

|                |             |             |             |             |
| **R^2** (within)| 0.64        | 0.57        | 0.53        | 0.45        |
| **σ_u**        | 0.56        | N/A         | 0.60        | N/A         |
| **σ_e**        | 0.25        | N/A         | 0.24        | N/A         |
| **ρ**          | 0.83        | N/A         | 0.86        | N/A         |
| **Observations**| 503         | 503         | 501         | 501         |

Notes: ***p<0.01 **p<0.05 *p<0.10. Cluster-robust (4) and Drisc/Kraay (5) standard errors are in parentheses. All regressions include country fixed effects. Year represents the time effect. The dependent variable and GDP per capita are expressed in log. The Gini index is not lagged.

6 Conclusion

In this paper, we investigate the nature of the interaction between the income distribution and the environmental policy. Recent data for European countries especially reveal that the relationship between the Gini index and environmental public expenditure may not be monotone. A result that cannot be explained by the existing literature.

Our contribution is first to develop an original political economy model that helps understanding the factors that shape this relationship. Among the key factors is the opportunity for people to choose between conventional and green consumption, and to vote for the environmental policy. Both decisions are dictated by individuals’ income capacities, while both green consumption and environmental public expenditure enhance environmental quality. Our analysis shows that a change in the level of inequality induces
variations in both the size and composition of the two groups of citizens, those who consume green and those who do not. Their respective importance in turn determines whether or not such a change stimulates the public policy. We provide some conditions under which it is possible to conclude that inequality impairs the environmental policy. But in general the outcome remains unclear.

Second, we conduct a full-fledged empirical investigation of the link between the income distribution and environmental policy. The main dependent variable corresponds to the public expenditure in environmental protection. Our data consist of a sample covering thirty-one European countries over the period 1996-2019. We analyze the impact of the Gini index on public environmental expenditure by means of a fixed-effect model with robust standard errors, accounting for the potential non-monotonicity. Results show the existence of an inverted J-shape relationship between inequality and environmental spending. Here we can refer to our theory to provide an intuition of this outcome. As mentioned above, a change in inequality causes a change in the size and composition of the group that consumes the green good, which in turn affects the social demand for public provision of environmental quality according to a substitution effect. When the level of inequality is initially low, an increase of it tends to increase green consumption originating from the richest people who become more numerous, which calls for less environmental spending by the government.

A promising future line of research, on theoretical ground, might consist of the inclusion of different political powers in the hands of the socio-economic groups. It would be interesting to understand how the heterogeneity in political power could affect the outcome of the electoral process and resulting public policy. Such analysis would contribute to the literature discussing how political power and conflict among opposing interest groups affect the design of the environmental policy.
References


A Appendix

A.1 Proof of Proposition 1

First and second derivatives of $Q(t)$ w.r.t to $t$:

$$Q'(t) = E[w] - \pi^{-1} \int_{\tilde{\omega}(t)}^\infty w f(w) dw > 0,$$
$$Q''(t) = \pi^{-1} \tilde{\omega}'(t) \tilde{\omega}(t) f(\tilde{\omega}(t)) > 0.$$

For the Pareto distribution, the marginal benefit and costs of the public policy are:

$$MB = \beta \left[ \mu - \frac{k \pi^{-1} \tilde{\omega}(t)}{k-1} \left( \frac{w_m}{\tilde{\omega}(t)} \right)^k \right],$$
$$MC^g = \frac{k \pi^{-1} \tilde{\omega}(t)}{k-1} \left( \frac{w_m}{\tilde{\omega}(t)} \right)^k,$$
$$MC^n = \frac{k \pi^{-1} \tilde{\omega}(t)}{k-\alpha} \left( \frac{w_m}{\tilde{\omega}(t)} \right)^k \left[ \left( \frac{w_m}{\tilde{\omega}(t)} \right)^{\alpha-k} - 1 \right].$$

The relative size of group $G$, $N^g$, is equal to: $N^g(\tilde{\omega}(t)) = \left( \frac{w_m}{\tilde{\omega}(t)} \right)^k$. So we observe that $MC^g$ is proportional to $N^g$, while the $MB$ is linearly decreasing in this size.

Using this expression, we get the expression of the marginal value, $W'(t) = MB - (MC^g + MC^n)$:

$$W'(t) = \mu \left[ \beta - \pi^{-1} \left( \frac{w_m}{\tilde{\omega}(t)} \right)^{k-1} \left( 1 + \beta - \frac{k-1}{k-\alpha} + \frac{k-1}{k-\alpha} \left( \frac{w_m}{\tilde{\omega}(t)} \right)^{\alpha-k} \right) \right]. \tag{11}$$

First, we have to check that the second order optimality condition holds, given that

$$W''(t) = -\frac{k w_m \pi^{-1}}{1-t} \left( \frac{\tilde{\omega}(t)}{w_m} \right)^{1-\alpha} \left[ \frac{1-\alpha}{k-\alpha} - \left( 1 + \beta - \frac{k-1}{k-\alpha} \right) \left( \frac{\tilde{\omega}(t)}{w_m} \right)^{\alpha-k} \right].$$
We obtain $W''(t) < 0 \iff \check{w}(t) > w_m \left(1 + \frac{\beta(k-\alpha)}{1-\alpha}\right)^{\frac{1}{\alpha-k}} \equiv \check{w}^c$, which is equivalent to

$$t > t^c \text{ with } t^c = 1 - w_m^{-1}(\gamma\pi) \frac{1}{1-\alpha} \left(1 + \frac{\beta(k-\alpha)}{1-\alpha}\right)^{\frac{1}{\alpha-k}},$$

and one may note that $t^c > (\leq)0$ if and only if

$$w_m > (\leq)(\gamma\pi) \frac{1}{1-\alpha} \left(1 + \frac{\beta(k-\alpha)}{1-\alpha}\right)^{\frac{1}{\alpha-k}}.$$

We can easily verify that $\lim_{t \to 1} W''(t) = -\infty$. Assuming $t^c > 0$ (which holds under condition (8)), a necessary and sufficient condition for existence is: $W'(t^c) > 0$. This is equivalent to:

$$\beta\pi > \left(1 + \frac{\beta(k-\alpha)}{1-\alpha}\right)^{\frac{1}{\alpha-k}}. \quad (12)$$

Rearranging, the existence condition can be stated as follows: $W'(t^c) > 0 \iff \pi > \pi(\beta)$, with

$$\pi(\beta) = \beta^{-1} \left(1 + \frac{\beta(k-\alpha)}{1-\alpha}\right)^{\frac{1}{k-\alpha}}.$$

A.2 Proof of Proposition 2

First we express the two parameters of the Pareto distribution in terms of the average, $\mu$, and standard deviation, $\sigma$:

$$w_m(\mu, \sigma) = \frac{(k(\mu, \sigma) - 1)\mu}{k(\mu, \sigma)} \text{ and } k(\mu, \sigma) = 1 + \sqrt{1 + \left(\frac{\mu}{\sigma}\right)^2},$$

and we obtain, after some computations, $\frac{\partial w_m}{\partial \mu}, \frac{\partial k}{\partial \mu} > 0$, and $\frac{\partial w_m}{\partial \sigma}, \frac{\partial k}{\partial \sigma} < 0$. 

45
Next, we differentiate the expression of $MB$, $MC^g$ and $MC^n$ w.r.t $w_m$, $k$, and $\mu$:

\[
\begin{align*}
    dMC^g &= \Psi \frac{dw_m}{k-1} \left[ 1 + \frac{w_m}{k^2} \left( k \ln \left( \frac{w_m}{w^*} \right) - \frac{1}{k-1} \right) \frac{dk}{dw_m} \right], \\
    dMB &= \beta \left( d\mu - dMC^g \right), \\
    dMC^n &= \Psi \frac{dw_m}{k-\alpha} \left[ \frac{\alpha}{k} \left( \frac{w_m}{w^*} \right)^{\alpha-k} - 1 - \frac{w_m}{k} \left( \frac{\alpha}{k(\alpha)} \right)^{\alpha-k} - 1 + \ln \left( \frac{w_m}{w^*} \right) \right] \frac{dk}{dw_m}. 
\end{align*}
\]

with $\Psi = k^2 \pi^{-1} \left( \frac{w_m}{w^*} \right)^{k-1} > 0$, and $\tilde{w}^* = \tilde{w}(t^*)$.

Hereafter, we conduct the analysis of the impact of a mean preserving spread (change in $d\sigma > 0$ taking $d\mu = 0$ as given). Then we turn to the analysis of a change in $\mu$ keeping $\sigma$ constant.

### A.2.1 Mean preserving spread

Under a mean preserving spread, the joint variation of $w_m$ and $k$ satisfies: $\frac{dk}{dw_m} = \frac{k(k-1)}{w_m}$.

Using this relationship in the expressions above, we get:

\[
\begin{align*}
    \frac{\partial MC^g}{\partial \sigma} &= \Phi^{\alpha} \frac{\sigma}{k-1} \left[ 1 + (k-1) \ln \left( \frac{w_m}{w^*} \right) - \frac{1}{k} \right], \\
    \frac{\partial MB}{\partial \sigma} &= -\beta \frac{\partial MC^g}{\partial \sigma}, \\
    \frac{\partial MC^n}{\partial \sigma} &= \Phi^{\alpha} \frac{\sigma}{k-\alpha} \left[ -1 + (k-1) \ln \left( \frac{w_m}{w^*} \right) + \frac{\alpha(k-1)}{k(\alpha)} + \frac{\alpha(1-\alpha)}{k(k-\alpha)} \left( \frac{w_m}{w^*} \right)^{\alpha-k} \right]. 
\end{align*}
\]

with $\Phi^{\alpha} = k^2 \pi^{-1} \left( \frac{w_m}{w^*} \right)^{k-1} \frac{\partial w_m}{\partial \sigma} < 0$.

We want to determine the sign of $\frac{\partial^2 W(t^*)}{\partial t \partial \sigma} = \frac{\partial MB}{\partial \sigma} - \frac{\partial MC^g}{\partial \sigma} - \frac{\partial MC^n}{\partial \sigma}$. Rearranging, we obtain:

\[
\begin{align*}
    \frac{\partial^2 W(t^*)}{\partial t \partial \sigma} = \Phi^{\alpha} \left( \frac{w_m}{w^*} \right)^{\alpha-k} \left[ G(t) - \frac{\alpha(1-\alpha)}{k(k-\alpha)^2} \right], 
\end{align*}
\]

with,

\[
\begin{align*}
    G(t) &= \left( \frac{\tilde{w}(t)}{w_m} \right)^{\alpha-k} \left[ 1 + \beta - \frac{k-1}{k} \left( \ln \left( \frac{\tilde{w}(t)}{w_m} \right) - \frac{1}{k} \right) + \frac{1-\alpha}{(k-\alpha)^2} \right].
\end{align*}
\]
As to the features of $G(\cdot)$: We get that $\lim_{t \to 1} G(t) = 0$, and

$$G(t^*) > \frac{\alpha(1-\alpha)}{k(k-\alpha)^2} \Leftrightarrow 1 - \frac{\alpha}{\beta(k-\alpha)} \left( 1 + \frac{\beta(k-\alpha)}{1-\alpha} \right) \ln \left( 1 + \frac{\beta(k-\alpha)}{1-\alpha} \right) > 1,$$

which always holds. Moreover, either $G(\cdot)$ is monotone decreasing on $(t^*, 1)$, or it is bell-shaped. So we can conclude that there exists a unique $t^* \in (t^*, 1)$ such that $G(t^*) = \frac{\alpha(1-\alpha)}{k(k-\alpha)^2}$ and $G(t) > 0 \Leftrightarrow t < t^*$. Put differently, we have shown that

$$\frac{\partial^2 W(t^*)}{\partial t \partial \sigma} < 0 \Leftrightarrow t^* < t^*, $$

which completes the first part of the proof.

A.2.2 Average income variation (constant standard deviation)

Here, it is more convenient to start from the expression of $W'(t)$ given in (11) once we observe that the coefficient in front of the squared brackets is simply $\mu$ and that the term between the squared brackets is equal to 0 for $t = t^*$. We have to combine $d\mu > 0$ and $d\sigma = 0$, the latter restriction imposing $\frac{dk}{d\mu} = \frac{k(k-1)(k-2)}{w_m(k-1+k(k-2))}$. After some calculations, we get

$$\frac{\partial^2 W(t^*)}{\partial t \partial \mu} = \Phi^\mu [H(t) - \Delta], $$

with $\Phi^\mu = \mu \pi^{-1} \left( \frac{\tilde{w}}{w_m} \right)^{1-\alpha} \frac{\partial k}{\partial \mu} > 0,$

$$H(t) = \left( \frac{\tilde{w}(t)}{w_m} \right)^{\alpha-k} \left[ \left( 1 + \beta - \frac{k-1}{k-\alpha} \right) \left( \ln \left( \frac{\tilde{w}(t)}{w_m} \right) - \frac{k-1+k(k-2)}{k(k-2)} \right) + \frac{1-\alpha}{(k-\alpha)^2} \right],$$

and

$$\Delta = \frac{(1-\alpha)(k(k-2) - (k-\alpha)(k-1+k(k-2)))}{k(k-2)(k-\alpha)^2} < 0.$$

We immediately observe that $G(t) > H(t)$ for all $t < 1$ and $\lim_{t \to 1} H(t) = 0$. In
addition, we have $H(t^c) > \Delta$ because

$$G(t^c) > \frac{\alpha(1 - \alpha)}{k(k - \alpha)^2} \iff H(t^c) > \Delta.$$ 

Given that $H(.)$ is either monotone decreasing or increasing then decreasing on $(t^c, 1)$, we finally conclude that

$$\frac{\partial^2 W(t^*)}{\partial t \partial \mu} > 0 \text{ for all } t,$$

which completes the second part of the proof.

A.3 Mean preserving spread: discussion

Here we provide some elements that help to understand the comparative statics results, for a change in $\sigma$. This change negatively affects both the lower bound of the support, $w_m$, and the parameter, $k$, of the Pareto distribution $F(w)$.

Differentiating the expression of group $G$’s relative size w.r.t $w_m$ and $\sigma$ yields:

$$dN^g = N^g dw_m \left[ \frac{k}{w_m} + \ln \left( \frac{w_m}{w^*} \right) \frac{dk}{dw_m} \right].$$

Considering a change in the parameters resulting from a change in $\sigma$, for a constant $\mu$, we have

$$\frac{\partial N^g}{\partial \sigma} = N^g \frac{\partial w_m}{\partial \sigma} \frac{k}{w_m} \left( 1 + (k - 1) \ln \left( \frac{w_m}{w^*} \right) \right),$$

and

$$\frac{\partial f(w)}{\partial \sigma} = \frac{\partial w_m}{\partial \sigma} \frac{k}{w_m} f(w) \left( 1 + (k - 1) \left( \frac{1}{k} + \ln \left( \frac{w_m}{w} \right) \right) \right).$$

From the last expression, we see that there exists a critical $w_f = w_m \exp \frac{1}{k(k-1)}$ such that

$$\frac{\partial f(w)}{\partial \sigma} > 0 \iff w > w_f.$$
Recall that:

\[ \frac{\partial^2 W(t^*)}{\partial t \partial \sigma} = \frac{\partial M B}{\partial \sigma} - \left[ \frac{\partial MC^g}{\partial \sigma} + \frac{\partial MC^n}{\partial \sigma} \right]. \]

We want to sign the expressions in (14) and see how it relates to \( t^* \geq t^* \). Focusing on the changes in \( MC^g \) and \( MC^n \), there are four possible cases:

1. \( \frac{\partial MC^g}{\partial \sigma}, \frac{\partial MC^n}{\partial \sigma} < 0 \) iff \( \frac{1}{k} < 1 + (k - 1) \ln \left( \frac{w_m}{\tilde{w}^*} \right) < \Psi \),

2. \( \frac{\partial MC^g}{\partial \sigma} < 0 \) and \( \frac{\partial MC^n}{\partial \sigma} > 0 \) iff \( \max \{ \frac{1}{k}, \Psi \} < 1 + (k - 1) \ln \left( \frac{w_m}{\tilde{w}^*} \right) \),

3. \( \frac{\partial MC^g}{\partial \sigma} > 0 \) and \( \frac{\partial MC^n}{\partial \sigma} < 0 \) iff \( 1 + (k - 1) \ln \left( \frac{w_m}{\tilde{w}^*} \right) < \min \{ \frac{1}{k}, \Psi \} \),

4. \( \frac{\partial MC^g}{\partial \sigma}, \frac{\partial MC^n}{\partial \sigma} > 0 \) iff \( \Psi < 1 + (k - 1) \ln \left( \frac{w_m}{\tilde{w}^*} \right) < \frac{1}{k} \),

with \( \Psi = \frac{\alpha(k-1)}{k(k-\alpha)} + \frac{\alpha(1-\alpha)}{k(k-\alpha)} \left( \frac{w_m}{\tilde{w}^*} \right)^{\alpha-k} > 0 \).

We don’t know much a priori about the signs and rankings between those different terms. To start with, let us determine whether \( 1 + (k - 1) \ln \left( \frac{w_m}{\tilde{w}^*} \right) \geq \frac{1}{k} \). We obtain \( 1 + (k - 1) \ln \left( \frac{w_m}{\tilde{w}^*} \right) > \frac{1}{k} \Leftrightarrow \tilde{w}^* < \tilde{w}^x \) with \( \tilde{w}^x = w_m \exp^{\frac{1}{k}} \). Evaluating the expression of \( W'(t) \) given by (11) at \( \tilde{w}(t) = \tilde{w}^x \), we get

\[ W'(t)|_{\tilde{w}(t) = \tilde{w}^x} < 0 \Leftrightarrow \beta (1 - \pi^{-1} \exp^{\frac{k-1}{k}}) < \frac{\pi^{-1} \exp^{\frac{k-1}{k}}}{k - \alpha} \left( 1 - \alpha + (k - 1) \exp^{\frac{\alpha-k}{\pi}} \right), \]

and imposing

\[ \pi < \exp^{\frac{k-1}{k}}, \]

is sufficient to conclude that \( \tilde{w}^* < \tilde{w}^x \). Under this (weak) restriction, we know that \( \frac{\partial MC^g}{\partial \sigma} < 0 \). This also implies \( \frac{\partial N^g(\tilde{w}^*, \mu, \sigma)}{\partial \sigma} < 0 \) as \( \frac{\partial N^g(\tilde{w}^*, \mu, \sigma)}{\partial \sigma} < 0 \Leftrightarrow 1 + (k - 1) \ln \left( \frac{w_m}{\tilde{w}^*} \right) > 0 \Leftrightarrow \tilde{w}^* < \tilde{w}^G = w_m \exp^{\frac{1}{k-1}} \) and \( \tilde{w}^x < \tilde{w}^G \).

So we know that the increase in \( \sigma \) translates into both a decrease in \( MC^g \) and an
increase in MB. In this situation, two possibilities remain regarding the evolution of $MC^n$. They correspond to the cases 1. and 2. listed above. We also know that the relative size of group $N$ increases while the density at each income levels in $(w_m, \tilde{w}^*)$ decreases since while $(\tilde{\tilde{w}}^* <) \tilde{\tilde{w}}^x < w^f$ implies that $\frac{\partial f(w)}{\partial \sigma} < 0$ for all $w < \tilde{\tilde{w}}^*$.

Suppose that equilibrium tax is pretty high so that $\tilde{\tilde{w}}^* > \tilde{\tilde{w}}^s$. A necessary condition for this to occur is $\tilde{\tilde{w}}^s < \tilde{\tilde{w}}^\sigma$. Either $MC^n$ decreases and we can directly conclude that $\frac{\partial^2 W(t^*)}{\partial t \partial \sigma} > 0$ (case 1.). Or, $MC^n$ increases (case 2.). But then, based on the results of Appendix A.2.1, we can conclude that

$$|\frac{\partial MC^n}{\partial \sigma}| < \left|\frac{\partial MC^g}{\partial \sigma} - \frac{\partial MB}{\partial \sigma}\right| \Leftrightarrow \frac{\partial^2 W(t^*)}{\partial t \partial \sigma} > 0.$$  

Consider next that the tax is low so that $\tilde{\tilde{w}}^* < \tilde{\tilde{w}}^\sigma$. Then we know that $MC^n$ necessarily increases, which places us in case 2. Relying on our previous results, we can furthermore conclude that:

$$|\frac{\partial MC^n}{\partial \sigma}| > \left|\frac{\partial MC^g}{\partial \sigma} - \frac{\partial MB}{\partial \sigma}\right| \Leftrightarrow \frac{\partial^2 W(t^*)}{\partial t \partial \sigma} < 0,$$

which ends the discussion.

### A.4 Proof of Corollary

This proof is based on the following observation. Unlike the equilibrium tax, the critical level $t^*$, that determines whether a mean preserving spread stimulates taxation, is independent of $\pi$. Noticing that the necessary and sufficient existence condition (12) can be rewritten as:

$$\pi > \pi(\beta) = \beta^{-1} \left(1 + \frac{\beta(k - \alpha)}{1 - \alpha}\right)^{\frac{1 - \alpha}{k - \alpha}}.$$

with $\pi'(\beta) < 0$ because $k > 1$, and $\pi(\beta) > 1$ for $\beta \in (0, 1)$. Actually, only for very high $\beta$ would the threshold fall below 1 (and then become irrelevant).
Under this condition, we know that there exists a unique $t^*$ satisfying $W'(t^*) = 0$. Now, simply observe that

$$\lim_{\pi \to \pi(\beta)} t^* = t^c,$$

which is by construction lower than $t^*$. Then, by a continuity argument, we can conclude that in situations where $\pi$ is close enough to $\pi(\beta)$, a mean preserving spread induces the policy maker to reduce the income tax and the public provision of environmental quality.
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