Time-indexed formulations for the Order Acceptance Scheduling problem under energy aspects

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ABSTRACT: *To comply with the new challenges of sustainability, the industrial sector is revising its means of supply and production. This entails, for instance, optimizing energy consumption and costs at the operational level. In this vein, this research presents an Order Acceptance Scheduling problem (OAS) on a single machine under electricity time-of-use tariffs and taxed carbon emission periods. The objective is to maximize the total profit. This problem arises when a company decides to select and process a subset of orders only if it is possible within a predetermined time-window. Therefore, the number of possible schedules grow at a factorial rate. To tackle this NP-hard problem, two time-indexed formulations are provided. Finally, to assess the performance of the proposed models, a comparative analysis against a classical formulation is conducted.*

KEYWORDS: *Energy, Order Acceptance Scheduling, Time-indexed formulation.*

1 INTRODUCTION

For centuries, industry has been a vector for social and economic prosperity, shimmering an indefinite growth. Although, new concerns about environmental issues, such as resource depletion and greenhouse gases (GHG) emission, cast doubt upon this vision. According to the International Energy Agency, the industrial sector accounted for more than a third of the energy used in the world in 2017, which is responsible for climate changes and jeopardizing social advances at the same time.

Industry 4.0 – through the development of smart decision tools and the modernizing of equipment – allows a rational and an accountable response to the sustainability stakes, which aim at maintaining environmental, social and economic viability. At the strategic level, this is embodied by Corporate Social Responsibility (CSR) that advocate for ethics and green sustainability from global reference frameworks. At the operational level, this includes optimizing energy consumption, costs and carbon footprint. To perform efficient demand management, energy suppliers developed preferential rate, designated as time-of-use (TOU) tariffs, at specific times of the day. In the meantime, industrials must abide by the rules on regulation of GHG emissions, which are reflected by the implementation of carbon emission taxes by governments. We can mention that the objectives for reducing GHG emissions were reinforced at COP22 in Marrakesh, Morocco.

In this context, we present two time-indexed models for the OAS problem with electricity TOU tariffs and taxed carbon emission periods. This research follows up the work of (Chen et al., 2019) which presented a disjunctive model.

The paper is organised as follows. Section 2 includes a review on OAS problems and scheduling under electricity TOU tariffs. Section 3 states the problem and presents the solution approach. Section 4 features the resolution method. Section 5 presents the computational results. The last section concludes the paper and draws perspectives.

2 STATE OF THE ART

The standard OAS problem is a double-decision problem that consists in the selection and the sequencing of a subset of orders – among n – with the objective to maximize the total profit. (Slotnick, 2011) proposes a literature review on this topic, indicating that OAS are studied for both single and multi-machines systems and with various job characteristics such as preemption, release date or setup. These problems are generally known to be NP-hard as demonstrated in (Palakiti et al., 2019). At worst the number of possible schedules is $\sum_{k=1}^{n} k!$, while in standard scheduling problems, all the orders are accepted and thus only $n!$ sequences – which are all the possible permutations of n elements without repetition – can be obtained. (Oğuz et al., 2010) address the OAS problem with release dates, setup times and time-related penalties using a disjunctive Mixed Integer Linear Program (MILP). For the same problem, (Cesaret et al., 2012) propose a Tabu Search and (Silva et al., 2018) provide an efficient arc-time-indexed model. However, in the literature, time-indexed formulations are not as developed for OAS problem.

Energy considerations are essential for both economic and environmental reasons. With energy prices increase, demanding specification and taxes, the operational level takes a crucial part in the efforts for sustainability. Pricing policies, especially TOU rate, are largely studied for single and more complex systems. For instance, (Aghelinejad et al., 2018) exploit machine states mechanism to minimize total energy costs – comprising idle, transition and processing energy – on a single machine with a predetermined sequence. The latter use an on/off time-indexed model. (Che et al., 2016) formulate a time-indexed MILP and designed a greedy heuristic for the single machine under TOU electricity tariffs in order to minimize total energy costs. The same authors in (Che et al., 2017) investigate the unrelated parallel machine under TOU tariffs with energy costs minimization. (Ho et al., 2020) jointly enhance energy costs and makespan for a two-machine flow shop under TOU tariffs. Finally, for a job-shop system with TOU tariffs and peak-power considerations, (Masmoudi et al., 2019) employ a time-indexed formulation for the problem of minimizing energy costs. In the literature, energy aspects are also integrated as constraints. In (Liao et al., 2017), weighted completion time and weighted tardiness are minimized on a single machine with a periodic threshold on energy consumption. (Fang et al., 2013) minimize the makespan of a flow shop under peak power consumption constraints.
GHG emission management is another major challenge at the operational level. (Foumani and Smith-Miles, 2019) provide a comprehensive study on the different carbon taxes policies applied to a flow shop. (Zhang et al., 2014) develop a time-indexed formulation for a flow shop under TOU tariffs and carbon emission periods with a trade-off between the low-carbon emission period and the TOU on-peak hours. In their work, peak demands are handled by natural gas and base energy is provided by coal-based sources, which emit more GHG.

Few studies have been done for the OAS under TOU tariffs, and even fewer have been focused on GHG emissions. The work of (Chen et al., 2019) is the first of its kind, while proposing a benchmark and an exact solving approach with a disjunctive formulation. On this basis and upon their study, Section 3 provides two time-indexed MILP for the OAS problem with release dates under energy aspects. In Section 4 and 5, a comparative analysis based on the number of feasible and optimal solutions found and the time spent to solve instances is carried out. In addition, the characteristics of the models in terms of number of constraints and variables are discussed.

3 PROBLEM FORMULATION AND SOLUTION APPROACH

We investigate an OAS problem on a single non-preemptive machine with release dates. Each order j is characterized by its processing time p_j, release date r_j, due date d_j, deadline $\bar{d}_j$, revenue e_j, tardiness penalty w_j and power consumption Ω_j.

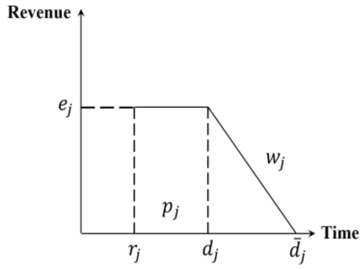

Figure 1: Profit calculation of an order j taken from (Chen et al., 2019)

Figure 1 presents the profit calculation of an order j according to its deadline, due date and tardiness penalties. An order j is accepted only if it can be finished before its deadline $\bar{d}_j$; besides , it earns the totality of its revenue e_j until its due date d_j, since the tardiness penalties w_j are applied after d_j.

In the initial work of (Chen et al., 2019), the horizon is divided into m and h intervals of electricity TOU and carbon emission respectively. Each electricity TOU interval $k = 1, \dots, m$ is characterized by an electricity cost EC_k and a starting time b_k and each carbon emission interval $l = 1, \dots, h$ is defined by its starting time g_l and an amount q_l of emitted carbon per kg, applying a Tax for each emitted kg of CO_2. For the sake of simplicity, in our formulations the time horizon is split into T periods determined in equation (1), where each period is characterized by its electricity TOU cost and the amount of emitted CO_2.

$$T = \max_{j=1,\dots,n} \bar{d}_j + 1 \qquad (1)$$

In this problem, idle times are considered but their energy consumption is neglected. The objective is to maximize the sum of the profit of the orders minus the environmental costs during processing time.

Two time-indexed MILP models are developed for this profit and time-driven problem. These formulations rely on the discretization of time, *i.e.* time is divided into unitary slots $t = 0, \dots, T$.

The energy cost c_{jt} is precomputed for each order $j = 1, \dots, n$ and at each time processing time $t = r_j, \dots, \bar{d}_j$:

$$c_{jt} = \frac{\Omega_j}{60}\left(\sum_{k=1}^{m} EC_k \mathbb{1}_{\substack{t \geq b_{k-1} \\ t < b_k}} + Tax \sum_{l=1}^{h} q_l \mathbb{1}_{\substack{t \geq g_{l-1} \\ t < g_l}}\right) \qquad (2)$$

In equation (2), $\mathbb{1}_x$ denotes the indicator function, which takes value 1 if condition x holds and 0 otherwise. For each order j at each time t, the power consumption – expressed into minutes – is multiplied by the cost of the respective TOU interval $k = 1, \dots, m$ of period $t \in [b_{k-1}, b_k[$ and the taxed CO_2 emission interval $l = 1, \dots, h$ of period $t \in [g_{l-1}, g_l[$.

3.1 Pulse formulation

The first time-indexed model is known as the pulse formulation, where the binary decision variables $x_{jt} = 1$ indicates that order $j = 1, \dots, n$ starts at time $t = 1, \dots, T$ or not ($x_{jt} = 0$). Note that the possible periods for the starting time of an order j are $t = r_j, \dots, \bar{d}_j - p_j + 1$.

The profit f_{jt} of an order $j = 1, \dots, n$ at time $t = r_j + p_j, \dots, \bar{d}_j$ is precomputed. Equation (3) corresponds to this calculation, which is the revenue minus the possible tardiness penalties when $t > d_j$.

$$f_{jt} = e_j - w_j \max\{0; t - d_j\} \tag{3}$$

The corresponding MILP is written as follows.

$$\text{maximize} \sum_{j=1}^{n} \sum_{t=r_j}^{\bar{d}_j - p_j + 1} x_{jt} \left(f_{jt} - \left(\sum_{t'=t}^{t+p_j-1} c_{jt'} \right) \right) \tag{4}$$

$$\sum_{j=1}^{n} x_{jt} \leq 1, \forall t = 0, \dots, T \tag{5}$$

$$\sum_{t=r_j}^{\bar{d}_j - p_j + 1} x_{jt} \leq 1, \forall j = 1, \dots, n \tag{6}$$

$$\sum_{t=0}^{r_j - 1} x_{jt} = 0, \forall j = 1, \dots, n \tag{7}$$

$$\sum_{t=(\bar{d}_j - p_j + 1)+1}^{T} x_{jt} = 0, \forall j = 1, \dots, n \tag{8}$$

$$x_{it} + \sum_{\substack{t'=t \\ r_j+1 \leq t' \leq \bar{d}_j}}^{t+p_i-1} x_{jt'} \leq 1, \forall i, j = 1, \dots, n; i \neq j, \forall t = r_i, \dots, \bar{d}_i - p_i + 1 \tag{9}$$

The objective (4) is the maximization of the sum of the total profit of each order, that is the profit f_{jt} including the tardiness penalties and environmental cost during the processing time, given by the sum of $c_{jt'}$ from the starting time t until completion $t + p_j - 1$. Constraints (5) specify that at each time t, the machine can start only one job. Constraints (6) restrict the starting time of each order to the interval defined from its release date to its deadline. In the same manner, constraints (7) and (8) prevent each order to be processed before its release date and after its deadline. Constraints (9) prevent any order j to overlap in the interval $[t, t + p_i - 1]$ when an order i starts at time $t = r_i, \dots, \bar{d}_i - p_i + 1$.

Table 1 presents the optimal solution of an example with $n = 4$ orders with their processing times $p = [5,3,2,4]$, release dates $r = [1,2,1,1]$, due dates $d = [6,5,12,7]$, deadlines $\bar{d} = [9,10,14,12]$, revenues $e = [10,10,6,10]$, power consumption $\Omega = [1,2,1,1]$ and weight penalties $w = [2,1,3,2]$. Finally, the starting times of TOU and carbon emission intervals $b = g = [0,5,8]$, the electricity price $EC = [2,10,2]$ and the amount of CO_2 emitted $q = [4,1,4]$ are defined.

j \ t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
4	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0

Table 1: Optimal solution represented by the values of the decision variables x, sequence is 4-2-3 and order 1 is rejected.

For this example, there are 180 constraints and 80 variables. The optimal solution has been found in 0.01 s.

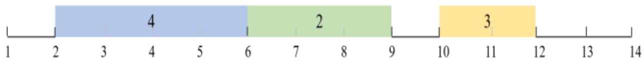

Figure 2: Gantt chart representation for the example.

3.2 On-off formulation

The second model presented is the on-off formulation. Each binary decision variable $x_{jt} = 1$ indicates whether the order j is processed at time $t = r_j, \dots, \bar{d}_j$, or not $x_{jt} = 0$. In addition, the binary decision variable $a_j = 1$ represents whether the order j is accepted.

$$\text{maximize} \sum_{j=1}^{n} \left(a_j e_j - w_j \max_{t = d_j, \dots \bar{d}_j} \{t - d_j\} x_{jt} - \sum_{t=r_j}^{\bar{d}_j} c_{jt} x_{jt} \right) \tag{10}$$

$$\sum_{j=1}^{n} x_{jt} \leq 1, \forall t = 0, \dots, T \tag{11}$$

$$\sum_{t=0}^{r_j - 1} x_{jt} = 0, \forall j = 1, \dots, n \tag{12}$$

$$\sum_{t=\bar{d}_j+1}^{T} x_{jt} = 0, \forall j = 1, \dots, n \tag{13}$$

$$p_j a_j = \sum_{t=r_j}^{\bar{d}_j} x_{jt}, \forall j = 1, \dots, n \tag{14}$$

$$\sum_{t'=r_j}^{t-p_j} x_{jt'} + \sum_{t'=t+p_j}^{\bar{d}_j} x_{jt'} \leq (1 - x_{jt}) p_j, \forall j = 1, \dots, n, \forall t = r_j, \dots, \bar{d}_j \tag{15}$$

The objective function (10) is the maximization of the total profit of the accepted orders, i.e. their revenues $a_j e_j$ minus their possible tardiness penalties w_j by retrieving the instant $t = d_j, \dots, \bar{d}_j$ when $x_{jt} = 1$ (completion time) and the environmental costs during processing time. Constraints (11) state that at each time the machine is either doing nothing or processing an order. Constraints (12) and (13) ensure that each order cannot be processed before its release date and after its deadline. Constraints (14) impose that each accepted order must be processed during the totality of its processing time. Constraints (15) guarantee non-preemption by forcing the continuity of the decision variables.

Table 2 presents the same solution as in 3.1 with the perspective of the on-off model.

j \ t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
4	0	0	1	1	1	1	0	0		0	0	0	0	0	0

Table 2: Optimal solution represented by the values of the decision variables x, sequence is 4-2-3.

For this solution and this formulation, there are 97 constraints and 111 variables. The optimal solution has been found in 0.03s.

4 RESOLUTION METHOD

The considered benchmark derives from the work of (Chen et al., 2019). Similar TOU tariffs and carbon emission periods are used. Instances with various number of orders $n = 5,\ 10,\ 15,\ 20,\ 50$ and with different characteristics are tested in order to have a diverse set of instances. Two coefficients are used to generate the parameters of an instance: the tardiness factor $\tau = 0.1, 0.3, 0.5$ and the due date ranges $R = 0.1, 0.5, 0.9$. This first comparative study involves 45 instances.

Processing time, release date and revenue are generated with an uniform distribution: $p_j, e_j \sim \mathcal{U}(1,20)$ and $r_j \sim \mathcal{U}(0, \tau p_T)$ with $p_T = \sum_{j=1}^{n} p_j$. Due dates, deadlines, tardiness penalties and power consumption are computed from the values of these generated parameters:

$$d_j = r_j + \max\{slack, p_j\} \tag{16}$$

$$slack \sim \mathcal{U}\left(p_T\left(1 - \tau - \frac{R}{2}\right), p_T\left(1 - \tau + \frac{R}{2}\right)\right) \tag{17}$$

$$\bar{d}_j = d_j + Rp_j \tag{18}$$

$$w_j = \frac{e_j}{\bar{d}_j - d_j} \tag{19}$$

$$\Omega_j \sim \mathcal{U}(1, e_j) \times \frac{1}{2} \tag{20}$$

This ensures coherent values for these parameters. For instance, the deadline must be greater or equal to the due date.

The tested models are the pulse formulation, the on/off formulation and the disjunctive model of (Chen et al., 2019). For the sake of comparability, each model has been implemented and solved in a commercial solver (IBM CPLEX Optimization Studio v12.9) on a desktop computer with processor Intel i5 2GHz CPU with 4GB RAM. Solving time is limited to 3600 seconds.

5 COMPUTATIONAL RESULTS

Table 3 resumes the benchmark results for each model and their average performances for 9 instances of same size and different values for τ and R. Average solving time ($\overline{cpu}$), average gap ($\overline{gap}$) and the number of feasible ($\#fea$) and optimal ($\#opt$) solutions found are reported. In our tests, the gap is retrieved from CPLEX relative MILP gap, which represent the gap between the best bound and the best integral solution found by the solver.
A summary of the performances of all formulation on the 45 instances is also provided.

The results clearly point out that time-indexed formulations outperform a standard disjunctive model on average. The pulse formulation find 38 optimal solutions among 44 in 711 seconds on average, which represent a success rate of 86%. In contrast, the disjunctive formulation takes on average more than twice the time to find half of the optimal solutions. The on/off formulation provides similar results to the pulse model, achieving a rate of 84% of optimal solutions found in 721 seconds on average. This model seems to be more performant than the pulse formulation for small to medium instances ($n = 5, 10, 15, 20$), finding all the optimal solutions in less time. However, its average gap is less tight than the pulse formulation.

The size of time-indexed formulations (constraints and variables) is their main weakness. Indeed, both time-indexed formulations cannot find all the feasible solutions of the benchmark due to either out-of-memory issues or low quality gap. At worst, the on-off formulation has $\mathcal{O}(nT) + \mathcal{O}(n)$ variables and $\mathcal{O}(nT)$ constraints, whereas the pulse formulation has $\mathcal{O}(nT)$ variables and $\mathcal{O}(n^2T)$ constraints. The disjunctive formulation has $\mathcal{O}(n^2) + \mathcal{O}(nm) + \mathcal{O}(nh)$ variables and constraints.

As seen in Section 3, the pulse formulation binary variables contain information for both acceptance and the instant where the order starts, whereas the on/off formulation differs in semantic. A binary variable x_{jt} in the pulse formulation corresponds to $p_j + 1$ variables in the on/off formulation. The pulse formulation effectiveness on average compared to the on/off model may reside in the use of less binary variables and thus imply a lower number of branching. Nevertheless, the number of constraints in the on/off model provides an advantage for small instances.

6 CONCLUSION AND PERSPECTIVES

In this paper, we presented two time-indexed formulations for the OAS with release dates under energy aspects.
Our proposed formulations are more performant on average than the disjunctive formulation described in (Chen et al., 2019). This can be explained by their LP-relaxation which provide good bound for medium instances, according to (van den Akker et al., 2000). Moreover, time-indexed models seem to be the most efficient formulations

for this problem since the objective is the maximization of a time-driven profit comprising time-related penalties and time-varying environmental costs. However, these formulations are limited by their spatial complexity. Our future work is focused on the development of dedicated cuts or other exact approaches that have the potential to improve the performances for large instances. Finally, time-indexed formulations for an extension of the proposed problem with setup-dependent sequence are under development.

	Disjunctive model				**Time-indexed pulse**				**Time-indexed on/off**			
n	#fea	#opt	$\overline{cpu}$	$\overline{gap}$	#fea	#opt	$\overline{cpu}$	$\overline{gap}$	#fea	#opt	$\overline{cpu}$	$\overline{gap}$
5	9	9	0.07	0.0	9	9	0.05	0.0	9	9	0.07	0.0
10	9	9	47.7	0.0	9	9	8.4	0.0	9	9	0.94	0.0
15	9	3	2416	4.2	9	8	471	0.08	9	9	23	0.0
20	9	3	2404	2.8	9	8	574	0.02	9	9	146	0.0
50	9	0	3600	5.9	8*	4	2503	0.1	8*	1	3434	6
summary	45	24	1693	2.5	44	38	711	0.05	44	37	721	1.2

Table 3: Models performances in terms of solving time, gap to the best-bound, feasible and optimal solutions (*out-of-memory status or found infinite gap).

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