Continuous versus Discrete Time in Dynamic Common Pool Resource Game Experiments

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Abstract

We study the impact of discrete versus continuous time on the behavior of agents in the context of a dynamic common pool resource game. To this purpose, we consider a linear quadratic model in which agents exploit a renewable resource with an infinite horizon and conduct a lab experiment. We use a differential game for continuous time and derive its discrete time approximation. When the agent is the sole owner of the resource, we fail to detect on a battery of indicators any difference between discrete and continuous time. Conversely, in the two-player setting, significantly more agents can be classified as myopic and end up with a low resource level in discrete time. Continuous time seems to allow for better cooperation and thus greater sustainability of the resource than does discrete time. Also, payoffs are more equally distributed in the continuous time setting.

Keywords : Common Pool Resource; Differential Games; Experimental Economics; Continuous Time; Discrete Time

JEL Codes : C01; C73; C91; C92; Q20

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1 Introduction

On many issues, we have the possibility of taking decisions at any moment in time, and asynchronously with other agents: sending a message, extracting water from a groundwater table, reducing prices, etc. Many of the interactions we engage in have a real-time aspect. How does this ability to rapidly and asynchronously adjust actions shape our behavior? This question has been of deep interest for behavioral and experimental economists over the past decade. Indeed, many questions that were initially analyzed in discrete time in laboratory experiments can today be analyzed using continuous time protocols that allow researchers to compare the behavior of agents in discrete versus continuous time.

Previous articles find that continuous time can foster cooperation, but only under certain conditions. When presenting prisoner's dilemma games to two-person groups in three treatments, one in continuous time, one in static time (one-shot) and one in discrete time, Friedman and Oprea (2012) find a higher median cooperation rate in continuous time. Bigoni et al. (2015) combine elements of the design of Bó (2005) and of Friedman and Oprea (2012) to study cooperation in repeated prisoner's dilemma. They find that contrary to previous results in discrete time, cooperation is easier to achieve in continuous time with a deterministic time horizon than with a stochastic time horizon. Oprea et al. (2014) study subjects' contributions in a public good game played in groups of five people. They find players contribute higher amounts in continuous time than in discrete time but only when a rich communication protocol among participants is included. Introducing new laboratory methods in order to eliminate inertia in a subject's decision in continuous time experiments, Calford and Oprea (2017) find strikingly different behaviors in continuous vs. discrete time in a simple timing game where two participants compete to enter a market. Finally, Leng et al. (2018) study the evolution of cooperation by crossing time protocols (continuous vs. discrete time) and information feedback (group minimum effort level vs. effort level of each member of the group) in a minimum effort coordination game played in groups of six people. Among the four treatments, the authors find that the average payoff increases only when continuous time is associated with the provision of information on the effort level of each member of the group.

Although studying interactions in the prisoner's dilemma, public good, timing, or minimum effort coordination games is extremely useful, these games abstract from a feature relevant to many economic applications, the presence of a state variable that makes the impact of any decision to persist through time, which is the case in common pool resource (CPR) games (Vespa 2020). The vast majority of the CPR literature that combines theory and experimentation is in discrete time. A possible explanation is that discrete time is easier to implement in the lab and can be compared to a static repeated game in which the state variable evolves from one period to the other (Herr et al. 1997, Gardner et al. 1997, Mason and Phillips 1997, Hey et al. 2009, Suter et al. 2012, for instance). Nevertheless, Tasneem et al. (2017) recently tested a CPR differential game in the lab using a continuous time protocol. Focusing on Markov's perfect equilibrium strategy, they tried to determine the relevance of the nonlinear equilibria in a two-player common property resource game. Janssen et al. (2010) have also studied the role of communication and punishment in a CPR game in continuous time. They find that punishment can foster cooperation only when combined with communication. The authors do not present the formal theoretical model underlying their experiment.[1]

In this paper we build on the previous literature to study the impact of the nature of time in a two-person common pool resource (CPR) problem. Several important differences with previously tested games (prisoner's dilemma, public good, timing, and minimum effort coordination games) can lead to a different impact of the nature of time. First, the presence of the state variable makes the impact of any decision to persist through time (Vespa 2020), which can, for instance, generate dynamic free riding (Battaglini et al. 2016).[2] Moreover, as opposed to the prisoner's dilemma, where payoffs can be directly read from a matrix, dynamic games are more difficult to handle. These two elements can make the optimal solution harder to reach in the case of CPR games. Reversely, infinite horizon can provide strategic opportunities to endogenously support cooperative outcomes (Battaglini et al. 2016). In addition, using

[1]Note also that some authors such as Noussair et al. (2015) conduct their experiments in discrete time, while their theoretical model is in continuous time, which poses the question of to what theoretical predictions should we compare lab results: those from discrete or those from continuous time models? Moreover, Tasneem et al. (2019) study the ability of a single economic agent to exploit a renewable resource efficiently. To do that they test in the laboratory an optimal control problem with an infinite horizon in continuous time and show that extraction behavior results in a steady state of the resource only 56% of the time.

[2]Battaglini et al. (2016) define dynamic free-riding this way: "an increase in current investment by one agent [which] typically triggers a reduction in future investment by all agents". In the context of a CPR, a decrease in extraction level can be seen as an investment to obtain a higher resource level.

dynamic CPR games allows us to explicitly derive equilibrium paths for three well identified types of behavior – myopic, feedback and optimal. How does the nature of time affect strategic interactions in this context? Can continuous time still foster cooperation? Does the nature of time affect the equilibrium path to which participants are the closest?

To analyze these questions, we consider a simple linear quadratic model, based on Gisser and Sanchez (1980), Negri (1989), and Rubio and Casino (2003), in which agents exploit a renewable resource with an infinite horizon. The resource can be assimilated to a groundwater basin but other interpretations of CPR are possible. We use a differential game for continuous time and propose a discretization of the CPR game so that the equilibrium paths for myopic, feedback and optimal behaviors are almost identical in the discrete and continuous time models. For the implementation in the lab we choose to lead a non-contextualized experiment in a between-subject design with four treatments. We cross the nature of time (discrete versus continuous) and the number of subjects exploiting the resource (one versus two). In the continuous time treatments, we follow the literature and mimic continuous time by allowing the agent to change his extraction level every second. In the discrete time treatments, the agent can change his extraction level every period. About one hundred subjects participated in each treatment.

Presenting subjects with the simplest setting, i.e., a single agent exploiting the resource, allows us to test whether the ability to manage a resource differs in continuous and discrete time. Indeed, the greater number of decisions potentially taken in continuous time could facilitate a trial and error process to reach optimal management of the resource. It is important to establish this baseline because, as explained earlier, dynamic situations are complex problems to handle, and it is important to understand the impact of the nature of time without interactions. Our estimates indicating that only 37% of the agents play optimally, confirms this statement. Our results also show that in all aspects tested, a subject's ability is not affected by the nature of time in a single agent setting. This allows us to deduce that the differences observed in the multiplayer setting are due to the impact of the nature of time on the interactions.

When running the experiment in a multiplayer setting, we find striking differences between continuous and discrete time. For example, the average resource level is significantly lower in discrete time. There is a larger proportion of agents that can be classified as myopic and a larger proportion of agents that end up with a low resource level in discrete time, while the proportion of optimal and feedback agents are not significantly different between the discrete and continuous time. Continuous time seems to favor a more sustainable exploitation of the resource. Our underlying intuition for this result is similar to Friedman and Oprea (2012), Oprea et al. (2014) and Leng et al. (2018). Continuous time allows subjects to briefly switch to cooperative behavior, such as a socially optimal extraction level, in order to incite the other player to do the same, or conversely to quickly increase extraction if the other player increases their extraction too much. The fact that we observe more stable extraction levels in continuous time and that extraction levels are more homogeneous within the group is consistent with this potential explanatory mechanism. It also results in less unequally distributed payoffs in continuous than in discrete time.

Through this work, we provide several contributions to the literature. We offer the first in-lab analysis of the impact of discrete versus continuous time in the lab in the case of CPR games. We contribute to the analysis of common pool resources using differential games, by being the first experimental paper to consider socially optimal and myopic strategies in a continuous time setting. We also make two secondary contributions. We clearly present the experimental protocol allowing the implementation in the laboratory of a continuous-time model with an infinite horizon. Finally, to compare the behavior of subjects in the lab to theoretical projections, we combine mean-squared deviation statistics and linear regressions.

The next section of this paper presents the theoretical setting. Section 3 describes the experimental design used to test the theoretical model. Section 4 is devoted to the empirical strategy, and results are analyzed in Section 5. The final section provides a discussion and conclusion.

2 The Model

We consider a simple linear quadratic model in which two agents, i, j exploit a renewable resource over an infinite horizon. The resource can be assimilated to a groundwater table. Water pumped provides agents revenue $B(w)$ depending only on the extraction w. Agents also incur a cost $C(H, w)$, which depends negatively on the level of the groundwater H. The parameters a, b, c_0 and c_1 are positive. An agent's instantaneous payoff is given by the difference between revenue and cost, as shown by equation (1):

$$\overbrace{aw - \frac{b}{2}w^2}^{B(w)} - \underbrace{\overbrace{\max(0,\ c_0 - c_1 H)}^{marginal\ cost\ (c(H))}\, w}_{C(H,w)} \tag{1}$$

We take into account the positivity of the marginal or unitary cost $c(H)$, so that it is important to adopt a piecewise marginal cost function (2) to prevent agents from perceiving the cost as a subsidy.

$$c(H) = \begin{cases} (c_0 - c_1 H) & \text{if } 0 \leq H < \dfrac{c_0}{c_1} \\ 0 & \text{if} \quad H \geq \dfrac{c_0}{c_1} \end{cases} \tag{2}$$

In the model, agents have to choose an extraction level that maximizes their instantaneous payoff. The problem differs between continuous time and discrete time. In continuous time, decisions are made at each instant t in real time and the resource evolves continuously, while in discrete time, decisions are made at each period n and the resource evolves from one period to the next. Whether in continuous or discrete time, the behavior of agents is analyzed in two settings. First, in an optimal control problem, where a sole agent exploits the groundwater, we characterize both the myopic and the optimal behaviors. Second, the behavior of agents can be analyzed in a game, where strategic interaction is introduced by considering two identical and symmetrical agents in the exploitation of the groundwater. A feedback equilibrium path can be defined, in addition to the myopic and optimal equilibrium paths in the game.

Social optimum can be defined as a behavior in which an agent's extraction decision allows him to maximize his discounted net payoffs in order to maintain the resource at an efficient level. The social optimum is also called the "cooperative solution" in the game. In that case the resource is maintained at an efficient level by maximizing the joint discounted net payoff of all agents. The myopic solution is where the agent is only interested in the maximization of his current payoff (equation (1)), regardless of the evolution of the groundwater. The feedback equilibrium can be seen as a scenario in which agents adopt non-cooperative behavior, maximizing their own discounted net payoffs while also taking into account the evolution of the groundwater.

In continuous time, the total discounted payoff (with r the discount rate) for player i is:

$$\int_0^\infty e^{-rt}\left[aw_i(t)-\frac{b}{2}w_i(t)^2-\max(0,\ c_0-c_1H(t))w_i(t)\right]dt \tag{3}$$

and the dynamics is given as:

$$\begin{cases}\dot{H}(t)=R-\alpha(w_i(t)+w_j(t))\\ H(0)=H_0 \ and \ H_0\geq 0,\ H_0\ \ given\\ H(t)\geq 0\\ w_i(t)\geq 0\end{cases}$$

In discrete time, the total discounted payoff (with $1-r\tau$ the discount factor) and the dynamics for player i are given as:

$$\sum_{n=0}^{\infty}(1-r\tau)^n\left[aw_i(n)-\frac{b}{2}w_i(n)^2-max\left(0,c_0-c_1H(n)\right)w_i(n)\right]\tau \tag{4}$$

$$\begin{cases}H(n+1)=H(n)+\tau\left(R-\alpha(w_i(n)+w_j(n))\right)\\ H(0)=H_0 \ and \ H_0\geq 0,\ H_0\ \ given\\ H(n)\geq 0\\ w_i(n)\geq 0\end{cases}$$

The discretization rate τ chosen in discrete time provides a good approximation of the continuous time problem, and optimal solutions can be found by means of the Hamiltonian operator. The Nash feedback equilibrium in continuous time can be found by means of the Hamilton Jacobi Bellman (HJB) equation, by applying the guessing method to guess a quadratic value function and in discrete time by means of the Bellman equation. Finally, myopic solutions are obtained by means of a simple first-order derivative. They can also provide a feedback representation of the solutions when considering the constraints.[3] When w_j is dropped from the dynamics, one is able to solve the optimal control maximization problem (the sole-agent setting).

3 The Experiment

The computer is naturally unable to implement "pure" continuous time. Thus, we proposed a discretization of our theoretical model in continuous time, and by varying the discretization step, we were able to implement both an experiment which approximates continuous time and one which approximates discrete time. The discretization procedure is detailed in Appendix A. In what follows, we present the implementation of the different components of our experiment: continuous time and discrete time with a sole player, continuous time and discrete time with multiple players, the infinite horizon, and the choice of the parameters.

3.1 Experimental Design

The experiment took place at the Experimental Economics Laboratory of Montpellier (LEEM). From December 2019 to February 2020, a total of 200 students from the University of Montpellier participated in the first part of the experiment. This part was devoted to data collection for the single agent condition. It included a total of 17 sessions, 11 for the continuous-time treatment and 6 for the discrete-time treatment.[4] From November to December 2020, a total of 190 subjects participated in the second part of the experiment, which was devoted to data collection for the two-players game. The experiment involved 20 sessions of continuous and discrete time treatments for groups of two players, so that we had 49 groups in continuous time and 46 groups in discrete time.[5] It was a non-contextualized experiment, using the oTree

[3]The feedback representation is obtained when the solution is written according to the state variable, instead of according to time.

[4]Since the continuous time condition involves higher network traffic, we limited the number of participants per session to a maximum of 14, which explains the greater number of sessions for this treatment.

[5]ORSEE (Greiner 2015) is the platform used by the LEEM to manage the subject pool.

platform (Chen et al. 2016), in which subjects participated in a ten-minute training phase of the game, followed by a ten-minute effective phase of the game which counted for their remuneration. The experimental currencies (ECU) accumulated by subjects in the experiment were converted into cash payments with the conversion rate of 10 ECUs to 0.5 euro.[6] Each experimental session lasted around an hour.

3.2 Experimental Procedures

3.2.1 Global Description

We used a between-subject design in which participants in the sole-agent treatments were different from the ones in the multiple-agent treatments. In the sole-agent treatments, instructions explained the dynamics of the resource, the decision-making process and its consequences on the available resource, the cost of extraction and the payoff. After an initial individual reading, an experimenter proceeded to an outloud reading of the instructions. Next, subjects answered a digital questionnaire to make sure they understood the evolution of the resource as well as the computation of payoffs. They were also invited to ask questions by raising their hands.

To familiarize subjects with the graphical interface, they participated in a 10-minute training phase before a 10-minute paid phase. At the beginning of each phase, subjects had to choose an initial extraction between 0 and 2.8 by moving their cursor on a graduated slider, which displayed values up to two decimal points. Due to the quadratic nature of our revenue function, any extraction level led to a positive revenue. Figure B.1 in the Appendix B shows a concave revenue curve with a maximum revenue reached for an extraction of 1.4. Figure B.2 in the Appendix B also shows the unitary cost function, which decreases as the available resource increases and vanishes when the level of the available resource is above 20.

In the continuous time instructions, the extraction refers to an extraction rate, while in the discrete time instructions it refers to an extraction level. In addition, a distinction is made between the differential equation representing the dynamics of the resource in continuous time and the difference equation representing the dynamics of the resource in discrete time.

[6] ECU means Experimental Currency Unit.

However, for the sake of simplification, we explain the dynamics in continuous time rather than writing the differential equation. Once the subjects chose an initial extraction level, a new screen appeared and subjects were able to see the dynamics of the resource along with their payoff, which included the cumulative and continuation payoffs, updated every second in the continuous time treatment and every period in the discrete time treatment.

Adapted instructions were provided to subjects in the multiple-agent treatments. Environments remained the same as in the sole agent treatments, except that subjects extracted the resource in groups of two. The layout of the user interface was slightly different from that of the sole agent treatments, with an additional curve showing the pair's total extraction. Complete instructions for the four treatments can be found in the online supplementary materials.

3.2.2 Parameters

Table 1 reports the parameters used. To get comparable results, parameters were the same in continuous time and discrete time for both the sole- and multiple-agent treatments.

Table 1 – Parameters for the experiment

Variable	**Description**	**Value**
a	Linear parameter in the revenue function	2.5
b	Quadratic parameter in the revenue function	1.8
c_0	Maximum average cost	2
c_1	Variable cost	0.1
$c_0 - c_1 H$	Marginal or unitary cost	$2 - 0.1H$
r	Discount rate in continuous time	0.005
$\beta = (1 - r\tau)$	Discount factor in discrete time	0.995
R	Natural recharge (rain)	0.56
α	Return flow coefficient	1
H_0	Initial resource level	15
τ	Discretization step	0.1 & 1

Figure 1 and 2 below show the theoretical time paths for the extraction and resource levels in continuous time for 100 seconds. As the theoretical time paths in discrete time are almost identical to those in continuous time, we do not show them.

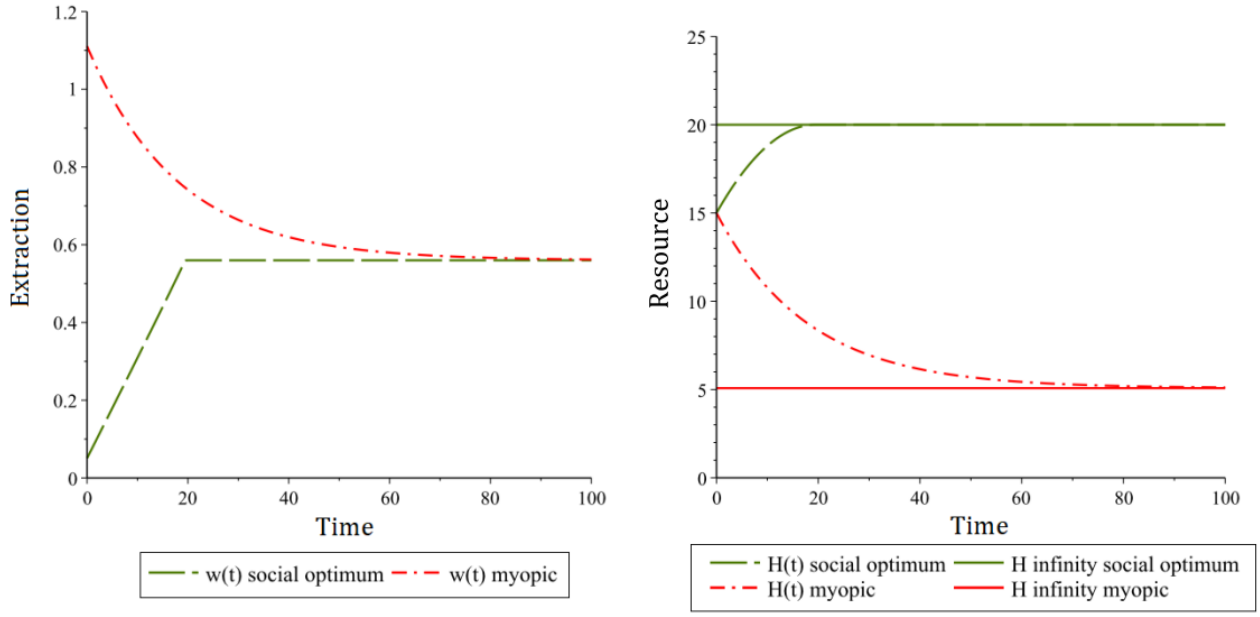


Figure 1 – Extraction behaviors and resource levels in sole-agent continuous time

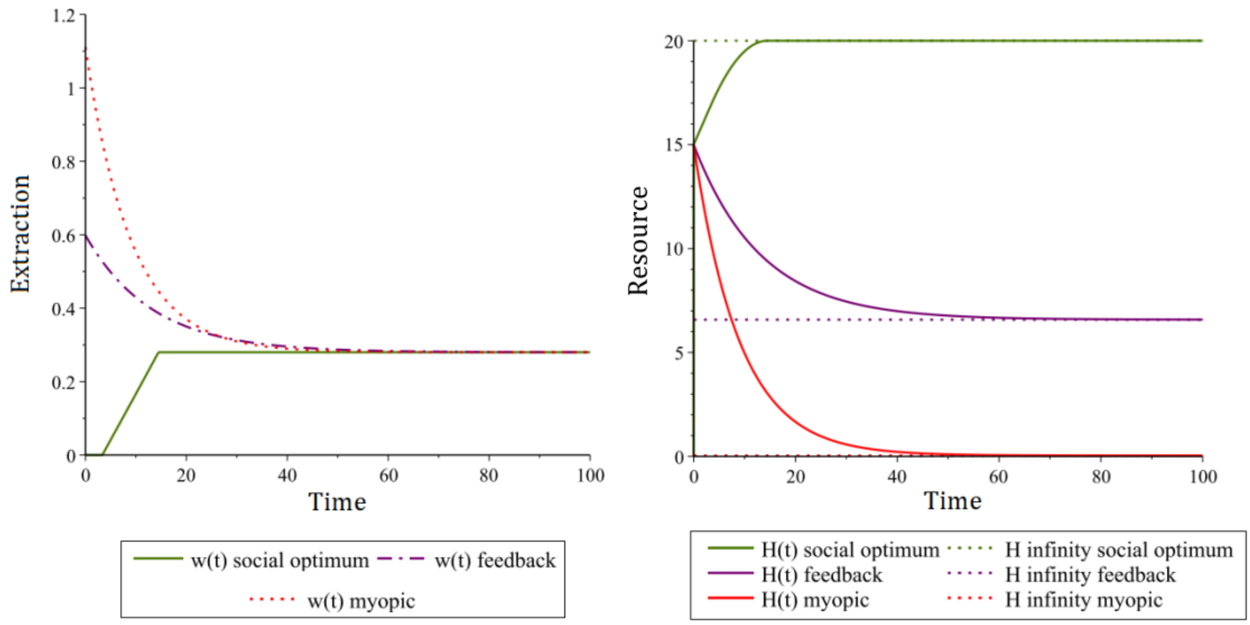


Figure 2 – Extraction behaviors and resource levels in multiple-agent continuous time

The infinite horizon requires us to set a small discount rate r to capture subjects' attention on the sustainability of the resource. The corresponding discount factor in discrete time is β. We also chose these parameters so that the steady state level of the resource in the socially optimal case is strongly separated from other cases. The socially optimal behavior leads to a high level of the groundwater, while the myopic behavior results in low groundwater levels (see the right sides of figures 1 and 2).

Both the natural recharge R and the return flow coefficient α were designated at a small enough size to capture the renewable nature of the resource, simulate real life conditions and avoid floods in the model.[7]

In situations where a subject's extraction is higher than the available resource, the rule was to set the extraction to zero until she changed her decision or until the amount of the resource increased enough to allow for a new extraction. This rule was chosen because it is easy to implement in the lab and because setting an allocation rule for the extraction in proportion to the available resource would have led to a multiplicity of equilibria, which would have greatly complicated the empirical strategy needed to compare lab results to equilibrium paths without revealing any (particularly) interesting information on the behavior of agents.

3.2.3 Decision Timing in Continuous and Discrete Time

In the sole agent continuous time treatment, subjects were able to change their extraction rate at any moment by simply moving the graduated slider displayed on their computer. Every second, the computer transmitted the slider value to the server, which then performed the computations (resource and payoff) and updated the values displayed on the computer's graph and text interfaces.

In the two-player continuous-time treatment, player 2's computer sent the cursor value to the server as soon as it changed, while player 1's computer transmitted the cursor value to the server every second, which triggered the server to continuously broadcast the updated values to both players. Thus, every second, the server took player 1's current extraction and player 2's most recent extraction (i.e. the last one transmitted by his computer). In this way, the time was synchronized between the two members of the group, since only one player was triggering the continuous updating of the information.[8]

[7]The return flow coefficient is the quantity of water returning to the groundwater after each extraction.

[8]This also reduced network traffic because as long as the second player did not change his extraction, his computer did not transmit a new value.

Since we have tried to provide an experiment that is as close as possible to continuous time, we have chosen a discretization step that is as small as possible, $\tau = 0.1$, to capture the specific characteristic of continuous time, i.e., its uninterrupted evolution. This means that in our continuous time treatment, one second of real time corresponds to 0.1 instant in the model. Thus, 10 minutes of experiment are equal to 600 seconds and equivalent to 60 instants. In the discrete time treatment, we have chosen a larger but reasonable discretization rate, $\tau = 1$. With this rate, 1 period equals 1 instant in the model. Therefore, subjects participated in a 60-period dynamic environment. In addition, in order to ensure a similar duration in both treatments, we gave the subject exactly 10 seconds in each period to take her decision, which means that the play time was also 10 minutes in discrete time.

The graphical user interface was divided into four areas. On the top left, a graph showed the evolution of the player's extraction. At the top right, a graph displayed the evolution of the resource, and at the bottom left there was a graph showing the evolution of the payoff. Finally, at the bottom right, a text box presented the same information as the graphs but in text form. Figure B.3 in Appendix B shows a screenshot of the user interface for the sole agent treatment in continuous time. In the multiple agent treatments, the user interface was identical except that an additional curve in the upper left graph showed the evolution of the group's total extraction.

3.2.4 Infinite Horizon

In both continuous and discrete time, the infinite horizon is implemented through the payoff, as in Tasneem et al. (2017) and Tasneem et al. (2019). The payoff is composed of two elements: (i) a cumulative payoff from the first instant of play ($t = 0$) to the present instant ($t = p$), and (ii) a continuation payoff, which is computed as an integral of payoffs from the present instant ($t = p$) to infinity ($t = \infty$), assuming that the player's extraction remains unchanged. In the two-player game, the continuation payoff was calculated assuming that both players' extraction remained unchanged.

The cumulative payoff in continuous time corresponds to the discounted integral of the instantaneous payoffs from the beginning of the experiment up to the present instant. Thus, the discount rate is $r = 0.5\%$ and means that the payoff of instant t is multiplied by $e^{-0.005\times t}$. The discounting principle allows subjects to understand that the same instantaneous payoff has a different discounted value according to the instant. In other words, as time goes on, the payoffs of the last instants have a lesser impact on the subject's total payoff for the experiment. Similarly, the cumulative payoff in discrete time corresponds to the discounted sum of each period's payoff from the beginning of the experiment up to the present period. Thus, the discount factor is $\beta = 0.995$ and means that the payoff of period n is multiplied by 0.995^n. The discounting principle allows subjects to understand that the same payoff has a different discounted value according to the period. In other words, in the experiment, the same instantaneous payoff contributes less to the total final payoff when it occurs in the later periods rather than in the earlier periods.

4 Empirical Strategy

Two hundred subjects participated in the sole-agent (optimal control) experiment and 190 in the multiple-player (game) experiments. They took (paid) extraction decisions for 600 seconds during each session. We use these extraction decisions data to understand whether agents take different decisions in continuous vs. discrete time, and in the control vs. in the game. Through the empirical analysis, we use standard tests such as the Mann-Whitney and the Fisher exact proportion tests to compare our indicators among the different treatments. Furthermore, to determine whether agents demonstrated myopic or optimal behavior (or feedback behavior in the game), we use the empirical strategy presented in this section. For ease of understanding, the empirical strategy is first explained in detail for the sole-agent setting.

To identify which theoretical extraction pattern an agent's extraction comes closest to, a widely used statistics is the mean squared deviations (MSD, e.g., Herr et al. 1997). The minimum MSD gives the agent type. The MSDs are calculated for each agent such that:

$$
\begin{aligned}
MSD_{my}^{th} &= \frac{\sum_{t=1}^{T}\left(w(t)-w(t)_{my}^{th}\right)^2}{T} \\
MSD_{op}^{th} &= \frac{\sum_{t=1}^{T}\left(w(t)-w(t)_{op}^{th}\right)^2}{T}
\end{aligned}
\tag{5}
$$

where $w(t)$ is the extraction of the agent at time t, $w(t)_{my}^{th}$ is the constrained myopic theoretical extraction at time t, and $w(t)_{op}^{th}$ is the optimal theoretical extraction at time t. Agents can be classified as myopic or optimal, depending on which MSD, MSD_{my}^{th} or MSD_{op}^{th} is the smallest. Comparing extractions of the agent to the theoretical constrained myopic and optimal extraction in this way is imperfect since an agent can make mistakes and begin adopting an optimal path after, say, 30 seconds, which will not be captured correctly by the method.

For instance, if an agent under-extracts for the first 30 seconds, the optimal extraction at time 31, given the observed groundwater level H (called conditional, $w(31)_{op}^{c}$) will be greater than the optimal extraction at time 31 if the agent behaved perfectly optimally from time 0 ($w(31)_{op}^{th}$). Thus, in order to correctly identify an agent's behavior type - myopic or optimal - we compare observed extraction to conditional extractions throughout the remainder of the paper. Conditional extractions are computed with respect to the $t-1$ actual groundwater level. Thus, we compute the following MSDs :

$$
\begin{aligned}
MSD_{my}^{c} &= \frac{\sum_{t=1}^{T}\left(w(t)-w(t)_{my}^{c}\right)^2}{T} \\
MSD_{op}^{c} &= \frac{\sum_{t=1}^{T}\left(w(t)-w(t)_{op}^{c}\right)^2}{T},
\end{aligned}
\tag{6}
$$

where $w(t)_{my}^{c}$ is the conditional constrained myopic extraction of the agent at each second (every ten seconds for discrete time), and $w(t)_{op}^{c}$ is the conditional optimal extraction of the agent. Agents are classified as myopic or optimal depending on which MSD, MSD_{my}^{c} or MSD_{op}^{c} is the smallest.

The inconvenient of a classification of agents based on the MSD alone is that an agent will always be classified, even if he doesn't follow the theoretical patterns studied at all.[9] To overcome this flaw, we add a second criteria based on a regression analysis. Supposing that for a given agent, we have:

$$\begin{aligned} & w(t)^c_{my} < w(t)^c_{op}, \quad or \\ & w(t)^c_{my} > w(t)^c_{op}, \end{aligned} \tag{7}$$

then we run the following regression:

$$\begin{aligned} & w(t) = \beta_0 + \beta_1 w(t)^c_{my} + \varepsilon_t, \quad or \\ & w(t) = \beta_0 + \beta_1 w(t)^c_{op} + \varepsilon_t. \end{aligned} \tag{8}$$

We consider an agent to be significantly myopic (or optimal) if β_1 is positive and significantly different from 0. This allows us to categorize the agents as: myopic, optimal, or undetermined.[10] Regarding the econometric time series treatments, we implement an augmented Dickey-Fuller test to detect the presence of unit roots in the series. In case of non-stationarity of the variables, we run our regressions on a differentiated series. Serial correlation of the error terms is dealt with using Newey-West standard errors, and sensitivity tests using 1, 5, and 10 lags are implemented.[11]

We follow exactly the same strategy to analyze experimental data for the game, but this time for three instead of two predicted behaviors, namely: myopic, optimal and feedback. Note that the continuous time framework provides us with 600 decisions per agent, while

[9] To take a concrete example, instead of comparing the agent's extraction $w(t)$ to the conditional constrained myopic and conditional optimal extraction, $w(t)^c_{my}$ and $w(t)^c_{op}$, we could compare it to the temperature in Moscow and Istanbul from day 1 to day 600, and we would find that our agent's extraction is closer to the temperature either in Moscow or in Istanbul, because one MSD will always be smaller than the other, even if completely irrelevant.

[10] An alternative is proposed by Suter et al. (2012), who run a similar regression (without the constant term) and consider that an agent follows a given behavior if the coefficient is not significantly different from 1. A natural way to do this is to implement a Wald test with:

$$\begin{cases} H_0 : \beta_1 = 1 \\ H_A : \beta_1 \neq 1, \end{cases} \quad and \;\; W = \frac{(\hat{\beta}_1 - 1)^2}{var(\hat{\beta}_1)} \to F_{(1,300)}$$

In this case, a very imprecisely estimated coefficient β_1 (very large $var(\hat{\beta}_1)$) will lead us to reject H_A and classify the agent as myopic or optimal, although he follows neither an optimal or myopic path. This is the reason why we propose the aforementioned alternative rule for classification.

[11] We present regression results using 1 lags. Results using 5 and 10 lags are available upon request.

the discrete time framework provides us with only 60. This greatly impacts our empirical strategy as β-coefficients would have more chances to be significant in continuous time - a greater number of observations leading to a lower minimum effect size. To avoid this issue, we keep only one observation every ten seconds when running the regressions in continuous time.

5 Results

Figure 3 presents an overview of our results. We plotted the mean resource by treatment along with the 95% confidence interval around the estimated mean. It seems we have close average resource levels in the two time treatments in the control, but different ones in the game. Also, the average resource level increases in the control and decreases in the game.

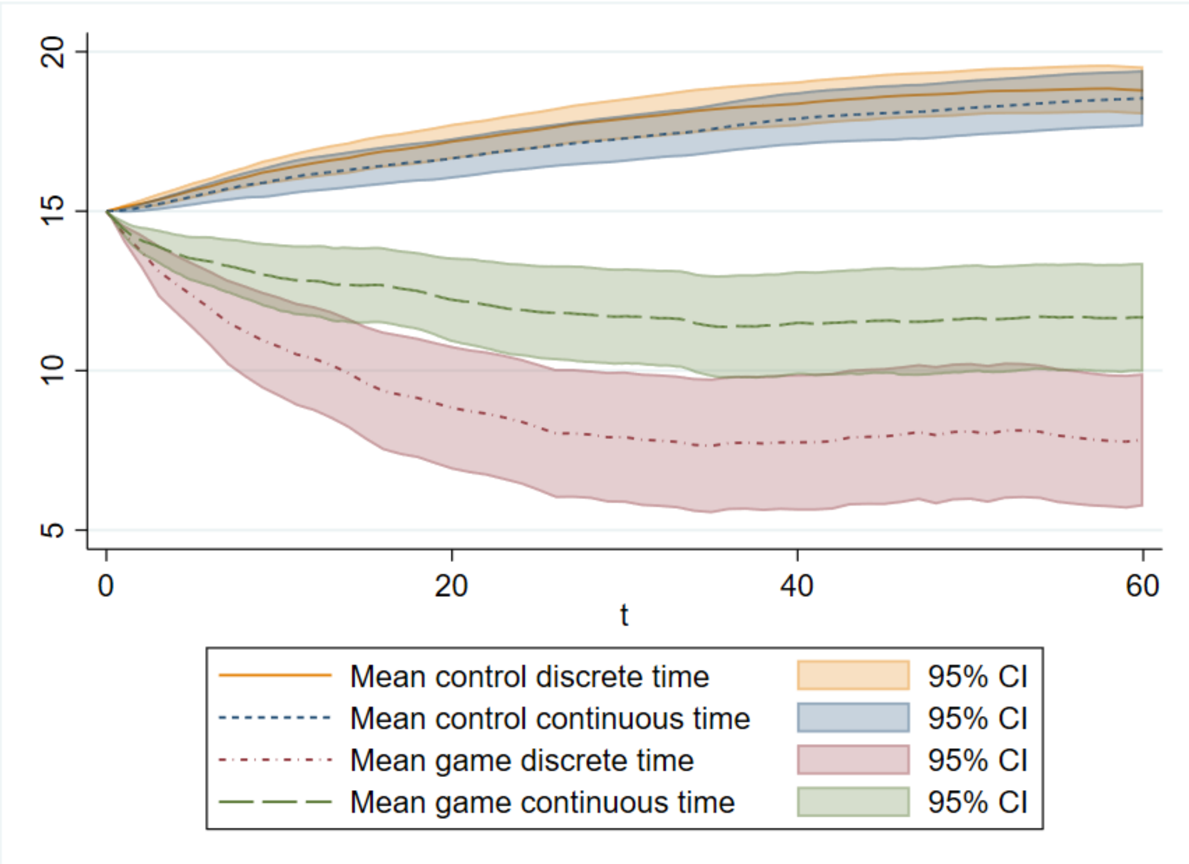


Figure 3 – Evolution of mean resource level by treatment

In the rest of the section we take a closer look at what happens within each treatment. We first compare the agents in the control setting. Second, we compare the average behaviors in the control and in the game. Third, we thoroughly study behaviors in the game. Finally, we build specific indicators to examine the potential mechanism at play. Note that through the rest of the paper, the term 'agents' is used to refer to subjects in the control, the term 'players' to subjects in the game, and the term 'groups' to groups of two subjects that were paired in the game.

5.1 Analysis of the Optimal Control

Table 2 compares continuous and discrete time over various indicators. The average resource level is not significantly different between the two treatments. About 40% of the players reach a resource level greater than 20 in each treatment (the optimal steady state resource level) and at approximately the same time. Only three agents in each treatment end up with a resource level below ten. Finally, the average extraction level is around 0.50 in both treatments and, perhaps more surprisingly, the number of times the players change their extraction level is not significantly different between the continuous and discrete time treatments, while in theory they had the possibility to change it 61 times in discrete time and 601 times in continuous time.

Table 2 – Continuous versus discrete time in the control

	Average agent's resource level			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Discrete time	17.572	2.639	98	-0.98	0.328
Continuous time	17.144	3.297	102	-	-
	Agents reaching R=20			Fisher exact test	
	Yes	No	N	Odds ratio	Exact prob
Discrete time	39	59	98	0.983	0.535
Continuous time	41	61	102	-	-
	Time agents reach R=20			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Discrete time	23.795	13.546	39	-0.563	0.577
Continuous time	23.115	15.460	41	-	-
	Agents ending up with R<10			Fisher exact test	
	Yes	No	N	Odds ratio	Exact prob
Discrete time	3	95	98	1.042	0.640
Continuous time	3	99	102	-	-
	Average agents extraction			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Discrete time	0.497	0.064	98	0.992	0.322
Continuous time	0.501	0.075	102	-	-
	Number of agents extraction change			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Discrete time	34.122	17.603	98	-0.304	0.762
Continuous time	44.902	47.515	102	-	-
	Agents with smaller MSD^c_{my} than MSD^c_{op}			Fisher exact test	
	Yes	No	N	Odds ratio	Exact prob
Discrete time	6	92	98	0.446	0.087
Continuous time	13	89	102	-	-

The fact that we observe a substantial share of agents reaching a resource level above 20 and very few ending up with a resource level below ten is consistent with the fact that the average resource level in the control observed in Figure 3 is closer to the optimal than to the myopic path. This is confirmed by the MSDs map Figure 4, which presents the location of agents with respect to the MSD^c_{op} on the y axis and the MSD^c_{my} on the x axis. Agents located above the bisector can be considered as more myopic ($MSD^c_{op} > MSD^c_{my}$) and vice versa. Very few agents have a greater MSD^c_{op} than the MSD^c_{my}, i.e., 19 over 200. This proportion is slightly lower in discrete than in continuous time (see the last test in Table 2).

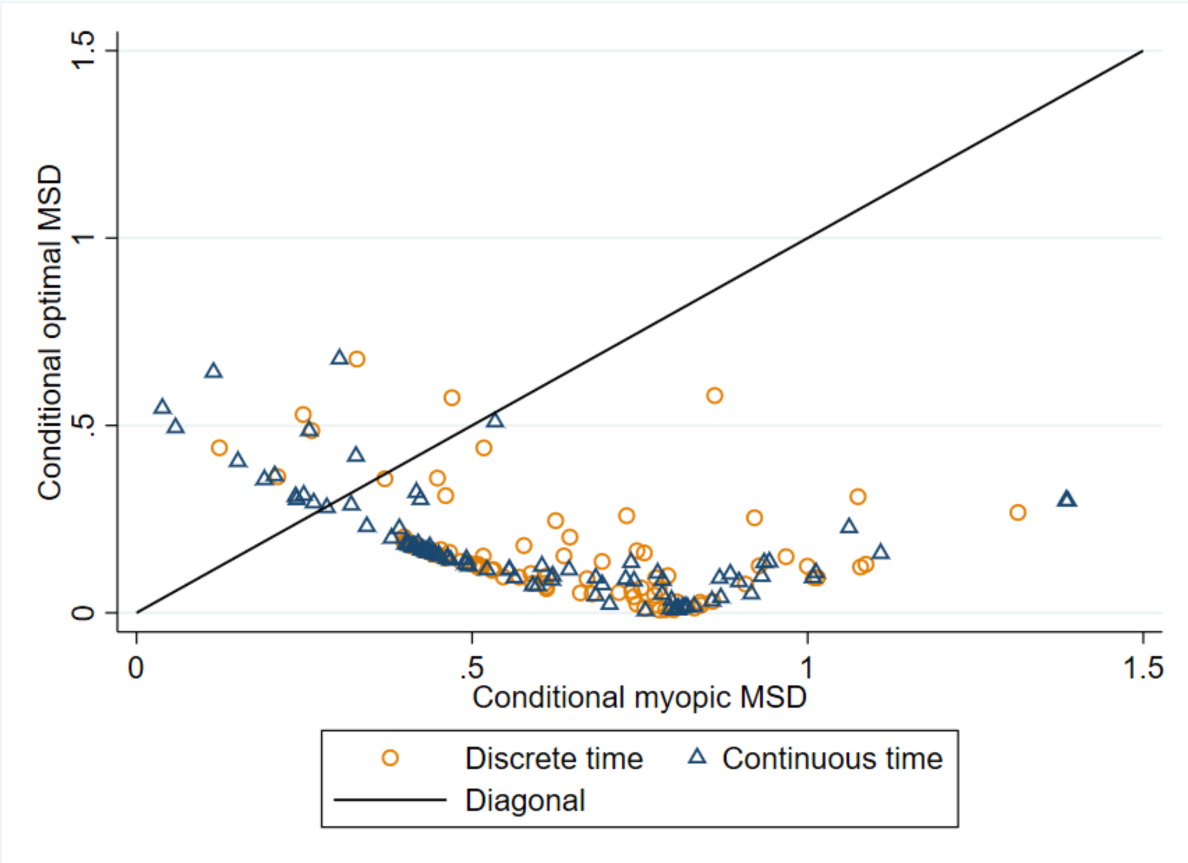


Figure 4 – Map of conditional MSDs in the control

As we explained in Section 4, using the MSD alone is unsatisfactory, because we want to know if agents are significantly optimal or myopic. Applying the regression filter presented in the previous section leads us to find that in discrete time 33 agents can be classified as significantly optimal and one as myopic, and 41 can be considered optimal and four as myopic in continuous time. Proportions of optimal and myopic agents are not significantly different between the two treatments. As expected, average payoffs are not significantly different either (see Table 3). The proportion of optimal agents seems comparable to the experiment of Tasneem et al. (2019) who found that extraction behavior results in a steady state of the resource 56% of the time, with the mode of the distribution being optimal.[12] Also, the average efficiency ratio (individual payoff over the optimal payoff, here 220 ECUs) is 83% in Tasneem et al. (2019)'s study while it is 88% in ours. Suter et al. (2012) found a slightly higher efficiency ratio in the optimal control in a discrete time experiment, about 95%.

Table 3 – Classification and payoffs in the control

	Proportion of optimal agents			Fisher exact test	
	Yes	No	N	Odds ratio	Exact prob
Discrete time	33	65	98	0.755	0.209
Continuous time	41	61	102	-	-
	Proportion of myopic agents			Fisher exact test	
	Yes	No	N	Odds ratio	Exact prob
Discrete time	1	97	98	0.253	0.198
Continuous time	4	98	102	-	-
	Average agents payoffs			Mann-Whitney test	
	Mean	S.D.	N	z-stat	Exact prob
Discrete time	191.370	38.497	98	-0.755	0.452
Continuous time	196.605	16.878	102	-	-

To summarize, in a control setting, both continuous and discrete times lead to similar choices by participants. Having made this first observation we now study how the nature of time affects strategic interactions between players.

5.2 The Control Versus the Game

The first observation that can be made by looking at Figure 3 is that the average level of the resource is lower in the game than in the control and decreases over time, whereas the resource level was increasing over time in the control. Mann-Whitney tests reported in Table 4 confirm that, compared to the control, the average resource level in the game is significantly lower and the average extraction level significantly higher. This is consistent with what one

[12] A more precise comparison of the results is not possible since the authors use a different empirical strategy.

would expect if agents had unlimited rationality, since they would play optimal in the control and feedback in the game. In addition, we observe that agents change their extraction levels more often in the game than in the control.

Table 4 – Control versus game

	Agent and group average resource levels			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Control	17.354	2.993	200	9.720	0.000
Game	1.653	5.406	95	-	-
	Agent and group average extraction levels			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Control	0.499	0.069	200	-10.025	0.000
Game	0.652	0.012	95	-	-
	Number of agents and groups extraction changes			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Control	39.62	36.415	200	-5.541	0.000
Game	60.658	56.041	190	-	-
	Agents and groups with smaller MSD^c_{my} than MSD^c_{op}			Fisher exact test	
	Yes	No	N	Odds ratio	Exact prob
Control	19	181	200	0.207	0.000
Game	32	63	95	-	-

Finally, the MSDs map reported in Figure 5 shows that, compared to Figure 4, significantly more agents have a smaller MSD^c_{my} than MSD^c_{op} in the game than in the control (32 groups over 95, see Fisher test in Table 4).

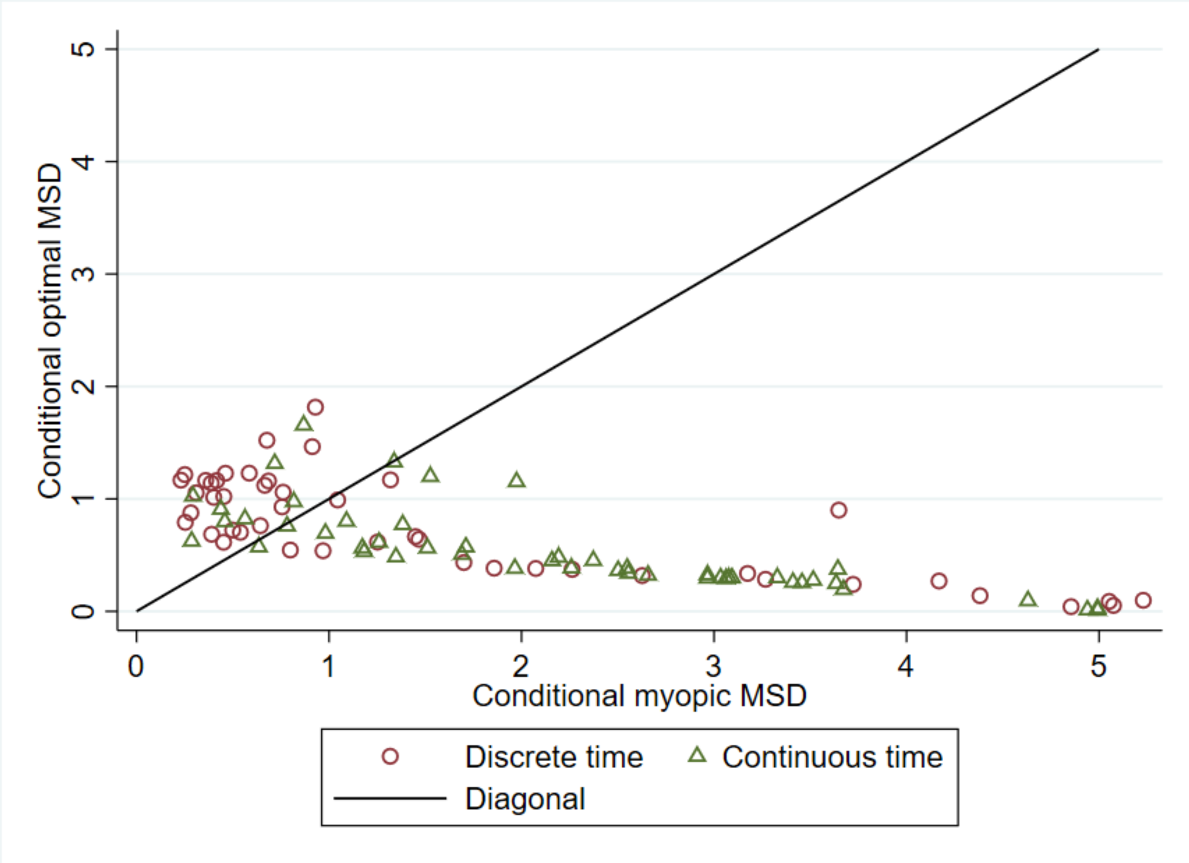


Figure 5 – Map of conditional group MSDs in the game

5.3 Analysis of Behaviors in the Game

Table 5 compares the decisions in discrete and continuous time in the game over various indicators. The average resource level is significantly lower in discrete time and the average extraction significantly higher. Very few groups reach a resource level greater than 20 – only five in each treatment, and at approximately the same time. The big difference with the control is that now a large number of groups end up with a resource level below ten and in a significantly larger proportion in discrete time. Introducing strategic interaction thus leads to an over-exploitation of the resource, as the theory predicted, but to a greater extent in discrete time, suggesting that continuous time allows for better cooperation between players. Finally, the number of times the agents change their extraction level is now significantly greater in continuous time.

Continuous time offers more opportunities to change one's extraction level. This possibility can be used to test the reaction of the other players and perhaps to try to induce a change in their behavior. For example, one player can temporarily lower his extraction level to see if the other player will do the same. This type of test is less expensive in continuous time than in discrete time. Indeed, in discrete time, the player can only make one decision per period and this corresponds to one instant, whereas in continuous time, the player can make one decision per second and this corresponds to only 0.1 of an instant. In other words, the opportunity cost of testing a strategy, in terms of payoff, is much lower in continuous time, because only a fraction of the payoff is given up during the temporary test strategy. This mechanism through which continuous time can foster cooperation was also advanced by Friedman and Oprea (2012), Oprea et al. (2014) and Leng et al. (2018). Oprea et al. (2014) calls this "pulse behavior" and sees it as a non-verbal form of communication. It can be used as a way to incite the other player to decrease extraction up to the optimal level or to retaliate if the other players increase their extraction level too much.

Table 5 – Continuous versus discrete time in the game

	Average group resource			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Discrete time	9.06	5.884	46	2.867	0.004
Continuous time	12.149	4.477	49	-	-
	Groups reaching R=20			Fisher exact test	
	Yes	No	N	Odds ratio	Exact prob
Discrete time	5	41	46	1.073	0.589
Continuous time	5	44	49	-	-
	Time required for groups to reach 20			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Discrete time	27.8	12.911	5	0.314	0.314
Continuous time	32.46	14.622	5	-	-
	Groups ending up with R<10			Fisher exact test	
	Yes	No	N	Odds ratio	Exact prob
Discrete time	31	15	46	3.89	0.001
Continuous time	17	32	49	-	-
	Average players extraction			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Discrete time	0.345	0.129	92	-2.352	0.019
Continuous time	0.308	0.114	98	-	-
	Number of extraction changes by players			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Discrete time	40.674	16.202	92	4.203	0.000
Continuous time	79.418	71.68	98	-	-

Applying the regression filter presented in Section 4 leads us to find that 14 groups (28 players) can be classified as significantly myopic in discrete time versus three groups in continuous time, making the proportion of myopic behavior significantly larger in discrete time. Six groups are classified as feedback in the two treatments, and we find only two optimal in discrete time and one in continuous time. Proportion of optimal and feedback agents are not significantly different between discrete and continuous time. Note that the presence of optimal groups is consistent with Battaglini et al. (2016)'s argument that infinite horizon can provide strategic opportunities to endogenously support cooperative outcomes.

As a result, we observe significantly higher average individual payoffs in continuous time than in discrete time. Efficiency ratios in the game are lower than in the control, and lower in discrete time (48%) than in continuous time (64%).[13]

Table 6 – Analysis of types in the game

	Proportion of optimal groups			Fisher exact test	
	Yes	No	N	Odds ratio	Exact prob
Discrete time	2	44	46	2.182	0.476
Continuous time	1	48	49	-	-
	Proportion of feedback groups			Fisher exact test	
	Yes	No	N	Odds ratio	Exact prob
Discrete time	6	40	46	1.075	0.575
Continuous time	6	43	49	-	-
	Proportion of myopic groups			Fisher exact test	
	Yes	No	N	Odds ratio	Exact prob
Discrete time	14	32	46	6.708	0.002
Continuous time	3	46	49	-	-
	Average individual payoffs			Mann-Whitney test	
	Mean	S.D.	N	Z-stat	Exact prob
Discrete time	57.987	46.233	92	3.184	0.002
Continuous time	76.806	41.897	98	-	-

Finally, Figure 6 provides an overview of the results of the classification by type by plotting the cumulative density functions (c.d.f.) of the resource levels. The distribution of the observed resource levels rank as expected, with the myopic groups experiencing the lowest resource levels, followed by the feedback and optimal groups. The undetermined group

[13]The maximum group payoff is 240 ECUs, so we computed the individual efficiency ratio by halving this value. Nevertheless, it is possible to get "more than your own share". Obviously, if one of the two members of the pair extracts a very small amount of groundwater, the other member can obtain more than 50% of the total maximum payoff.

displays a high level of heterogeneity, which could be of interest in further research.

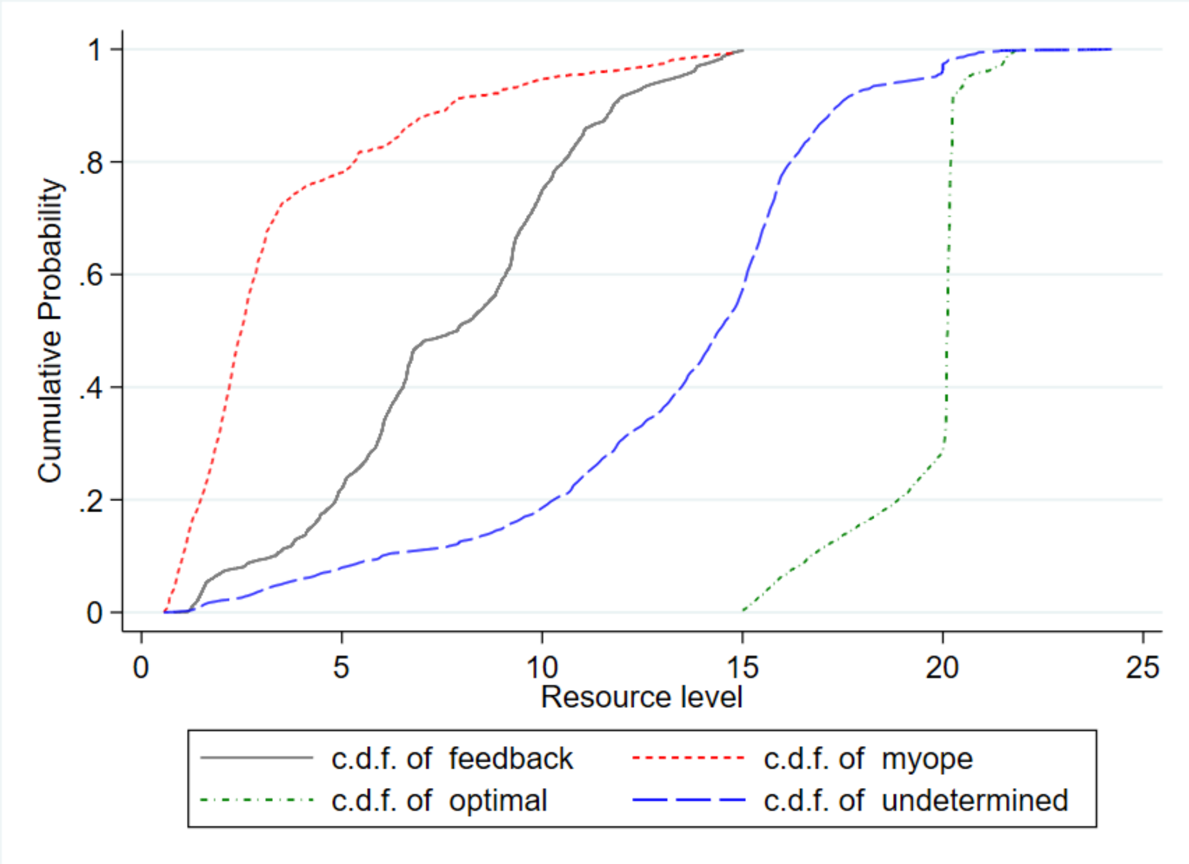
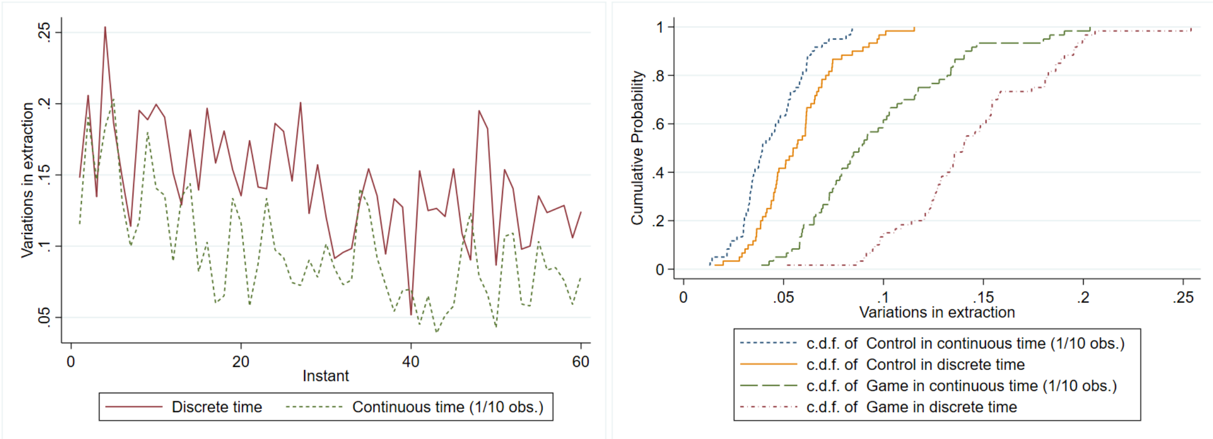


Figure 6 – Cumulative density functions of the resource levels by type

5.4 Potential Mechanism at Play

Our results show that continuous time fosters cooperation and allows for more sustainable management of the resource than does discrete time. Our intuition is that continuous time offers the possibility to induce cooperation at a lower opportunity cost, by lessening one's own extraction to incite the other player to do the same or to retaliate against them for over-extracting. If this mechanism actually applies, the threat of immediate sanction should make extraction patterns more stable and extraction levels should be more homogeneous, resulting in a more even distribution of payoffs within groups. To test this reasoning, we compute several statistics.

First, for each player we compute the absolute value of the difference of extraction between two consecutive instants ($|E_t - E_{t-1}|$) and calculate the average value over time by treatment, as did Oprea et al. (2014).[14] As shown by Figures 7.a and 7.b, continuous time leads to greater stability than does discrete time, and, not surprisingly, playing alone leads to greater stability than playing with someone else.[15]



(a) Evolution through time in the game

(b) Cumulative density functions

Figure 7 – Variations in players' extraction levels (w)

Second, we compute the absolute value of the difference in extraction levels between two players (A and B) of the same group at each point in time ($|E_{tA} - E_{tB}|$). We then take the average value over each period of time, by treatment.[16]

Figure 8.a shows that the average difference in extraction inside groups is almost always greater in discrete time, which is confirmed by the c.d.f. displayed in Figure 8.b.[17] Also, although extraction level differences decrease over the course of the game, it remains an issue until the end. Indeed, at the last instant the average difference in extraction levels still represents two-thirds of the average player's extraction.[18]

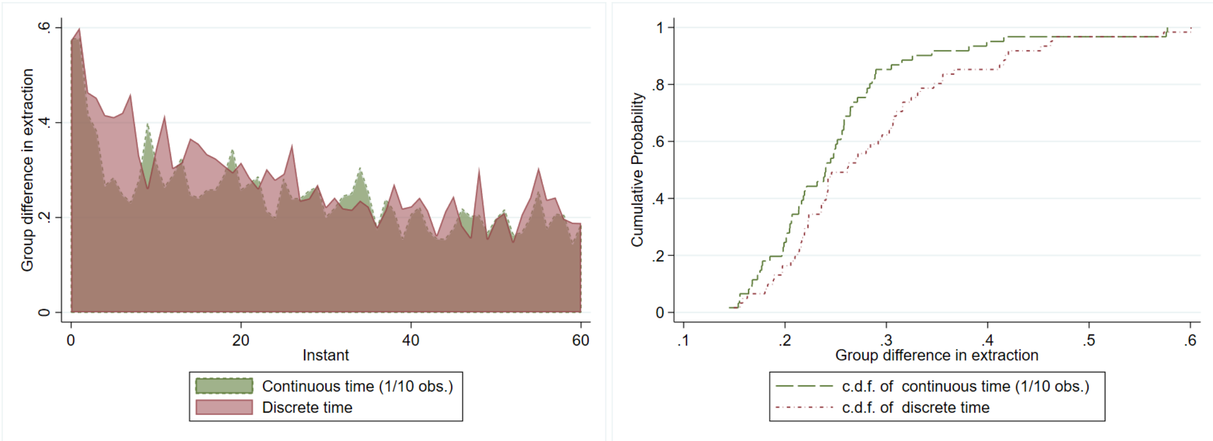
[14]To make continuous and discrete time comparable, we take the difference between two decisions separated by ten seconds in continuous time.

[15]In Figure 7.a we also see an increase in stability over time for both treatments. Note, however, that the greater instability in the beginning of the play time can be explained by the game setting. Indeed, players need first to either let the resource grow or deplete it before reaching a steady state, depending on their preferred equilibrium.

[16]To make continuous and discrete time comparable, we use only one decision every ten seconds in continuous time.

[17]The c.d.f. are statistically different according to the Kolmogorov–Smirnov test (p-value < 0.05).

[18]The average difference in extraction between players of the same group at the end of the game equals 0.18, while the average player's extraction level equals 0.27.



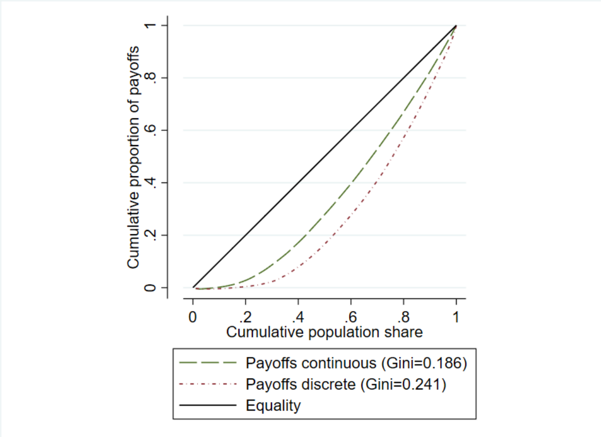
(a) Evolution through time

(b) Cumulative density functions

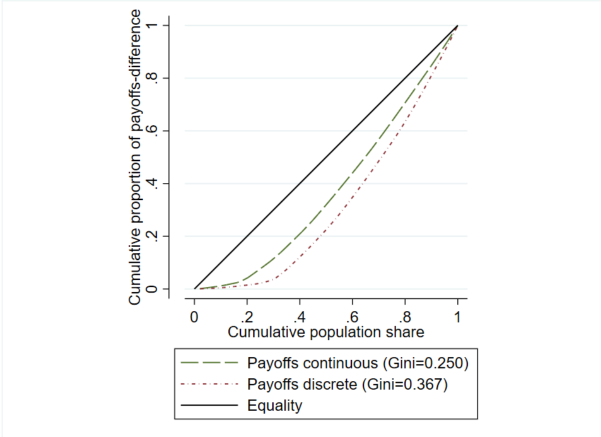
Figure 8 – Difference of extraction levels (w) within groups

To see whether or not within-group differences in extraction levels results in more unequal distribution of payoffs, we compute the Lorenz curves of individual final payoffs in the game. We can see in Figure 9.a that final payoffs are more unequally distributed in discrete time. More precisely, 50% of the poorest players share 28% of the payoffs in continuous time while they share 17% in discrete time. The Lorenz curves in Figure 9.a are easily readable but here unequal distribution can come from between-group inequalities and within-group inequalities. To take a closer look at within-group inequalities we compute the difference between individual final payoffs within a group and plot the corresponding Lorenz curves (Figure 9.b). Payoff distribution is more unequal in the discrete time setting. If within-group payoff differences were the same for all groups, the Lorenz curves would be confounded with the diagonal. Here we see that large payoff-differences represent a greater proportion of total payoff differences in discrete time than in continuous time, as the Lorenz curve for discrete time is further from the diagonal than the Lorenz curve for continuous time.[19]

[19]Concentration (Gini) indexes are significantly different whether we use the standard, Erreygers or Wagstaff indexes (O'Donnell et al. 2016).



(a) Individual final payoffs



(b) Within-group difference in final payoffs

Figure 9 – Lorenz curves

To summarize, even if we cannot prove the mechanism at play, the fact that extractions are more stable and that within-group differences in final payoffs are lower in continuous time is consistent with the fact that continuous time offers a less costly opportunity to influence the other player's decisions. As a result, continuous time seems to reduce inequality in payoff distribution, in addition to favoring more sustainable resource exploitation.

6 Discussion and conclusion

In this paper, we intended to determine the impact of the nature of time, discrete or continuous, on the behavior of agents in the context of a dynamic CPR game. To this end, we considered a simple linear quadratic model in which agents exploit a renewable resource over an infinite time horizon. Starting from a differential game, we proposed a discretization such that the equilibrium paths for the myopic, feedback and optimal behaviors are almost identical in discrete and continuous time. We then took on the challenge of implementing continuous time and infinite horizon in the lab, allowing participants to make extraction decisions every second, and adding continuation payoffs to cumulative payoffs to simulate an infinite horizon.

To determine whether the nature of time has an impact on the ability of agents to manage a resource, we first looked at the situation where the resource is owned by a single agent. Observations showed no difference between discrete and continuous time, based on a battery of indicators, including the average level of the resource, the average level of extraction, the proportion of myopic agents, and the proportion of optimal agents. Furthermore, about 35% of the subjects could be classified as significantly optimal and the average resource level increased over time, as is the case with the optimal solution.

In the context of a two-player game, the results were dramatically different. First, unlike what we observed with a single agent, the average resource level decreased over time, as is the case with the myopic and feedback equilibrium paths. Furthermore, only 2% of the groups behaved according to the optimal (cooperative) path. The competitive nature of the game when multiple players simultaneously extract on the same resource explains the difficulty in adopting a sustainable path. Second, we observed significant differences between discrete and continuous time settings. In particular, the discrete time setting led to the observation of a larger number of agents exhibiting myopic behavior, thus leading to a much lower average resource level than that observed in the continuous time setting. The continuous time environment seems to allow for better cooperation within groups and thus greater resource sustainability. Although our experimental design does not allow us to prove the exact mechanism at play, our intuition is consistent with Friedman and Oprea (2012), Oprea et al. (2014) or Leng et al. (2018): compared to discrete time, continuous time allows for rapid and adaptive strategic choices that promote the emergence of cooperation, either by attempting to influence the other or by retaliating against their tendency to over-exploit the resource. The observed greater stability of continuous-time extraction, as well as the greater homogeneity within groups in this environment, is consistent with this explanatory mechanism.

We voluntarily used a very simple design, as to our knowledge we are the first paper to test the impact of the nature of time in dynamic CPR games. Consequently, many extensions are possible. We hope our work can offer a basis for future works examining, for instance, whether continuous time can still foster cooperation when increasing the group size, as the continuous time frame by itself was able to induce cooperation compared to the discrete time frame in a two-person prisoner's dilemma in Friedman and Oprea (2012), but not in a five-person public good game as in Oprea et al. (2014) or a six-person minimum effort game as in Leng et al. (2018). Also, many refinements of the underlying theoretical model and of the game setting are possible. In particular, the role of major mechanisms such as rewards, punishments and communication settings in the continuous versus the discrete time frame remain to be examined.

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Appendices

A The Discretization of the Continuous Time Model

This section presents the procedure adopted to discretize the continuous time model. Let's consider the following continuous time model:

$$\max_{w(t)} \int_0^\infty e^{-rt} f(w(t), H(t))dt \tag{9}$$

$$s.t \begin{cases} \dot{H}(t) = R - \alpha w(t) \\ H(0) = H_0 \geq 0,\ H_0\ \ given \\ H(t) \geq 0 \\ w(t) \geq 0 \end{cases}$$

For the discretization of the model above, let's consider τ as the discretization step and n as a period. Time is discretized into intervals of length τ, such that the differential equation and the payoff are approximated in each interval $n\tau$, $(n+1)\tau$. Thus, the discretization of the objective function gives:

$$\begin{aligned}
\int_{n\tau}^{(n+1)\tau} e^{-rt} f(w(t), H(t))dt &= \left[-\frac{e^{-rt}}{r} f(w(n), H(n)) \right]_{n\tau}^{(n+1)\tau} \\
&= -\frac{e^{-r(n+1)\tau}}{r} f(w(n), H(n)) - \left(-\frac{e^{-rn\tau}}{r} \right) f(w(n), H(n)) \\
&= \frac{e^{-rn\tau}}{r} \left(-e^{-r\tau} f(w(n), H(n)) \right) + \frac{e^{-rn\tau}}{r} f(w(n), H(n)) \\
&= f(w(n), H(n)) \frac{e^{-rn\tau}}{r} \left(-e^{-r\tau} + 1 \right) \\
\int_{n\tau}^{(n+1)\tau} e^{-rt} f(w(t), H(t))dt &= f(w(n), H(n)) e^{-rn\tau} \left(\frac{1 - e^{-r\tau}}{r} \right)
\end{aligned}$$

Using Taylor's first order limited development of $e^{-r\tau}$ gives :

$$e^{-r\tau} \simeq 1 - r\tau$$

Thus, the objective function becomes:

$$\begin{aligned}
\int_{n\tau}^{(n+1)\tau} e^{-rt} f(w(t), H(t))dt &\simeq f(w(n), H(n))(1 - r\tau)^n \left(\frac{1 - (1 - r\tau)}{r} \right) \\
&= f(w(n), H(n))(1 - r\tau)^n \left(\frac{1 - 1 + r\tau}{r} \right) \\
\int_{n\tau}^{(n+1)\tau} e^{-rt} f(w(t), H(t))dt &= f(w(n), H(n))(1 - r\tau)^n \tau
\end{aligned}$$

The discretization of the dynamics gives:

$$H(n+1) = H(n) + (R - \alpha w(n))\,\tau$$

The discrete time problem can be defined as:

$$\max_{w(n)} \sum_{n=0}^{\infty} (1 - r\tau)^n \left[a w(n) - \frac{b}{2} w(n)^2 - max\,(0, c_0 - c_1 H(n))\, w(n) \right] \tau \tag{10}$$

$$s.t \begin{cases} H(n+1) = H(n) + \tau\,(R - \alpha w(n)) \\ H(0) = H_0 \geq 0,\ H_0\ \ given \\ H(n) \geq 0 \\ w(n) \geq 0 \end{cases}$$

The discrete time model therefore converges towards the continuous time model when the discretization step τ tends toward zero.

B Figures from Experimental Instructions

Figure B.1 – Total revenue from extraction

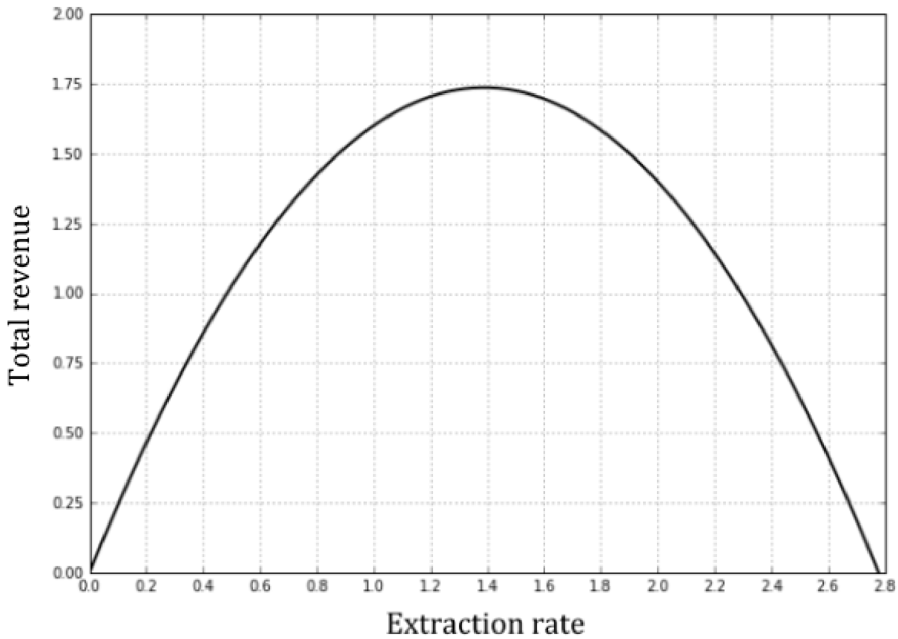


Figure B.2 – Unitary cost of extraction

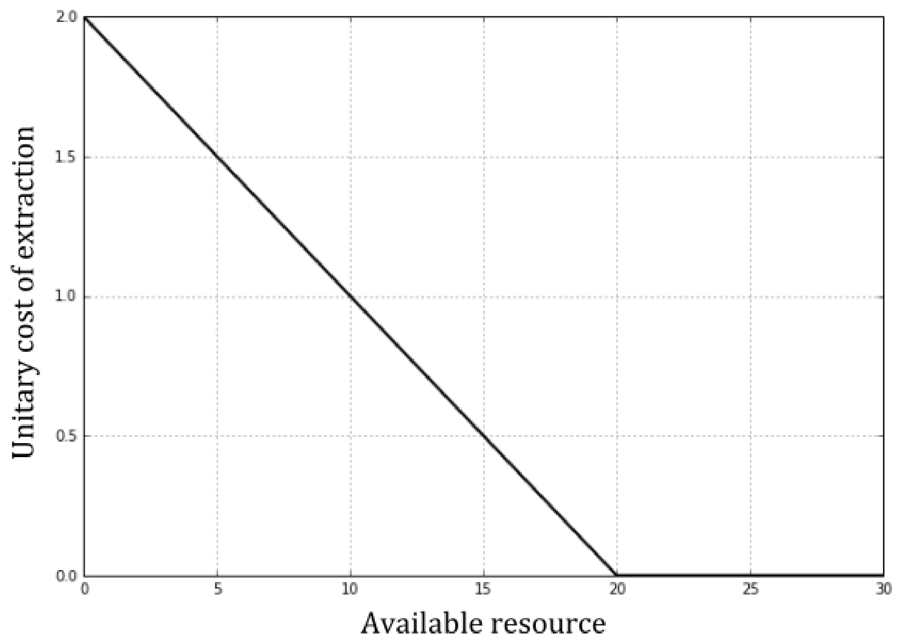
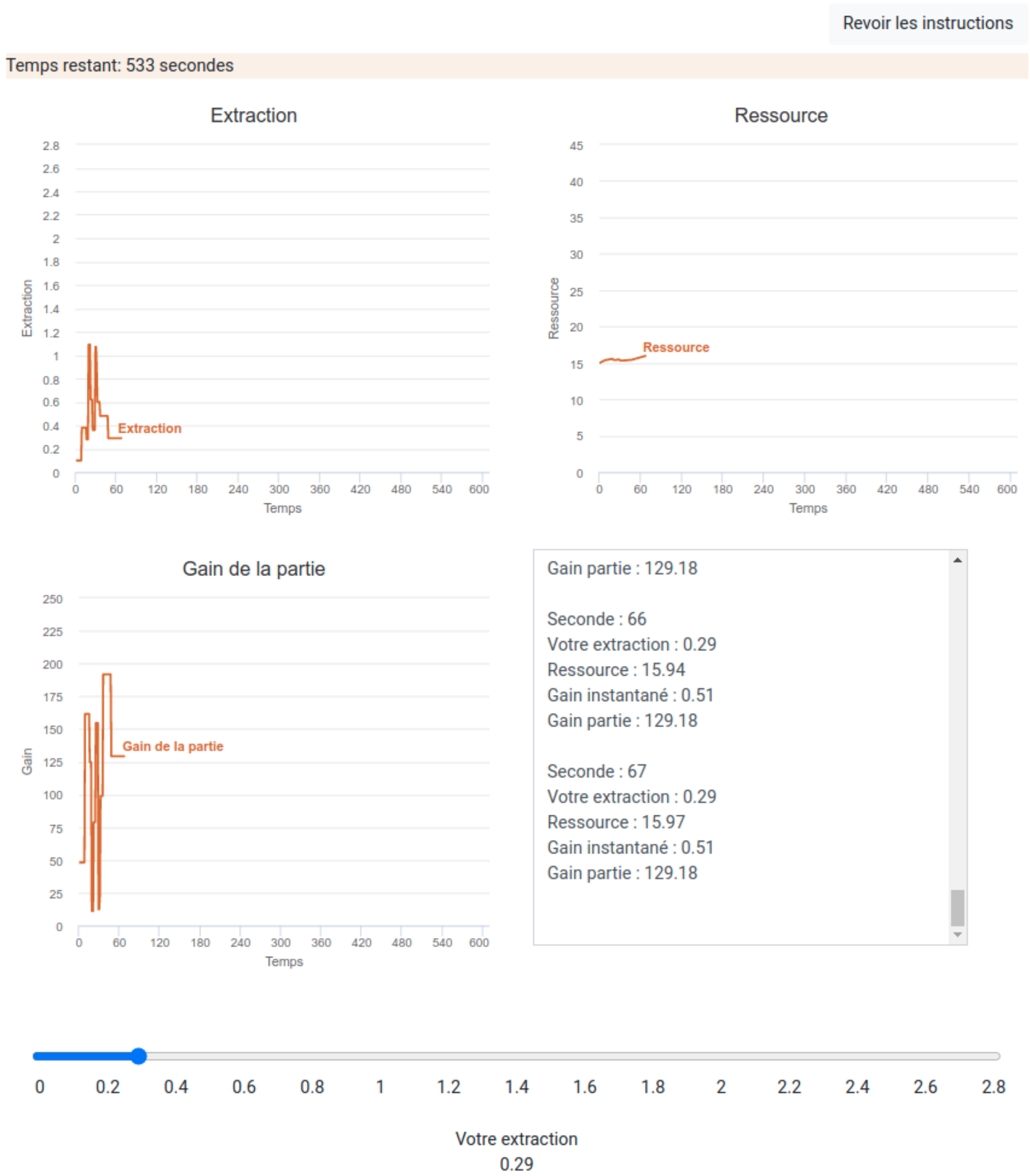


Figure B.3 – Decision-making screen shot. We follow a hypothetical subject who chooses his extraction rate at random



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