Intragenerational inequality aversion and intergenerational equity

Robert D. Cairns, Stellio del Campo, Vincent Martinet

Intragenerational inequality aversion and intergenerational equity

Vincent Martinet*†   Stellio Del Campo‡   Robert D. Cairns§

Abstract

We study the interplay between intragenerational and intergenerational equity in an economy with two countries producing and consuming from national capital stocks. We characterize the sustainable development path that a social planner would implement to achieve intertemporal egalitarianism. If intergenerational equity is defined with respect to the global consumption of each generation regardless of its distribution between countries, consumption in the poor country should be set as low as possible to maximize investment and hasten convergence, resulting in important intragenerational inequality. When social welfare accounts for intragenerational equity, the larger the intragenerational inequality aversion (IIA), the smaller the sacrifice asked of the poor country, but the lower the sustained level of generational welfare. Along the intertemporal welfare-egalitarian path with IIA, consumption in the poor country increases, while it decreases in the rich country, resulting in a global degrowth.

Key words: sustainable development; intergenerational egalitarianism; maximin; intragenerational inequality; differentiated degrowth.

JEL Code: O44; Q56.

*Corresponding author. Université Paris-Saclay, INRAE, AgroParisTech, Paris-Saclay Applied Economics, F-91120, Palaiseau, France. Email: vincent.martinet@inrae.fr. ORCID: 0000-0002-7135-8849

†CEPS, ENS Paris-Saclay, France.

‡Mercator Research Institute on Global Commons and Climate Change, Torgauer Str. 12-15, Berlin 10829, Germany. Email: delcampo@mcc-berlin.net. ORCID: 0000-0001-8986-1170

§Department of Economics and CIREQ, McGill University, Montreal, Quebec, Canada H3A 2T7, and CESifo, Munich, Germany. Email: robert.cairns@mcgill.ca

1   Introduction

The idea of perpetual growth is challenged in the sustainability literature, which puts a strong emphasis on intergenerational equity (Heal, 1998; Martinet, 2012). By depleting natural resources and inducing pollution, growth may jeopardize future generations' ability to enjoy a livable planet earth (Arrow et al., 1995; Rockström et al., 2009), especially when environmental externalities are not accounted for (Arrow et al., 2004), or when departing from the optimal path, even slightly (Bretschger, 2017). Significant environmental policies, such as a carbon tax, are justified to overcome intergenerational environmental issues, but face rejection from developing countries as well as from many citizens of developed countries (Sterner, 2012). Just as in the climate change example (Heal, 2009), inequality within generations can get in the way of intergenerational equity.

Part of current inequality rely on unequal access to resources, broadly speaking, be it at individual, national, or continental levels. Productive assets, including financial and physical capital, knowledge, and environmental assets, are unequally distributed within a generation, inducing inequality in production and consumption. Growth and capital accumulation could reduce inequality gradually. Inclusive economic growth is one of the United Nations' sustainable development goals (SDG8), in particular for the least developed countries, and is presented as a way of reducing poverty (SDG1) and inequality (Boarini et al., 2018). As such, the call for a stationary state, or even degrowth, for intergenerational equity purposes may seem unfair to the less endowed of a generation. There is a need to account for intragenerational inequality when defining a sustainable development path. How do sustainability policies affect intragenerational inequality? How does intragenerational inequality aversion affect sustainable development paths?

In this paper, we investigate the theoretical interplay of intragenerational and intergenerational equity in the design of sustainable development paths. We examine how the optimal development path for a social planner aiming at implementing intergenerational equity is modified when taking into account intragenerational inequality aversion. This

allows us to begin to answer the previous questions. First, we show how the pursuing of intergenerational equity affects inequality within generations in the short and the long runs. Second, we examine how greater inequality aversion within generations impacts the prospects for a sustainable development path.

Intragenerational and intergenerational equity issues have mostly been studied formally in different branches of the economics literature.[1] An exception is the literature on climate change,[2] which mainly adopts the discounted utility framework. This framework has been strongly criticized in the sustainability literature as poorly accounting for intergenerational equity concerns, leading to the definition of alternative criteria, such as maximin (Solow, 1974; Burmeister and Hammond, 1977; Cairns and Long, 2006; d'Autume and Schubert, 2008a; Cairns and Martinet, 2014; Fleurbaey, 2015a; Cairns et al., 2019), undiscounted utilitarianism (Ramsey, 1928; Dasgupta and Heal, 1979; d'Autume and Schubert, 2008b; d'Autume et al., 2010), the Chichilnisky criterion (Chichilnisky, 1996), the weighting of the worst-off generation (Alvarez-Cuadrado and Long, 2009; Adler and Treich, 2015; Adler et al., 2017), sustainable discounted utilitarianism (Asheim and Mitra, 2010; Dietz and Asheim, 2012), as well as intergenerational egalitarianism (Piacquadio, 2014). Botzen and Bergh (2014) report that applying (some of) these alternative criteria to the climate change issue would result in more stringent climate policies than under discounted utilitarianism. Imposing stronger requirements for intergenerational equity may thus induce greater tensions regarding intragenerational equity.

The literature on intergenerational equity is mostly axiomatic, and uses growth models to explore the consequences of the studied criteria (see Asheim, 2010, for a review).

---

[1] Some papers discuss explicitly the interplay of the two dimensions of equity. For example, Baumgärtner et al. (2012) offer an informal discussion of the issue. Baumgärtner and Glotzbach (2012) discuss these links for ecosystem services, without a formal model. Berger and Emmerling (2020) offer a theoretical framework that could lead to interesting discussions combining both equity issues.

[2] Schelling (1992) emphasizes the interplay of the two dimensions of equity in the climate change issue. Anthoff et al. (2009) introduce equity-weights to account for the different impacts of climate change in different countries with different levels of development. Kverndokk et al. (2014) study the effect of inequality aversion on optimal climate policies, using the Fehr and Schmidt (1999) framework for intragenerational equity, and discounted utility as an intertemporal welfare function. Yamaguchi (2019) studies the effect of inequality aversion on the consumption discount rate in a climate economy. Several empirical works based on Integrated Assessment Models introduce a concern for intragenerational inequality too. Dennig et al. (2015) develop a nested-inequalities model that accounts for inequality within and between regions. Scovronick et al. (2017) examine the effect of population growth on optimal climate mitigation policies, comparing Total Utilitarianism (which is sensitive to the population size) and Average Utilitarianism (which is not). Anthoff and Emmerling (2019) disentangle the effect of intragenerational and intergenerational inequality aversions on the social cost of carbon.

The key models used in this framework are Ramsey's one sector growth model[3] and the Dasgupta-Heal-Solow model (Dasgupta and Heal, 1974; Solow, 1974). In most cases, the analysis is based on a single representative agent,[4] overlooking the intragenerational equity issue.

We move away from the single representative agent model and consider a dynamic economy with two agents (called countries for simplicity), each endowed with a stock of productive capital. We use intergenerational egalitarianism to represent intergenerational equity.[5] Equality as an ideal of justice has a long tradition in philosophy and economics (Temkin, 1993; Parfit, 1995). It is based on the idea that "unequal distributions have something bad that equal distributions do not have" (Fleurbaey, 2015b, p. 205). Egalitarianism may not be the criterion selected for intergenerational equity and sustainable development, but it is an interesting benchmark for evaluating intertemporal inequality in different theoretical frameworks (Piacquadio, 2014). Equality may result both from (intrinsic) egalitarianism or prioritarianism (which implies instrumental egalitarianism). When there are no trade-offs between individuals' utility, prioritarianism corresponds to the maximin criterion (Rawls, 1971; Myerson, 1981; Epstein, 1986), which leads to an egalitarian outcome under many circumstances. Asheim (2010, p. 206) emphasizes that "maximin, the principle of maximizing the well-being of the worst-off generation, [...] satisfies the finite anonymity axiom and is thus an alternative way of treating generations equally." According to Fleurbaey (2015b, p. 214), "in welfare economics, [...], the maximin (or leximin) criterion has then been adopted as yielding the most egalitarian among reasonable (that is, Paretian) social rankings."[6] Asheim and Nesje (2016) use the maximin path as a benchmark for intergenerational equity, as the maximin welfare level offers a lower bound for welfare under other, more sophisticated criteria, such as the Calvo criterion, Sustainable Discounted Utility, and Rank Discounted Utility, whose solutions may not be easy to compute in particular problems. Also, more sophisticated forms of intergenerational equity imply time-inconsistency, which has to be dealt with using

---

[3]See Asheim and Ekeland (2016) for a discussion of the interest of this model to study sustainability issues, and Asheim and Nesje (2016) for the analysis of the optimal path in this model under various criteria of intergenerational equity. See also Asheim et al. (2020) who discuss time-consistency issues for criteria that do not satisfy stationarity, and illustrate their results in the Ramsey model.

[4]There are a few exceptions, including the axiomatic work on intergenerational equity with varying population, which mainly focuses on population ethics (see, e.g., Asheim and Zuber, 2014).

[5]Other approaches could be considered. For example, Del Campo (2019) studies redistribution effects along the optimal growth path under undiscounted utilitarianism.

[6]Fleurbaey and Tungodden (2010) show that, as soon as one rejects the idea that a large sacrifice of the worst-off is justified if it results in a tiny gain for sufficiently many of the better-off (along with a condition of replication invariance), then one is forced to accept the maximin principle.

sophisticated game-theoretic equilibrium selection (Asheim et al., 2020). Intertemporal egalitarianism is thus for us a natural first step. It also allows us to avoid the delicate exercise of comparing growing economies (Asheim, 2011; Llavador et al., 2011). We point out that, even in this egalitarian case, there are interactions between the two dimensions of equity that have far reaching consequences.

Regarding intragenerational equity, we consider a welfarist approach with inequality aversion. The welfarist approach allows us to deal with a continuum of cases, especially the two polar ones: no aversion and infinite aversion (intragenerational maximin).[7] The effective distribution of consumption, both within a generation and across time, is ultimately linked to the chosen degree of inequality aversion. This simple model allows us i) to examine the dynamic effect of intergenerational equity (egalitarianism) on intragenerational inequality, and ii) to study the effect of inequality aversion on sustainability strategies. We show that, when the technology is the same in the two countries, pursuing intergenerational egalitarianism increases intragenerational inequality in the short run. In the long-run, however, intragenerational inequality vanishes as the two countries converge. These results are mitigated by inequality aversion. A larger intragenerational inequality aversion reduces inequality in the short run, but at the cost of a lower intergenerational level of sustained welfare. It also induces a decreasing global consumption, even if welfare is constant over time. This development path corresponds to overall degrowth, but with growth in the poor country. When the two countries have different technologies, the interplay of the two dimensions of equity is more complex, and depends on the technologies as well as on the initial state of the economy.

We present our model in Section 2, examine the case of a nil inequality aversion in Section 3, and study the effect of inequality aversion in Section 4. We relax some of the simplifying assumptions of the main model in Section 5, and conclude in Section 6. All the mathematical details are in the appendix.

---

[7]Note that comparing unequal situations among a finite number of individuals within a generation is much less sensitive than comparing unequal situations in the intertemporal setting of an infinite number of generations (Asheim, 2010).

2 Modeling framework

2.1 The economy

Consider an economy composed of two entities, R and P, that we shall call countries for simplicity. The population sizes of the two countries are equal and normalized to unity, for simplicity. In our dynamic, continuous-time framework, we assume that at each time t each country has a single representative agent that lives for that time only. Country R (respectively, P) is endowed with an aggregate stock $X_R(t)$ (respectively, $X_P(t)$) of comprehensive productive assets (including natural resources), capital for short, that can evolve over time. These capital stocks correspond to national wealth. The aggregate capital of a generation, $X_R(t) + X_P(t)$, corresponds to global wealth. Wealth is unequally distributed at initial time 0, with $X_R(0) > X_P(0)$, so that we call R the rich country, and P the poor country.

Capital stocks are productive, according to a production function $F(X_i)$ that is identical for both countries, meaning that countries differ only in their wealth endowment and related country-specific production.[8] A similar assumption is made in Kverndokk et al. (2014) in a climate change economy with two countries. In Section 5, we relax this assumption and consider country-specific production functions, but for now we consider the *symmetric case*. Technology can be interpreted as the countries' capacity to derive output from national endowments. We assume that the production function is strictly increasing and strictly concave, that marginal product $F'(X)$ is bounded from above,[9] and that production is bounded from above, i.e., $\lim_{X \to \infty} F(X) = \bar{F} < \infty$.[10] Under the assumption of identical production functions, a lower capital stock entails lower production, and the

---

[8]By defining national production as a function of national capital only, we overlook the possible interactions between the two capital stocks. This could be the case if the two countries are involved in trade, or if the model was interpreted as representing two individuals whose productivity in the global economy depends on own endowment as well as on the endowment of the other individual. A way to account for such interactions would be to assume that both production levels depend on both capital stocks, but the solution would depend on the specific type of interactions considered. For example, trade would require to consider at least two different goods in two locations, and thus at least four decision variables. An easier way to include interactions would be to consider two dynasties that own a share of the same capital stock, as in Asheim and Nesje (2016). As we want to keep results as generic as possible, we avoid such specification. Note that when capital stocks correspond to local natural resources, there is no such concern if the two stocks are independent. This is the case, for example, in Quaas et al. (2013).

[9]When this technical assumption is satisfied, the transversality conditions are necessary conditions (see Cairns et al., 2019, footnote 18).

[10]This condition is used to prove the existence of an efficient egalitarian path (see Cairns et al., 2019, Proposition 2). Our intuition is that it could be relaxed for maximin problems in which state trajectories are converging to a finite limit.

poor country produces less initially, i.e., $X_P(0) < X_R(0)$ implies $F(X_P(0)) < F(X_R(0))$.

For simplicity, we assume at first that each country can invest only in its own productive capacities, and that no transfer of capital or consumption occurs. We relax this assumption and discuss the consequences of the possibility of such transfers in Section 5. Output can be either consumed by the producing country or invested in its capital stock, so that country-specific capital dynamics is given by

$$\dot{X}_i = F(X_i) - c_i, \quad i = R, P \ .$$

Output can be interpreted as Net National Product, which is shared between national consumption and national investment. These measures are used in national accounts and for international comparisons.

2.2   Implementing intergenerational egalitarianism

Our objective is to determine the development path that a social planner aiming at intergenerational egalitarianism would implement. To combine sustainability and Pareto-efficiency concerns, we consider the "highest" such intertemporal egalitarian path.[11]

There are several ways to think about, and implement, intergenerational egalitarianism in this economy. One could consider intergenerational egalitarianism at an individual (country) level or at a generational (global) level. As sustainability is usually interpreted as the ability to sustain the global welfare of each generation – welfare being a function of current consumption levels – we shall adopt a generational perspective. We, however, briefly discuss the consequences of the individual perspective.

The individual perspective   A first possibility would be to define intergenerational egalitarianism at the individual (country) level, considering each country as an infinitely-lived agent seeking a sustainable level of consumption. This amounts to defining, for each country, the highest constant consumption level that can be sustained, i.e., $\max_{c_i(\cdot)}(\min_t c_i(t))$ for $i = \{R, P\}$.

The sustainability literature emphasizes that, along efficient egalitarian paths, net investment is nil and welfare (or consumption when it is the only source of welfare) is

---

[11] Note that, in some models, this egalitarian path may not be strongly Pareto-efficient. The maximin criterion only satisfies weak Pareto efficiency (Lauwers, 1997). To retrieve strong Pareto efficiency in infinite horizon problems, one needs to consider (some versions of) the leximin criterion (see, e.g., Asheim and Tungodden, 2004; Bossert et al., 2007, as well as the introduction in Asheim and Zuber, 2013). We will discuss this issue when considering infinite intragenerational inequality aversion.

constant over time.[12] Applying intergenerational egalitarianism at an individual level would mean that each country consumes its national sustainable income, i.e., its whole production (Cairns and Long, 2006; Cairns and Martinet, 2014). As such, one would get a stationary state in which capital stocks remain constant, with $c_P(t) = F(X_P(0)) < F(X_R(0)) = c_R(t)$, inequality perpetuating forever. Implementing intergenerational egalitarianism at an individual level would thus ensure equity between generations (both at a global and country level), but not equity within generations.

Considering an aversion to the inequality between the sustained levels of consumption of the two countries with, for example, a welfare function of the form[13]

$$\max_{c_R(\cdot),c_P(\cdot)} \left(\left(\min_t c_R(t) \right)^{1-\theta} + \left(\min_t c_P(t) \right)^{1-\theta} \right)^{\frac{1}{1-\theta}}, \ \theta > 0, \ \theta \neq 1 \tag{1}$$

would not change the outcome. Both the maximin consumption levels for each country, which correspond to the consumption of the total national production, can be jointly achieved by consuming national income and keeping the capital stocks stationary. Any deviation from this stationary state would induce a lower minimal level of consumption for at least one of the countries, with no possibility to compensate it by increasing the minimal level of consumption in the other country.[14]

The generational perspective Defined at a generational (global) level, an intergenerationally egalitarian path may depart from such a stationary state. For example, in economies with a single representative agent and two reproducible assets, the efficient egalitarian path may not correspond to a stationary state (Burmeister and Hammond, 1977; Asako, 1980; Cairns et al., 2019). We will investigate this possibility in our economy with two countries. We will show that the results differ depending on the way the outcome of a generation is defined – global consumption versus a welfare measure accounting for consumption inequality.

---

[12]See the literature on Hartwick's rule and its generalization (Hartwick, 1977; Dixit et al., 1980; Mitra, 2002; Asheim, 2013) and on the characterization of egalitarian paths (Burmeister and Hammond, 1977; Cairns et al., 2019).

[13]This amounts to first aggregating with respect to time (with an intertemporal min) and then aggregating individual outcomes with an inequality aversion parameter θ. See the interesting discussion on the disaggregation of welfare problems in Berger and Emmerling (2020, sect. 4.3.3, pp. 740–741).

[14]More extreme forms of inequality aversion than that represented by the welfare function (1), for example strongly penalizing inequality, could result in Pareto dominated outcomes, the sustained level of consumption of the rich country being reduced only to reduce inequality, without offering any prospect to increasing the sustained level of consumption in the poor country.

Whether one should equalize welfare, income, resources, *functionings* or opportunities is source of a rich debate in philosophy and economics theories of distributive justice (Putterman et al., 1998), which often oppose 'resource egalitarianism' to 'welfare egalitarianism.' The two conceptions share common grounds, however (Moreno-Ternero and Roemer, 2012). Sen (1997) emphasizes that "the identification of economic inequality with income inequality is fairly standard" (p. 384) and that "in traditional welfare economics, there has been interest both in individual utilities and in individual incomes. When individuals are taken to be symmetrical, the two are closely linked" (p. 393). Using income as a proxy refers back to the pioneering work of Atkinson (1970) on the measurement of inequality. It focuses on objective means without making assumptions on individual capacities to turn them into outcomes such as achievements or happiness.

We shall overlook the relationship between income (and other resources) and individual well-being, and consider inequality in consumption levels and not in individual utility. As long as one considers symmetrical individuals and similar non-income circumstances, this is not a crazy assumption. Moreover, using the levels of consumption as a metric to compare outcomes within a generation and/or between generations is very similar to what is done in practice, with the use of monetary metrics to compare consumption expenditures, earnings or income, or even production levels (e.g., GDP). A practical reason to do so is that well-being is difficult to define and measure, whereas consumption or income are more easy to measure and "may be a good proxy for [a person's] level of functioning, resource control, and opportunities" (Putterman et al., 1998, p. 866).

In our analysis, we shall start by the case of a social planner aiming at sustaining the highest possible global consumption level for successive generations (Section 3). From a modeling perspective, this approach is equivalent to a single-agent model approach, a generation being reduced to its global consumption level. We shall see that this option is not ethically attractive as it induces important intragenerational inequality.

We then examine how the egalitarian path is modified when aversion to intragenerational inequality is introduced (Section 4). Whenever the two countries have the same technology, i.e., the same ability to produce, inequality is fully encompassed in the current capital stock. There may be several reasons for the two countries to have unequal capital stocks at the initial time, which may influence the normative justification of intragenerational inequality aversion.[15] On the one hand, the countries may be endowed with different natural resources (e.g., pedoclimatic conditions, nonrenewable or renewable re-

---

[15]When the technologies differ, the fact that the two countries do not have the same opportunities may justify stronger intragenerational inequality aversion.

sources, etc.), or they may be at different stages of development and capital accumulation due to a later access to the technology. In that case, equity considerations would justify inequality aversion, perhaps up to targeting intragenerational egalitarianism. If, on the other hand, the two countries have had equal opportunities (i.e., if, at some point in the past, the two countries had the same endowment and same technology), one could argue that current capital inequality is the responsibility of each country, and intragenerational inequality may be acceptable from a libertarian point of view (Fleurbaey, 2019).[16] For example, the two countries may have different time preferences that drove the economy to different stationary states according to the modified golden rule (i.e., $F'(X_i(0)) = \delta_i$), with a larger discount rate in P (i.e., $\delta_P > \delta_R$) so that $F'(X_P(0)) > F'(X_R(0))$ and thus $X_R(0) > X_P(0)$.[17] Current inequality is then the result of past choices, and provide less justification to high intragenerational inequality aversion. Even in this scenario, our analysis would amount to studying the transition from an "individual-based" approach of intertemporal decisions to a global sustainable development taking the form of a "generation-based" equity.

Depending on the reasons for unequal endowment at time $t = 0$, one may justify a stronger or milder intragenerational inequality aversion. Even if we overlook the question of how the initial state was reached, we consider the full range of inequality aversion, so that our analysis remains flexible on that point. In particular, we do not impose egalitarianism between the two countries. We impose intergenerational egalitarianism, though, which may be justified as no generation is responsible for its date of birth.

To account for intragenerational inequality aversion, we consider an aggregator of current consumption that accounts for inequality for global welfare, and interpret sustainability as maintaining the largest possible level of equally-distributed-equivalent consumption. The sum of consumption levels is a limiting case of this criterion when the social planner is not concerned with intragenerational inequality. At the opposite limit, the welfare of a generation is given by the consumption level of the worst-off. We shall discuss these limiting cases too.

---

[16]Even in this case, though, past events due to brute luck could explain the inequality and justify some inequality aversion (Dworkin, 2000).

[17]Ramsey (1928, section III (γ)) considers such family-differentiated discount rates and their consequences on the economic dynamics.

3 Sustaining global consumption

We first examine what the path of the economy would be if a social planner aimed at maximizing the level of global consumption to be sustained over time by following an efficient and intergenerationally egalitarian path. This corresponds to a case in which one is concerned only with the total consumption of a generation, without considerations for intragenerational inequality. This is also equivalent to maximizing the sustained consumption level in a single-agent model.

We base our inequality analysis on consumption, and not on utility, for two reasons. First, as stated before, it has the advantage of being more concrete than the consideration of individual utility levels. It requires no assumption on how utility is derived from consumption, nor interpersonal utility comparison, and is consistent with the practice of empirical inequality analysis, mostly based on income or consumption data. Second, it allows us to establish a clear link with the single-agent approach, which considers aggregate consumption as a source of utility. Introducing individual utility functions with decreasing marginal utility would induce a motive to smooth consumption within a generation for efficiency purposes, irrespective of the social planner's motive to account for intragenerational inequality. The results would then be close to that of Section 4, but would not make the analysis of this section possible.

To characterize the global-consumption egalitarian path, we solve a maximin problem.[18] The maximin consumption value $m^0(X_R, X_P)$ is the highest level of global consumption that can be sustained forever from the initial state of the economy:[19]

$$m^0(X_R, X_P) = \max_{\underline{c}, c_R(\cdot), c_P(\cdot)} \underline{c} \,, \tag{2}$$

$$\text{s.t. } (X_R(0), X_P(0)) = (X_R, X_P) \,;$$

$$\dot{X}_i(t) = F(X_i(t)) - c_i(t), \ i = R, P, \ \text{ and}$$

$$c_R(t) + c_P(t) \geq \underline{c} \text{ for all } t \geq 0 \,. \tag{3}$$

---

[18]The objective of a maximin problem is to define an equitable development path by maximizing the utility of the worst-off generation, through intergenerational redistribution. It guarantees a procedural equity (finite anonymity) since all generations are treated equally (Lauwers, 1997; Asheim, 2010). Whenever it is possible, i.e., in so-called *regular* maximin problems (see Solow, 1974; Burmeister and Hammond, 1977; Cairns and Long, 2006; Cairns and Martinet, 2014), such a redistribution results in a strongly Paretian egalitarian allocation. In such a case, even if this is not formally the objective of a maximin problem, it characterizes an egalitarian and efficient path. This is the case in our model, which shares technical similarities with the model in Cairns et al. (2019).

[19]The superscript ^{0} refers to the fact that intragenerational inequality aversion is nil in this case.

We follow Cairns and Long (2006) and Cairns et al. (2019) to solve this maximization problem.[20] The details of the resolution are in Appendix A.1. The main results are the following.

Proposition 1 (Sustaining global consumption: stationary states). *The efficient egalitarian path is a constant consumption path with $c_i(t) = F(X_i^*)$ and $\dot{X}_i = 0$ if and only if the capital stocks (X_P^*, X_R^*) satisfy $F'(X_P^*) = F'(X_R^*)$, i.e., $X_P^* = X_R^*$ in the symmetric case.*

This result means that, when the objective is to maximize the level of global consumption sustained over time, the optimal path is a stationary path with constant consumption levels at country-specific production levels and constant capital stocks (if and) only if the marginal product of capital in both countries is equal, which corresponds to equal capital stocks under the assumption of identical production functions. In that case, consumption levels are equal in the two countries, and there is no inequality, neither within a generation nor between generations. If the capital stocks do not satisfy the restrictive condition, i.e., if endowments are unequal, the optimal path is not stationary. Apart from a stationary state, the dynamics is as follows.

Proposition 2 (Sustaining global consumption: Transition). *For any state such that $F'(X_P) > F'(X_R)$, i.e., $X_P < X_R$ in the symmetric case, the constant consumption path with $c_R(t) = m^0(X_R, X_P) > F(X_R)$ and $c_P(t) = 0 < F(X_P)$ is an optimal maximin path. Stock X_R decreases while stock X_P increases over time.*

This dynamic path corresponds to a corner solution in terms of country-specific consumption levels. This result is due to the fact that the two consumption levels are perfect substitutes in global consumption. The country with the higher marginal product of capital, i.e., P in the case of identical production functions, has a higher return on investment. From a global point of view, it is more efficient to reduce that country's consumption as much as possible to invest where capital has the higher return. At the same time, global consumption is sustained through a large consumption of the rich country, where capital stock is larger and marginal product of capital lower. In the absence of inequality aversion, sustaining global consumption consists in adopting the same strategy as an annuitant owning two financial accounts with different returns: let the asset with the higher return grow, and withdraw capital to sustain consumption from the other asset.

---

[20]See the discussion in Cairns et al. (2019), and especially their footnotes 9 and 10, for an overview of the different mathematical approaches to the characterization of an efficient and egalitarian development path, their equivalence in most cases, and the differences in their interpretations.
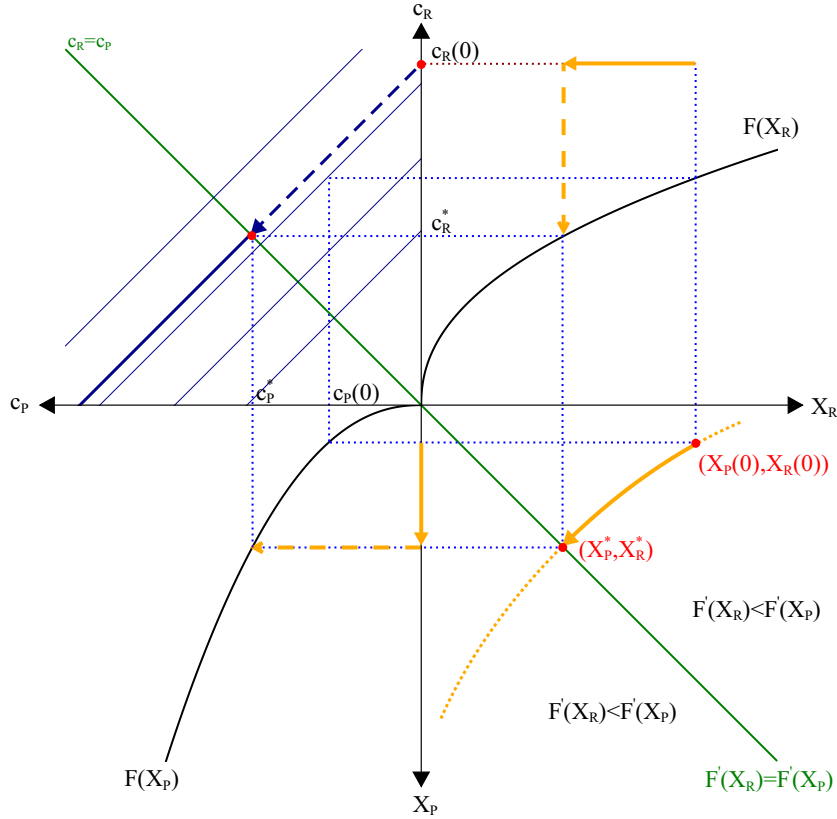
Figure 1: Sustaining global consumption

These results are illustrated in Fig. 1. It is a four-quadrant graph in which the east axis represents X_R, the south axis X_P, the north axis c_R, and the west axis c_P. The north-east quadrant represents production $F(X_R)$ and the south-west quadrant production $F(X_P)$. The north-west quadrant plots social welfare indifference curves in the consumption map (c_R, c_P), and the south-east quadrant is the state map (X_P, X_R) in which state trajectories can be drawn. Efficient and egalitarian paths converge to stationary states that lie on the "wealth-equality line" $X_R^* = X_P^*$ in the state map. The equilibrium consumption levels $c_R^* = F(X_R^*) = F(X_P^*) = c_P^*$ are characterized by an egalitarian consumption within a generation, and correspond to a sustained global consumption $c_R^* + c_P^*$. During the transition, global consumption is sustained at this level, but with only the rich country consuming. All the production in P is invested to have X_P growing as fast as possible, while X_R is depleted. This situation lasts as long as the marginal products of capital differ in the two countries, i.e., until the capital stocks converge.[21]

---

[21]For a given production function, the trajectory leading to a given stationary state (X_R^*, X_P^*) can be

One could impose a constraint on the minimal consumption of the poor country, e.g., $c^\flat \leq c_P(t) \leq F(X_P(t))$, without changing qualitatively the result. The optimal solution to maximize sustainable global consumption is to consume as little as possible in the poor country, which has higher marginal product of capital, and invest as much as possible in this high return capital stock.

The consequences of these results are twofold. First, if a policy was put into place to maximize the level of global consumption sustained over time, it would require high savings from the poor country and higher consumption in the rich country, and thus an increase of inequality in the short-run with respect to a situation in which each country consumes its (unequal) sustainable income. Second, such a policy would lead to capital accumulation in the poor country and capital depletion in the rich county until the equalization of marginal productivity, capital and production. Inequality vanishes in the long-run.

The case examined in this section corresponds to a social planner indifferent to intragenerational inequality. We now turn to the case in which intragenerational inequality matters.

4   Intragenerational inequality aversion

To introduce aversion to intragenerational inequality, we use a social welfare function (SWF) valuing the allocation of individual consumption. The social planner is assumed to have an *intragenerational inequality aversion* (IIA) measured by a parameter θ (Atkinson, 1970). This parameter restricts the substitutability of the consumption of the two countries in the definition of the welfare of a generation. We use the following welfare function:

$$W^\theta(c_R, c_P) = 2 \left(\frac{1}{2} c_R^{1-\theta} + \frac{1}{2} c_P^{1-\theta} \right)^{\frac{1}{1-\theta}}, \; \theta > 0, \; \theta \neq 1 \; .$$

The chosen welfare function allows us to represent a range of cases. (Intragenerational) maximin (Rawls, 1971) depicts an infinite IIA ($\theta = \infty$), and no trade-off between individual consumption is allowed. The previous case considering only global consumption corresponds to $\theta = 0$, a situation with no inequality aversion. Finite positive values for θ correspond to intermediate cases.

---

integrated backward from that state, with constant controls $c_P(t) = 0$ and $c_R(t) = F(X_R^*) + F(X_P^*)$.

With this SWF, in case of equality within a generation (i.e., $c_R = c_P = c$), welfare is equal to global consumption: $W^\theta(c, c) = 2c$.[22] This means that the level of welfare is directly expressed in terms of *equally-distributed-equivalent* (e.d.e.) global consumption.[23] It can thus be compared to the actual global consumption, offering a measure of the effect of intragenerational inequality, and making the comparison with the previous case easy. As $W^\theta(c_R, c_P) \leq c_R + c_P$, the difference between the actual global consumption of a generation $c_R + c_P$ and its e.d.e. level $W^\theta(c_R, c_P)$ is a measure of the extra consumption needed to compensate for the inequality within a generation for a given welfare level.

The maximin problem is now

$$m^\theta(X_R, X_P) = \max_{\underline{w}, c_R(\cdot), c_P(\cdot)} \underline{w} \,, \tag{4}$$

$$\text{s.t. } (X_R(0), X_P(0)) = (X_R, X_P) \,;$$

$$\dot{X}_i(t) = F(X_i(t)) - c_i(t), \ i = R, P, \text{ and}$$

$$W^\theta(c_R(t), c_P(t)) \geq \underline{w} \text{ for all } t \geq 0 \,. \tag{5}$$

We conduct the same analysis as before to determine the maximin path. The details of the resolution are in Appendix A.2. The main results are the following.

Proposition 3 (IIA: Stationary states)**.** *The efficient egalitarian path is a constant consumption path with $c_i(t) = F(X_i^*)$ and $\dot{X}_i = 0$ if and only if the capital stocks (X_P^*, X_R^*) satisfy $F'(X_P^*) = F'(X_R^*)$, i.e., $X_P^* = X_R^*$ in the symmetric case.*

The potential stationary states are the same as in the case maximizing the level of sustained global consumption. In the absence of inequality within a generation (equal endowments), IIA does not affect the intergenerational egalitarian path.

---

[22]Such a construct is discussed in Fleurbaey (2015b, p. 208) and is very convenient to relate welfare to an inequality index, either based on average or total consumption levels.

[23]In his renowned article on inequality measurement, Atkinson (1970) coined the e.d.e. concept. (A similar concept was independently presented by S.G. Kolm in 1968; see Lambert, 2007.) It represents the level of consumption (or income, utility, etc.) which, if equally distributed, would allow to reach the same level of social welfare as the actual distribution. It leads to the definition of the inequality index $\mathcal{A}_\theta = 1 - \frac{\text{e.d.e.}}{\text{mean level}}$. In our two-agent economy, for given consumption levels (c_R, c_P), the e.d.e. consumption $\bar{c}$ is defined as the level of *per capita* consumption which, if consumed by all agents, would result in the same level of welfare, i.e., $W^\theta(\bar{c}, \bar{c}) = W^\theta(c_R, c_P)$. We shall here often refer to the corresponding *global* consumption $2\bar{c} = W^\theta(\bar{c}, \bar{c})$. This presentation is convenient to compare the results with that of the single-agent case without modifying the interpretation of the θ parameter nor the related inequality index, as the ratio of the e.d.e. consumption $\bar{c}$ to the mean consumption $(c_R + c_P)/2$ is equal to the ratio of the e.d.e. global consumption $2\bar{c}$ to the global consumption $c_R + c_P$. Berger and Emmerling (2020) discuss how the e.d.e. concept can be applied jointly to inequality across individuals, time, and states of the world, in a utilitarian framework restricted to nested additively separable welfare functional forms.

Apart from a stationary state, the dynamics is as follows.

Proposition 4 (IIA: Transition). *For any state such that $F'(X_P(t)) > F'(X_R(t))$, i.e., $X_P(t) < X_R(t)$ in the symmetric case, the efficient egalitarian path satisfies $0 < c_P(t) < F(X_P(t))$ and $F(X_R(t)) < c_R(t)$, such that capital X_P increases and capital X_R decreases toward a stationary state characterized by $F'(X_P^*) = F'(X_R^*)$, i.e., $X_R^* = X_P^*$. The consumption levels are characterized by $W^\theta(c_R(t), c_P(t)) = W^\theta(F(X_R^*), F(X_P^*))$, which is equal to $F(X_R^*) + F(X_P^*)$ when production levels are equal in the symmetric case.*

When the social planner is averse to intragenerational inequality, the consumption in the poor country is not reduced to a minimum, and investment is lower than in the case with no IIA. As a consequence, intragenerational inequality is reduced with respect to the situation in which the social planner aims at sustaining global consumption.

The investment pattern is related to the IIA as follows.

Proposition 5 (IIA: Investment pattern). *Along an efficient egalitarian path, at state (X_P, X_R), the convergence pattern (relative investment) is characterized by the relationship* $-\dfrac{\dot{X}_P}{\dot{X}_R} = \dfrac{W^\theta_{c_R}}{W^\theta_{c_P}}\Big|_{W^\theta = \underline{w}} = \left(\dfrac{c_P}{c_R}\right)^\theta$.

The relationship in Prop. 5 relates the relative investment $-\dfrac{\dot{X}_P}{\dot{X}_R}$ (how many units of capital are invested in P for each unit of capital depleted in R), and thus the investment velocity, to the shape of the social welfare indifference curves. The latter are steeper when the IIA is higher. The investment pattern is also related to the level of inequality through the term $\left(\dfrac{c_P}{c_R}\right)^\theta$, which depends on the relative consumption (how much P consumes less than R) and the IIA parameter θ.

These results are illustrated in Fig. 2. The shape of the capital stocks trajectory is related to the shape of the social welfare indifference curves and thus to the inequality aversion. All the points on the state trajectory correspond to as-good-as-stationary-state loci, and could be interpreted as indifference curves in terms of endowments regarding intergenerational equity.

We can discuss the interplay between IIA and intergenerational equity. The degree of IIA influences the level of sustained welfare in a subtle way. The higher the IIA, the lower the substitutability of consumption levels in the SWF, and the larger the consumption of the rich country has to be to compensate a low consumption by the poor country. This induces a larger global consumption to achieve a given welfare level when there is inequality. When intragenerational inequality decreases, the global consumption gets

Figure 2: Sustaining IIA social welfare

closer to its e.d.e. level. As a consequence, along a given efficient egalitarian path with IIA, the level of global consumption decreases, in spite of a constant e.d.e. level. Accounting for IIA results in unequal global consumption over time. Successive generations have the same welfare but decreasing global consumption (and decreasing inequality).

We can also examine the effect of the degree of IIA on the intergenerational egalitarian path. For this purpose, let us consider the paths that would start from a given state under two scenarios of IIA, i.e., corresponding to two SWFs with different levels of θ. Given our definition of welfare and the interpretation of the e.d.e., we can compare the two trajectories through the corresponding egalitarian consumption at stationary state. The optimal stationary states (satisfying $F'(X_R^*) = F'(X_P^*)$) fall on the line $X_R = X_P$. The corresponding stationary state consumption levels are on the line $c_R = c_P$ in the symmetric case. Consider two different sets of social welfare indifference curves corresponding to different degrees of IIA. For any point on the equal-consumption line, the indifferent curves passing through that point for both sets are tangent one to the other at that point.

According to Proposition 5, the trajectories converging to a given stationary state are also tangent in the state map along the optimal stationary states line $X_R = X_P$, the one with the higher IIA having greater curvature and thus lying south-east of the one for the lower IIA. Such trajectories are illustrated in Fig. 3. For an endowment $(X_P(0), X_R(0))$

Figure 3: Effect of intragenerational inequality aversion on intergenerational equity

away from the stationary state, a lower IIA implies higher stationary state levels of the stocks and of consumption. The IIA affects negatively the level of sustained welfare. The lower the degree of IIA, the higher the level of welfare sustained over time, except if the initial state corresponds to a stationary state with no inequality, in which case IIA plays no role. The farther from the optimal stationary states line, the stronger the effect of IIA on the sustained level of welfare. The degree of inequality aversion strongly influences the optimal stationary state of an economy pursuing an egalitarian efficient path, and thus the level of e.d.e. global consumption. It creates an interplay between intragenerational and intergenerational equity issues.

The fact that the IIA affects the whole development trajectory, including the equi-

librium, is a feature of intergenerational egalitarianism. In a maximin problem, the stationary state reached by a trajectory depends on the initial state of the economy (Cairns et al., 2019). In the discounted-utilitarian framework, on the other hand, the stationary state is determined by the (constant) discount rate, and is usually the same whatever the initial state of the economy. For example, in the climate economy with two countries of Kverndokk et al. (2014), higher inequality aversion "generally lifts the consumption path of the poor region, while the rich region must take a greater share of the climate burden" but without modifying the long-run socially optimal consumption levels and capital stocks.[24]

Interestingly, inequality aversion within generations generates an egalitarian generational welfare, but an unequal global consumption over time. Earlier generations consume more than their distant heirs. The larger the IIA, the larger that effect.

Infinite inequality aversion. Let us now briefly consider the case in which the social planner is infinitely averse to inequality within a generation. As the social planner focuses only on the worst-off individual, the SWF is of the form $W(c_R, c_P) = 2 \min \{c_R, c_P\}$, corresponding to the limiting case $\theta = \infty$. In this case, total, not marginal, country-specific production levels drive the solution. The level of sustainable welfare depends on the level of production within each country and is given by $m(X_R, X_P) = 2 \min \{F(X_R), F(X_P)\}$. The social planner would not request the poor country to reduce consumption with respect to the sustainable national income level. No substitution in social welfare is tolerated. No decision in the rich country (especially a high consumption) can justify a sacrifice in the poor country.[25] There is no savings and no growth in P. Inequality perpetuates forever (unless if one drives the rich country down), just as if an intertemporal egalitarian path were implemented at the country level. The only way to escape from this situation is to consider transfers (of capital or consumption) from the rich to the poor country, an

---

[24]This pattern can also be illustrated by the results in Baumgärtner et al. (2017), who consider a single-agent model with several goods and a CES utility function. Modifying the substitutability between goods in utility (which is, from a technical point of view, close to modifying IIA in a model with several consumers) does not change the optimal stationary state, but only the transition path to it. In the discounted-utility framework, the degree of substitutability between goods affects the social discount rate and is central to the debate between proponents of weak and strong intergenerational equity, as discussed in Traeger (2011) and Drupp (2018), but it does not affect the stationary state of the economy.

[25]Note that the maximin criterion would not prescribe anything for the rich country, except a consumption at least equal to that of the poor country and a long-run constraint that the capital stock does not decline below the level that makes it possible to produce this level. Among the possible maximin paths, many may be inefficient in a strong Pareto sense. To get strong Pareto-efficiency, one may consider a *leximin* criterion (see the discussion in footnote 11). In that case, the consumption in the rich country would be set at the production level, i.e., $c_R = F(X_R)$.

option we shall analyze in next section.

This "poverty trap" is a classical outcome of the extreme version of intergenerational equity imposed by intertemporal egalitarianism. It has been the main argument opposing the use of maximin or leximin criteria to represent intergenerational equity (Asheim, 2010). The results in this section offer an interesting sideline to the story, though. This outcome of intertemporal egalitarianism is critical in the single-agent model in which, if the initial capital stock is low, sustaining poverty is not an interesting option for sure. It is usually recognized, however, that the absence of growth would be an acceptable outcome if the level of sustained welfare was high enough (with the underlying argument that this is not currently the case, especially in developing countries). We have shown that, in an economy with several agents, as soon as society is not totally averse to intragenerational inequality and there is a rich country, targeting an intertemporal egalitarian path for intergenerational equity purposes calls for growth in the poor country, and a way out of the poverty trap. As such, the "problem" of the poverty-trap may be less stringent in a model with several agents than in a single-agent model.

5 Extensions

In this section, we relax two of the simplifying assumptions of the main model. First, we consider the case of heterogeneous production capacities. Second, we examine the question of capital transfers.

5.1 Heterogeneous production capacities

Let us relax the assumption that the two countries have the same technology to generate income from capital, and consider the heterogeneous case $F_R \neq F_P$. In this case, it becomes difficult to say which country is rich and which is poor, as $X_P < X_R$ does not necessarily entail $F_P(X_P) < F_R(X_R)$ for all capital stocks. We shall, however, consider that this is the case, to ease interpretation, and assume that country P is having initially less capital and a lower production. In this case, the previous results are modified as follows:

- The stationary states are still characterized by the equality of the marginal products of capital, i.e., $F'_P(X^*_P) = F'_R(X^*_R)$, but this no longer implies an equality of wealth at equilibrium. Depending on the technologies *and* on the initial wealth levels, we may either have $X^*_P < X^*_R$ or the opposite. The condition $F'_P(X^*_P) = F'_R(X^*_R)$

defines a curve of potential equilibria in the state map. The location of a particular equilibrium with respect to the "wealth-equality line" $X_P = X_R$ characterizes which country has more capital in the long-run.

- The equilibrium condition on marginal productivity tells nothing about the consumption inequality in the long-run, as we may have $F_P(X_P^*) < F_R(X_R^*)$ or the opposite. The curve defined by the condition $F_P(X_P) = F_R(X_R)$ corresponds to situations of equal production. The location of a particular equilibrium with respect to that curve characterizes which country consumes more in the long-run.

- The transition path is still characterized by positive savings (consumption is lower than production) in the country with the larger marginal product of capital, and negative savings (consumption is larger than production) in the country with the lower marginal product of capital. Interestingly, this pattern does not necessarily mean that the poor country grows as the rich country declines. The opposite occurs if $F_P'(X_P(0)) < F_R'(X_R(0))$, so that the rich country saves more when its marginal product of capital is larger, in spite of larger capital stock and production. Implementing an intertemporal egalitarian path may even require that the poor country 'optimally' exhausts its capital stock in the case of nil IIA (sustaining global consumption), if $F_P'(0)$ is lower than the marginal product of capital in the rich country.[26]

These results underline the complex interplay between intragenerational and intergenerational equity concerns when production capacities are heterogeneous. Fig. 4 illus-

---

[26]The absence of IIA can justify the sacrifice of a country for the sustaining of global consumption. This result is endogenous to the initial stocks. It occurs when the marginal products of capital in the two countries cannot be equalized, i.e., when the poor country's capital stock is still relatively unproductive at the margin when it declines toward zero whereas the marginal product of capital in the rich country does not fall too much as it is built up. This result is in striking contrast to the result of Quaas et al. (2013) in the discounted-utilitarian framework. They study a model with a manufactured good and two renewable natural resources. They investigate the resilience of this economy to a one-time shock. Solving the post-shock optimum, they show that when a stock is low (and thus has a higher marginal product of capital), building it up is optimal only if the two resources are substitutable enough. The transition requires limiting the consumption of the more productive stock to build it up. If the resources are substitutes, this pattern has only a limited effect on utility. On the other hand, if the resources are complements, building up the stock of the scarce resource has a high utility cost. It may even be optimal to exhaust this stock if the discount rate is high. Exhaustion of the stock with the *higher* marginal product may be optimal when resources are perfect *complements*. In our case, we may exhaust the stock with the lower marginal product when the consumption levels are perfect *substitutes*. When they are complements, exhaustion is not an option because it would lead to an overall collapse. This difference is due to the unequal treatment of generations under discounting. When the very long run matters, as in our intergenerational egalitarian case, overall collapse is not an option.

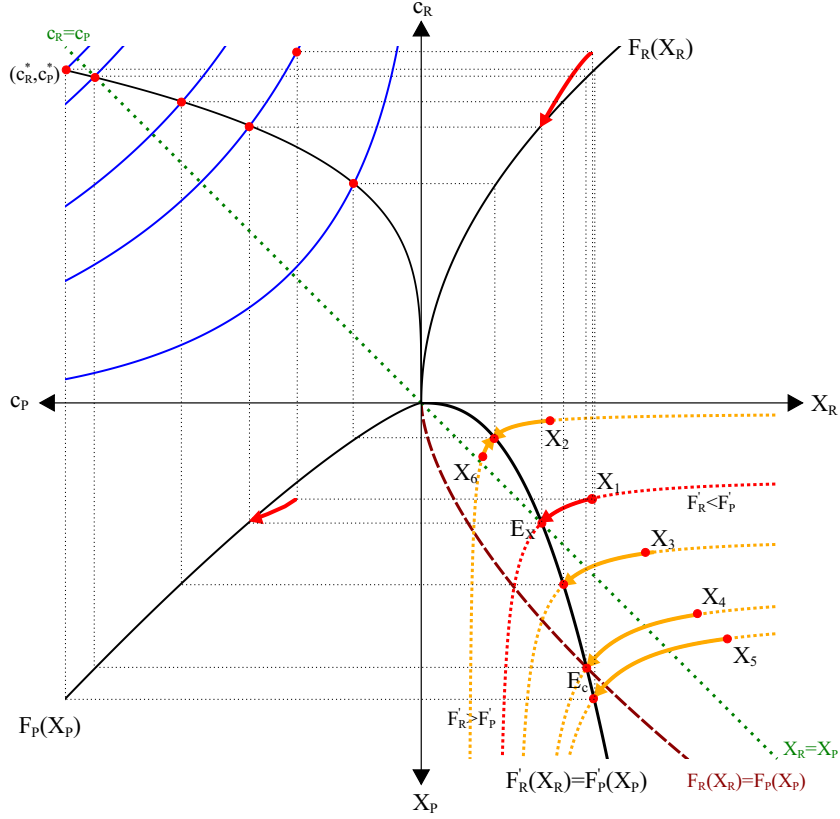trates these results. The figure exhibits trajectories starting from different initial states

Figure 4: Welfare egalitarian paths with different technologies

and resulting in optimal intertemporal egalitarian paths with contrasted effects on intragenerational inequality. The trajectory starting from the initial state X_1 reaches the wealth-egalitarian state E_X, at the intersection of the equilibrium curve and the wealth-equality line. At this equilibrium, capital stocks are equal but consumption levels are not. Intragenerational inequality diminishes along the intertemporal egalitarian path, but inequality does not vanish in the long run. The trajectory starting from initial state X_2 follows the same pattern regarding consumption, but leads to a situation in which even capital stocks are not equalized. The trajectory starting from X_3 leads to situations in which wealth inequality is reversed at some point in time, with country P ending up with more capital than country R, but still a lower consumption. The trajectory starting from state X_4 is interesting in that it tends towards the consumption-egalitarian state E_c, at the intersection of the equilibrium curve and the equal-production curve. Consumption levels are equal in the long run, and intragenerational (consumption) inequality vanishes.

The consumption inequality could even reverse at some point in time for trajectories that reach an equilibrium on the other side of the equal-production curve, as the trajectory starting from state X_5. The trajectory starting from the initial state X_6, which is characterized by a larger capital stock and a larger production level in country R along with a larger marginal product of capital $(F'_R(X_R(0)) > F'_P(X_P(0)))$, illustrates the fact that, with different production functions, implementing an intertemporal egalitarian path could even lead to an increase of intragenerational inequality of wealth and consumption in particular circumstances.

When technologies differ, the effect of the degree of IIA on the intergenerational egalitarian path is more complex than in the symmetric case.

First of all, let us emphasize that comparing the results for two degrees of IIA in the asymmetric case is less straightforward than in the symmetric case, because the equilibria are no longer on the equal-consumption line. In the symmetric case, the welfare level along a trajectory corresponds to the actual global consumption reached in the long-run, which can serve as a benchmark to assess what is "sustained" objectively, as two social planners with different degrees of IIA would assign the same welfare value, $W(c, c) = 2c$, to a given equilibrium. This is no longer the case in the asymmetric case, in which the equilibria do not correspond to equal consumption levels and would not be "valued" the same by the two planners. They would, however, agree that the farther from the origin the equilibrium is, the larger the sustained global consumption.

In the symmetric case, a larger IIA induces a lower sustained global consumption level. In the asymmetric case $F_R \neq F_P$, a higher IIA still induces less inequality within generations, but the effect on the long-run global consumption depends on the initial state (and on the technologies). This is due to the fact that i) the path can result either in decreasing or increasing inequality, and ii) the indifference curves, whose shape determines state trajectories (Proposition 5), are not tangent one to another and symmetric at the equilibria, but cross. As such, the effect of the degree of IIA will depend on whether the equilibrium is reached from above or from below, i.e. whether $F'_R(X_R) < F'_P(X_P)$ or the opposite. Fig. 5 illustrates these two cases. For initial state X_3, satisfying $F'_R(X_R) < F'_P(X_P)$, a larger IIA results in a lower level of sustained global consumption, just as in the symmetric case. This is due to the fact that the convergence is based on sacrifice and savings in the poor country and over-consumption in the rich country, inducing important intragenerational inequality. With a higher IIA, growth in the poor country is slower and convergence is achieved at a "smaller" stationary state. For initial state X_6
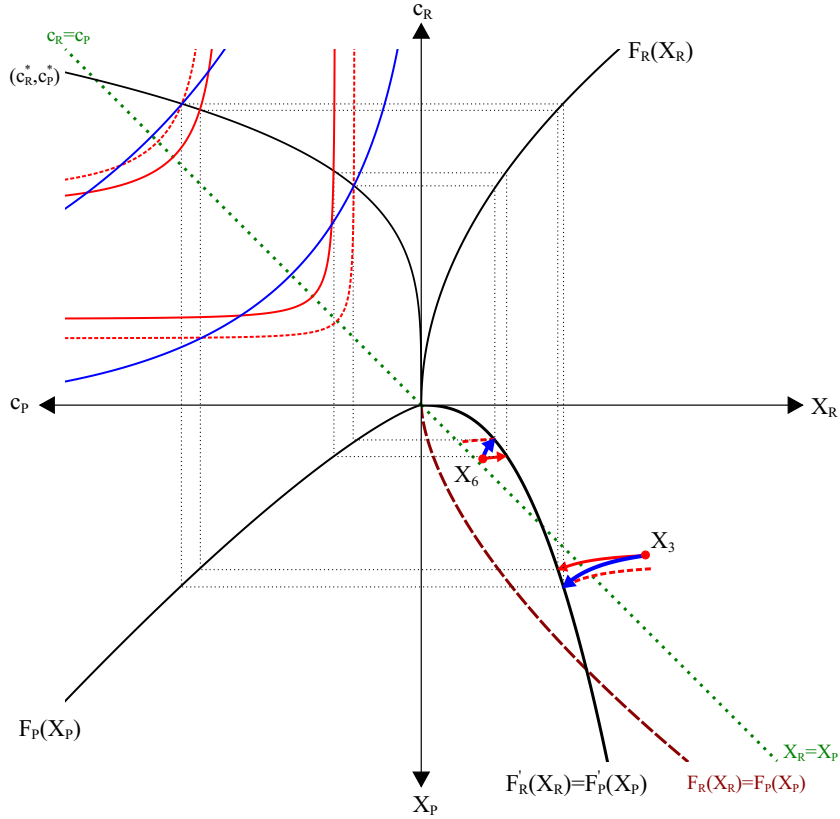
Figure 5: Effect of IIA on intergenerational equity with different technologies

(satisfying $F'_R(X_R) > F'_P(X_P)$), however, a larger IIA results in a higher level of sustained global consumption, and thus benefits future generations. This is due to the fact that intragenerational inequality increases along the trajectories starting from this state. A larger IIA induces a lower consumption in the rich country (and thus a larger investment level) combined with a minimal over-consumption in the poor country (and thus a lower capital depletion). Current inequality is reduced with respect to a trajectory with lower IIA, and the global consumption level reached in the long-run is higher due to the resulting investment pattern. These two cases illustrate that, even in our simple two-agent model, intragenerational equity and intergenerational equity can conflict in some ways.

The effect of IIA on the intertemporal trajectory depends on the degree of IIA in the SWF of course but also, and perhaps less intuitively, on the shape of the production functions. Even though this result may seem surprising, it is not unheard of in the maximin literature. An ambiguous effect of substitutability on intergenerational equity occurs for substitutability in production in the Dasgupta-Heal-Solow model too (Solow,

24

1974; Dasgupta and Heal, 1979; Martinet and Doyen, 2007). Martinet (2012, pp. 145-146) shows that, for low capital stocks, a higher elasticity between inputs reduces the level of utility that can be sustained from a given state, whereas for larger capital stocks a higher elasticity increases the level of utility that can be sustained.

5.2 The case of capital transfers through foreign investment

If inequality within a generation is due to unequal endowments of productive assets, a possibility is to reduce the wealth gap through transfers of capital. Of course, some forms of capital do not move easily. This is the case for natural resources, land, infrastructures, human capital, etc., but there is a possibility to transfer some of it through specific investment, including knowledge.[27]

In the economic model of Section 2, the two capital dynamics are independent. We now relax this assumption by making it possible for R to invest in P. We do so by introducing a new state variable, T, corresponding to the cumulative amount of capital transferred, with an instantaneous transfer flow τ, with $\tau(t) \in [-\bar{\tau}, +\bar{\tau}]$ where $\bar{\tau}$ is a positive constant for simplicity. We assume that this transfer occurs through investment (forgone consumption in R) in the aggregate capital stock in P (e.g., through investment in manufactured capital, infrastructures, education and human capital enhancement, etc.).[28] This investment may generate a return $\pi \in [0; \bar{\pi}]$, which may go from nothing (pure capital transfer without compensation) to a positive return on foreign investment, and even spoliation of income (very high capture of the production in P). We assume that the planner cannot relocate consumption between the countries.[29]

The model reads as follows:

$$\dot{X}_R = F(X_R) - c_R - \tau + \pi \,, \tag{6}$$

$$\dot{X}_P = F(X_P + T) - c_P - \pi \,, \tag{7}$$

$$\dot{T} = \tau \,. \tag{8}$$

We assume that the initial state of the economy is $(X_P(0), X_R(0), T(0))$, with $0 < X_P(0) < X_R(0)$ and no foreign capital $T(0) = 0$.

---

[27] As suggested by a referee, another possibility would be to consider investment in a technology that produces some common good, for example, a technology mitigating climate change. This is an interesting question for future research.

[28] Formally, the model could also encompass the case of capital spoliation, when T is negative.

[29] Formally, the model could also encompass consumption transfer from R to P, with $\pi < 0$.

This model could be studied under different settings, including the maximization of countries' intertemporal consumption. For consistency with the previous analysis, we will consider the case of a social planner aiming at sustaining generational welfare, with IIA. We first consider the unconstrained problem in which the social planner can direct investments to be made in the other country, choosing the transfer and return levels optimally. We then turn to the more realistic case in which foreign direct investment is made voluntarily with the investor getting market return. The return corresponds to the marginal product of capital in P times the amount of foreign investment.

5.2.1 The unconstrained transfer problem

We first solve the following maximin problem, in which the social planner is free to choose the transfer amount and the associated return.

$$m^\tau(X_R, X_P, 0) \qquad = \max_{\underline{w}, c_R(\cdot), c_P(\cdot), \tau(\cdot), \pi(\cdot)} \underline{w} \ , \qquad (9)$$
$$\text{s.t.} \quad (X_R(0), X_P(0), T(0)) = (X_R, X_P, 0) \ ;$$
$$\text{dynamics} \ (7-6-8) \ ;$$
$$W^\theta(c_R, c_P) \geq \underline{w} \ \text{for all} \ t \geq 0 \ . \qquad (10)$$

We derive two results from this model. Mathematical details are in Appendix A.3.

Proposition 6 (Maximal capital transfer). *Implementing an efficient egalitarian path would require a maximal transfer of capital ($\tau = \bar{\tau}$) from the country with the lower marginal product of capital to the country with the higher marginal product of capital, until converge.*

In the symmetric case, it means that a social planner would require the rich country to transfer capital to the poor country as fast as possible, to accelerate the convergence.

Proposition 7 (Minimal return on foreign investment). *Implementing an efficient egalitarian path would require that the return on foreign investment be minimized ($\pi = 0$).*

In the symmetric case, it means that the rich country should not get a return on capital transfer, to accelerate the convergence.[30]

---

[30]Note that if the return is allowed to be negative, it is possible for the social planner to proceed to consumption transfer directly. One can then follow the paths of capital accumulation as in the case of no inequality aversion, maximizing the level of global consumption and splitting it equally between the two countries.

Note that these results point in the same direction as the analysis of the role of international transfers in the discounted-utility, two-country climate economy of Kverndokk et al. (2014). In their setting, the optimal transfer policy is a most rapid approach toward equality too. While such a particular solution is the result of the same mathematical pattern in both approaches (the Lagrangean/Hamiltonian is linear in the transfer control variable), one should not conclude that this result is a technical artifact. This pattern emerges from the structure of the model, which is supported by the economic stylized facts it represents. If one is really concerned with inequality (or with efficiency in a utilitarian approach with decreasing marginal utility of consumption), any cost-free transfer should be implemented right away. Such an unconstrained transfer of capital with no return is highly unlikely, though, and we now turn to the case in which the transfer generates a return.

5.2.2 Foreign investment with return

We now consider the case in which the foreign investment generates a return $\pi = TF'(X_P + T)$ corresponding to the product of the invested amount and the marginal product of capital in P. The return is thus no longer a control variable. Moreover, as the foreign investment generates a return, it is no longer neutral in terms of consumption and welfare sustainment. The following propositions summarize the features of the optimal path. Mathematical details are in Appendix A.3.

Proposition 8 (Foreign investment with return: stationary state). *When foreign investment generates a return equal to the amount of foreign capital T times marginal product of capital $F'(X_P + T)$ in the poor country, the egalitarian path reaches an equilibrium with equal capital stocks in both countries, no foreign investment and equal consumption levels.*

At a stationary state, there is no more inequality and the level of foreign investment is nil, meaning that capital transfer is transitory when there is a return on investment.

Proposition 9 (Foreign investment with return: transition). *The stationary state is reached after a transition characterized by two phases. First, the social planner increases foreign investment at the highest possible rate, with $\tau = \bar{\tau}$. Then, investment is withdrawn at the highest possible rate, with $\tau = -\bar{\tau}$, to reach the stationary state with equal capital stocks in the two countries and no foreign investment.*

Along this transition path, the flow of foreign investment is dictated by the sign of the difference between the shadow value of the capital stock in R and that of the foreign

capital T invested in the poor country. At the initial time, the shadow value of a unit of foreign investment T is higher than that of the capital stock in the rich country X_R, and investment flows from the rich country to the poor country. The two shadow values decrease along the optimal path, but that of T decreases initially faster than that of X_R. When the shadow value of T crosses that of X_R downward, foreign investment peaks and starts to decline to zero. At some point along the path, the marginal product of capital in the poor country decreases below that of the rich country, i.e., $F'(X_P + T) < F'(X_R)$, meaning that foreign investment allows the poor country to produce more than the rich country $(F(X_P+T) > F(X_R))$, but with a share $TF'(X_P+T)$ of P's production benefiting the rich country due to the return on investment. Such an investment pattern would not emerge in a decentralized economy in which the rich country would invest only up to the point at which the marginal product of capital in the poor country $F'(X_P + T)$ equalizes the marginal product of capital in the rich country $F'(X_R)$. To implement the social planner's optimal path, one would need to modify instantaneous private incentives, either by considering the intertemporal value of investment along with a commitment on the investment duration, or to accept a lower return on investment abroad with respect to investment in the home-country, with for example a non-monetary benefit of supporting development.

6   Conclusion

In the introduction, we asked two questions on the link between intragenerational inequality and intergenerational equity. (1) How do sustainability policies affect intragenerational inequality? (2) How does inequality aversion affect sustainable development paths? We studied the interplay between intra- and intergenerational equity in the definition of an intertemporal egalitarian path. For this purpose, we developed a dynamic economic model with two countries having different endowments. In a situation in which a poor country is endowed with less productive assets, has a lower production but a higher marginal product of capital, the following results hold.

Targeting intergenerational egalitarianism while considering only the global consumption of a generation – which is equivalent to considering a single-agent model – would imply extreme intragenerational inequality. This result urges moving away from the single-agent model to consider heterogeneous agents within generations, with inequality aversion, when examining the consequences of criteria aiming at representing intergenerational equity.

When the social planner has an aversion to intragenerational inequality, implementing an intertemporal egalitarian path sustaining welfare generates inequality in the short run, but inequality is reduced over time and vanishes in the long run. As such, targeting intergenerational egalitarianism affects intragenerational inequality in a dynamic way, beginning to answer our first question. More specifically, when the two countries have the same technology, implementing an egalitarian welfare path induces a convergence toward equal wealth, income, and consumption. The stock of productive assets declines in the rich country while it grows in the poor country. Along that convergence path, consumption in the poor country increases while consumption in the rich country decreases, resulting in a constant welfare over time. Such a pattern is followed as long as the marginal products of capital in the two countries differ. As intragenerational inequality decreases over time, global consumption also decreases without reducing the welfare level. The egalitarian path with intragenerational inequality aversion thus corresponds to a differentiated degrowth path, with actual degrowth in the rich country, but growth in the poor country. Such a path may appear consistent with the needed growth in developing countries and the perhaps needed global degrowth for environmental sustainability.

Regarding our second question, we showed that the degree of intragenerational inequality aversion affects both the level of sustainable welfare and the path of global consumption. Along an intertemporal egalitarian path, welfare is constant but the global consumption decreases over time as intragenerational inequality decreases. The larger the intragenerational inequality aversion, the lower the sustainable level of welfare, and the lower the level of global consumption reached in the long-run. The intuition is that one can take a greater advantage of disparities between endowments and capacities of countries to make trade-offs to increase welfare when inequality aversion is lower. In the extreme case of an infinite aversion to inequality within generations, no sacrifice can be asked of the poor country, which does not grow. Initial inequality is perpetuated.

Considering several agents and the intragenerational equity issue modifies the optimal path of an intertemporal egalitarian criterion. The larger the intragenerational inequality aversion, the lower the sustained level of (equally distributed equivalent) global consumption. There is thus, here again, a tension between the two types of equity issues. Of course, allowing for transfers of capital (or consumption) from the rich country to the poor country solves the problem. Any policy that makes such transfers easier would reduce the tension between the two equity requirements. Just as in the discounted-utility case examined in the two-country climate economy of Kverndokk et al. (2014), stronger inequality aversion leads to a reduction of the consumption gap, and thus to a larger

consumption in the poor country and a lower consumption in the rich country compared to the case with a lower inequality aversion. In the discounted utility case, however, it may be optimal to reduce consumption inequality in the short run by reducing investment in the poor country and increasing it in the rich country, which can lead to higher consumption inequality in the future. In our intertemporal egalitarian set-up, an higher intragenerational inequality aversion results in a reduction of consumption inequality at all times.

Of course, all our results rely on a social planner approach. One may wonder who would be legitimate (and able) to implement an intertemporal egalitarian path. But the problem is the same for any global issue. Boarini et al. (2018) offer an interesting discussion on the role of national governance on global issues, which we here briefly summarize and quote. Countries could easily pursue their individual interest of growth, disregarding global environmental issues, but global objectives should sometimes get priority over national ones, and even be incorporated in national policy agendas. "These objectives entail a net transfer of resources from high-income countries toward low-income countries. As such, they imply a cost for high-income countries that, strictly speaking, may hinder the achievement of national objectives, at least in the short run. In the long run, however, there is a strong common interest in achieving convergence of living standards over the world in order to construct a peaceful, cooperative and flourishing global society" (p. 18-19). Intergenerational equity may be such an issue, worth implementing at a global level rather than at a national level (notwithstanding the interest of net transfers we emphasized in Section 5). Applied at a national level, intergenerational equity would result in perpetual, and perhaps unbearable intragenerational inequality.

Our results suggest that combining intragenerational inequality aversion with intergenerational equity concerns could provide arguments for a differentiated degrowth, in addition to what Bretschger (2017) exhibits for reasons for a degrowth of material consumption, with possible compensation if natural conditions improve. Studying the effect of such a degrowth on individual utilities when environmental assets are explicitly accounted for would be of great interest.

Putterman et al. (1998) provide an interesting discussion on ways to implement egalitarianism. Given the convergence pattern described in our model, the following policy instruments could be used to decentralize the efficient egalitarian development path. Capital decrease in the rich country could be achieved by wealth or inheritance taxation, which would result in a dynamic diminution of the capital stock. Capital accumulation in the

poor country could be fostered by promoting savings. Such tools may, however, be ineffective in the long run if inequality is endogenous, for example due to different discount rates. As capital transfer increases efficiency, reducing the friction in international capital markets would hasten convergence too. If inequality is due to non-material factors such as human capital, policies could focus on redistribution in support of education.

Our work could be extended in several ways. First, one could examine how the results are modified when the two countries interact, for example through trade. Second, our results were obtained in the quite extreme version of intergenerational equity implied by intertemporal egalitarianism. Asheim (2010) stresses that such a criterion may have "undesirable consequences," such as no growth at all. It, however, offers a starting point to identify interactions between the intra- and intergenerational dimensions of equity. Analyzing theses interactions for other forms of intergenerationally equitable criteria would richen our analysis.[31] Last, studying the interaction of the two equity dimensions in an economic model of interest (e.g., a climate change economy) would be in line with Rawls' reflective equilibrium (Asheim, 2010).

References

Adler, M., Anthoff, D., Bosetti, V., Garner, G., Keller, K., and Treich, N. (2017). "Priority for the worse-off and the social cost of carbon." *Nature Climate Change* 7, pp. 443–449.

Adler, M. and Treich, N. (2015). "Prioritarianism and Climate Change." *Environmental and Resource Economics* 62(2), pp. 279–308.

Alvarez-Cuadrado, F. and Long, N. (2009). "A mixed Bentham-Rawls criterion for intergenerational equity: Theory and implications." *Journal of Environmental Economics and Management* 58 (2), pp. 154–168.

Anthoff, D. and Emmerling, J. (2019). "Inequality and the Social Cost of Carbon." *Journal of the Association of Environmental and Resource Economists* 6 (2), pp. 29–59.

Anthoff, D., Hepburn, C., and Tol, R. S. J. (2009). "Equity weighting and the marginal damage costs of climate change." *Ecological Economics* 68 (3), pp. 836–849.

---

[31] As suggested by a referee, one could address jointly intragenerational and intergenerational equity issues by considering equity at the individual level, giving more weight to the poorer individuals, independently of their location and time, using a rank-dependent utilitarian criterion.

Arrow, K., Bolin, B., Costanza, R., Dasgupta, P., Folke, C., Holling, C. S., Jansson, B.-O., Levin, S., Mäler, K.-G., Perrings, C., and Pimentel, D. (1995). "Economic Growth, Carrying Capacity, and the Environment." *Ecological Economics* 15 (2), pp. 91–95.

Arrow, K., Dasgupta, P., Goulder, L., Daily, G., Ehrlich, P., Heal, G., Levin, S., Mäler, K.-G., Schneider, S., Starrett, D., and Walker, B. (2004). "Are We Consuming Too Much?" *Journal of Economic Perspectives* 18 (3), pp. 147–172.

Asako, K. (1980). "Economic growth and environmental pollution under the max-min principle." *Journal of Environmental Economics and Management* 7 (3), pp. 157–183.

Asheim, G. B. (2013). "Hartwick's rule." In: *Shogren, J. (Ed.), Encyclopedia of Energy, Natural Resources and Environmental Economics, Elsevier Science.*

Asheim, G. B. and Ekeland, I. (2016). "Resource conservation across generations in a Ramsey-Chichilnisky model." *Economic Theory* 61, pp. 611–639.

Asheim, G. B. and Mitra, T. (2010). "Sustainability and discounted utilitarianism in models of economic growth." *Mathematical Social Sciences* 59 (2), pp. 148–169.

Asheim, G. B. and Zuber, S. (2013). "A complete and strongly anonymous leximin relation on infinite streams." *Social Choice and Welfare* 41 (4), pp. 819–834.

Asheim, G. B. (2010). "Intergenerational Equity." *Annual Review of Economics* 2 (1), pp. 197–222.

— (2011). "Comparing the welfare of growing economies." *Revue d'économie politique* 121 (1), pp. 59–72.

Asheim, G. B., Banerjee, K., and Mitra, T. (2020). "How stationarity contradicts intergenerational equity." *Economic Theory*, pp. 1–22.

Asheim, G. B. and Nesje, F. (2016). "Destructive Intergenerational Altruism." *Journal of the Association of Environmental and Resource Economists* 3 (4), pp. 957–984.

Asheim, G. B. and Tungodden, B. (2004). "Resolving distributional conflicts between generations." *Economic Theory* 24, pp. 221–230.

Asheim, G. B. and Zuber, S. (2014). "Escaping the repugnant conclusion: Rank-discounted utilitarianism with variable population." *Theoretical Economics* 9 (3), pp. 629–650.

Atkinson, A. B. (1970). "On the Measurement of Inequality." *Journal of Economic Theory* 2 (3), pp. 244–263.

Baumgärtner, S., Drupp, M., and Quaas, M. (2017). "Subsistence, Substitutability and Sustainability in Consumption." *Environmental and Resource Economics* 67 (1), pp. 47–66.

Baumgärtner, S. and Glotzbach, S. (2012). "The relationship between intragenerational and intergenerational ecological justice." *Environmental Values* 21 (3), pp. 331–355.

Baumgärtner, S., Glotzbach, S., Hoberg, N., Quaas, M., and Stumpf, K. (2012). "Economic Analysis of Trade-offs between Justices." *Intergenerational Justice Review* 1, pp. 4–9.

Berger, L. and Emmerling, J. (2020). "Welfare as equity equivalents." *Journal of Economic Surveys* 34 (4), pp. 727–752.

Boarini, R., Causa, O., Fleurbaey, M., Grimalda, G., and Woolard, I. (2018). "Reducing inequalities and strengthening social cohesion through inclusive growth: a roadmap for action." *Economics* 12 (2018-63), pp. 1–26.

Bossert, W., Sprumont, Y., and Suzumura, K. (2007). "Ordering infinite utility streams." *Journal of Economic Theory* 135.1, pp. 579–589.

Botzen, W. J. W. and Bergh, J. C. J. M. van den (2014). "Specifications of Social Welfare in Economic Studies of Climate Policy: Overview of Criteria and Related Policy Insights." *Environmental and Resource Economics* 58 (1), pp. 1–33.

Bretschger, L. (2017). "Is the Environment Compatible with Growth? Adopting an Integrated Framework for Sustainability." *Annual Review of Resource Economics* 9 (1), pp. 185–207.

Burmeister, E. and Hammond, P. J. (1977). "Maximin Paths of Heterogeneous Capital Accumulation and the Instability of Paradoxical Steady States." *Econometrica* 45 (4), pp. 853–870.

Cairns, R. and Long, N. (2006). "Maximin: a direct approach to sustainability." *Environment and Development Economics* 11 (3), pp. 275–300.

Cairns, R. and Martinet, V. (2014). "An environmental-economic measure of sustainable development." *European Economic Review* 69, pp. 4–17.

Cairns, R. D., Del Campo, S., and Martinet, V. (2019). "Sustainability of an economy relying on two reproducible assets." *Journal of Economic Dynamics and Control* 101, pp. 145–160.

Chichilnisky, G. (1996). "An axiomatic approach to sustainable development." *Social Choice and Welfare* 13 (2), pp. 231–257.

Dasgupta, P. and Heal, G. (1974). "The Optimal Depletion of Exhaustible Resources." *Review of Economic Studies* 41, pp. 3–28.

— (1979). *Economic Theory and Exhaustible Resources*. Cambridge University Press.

d'Autume, A., Hartwick, J. M., and Schubert, K. (2010). "The zero discounting and maximin optimal paths in a simple model of global warming." *Mathematical Social Sciences* 59 (2), pp. 193–207.

d'Autume, A. and Schubert, K. (2008a). "Hartwick's rule and maximin paths when the exhaustible resource has an amenity value." *Journal of Environmental Economics and Management* 56 (3), pp. 260–274.

d'Autume, A. and Schubert, K. (2008b). "Zero discounting and optimal paths of depletion of an exhaustible resource with an amenity value." *Revue d'Économie Politique* 118 (6), pp. 827–845.

Del Campo, S. (2019). "Distributional considerations during growth toward the golden rule." *EAERE*, 25 p.

Dennig, F., Budolfson, M. B., Fleurbaey, M., Siebert, A., and Socolow, R. H. (2015). "Inequality, climate impacts on the future poor, and carbon prices." *Proceedings of the National Academy of Sciences* 112 (52), pp. 15827–15832.

Dietz, S. and Asheim, G. (2012). "Climate policy under sustainable discounted utilitarianism." *Journal of Environmental Economics and Management* 63, pp. 321–335.

Dixit, A., Hammond, P., and Hoel, M. (1980). "On Hartwick's Rule for Regular Maximin Paths of Capital Accumulation and Resource Depletion." *Review of Economic Studies* 47 (3), pp. 551–556.

Drupp, M. (2018). "Limits to Substitution Between Ecosystem Services and Manufactured Goods and Implications for Social Discounting." *Environmental and Resource Economics* 69 (1), pp. 135–158.

Dworkin, R. (2000). *Sovereign Virtue: The Theory and Practice of Equality.* Harvard University Press.

Epstein, L. G. (1986). "Intergenerational consumption rules: An axiomatization of utilitarianism and egalitarianism." *Journal of Economic Theory* 38 (2), pp. 280–297.

Fehr, E. and Schmidt, K. M. (1999). "A theory of fairness, competition, and cooperation." *Quarterly Journal of Economics* 114 (3), pp. 817–868.

Fleurbaey, M. (2015a). "On sustainability and social welfare." *Journal of Environmental Economics and Management* 71, pp. 34 –53.

Fleurbaey, M. (2015b). "Equality versus Priority: How Relevant is the Distinction?" *Economics and Philosophy* 31 (2), pp. 203–217.

— (2019). "Economic Theories of Justice." *Annual Review of Economics* 11, pp. 665–684.

Fleurbaey, M. and Tungodden, B. (2010). "The tyranny of non-aggregation versus the tyranny of aggregation in social choices: a real dilemma." *Economic Theory* 44, pp. 399–414.

Hartwick, J. (1977). "Intergenerational equity and the investing of rents from exhaustible resources." *American Economic Review* 67 (5), pp. 972–974.

Heal, G. (1998). *Valuing the Future: Economic Theory and Sustainability.* Columbia University Press, New York.

— (2009). "Climate Economics: A Meta-Review and Some Suggestions for Future Research." *Review of Environmental Economics and Policy* 3 (1), pp. 4–21.

Kverndokk, S., Nævdal, E., and Nøstbakken, L. (2014). "The trade-off between intra- and intergenerational equity in climate policy." *European Economic Review* 69, pp. 40–58.

Lambert, P. (2007). "Serge Kolm's "The Optimal Production of Social Justice"." *Journal of Economic Inequality* 5 (2), pp. 213–234.

Lauwers, L. (1997). "Rawlsian equity and generalised utilitarianism with an infinite population." *Economic Theory* 9, pp. 143–150.

Llavador, H., Roemer, J., and Silvestre, J. (2011). "A dynamic analysis of human welfare in a warming planet." *Journal of Public Economics* 95 (11), pp. 1607–1620.

Martinet, V. (2012). *Economic Theory and Sustainable Development: What Can We Preserve for Future Generations?* Routledge.

Martinet, V. and Doyen, L. (2007). "Sustainability of an economy with an exhaustible resource: A viable control approach." *Resource and Energy Economics* 29 (1), pp. 17–39.

Mitra, T. (2002). "Intertemporal Equity and Efficient Allocation of Resources." *Journal of Economic Theory* 107 (2), pp. 356–376.

Moreno-Ternero, J. D. and Roemer, J. E. (2012). "A common ground for resource and welfare egalitarianism." *Games and Economic Behavior* 75 (2), pp. 832–841.

Myerson, R. B. (1981). "Utilitarianism, egalitarianism, and the timing effect in social choice problems." *Econometrica*, pp. 883–897.

Parfit, D. (1995). "Equality or Priority?" *The Lindley Lecture, University of Kansas.*

Piacquadio, P. (2014). "Intergenerational egalitarianism." *Journal of Economic Theory* 153, pp. 117–127.

Putterman, L., Roemer, J. E., and Silvestre, J. (1998). "Does Egalitarianism Have a Future?" *Journal of economic literature* 36.2, pp. 861–902.

Quaas, M., Soest, D. van, and Baumgärtner, S. (2013). "Complementarity, impatience, and the resilience of natural-resource-dependent economies." *Journal of Environmental Economics and Management* 66 (1), pp. 15–32.

Ramsey, F. P. (1928). "A Mathematical Theory of Saving." *Economic Journal*, pp. 543–559.

Rawls, J. (1971). *A Theory of Justice.* Harvard University Press, Cambridge.

Rockström, J., Steffen, W., Noone, K., Persson, A., Chapin, F. S. I., Lambin, E., Lentonm, T. M., Scheffer, M., Folke, C., Schellnhuber, H. J., Nykvist, B., Wit, C. A. d., Hughes,

T., Leeuw, S. van der, Rodhe, H., Sörlin, S., Snyder, P. K., Costanza, R., Svedin, U., Falkenmark, M., Karlberg, L., Corell, R. W., Fabry, V. J., Hansen, J., Walker, B., Liverman, D., Richardson, K., Crutzen, P., and Foley, J. (2009). "Planetary Boundaries: Exploring the Safe Operating Space for Humanity." *Ecology and Society* 14 (2).

Schelling, T. (1992). "Some Economics of Global Warming." *American Economic Review* 82 (1), pp. 1–14.

Scovronick, N., Budolfson, M. B., Dennig, F., Fleurbaey, M., Siebert, A., Socolow, R. H., Spears, D., and Wagner, F. (2017). "Impact of population growth and population ethics on climate change mitigation policy." *Proceedings of the National Academy of Sciences* 114 (46), pp. 12338–12343.

Sen, A. K. (1997). "From Income Inequality to Economic Inequality." *Southern Economic Journal* 64.2, pp. 383–401.

Solow, R. M. (1974). "Intergenerational Equity and Exhaustible Resources." *Review of Economic Studies* 41, pp. 29–45.

Sterner, T. (2012). *Fuel Taxes and the Poor: the Distributional Effects of Gasoline Taxation and Their Implications for Climate Policy.* Routledge.

Temkin, L. S. (1993). *Inequality.* Oxford University Press.

Traeger, C. (2011). "Sustainability, limited substitutability, and non-constant social discount rates." *Journal of Environmental Economics and Management* 62 (2), pp. 215–228.

Yamaguchi, R. (2019). "Intergenerational discounting with intragenerational inequality in consumption and the environment." *Environmental and Resource Economics* 73, pp. 957–972.

A Appendix

A.1 Sustaining global consumption: Mathematical details

This section solves the problem of Section 3. Problem (4) can be converted into an usual optimal control problem under the constraint (5), the sustained consumption level $\underline{c}$ being a control parameter. From a technical point of view, our problem is close to the problems treated in Burmeister and Hammond (1977) and Cairns et al. (2019).[32] We denote the costate variables of the stocks by μ_i. The Hamiltonian is

$$\mathcal{H}(X, c, \mu) = \mu_R \left(F(X_R) - c_R \right) + \mu_P \left(F(X_P) - c_P \right) .$$

Denoting the multiplier associated with the constraint (3) by ρ, we get the Lagrangean associated with the maximin problem:

$$\mathcal{L}(X, c, \mu, \underline{c}, \rho) = \mu_R \left(F(X_R) - c_R \right) + \mu_P \left(F(X_P) - c_P \right) + \rho \left(c_R + c_P - \underline{c} \right) .$$

This Lagrangean is linear in the decisions, which implies that corner solutions for the controls are possible.

The necessary conditions are, for $i = R, P$:

$$\frac{\partial \mathcal{L}}{\partial c_i} = -\mu_i + \rho \leq 0 \ , \ c_i \geq 0 \ , \ c_i \frac{\partial \mathcal{L}}{\partial c_i} = 0 \ ; \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial X_i} = -\dot{\mu}_i \ . \tag{12}$$

The complementary slackness conditions are

$$\rho \geq 0 \ , \quad c_R + c_P - \underline{c} \geq 0 \ , \quad \rho \left(c_R + c_P - \underline{c} \right) = 0 \ .$$

By Lemma 1 of Cairns et al. (2019), $c_R = c_P = 0$ cannot be solution of the problem. There are thus three cases to consider: (i) $c_R > 0$ and $c_P > 0$; (ii) $c_R > 0$ and $c_P = 0$; (iii) $c_R = 0$ and $c_P > 0$. We start with the case (i) of positive consumption in both countries. In this case, condition (11) corresponds to $\frac{\partial \mathcal{L}}{\partial c_i} = -\mu_i + \rho = 0$ for both countries. It implies $\mu_R = \mu_P = \rho$. Taking the time derivative of ρ, one gets $\frac{\dot{\rho}}{\rho} = \frac{\dot{\mu}_R}{\mu_R} = \frac{\dot{\mu}_P}{\mu_P}$. Moreover,

---

[32]See Cairns et al. (2019, Proposition 2) for a proof of the existence of the optimal solution.

from conditions (12), we get for $i = R, P$

$$-\frac{\partial \mathcal{L}}{\partial X_i} = -\mu_i F'(X_i) = \dot{\mu}_i, \quad \Leftrightarrow \quad \frac{\dot{\mu}_i}{\mu_i} = -F'(X_i) \, .$$

Combining these conditions, we can state that an internal solution with $c_R > 0$ and $c_P > 0$ is possible only for states (X_R^*, X_P^*) such that $F'(X_R^*) = F'(X_P^*)$. This is dynamically possible only for a stationary state, with $c_R = F(X_R^*)$ and $c_P = F(X_P^*)$, *proving Proposition 1*. Apart from those stationary states, one must have either $c_P = 0$ or $c_R = 0$ along the maximin path.

We now characterize the transition described in Proposition 2. Consider a state $(X_P, X_R) >> (0, 0)$ such that $F'(X_P) > F'(X_R)$. We demonstrate that, under these conditions, stock X_R is consumed alone while stock X_P builds up as long as the previous inequality holds by proving that the opposite is not possible. For a regular maximin path, $(\mu_R, \mu_P, \rho) \neq (0, 0, 0)$, $\rho > 0$ and thus $c_R + c_P = m^0(X_R, X_P)$. Assume that $c_P > 0$ and $c_R = 0$. Given the maximin value m^0, along the maximin path, one would have $c_P = m^0$, which is constant. By Lemmata 1 and 2 of Cairns et al. (2019), $m^0(X_P, X_R) > F(X_P) + F(X_R)$. This implies that $c_P > F(X_P)$. Therefore, $\frac{dX_P}{dt} = F(X_P) - c_P < 0$. Also, $\frac{d^2 X_P}{(dt)^2} = F'(X_P)\frac{dX_P}{dt} - \frac{dc_P}{dt} = F'(X_P)\frac{dX_P}{dt} < 0$. Therefore, stock X_P would be exhausted in a finite time ν. After that time, global consumption would correspond to the sustained production of stock X_R. At time ν, stock X_R would have increased to some level $X_R^* \equiv X_R(\nu)$ such that $F(X_R^*) = m^0$, allowing consumption c_R to sustain exactly the maximin utility. The stationary state would be $(0, X_R^*)$. Making a step backward to examine the states through which such a path goes just prior to exhaustion, the dynamics before exhaustion would be

$$\dot{X}_P = F(X_P) - c_P \quad \Leftrightarrow \quad dX_P = \left(F(X_P) - m^0\right) dt \, ,$$
$$\dot{X}_R = F(X_R) \quad \Leftrightarrow \quad dX_R = F(X_R)dt \, .$$

Consider an infinitesimal time lapse dt. At time $\nu - dt$, stock X_P is equal to $\tilde{X}_P = dX_P = m^0 dt$. Stock X_R is equal to $\tilde{X}_R = X_R^* - dX_R = X_R^* - m^0 dt$. By Lemma 1 of Cairns et al. (2019), we know that the maximin value at time $\nu - dt$ is greater than or equal to the equilibrium consumption of state $(\tilde{X}_P, \tilde{X}_R)$. Let us denote this consumption level by $\tilde{c} = F(m^0 dt) + F(X_R^* - m^0 dt)$. We have $m^0(\nu - dt) \geq \tilde{c}$. By subtracting $m^0(\nu)$

from both sides of the equation, we obtain the following.

$$
\begin{aligned}
m^0(\nu - \mathrm{d}t) - m^0(\nu) \;\; &\geq\;\; \tilde{c} - m^0(\nu) \; ; \\
&\geq\;\; F\left(m^0\mathrm{d}t\right) + F\left(X_R^* - m^0\mathrm{d}t\right) - F(X_R^*) \; ; \\
&\geq\;\; \left(F\left(0 + m^0\mathrm{d}t\right) - F(0)\right) + \left(F\left(X_R^* - m^0\mathrm{d}t\right) - F(X_R^*)\right) \; ; \\
&\geq\;\; m^0\mathrm{d}t\frac{F\left(0 + m^0\mathrm{d}t\right) - F(0)}{m^0\mathrm{d}t} - m^0\mathrm{d}t\frac{F\left(X_R^* - m^0\mathrm{d}t\right) - F(X_R^*)}{-m^0\mathrm{d}t} \; .
\end{aligned}
$$

Let $\epsilon = m^0\mathrm{d}t$ and $\tilde{\nu} = \nu - \mathrm{d}t$ (thus $\nu = \tilde{\nu} + \mathrm{d}t$). We get

$$
\begin{aligned}
m^0(\tilde{\nu}) - m^0(\tilde{\nu} + \mathrm{d}t) \;\; &\geq m^0\mathrm{d}t\frac{F(0 + \epsilon) - F(0)}{\epsilon} - m^0\mathrm{d}t\frac{F(X_R^* - \epsilon) - F(X_R^*)}{-\epsilon} \; ; \\
\Leftrightarrow \;\; \frac{1}{m^0}\left(\frac{m^0(\tilde{\nu}+\mathrm{d}t)-m^0(\tilde{\nu})}{\mathrm{d}t}\right) \;\; &\leq \frac{F(X_R^* - \epsilon) - F(X_R^*)}{-\epsilon} - \frac{F(0 + \epsilon) - F(0)}{\epsilon} \; .
\end{aligned}
$$

By taking the limits $\epsilon, \mathrm{d}t \to 0$, we obtain

$$
\frac{\dot{m^0}}{m^0} \leq F'(X_R^*) - F'(0) < 0 \; .
$$

As the maximin value cannot decrease along a maximin path, we have a contradiction. We thus can assert that if $F'(X_P) > F'(X_R)$, then $c_P = 0$ and $c_R > 0$. By regularity, $m^0 = c_R + c_P$. Thus, $c_R = m^0(X_R, X_P)$. *This proves Proposition 2.*

A.2 Intragenerational inequality aversion: Mathematical details

This section solves the problem of Section 4. To make the analysis more general and cover the results presented in the extension with different production capacities of Section 5, we distinguish the technologies of the two countries. The symmetric case is retrieved by setting $F_P \equiv F_R \equiv F$.

The resolution of the maximin problem is similar to that of the previous section, except that both consumption levels will be positive when $\theta > 0$. The Lagrangian is

$$
\mathcal{L}(X, c, \mu, \underline{w}, \rho) = \mathcal{H}(X, c, \mu) + \rho\left(W^\theta(c_R, c_P) - \underline{w}\right) \; .
$$

The necessary conditions include, for $i = R, P$, and for any time t (Cairns and Long,

2006; Cairns et al., 2019):

$$\frac{\partial \mathcal{L}}{\partial c_i} \;=\; 0 \quad \Leftrightarrow \quad \mu_i = \rho W_{c_i}^{\theta} \; ; \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial X_i} \;=\; -\dot{\mu}_i \quad \Leftrightarrow \quad -\frac{\dot{\mu}_i}{\mu_i} = F_i'(X_i) \;. \tag{14}$$

As $W_{c_i=0}^{\theta} = +\infty$ whatever $\theta > 0$, given that the costate variables are continuous functions and cannot take infinite value at any finite time, one must have $c_i(t) > 0$ at all times.

Deriving an expression for $\frac{\dot{\mu}_i}{\mu_i}$ from eq. (13) and combining the two equations gives

$$-\frac{\dot{\rho}}{\rho} = F_i'(X_i) + \frac{\dot{W}_{c_i}^{\theta}}{W_{c_i}^{\theta}} \;. \tag{15}$$

Equation (15) holds for both countries, so we can write

$$\frac{\dot{W}_{c_R}^{\theta}}{W_{c_R}^{\theta}} - \frac{\dot{W}_{c_P}^{\theta}}{W_{c_P}^{\theta}} = F_P'(X_P) - F_R'(X_R) \;.$$

Since $W_{c_R}/W_{c_P} = (c_P/c_R)^{\theta}$, log-differentiation gives

$$\frac{\dot{c}_P}{c_P} - \frac{\dot{c}_R}{c_R} = \frac{F_P'(X_P) - F_R'(X_R)}{\theta} \;.$$

The growth gap depends positively on the productivity gap and negatively on the intra-generational inequality aversion.

Stationary states, with $\frac{\dot{c}_i}{c_i} = 0$ and $c_i = F_i(X_i^*)$, are characterized by the no-arbitrage condition $F_P'(X_P^*) = F_R'(X_R^*)$. *This constitutes the proof of Proposition 3.*

Apart from the stationary state, $F_P'(X_P) > F_R'(X_R)$ and $\frac{\dot{c}_P}{c_P} > \frac{\dot{c}_R}{c_R}$. Along an egalitarian path, both consumption levels cannot decrease or increase at the same time, implying $\frac{\dot{c}_P}{c_P} > 0$ and $0 > \frac{\dot{c}_R}{c_R}$. Then, $c_P < F_P(X_P)$ and $c_R > F_R(X_R)$. The welfare level during the transition equals its value at the stationary state, namely $W^{\theta}(c_R, c_P) = F_R(X_R^*) + F_P(X_P^*)$. *This constitutes the proof of Proposition 4.* For a more detailed analysis of the dynamics, see the indications in Cairns et al. (2019, Proof of Prop. 5).

Along the efficient egalitarian path, welfare is constant, i.e., $dW^{\theta}/dt = 0 = \dot{c}_R W_{c_R}^{\theta} + \dot{c}_P W_{c_P}^{\theta}$. This implies $-\frac{\dot{c}_P}{\dot{c}_R} = \frac{W_{c_R}^{\theta}}{W_{c_P}^{\theta}}\Big|_{W^{\theta} = \underline{w}}$. Also, we get from the Hartwick rule (Cairns and Long, 2006) and condition (13) that $-\frac{\dot{X}_P}{\dot{X}_R} = \frac{W_{c_R}^{\theta}}{W_{c_P}^{\theta}}\Big|_{W^{\theta} = \underline{w}}$. Combining these two conditions,

and noting that $\left.\frac{W^\theta_{c_R}}{W^\theta_{c_P}}\right|_{W^\theta = \underline{w}} = \left(\frac{c_P}{c_R}\right)^\theta$, *we get the result of Proposition 5.*

A.3 Extensions

A.3.1 Capital transfers without constraints

When the social planner can implement transfers, the Lagrangian of the optimal control problem becomes

$$\mathcal{L} = \mu_R \left(F(X_R) - c_R\right) + \mu_P \left(F(X_P + T) - c_P\right) + (\mu_R - \mu_P)\pi + (\mu_T - \mu_R)\tau + \rho \left(W^\theta - \underline{w}\right) \ ,$$

which is linear in π and τ. Optimal transfer levels will then respectively depend on the sign of

$$\frac{\partial \mathcal{L}}{\partial \pi} = \mu_R - \mu_P \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial \tau} = \mu_T - \mu_R \ .$$

Two new FOCs give $-\frac{\dot{\mu}_P}{\mu_P} = F'\left(X_P + T\right)$ and $-\frac{\dot{\mu}_T}{\mu_P} = F'\left(X_P + T\right)$, and thus $\dot{\mu}_T = \dot{\mu}_P$. Given that $\mu_T^* = \mu_P^*$ at the steady state, we then have $\mu_T = \mu_P$ at any date.

Also, $\frac{\mu_P}{\mu_R} = \frac{W^\theta_{c_P}}{W^\theta_{c_R}} = \left(\frac{c_R}{c_P}\right)^\theta$, and $c_R \geq c_P$, $\mu_P \geq \mu_R$. Therefore, during the transition to the steady state, $\mu_T = \mu_P > \mu_R$. It leads to $\frac{\partial \mathcal{L}}{\partial \tau} \geq 0$, and $\frac{\partial \mathcal{L}}{\partial \pi} \leq 0$. The optimal trasfer τ^* equals its upper limit (*Proposition 6*), and the optimal return is $\pi^* = 0$ (*Proposition 7*).[33]

A.3.2 Capital transfers with return

When return on foreign investment T takes the form $\pi = TF'(X_P + T)$, the dynamics of the economy reads

$$\begin{aligned}
\dot{X}_R &= F(X_R) - c_R - \tau + TF'(X_P + T) \\
\dot{X}_P &= F(X_P + T) - c_P - TF'(X_P + T) \\
\dot{T} &= \tau
\end{aligned}$$

The Lagrangian becomes

$$\begin{aligned}
\mathcal{L} = \mu_R \left(F(X_R) - c_R\right) + \mu_P \left(F(X_P T) - c_P\right) + (\mu_T - \mu_R)\tau + \\
(\mu_R - \mu_P)TF'(X_P + T) + \rho \left(W^\theta - \underline{w}\right)
\end{aligned}$$

---

[33]The optimal return could take a negative value if consumption transfer is possible.

The optimality conditions on the consumption levels give $\mu_i = \rho W_{c_i}^\theta$, for $i = R, P$, which means that the shadow value of each capital stock is still equal to the product of the equity constraint multiplier and the marginal welfare derived from the consumption in that country. The possibility to invest abroad does not modify the equity pattern. Consumption will follow indifference curves. Only the level of sustained welfare will be higher.

As the Lagrangean is linear in the capital transfer variable τ, we have $\frac{\partial \mathcal{L}}{\partial \tau} = \mu_T - \mu_R$, implying bang-bang controls, just as in the previous case of unconstrained transfers. The return, on the other hand, is no more a control, and it will influence the dynamics trough the state variable T and the marginal product $F'(X_P + T)$.

The optimality conditions associated with the evolution of the costate variables are

$$-\dot{\mu}_R = \frac{\partial \mathcal{L}}{\partial X_R} = \mu_R F'(X_R) \tag{16}$$

$$-\dot{\mu}_P = \frac{\partial \mathcal{L}}{\partial X_P} = \mu_P F'(X_P + T) + (\mu_R - \mu_P)TF''(X_P + T) \tag{17}$$

$$-\dot{\mu}_T = \frac{\partial \mathcal{L}}{\partial T} = \mu_R F'(X_P + T) + (\mu_R - \mu_P)TF''(X_P + T) \tag{18}$$

The pattern for return on investment modifies the dynamics of the costate variables for X_P and T with respect to the cases with no transfer and with unconstrained transfers.

A first analysis of this dynamic system consists in characterizing the conditions for a stationary state. When $(\dot{X}_R, \dot{X}_P, \dot{T}) = (0, 0, 0)$, controls must satisfy $c_R = F(X_R) + TF'(X_P + T)$ and $c_P = F(X_P) - TF'(X_P + T)$, as well as $\tau = 0$. This last condition implies $\mu_T = \mu_R$ (to make the interior solution $\tau = 0$ possible), and thus $\dot{\mu}_T = \dot{\mu}_R = -\mu_R F'(X_R)$. One also has $\frac{\dot{\mu}_P}{\mu_P} = \frac{\dot{\mu}_R}{\mu_R} = -F'(X_R)$ (from the fact that $\frac{\mu_P}{\mu_R} = \frac{W_P^\theta}{W_R^\theta}$ and given that consumption levels are constant at a stationary state), and thus $\dot{\mu}_P = -\mu_P F'(X_R)$. Taking the difference between these two expressions gives $\dot{\mu}_T - \dot{\mu}_P = -(\mu_R - \mu_P)F'(X_R)$. Taking the difference between eq. (17) and eq. (18) leads to $\dot{\mu}_T - \dot{\mu}_P = -(\mu_R - \mu_P)F'(X_P + T)$. Combining the two expressions implies $F'(X_R) = F'(X_P + T)$ for a stationarity state. Considering the difference between eq. (16) and (18), we get $\dot{\mu}_R - \dot{\mu}_T = \mu_R(F'(X_P + T) - F'(X_R)) + (\mu_R - \mu_P)TF''(X_P + T)$, which has to be nil given $\mu_T = \mu_R$. Given the equality of marginal products of capital in the two countries, we must have $(\mu_R - \mu_P)TF''(X_P + T) = 0$ for a stationary state. This condition is met either if $T = 0$, which implies $X_P = X_R$, as well as $c_P = c_R$ and $\mu_P = \mu_R$, or if $\mu_P = \mu_R$, which implies $c_P = c_R$ and thus $T = 0$. This proves that, at a stationary state, $X_R^*(t) = X_P^*(t)$ and $T^*(t) = 0$ (*Proposition 8*).

Regarding the transition path, at the initial state, $T(0) = 0$ and $X_R(0) > X_P(0)$,

which implies $F'(X_R(0)) < F'(X_P(0) + T(0))$. Moreover, whenever $c_R(t) > c_P(t)$, we have $\mu_R(t) < \mu_P(t)$. From the set of eqs. (16-17-18), we can deduce that all costate variables are monotonically decreasing, and that $\dot{\mu}_P < \dot{\mu}_T < \dot{\mu}_R < 0$ at the beginning of the optimal path. Given the stationary state condition $\mu_R = \mu_P = \mu_T$, this is consistent only with initial conditions $0 < \mu_R(0) < \mu_T(0) < \mu_P(0)$. During a first phase, $\mu_R(0) < \mu_T(0)$ and $\tau(t) = \bar{\tau}$. This investment pattern lasts as long as $\mu_T(t) > \mu_R(t)$. Once the costate variables cross each other, foreign investment reaches a peak after which the investment pattern reverses, with $\tau(t) = -\bar{\tau}$. At some point after the investment peak, as $F'(X_P(t) + T(t))$ becomes sufficiently smaller than $F'(X_R(t))$, the inequality between $\dot{\mu}_T$ and $\dot{\mu}_R$ reverses and the costate variable of capital in the rich country decreases faster than that of foreign capital in the poor country, allowing all costate variables to converge to a single value at equilibrium when the capital stocks in both countries equalize $(X_R^*(t) = X_P^*(t))$ and the foreign investment vanishes $(T^*(t) = 0)$ (*Proposition 8*).