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Abstract. This paper analyzes how pro-social motivations shape the relationship between incentives and inequality. I consider a principal who offers individual rewards to a group of agents to induce them to exert effort and to coordinate at least-cost. The agents value the payoffs of the other agents, and they are averse to inequality. My analysis highlights that pro-social motivations have an a priori ambiguous effect on inequality in the reward distribution. Despite this initial ambiguity, I show that the rewards are more unequal and lower when the agents have pro-social preferences. The model delivers empirical implications for intervention programs supporting the adoption of new health or agricultural technologies.

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1 Introduction

Pro-social motivations are at the center of many important economic problems. For example, people adopt protection technologies for the Covid-19 virus because they care about the welfare of others (Kar Keung Cheng, 2020; Barzilay et al., 2020). While these motivations often determine how individuals respond to incentives in practice (Bandiera et al., 2005), little is known on how they should theoretically affect optimal incentives and the resulting distribution of wealth.

In this paper, I examine how pro-social preferences shape the relationship between incentives and inequality. My analysis considers the optimal incentives that will induce a group of agents with pro-social preferences to make complementary efforts (i.e., positive peer effects). Pro-social motivations are conceptually different from peer effects because pro-social motivations depend on both the actions and the rewards of others. In contrast, peer effects depend only on the actions of others. Here I consider a principal who offers individual rewards, or bilateral contracts, to a group of agents to induce them to coordinate and to exert effort at least cost. The optimal reward scheme is called a least-cost unique implementation scheme. An agent's social utility is a linear combination of their private utility, which depends only on their own material payoff, and a function of the other agents' individual material payoffs. The pro-social function is assumed to increase in each of the other agents' material payoffs. It is also assumed to be concave and supermodular, meaning the agents are averse to inequality. These assumptions encompass, among others, the constant elasticity of substitution (CES) function.

I perform my analysis in three steps. First, I study specific incentive schemes (global ranking schemes) wherein agents are ranked and each agent is indifferent between exerting and not exerting effort while the preceding agents exert effort and the remaining agents do not. I show that within these schemes, agents with pro-social preferences get lower and more unequal rewards. Second, I show that any least-cost unique implementation scheme is necessarily a global ranking scheme. This implies that a least-cost unique implementation scheme can never lead to an increase in the rewards of an agent nor to less inequality in the reward distribution. Third, I show that any global ranking scheme is a unique implementation scheme if the agents are not too averse to inequality, and thus that the solution exists in this case.

The fact that pro-social preferences lead to more inequality in the reward distribution is not straightforward. If the agents have no pro-social preferences, as more agents exert effort, fewer rewards are needed to induce the remaining agents to exert effort. In this case of the least-cost implementation scheme, identical agents get different rewards. Pro-social motivations may lead to more or less inequality in the reward distribution. Indeed, pro-social motivations have two opposing effects on inequality: an extensive margin effect and an intensive margin effect. With the extensive marginal effect, each additional agent who decides to make effort generates a positive externality for the previous agent. Since the additional agent values this externality, the principal can give them fewer rewards. Therefore, the extensive margin effect tends to increase inequality in the reward distribution. However, with the intensive margin effect, each additional agent who decides to make effort generates positive externalities for all the previous agents, who already benefit from the positive externalities they generate for each other. Since the agents are averse to inequality, the additional agent's marginal contribution to the pro-social motivation is lower when more agents make effort. Thus, when more agents exert effort, the principal must increase the rewards given to the remaining agents to induce them to also exert effort. This shows how the intensive margin effect tends to decrease inequality in the reward distribution. Moreover, the extensive margin effect applies to the agent who obtains the lowest material payoff. The intensive margin effect applies to the other agents who exert effort and who receive higher rewards. Since the agents are averse to inequality, the extensive margin effect is always stronger than the intensive margin effect. Finally, inequality in the reward distribution is higher when the agents have pro-social preferences.

This paper introduces the analysis of pro-social preferences in the literature on contracts with externalities (Segal, 1999, 2003; Bernstein and Winter, 2012). Here I consider a situation where these agents make binary decisions, and their actions are strategic complements (as in Winter, 2004). The literature typically characterizes optimal incentive schemes as generating heterogeneity among symmetric agents. My analysis reveals that pro-social preferences, which play an important role in current major issues, lead to an increase in inequality, while this was a priori unclear.[1]

The remainder of the paper is organized as follows: the model is introduced in Section 2, the main results are delivered in Section 3, a discussion on heterogeneity is provided in Section 4 and conclusions are summarized in Section 5. All proofs are provided in the appendix.

2 The Model

A principal offers individual bilateral contracts to several agents in an environment characterized by positive externalities between the agents. First, the principal proposes a publicly observable incentive scheme to a set of agents. Second, the agents observe the principal's proposition and simultaneously decide whether to exert effort at their individual price.

An agent who decides to exert effort generates a positive externality $w \geq 0$ to the other agents that exert effort and no externality to the agents who do not exert effort. An agent who decides not to exert effort receives an outside option c. The principal aims to induce effort from all agents at the lowest possible cost. To reach this goal, the principal proposes an incentive scheme $v = (v_1, v_2, ..., v_n) \in R^n$ to the agents in the set of agents N, with $i = 1, 2, ..., n$, to incentivize them to exert effort. The reward is conditional on the agent exerting effort: agent i receives v_i from the principal if he exerts effort and 0 otherwise. The incentive scheme v is designed such that each agent receives a unique offer v_i, $i \in N$, meaning the principal is able to use individualized rewards. The vector of agents' decisions is $x = (x_1, ..., x_n) \in \{0, 1\}^n$, where $x_i = 1$ means that agent i chooses to exert effort and $x_i = 0$ means that that agent decides not to exert effort. I will denote $\boldsymbol{y}_{y_i=a}$ vector $\boldsymbol{y} = (y_1, ..., y_n) \in R^n$ when its ith component is set to a. I will denote $\boldsymbol{y}_{-i}$ vector $\boldsymbol{y}$ when the ith component is removed. $\boldsymbol{y}^k = (y, ..., y)$ denotes the vector of dimension k with each component set to $y \in R$. Finally, $\boldsymbol{t} = (\boldsymbol{z}, \boldsymbol{y})$ denotes the vector whose first elements are those of vector $\boldsymbol{z}$ followed by those of vector $\boldsymbol{y}$.

[1]See Halac et al. (2020) for an analysis of discriminatory incentives with heterogeneous agents and their implications in terms of inequality. Another related paper is Gueye et al. (2021). They consider inequality aversion à la Fehr and Schmidt (1999) and show that inequality can increase or decrease when the degree of inequality aversion increases. The present paper differs substantially since, by definition of pro-social preferences, the utility of an agent cannot decrease when the payoff of another agent increases, even if the agents dislike inequality.

Agent i's material payoff is:

$$\pi_i(\boldsymbol{x}, v_i) = x_i \left(v_i + w \sum_{j \neq i} x_j \right) + (1 - x_i)c. \tag{1}$$

I assume that the agents have social preferences and that they give weight to their own payoff (a "selfish" motive) and the payoffs of all other agents (a pro-social motive). Formally, the social utility of agent i is:

$$U_i(\boldsymbol{x}, \boldsymbol{v}) = u_i(\pi_i(\boldsymbol{x}, v_i)) + \theta W_i(\boldsymbol{\pi_{-i}}(\boldsymbol{x}, \boldsymbol{v_{-i}})), \tag{2}$$

where $\boldsymbol{\pi_{-i}}(\boldsymbol{x}, v_{-i})$ is the vector of the agents' payoffs but i, u_i is the selfish part of the utility function and W_i is the agent's pro-social function. Parameter $\theta \geq 0$ indicates the strength of the pro-social motivations for the agents. When $\theta = 0$, the agents are purely selfish and when $\theta > 0$ the agents have pro-social preferences. The individual utility function u_i is strictly increasing and the pro-social function W_i is increasing in the payoff of each of the other agents.

In the rest of the paper, I make the two following assumptions:

Assumption C-SupM: The pro-social functions W_i are concave and supermodular.

Assumption S: The agents are symmetric, that is $U_i \equiv U$ and then $u_i \equiv u$ and $W_i \equiv W$ for all i.

Notice that Assumption S implies that the level of the pro-social component W_i is the same for any permutation of the material payoff of the other agents (Anonymity).

The two assumptions hold for the following example of pro-social function:

Example 1: *If u is concave, then the constant elasticity of subsitution (CES) pro-social function $W_i(\boldsymbol{\pi_{-i}}) = \left[\sum_{k \neq i} u(\pi_k)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}$, where $s \in]0, +\infty[$ and $s \neq 1$, which is increasing in its arguments, satisfies assumptions S and C-SupM.*

The main results of this paper are that pro-social motivations lead to a decrease in the rewards of the agents and cannot lead to a decrease in inequality. These results hold for this CES specification of the pro-social function.

In the following, I study the least-cost scheme that implements effort by all agents as a unique Nash equilibrium (i.e., the least-cost unique implementation scheme). I require schemes to induce a unique equilibrium only when the individual rewards are increased by a positive amount.[2] More precisely, I characterize the reward vector v^* that solves the following optimization problem:

$$Min_{\boldsymbol{v} \in \Re^n} \sum_{j \in N} v_j \tag{3}$$

s.t. for all $\epsilon > 0$, full effort ($\boldsymbol{x} = \mathbf{1}^n$) is a Nash equilibrium:

$$U_i(\mathbf{1}^n, \boldsymbol{v} + \boldsymbol{\epsilon^n}) > U_i(\mathbf{1}^n_{x_i=0}, \boldsymbol{v} + \boldsymbol{\epsilon^n}), \tag{NE}$$

[2]This solution concept corresponds to that of Winter (2004) and Halac et al. (2020). Halac et al. (2021) use a similar concept in a Bayesian game.

for all $i \in N$, and, there is no other Nash equilibrium, that is, for all $\boldsymbol{x} \neq \mathbf{1}^n$, $\exists i \in N$ such that $x_i = a$, $a \in \{0,1\}$ and:

$$U_i(\boldsymbol{x}_{x_i=1-a}, \boldsymbol{v} + \boldsymbol{\epsilon^n}) > U_i(\boldsymbol{x}, \boldsymbol{v} + \boldsymbol{\epsilon^n}). \tag{UC}$$

The set of constraints (NE) ensures that each agent has an incentive to exert effort when all other agents exert effort, and the set of constraints (UC) ensures that for each for each of the other outcomes, at least one agent has an incentive to deviate.

3 Optimal Incentive Scheme

In this section, I present the results of the analysis of the problem of the principal.

3.1 Ranking Agents

It will be useful for the analysis to define when I consider a class of incentive schemes to be a ranking scheme for an agent:

Definition 1: *An incentive scheme $\boldsymbol{v}$ is a* ***ranking scheme for an agent*** *if the agents are ranked in a given order and agent i is indifferent to exerting or not exerting effort when the previous agents also exert effort and the subsequent agents do not.*

Formally, if the agents are ranked from 1 to n (without loss of generality), and $\boldsymbol{v}$ is a ranking scheme for agent i, we must have:

$$u_i(c) - u_i(v_i + (i-1)w) = \theta \left[W_i(\boldsymbol{\pi_{-i}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-i}})) - W_i(\boldsymbol{\pi_{-i}}((\mathbf{1}^{i-1}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-i}}))\right] \tag{4}$$

'

Proposition 1: *If $\boldsymbol{v}$ is a ranking scheme for agent i and all previous agents exert effort while the subsequent agents do not, then the material payoff of this agent is lower than the opportunity cost, $v_i + (i-1)w \leq c$.*

The intuition of this result is clear since when an agent exerts effort, he generates positive externalities for other agents who also exert effort, which increases the level of his pro-social function. It is thus not necessary to compensate the full opportunity cost to induce this agent to exert effort.

I now show a less intuitive result:

Proposition 2: *If assumptions S and C-SupM hold and $\boldsymbol{v}$ is a ranking scheme for the i first agents according to a common ranking (1 to n without loss of generality), then $v_{k-1} \geq v_k + w$ for all $2 \leq k \leq i$.*

Therefore, agents ranked earlier obtain a higher material payoff. This inequality is binding when the agents have no pro-social motivations and strict when they have pro-social motivations. I explain the intuition in both cases below.

If the agents have no pro-social preferences ($\theta = 0$), the first agent must be indifferent between exerting and not exerting effort when no other agents exert effort. In this case, the agent does not benefit from positive externalities. The second agent has to be indifferent between exerting and not exerting effort when the first agent also exerts effort. The second agent thus benefits from a positive externality and has a greater incentive to exert effort. The second agent thus obtains a lower reward than the first agent. This reasoning holds for any two subsequent agents.

Now, consider the case where the agents have pro-social preferences ($\theta > 0$). The difference between the utility levels agent $i+1$ and $i+2$ obtain in the situation where the agents ranked before them exert effort while the agents that are ranked after them do not is such that:

$$\begin{aligned}\frac{1}{\theta}&\left[u(v_{i+1}+iw)-u(v_{i+2}+(i+1)w)\right]\\ &=W(\boldsymbol{\pi}_{-(i+2)}((\mathbf{1}^{i+2},\mathbf{0}^{n-i-2}),\boldsymbol{v}_{-(i+2)}))-W(\boldsymbol{\pi}_{-(i+2)}((\mathbf{1}^{i+1},\mathbf{0}^{n-i-1}),\boldsymbol{v}_{-(i+2)}))\\ &\quad-\left[W(\boldsymbol{\pi}_{-(i+1)}((\mathbf{1}^{i+1},\mathbf{0}^{n-i-1}),\boldsymbol{v}_{-(i+1)}))-W(\boldsymbol{\pi}_{-(i+1)}((\mathbf{1}^{i},\mathbf{0}^{n-i}),\boldsymbol{v}_{-(i+1)}))\right].\end{aligned} \tag{5}$$

The right side of condition (5) captures the difference between the increase of the pro-social component that agent $i+2$ and $i+1$ obtain when they exert effort. This difference can be written more explicitly as follows:

$$\begin{aligned}&W(v_1+(i+1)w,v_2+(i+1)w,...,v_{i+1}+(i+1)w,\boldsymbol{c}^{n-i-2})-W(v_1+iw,v_2+iw,...,v_{i+1}+iw,\boldsymbol{c}^{n-i-2})\\ &\quad-\left[W(v_1+iw,...,v_i+iw,\boldsymbol{c}^{n-i-1})-W(v_1+(i-1)w,...,v_i+(i-1)w,\boldsymbol{c}^{n-i-1})\right].\end{aligned} \tag{6}$$

The sign of this difference is a priori ambiguous. There are two diverging effects: a positive effect at the extensive margin and a negative effect at the intensive margin. First, when agent $i+2$ decides to exert effort, he generates a positive externality for an additional agent (agent $i+1$) compared to when agent $i+1$ decides to exert effort. This extensive margin effect suggests that the difference should be positive. But when $i+1$ agents exert effort, the material payoff of the i first agents is greater than when the i first agents only exert effort. Since W is concave, the increase in utility for the i first agents is greater when both the i first agents and agent $i+1$ decide to exert effort than when the $i+1$ first agents and agent $i+2$ decide to exert effort. This second effect is an intensive margin effect, and it suggests the difference should be negative.

Let me illustrate this intuition using the utilitarian example, where $W_i(\boldsymbol{\pi_{-i}}) \equiv \sum_{k\neq i} u(\pi_k)$. In this case, the two effects can be easily separated. Notice that since W must be concave, u must also be concave. The proof works by induction. First notice that $v_1 = c$, and by Proposition 1, the result holds for $i = 1$ ($v_1 \geq v_2 + w$). Assume that the result of Proposition 2 holds for all $k \leq i+1$. Using the utilitarian specification, condition (6) can be written as follows:

$$\begin{aligned}&\underbrace{u(v_{i+1}+(i+1)w)-u(v_{i+1}+iw)}_{\text{Extensive margin (+)}}\\ &\qquad+\underbrace{\sum_{1\leq k\leq i}\left([u(v_k+(i+1)w)-u(v_k+iw)]-[u(v_k+iw)-u(v_k+(i-1)w)]\right)}_{\text{Intensive margin (-)}}\end{aligned} \tag{7}$$

Using (7) and concavity of u I can conclude that the extensive margin is larger than the intensive margin. This can be easily shown by rewriting condition (7) as follows:

$$
\begin{aligned}
&u(v_1+(i+1)w)-u(v_1+iw)\\
&\quad+\sum_{2\le k\le i+1}\left([u(v_k+(i+1)w)-u(v_k+iw)]-[u(v_{k-1}+iw)-u(v_{k-1}+(i-1)w)]\right)\ge 0 \quad (8)
\end{aligned}
$$

The condition above proves that the result of Proposition 2 holds in the utilitarian case with concave utility u. The extensive margin applies to the agent with the lowest material payoff, while the intensive margin applies to the agents who also exert effort and whose material payoffs are higher. The concavity of W implies that the extensive margin is larger than the intensive margin.

Proposition 2 can be applied to the CES function of Example 1 and more general functions. As in the utilitarian case, the general proof uses the symmetry assumption (where the payoff of two agents can be permuted without affecting the level of W) and the concavity of W. Supermodularity enables one to prove the result in the general case.

3.2 Optimal Incentive Scheme and Inequality

I first prove a preliminary result that will be used to analyze the optimal scheme:

Lemma 1: *If assumption S holds and $\widetilde{\boldsymbol{v}}$ is such that each agent $k \le i$ prefers to exert effort when the previous agents (according to a common ranking,* 1 *to n without loss of generality) also exert effort and the remaining agents do not, then each agent k also prefers to make an effort when j agents, with $k \le j \le i$, exert effort.*

The intuition of this result is simple if the agents have no pro-social preferences. As more agents exert effort, the agents who make effort benefit from more positive externalities and thus they are more likely to exert effort.

If the agents have pro-social preferences, the result is less straightforward. The monotonicity result of Proposition 2 is needed to prove the result. The proof works as follows: if $\widetilde{\boldsymbol{v}}$ is a ranking scheme for the i first agents according to a common ranking, then their rewards are characterized by condition (4). Consider the situation where the i first agents exert effort. If i or k decide not to exert effort, the material payoff of each agent ranked before i is reduced by w. Moreover, if i does not exert effort, the payoff of k is reduced by w and if k does not exert effort, the payoff of i is reduced by w. Given that $v_k > v_i$ and W is concave, the pro-social component of agent k's utility decreases more when k decides not to exert effort than the pro-social component of agent i's utility when i decides not to exert effort. Thus, agent k has fewer incentives to deviate than agent i. Since agent i is indifferent between exerting and not exerting effort, agent k strictly prefers to exert effort.

It is helpful to define a class of incentive schemes, or global ranking schemes, to characterize the least-cost unique implementation scheme:

Definition 2: *An incentive scheme $\boldsymbol{v}$ is a* ***global ranking scheme*** *if it is a ranking scheme for all the agents according to a common ranking.*

Notice that this scheme is unique for a given ranking. Each agent is indifferent between exerting and not exerting effort when the agents ranked before him also exert effort, and the remaining agents do not. This definition is used to provide the necessary conditions for an incentive scheme to be optimal:

Theorem 1: *Assuming that assumptions S and C-SupM hold, any least-cost unique implementation scheme is a global ranking scheme.*

This Theorem proves that the optimal scheme is necessarily a global ranking scheme. Notice that the global ranking scheme is unique, up to a reordering of the (ex-ante identical) agents. Using the results from Propositions 1 and 2, I conclude that individual rewards decrease and inequality in the reward distribution increases when the agents have pro-social motivations.

To prove sufficiency, I need an additional assumption:

Assumption BIA: *An utility function $U \equiv u + \theta W$ exhibits* ***bounded inequality aversion*** *over I if, for all $i \in N$, $\boldsymbol{\pi} \in I \subset R^n$ and $\delta > 0$*

$$\theta\left(\left[W(\boldsymbol{\pi_{-i}} + \boldsymbol{w^{n-1}}) - W(\boldsymbol{\pi_{-i}})\right] - \left[W(\boldsymbol{\pi_{-i}} + \boldsymbol{w^{n-1}} + \boldsymbol{\delta^{n-1}}) - W(\boldsymbol{\pi_{-i}} + \boldsymbol{\delta^{n-1}})\right]\right) < u(\pi_i + w + \delta) - u(\pi_i + w)$$

Let me illustrate this assumption with the two examples:

Example 2: *The* ***quasi-maximin*** *utility function, which is such that $u(\pi) \equiv \pi$ and $W(\boldsymbol{\pi}) \equiv \gamma min\{\{\pi_j\}_{j\neq i}\} + (1-\gamma)\sum_{j\neq i}\pi_j$, $\gamma \in [0,1]$ satisfies assumption BIA.*

This utility function introduced in Charness and Rabin (2002) specifies that pro-social motivations such as altruism -captured by the sum of all the agents' payoffs- and inequality aversion -captured by the Rawlsian component- are additively separable. Assumption BIA holds for this example.

Example 3: *The* ***quadratic utilitarian*** *utility function, which is such that $u(\pi) = a\pi_i - b(\pi_i)^2$, where $\pi_i \in [0, a/2b]$ and $W(\boldsymbol{\pi}) \equiv \sum_{j\neq i} u(\pi_j)$, $\gamma \in [0,1]$ satisfies assumption BIA if and only if $b < \frac{a}{2\theta(n-1)w}$.*

This second example illustrates that, for assumption BIA to hold, the agents cannot be too averse to inequality, meaning the utility function cannot be too concave.

Theorem 2: *Assuming that assumptions S, C-SupM, and BIA hold, any global ranking scheme is a least-cost unique implementation scheme.*

Theorem 2 proves sufficiency under an additional assumption. Namely, if the agents are not too averse to inequality, then full effort is a unique Nash equilibrium under any global ranking scheme. Suppose assumption BIA does not hold, and the agents are extremely inequality averse. An agent may prefer not to exert effort when the preceding agents exert effort and the remaining agents do not if the rewards of each agent are increased by a positive amount. Indeed, an increase in all the agents' rewards increases the marginal effect of exerting effort on the selfish part of the utility function and decreases the marginal effect of exerting effort on the pro-social part of the utility function. Thus, full effort is a unique Nash Equilibrium under a global ranking scheme if the latter effect is larger than the former for each agent when the preceding agents exert effort while the remaining agents do not.

4 Discussion

Up to now, I have focused on the case identical agents and homogenous externalities. One may wonder whether the main results of the paper hold in a situation where the agents have heterogeneous preferences and generate heterogeneous externalities.

Let me analyze the role of heterogeneity using a simple form of pro-social preference. Assume that the agents have heterogeneous preferences for the payoffs of the other agents and that the pro-social function of an agent is a weighted form of the other agents' payoffs:

$$W_i(\boldsymbol{x}) = \sum_{j \neq i} \gamma_{ij} \pi_j, \tag{9}$$

where $\gamma_{ij} \geq 0$ represents the intensity of the preference of agent i for the payoff of agent j.

Also assume that externalities are heterogeneous: when agents i and j make effort, agent i benefits from a positive externality from agent j denoted $w_{ij} \geq 0$.

To keep things simple, also assume that the selfish part of the utility is such that $u_i(\pi_i) \equiv \pi_i$. Using all the assumptions above, the utility of agent i can be written as:

$$U_i(\boldsymbol{x}, \boldsymbol{v}) = \pi_i + \theta \sum_{j \neq i} \gamma_{ij} \pi_j, \tag{10}$$

where

$$\pi_i = x_i \left(v_i + \sum_{j \neq i} x_j w_{ij} \right) + (1 - x_i) c. \tag{11}$$

Thanks to the linearity of the utility function, I can derive the optimal reward scheme using similar proofs as in Bernstein and Winter (2012). Indeed, if each agent i_j with $j < k$ make effort, then agent k also prefers to make effort if:

$$v_{i_k} + \sum_{j<k} w_{i_k i_j} + \theta \sum_{j<k} \gamma_{i_k i_j} w_{i_j i_k} > c. \tag{12}$$

Thus, the least cost unique implementation scheme is such that the agents are ranked from i_1 to i_n and the optimal reward of agent i_k is given by:

$$v^*_{i_k} = c - \sum_{j<k} (w_{i_k i_j} + \theta \gamma_{i_k i_j} w_{i_j i_k}), \tag{13}$$

Hence, the agents still obtain lower rewards when they have pro-social preferences ($\theta > 0$) than when they do not ($\theta = 0$).

The optimal ranking depends on the virtual popularity tournament described in section A in Bernstein and Winter (2012). In the context of the present model, agent j beats agent k if:

$$(1-\theta\gamma_{i_j i_k})w_{i_k i_j} < (1-\theta\gamma_{i_k i_j})w_{i_j i_k} \tag{14}$$

This result is interesting since it implies that pro-social preferences affect the optimal ranking of the agents. Indeed, one may have $w_{i_k i_j} > w_{i_j i_k}$ and $(1-\theta\gamma_{i_j i_k})w_{i_k i_j} < (1-\theta\gamma_{i_k i_j})w_{i_j i_k}$. This occurs for instance when agent k receives large externalities from agent j and agent j gives a strong weight to the payoff of agent k.

How the introduction of pro-social preferences ($\theta > 0$) affect inequality is not straightforward. Let me first consider the spread of the rewards distribution:

$$\max_{k\in N} v_{i_k} - \min_{k\in N} v_{i_k} = \sum_{j<l}(w_{i_l i_j} + \theta\gamma_{i_l i_j} w_{i_j i_l}), \tag{15}$$

where l is such that $\min_{k\in N} v_{i_k} = v_l$. Notice that we have $l > 1$.

The spread is clearly larger when the agents have pro-social preferences. Thus, in this sense, pro-social preferences also lead to more inequality when one introduces heterogeneity in the model.

It is however important to notice that the results of this section are limited to the case where the utility is linear in the actions of the players. They cannot be easily extended to the more general case of non linear preferences (that encompasses inequality aversion) considered in the rest of the paper.

5 Conclusion

In this paper, I examine a situation where a principal designs a least-cost unique implementation scheme by offering a group of agents with pro-social preferences rewards to exert effort. I show that the least-cost unique implementation scheme is necessarily a global ranking scheme. Additionally, I show that the agents get lower and more unequal rewards when they have pro-social preferences. The least-cost unique implementation scheme exists if the agents are not too averse to inequality. These results have practical implications for implementing schemes to induce agents to adopt new technology. When the agents have pro-social preferences, the optimal scheme must be more unequal and its cost is reduced. This is especially relevant when relatives or peers have to decide whether to adopt new technologies such as health (Kremer and Miguel, 2007)[3] or agricultural technologies (Foster and Rosenzweig, 1995).[4]

[3]See also Oster and Thornton (2012); Dupas (2014); Tarozzi et al. (2014); Adhvaryu (2014)

[4]See also Munshi (2004); Bandiera and Rasul (2006); Conley and Udry (2010); Carter et al. (2021).

Appendix

Proof of Proposition 2:

Assume that $\boldsymbol{v}$ is a ranking scheme for the first i agents. Using (4) for $i=1$, we have

$$u(c) - u(v_1) = \theta \left[W(\boldsymbol{c}^{n-1}) - W(\boldsymbol{c}^{n-1}) \right] = 0, \tag{16}$$

and then $v_1 = c$.

Now I use induction to show that $v_k + w \leq v_{k-1}$ for all $k \leq i$. Assume that this inequality holds for all $k \leq j+1$, where $j+2 \leq i$. Using condition (4) for agents $j+1$ and $j+2$, we have:

$$\begin{aligned} & u(v_{j+1} + jw) - u(v_{j+2} + (j+1)w) \\ & \quad = \theta \left[W(\boldsymbol{\pi}_{-(\boldsymbol{j}+\boldsymbol{2})}((\boldsymbol{1}^{j+2}, \boldsymbol{0}^{n-j-2}), \boldsymbol{v}_{-(\boldsymbol{j}+\boldsymbol{2})})) - W(\boldsymbol{\pi}_{-(\boldsymbol{j}+\boldsymbol{2})}((\boldsymbol{1}^{j+1}, \boldsymbol{0}^{n-j-1}), \boldsymbol{v}_{-(\boldsymbol{j}+\boldsymbol{2})})) \right] \\ & \qquad - \theta \left[W(\boldsymbol{\pi}_{-(\boldsymbol{j}+\boldsymbol{1})}((\boldsymbol{1}^{j+1}, \boldsymbol{0}^{n-j-1}), \boldsymbol{v}_{-(\boldsymbol{j}+\boldsymbol{1})})) - W(\boldsymbol{\pi}_{-(\boldsymbol{j}+\boldsymbol{1})}((\boldsymbol{1}^{j}, \boldsymbol{0}^{n-j}), \boldsymbol{v}_{-(\boldsymbol{j}+\boldsymbol{1})})) \right]. \end{aligned} \tag{17}$$

It is sufficient to show that the right hand side of condition (17) is positive. Let $\boldsymbol{y}, \boldsymbol{\Delta}, \boldsymbol{\delta} \in R^{n-1}$. If $\delta_k \leq \Delta_k$ for $1 \leq k \leq n-1$, concavity of W implies $W(\boldsymbol{y}+\boldsymbol{\Delta}) + W(\boldsymbol{y}-\boldsymbol{\Delta}) \leq W(\boldsymbol{y}+\boldsymbol{\delta}) + W(\boldsymbol{y}-\boldsymbol{\delta})$ (Rothschild and Stiglitz, 1970). Letting $y_k = \frac{v_k + v_{k+1}}{2} + jw$ for $1 \leq k \leq j$ and $y_k = 0$ for $j+1 \leq k \leq n-1$, $\Delta_k = \left| \frac{v_k - v_{k+1}}{2} - w \right|$ for $1 \leq k \leq j$ and $\Delta_k = c$ for $j+1 \leq k \leq n-1$, and $\delta_k = \frac{v_k - v_{k+1}}{2}$ for $1 \leq k \leq j$ and $\Delta_k = c$ for $j+1 \leq k \leq n-1$, we must have:

$$\begin{aligned} & W(v_2 + (j+1)w, ..., v_{j+1} + (j+1)w, \boldsymbol{c}^{n-j-1}) + W(v_1 + (j-1)w, ..., v_j + (j-1)w, \boldsymbol{c}^{n-j-1}) \\ & \qquad \geq W(v_1 + jw, ..., v_j + jw, \boldsymbol{c}^{n-j-1}) + W(v_2 + jw, ..., v_{j+1} + jw, \boldsymbol{c}^{n-j-1}). \end{aligned} \tag{18}$$

Since the agents are symmetric, we can permute the material payoffs without affecting the level of W. Hence, I obtain:

$$\begin{aligned} & W(c, v_2 + (j+1)w, ..., v_{j+1} + (j+1)w, \boldsymbol{c}^{n-j-2}) - W(c, v_2 + jw, ..., v_{j+1} + jw, \boldsymbol{c}^{n-j-2}) \\ & \qquad \geq W(v_1 + jw, ..., v_j + jw, \boldsymbol{c}^{n-j-1}) - W(v_1 + (j-1)w, ..., v_j + (j-1)w, \boldsymbol{c}^{n-j-1}). \end{aligned} \tag{19}$$

Moreover, using supermodularity of W and $v_1 = c$, we have

$$\begin{aligned} & W(v_1 + jw, v_2 + (j+1)w, ..., v_{j+1} + (j+1)w, \boldsymbol{c}^{n-j-2}) + W(c, v_2 + jw, ..., v_{j+1} + jw, \boldsymbol{c}^{n-j-2}) \\ & \geq W(c, v_2 + (j+1)w, ..., v_{j+1} + (j+1)w, \boldsymbol{c}^{n-j-2}) + W(v_1 + jw, v_2 + jw, ..., v_{j+1} + jw, \boldsymbol{c}^{n-j-2}). \end{aligned} \tag{20}$$

Combining (19) and (20), we find:

$$\begin{aligned} & W(v_1 + jw, v_2 + (j+1)w, ..., v_{j+1} + (j+1)w, \boldsymbol{c}^{n-j-2}) - W(v_1 + jw, v_2 + jw, ..., v_{j+1} + jw, \boldsymbol{c}^{n-j-2}) \\ & \qquad \geq W(v_1 + jw, ..., v_j + jw, \boldsymbol{c}^{n-j-1}) - W(v_1 + (j-1)w, ..., v_j + (j-1)w, \boldsymbol{c}^{n-j-1}). \end{aligned} \tag{21}$$

Since W is an increasing function, condition (21) implies that the right hand side in (17) is indeed positive.$\square$

Proof of Lemma 1: Assume that Assumption S holds. Assume that agent i prefers to make effort when the previous agents also exert effort while the remaining do not (assuming they are ranked from 1 to n, without loss of generality). Hence:

$$u(v_i + (i-1)w) + \theta W(\boldsymbol{\pi_{-i}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-i}})) \geq u(c) + \theta W(\boldsymbol{\pi_{-i}}((\mathbf{1}^{i-1}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-i}})) \quad (22)$$

Assume that agent $k < i$ weakly prefers not to exert effort when each agent $j \leq i, j \neq k$ exerts effort while the remaining agents do not. Hence, we must have

$$u(c) + \theta W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i_{x_k=0}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-k}})) \geq u(v_k + (i-1)w) + \theta W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-k}})) \quad (23)$$

Using (22), I obtain:

$$\begin{aligned} &u(v_i + (i-1)w) + \theta\left[W(\boldsymbol{\pi_{-i}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-i}})) - W(\boldsymbol{\pi_{-i}}((\mathbf{1}^{i-1}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-i}}))\right] \\ &\quad \geq u(v_k + (i-1)w) + \theta\left[W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-k}})) - W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i_{x_k=0}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-k}}))\right] \quad (24) \end{aligned}$$

which is equivalent to

$$\begin{aligned} &\theta\left[W(\boldsymbol{\pi_{-i}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-i}})) - W(\boldsymbol{\pi_{-i}}((\mathbf{1}^{i-1}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-i}}))\right] \\ &\qquad - \theta\left[W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-k}})) - W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i_{x_k=0}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-k}}))\right] \\ &\qquad\qquad \geq u(v_k + (i-1)w) - u(v_i + (i-1)w) \quad (25) \end{aligned}$$

The second term in brackets in the left hand side of (25) is identical to the first term in brackets except that v_k is replaced by v_i, which is smaller. Thus, concavity of W implies that the left hand side in (25) is negative. Hence:

$$u(v_k + (i-1)w) - u(v_i + (i-1)w) \leq 0 \quad (26)$$

Thus, we must have $v_k \leq v_i$, where $k < i$. This contradicts Proposition 2.□

Proof of Theorem 1: Assume that $\boldsymbol{v}$ is a least cost unique implementation scheme. Hence, if no agent exert effort, one agent, say 1, must prefer to exert effort (otherwise $\mathbf{0}^n$ is a Nash equilibrium). Assume that each agent $k \leq i$ prefers to exert effort when the agents ranked before this agent (according to a common ranking, 1 to n without loss of generality) exert effort while the remaining agents do not. When the i first agents exert effort, none of these agents has an incentive to deviate. Hence, another agent, say $i+1$ has an incentive to exert effort, otherwise $(\mathbf{1}^i, \mathbf{0}^{n-i})$ is a Nash equilibrium. Hence, the set of constraints (UC) can be reduced to:

$$\begin{aligned} &u(v_{i+1} + iw + \epsilon) + \theta W(\boldsymbol{\pi_{-(i+1)}}((\mathbf{1}^{i+1}, \mathbf{0}^{n-i-1}), \boldsymbol{v_{-(i+1)}} + \boldsymbol{\epsilon^{n-1}})) \\ &\qquad > u(c) + \theta W(\boldsymbol{\pi_{-(i+1)}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-(i+1)}} + \boldsymbol{\epsilon^{n-1}})), \quad (27) \end{aligned}$$

for all $i+1$.

Given that the objective of the principal is linear, the optimal level v_{i+1} is either characterized by the following condition:

$$u(v_{i+1}+iw)-u(c) = \theta\left[W(\boldsymbol{\pi}_{-(i+1)}((\mathbf{1}^{i},\mathbf{0}^{n-i}),\boldsymbol{v}_{-(i+1)})) - W(\boldsymbol{\pi}_{-(i+1)}((\mathbf{1}^{i+1},\mathbf{0}^{n-i-1}),\boldsymbol{v}_{-(i+1)}))\right], \tag{28}$$

or by $U(\mathbf{1}^n,\boldsymbol{v}) = U(\mathbf{1}^n_{x_{i+1}=0},\boldsymbol{v})$, which is equivalent to

$$u(v_{i+1}+(n-1)w)-u(c) = \theta\left[W(\boldsymbol{\pi}_{-(i+1)}(\mathbf{1}^n_{x_{i+1}=0},\boldsymbol{v}_{-(i+1)})) - W(\boldsymbol{\pi}_{-(i+1)}(\mathbf{1}^{n},\boldsymbol{v}_{-(i+1)}))\right]. \tag{29}$$

Let me denote v^*_{i+1} the solution to (28) and v'_{i+1} the solution to (29). We must have $v'_{i+1} \leq v^*_{i+1}$, which means that (28) is more constraining than (29). Hence $\boldsymbol{v}$ is a global ranking scheme.□

Proof of Theorem 2: Let $\boldsymbol{v}$ be a global ranking scheme. Theorem 1 ensures that the solution cannot be a scheme that has is not a global ranking scheme. Lemma 1 (see the proof) ensures that constraints (NE) are not violated for $i = 1, ..., n-1$. It is thus sufficient to check that each agent strictly prefers to exert effort under scheme $\boldsymbol{v}+\boldsymbol{\epsilon}$ where $\epsilon > 0$ and infinitesimal when all the previous agents (according to the ranking 1,...,n without loss of generality) also exert effort while the remaining agents do not. This is immediate from Assumption BIA.□

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