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# Evolutionarily stable in-group altruism in intergroup conflict over (local) public goods

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**Abstract:** We provide an evolutionary explanation for the well-established evidence of the existence of in-group favoritism in intergroup conflict. Using a model of group contest for a (local) public good, we show that the larger the number of groups competing against one another, or the larger the degree of complementarity between individual efforts, the more likely are group members altruistic towards their teammates under preference evolution.

Keywords: Indirect Evolutionary Approach; Evolutionary Stability; Altruism; Group Contest; Public Goods

JEL classification: C73; D64; D74

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# 1 Introduction

Since ancient times, various economic, social, and political activities involve groups that oppose one another. Thus, intergroup conflicts have been extensively studied within different disciplines, and especially, in social psychology. In particular, the social identity theory originally formulated by Tajfel and Turner (1979) emphasizes that individuals have a tendency to discriminate between "in-group" and "out-group" members, although "groups" are not formed according to some intrinsic characteristics or preferences but by random assignment (see Turner et al., 1979). More recently, social psychologists have conducted experimental studies in the laboratory, which confirm that inter-group competition tends to improve intra-group coordination in simple team games (see, e.g., Bornstein et al., 2002; Bornstein, 2003). As for the economic literature, the analysis of group conflicts is principally based on contest or rent-seeking games between groups (see Katz et al. 1990; Nitzan, 1991; and Konrad, 2009, for a review). Several relatively recent experimental studies aim at testing this type of game, and generally conclude that subjects over-contribute to group effort relative to the predictions of the *Homo economicus* model, thus, reflecting some form of cooperative behavior within groups (see, e.g., Abbink et al., 2010, 2012; Ahn et al., 2011; and Sheremeta, 2018, for a review).

In this study, we provide an evolutionary and theoretical analysis of the emergence and stability of in-group favoritism in intergroup conflict using a model of group contest. We consider a large population of players who are randomly matched into several groups of two members that compete against each other for an exogenous prize. Each group's probability of winning the contest is given by a *contest success function*, which depends on the group members' efforts relative to those of competing groups. The contested prize has the characteristics of a public good, and thus, there exists a "free-rider" problem in collective effort within groups.

An important feature of this present analysis is that, in contrast to most studies on group contests that use a "summation technology" for aggregating group members' efforts, we consider that the effective level of group effort – its *impact function* – is given by a technology featuring a varying degree of complementarity between individual efforts. Indeed, as first noted by Alchian and Demsetz (1972), team or group production exists to the extent that it can exploit complementarities of inputs, and this might be especially true in intergroup conflicts (see, also, Kolmar and Rommeswinkel, 2013; Brookins et al., 2015).

Another crucial feature of this analysis is that each player has *other-regarding preferences* that depend not only on her own material payoff but also on the material payoff of her teammate. We first remain agnostic whether this concern is altruistic or spiteful, that is, if each member puts a positive or negative weight on the payoff of her teammate when deciding her contribution to collective effort. We then characterize the (pure strategy) Nash equilibria of this group contest game when group members have heterogeneous other-regarding preferences within and across groups. Next, we use the *indirect evolutionary approach* pioneered

by Güth and Yaari (1992) to endogenize players' preferences (see, also, Bester and Güth, 1998; Sethi and Somanathan, 2001). This means that evolution does not play directly at the level of strategies as in standard evolutionary game theory (see Maynard Smith, 1982) but indirectly at the level of preferences, while players act rationally. In other words, preferences determine the players' actions, which in turn determine their material payoffs – or *fitness* – and ultimately, the evolutionary survival of certain preference types. Thus, applying the concept of evolutionary stability (Maynard Smith and Price, 1973) to preferences – rather than to strategies – allows us to endogenize the (other-regarding) preference parameters.

We suppose that preferences are observable, and thus, agents with "biased" preferences, such as altruism or spite, can be more successful in terms of material payoff. This is because a change in the preference type, and therefore, in the optimal choice of one player has strategic effects, in that it implies different optimal replies for all other players.<sup>1</sup> The difficulty in this present analysis stems from the dual level of strategic interactions between players. The first one occurs within groups. Each group member decides on her contribution to group effort according to her preferences, given the preference type, and the resulting action of her teammate. The second level of strategic interactions occurs across groups. A change in the preference type of one or both members of the same group is passed on group effort. This in turn leads to different effort choices by all members of the competing groups, thus, changing the winning probabilities of all groups. The evolutionary success of a certain preference type is the product of this dual level of strategic interactions.

Using a notion of local evolutionary stability (Alger and Weibull, 2010), we can demonstrate the existence of a unique evolutionarily stable preference parameter, and determine the evolutionarily stable degree to which a group member cares about the material payoff of her teammate. Clearly, a given group would be more successful in the group contest with more in-group *altruism*. However, in the process of evolution, is a certain degree of altruism within groups immune against *mutant* members with lower degrees of altruism? In fact, we show that in-group *altruism* and in-group *spite* can be both (locally) evolutionarily stable under preference evolution. In the first case, players exert more effort than if they were maximizing their material payoffs only, while they exert less effort in the second case. Yet, we show that the larger the degree of complementarity between individual efforts or the larger the number of opponents, the more likely that group members are altruistic towards each other. Also, a further increase in the degree of complementarity between partners' efforts or in the intensity of competition usually tends to reinforce group cohesion, in that it increases

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<sup>1</sup>Heifetz et al. (2007a, 2007b) show that in a large class of games, agents with "biased" preferences may actually be more successful in terms of material payoff, although agents' preferences are not fully observable. However, if preferences are completely unobservable, payoff-maximization is generally evolutionarily stable (Ok and Vega-Redondo, 2001; Dekel et al., 2007). More recently, Alger and Weibull (2013) show that, under incomplete information, selfishness will indeed prevail if there is no assortative matching at all; however, with some degree of assortativity, preference evolution leads to a certain degree of *Kantian* morality. For a recent review of the literature on preference evolution, with a particular emphasis on the role of the informational structure and of the matching rule, see Alger and Weibull (2019).

(locally) evolutionarily stable in-group altruism.

Despite several evolutionary analysis of contests between single players,<sup>2</sup> very few theoretical analysis apply the (direct or indirect) evolutionary approach to group contests. The first is Eaton et al. (2011), who consider a production and conflict model with a large population of players. In each period, players are randomly matched to form two groups of two members that compete for a common access resource. After appropriating some of the common resource, the members of each group can spend some processing efforts to produce a consumption good. While the model is specific, they have to rely on numerical simulations for endogenizing the preference parameters. The numerical results show that the evolutionarily stable weight on the payoff of one's teammate is positive – featuring in-group altruism – while that on the payoffs of the out-group members is negative – featuring out-group spite.<sup>3</sup>

Another theoretical analysis is by Konrad and Morath (2012). They consider a contest game with two groups of equal size fighting for an exogenous prize, and where group effort is given by the sum of individual efforts. Each player cannot observe the preference types of other players, and thus, Konrad and Morath (2012) introduce the concept of *robust beliefs*, such that any player with a certain preference type believes that all other players are of the same type (and have the same robust beliefs). This assumption greatly simplifies the analysis, as it eliminates all strategic effects on the behaviors of a change in preference types, that is, a change in the weights attributed to others' payoffs. The authors then characterize the set of evolutionarily stable preferences, which involve a linear combination of in-group favoritism and out-group spite with the two traits being perfect substitutes. Consequently, the players always exert more effort in the evolutionarily stable equilibrium than if they were endowed with selfish preferences.

There is also a large literature in evolutionary biology that studies the evolution of altruism and spite. These behavioral traits have been explained by kin selection and inclusive fitness (Hamilton, 1964), reciprocity with repeated interactions (Trivers, 1971; Axelrod and Hamilton, 1981)<sup>4</sup>, or "haystack models" of group selection (Maynard Smith, 1964).<sup>5</sup> More

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<sup>2</sup>The evolutionary analysis of contests between single players started with Schaffer (1988), who adapted the notion of an Evolutionarily Stable Strategy (ESS) by Maynard Smith and Price (1973) to a finite population of players. Following Schaffer (1988), Hehenkamp et al. (2004) compare behaviors induced by an ESS to behaviors in Nash equilibrium, and show that an ESS involves spiteful efforts in the contest between individuals. This in turn involves the overdissipation of the rent compared to Nash equilibrium. Yet, in an infinite population, ESS behavior coincides with Nash equilibrium behavior. Finally, Leininger (2009) applies the *indirect evolutionary approach* to contests between single players, and shows that it generates spiteful preferences that induce the same aggressive behavior than in an ESS.

<sup>3</sup>In the second part of their study, they analyze a contest game for a public prize with interdependent preferences but without effort complementarity within groups. Most importantly, they do not try to endogenize the preference parameters of the players in this part of the analysis.

<sup>4</sup>In another and related study (Cheikbossian, 2012), we investigate the potential of "trigger strategies" to induce mutual cooperation within groups in an infinitely repeated contest between two groups. We show that the set of parameters for which cooperation can be sustained within the larger group as a subgame perfect outcome is as large as that for which cooperation can be sustained in the smaller group.

<sup>5</sup>Maynard Smith (1964) proposed a simple model for describing mice living and replicating during the

closely related to this present study, several theoretical studies in human evolutionary biology explain (often using dynamic games and simulations) how genetic relatedness can result in *parochial* altruism, that is, altruistic behaviors within groups in war contexts (see, e.g., Choi and Bowles, 2007; Lehmann and Feldman, 2008; and Rush, 2014, for a review). Finally, altruism or spite can result from a "gene-culture coevolutionary" process, that is, from the interaction between genes and culture over long periods of time (see, e.g., Bowles, 2006; Gintis, 2011). Similarly, Alger et al. (2020) investigate both genetic and cultural transmission of preferences in populations structured in small groups with interactions across groups due to (limited) migration. They distinguish between preferences at the material payoff level and preferences at the fitness – or reproductive – level, and show that conflicts, by reducing fitness interdependence, can lead to altruism towards neighbors at the material payoff level.

Our contribution is to demonstrate the existence of and obtain an analytical solution for the evolutionarily stable degree of non-kin altruism or spite within groups in a complete information game featuring strategic interactions within and across groups. Furthermore, we show that a stronger adversity, as measured by the number of groups competing, and a stronger within-group effort complementarity go hand-in-hand to promote and reinforce in-group altruism under preference evolution.

## 2 The framework

### 2.1 A simple group contest game

We consider an infinitely large population of players. In each period, players are randomly matched to pairs playing a group contest game. As in Eaton et al. (2011), we consider that group contests are located and isolated from each other, in that each contest involves a certain number  $n$  of groups. Thus, we consider a normal-form game that involves  $2n$  players.

Groups compete against each other for an exogenous prize, which has the characteristics of a (local) public good. One can think of the collective prize as the location of a public facility (e.g. sanitation or water management infrastructures) or funding for certain group activities (e.g. sports and cultural associations).<sup>6</sup> The value of the prize for each member of the winning group is normalized to 1.

Let  $(i, j)$ , for  $i = 1, 2$  and  $j = 1, 2, \dots, n$ , denote member  $i$  of group  $j$ .  $e_{ij} \in \mathbb{R}_+$  is the amount of effort expended by player  $(i, j)$  and  $\mathbf{e}_j = (e_{1j}, e_{2j}) \in \mathbb{R}_+^2$  is the vector of efforts in

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summer within separate haystacks. At harvest time, the haystacks are cleared, and mice scramble out in the meadow. In the next summer, the new haystacks are colonized randomly by the survivors. A crucial feature of "haystack models" is that groups of small size are randomly formed from the infinite population of players (see Bergstrom, 2002, or Salomonsson, 2010, for a review). Regarding the formation of groups, our model can be viewed as a "haystack model". However, in our analysis, there are interactions across groups.

<sup>6</sup>Thus, we ignore the source of the funds with which the "prize" can be financed to focus on contest efforts. For "production and conflict" (or "guns-versus-butter") models, where agents allocate their resources between producing the collective prize and fighting for it, see, for instance, Garfinkel and Skaperdas (2007).

group  $j$ . The effort of group  $j$  depends on group members' efforts, according to an *impact function*  $G_j : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ , which has, for  $j = 1, 2, \dots, n$ , the CES form:

$$G_j(\mathbf{e}_j) = [e_{1j}^\sigma + e_{2j}^\sigma]^{\frac{1}{\sigma}}, \quad (1)$$

where  $\sigma \in (-\infty, 0) \cup (0, 1]$  measures the degree of complementarity between individual efforts. The elasticity of substitution is  $1/(1 - \sigma)$ . Thus, the lower  $\sigma$ , the lower is the elasticity of substitution or the higher is the degree of complementarity between individual efforts within groups. For  $\sigma = 1$ , we have perfect substitutability between individual efforts and Eq. (1) becomes the standard "summation technology" used in most analysis of group contests, that is  $G_j(\mathbf{e}_j) = \sum_i e_{ij}$ . For  $\sigma \rightarrow -\infty$ , we have perfect complementarity, that is  $G_j(\mathbf{e}_j) = \text{Min}\{e_{ij}\}$ . Finally, for  $\sigma < 0$  and  $e_{ij} = 0$ , the impact function of group  $j$  is not well-defined. Hence, we take the limit of (1) as  $e_{ij} \rightarrow 0$ , which means  $G_j(\mathbf{e}_j) = 0$ , in this case.<sup>7</sup>

The winning probability of each group is given by a Contest Success Function (CSF), which has the ratio-form. This means that each group's probability of winning the prize is equal to the proportion of its collective effort out of the sum of collective efforts by all groups involved in the contest. Thus, the winning probability of group  $j$ , for  $j = 1, 2, \dots, n$ , is given by  $p_j : \mathbb{R}_+^{2n} \rightarrow [0, 1]$  with

$$p_j(\mathbf{e}_j, \mathbf{e}_{-j}) = \begin{cases} \frac{G_j(\mathbf{e}_j)}{\sum_{k=1}^n G_k(\mathbf{e}_k)} & \text{if } \sum_{k=1}^n G_k(\mathbf{e}_k) \neq 0, \\ \frac{1}{n} & \text{otherwise,} \end{cases} \quad (2)$$

where  $\mathbf{e}_{-j} = ((e_{11}, e_{21}), \dots, (e_{1j-1}, e_{2j-1}), (e_{1j+1}, e_{2j+1}), \dots, (e_{1n}, e_{2n}))$ .<sup>8</sup>

We further assume that the marginal cost of individual effort is constant and that the expected *material payoff* to member  $(i, j)$ , for  $i = 1, 2$  and  $j = 1, 2, \dots, n$ , is given by an additively separable function  $\Pi_{ij} : \mathbb{R}_+ \times [0, 1] \rightarrow \mathbb{R}_+$ , that is,

$$\Pi_{ij}(\mathbf{e}_j, \mathbf{e}_{-j}) = p_j(\mathbf{e}_j, \mathbf{e}_{-j}) - e_{ij}, \quad (3)$$

by recalling that the value of the prize for each member of the winning group is normalized to 1.

Collective effort is a public good to the group members, and furthermore, the contested prize is also a (local) public good. Consequently, a *free-rider* or *collective action* problem exists. Each group member bears the full cost of his effort, whereas the benefits are enjoyed by the two group members if they win the contest. In other words, when choosing his

<sup>7</sup>Note that (1) is also discontinuous at  $\sigma = 0$ . We exclude this from our analysis.

<sup>8</sup>See Munster (2009) for an axiomatization of group contest success functions. For an analysis of group contests – but without other-regarding preferences – where the *impact function* is given by a CES technology, see Kolmar and Rommeswinkel (2013), Brookins et al. (2015), and Cheikbossian and Fayat (2018).

contribution to group effort, a selfish member – maximizing his own material payoff – does not take into account that his individual contribution confers a benefit to his teammate because of an increase in the group’s probability of success.<sup>9</sup> Thus, the *free-rider problem* is defined as the failure to internalize this positive externality, implying that group effort is suboptimally low from the perspective of the group.

## 2.2 Equilibrium with preference interdependence

Let us consider that each player has *other-regarding preferences*, or a *utility*, which depends not only on her own material payoff but also on the material payoff of her teammate. The expected utility of member  $(i, j)$ , for  $i = 1, 2$  and  $j = 1, 2, \dots, n$ , is given by an additively separable function  $V_{ij} : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ , that is,

$$V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j}) = \Pi_{ij}(\mathbf{e}_j, \mathbf{e}_{-j}) + \theta_{ij}\Pi_{-ij}(\mathbf{e}_j, \mathbf{e}_{-j}), \quad (4)$$

where the subscript  $-ij$  stands for the member other than  $i$  in team  $j$ .  $\theta_{ij} \in (-1, R]$ , with  $R \geq 1$  and for  $i = 1, 2$  and  $j = 1, 2, \dots, n$ , is the utility weight given by member  $i$  in group  $j$  to the material payoff of her teammate – with positive values representing "altruism" and negative values representing "spite".<sup>10</sup> Let  $\Theta \in (-1, R]^{2n}$  be the vector of preference parameters in a contest that involves  $n$  groups of two members. Each group member observes the degree of altruism/spite of her teammate as well as those of all members of opposing groups.<sup>11</sup>

Player  $(i, j)$ , for  $i = 1, 2$  and  $j = 1, 2, \dots, n$  chooses her effort level  $e_{ij}$  to maximize her utility given by (4), which can be rewritten with (3) as

$$V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j}) = (1 + \theta_{ij})p_j(\mathbf{e}_j, \mathbf{e}_{-j}) - (e_{ij} + \theta_{ij}e_{-ij}). \quad (5)$$

We have the following Lemma:<sup>12</sup>

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<sup>9</sup>We could equivalently consider that the prize has the characteristics of a fully private good and that it is redistributed equally among the group members of the winning group. The crucial feature is that the prize has the same value for the two members of the winning group, independently of their efforts. If, for instance, the prize had the characteristics of a private good and was redistributed according to the relative effort of group members, then there would be no free-rider problem within groups, and the multi-group contest would reduce to a contest between individual players (see Nitzan, 1991).

<sup>10</sup>We exclude the case  $\theta_{ij} \leq -1$  since then it would prevent positive levels of efforts. However, degrees of altruism  $\theta_{ij} \geq 1$  may exist, for instance, between parents or couples.

<sup>11</sup>Observe also that, in contrast to Eaton et al. (2011) and Konrad and Morath (2012), the utility of a player does not depend – presumably negatively – on the material payoffs of out-group members. Here, ignoring preferences towards out-group members can be justified because players’ utilities are linear in payoffs, and because they exert just one level of effort for obtaining an exogenous prize. An increase in individual effort as a result of more in-group altruism reduces the probability of success of competing groups, and therefore, reduces the payoffs of out-group members. Thus, the effect of in-group altruism could also be viewed as induced by out-group spite. We leave the complete and careful examination of in-group preferences together with out-group preferences for future research.

<sup>12</sup>We provide all the proofs in the Appendix.

**Lemma 1:** (i) Given  $\Theta \in (-1, R]^{2n}$ , there exists a pure-strategy Nash equilibrium. The equilibrium effort levels  $e_{ij}^*$ ,  $i = 1, 2$  and  $j = 1, 2, \dots, n$ , must satisfy the following set of first-order conditions

$$(1 + \theta_{ij}) \frac{p_j(\mathbf{e}_j^*, \mathbf{e}_{-j}^*) [1 - p_j(\mathbf{e}_j^*, \mathbf{e}_{-j}^*)]}{e_{ij}^{*1-\sigma} (e_{ij}^{*\sigma} + e_{-ij}^{*\sigma})} \leq 1, \quad (6)$$

and if strict inequality holds,  $e_{ij}^* = 0$ ;

(ii) In any equilibrium, a given group  $j$  is either fully active, that is  $e_{ij}^* > 0$  for  $i = 1, 2$  or fully inactive, that is  $e_{ij}^* = 0$  for  $i = 1, 2$ .

The proof consists of three steps. We first show that  $V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j})$  is strictly concave in  $e_{ij}$ , so that the first-order condition is necessary and sufficient for maximization of *utility*. The second step is to show that there cannot exist an equilibrium in which a corner solution holds for a member of one group, while an interior solution holds for the other member of the *same* group (property (ii)). This important property allows us to reduce the group contest to a lottery contest between individual players with heterogeneous preferences. Consequently, in the final step of the proof, we can use the existence theorems of Cornes and Hartley (2005).

Notably, in our framework, there are two reasons why group  $j$  might not participate in the contest, that is  $e_{ij}^* = 0$  for  $i = 1, 2$ . The first is that the utility weights that group members give to each other are too low, relative to other groups. The second reason is that if  $\sigma < 0$  and player  $(i, j)$  chooses  $e_{ij} = 0$ , then  $G_j(\mathbf{e}_j) = 0$ , and thus, player  $(-i, j)$  cannot do better than choosing  $e_{-ij} = 0$ . In other words, the members of group  $j$  can "coordinate" on participating *or not* participating in the group contest independently of  $(\theta_{ij}, \theta_{-ij}) \in (-1, R]^2$  when  $\sigma < 0$ . Thus, multiple equilibria exist in this case (see also, Kolmar and Rommeswinkel, 2013). Yet, ignoring these equilibria, the set of active groups is uniquely determined and equilibrium is unique for a given vector of preference parameters  $\Theta \in (-1, R]^{2n}$ .

The equilibrium material payoff – or *fitness* – of player  $(i, j)$ , for  $i = 1, 2$  and  $j = 1, 2, \dots, n$ , can be written as

$$\Pi_{ij}^*(\Theta) = p_j(\mathbf{e}_j^*(\Theta), \mathbf{e}_{-j}^*(\Theta)) - e_{ij}^*(\Theta), \quad (7)$$

where, again,  $\Theta \in (-1, R]^{2n}$  is the  $2n$ -dimensional vector of preference types of all players. Indeed, with observed preference parameters, the equilibrium material payoff of each player depends on his own preference type, and on all other players' types.

### 3 Evolutionarily stable preferences

To study the evolutionary stability of altruism or spite, we employ the *indirect evolutionary approach* pioneered by Güth and Yaari (1992). All players choose effort levels that maximize

their utility and evolution pressure ensures the survival of preference parameters that induce equilibrium behavior providing the highest level of material payoff.

Initially, all players have the same preference parameter  $\theta$ , and the question is whether this preference parameter is immune against invading "mutant" players with a different preference parameter. Hence, let us suppose that, at a certain period, a "mutation" occurs with a *small* share  $\varepsilon$  of the total population characterized by the preference parameter  $\theta_m \neq \theta$ . Players are still randomly matched into pairs to play a group contest game between  $n$  groups.

To simplify the exposition, let us consider the situation of player (1, 1), that is player 1 of group 1. If this player is an incumbent player, he has a probability  $(1 - \varepsilon)$  of being matched with another incumbent player and a probability  $\varepsilon$  of being matched with a mutant player. Furthermore, the members of group 1 compete against  $(n - 1)$  other groups, each of which may have two incumbent players, or one incumbent player and one mutant player, or two mutant players. Thus, the probability that group 1 compete against  $(n - 1)$  other groups, with a total number of  $k$  mutant players in these other groups, is given by the following binomial distribution with parameters  $2(n - 1)$  and  $\varepsilon$ :<sup>13</sup>

$$\Phi_k(\varepsilon) = \binom{2(n-1)}{k} (1-\varepsilon)^{2(n-1)-k} \varepsilon^k. \quad (8)$$

Let  $\boldsymbol{\theta}_1(l) \in (-1, R]^2$  be the vector of preference parameters in group 1, where  $l = \{0, 1, 2\}$  is the number of mutant players in this group, and let  $\boldsymbol{\theta}_{-1}(k) \in (-1, R]^{2(n-1)}$  be the vector of preference parameters of all players, except the two players of group 1, where  $k = \{0, 1, 2, \dots, 2(n-1)\}$  is the total number of mutant players in the  $(n - 1)$  remaining groups. Thus, if player (1, 1) is an incumbent player, his expected material payoff is given by

$$\begin{aligned} \Pi_{11}^I(\boldsymbol{\theta}_1(l), \boldsymbol{\theta}_{-1}(k), \varepsilon) &= (1 - \varepsilon) \sum_{k=0}^{2(n-1)} \Phi_k(\varepsilon) \Pi_{11}^*(\boldsymbol{\theta}_1(0), \boldsymbol{\theta}_{-1}(k)) \\ &\quad + \varepsilon \sum_{k=0}^{2(n-1)} \Phi_k(\varepsilon) \Pi_{11}^*(\boldsymbol{\theta}_1(1), \boldsymbol{\theta}_{-1}(k)). \end{aligned} \quad (9)$$

However, if player (1, 1) is a mutant player, his expected material payoff is given by

$$\Pi_{11}^M(\boldsymbol{\theta}_1(l), \boldsymbol{\theta}_{-1}(k), \varepsilon) = (1 - \varepsilon) \sum_{k=0}^{2(n-1)} \Phi_k(\varepsilon) \Pi_{11}^*(\boldsymbol{\theta}_1(1), \boldsymbol{\theta}_{-1}(k))$$

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<sup>13</sup>Suppose, for example, that there are three groups. Then, the probability that group 1 faces two groups with no mutant players in the two groups – that is  $k = 0$  – is  $(1 - \varepsilon)^4$ . The probability that one opposing group has one mutant player and one incumbent player, while the remaining group has two incumbent players – that is  $k = 1$  – is  $4\varepsilon(1 - \varepsilon)^3$ . The probability that  $k = 2$  is given by the probability that the two opposing groups have one mutant player and one incumbent player – that is  $4\varepsilon^2(1 - \varepsilon)^2$  – plus the probability that one opposing group has two mutant players, while the remaining group has two incumbent players – that is  $2\varepsilon^2(1 - \varepsilon)^2$ . The probability that  $k = 3$ , or that one opposing group has two mutant players, while the remaining group has one mutant player and one incumbent player, is  $4\varepsilon^3(1 - \varepsilon)$ . Finally, the probability that the two opposing groups have two mutant players – that is  $k = 4$  – is  $\varepsilon^4$ .

$$+\varepsilon \sum_{k=0}^{2(n-1)} \Phi_k(\varepsilon) \Pi_{11}^*(\boldsymbol{\theta}_1(2), \boldsymbol{\theta}_{-1}(k)). \quad (10)$$

Again, these expected material payoffs show that each player of group 1, whether an incumbent or a mutant player, can be matched with an incumbent or a mutant player with his group engaged in a group contest involving  $(n-1)$  other groups, each of which can have two incumbent players, or one incumbent player and a mutant player, or two mutant players.

We now adapt Maynard Smith and Price's (1973) notion of *evolutionarily stable strategy* to preference types in our group contest game.

**Definition 1:** Consider two preference parameters  $\theta_m \in (-1, R]$  and  $\theta \in (-1, R]$ , and let  $\boldsymbol{\theta}_1(l) \in (-1, R]^2$  and  $\boldsymbol{\theta}_{-1}(k) \in (-1, R]^{2(n-1)}$  be the vectors of players' preference types in group 1 and in the  $(n-1)$  other groups, where  $l = \{0, 1, 2\}$  and  $k = \{1, 2, \dots, 2(n-1)\}$  are the numbers of  $\theta_m$ -players. Then,  $\theta$  is **evolutionarily stable** if there exists  $\bar{\varepsilon} > 0$ , such that for any  $\theta_m \neq \theta$  and for all  $\varepsilon \in (0, \bar{\varepsilon})$ , we have

$$\Pi_{11}^I(\boldsymbol{\theta}_1(l), \boldsymbol{\theta}_{-1}(k), \varepsilon) > \Pi_{11}^M(\boldsymbol{\theta}_1(l), \boldsymbol{\theta}_{-1}(k), \varepsilon). \quad (11)$$

If the population of mutants  $\varepsilon$  is close to 0, then an incumbent or a mutant player in group 1 is almost surely matched with an incumbent player, and the members of group 1 compete against  $(n-1)$  other groups, each of them having almost surely two incumbent players. Formally, when  $\varepsilon$  tends to 0, we have

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0} \Pi_{11}^I(\boldsymbol{\theta}_1(l), \boldsymbol{\theta}_{-1}(k), \varepsilon) &= \Pi_{11}^*(\boldsymbol{\theta}_1(0), \boldsymbol{\theta}_{-1}(0)), \\ \lim_{\varepsilon \rightarrow 0} \Pi_{11}^M(\boldsymbol{\theta}_1(l), \boldsymbol{\theta}_{-1}(k), \varepsilon) &= \Pi_{11}^*(\boldsymbol{\theta}_1(1), \boldsymbol{\theta}_{-1}(0)). \end{aligned} \quad (12)$$

where  $\boldsymbol{\theta}_1(0) = (\theta, \theta)$ ,  $\boldsymbol{\theta}_1(1) = (\theta_m, \theta)$ , and  $\boldsymbol{\theta}_{-1}(0)$  is the  $2(n-1)$ -dimensional vector whose components all equal  $\theta$ .

Thus, a necessary condition for (11) to be satisfied is that for every  $\theta_m \neq \theta$ ,  $\Pi_{11}^*(\boldsymbol{\theta}_1(0), \boldsymbol{\theta}_{-1}(0)) \geq \Pi_{11}^*(\boldsymbol{\theta}_1(1), \boldsymbol{\theta}_{-1}(0))$ , and a sufficient condition is

$$\Pi_{11}^*(\boldsymbol{\theta}_1(0), \boldsymbol{\theta}_{-1}(0)) > \Pi_{11}^*(\boldsymbol{\theta}_1(1), \boldsymbol{\theta}_{-1}(0)). \quad (13)$$

To summarize, a preference parameter  $\theta$  is *evolutionarily stable* if the expected material payoff of the incumbent players is strictly larger than the expected material payoff of the rare mutant players with a preference parameter  $\theta_m$ , for every  $\theta_m \neq \theta$ . Furthermore, when the proportion of mutants  $\varepsilon$  is close to 0, a sufficient condition for  $\theta$  to be evolutionarily stable reduces to (13).

### 3.1 Equilibrium with one mutant player

We now characterize the equilibrium of the group contest game with one mutant player. Again, to simplify the exposition, let us consider that the mutant is player (1,1), that is

member 1 of group 1, with  $\theta_m \in (-1, R]$  being his degree of altruism/spite towards his teammate. All other players have a common degree of altruism/spite  $\theta \in (-1, R]$  towards their teammates. These parameters are exogenous to the players.

The mutant with the preference parameter  $\theta_m$  chooses  $e_{11}$  to maximize his utility given by

$$V_{11}(\mathbf{e}_1, \mathbf{e}_{-1}) = (1 + \theta_m)p_1(\mathbf{e}_1, \mathbf{e}_{-1}) - (e_{11} + \theta_m e_{21}). \quad (14)$$

Member 2 of group 1, with the preference parameter  $\theta$ , chooses  $e_{21}$  to maximize his or her utility given by

$$V_{21}(\mathbf{e}_1, \mathbf{e}_{-1}) = (1 + \theta)p_1(\mathbf{e}_1, \mathbf{e}_{-1}) - (e_{21} + \theta e_{11}). \quad (15)$$

Finally, the utility of all other players are symmetric, as they all belong to a group where the two members have the same preference parameter  $\theta$ . Thus, player  $(i, j)$ , for  $i = 1, 2$ , and  $j = 2, 3, \dots, n$ , chooses  $e_{ij}$  to maximize his or her utility given by

$$V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j}) = (1 + \theta)p_j(\mathbf{e}_j, \mathbf{e}_{-j}) - (e_{ij} + \theta e_{-ij}). \quad (16)$$

We have the following Proposition:

**Proposition 1:** *Let  $\Delta(\theta_m, \theta) = (1 + \theta_m)^{\frac{1-\sigma}{1-\sigma}} + (1 + \theta)^{\frac{1-\sigma}{1-\sigma}}$ , where  $\theta_m \in (-1, R]$  is the preference parameter of player  $(1, 1)$ , and where  $\theta \in (-1, R]$  is the incumbent preference parameter of all other players, then:*

(i) *There exists a unique interior pure strategy Nash equilibrium where all players exert a strictly positive level of effort if and only if*

$$\left[ \frac{\Delta(\theta_m, \theta)}{2} \right]^{\frac{1-\sigma}{\sigma}} \geq \frac{(n-2)(1+\theta)}{(n-1)}; \quad (17)$$

(ii) *If (17) holds, the equilibrium effort levels are given by*

$$e_{11}^*(\theta_m, \theta) = \frac{2^{\frac{1-\sigma}{\sigma}}(n-1)(1+\theta)(1+\theta_m)^{\frac{1}{1-\sigma}}[(n-1)[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}} - 2^{\frac{1-\sigma}{\sigma}}(n-2)(1+\theta)]}{\Delta(\theta_m, \theta)[2^{\frac{1-\sigma}{\sigma}}(1+\theta) + (n-1)[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}}]^2}, \quad (18.1)$$

$$e_{21}^*(\theta_m, \theta) = \frac{2^{\frac{1-\sigma}{\sigma}}(n-1)(1+\theta)^{\frac{2-\sigma}{1-\sigma}}[(n-1)[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}} - 2^{\frac{1-\sigma}{\sigma}}(n-2)(1+\theta)]}{\Delta(\theta_m, \theta)[2^{\frac{1-\sigma}{\sigma}}(1+\theta) + (n-1)[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}}]^2}, \quad (18.2)$$

$$e_R^*(\theta_m, \theta) = \frac{2^{\frac{1-\sigma}{\sigma}}(n-1)(1+\theta)^2[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}}}{2[2^{\frac{1-\sigma}{\sigma}}(1+\theta) + (n-1)[\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}}]^2}, \text{ for } i = 1, 2 \text{ and } j = 2, 3, \dots, n; \quad (18.3)$$

(iii) If (17) does not hold, then group 1 is fully inactive, that is,  $e_{11}^* = e_{21}^* = 0$  in any pure strategy Nash equilibrium.

When all players have the same preference parameter except one player – in this case player (1, 1) – and when (17) is satisfied, there exists a unique interior pure strategy equilibrium with all players of all groups exerting a strictly positive level of effort. In this equilibrium, there are three distinct levels of individual effort: two different levels of effort for the two members of group 1 to which player (1, 1) belongs, and a third level of effort, which is the same for all members of group 1's competitors.

If the inequality (17) is not satisfied, then group 1 is fully inactive in any equilibrium, that is, even if we exclude equilibria in which group members "coordinate" on *not* participating when  $\sigma < 0$ . Observe that (17) is always verified for  $\theta_m = \theta$ , and that its left-hand term is increasing in  $\theta_m$ . Therefore, the inequality is always verified for  $\theta_m \geq \theta$ . However, if given  $\sigma$  and  $n$ ,  $\theta_m$  is sufficiently small relative to  $\theta$ , the inequality (17) can be reversed, in which case, group 1 is fully inactive in equilibrium. The explanation is that the *total* valuation of the prize by player (1, 1), that is, his own valuation plus the valuation of the prize for his teammate as a result of his altruism (or spite), is too low, relative to the total valuation of the prize by competing groups, for making profitable a positive contribution. Thus, player (1, 1) does not participate in the contest. As mentioned, complete free-riding by the mutant player implies that his teammate is also inactive. He or she cannot bear alone the cost of participating in the group contest. Consequently, when (17) does not hold, the group with the mutant player is fully inactive in equilibrium.<sup>14</sup>

It would be useful to determine how equilibrium levels of effort described in Proposition 1 change with the degree of altruism/spite  $\theta_m$  of player (1, 1). As shown by Lemma 1, each player's equilibrium level of effort depends on the vector of all players' preference parameters as a result of the observability of preference types. Even when all players have the same preference parameter except one member of a given group, the resulting three distinct equilibrium levels of effort (18.1)-(18.3) described in Proposition 1 depend in a non-monotonic way on  $\theta_m$ .<sup>15</sup> Therefore, in Proposition 2, we analyze the impact of a marginal increase in the degree of altruism/spite  $\theta_m$  on the equilibrium levels of effort at the point where all players are of the same type, that is  $\theta_m = \theta$ .

We have the following Proposition:

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<sup>14</sup>This result can be related to previous works on asymmetric rent-seeking contests between single players. In particular, Hillman and Riley (1989) and Stein (2002) show that players with the lowest valuations of the prize may stay inactive in equilibrium (in contests with more than two asymmetric players). As the proof of Lemma 1 shows, the present model of group contest can be reduced to a lottery contest between individual players and a low degree of in-group altruism relative to other groups can be interpreted as a relatively low aggregate valuation of the prize. Thus, our result that group 1 is inactive if (17) does not hold aligns with previous studies on contests between asymmetric single players.

<sup>15</sup>This is mainly, but not completely, due to the power  $\sigma$ . For instance, with  $\sigma = 0.5$  and  $\theta = 0$ , the sign of the derivative of  $e_{11}^*(\theta_m, \theta)$  with respect to  $\theta_m$  can be positive or negative depending on the number of competing groups  $n$ , and on the value of  $\theta_m$  itself.

**Proposition 2:** Suppose that (17) holds, then for  $n \geq 2$ ,  $\sigma \leq 1$ , and  $(\theta_m, \theta) \in (-1, R]^2$  :

$$(i) \quad \frac{\partial e_{11}^*(\theta_m, \theta)}{\partial \theta_m} \Big|_{\theta_m=\theta} = \frac{(n-1)[n^2 - n + 2 - \sigma(n^2 - 2n + 2)]}{4(1-\sigma)n^3} > 0; \quad (20.1)$$

$$(ii) \quad \frac{\partial e_{21}^*(\theta_m, \theta)}{\partial \theta_m} \Big|_{\theta_m=\theta} = \frac{(n-1)[(n-1)(n-2) - \sigma(n^2 - 2n + 2)]}{4(1-\sigma)n^3} \begin{matrix} \geq \\ \leq \end{matrix} 0 \Leftrightarrow \sigma \begin{matrix} \leq \\ \geq \end{matrix} \tilde{\sigma}, \quad (20.2)$$

$$\text{where } \tilde{\sigma} = \frac{(n-1)(n-2)}{n^2 - 2n + 2} < 1; \quad (20.3)$$

$$(iii) \quad \frac{\partial e_R^*(\theta_m, \theta)}{\partial \theta_m} \Big|_{\theta_m=\theta} = -\frac{(n-2)(n-1)}{4n^3} \leq 0. \quad (20.4)$$

Thus, starting from a situation where all players in all groups have the same degree of altruism/spite, an increase (decrease) in the mutant's degree of altruism (spite) increases his own equilibrium level of effort (property (i) of Proposition 2). However, it can increase or decrease the equilibrium level of effort of his teammate depending on the degree of complementarity between individual efforts (property (ii) of Proposition 2). If the degree of effort complementarity is sufficiently large, that is if  $\sigma$  is *lower* than  $\tilde{\sigma}$ , then an increase (decrease) in the mutant's degree of altruism (spite) *mitigates* the free-riding incentive of his teammate by inducing him to increase his equilibrium level of effort; otherwise, if  $\sigma$  is *larger* than  $\tilde{\sigma}$ , it *exacerbates* the free-riding incentive of his teammate by inducing him to decrease his contribution to group effort.<sup>16</sup> Yet, the mutant's preference parameter has a greater impact on his own effort choice than on the effort choice of his partner. We indeed have  $[\partial(e_{11}^*(\theta_m, \theta) + e_{21}^*(\theta_m, \theta))/\partial \theta_m] \Big|_{\theta_m=\theta} > 0$  independently of  $\sigma$ , and hence, a marginal increase in the degree of altruism/spite  $\theta_m$  – starting from  $\theta_m = \theta$  – always raises the equilibrium level of *impact* of the mutant's group.<sup>17</sup>

Finally, an increase (decrease) in the mutant's degree of altruism (spite) – starting from  $\theta_m = \theta$  – decreases the equilibrium level of effort of all the members of the rival groups (property (iii) of Proposition 2). Indeed, the increased collective *impact* of the mutant's group (due to a slight increase in  $\theta$ ) diminishes the marginal effect of the out-group members' contributions on the probability of success of their groups. In other words, it reduces the marginal benefit of exerting individual effort in the rival groups. The marginal cost of effort being constant, out-group members exert lower levels of efforts when the mutant's degree of altruism (slightly) increases. Overall, an increase in the mutant's degree of altruism increases the equilibrium probability of success of the mutant's group to the detriment of the rival groups.

<sup>16</sup> Observe also that  $\tilde{\sigma}$  is increasing in the number of competing groups  $n$  and that  $\tilde{\sigma} \in [0, 1)$ . For  $n = 2$  and  $\sigma \rightarrow 0$ , a change in  $\theta_m$  (from  $\theta_m = \theta$ ) has no impact on the equilibrium level of effort exerted by the mutant's teammate. Furthermore, with  $n = 2$ , a change in  $\theta_m$  (from  $\theta_m = \theta$ ) does not modify either the effort choices of out-group members independently of  $\sigma$ , as (20.4) shows.

<sup>17</sup> At the point where all players are of the same type  $\theta_m = \theta$ , they exert the same level of effort  $e^*$ . It follows that  $[\partial G_1(e_1^*)/\partial \theta_m] \Big|_{\theta_m=\theta} = 2 \frac{1-\sigma}{\sigma} [\partial(e_{11}^*(\theta_m, \theta) + e_{21}^*(\theta_m, \theta))/\partial \theta_m] \Big|_{\theta_m=\theta} > 0$ .

To conclude this Section, one can also observe that, in a monomorphic population where all players are of the same type  $\theta$ , the equilibrium level of individual effort is – using (18) – given by  $e^* = (1+\theta)(n-1)/2n^2$ . This equilibrium effort level is decreasing in the number  $n$  of competing groups, as an increased number of contenders decreases each group’s probability of success for any level of individual and group effort. Note also that  $e^*$  does not depend on the degree of complementarity between individual efforts, as measured by  $\sigma$ . This is because each group’s impact function has constant returns to scale, and that the contest success function is homogeneous of degree 0. Yet, the level of within-group effort complementarity does have an important role in the evolution of preferences, as we shall now see.

### 3.2 Local evolutionary stability

We are now prepared to analyze the evolutionary stability of in-group altruism/spite. Unfortunately, we are unable to obtain results on globally stable preferences, which would have necessitated that the inequality (13) holds *for all*  $\theta_m \neq \theta$ . Hence, following Alger and Weibull (2010), we use the concept of local evolutionary stability.

**Definition 2:**  $\theta \in (-1, R]$  is *locally evolutionarily stable* if (13) holds for  $\theta_m \neq \theta$  in some neighborhood of  $\theta$ .

The requirement of local stability is reasonable in an evolutionary context, where mutants enter the field with a similar – but slightly deviant – preference parameter relative to the incumbent preference parameter of the whole population.

When the preference parameter of the mutant player  $\theta_m$  is in a neighborhood of  $\theta$ , inequality (17) holds, and all players exert a strictly positive level of effort. The equilibrium material payoff of the mutant player is

$$\Pi_{11}^*(\theta_m, \theta) = p_1(\mathbf{e}_1^*(\theta_m, \theta), \mathbf{e}_R^*(\theta_m, \theta)) - e_{11}^*(\theta_m, \theta), \quad (21)$$

where  $\mathbf{e}_1^*(\theta_m, \theta) = (e_{11}^*(\theta_m, \theta), e_{21}^*(\theta_m, \theta))$ , with  $e_{11}^*(\theta_m, \theta)$  and  $e_{21}^*(\theta_m, \theta)$  given by (18.1) and (18.2), and where  $\mathbf{e}_R^*(\theta_m, \theta)$  is the  $2(n-1)$ -dimensional vector whose components all equal to  $e_R^*(\theta_m, \theta)$  given by (18-3).

Thus, we obtain a Proposition similar to Proposition (15) in Alger and Weibull (2010).

**Proposition 3:** *A necessary and sufficient condition for a degree of altruism/spite  $\theta \in (-1, R]$  to be locally evolutionarily stable is (i)-(ii), where*

$$(i) \frac{\partial \Pi_{11}^*(\theta_m, \theta)}{\partial \theta_m} \Big|_{\theta_m=\theta} = 0;$$

$$(ii) \frac{\partial^2 \Pi_{11}^*(\theta_m, \theta)}{\partial \theta_m^2} \Big|_{\theta_m=\theta} < 0.$$

In other words,  $\theta$  is *locally* evolutionarily stable if and only if  $\Pi_{11}^*(\theta_m, \theta)$ , given by (21), has a strict local maximum at  $\theta_m = \theta$ .

We can then obtain the following Proposition:

**Proposition 4:** *There exists a unique locally evolutionarily stable preference parameter  $\theta^* \in (-1, R]$  if and only if  $\sigma < \bar{\sigma}$ , where*

$$\theta^* = \frac{(n^2 - 2)(1 - \sigma) - n}{(n^2 + 2)(1 - \sigma) - n(1 - 2\sigma)}, \quad (22.1)$$

and

$$\bar{\sigma} = \frac{2n^3 + 3n^2 - 6n + 4 - n\sqrt{4n^3(n-1) + 17n^2 - 20n + 12}}{4(n-1)^2} < 1. \quad (22.2)$$

Recall that the lower  $\sigma$ , the greater is the complementarity between group members' contributions. Thus, according to Proposition 4, a necessary and sufficient condition for the existence of a locally evolutionarily stable preference parameter is that group members' efforts are sufficiently complementary. It is useful to carry out a comparative-statics analysis of the threshold value  $\bar{\sigma}$ , and to compare it with the threshold value  $\tilde{\sigma}$  given by (20.3) in Proposition 2. We have the following Proposition:

**Proposition 5:** *For any  $n \geq 2$  : (i)  $\bar{\sigma}$  given by (22.2) is increasing in  $n$ , with  $\bar{\sigma}|_{n=2} = 5 - 3\sqrt{2} \approx 0.76$  and  $\lim_{n \rightarrow \infty} \bar{\sigma} = 1$ ; (ii)  $\bar{\sigma} > \tilde{\sigma}$ , where  $\tilde{\sigma}$  is given by (20.3).*

Thus, according to property (i) of Proposition 5, the minimum degree of complementarity that is required for the existence of locally evolutionarily stable preferences becomes less stringent as the number of competing groups increases. Furthermore, according to property (ii) of Proposition 5, the threshold value  $\bar{\sigma}$  is strictly larger than the threshold value  $\tilde{\sigma}$ , above which an increase in the mutant's degree of altruism induces the mutant's teammate to decrease her contribution to group effort. Therefore, the increased *free-rider problem* due to the other's altruism, when  $\sigma \geq \tilde{\sigma}$ , does not prevent the existence of locally evolutionarily stable preferences. However, if group members' efforts are too substitutable, an equilibrium in evolutionarily stable preferences fails to exist, even using the weaker concept of local stability.

Before explaining these results in more detail, we investigate whether locally evolutionarily stable preferences exhibit altruism or spite within groups, that is, whether  $\theta^*$  given by (22.1) is positive or negative. We have the following Proposition:

**Proposition 6:** *Let  $\sigma < \bar{\sigma}$  and  $n \geq 2$ , then*

$$\theta^* \begin{matrix} \geq \\ \leq \end{matrix} 0 \Leftrightarrow \sigma \begin{matrix} \leq \\ \geq \end{matrix} \hat{\sigma}, \text{ where} \quad (23.1)$$

$$\hat{\sigma} = \frac{n^2 - n - 2}{n^2 - 2} < \bar{\sigma}. \quad (23.2)$$

This Proposition states that the locally evolutionarily stable preference parameter  $\theta^*$  can differ from zero, depending on the degree of complementarity between individual efforts, and on the number of competing groups. In other words, "biased" preferences can be evolutionarily stable, with positive values of  $\theta^*$  corresponding to in-group *altruism*, and with negative values of  $\theta^*$  corresponding to in-group *spite*. "Biased" preferences can be profitable in terms of material payoffs because a change in a player's type can modify other players' equilibrium actions.<sup>18</sup> In other words, type observability has strategic implications within and across groups. A change in the optimal choice of one player as a result of a change in his own preference induces not only a different optimal reply for his teammate but also for all members of competing groups. In turn, it changes the winning probabilities of all groups involved in the group contest, and hence, the player's fitness. Consequently, altruistic or spiteful preferences can be evolutionarily stable.<sup>19</sup>

Now, let us explain the sign of  $\theta^*$ , according to whether the degree of complementarity between individual efforts  $\sigma$  is larger or lower than the threshold value  $\hat{\sigma}$ , given by (23.2) in Proposition 6. Consider a population where all players are selfish and a mutation occurs with one player being (slightly) altruistic towards his partner. Then, this mutant player exerts a higher level of effort (property (i) of Proposition 2), while all members of competing groups exert a lower level of effort (property (iii) of Proposition 2). As for the impact of the mutation on the equilibrium level of effort of the mutant's teammate, it depends on whether the degree of complementarity between individual efforts, for a given number of competing groups, is larger or lower than the threshold value  $\tilde{\sigma}$  given by (20.3) (property (ii) of Proposition 2). When  $\sigma$  is larger than  $\tilde{\sigma}$ , a positive degree of altruism leads the mutant's teammate to reduce his effort level, increasing it when  $\sigma$  is lower than  $\tilde{\sigma}$ . Therefore, it is instructive to compare the threshold value  $\hat{\sigma}$ , which determines the sign of  $\theta^*$ , with  $\tilde{\sigma}$ . The following Proposition provides such a comparison together with a comparative-static analysis of  $\hat{\sigma}$ :

**Proposition 7:** *For any  $n \geq 2$  : (i)  $\hat{\sigma}$  given by (23.2) is increasing in  $n$ , with  $\hat{\sigma}|_{n=2} = 0$*

<sup>18</sup>As stated in Footnote 16, a change in  $\theta_m$  (from  $\theta_m = \theta$ ) does not modify the effort choices of out-group members when  $n = 2$ . If, in addition,  $\sigma \rightarrow 0$ , it does not modify either the equilibrium level of effort exerted by the mutant's teammate. Consequently, with  $n = 2$  and  $\sigma \rightarrow 0$ , selfishness – that is  $\theta^* = 0$  – is (locally) evolutionarily stable. It confirms that a necessary condition for "biased" preferences to be evolutionarily stable is that a change in a player's type has strategic implications by modifying other players' effort choices (within and across groups). Yet, in the present analysis, selfishness can also be locally evolutionarily stable, although a change in a player's type induces strategic effects. It happens in the specific case where, given the number of competing groups, the degree of effort complementarity within groups is precisely equal to  $\hat{\sigma}$  given by (23.2) in Proposition 6.

<sup>19</sup>In general, the nature of strategic interactions between players, that is, strategic substitutes versus strategic complements in the sense of Bulow et al., (1985), is not decisive for determining the "sign" of other-regarding preferences parameters, that is, altruism versus spite. For instance, Possajennikov (2000) shows in a simple two-player game that strategic complementarity in effort choices can lead to an evolutionarily stable utility weight on the other's payoff, which is either positive (altruism) or negative (spite), according to "the degree of [positive] interdependence between players' strategies".

and  $\lim_{n \rightarrow \infty} \hat{\sigma} = 1$ ; (ii)  $\hat{\sigma} \in [\tilde{\sigma}, \bar{\sigma})$ , where  $\tilde{\sigma}$  and  $\bar{\sigma}$  are given by (20.3) and (22.2), respectively.

Let us first explain property (ii) of Proposition 7. It implies that one can have  $\sigma \geq \tilde{\sigma}$  and  $\sigma \leq \hat{\sigma} < \bar{\sigma}$ , in which case, in-group altruism is locally evolutionarily stable (Proposition 6), although mutating towards in-group altruism *exacerbates* the free-riding incentive of the mutant's teammate (Proposition 2). Group effort, nevertheless, slightly increases, and more importantly, the mutation induces all members of opposing groups to exert a lower level of effort. This results in the probability of success of the group with the altruistic mutant player increasing to the detriment of all other groups. Overall, the increase in the probability of success is sufficiently large for overcoming the fitness cost of exerting a higher level of effort than that corresponding to selfish behavior, thus, making in-group altruism locally evolutionarily stable. This is even more so when the mutation towards in-group altruism *mitigates* the free-riding incentive of the mutant's teammate by inducing him to increase his individual contribution to group effort, that is when  $\sigma \leq \tilde{\sigma} \leq \hat{\sigma}$ .

When the degree of complementarity between individual efforts is rather low (but not too low), that is when  $\sigma \in [\hat{\sigma}, \bar{\sigma})$ , there still exists a locally evolutionarily stable preference parameter; however, in this case, it is negative, thus, featuring in-group spite. In other words, if individual efforts are quite substitutable, the increased free-riding by the other group member in response to the mutant's altruism is too strong for making in-group altruism profitable in terms of material payoff. On the contrary, a mutant player who is (slightly) spiteful towards his teammate can invade a population of selfish players. It leads his teammate and all members of competing groups to increase their efforts. Ultimately, the probability of success of the group with the mutant player decreases but the fitness benefit of exerting a lower level of effort (because of spite) more than compensates the (slight) decrease in the probability of success. It follows that in-group spite is locally evolutionarily stable when individual efforts are quite substitutable, that is when  $\sigma \in [\hat{\sigma}, \bar{\sigma})$ . Finally, if group members' efforts are too substitutable – if  $\sigma \geq \bar{\sigma}$  – preference evolution leads to ever more spite within groups, which in turn prevents the existence of locally evolutionarily stable preferences (Proposition 4).<sup>20</sup>

Finally, according to property (i) of Proposition 7, the requirement about the minimum degree of effort complementarity, for making in-group altruism locally evolutionarily stable, becomes less stringent as the number of competing groups increases. In particular, if the number of competing groups becomes very large, in-group altruism is locally evolutionarily stable for any  $\sigma < 1$  (recall that  $\bar{\sigma}$  also converges to 1 in this case). We shall return to the impact of the number of competing groups on preference evolution in detail later.

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<sup>20</sup>Observe that in the extreme case of perfect substitutability, that is when  $\sigma = 1$ , the only candidate for a locally evolutionarily stable preference parameter would be  $\theta = -1$ . In this case, no player would exert a positive level of effort, and thus, each player's material payoff would be equal to  $1/n$  (as each group would win the public prize of value 1 with a probability  $1/n$ : see Eq. (2)). Clearly, it cannot be a (local) equilibrium in evolutionarily stable preferences because an individual player could make his own group win the prize with certainty (and could enjoy the prize of value 1 with certainty) by investing an arbitrarily small amount because of an arbitrarily smaller degree of spite than  $\theta = -1$ .

The above discussion suggests a monotonicity result for the impact of the degree of complementarity between individual efforts on the locally evolutionarily stable degree of altruism/spite. The derivative of  $\theta^*$  with respect to  $\sigma$  is indeed given by

$$\frac{\partial \theta^*}{\partial \sigma} = -\frac{2n^2(n-1)}{[(n^2+2)(1-\sigma) - n(1-2\sigma)]^2}. \quad (24)$$

Thus, we have the following Proposition.

**Proposition 8:** *Let  $n \geq 2$  be given and  $\sigma < \bar{\sigma}$ . Then, the higher the degree of complementarity between individual efforts – the lower the  $\sigma$  – the higher (lower) is the locally evolutionarily stable degree of altruism (spite), that is, the higher is  $\theta^*$ .*

Thus, an increase in the degree of complementarity between partners' efforts always reinforces group cohesion, in that it decreases spite or increases altruism towards one's teammate. If starting with a monomorphic population of altruistic (spiteful) players, one player becomes slightly more altruistic (less spiteful), then the mutant player exerts a higher level of effort, and all members of opposing groups decrease their efforts. When the degree of complementarity is relatively low, that is when  $\sigma \geq \tilde{\sigma}$ , it also leads the mutant's teammate to decrease his contribution to the collective effort; however, *the extent* of free-riding on the (additional) effort exerted by the mutant becomes less severe as the degree of complementarity increases. If this last is already relatively large, that is if  $\sigma \leq \tilde{\sigma}$ , then the mutant's teammate also exerts a higher level of effort, all the more so as the degree of effort complementarity increases, that is, as  $\sigma$  decreases. Formally, the derivative of  $[\partial e_{21}^*(\theta_m, \theta) / \partial \theta_m] |_{\theta_m=\theta}$  given by (20.2) with respect to  $\sigma$  is always negative.<sup>21</sup> As already stated, an increase in  $\theta_m$  also leads the members of opposing groups to decrease their effort; however, the extent of this strategic effect is independent of the degree of effort complementarity (as (20.4) shows).

In sum, the marginal fitness benefit of mutating towards a higher (lower) degree of altruism (spite) always increases with the degree of complementarity between individual efforts, independently of the number of competing groups. The explanation is as follows: in general, higher degrees of complementarity discourage free-riding because it makes collective effort more sensitive to each of the individual contributions.<sup>22</sup> In intergroup conflict, stronger complementarity implies that one's level of effort has a greater impact on the probability of success of the group, and consequently, on one's individual fitness. Consequently, the mutant's teammate free-rides to a smaller extent if  $\sigma \geq \tilde{\sigma}$ , or increases even more his effort level if  $\sigma \leq \tilde{\sigma}$ , when the mutant exerts more effort because of stronger in-group altruism.

<sup>21</sup>We indeed have  $\partial([\partial e_{21}^*(\theta_m, \theta) / \partial \theta_m] |_{\theta_m=\theta}) / \partial \sigma = -(n-1) / [2n(1-\sigma)]^2 < 0$ .

<sup>22</sup>The idea that complementarity between individual efforts can help solve the collective action problem goes back to Hirshleifer (1983). In the private provision of public goods model, he showed that "*underprovision of the public good tends to considerably moderated*" with perfect complementarity between players' contributions. Cornes and Hartley (2007) have generalized his analysis, considering a CES function for aggregating individual contributions.

Thus, a larger degree of complementarity between group members' effort strengthens in-group altruism under preference evolution.

Do a larger number of competing groups increase the degree of in-group altruism (or decrease the degree of in-group spite)? Calculating the derivative of  $\theta^*$  with respect to  $n$ , we obtain,

$$\frac{\partial \theta^*}{\partial n} = \frac{2(1 - \sigma) [2(2n - 1) + \sigma(n^2 - 4n + 2)]}{[(n^2 + 2)(1 - \sigma) - n(1 - 2\sigma)]^2}. \quad (25)$$

We have the following Proposition:

**Proposition 9:** *Let  $\sigma < \bar{\sigma}$  be given and  $n \geq 2$ .*

(i) *If  $\sigma \in (0, \bar{\sigma})$ , then the locally evolutionarily stable preference parameter  $\theta^*$  is increasing in  $n$  and is negative for  $n \leq \lfloor \hat{n} \rfloor$  and positive for  $n \geq \lceil \hat{n} \rceil$ , where  $\hat{n} = [1 + \sqrt{9 - 8\sigma(2 - \sigma)}] / [2(1 - \sigma)] > 2$ .*

(ii) *If  $\sigma \in (-\infty, 0)$ , then the locally evolutionarily stable preference parameter  $\theta^*$  is always positive and is first increasing in  $n$  for  $n \leq \lfloor \bar{n} \rfloor$ , and then decreasing in  $n$  for  $n \geq \lceil \bar{n} \rceil$ , where  $\bar{n} = -[2(1 - \sigma) + \sqrt{2(1 - \sigma)(2 - \sigma)}] / \sigma > 2$ .*

Similar to an increase in the degree of effort complementarity, an increase in the number of competing groups changes the *extent* of the strategic impact of higher altruism on the effort choice exerted by the mutant's teammate. Again, depending on whether the degree of effort complementarity is relatively low or large, the mutation induces the mutant's teammate to either contribute more or less to group effort. Yet, as the number of competing groups increases, the mutant's teammate decreases her effort less strongly or increases it more strongly in response to the mutant's increased altruism. Formally, the derivative of  $[\partial e_{21}^*(\theta_m, \theta) / \partial \theta_m] |_{\theta_m = \theta}$ , given by (20.2), with respect to  $n$  is always positive.<sup>23</sup> In isolation, this effect increases the marginal benefit of increased altruism.

The mutation also causes all members of competing groups to decrease their effort levels and the extent of this strategic effect depends on the number of competing groups. Formally, the derivative of  $[\partial e_R^*(\theta_m, \theta) / \partial \theta_m] |_{\theta_m = \theta}$ , given by (20.4), with respect to  $n$ , is first negative, and becomes rapidly positive.<sup>24</sup> This latter effect contributes in reducing the marginal benefit of increased altruism. Thus, when mutating towards higher degrees of in-group altruism, an increase in the number of competing groups has in general, a positive impact on the extent of the strategic effect within the group, and a negative impact on the extent of the strategic effect across groups.

Moreover, in contrast to the degree of within-group effort complementarity, the number of competing groups directly affects the equilibrium level of individual effort for any

<sup>23</sup>We indeed have  $\partial([\partial e_{21}^*(\theta_m, \theta) / \partial \theta_m] |_{\theta_m = \theta}) / \partial n = [2(2n^2 - 5n + 3) - \sigma(3n^2 - 8n + 6)] / [4(1 - \sigma)n^4]$ , which is strictly positive for any  $\sigma < \bar{\sigma} < 1$ .

<sup>24</sup>The derivative of  $[\partial e_R^*(\theta_m, \theta) / \partial \theta_m] |_{\theta_m = \theta}$  with respect to  $n$  becomes positive from  $n \geq 5$ .

monomorphic population of players with a common preference parameter  $\theta$ . Indeed, all players' equilibrium efforts decrease with the number of competing groups, because of the lower equilibrium probability of victory, and because of the fixed-prize value. All these features can explain why the locally evolutionarily stable degree of in-group altruism/spite is not (always) monotonous in the number of competing groups, according to whether  $\sigma$  is positive or negative.

If the degree of complementarity between individual efforts is relatively low, that is if  $\sigma$  is positive, the (locally) evolutionarily stable degree of in-group altruism (spite) is increasing (decreasing) in the number of competing groups. In this case, preference evolution can lead to in-group spite if few groups compete against each other. However, as the number of groups increases, it leads to the emergence of in-group altruism, and this preference trait becomes increasingly strong as the number of competing groups keeps increasing (property (i) of Proposition 9). When the degree of effort complementarity is relatively large, that is when  $\sigma$  is negative, the (locally) evolutionarily stable degree of in-group altruism first increases and then decreases in the number of competing groups (property (ii) of Proposition 9). It remains that  $\sigma < 0$  always leads to stronger in-group altruism under preference evolution than  $\sigma > 0$ , for any given number of competing groups (Proposition 8).

Propositions 8 and 9 together show that stronger complementarity between group members' efforts within groups and a larger number of competing groups reinforce evolutionary pressure towards higher degrees of in-group altruism. Intuitively, a player's fitness is increasingly merged with the success of his own group with increased number of competing groups, and increased within-group effort complementarity. In turn, preference evolution leads to stronger in-group altruism and group cohesiveness in intergroup conflict.

Finally, we can characterize the equilibrium level of individual effort and of individual fitness when players have locally evolutionarily stable preferences. When all players have the same preference parameter  $\theta^*$ , they all exert the same level of individual effort, that is,  $e^* = (1 + \theta^*)(n - 1)/2n^2$ . Substituting  $\theta^*$  given by (22.1) into this expression, we obtain

$$e^* = \frac{(1 - \sigma)(n - 1)^2}{n[(n^2 + 2)(1 - \sigma) - n(1 - 2\sigma)]}. \quad (26)$$

Furthermore, all groups have the same probability of success, that is  $1/n$ ; therefore, each player's material payoff induced by evolutionarily stable preferences is  $\Pi^* = (1/n) - e^*$  or

$$\Pi^* = \frac{n + 1 - \sigma}{n[(n^2 + 2)(1 - \sigma) - n(1 - 2\sigma)]}. \quad (27)$$

We can easily verify that an increase in the degree of complementarity between individual efforts, that is, a decrease in  $\sigma$ , unambiguously decreases material payoff. Indeed, the lower the  $\sigma$ , the higher (lower) is the locally evolutionarily stable degree of in-group altruism (spite), and thus, the higher is the level of individual effort. Consequently, the conflict between groups becomes more severe. A larger number of competing groups decreases the

equilibrium winning probability and can actually increase or decrease the stable degree of in-group altruism, depending on the constellation of parameters. Consequently, the equilibrium level of individual effort is not monotonous in the number of competing groups. Nevertheless, a larger number of groups also always decreases individual payoffs, simply because a larger number of players are involved in a group contest with a fixed-prize value.

## 4 Conclusion

The inclination of people to pull together when facing of a common enemy appears to be a universal trait of human behavior, which is confirmed by several experimental studies in economics and social psychology. In this study, we provide an evolutionary foundation for the emergence and stability of in-group altruism when several groups compete against each other for an exogenous prize. We demonstrate that both a strong degree of complementarity between individual efforts and a large number of competing groups reinforce in-group altruism under preference evolution. Indeed, the success of a group crucially depends on its ability to contain the tendency to free-riding or selfishness. Thus, there is an evolutionary pressure towards in-group altruism, because it increases the probability of success of one's own group, which in turn increases individual fitness. The downside of this behavior is that it makes the conflict between groups more severe and potentially more destructive.

One important feature of the present analysis is that the contested prize has the characteristics of a public good. Thus, a potential direction for future research would be to consider a group contest for a private prize, which must be divided between group members if they win the contest. How would the prize be distributed among the members of the winning group? As a first step, as proposed by Nitzan (1991), we could consider a linear sharing rule where a certain proportion of the prize is shared equally while the residual proportion is distributed according to relative effort. We could then analyze how the sharing rule parameter affects the evolutionarily stable degree of altruism/spite within groups. The next, and more ambitious, step would be to determine the evolutionarily stable sharing rule. However, in this case, characterizing an evolutionarily stable equilibrium would require studying the effects of a two-dimensional level of mutations: mutations in preferences at the individual level, and mutations in sharing rules at the group level.

## Appendix

### A.1. Proof of Lemma 1

The proof of this Lemma is conducted in three steps. The first one is to show that the first-order conditions given by (6) are necessary and sufficient for maximization.

The first derivative of  $V_{ij}(e_j, e_{-j})$ , given by (5), with respect to  $e_{ij}$  is given by

$$\frac{\partial V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial e_{ij}} = (1 + \theta_{ij}) \frac{\partial p_j(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial G_j(\mathbf{e}_j)} \frac{\partial G_j(\mathbf{e}_j)}{\partial e_{ij}} - 1. \quad (\text{A1})$$

Using (1) and (2), we have

$$\frac{\partial p_j(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial G_j(\mathbf{e}_j)} = \frac{\sum_{k \neq j} G_k(\mathbf{e}_k)}{(\sum_{l=1}^n G_l(\mathbf{e}_l))^2}, \quad (\text{A2})$$

and

$$\frac{\partial G_j(\mathbf{e}_j)}{\partial e_{ij}} = [e_{ij}^\sigma + e_{-ij}^\sigma]^{\frac{1}{\sigma}-1} e_{ij}^{\sigma-1} = \frac{G_j(\mathbf{e}_j) e_{ij}^{\sigma-1}}{e_{ij}^\sigma + e_{-ij}^\sigma}. \quad (\text{A3})$$

Thus, (A1) can be rewritten as

$$(1 + \theta_{ij}) \frac{p_j(\mathbf{e}_j, \mathbf{e}_{-j}) \left[ \sum_{k \neq j} p_k(\mathbf{e}_k, \mathbf{e}_{-k}) \right] e_{ij}^{\sigma-1}}{(e_{ij}^\sigma + e_{-ij}^\sigma)} - 1. \quad (\text{A4})$$

Hence, the first-order condition for  $i = 1, 2$  and  $j = 1, 2, \dots, n$  is given by (6), since  $p_j(\mathbf{e}_j, \mathbf{e}_{-j}) + \sum_{k \neq j} p_k(\mathbf{e}_k, \mathbf{e}_{-k}) = 1$ .

The second derivative of  $V_{ij}(e_j, e_{-j})$  with respect to  $e_{ij}$  is given by

$$\frac{\partial^2 V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial e_{ij}^2} = (1 + \theta_{ij}) \left\{ \begin{array}{l} [1 - 2p_j(\mathbf{e}_j, \mathbf{e}_{-j})] \frac{\partial p_j(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial G_j(\mathbf{e}_j)} \frac{\partial G_j(\mathbf{e}_j)}{\partial e_{ij}} \frac{e_{ij}^{\sigma-1}}{(e_{ij}^\sigma + e_{-ij}^\sigma)} \\ p_j(\mathbf{e}_j, \mathbf{e}_{-j}) [1 - p_j(\mathbf{e}_j, \mathbf{e}_{-j})] \frac{e_{ij}^{\sigma-2} [(\sigma - 1) e_{-ij}^\sigma - e_{ij}^\sigma]}{(e_{ij}^\sigma + e_{-ij}^\sigma)^2} \end{array} \right\}. \quad (\text{A5})$$

From (A2) and (A3), we also have

$$\frac{\partial p_j(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial G_j(\mathbf{e}_j)} \frac{\partial G_j(\mathbf{e}_j)}{\partial e_{ij}} = \frac{p_j(\mathbf{e}_j, \mathbf{e}_{-j}) [1 - p_j(\mathbf{e}_j, \mathbf{e}_{-j})] e_{ij}^{\sigma-1}}{e_{ij}^\sigma + e_{-ij}^\sigma}. \quad (\text{A6})$$

Substituting (A6) into (A5), we obtain

$$\frac{\partial^2 V_{ij}(\mathbf{e}_j, \mathbf{e}_{-j})}{\partial e_{ij}^2} = \frac{(1 + \theta_{ij}) p_j(\mathbf{e}_j, \mathbf{e}_{-j}) [1 - p_j(\mathbf{e}_j, \mathbf{e}_{-j})] [-2p_j(\mathbf{e}_j, \mathbf{e}_{-j}) e_{ij}^\sigma + (\sigma - 1) e_{-ij}^\sigma] e_{ij}^{\sigma-2}}{(e_{ij}^\sigma + e_{-ij}^\sigma)^2}, \quad (\text{A7})$$

which is always negative for  $i = 1, 2$  and  $j = 1, 2, \dots, n$ , since  $\sigma \leq 1$ .

Consequently,  $V_{ij}(e_j, e_{-j})$ , for  $i = 1, 2$  and  $j = 1, 2, \dots, n$ , is strictly concave and continuous in  $e_{ij}$  for  $\sigma \in (-\infty, 0)$  or  $\sigma \in (0, 1]$ . Thus, the first-order conditions given by (6) are both necessary and sufficient for maximization.

The second step of the proof is to show that if a group participates to the contest, then its two members produce positive levels of effort. In other words, a corner solution for player  $(i, j)$  – that is  $e_{ij} = 0$  – and an interior solution for player  $(-i, j)$  – that is  $e_{-ij} > 0$  – cannot be mutual best responses for the two players of group  $j$ . Indeed, suppose first that  $\sigma \in (0, 1]$  with  $e_{ij} = 0$  and  $e_{-ij} > 0$ , then we would have  $p_j(\mathbf{e}_j, \mathbf{e}_{-j}) > 0$  but the denominator of the LHT of (6) would tend to 0, so that the LHT would approach infinity. Hence, (6) cannot be satisfied for  $e_{ij} = 0$  and  $e_{-ij} > 0$  when  $\sigma \in (0, 1]$ . Now suppose that  $\sigma \in (-\infty, 0)$  with  $e_{ij} = 0$ , then  $G_j(\mathbf{e}_j) = 0$  and thus  $V_{-ij}(e_j, e_{-j})$  is strictly decreasing in  $e_{-ij}$  so that player  $(-i, j)$  cannot do better than  $e_{-ij} = 0$ , in this case. To conclude, if a group participates to the contest, it fully participates with both members being active (see also Kolmar and Rommeswinkel, 2003). Furthermore, there cannot exist an equilibrium where all groups do not participate to the contest for  $\sigma \in (0, 1]$ . Indeed, if all other groups do not enter the contest, the members of group  $j$  could win the prize with probability 1 in return for an arbitrarily small effort (exerted by both group members). As a result for  $\sigma \in (0, 1]$ , at least one group is fully active. If  $\sigma \in (-\infty, 0)$ ,  $e_{ij} = 0$  and  $e_{-ij} = 0$  are mutually best responses independently of the behavior of other groups. Hence, there may exist an equilibrium – that we ignore – in which none the  $n$  groups participate to the contest.

The final step for demonstrating the existence of a pure strategy Nash equilibrium characterized by (6), is to reduce the group contest to a contest among heterogeneous individual players,<sup>25</sup> and then, use Theorem 1 of Cornes and Hartley (2005). Suppose that there are  $m$  active groups and consider the system of  $2m$  first-order conditions (6), holding with equality. This system can be rewritten as (with (A1), (A2) and (A3))

$$\frac{\left(\sum_{k \in M \setminus \{j\}} G_k\right) G_j^{1-\sigma}}{\left(\sum_{l \in M} G_l\right)^2} = \frac{e_{ij}^{1-\sigma}}{(1 + \theta_{ij})}, \quad (\text{A8})$$

for  $i = 1, 2$  and  $j$  being an element of the set of active groups, denoted by  $M$ .

For a given group  $j$ , the LHS of (A8) is the same for  $i = 1, 2$  and hence  $e_{1j}(1 + \theta_{2j})^{1/(1-\sigma)} = (1 + \theta_{1j})^{1/(1-\sigma)} e_{2j}$ . The aggregate output of group  $j \in M$  can thus be written as a function of the effort of player 1, that is

$$G_j = \left[ \frac{(1 + \theta_{1j})^{\frac{\sigma}{1-\sigma}} + (1 + \theta_{2j})^{\frac{\sigma}{1-\sigma}}}{(1 + \theta_{1j})^{\frac{\sigma}{1-\sigma}}} \right]^{\frac{1}{\sigma}} e_{1j} \equiv \Psi_{1j}(\theta_{1j}, \theta_{2j}) e_{1j}. \quad (\text{A9})$$

We thus have  $e_{1j} = [\Psi_{1j}(\theta_{1j}, \theta_{2j})]^{-1} G_j$ .

The system of  $2m$  first-order conditions (A8) can thus be reduced to a system of  $m$  equations in  $G_j$  for  $j = 1, 2, \dots, m$ , that is

$$\frac{\sum_{k \neq j} G_k}{\left(\sum_{l \in M} G_l\right)^2} = \frac{1}{\Phi_j(\theta_{1j}, \theta_{2j})}, \quad (\text{A10})$$

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<sup>25</sup>See also Brookins et al. (2015) in a model of group contest with CES impact functions and heterogeneous and convex cost functions.

where

$$\Phi_j(\theta_{1j}, \theta_{2j}) = (1 + \theta_{1j}) [\Psi_{1j}(\theta_{1j}, \theta_{2j})]^{1-\sigma} = \left[ (1 + \theta_{1j})^{\frac{\sigma}{1-\sigma}} + (1 + \theta_{2j})^{\frac{\sigma}{1-\sigma}} \right]^{\frac{1-\sigma}{\sigma}}. \quad (\text{A11})$$

In other words, the system of first-order conditions (A4) for the group contest with heterogeneous group members' preferences can be reduced to a system that is induced by a lottery contest of  $m$  individual players choosing  $G_j$  with heterogeneous (and constant) marginal costs given by the RHT of (A10). Applying Theorem 1 of Cornes and Hartley (2005), we can conclude that there exists a unique pure strategy Nash equilibrium  $(G_1^*, G_2^*, \dots, G_n^*)$ . In turn,  $e_{ij}^* = [\Psi_{1j}(\theta_{1j}, \theta_{2j})]^{-1} G_j^*$  satisfying (6) constitute an equilibrium in the contest between groups.

## A.2. Proof of Proposition 1

(i) From the proof of Lemma 1, we know that if a group participates to the contest, then its two members exert positive levels of effort. Furthermore, the group contest with heterogeneous "other-regarding" preferences can be reduced to a lottery contest between individual players choosing  $G_j$  with heterogeneous marginal costs. Let  $G = \sum_k G_k$ . From (A10), the first-order condition for player  $j$  can be rewritten as

$$\frac{G - G_j}{G^2} - \frac{1}{\Phi_j(\theta_{1j}, \theta_{2j})} \leq 0. \quad (\text{A12})$$

It is non-positive at  $G_j = 0$  for  $\Phi_j(\theta_{1j}, \theta_{2j}) \leq G$ . Thus, player/group  $j$  is fully inactive if  $\Phi_j(\theta_{1j}, \theta_{2j}) \leq G$ . If, however  $\Phi_j(\theta_{1j}, \theta_{2j}) > G$ , then player  $j$  is active and thus  $G_j = G - (G^2/\Phi_j(\theta_{1j}, \theta_{2j}))$ . Again, let  $M$  be the set of the  $m$  active players in equilibrium. We have  $G = \sum_{j \in M} G_j$ , and hence

$$G = \frac{m - 1}{\sum_{j \in M} (1/\Phi_j(\theta_{1j}, \theta_{2j}))}. \quad (\text{A13})$$

Now, suppose that all group members have the same preference parameter  $\theta$ , except player (1, 1) who has the preference parameter  $\theta_m$ . Thus, we have (from (A11))  $\Phi_{-1}(\theta) = 2^{\frac{1-\sigma}{\sigma}} (1 + \theta)$  for all  $j \neq 1$  and  $\Phi_1(\theta_m, \theta) = \left[ (1 + \theta_m)^{\frac{\sigma}{1-\sigma}} + (1 + \theta)^{\frac{\sigma}{1-\sigma}} \right]^{\frac{1-\sigma}{\sigma}}$ .

We first show that there cannot exist an equilibrium in which player (group) 1 is active, and some, but not all, other players (groups) are inactive. Let denote by  $x$ , the number of inactive players, with  $1 < x < n - 1$ . If it were the case, we would have

$$G = \frac{n - x - 1}{[1/\Phi_1(\theta_m, \theta)] + (n - x - 1)[1/\Phi_{-1}(\theta)]} = \frac{(n - x - 1)\Phi_1(\theta_m, \theta)\Phi_{-1}(\theta)}{(n - x - 1)\Phi_1(\theta_m, \theta) + \Phi_{-1}(\theta)}. \quad (\text{A14})$$

However, for a player  $j \neq 1$  to be inactive, we must also have  $\Phi_{-1}(\theta) \leq G$ , which is in contradiction with (A14). Consequently, there are two possibilities. All groups  $j \neq 1$  are either fully active or fully inactive. However, there cannot exist an equilibrium where all these groups are fully inactive while group 1 is fully active. Indeed, in that case, the LHT of (A12) would be strictly negative for

any positive level of  $G_1 > 0$ , so that this group would not play its best response. As a consequence, if player (i.e. group) 1 is active, then all players (groups) are active in equilibrium.

Now, suppose that player 1 is fully inactive, and that some, but not all, other players (groups) are also fully inactive. Let, again, denote by  $x$  the number of inactive players in addition to player 1, with  $1 < x < n - 1$ . In this case, we would have

$$G = \frac{(n - x - 2) \Phi_{-1}(\theta)}{(n - x - 1)}. \quad (\text{A15})$$

However, again, for a player  $j \neq 1$  to be inactive, we must also have  $\Phi_{-1}(\theta) \leq G$ , which is in contradiction with (A15). Thus, when player 1 is fully inactive, all groups  $j \neq 1$  are either fully active or fully inactive. As already mentioned, we ignore the equilibrium where all groups are fully inactive. Thus, suppose that all players  $j \neq 1$  are fully active (still with player 1 being inactive). In this case,  $G$  is given by (A15) with  $x = 0$ . This is an equilibrium if  $\Phi_1(\theta_m, \theta) \leq G$ , or

$$\frac{\Phi_1(\theta_m, \theta)}{\Phi_{-1}(\theta)} \leq \frac{(n - 2)}{(n - 1)}. \quad (\text{A16})$$

To conclude, there are two types of equilibrium. If (A16) holds, group 1 is fully inactive while all other groups are active. If (A16) does not hold, then all groups are fully active. Let define

$$\Delta(\theta_m, \theta) = (1 + \theta_m)^{\frac{\sigma}{1-\sigma}} + (1 + \theta)^{\frac{\sigma}{1-\sigma}}, \quad (\text{A17})$$

so that  $\Phi_1(\theta_m, \theta) = [\Delta(\theta_m, \theta)]^{\frac{1-\sigma}{\sigma}}$ . Since  $\Phi_{-1}(\theta) = 2^{\frac{1-\sigma}{\sigma}} (1 + \theta)$ , the necessary and sufficient condition for all groups – and thus all group members – being active is given by (17).

(ii) Suppose (17) holds and that  $\theta_{ij} = \theta$  for  $i = 1, 2$  and  $j = 2, \dots, n$ . Then, the first-order conditions given by (6) hold with equality and are symmetric for all  $j \neq 1$ . Thus, all players – except the members of group 1 – exert the same level of individual effort. To simplify the exposition, let denote by  $e_R^*$  this equilibrium level of effort, i.e.  $e_R^* \equiv e_{ij}^*$  for  $i = 1, 2$  and  $j = 2, 3, \dots, n$ . Hence, we write  $p_j(\mathbf{e}_j^*, \mathbf{e}_{-j}^*) \equiv p_R(\mathbf{e}_1^*, \mathbf{e}_R^*)$  for all  $j = 2, 3, \dots, n$  and where  $\mathbf{e}_R^* = \underbrace{((e_R^*, e_R^*), \dots, (e_R^*, e_R^*))}_{n-1 \text{ times}}$ .

The two members of group 1 differ in terms of (other-regarding) preferences, and thus, exert different levels of effort in equilibrium. This results in a probability of success for group 1, that is  $p_1(\mathbf{e}_1^*, \mathbf{e}_R^*)$ , which differs from  $p_R(\mathbf{e}_1^*, \mathbf{e}_R^*)$ .

From these observations, there are three distinct equilibrium levels of effort that must satisfy the following first-order conditions,

$$(1 + \theta_m) \frac{p_1(\mathbf{e}_1^*, \mathbf{e}_R^*) [(n - 1)p_R(\mathbf{e}_1^*, \mathbf{e}_R^*)] e_{11}^{*\sigma-1}}{e_{11}^{*\sigma} + e_{21}^{*\sigma}} = 1, \quad (\text{A18})$$

for player (1, 1), and

$$(1 + \theta) \frac{p_1(\mathbf{e}_1^*, \mathbf{e}_R^*) [(n - 1)p_R(\mathbf{e}_1^*, \mathbf{e}_R^*)] e_{21}^{*\sigma-1}}{e_{11}^{*\sigma} + e_{21}^{*\sigma}} = 1, \quad (\text{A19})$$

for player (2, 1), who is the partner of player (1, 1), and

$$(1 + \theta) \frac{p_R(\mathbf{e}_1^*, \mathbf{e}_R^*) [p_1(\mathbf{e}_1^*, \mathbf{e}_R^*) + (n - 2)p_R(\mathbf{e}_1^*, \mathbf{e}_R^*)]}{2e_R^*} = 1, \quad (\text{A20})$$

for all players of group  $j \neq 1$ .

Thus, using (A18) and (A19), we have

$$e_{21}^* = \left[ \frac{1 + \theta}{1 + \theta_m} \right]^{\frac{1}{1-\sigma}} e_{11}^*. \quad (\text{A21})$$

With (A19) and (A20), we have

$$2e_R^* p_1(\mathbf{e}_1^*, \mathbf{e}_R^*) (n - 1) e_{21}^{*\sigma-1} = [p_1(\mathbf{e}_1^*, \mathbf{e}_R^*) + (n - 2)p_R(\mathbf{e}_1^*, \mathbf{e}_R^*)] (e_{11}^{*\sigma} + e_{21}^{*\sigma}). \quad (\text{A22})$$

Using (1) and (2) and the fact that  $G_j(\mathbf{e}_j^*) = G_R(\mathbf{e}_R^*) = 2^{1/\sigma} e_R^*$  for  $j = 2, 3, \dots, n$  and that  $p_1(\mathbf{e}_1^*, \mathbf{e}_R^*)$  and  $p_R^*(\mathbf{e}_1^*, \mathbf{e}_R^*)$  have the same denominator, (A22) becomes

$$e_R^* [2G_1(\mathbf{e}_1^*) (n - 1) e_{21}^{*\sigma-1} - 2^{1/\sigma} (n - 2) (e_{11}^{*\sigma} + e_{21}^{*\sigma})] = G_1(\mathbf{e}_1^*) (e_{11}^{*\sigma} + e_{21}^{*\sigma}). \quad (\text{A23})$$

Since  $G_1(\mathbf{e}_1^*) = (e_{11}^{*\sigma} + e_{21}^{*\sigma})^{1/\sigma}$ , (A23) can be rewritten as

$$e_R^* \left[ 2(n - 1) e_{21}^{*\sigma-1} - 2^{1/\sigma} (n - 2) (e_{11}^{*\sigma} + e_{21}^{*\sigma})^{(\sigma-1)/\sigma} \right] = (e_{11}^{*\sigma} + e_{21}^{*\sigma}). \quad (\text{A24})$$

Let

$$\Delta(\cdot) \equiv \Delta(\theta_m, \theta) = (1 + \theta_m)^{\frac{\sigma}{1-\sigma}} + (1 + \theta)^{\frac{\sigma}{1-\sigma}}. \quad (\text{A25})$$

Then, using (A21) and (A25), (A24) can be rewritten as

$$e_R^* \left[ 2(n - 1) [\Delta(\cdot)]^{\frac{1-\sigma}{\sigma}} - 2^{\frac{1}{\sigma}} (1 + \theta) (n - 2) \right] (1 + \theta_m)^{\frac{1}{1-\sigma}} = (1 + \theta) [\Delta(\cdot)]^{\frac{1}{\sigma}} e_{11}^*. \quad (\text{A26})$$

The first-order condition (A18) can also be rewritten as

$$\frac{(1 + \theta_m) (n - 1) G_1(\mathbf{e}_1^*) G_R(\mathbf{e}_R^*) e_{11}^{*\sigma-1}}{(e_{11}^{*\sigma} + e_{21}^{*\sigma}) [G_1(\mathbf{e}_1^*) + (n - 1) G_R(\mathbf{e}_R^*)]^2} - 1 = 0. \quad (\text{A27})$$

Since  $G_R(\mathbf{e}_R^*) = 2^{1/\sigma} e_R^*$  and  $G_1(\mathbf{e}_1^*) = (e_{11}^{*\sigma} + e_{21}^{*\sigma})^{1/\sigma}$ , (A27) can be rewritten as

$$2^{\frac{1}{\sigma}} (n - 1) (1 + \theta_m) (e_{11}^{*\sigma} + e_{21}^{*\sigma})^{\frac{1-\sigma}{\sigma}} e_{11}^{*\sigma-1} e_R^* = \left[ (e_{11}^{*\sigma} + e_{21}^{*\sigma})^{\frac{1}{\sigma}} + 2^{\frac{1}{\sigma}} (n - 1) e_R^* \right]^2. \quad (\text{A28})$$

Using (A21) and (A25), (A28) becomes

$$2^{\frac{1}{\sigma}} (n - 1) (1 + \theta_m)^{\frac{2}{1-\sigma}} [\Delta(\cdot)]^{\frac{1-\sigma}{\sigma}} e_R^* = \left[ [\Delta(\cdot)]^{\frac{1}{\sigma}} e_{11}^* + 2^{\frac{1}{\sigma}} (n - 1) (1 + \theta_m)^{\frac{1}{1-\sigma}} e_R^* \right]^2. \quad (\text{A29})$$

Substituting  $e_R^*$  given by (A26) into (A29), we find after some tedious rearrangements, the equilibrium level of effort  $e_{11}^*(\theta_m, \theta)$  given in (18.1). The equilibrium level of effort  $e_{21}^*(\theta_m, \theta)$ , given in (18.2), is obtained by using (A21). Finally, the common equilibrium level of effort for any player  $i = 1, 2$  of group  $j = 2, 3, \dots, n$ , that is  $e_R^*(\theta_m, \theta)$  in (18.3), is obtained by substituting  $e_{11}^*(\theta_m, \theta)$  into (A26).

### A.3. Proof of Proposition 2

Using (18.1), one can obtain (after long and tedious calculations),

$$\begin{aligned} \frac{\partial e_{11}^*}{\partial \theta_m} &= \frac{2^{\frac{1-\sigma}{\sigma}}(n-1)(1+\theta)(1+\theta_m)^{\frac{\sigma}{1-\sigma}} [\Gamma_1(\cdot) + \Gamma_2(\cdot) + \Gamma_3(\cdot)]}{(1-\sigma) [\Delta(\cdot)]^2 \left[ 2^{\frac{1-\sigma}{\sigma}}(1+\theta) + (n-1) [\Delta(\cdot)]^{\frac{1-\sigma}{\sigma}} \right]^3} \text{ where} \\ \Gamma_1(\cdot) &\equiv \Gamma_1(\theta_m, \theta) = -4^{\frac{1-\sigma}{\sigma}}(n-2)(1+\theta)^2 \left[ \Delta(\cdot) - \sigma(1+\theta_m)^{\frac{\sigma}{1-\sigma}} \right], \\ \Gamma_2(\cdot) &\equiv \Gamma_2(\theta_m, \theta) = (n-1)^2 [\Delta(\cdot)]^{\frac{2(1-\sigma)}{\sigma}} \left[ \Delta(\cdot) - (1+\theta_m)^{\frac{\sigma}{1-\sigma}} \right], \\ \Gamma_3(\cdot) &\equiv \Gamma_3(\theta_m, \theta) = -2^{\frac{1-\sigma}{\sigma}}(n-1)(1+\theta) [\Delta(\cdot)]^{\frac{1-\sigma}{\sigma}} \left[ (n-3)\Delta(\cdot) - [n(2-\sigma) - 3](1+\theta_m)^{\frac{\sigma}{1-\sigma}} \right]. \end{aligned} \tag{A30}$$

Evaluating this expression at  $\theta_m = \theta$ , we obtain

$$\frac{\partial e_{11}^*}{\partial \theta_m} \Big|_{\theta_m=\theta} = \frac{(n-1) [n^2 - n + 2 - \sigma(n^2 - 2n + 2)]}{4(1-\sigma)n^3}, \tag{A31}$$

which is strictly positive for any  $\sigma \in (-\infty, 0) \cup (0, 1]$ .

From (A21), we also have

$$\frac{\partial e_{21}^*}{\partial \theta_m} = \left( \frac{1+\theta}{1+\theta_m} \right)^{\frac{1}{1-\sigma}} \left[ -\frac{e_{11}^*}{(1-\sigma)(1+\theta_m)} + \frac{\partial e_{11}^*}{\partial \theta_m} \right], \tag{A32}$$

and, thus,

$$\frac{\partial e_{21}^*}{\partial \theta_m} \Big|_{\theta_m=\theta} = -\frac{e^*}{(1-\sigma)(1+\theta)} + \frac{\partial e_{11}^*}{\partial \theta_m} \Big|_{\theta_m=\theta}, \tag{A33}$$

where  $e^*$  is the equilibrium level of effort in the symmetric equilibrium with all players having the same preference parameter  $\theta$ . Using (18), we obtain for  $\theta_m = \theta$ :

$$e^* = \frac{(1+\theta)(n-1)}{2n^2}. \tag{A34}$$

Substituting (A34) and (A31) into (A33), we have

$$\frac{\partial e_{21}^*}{\partial \theta_m} \Big|_{\theta_m=\theta} = \frac{(n-1) [(n-1)(n-2) - \sigma(n^2 - 2n + 2)]}{4(1-\sigma)n^3}, \tag{A35}$$

which is positive (negative) for  $\sigma \leq \tilde{\sigma}$  ( $\sigma \geq \tilde{\sigma}$ ), where  $\tilde{\sigma} = [(n-1)(n-2)] / (n^2 - 2n + 2)$ . Observe that if  $n = 2$ , this derivative is *strictly* positive (negative) for  $\sigma < 0$  ( $\sigma > 0$ ) and it is equal to 0 when  $\sigma$  goes to 0. In any case, we have  $[\partial e_{11}^* / \partial \theta_m] \Big|_{\theta_m=\theta} > [\partial e_{21}^* / \partial \theta_m] \Big|_{\theta_m=\theta}$  and  $[\partial (e_{11}^* + e_{21}^*) / \partial \theta_m] \Big|_{\theta_m=\theta} > 0$ .

Finally, using (18.3), we can find that

$$\frac{\partial e_R^*}{\partial \theta_m} = \frac{2^{\frac{1-\sigma}{\sigma}}(n-1)(1+\theta)^2 [\Delta(\cdot)]^{\frac{1-2\sigma}{\sigma}} \left[ 2^{\frac{1-\sigma}{\sigma}}(1+\theta) - (n-1) [\Delta(\cdot)]^{\frac{1-\sigma}{\sigma}} \right]}{2(1+\theta_m)^{\frac{1-2\sigma}{1-\sigma}} \left[ 2^{\frac{1-\sigma}{\sigma}}(1+\theta) + (n-1) [\Delta(\cdot)]^{\frac{1-\sigma}{\sigma}} \right]^3}. \tag{A36}$$

Consequently, we have

$$\frac{\partial e_R^*}{\partial \theta_m} \Big|_{\theta_m=\theta} = -\frac{(n-2)(n-1)}{4n^3}, \quad (\text{A37})$$

which is equal to 0 for  $n = 2$ , and is *strictly* negative for any  $n \geq 3$ .

#### A.4. Proof of Proposition 4

First, we need to verify condition (i) of Proposition 3. Using (21), we have

$$\begin{aligned} \frac{\partial \Pi_{11}^*}{\partial \theta_m} &= \frac{\partial p_1(\mathbf{e}_1^*, \mathbf{e}_R^*)}{\partial G_1(\mathbf{e}_1^*)} \left[ \frac{\partial G_1(\mathbf{e}_1^*)}{\partial e_{11}^*} \frac{\partial e_{11}^*}{\partial \theta_m} + \frac{\partial G_1(\mathbf{e}_1^*)}{\partial e_{21}^*} \frac{\partial e_{21}^*}{\partial \theta_m} \right] \\ &+ \sum_{j \neq 1} \frac{\partial p_1(\mathbf{e}_1^*, \mathbf{e}_R^*)}{\partial G_j(\mathbf{e}_j^*)} \left[ \frac{\partial G_j(\mathbf{e}_j^*)}{\partial e_{1j}^*} \frac{\partial e_{1j}^*}{\partial \theta_m} + \frac{\partial G_j(\mathbf{e}_j^*)}{\partial e_{2j}^*} \frac{\partial e_{2j}^*}{\partial \theta_m} \right] - \frac{\partial e_{11}^*}{\partial \theta_m}. \end{aligned} \quad (\text{A38})$$

Using the first-order conditions for the effort levels of the two members of group 1, that is, (A1) with  $\theta_{11} = \theta_m$  and  $\theta_{21} = \theta$ , we obtain

$$\begin{aligned} \frac{\partial \Pi_{11}^*}{\partial \theta_m} &= -\frac{\theta_m}{1+\theta_m} \frac{\partial e_{11}^*}{\partial \theta_m} + \frac{1}{1+\theta} \frac{\partial e_{21}^*}{\partial \theta_m} \\ &+ \sum_{j \neq 1} \frac{\partial p_1(\mathbf{e}_1^*, \mathbf{e}_R^*)}{\partial G_j(\mathbf{e}_j^*)} \left[ \frac{\partial G_j(\mathbf{e}_j^*)}{\partial e_{1j}^*} \frac{\partial e_{1j}^*}{\partial \theta_m} + \frac{\partial G_j(\mathbf{e}_j^*)}{\partial e_{2j}^*} \frac{\partial e_{2j}^*}{\partial \theta_m} \right]. \end{aligned} \quad (\text{A39})$$

We have

$$\frac{\partial p_1(\mathbf{e}_1^*, \mathbf{e}_R^*)}{\partial G_j(\mathbf{e}_j^*)} = -\frac{G_1(\mathbf{e}_1^*)}{\left[ G_1(\mathbf{e}_1^*) + \sum_{j \neq 1} G_j(\mathbf{e}_j^*) \right]^2} \text{ for } j \neq 1. \quad (\text{A40})$$

From (A3), we also have

$$\frac{\partial G_j(\mathbf{e}_j^*)}{\partial e_{ij}^*} = \left[ e_{ij}^{*\sigma} + e_{-ij}^{*\sigma} \right]^{\frac{1-\sigma}{\sigma}} e_{ij}^{*\sigma-1}. \quad (\text{A41})$$

Let again  $e_R^* \equiv e_{ij}^*$  for  $i = 1, 2$  and  $j \neq 1$ , we have  $G_j(\mathbf{e}_j^*) = G_R(\mathbf{e}_R^*)$  and  $\partial G_j(\mathbf{e}_j^*) / \partial e_{ij}^* = 2^{(1-\sigma)/\sigma}$  for  $i = 1, 2$  and  $j \neq 1$ , and thus, (A39) reduces to

$$\begin{aligned} \frac{\partial \Pi_{11}^*}{\partial \theta_m} &= -\frac{\theta_m}{1+\theta_m} \frac{\partial e_{11}^*}{\partial \theta_m} + \frac{1}{1+\theta} \frac{\partial e_{21}^*}{\partial \theta_m} \\ &- \frac{2^{1/\sigma}(n-1)G_1(\mathbf{e}_1^*)}{\left[ G_1(\mathbf{e}_1^*) + (n-1)G_R(\mathbf{e}_R^*) \right]^2} \frac{\partial e_R^*}{\partial \theta_m}. \end{aligned} \quad (\text{A42})$$

We now evaluate this expression at  $\theta_m = \theta$ .

Substituting (A33) and (A37) into (A42), we have

$$\frac{\partial \Pi_{11}^*}{\partial \theta_m} \Big|_{\theta_m=\theta} = \frac{1-\theta}{1+\theta} \frac{\partial e_{11}^*}{\partial \theta_m} \Big|_{\theta_m=\theta} - \frac{e^*}{(1-\sigma)(1+\theta)^2} + \frac{(n-1)^2(n-2)}{4n^5 e^*}, \quad (\text{A43})$$

because  $G_1(\mathbf{e}_1^*) = G_R(\mathbf{e}_R^*) = 2^{\frac{1}{\sigma}} e^*$  for  $\theta_m = \theta$ .

Substituting (A31) and (A34) into (A43) and setting it to 0, yields a unique value for  $\theta^*$  given by (21). One can also easily verify that  $\theta^* \geq -1$ , since this inequality reduces to  $n \geq -\sigma/(1-\sigma)$ , which holds for any  $\sigma \in (-\infty, 0) \cup (0, 1]$ .

Now, we verify condition (ii) of Proposition 3. (A42) can be rewritten as

$$\begin{aligned} \frac{\partial \Pi_{11}^*}{\partial \theta_m} &= \Lambda_1(\cdot) + \Lambda_2(\cdot) + \Lambda_3(\cdot) \text{ where} \\ \Lambda_1(\cdot) &\equiv \Lambda_1(\theta_m, \theta) = -\frac{\theta_m}{1+\theta_m} \frac{\partial e_{11}^*}{\partial \theta_m}, \\ \Lambda_2(\cdot) &\equiv \Lambda_2(\theta_m, \theta) = \frac{1}{1+\theta} \frac{\partial e_{21}^*}{\partial \theta_m}, \\ \Lambda_3(\cdot) &\equiv \Lambda_3(\theta_m, \theta) = \frac{-2^{1/\sigma}(n-1)G_1(\mathbf{e}_1^*)}{[G_1(\mathbf{e}_1^*) + (n-1)G_R(\mathbf{e}_R^*)]^2} \frac{\partial e_R^*}{\partial \theta_m}. \end{aligned} \quad (\text{A44})$$

We have

$$\frac{\partial \Lambda_1(\cdot)}{\partial \theta_m} = -\frac{1}{(1+\theta_m)^2} \frac{\partial e_{11}^*}{\partial \theta_m} - \frac{\theta_m}{1+\theta_m} \frac{\partial^2 e_{11}^*}{\partial \theta_m^2}. \quad (\text{A45})$$

Now, let evaluate this expression at  $\theta_m = \theta$ . We have

$$\left. \frac{\partial \Lambda_1(\cdot)}{\partial \theta_m} \right|_{\theta_m=\theta} = -\frac{1}{(1+\theta)^2} \left. \frac{\partial e_{11}^*}{\partial \theta_m} \right|_{\theta_m=\theta} - \frac{\theta}{1+\theta} \left. \frac{\partial^2 e_{11}^*}{\partial \theta_m^2} \right|_{\theta_m=\theta}. \quad (\text{A46})$$

We also have

$$\left. \frac{\partial \Lambda_2(\cdot)}{\partial \theta_m} \right|_{\theta_m=\theta} = \frac{1}{1+\theta} \left. \frac{\partial^2 e_{21}^*}{\partial \theta_m^2} \right|_{\theta_m=\theta}. \quad (\text{A47})$$

Using (A32), the second derivative of  $e_{21}^*$  with respect to  $\theta_m$  is given by

$$\left. \frac{\partial^2 e_{21}^*}{\partial \theta_m^2} \right|_{\theta_m=\theta} = \left( \frac{1+\theta}{1+\theta_m} \right)^{\frac{1}{1-\sigma}} \left[ -\frac{2}{(1-\sigma)(1+\theta_m)} \left. \frac{\partial e_{11}^*}{\partial \theta_m} \right|_{\theta_m=\theta} + \frac{(2-\sigma)e_{11}^*}{(1-\sigma)^2(1+\theta_m)^2} + \left. \frac{\partial^2 e_{11}^*}{\partial \theta_m^2} \right|_{\theta_m=\theta} \right]. \quad (\text{A48})$$

Substituting (A48) into (A47), and evaluating this last expression at  $\theta_m = \theta$ , we obtain

$$\left. \frac{\partial \Lambda_2(\cdot)}{\partial \theta_m} \right|_{\theta_m=\theta} = \frac{1}{1+\theta} \left[ -\frac{2}{(1-\sigma)(1+\theta)} \left. \frac{\partial e_{11}^*}{\partial \theta_m} \right|_{\theta_m=\theta} + \frac{(2-\sigma)e^*}{(1-\sigma)^2(1+\theta)^2} + \left. \frac{\partial^2 e_{11}^*}{\partial \theta_m^2} \right|_{\theta_m=\theta} \right]. \quad (\text{A49})$$

Recalling that  $e^* = (1+\theta)(n-1)/2n^2$  and using (A46) and (A49), we obtain

$$\begin{aligned} \left. \frac{\partial (\Lambda_1(\cdot) + \Lambda_2(\cdot))}{\partial \theta_m} \right|_{\theta_m=\theta} &= \frac{(2-\sigma)(n-1)}{2n^2(1-\sigma)^2(1+\theta)^2} - \frac{(3-\sigma)}{(1-\sigma)(1+\theta)^2} \left. \frac{\partial e_{11}^*}{\partial \theta_m} \right|_{\theta_m=\theta} \\ &\quad + \left( \frac{1-\theta}{1+\theta} \right) \left. \frac{\partial^2 e_{11}^*}{\partial \theta_m^2} \right|_{\theta_m=\theta}. \end{aligned} \quad (\text{A50})$$

Substituting (A31) into (A50), we have

$$\frac{\partial (\Lambda_1(\cdot) + \Lambda_2(\cdot))}{\partial \theta_m} \Big|_{\theta_m=\theta} = -\frac{(n-1)[3n^2 - 7n + 6 - \sigma(n^2 - 2n + 2)]}{4n^3(1-\sigma)(1+\theta)^2} + \left(\frac{1-\theta}{1+\theta}\right) \frac{\partial^2 e_{11}^*}{\partial \theta_m^2} \Big|_{\theta_m=\theta}. \quad (\text{A51})$$

Now, we calculate the derivative of  $\Lambda_3(\cdot)$  with respect to  $\theta_m$ . From (A44),  $\Lambda_3(\cdot)$  can be rewritten as

$$\begin{aligned} \Lambda_3(\cdot) &= -2^{1/\sigma}(n-1)p_1(\mathbf{e}_1^*, \mathbf{e}_R^*) \Psi(\mathbf{e}_1^*, \mathbf{e}_R^*) \frac{\partial e_R^*}{\partial \theta_m} \\ &\text{where} \\ \Psi(\mathbf{e}_1^*, \mathbf{e}_R^*) &= \frac{1}{G_1(\mathbf{e}_1^*) + (n-1)G_R(\mathbf{e}_R^*)}. \end{aligned} \quad (\text{A52})$$

We obtain

$$\frac{\partial \Lambda_3(\cdot)}{\partial \theta_m} = -2^{1/\sigma}(n-1) \left\{ \begin{array}{l} \frac{\partial p_1(\cdot)}{\partial \theta_m} \frac{\partial e_R^*}{\partial \theta_m} \Psi(\cdot) + p_1(\cdot) \Psi(\cdot) \frac{\partial^2 e_R^*}{\partial \theta_m^2} \\ + p_1(\cdot) \frac{\partial e_R^*}{\partial \theta_m} \left[ \begin{array}{l} \frac{\partial \Psi(\cdot)}{\partial G_1(\cdot)} \left( \frac{\partial G_1(\cdot)}{\partial e_{11}^*} \frac{\partial e_{11}^*}{\partial \theta_m} + \frac{\partial G_1(\cdot)}{\partial e_{21}^*} \frac{\partial e_{21}^*}{\partial \theta_m} \right) \\ + \frac{\partial \Psi(\cdot)}{\partial G_R(\cdot)} \frac{\partial G_R(\cdot)}{\partial e_R^*} \frac{\partial e_R^*}{\partial \theta_m} \end{array} \right] \end{array} \right\}. \quad (\text{A53})$$

We have

$$\frac{\partial \Psi(\cdot)}{\partial G_R(\cdot)} = (n-1) \frac{\partial \Psi(\cdot)}{\partial G_1(\cdot)} = -\frac{n-1}{[G_1(\cdot) + (n-1)G_R(\cdot)]^2}. \quad (\text{A54})$$

Furthermore, from condition (i) of Proposition 3, we also have  $\partial p_1(\cdot)/\partial \theta_m = \partial e_{11}^*/\partial \theta_m$ . Thus (A53) becomes

$$\frac{\partial \Lambda_3}{\partial \theta_m} = -2^{1/\sigma}(n-1) \left\{ \begin{array}{l} \frac{\partial e_{11}}{\partial \theta_m} \frac{\partial e_R^*}{\partial \theta_m} \Psi(\cdot) + p_1(\cdot) \Psi(\cdot) \frac{\partial^2 e_R^*}{\partial \theta_m^2} \\ + p_1(\cdot) \frac{\partial e_R^*}{\partial \theta_m} \frac{\partial \Psi(\cdot)}{\partial G_1(\cdot)} \left[ \begin{array}{l} \frac{\partial G_1(\cdot)}{\partial e_{11}^*} \frac{\partial e_{11}^*}{\partial \theta_m} + \frac{\partial G_1(\cdot)}{\partial e_{21}^*} \frac{\partial e_{21}^*}{\partial \theta_m} \\ + (n-1) \frac{\partial G_S(\cdot)}{\partial e_R^*} \frac{\partial e_R^*}{\partial \theta_m} \end{array} \right] \end{array} \right\}. \quad (\text{A55})$$

In a symmetric equilibrium, we have

$$\begin{aligned} p_1(\cdot) \Big|_{\theta_m=\theta} &= \frac{1}{n}; \quad \Psi(\cdot) \Big|_{\theta_m=\theta} = \frac{1}{2^{\frac{1}{\sigma}} n e^*}; \quad \frac{\partial \Psi(\cdot)}{\partial G_1(\cdot)} \Big|_{\theta_m=\theta} = -\frac{1}{[nG(e^*)]^2} = -\frac{1}{\left[2^{\frac{1}{\sigma}} n e^*\right]^2}; \\ \frac{\partial G_R(\cdot)}{\partial e_R^*} \Big|_{\theta_m=\theta} &= 2^{\frac{1}{\sigma}}; \quad \frac{\partial G_1(\cdot)}{\partial e_{11}^*} \Big|_{\theta_m=\theta} = \frac{\partial G_1(\cdot)}{\partial e_{21}^*} \Big|_{\theta_m=\theta} = 2^{\frac{1-\sigma}{\sigma}}. \end{aligned} \quad (\text{A56})$$

Consequently, we have

$$\frac{\partial \Lambda_3(\cdot)}{\partial \theta_m} \Big|_{\theta_m=\theta} = -\frac{(n-1)}{ne^*} \left\{ \begin{array}{l} \frac{\partial e_{11}^*}{\partial \theta_m} \Big|_{\theta_m=\theta} \frac{\partial e_R^*}{\partial \theta_m} \Big|_{\theta_m=\theta} + \frac{1}{n} \frac{\partial^2 e_R^*}{\partial \theta_m^2} \Big|_{\theta_m=\theta} \\ -\frac{1}{2n^2 e^*} \frac{\partial e_R^*}{\partial \theta_m} \Big|_{\theta_m=\theta} \left[ \begin{array}{l} \frac{\partial e_{11}^*}{\partial \theta_m} \Big|_{\theta_m=\theta} + \frac{\partial e_{21}^*}{\partial \theta_m} \Big|_{\theta_m=\theta} \\ +2(n-1) \frac{\partial e_R^*}{\partial \theta_m} \Big|_{\theta_m=\theta} \end{array} \right] \end{array} \right\}. \quad (\text{A57})$$

Substituting (A33) into (A57), we obtain

$$\begin{aligned} \frac{\partial \Lambda_3(\cdot)}{\partial \theta_m} \Big|_{\theta_m=\theta} &= -\frac{(n-1)}{ne^*} \frac{\partial e_R^*}{\partial \theta_m} \Big|_{\theta_m=\theta} \left\{ \begin{array}{l} \left[ 1 - \frac{1}{n^2 e^*} \right] \frac{\partial e_{11}^*}{\partial \theta_m} \Big|_{\theta_m=\theta} \\ + \frac{1}{2n^2(1-\sigma)(1+\theta)} - \frac{(n-1)}{n^2 e^*} \frac{\partial e_R^*}{\partial \theta_m} \Big|_{\theta_m=\theta} \end{array} \right\} \\ &\quad - \frac{(n-1)}{n^2 e^*} \frac{\partial^2 e_R^*}{\partial \theta_m^2} \Big|_{\theta_m=\theta}. \end{aligned} \quad (\text{A58})$$

Substituting  $e^*$  given by (A34) into (A58) yields

$$\begin{aligned} \frac{\partial \Lambda_3(\cdot)}{\partial \theta_m} \Big|_{\theta_m=\theta} &= -\frac{2n}{(1+\theta)} \frac{\partial e_R^*}{\partial \theta_m} \Big|_{\theta_m=\theta} \left\{ \begin{array}{l} \left[ \frac{(1+\theta)(n-1)-2}{(1+\theta)(n-1)} \right] \frac{\partial e_{11}^*}{\partial \theta_m} \Big|_{\theta_m=\theta} \\ + \frac{1}{2n^2(1-\sigma)(1+\theta)} - \frac{2}{(1+\theta)} \frac{\partial e_R^*}{\partial \theta_m} \Big|_{\theta_m=\theta} \end{array} \right\} \\ &\quad - \frac{2}{(1+\theta)} \frac{\partial^2 e_R^*}{\partial \theta_m^2} \Big|_{\theta_m=\theta}. \end{aligned} \quad (\text{A59})$$

Substituting (A31) and (A37) into (A59), we finally obtain after some trivial (but tedious) calculations

$$\begin{aligned} \frac{\partial \Lambda_3(\cdot)}{\partial \theta_m} \Big|_{\theta_m=\theta} &= \frac{(n-1)(n-2)}{8(1-\sigma)(1+\theta)^2 n^5} \left[ \begin{array}{l} (n^3 - 2)(1-\sigma)(1+\theta) - n^2(2-3\sigma)(1+\theta) \\ + n(1-2\sigma + \theta(3-4\sigma)) \end{array} \right] \\ &\quad - \frac{2}{(1+\theta)} \frac{\partial^2 e_R^*}{\partial \theta_m^2} \Big|_{\theta_m=\theta}. \end{aligned} \quad (\text{A60})$$

The second derivative of  $\Pi_{11}^*(\theta_m, \theta)$  with respect to  $\theta_m$  evaluated at  $\theta_m = \theta$  is given by the sum of (A51) and (A60).

Thus, to complete the proof, we need to calculate the second derivative of  $e_{11}^*$  and of  $e_R^*$  with respect to  $\theta_m$ .<sup>26</sup> Calculating the derivative of  $\partial e_{11}^*/\partial\theta_m$ , given by (A30), with respect to  $\theta_m$ , and evaluating the resulting expression at  $\theta_m = \theta$ , we can obtain

$$\frac{\partial^2 e_{11}^*}{\partial\theta_m^2} \Big|_{\theta_m=\theta} = -\frac{(n-1)[n^3 - 5n^2 + 10n - 6 - 2\sigma(2n^3 - 6n^2 + 8n - 3)]}{8(1-\sigma)(1+\theta)n^4}. \quad (\text{A61})$$

Similarly, calculating the derivative of  $\partial e_R^*/\partial\theta_m$ , given by (A36), with respect to  $\theta_m$ , and evaluating the resulting expression at  $\theta_m = \theta$ , we can obtain

$$\frac{\partial^2 e_R^*}{\partial\theta_m^2} \Big|_{\theta_m=\theta} = \frac{(n-1)[3n^2 - 10n + 6 - 2\sigma(2n^2 - 6n + 3)]}{8(1-\sigma)(1+\theta)n^4}. \quad (\text{A62})$$

Substituting (A61) into (A51) to obtain  $[\partial(\Lambda_1(\cdot) + \Lambda_2(\cdot))/\partial\theta_m] \Big|_{\theta_m=\theta}$  and (A62) into (A60) to obtain  $[\partial\Lambda_3(\cdot)/\partial\theta_m] \Big|_{\theta_m=\theta}$  and adding the two terms, we obtain  $[\partial^2\Pi_{11}^*/\partial\theta_m^2] \Big|_{\theta_m=\theta}$  (see (A44)), that is,

$$\frac{\partial^2\Pi_{11}^*}{\partial\theta_m^2} \Big|_{\theta_m=\theta} = -\frac{(n-1) \left\{ \begin{array}{l} (n-1)[6n^3 - 3n^2 - 6n + 4 - \theta(2n^3 - 7n^2 + 10n - 4)] \\ -\sigma[5n^4 - 3n^3 - 12n^2 + 12n - 4 - \theta(5n^4 - 17n^3 + 26n^2 - 16n + 4)] \end{array} \right\}}{8(1-\sigma)(1+\theta)^2n^5}. \quad (\text{A63})$$

Substituting  $\theta$  by  $\theta^*$  given by (22.1) into this expression, we finally obtain

$$\frac{\partial^2\Pi_{11}^*}{\partial\theta_m^2} \Big|_{\theta_m=\theta^*} = \frac{[n^2 - n + 2 - \sigma(n^2 - 2n + 2)] \left[ \begin{array}{l} -2(n^3 - n + 1) - 2\sigma^2(n-1)^2 \\ +\sigma(2n^3 + 3n^2 - 6n + 4) \end{array} \right]}{16(1-\sigma)^3n^5}. \quad (\text{A64})$$

The sign of  $(\partial^2\Pi_{11}^*/\partial\theta_m^2) \Big|_{\theta_m=\theta^*}$  is the same as the sign of its numerator. The first term in  $[\cdot]$  in the numerator is strictly positive for any  $\sigma < 1$ . Thus, the sign of  $(\partial^2\Pi_{11}^*/\partial\theta_m^2) \Big|_{\theta_m=\theta^*}$  is the same as the sign of the second term in  $[\cdot]$  in the numerator. Let denote  $\Lambda(n, \sigma)$  this term, i.e.  $\Lambda(n, \sigma) = -2\sigma^2(n-1)^2 + \sigma(2n^3 + 3n^2 - 6n + 4) - 2(n^3 - n + 1)$ . For  $\sigma < 0$ , we have  $\Lambda(n, \sigma) < 0$ . Now, let us consider that  $\sigma > 0$ . The quadratic equation  $\Lambda(n, \sigma) = 0$  has two positive roots in  $\sigma$ , that is

$$\begin{aligned} \sigma_1 &= \frac{2n^3 + 3n^2 - 6n + 4 - n\sqrt{4n^3(n-1) + 17n^2 - 20n + 12}}{4(n-1)^2}, \\ \sigma_2 &= \frac{2n^3 + 3n^2 - 6n + 4 + n\sqrt{4n^3(n-1) + 17n^2 - 20n + 12}}{4(n-1)^2}. \end{aligned} \quad (\text{A65})$$

It can be verified that  $\sigma_2$  is strictly larger than 1 for any  $n \geq 2$ . Furthermore,  $\Lambda(n, \sigma)$  is concave in  $\sigma$  and hence property (ii) of Proposition 3 is verified if and only if  $\sigma < \sigma_1 \equiv \bar{\sigma}$ .

<sup>26</sup>Since the mathematical expressions are very long, we only write the values of these expressions evaluated at  $\theta_m = \theta$ . We also used the Mathematical software to verify the computation of these derivatives.

## A.5. Proof of Proposition 9

Part (i): The derivative of  $\theta^*$  with respect to  $n$ , given by (25), is clearly positive for  $\sigma > 0$ . Furthermore, the denominator of  $\theta^*$  given by (22.1) is strictly positive for any  $\sigma > 0$ . Thus, solving the equation  $\theta^* = 0$  reduces to solving  $(n^2 - 2)(1 - \sigma) - n = 0$ , which admits a negative root and a positive positive root, this last being given by  $\hat{n}$  in part (i) of Proposition 9. Since the above polynomial is convex in  $n$ ,  $\theta^*$  is negative (positive) for  $n \leq \lfloor \hat{n} \rfloor$  ( $n \geq \lceil \hat{n} \rceil$ ).

Part (ii): The sign of  $\partial\theta^*/\partial n$  is the same as the sign of the polynomial in  $n$  in [.] in the numerator of (25), which can be rewritten as  $\Gamma(n, \sigma) = n^2\sigma + 4n(1 - \sigma) - 2(1 - \sigma)$ . The quadratic equation  $\Gamma(n, \sigma) = 0$  has two positive roots in  $n$ , namely

$$\underline{n} = \frac{-2(1 - \sigma) + \sqrt{2(1 - \sigma)(2 - \sigma)}}{\sigma} \quad \text{and} \quad \bar{n} = -\frac{2(1 - \sigma) + \sqrt{2(1 - \sigma)(2 - \sigma)}}{\sigma}. \quad (\text{A66})$$

One can easily verify that  $\bar{n} > 2 > \underline{n} > 0$  for any  $\sigma \in (-\infty, 0)$ . Furthermore, the second derivative of  $\Gamma(n, \sigma)$  with respect to  $n$  is given by  $2\sigma$ . Thus, when  $\sigma < 0$ ,  $\Gamma(n, \sigma)$  is concave in  $n$  (for  $n \geq 2$ ), and is positive (negative) for  $n \leq \lfloor \bar{n} \rfloor$  ( $n \geq \lceil \bar{n} \rceil$ ). It follows that  $\theta^*$  is increasing (decreasing) in  $n$  for  $n \leq \lfloor \bar{n} \rfloor$  ( $n \geq \lceil \bar{n} \rceil$ ).

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